Galilean fermions: Classical and quantum aspects

Aditya Sharma[*]

BITS-Pilani, KK Birla Goa Campus, NH 17B, Bypass Road, Zuarinagar, Goa 403726, India

(Received 15 February 2023; accepted 23 May 2023; published 9 June 2023)

We study the classical and quantum "properties" of Galilean fermions in $3+1$ dimensions. We have taken the case of massless Galilean fermions minimally coupled to the scalar field. At the classical level, the Lagrangian is obtained by null reducing the relativistic theory in one higher dimension. The resulting theory is found to be invariant under infinite Galilean conformal symmetries. Using Noether's procedure, we construct the corresponding infinite conserved charges. Path integral techniques are then employed to probe the quantum properties of the theory. The theory is found to be renormalizable. A novel feature of the theory is the emergence of *mass* scale at the first order of quantum correction. The conformal symmetry of the theory breaks at quantum level. We confirm this by constructing the beta function of the theory.

DOI: 10.1103/PhysRevD.107.125009

I. INTRODUCTION

Symmetries are essential in the study of any physical system, in that they are responsible for conservation laws. For example, symmetry in space and time translation leads to momentum and energy conservation laws, respectively. It is well established that Lorentz symmetry is essential to describe the physics of fundamental particles and their interactions. However, for systems where the speed of objects (v) involved is much less than the speed of light (c) (i.e., $v \ll c$, also known as the nonrelativistic limit), Galilean symmetry is better suited. The emergence of nonrelativistic symmetries in the study of cold atoms, Fermi condensates, the Efimov effect, etc. [1–3], has further fuelled the validity to consider nonrelativistic limits.

Recently, there has been an upsurge to construct field theories consistent with Galilean symmetry. This is because Galilean symmetry has paved its way in describing condensed matter systems such as the quantum hall effect, nonrelativistic fluid dynamics, and magnetohydrodynamics [4–6]. Galilean symmetry is characterized by unequal scaling of space and time (also known as the Galilean limit[1]), i.e.,

$$t \to t, \qquad x_i \to \epsilon x_i, \qquad \epsilon \to 0,$$

and is described by a set of symmetry generators viz. spatial and temporal translations (P_i, H), homogeneous spatial rotations (J_{ij}), and Galilean boosts (B_i). In addition, Galilean symmetry can also be conformally extended by including spatial conformal transformations (K_i), temporal conformal transformations (K), and dilatations (D). Together, they form a closed Lie algebra known as finite Galilean conformal algebra (fGCA) [7,8]. The Galilean conformal symmetry generators can be obtained either by taking the Galilean limit of Poincaré symmetry generators [8] or by finding the conformal isometries of Newton-Cartan manifolds. (A brief discussion is given in Sec. II A. For more details see [9,10].) A remarkable feature of fGCA is that it can be given an infinite lift to construct an infinite Lie algebra (2) known as infinite Galilean conformal algebra (GCA) [7,8,11,12].

The study of Galilean conformal field theories has recently seen a revival [7,8,11–13]. This is mainly because field theories consistent with GCA admit an infinite number of conserved charges at classical level (see [11,12] and references therein). Surprisingly, not much heed has been paid to understand the quantum "properties" of these conformal theories except for some recent work in [14]. Addressing the issue of quantization of Galilean conformal field theories is also important because of its application in many physics systems [15–17]. In this paper we present both the classical and quantum field descriptions of massless Galilean fermions minimally coupled to the Galilean scalar in $3+1$ dimensions. Owing to the interaction between a scalar field and a fermionic field, we call the resultant theory the Galilean Yukawa theory.

Some of the early work on the Galilean fermion was carried out by Lévy-Leblond in 1967 [18] where a Galilei

[*]p20170442@goa.bits-pilani.ac.in
adityasharma.theory@gmail.com

[1]The Galilean limit is the same as the nonrelativistic limit. They are often used interchangeably for each other in the literature.

invariant analog of the Dirac equation was constructed. An interesting finding was that the spin magnetic moment, with its Landé factor $g = 2$, is not a relativistic property. Galilean fermions have also been studied in [19] where a massless Dirac equation was shown to exhibit the Schrödinger symmetry (which we know is a conformal extension of the Galilean group [9]). Recent investigation of the Galilean fermion are considered in [14,20,21]. The Galilean Yukawa theory is the simplest example of an interacting conformal field theory admitting fermionic degrees of freedom, consistent with GCA. The theory becomes even more captivating at quantum level because the *mass* term surfaces at the first order of quantum correction. Admittance of the *mass* scale in a pure Galilean field theory upon renormalization, to our knowledge, has never been addressed before in the literature.

A recent study carried out with Galilean quantum electrodynamics in $(3+1)$ dimensions [14] suggests the presence of global conformal anomalies in the theory at quantum level, which is quite different from the case of Galilean electrodynamics coupled to the Schrödinger scalar (sGED) in $(2+1)$ dimensions [22] where the beta function vanishes identically leading to a family of nonrelativistic conformal fixed points. The $\mathcal{N} = 2$ supersymmetric extension of Galilean electrodynamics in $(2+1)$ dimensions constructed has also been studied in [23]. It must be noted that the free Galilean scalar field theory does not admit any dynamical degrees of freedom. This is because the Galilean limit kills the kinetic part of the theory. Coupling Galilean fermions to the Galilean scalar field introduces the dynamical degrees of freedom into the theory. This makes for an interesting case of a Galilean invariant conformal field theory that admits an infinite number of conserved charges at the classical level. Thus, this paper is an attempt to present both the classical and the quantum field descriptions of the Galilean Yukawa theory.

The Galilean Yukawa theory is constructed in this paper by null reducing the relativistic Yukawa theory in one higher dimension. This method is well known in the literature and goes by the name of "null reduction" [24–27]. At the classical level, the theory admits an infinite number of conserved charges. To describe the quantum field description, we employ path integral techniques. The method of cutoff regularization has been employed to regulate the UV divergences, and the theory is then renormalized up to one-loop. Interestingly, the *mass* scale in the scalar sector appears at the first order of quantum correction. The admittance of the *mass* term assures one that the theory is no longer scale invariant at the quantum scale, albeit exhibiting conformal invariance at the classical level. This is suggestive of a global conformal anomaly in the theory [28,29], which is further guaranteed by the non-vanishing nature of the beta function.

This paper is organized as follows: In Sec. II we present the classical field description of the Galilean Yukawa theory. We briefly discuss the Galilean conformal symmetry in Sec. II A and present some of the well-known results in the literature. In Sec. II B, we construct the Lagrangian for the Galilean Yukawa theory using null reduction. We address the symmetries of the theory in Sec. II C followed by the construction of conserved charges. We delve further into the theory in Sec. III by presenting the quantum field description of the Galilean Yukawa theory. We have employed functional techniques to develop the quantum field description of the theory. To bring out the nature of divergences in the theory we employ the method of cutoff regularization. To this end, we evaluate the one-loop corrections to the propagators and vertex in Sec. III B. The issue of renormalization is addressed in Sec. III C followed by the summary and discussions in Sec. IV.

II. CLASSICAL FIELD DESCRIPTION OF GALILEAN YUKAWA THEORY

A. Galilean conformal symmetry

Galilean conformal symmetry of a $(d+1)$-dimensional spacetime is described by the set of symmetry generators-time translations (H), space translations (P_i), homogeneous rotations (J_{ij}), Galilean boosts (B_i), dilatation (D), and spatial and temporal conformal transformations (K_i, K). In an adapted coordinate system $x^\mu \equiv (t, x^i)$ we have

$$\begin{aligned} &H = -\partial_t, \qquad P_i = \partial_i, \qquad J_{ij} = x_i\partial_j - x_j\partial_i, \\ &B_i = t\partial_i, \qquad D = -(t\partial_t + x^i\partial_i), \qquad K_i = t^2\partial_i, \\ &K = -(t^2\partial_t + 2x_i t\partial_i). \end{aligned} \tag{1}$$

The symmetry generators (1) except J_{ij} can be cast into a compact notation, i.e.,

$$\begin{aligned} L^{(n)} &= -t^{n+1}\partial_t - (n+1)t^n x_i\partial_i, \\ M_i^{(n)} &= t^{n+1}\partial_i, \end{aligned}$$

where H, D, K can be recovered by setting $n = -1, 0, 1$ in $L^{(n)}$ and P_i, B_i, K_i are recovered by setting $n = -1, 0, 1$ in $M_i^{(n)}$. The generators $L^{(n)}$, $M_i^{(n)}$, J_{ij} form a closed Lie algebra called fGCA given by

$$\begin{aligned} [L^{(n)}, L^{(m)}] &= (n-m)L^{(n+m)}, \\ [L^{(n)}, M_i^{(m)}] &= (n-m)M_i^{(n+m)}, \\ [M_i^{(n)}, M_j^{(m)}] &= 0, \\ [L^{(n)}, J_{ij}] &= 0, \\ [J_{ij}, M_k^{(n)}] &= M_{[j}^{(n)}\delta_{i]k}. \end{aligned} \tag{2}$$

A striking feature of (2) is that the algebra closes $\forall\ n, m \in \mathbb{Z}$. This gives fGCA an infinite lift. The resulting Lie algebra is called infinite GCA. The reason for the infinite lift is captured in the underlying geometry. Precisely, the Galilean conformal symmetries are related to the conformal isometries of a "flat" Newton-Cartan spacetime (see [9,10,30–32]). A Newton-Cartan (NC) spacetime is a $(d+1)$-dimensional smooth manifold equipped with a degenerate contravariant metric g along with a nonvanishing one-form θ which also happens to be in the kernel of g.

The conformal isometries of NC spacetime are those vector field X that preserve θ and g up to a nontrivial conformal factor λ [9,33], i.e.,

$$\pounds_X g = \lambda g, \qquad \pounds_X \theta = -\frac{1}{2}\lambda\theta. \tag{3}$$

For a flat NC spacetime $(\mathbb{R}\times\mathbb{R}^d)$, in an adaptive coordinate chart $x^\mu \equiv (t, x^i)$,

$$g = g^{\mu\nu}\partial_\mu \otimes \partial_\nu, \qquad \theta = dt \quad \text{where } g^{\mu\nu} = \mathrm{diag}(0, I).$$

Equation (3) reduces to

$$X = \alpha(t)\frac{\partial}{\partial t} + \left(\omega_{ij}(t)x^j + x^i\beta(t) + \xi^i(t)\right)\frac{\partial}{\partial x^i}, \tag{4}$$

where $\omega \in SO(d)$, $\alpha, \beta \in \mathbb{R}$, and $\xi \in \mathbb{R}^d$ are the arbitrary functions of time, explaining the infinite lift of GCA.

The generators $\{L^{(n)}, M_i^{(n)}, J_{ij}\}$ can be used to construct the action of symmetry generators on the local fields. This can be done either by looking at scale-boost representations or scale-spin representations of GCA. In this paper we shall employ the scale-spin representation of GCA [34] (for scale-boost representations of GCA we request the reader to see [35,36]). For some general field $\Phi = (\varphi, \phi, A_i, \ldots)$, where φ is some scalar field, ϕ is a two-component spinor, A_i is a vector field, and the dots represent higher spin fields, we can write

$$\delta_{L^{(n)}}\Phi = (t^{n+1}\partial_t + (n+1)t^n(x^l\partial_l + \Delta))\Phi - t^{n-1}n(n+1)x^k\delta_{B_k}\Phi, \tag{5}$$

$$\delta_{M_l^{(n)}}\Phi = -t^{n+1}\partial_l\Phi + (n+1)t^n\delta_{B_l}\Phi, \tag{6}$$

$$\delta_{J_{ij}}\Phi = (x_i\partial_j - x_j\partial_i)\Phi + \Sigma_{ij}\Phi, \tag{7}$$

where Δ is the scaling dimension, $\delta_{B_l}\Phi$ is the action of the boost generator on the field Φ, and $\Sigma_{ij} = \frac{1}{4}[\sigma_i, \sigma_j] = \frac{i}{2}\epsilon_{ijk}\sigma_k$. For scalar field φ, $\Sigma_{ij}\varphi = 0$. We will employ (5)–(7) to establish the invariance of the Galilean Yukawa theory (16) under GCA and later again to construct the conserved charges for the theory.

B. Lagrangian formulation

We shall employ the method of null reduction to construct the Lagrangian for the Galilean Yukawa theory. The method of null reduction has been widely used in the literature to construct the Lagrangians for nonrelativistic field theories [24–27]. We start with a relativistic theory in one higher dimension. In an adaptive coordinate chart $x^\mu = (u, t, x^i)$, where u, t are the two real null coordinates and $x^i = (x, y, z)$ are the spatial coordinates, we write

$$\tilde{\mathcal{L}} = \frac{1}{2}\eta^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) + i\bar{\psi}\gamma^\mu\partial_\mu\psi - g\bar{\psi}\psi\varphi, \tag{8}$$

where $\eta^{\mu\nu}$ is the metric tensor for the Minkowski line element in the coordinate chart x^μ, i.e.,

$$ds^2 = du\otimes dt + dt\otimes du + \delta_{ij}dx^i \otimes dx^j = \eta_{\mu\nu}dx^\mu \otimes dx^\nu, \tag{9}$$

φ is the scalar field, ψ is a four-component spinor, g is the coupling strength, and γ^μ are the Dirac matrices whose explicit form in coordinate chart x^μ is taken to be

$$\gamma^u = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \tag{10}$$

$$\gamma^t = \begin{pmatrix} 0 & 0 \\ -\sqrt{2} & 0 \end{pmatrix}, \tag{11}$$

$$\gamma^i = \begin{pmatrix} i\sigma^i & 0 \\ 0 & -i\sigma^i \end{pmatrix}, \tag{12}$$

where σ^i are the usual Pauli matrices. The γ matrices obey the standard Clifford algebra, $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$, where $\eta^{\mu\nu}$ is the metric tensor associated with the Minkowski line element (9). The γ matrices allow us to define the adjoint of the ψ, i.e., $\bar{\psi} = \psi^\dagger G$, where[2]

$$G = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{13}$$

Now to write down the Lagrangian for the Galilean field theory we null reduce (8) along the null direction u; i.e., we demand

$$\partial_u\varphi = \partial_u\psi = 0.$$

This leads to the Lagrangian $\mathcal{L}$ for the Galilean Yukawa theory

$$\mathcal{L} = \frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) + i\bar{\psi}(\gamma^t\partial_t + \gamma^i\partial_i)\psi - g\bar{\psi}\psi\varphi. \tag{14}$$

[2]For a step-by-step construction of γ matrices, G and $\bar{\psi}$, we request the reader to check our previous work [14].

Note that the "leftover" null coordinate t has now acquired the status of the nonrelativistic time. The gamma matrices for (14) are now given by $\gamma^I = (\gamma^t, \gamma^i)$, and they obey the degenerate Clifford algebra given by

$$\{\gamma^I, \gamma^J\} = -2g^{IJ}, \tag{15}$$

where g^{IJ} is the degenerate metric, i.e., $g^{IJ} = \mathrm{diag}\,(0,1,1,1)$ on the Newton-Cartan spacetime. We can further reduce (14) in terms of the components of ψ; i.e., we write $\psi = \binom{\phi}{\chi}$ where ϕ and χ are the two-component spinors themselves, allowing us to write (14) as

$$\mathcal{L} = \frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) - \sqrt{2}i\phi^\dagger\partial_t\phi - \chi^\dagger\sigma^i\partial_i\phi + \phi^\dagger\sigma^i\partial_i\chi - g\varphi(\chi^\dagger\phi + \phi^\dagger\chi). \tag{16}$$

Using (16), we can write $\mathbb{L} = \int d^3x\mathcal{L}$, i.e.,

$$\mathbb{L} = \int d^3x\left\{\frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) - \sqrt{2}i\phi^\dagger\partial_t\phi - \chi^\dagger\sigma^i\partial_i\phi + \phi^\dagger\sigma^i\partial_i\chi - g\varphi(\chi^\dagger\phi + \phi^\dagger\chi)\right\}. \tag{17}$$

Note that in the absence of the spinor field, the theory reduces to that of a real Galilean scalar field that does not exhibit any dynamics. Thus, the coupling of spinor field can also be understood as the introduction of the matter degree of freedom into the Galilean scalar field theory. Variation of the Lagrangian results in the following equations of motion[3]:

$$-\sqrt{2}i\partial_t\phi + \sigma^i\partial_i\chi - g\varphi\chi = 0, \tag{18}$$

$$\partial^2\varphi + g(\chi^\dagger\phi + \phi^\dagger\chi) = 0, \tag{19}$$

$$\sigma^i\partial_i\phi + g\varphi\phi = 0. \tag{20}$$

For completeness, we mention the canonical[4] Hamiltonian H for the Galilean Yukawa theory,

$$H = -\int d^3x\left(\frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) - \chi^\dagger\sigma^i\partial_i\phi + \phi^\dagger\sigma^i\partial_i\chi - g\varphi(\chi^\dagger\phi + \phi^\dagger\chi)\right). \tag{21}$$

[3]For the full spectrum of equations of motion, one needs to include the complex conjugates of (18)–(20).

[4]Note that the canonical momentum (π_φ) for the Galilean scalar φ does not appear in (21). This is because π_φ is a primary constraint in the theory and shall only appear in the expression for the total Hamiltonian. The total Hamiltonian becomes essential if we were to address quantization via canonical techniques. However, in this paper we are interested in path integral quantization. For more details on canonical quantization of systems with constraints see [37].

C. Symmetries and conserved charges

Let us now analyze the symmetries of the Galilean Yukawa theory constructed above. To do that, we first have to write down the action of Galilean symmetry generators $\{L^{(n)}, M_i^{(n)}, J_{ij}\}$ on the fields (φ, ϕ, χ). We employ (5) and (6) to achieve that. The action of $M_i^{(n)}$ on the fields takes on the following form:

$$\delta_{M_i^{(n)}}\phi = -t^{n+1}\partial_i\phi, \tag{22}$$

$$\delta_{M_i^{(n)}}\chi = -t^{n+1}\partial_i\chi - \frac{i(n+1)}{\sqrt{2}}t^n\sigma_i\phi, \tag{23}$$

$$\delta_{M_i^{(n)}}\varphi = -t^{n+1}\partial_i\varphi, \tag{24}$$

and the action of $L^{(n)}$ on the fields reads

$$\delta_{L^{(n)}}\phi = t^{n+1}\partial_t\phi + (n+1)t^n(x^j\partial_j + \Delta_1)\phi, \tag{25}$$

$$\delta_{L^{(n)}}\chi = t^{n+1}\partial_t\chi + (n+1)t^n(x^j\partial_j + \Delta_1)\chi + \frac{i}{\sqrt{2}}n(n+1)t^{n-1}x^k\sigma_k\phi, \tag{26}$$

$$\delta_{L^{(n)}}\varphi = t^{n+1}\partial_t\varphi + (n+1)t^n(x^j\partial_j + \Delta_2)\varphi, \tag{27}$$

where $\Delta_1 = 3/2$ and $\Delta_2 = 1$. The stage is now set to address the symmetries of the Galilean Yukawa theory. We shall address the symmetries from the Lagrangian perspective.

1. Symmetries of Lagrangian

We begin by varying the Lagrangian of the theory (17) by an arbitrary variation δ, i.e.,

$$\delta\mathbb{L} = \int d^3x\{(\partial_i\varphi)(\partial_i\delta\varphi) - \sqrt{2}i\delta\phi^\dagger\partial_t\phi - \sqrt{2}i\phi^\dagger(\partial_t\delta\phi) - \delta\chi^\dagger\sigma^i\partial_i\phi - \chi^\dagger\sigma^i\partial_i\delta\phi + \delta\phi^\dagger\sigma^i\partial_i\chi + \phi^\dagger\sigma^i\partial_i\delta\chi - g\delta\varphi(\chi^\dagger\phi + \phi^\dagger\chi) - g\varphi(\delta\chi^\dagger\phi + \chi^\dagger\delta\phi + \phi^\dagger\delta\chi + \delta\phi^\dagger\chi)\}. \tag{28}$$

To arrive at the symmetries of the Lagrangian under Galilean conformal generators, we restrict δ to $\delta_{M_i^{(n)}}$ and $\delta_{L^{(n)}}$. Now upon using (22)–(24) we arrive at

$$\delta_{M_i^{(n)}}\mathbb{L} = 0. \tag{29}$$

Also, using (25)–(27), we can end up with

$$\delta_{L^{(n)}}\mathbb{L} = \int d^3x\partial_t\left(\frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) - \sqrt{2}i\phi^\dagger\partial_t\phi - \chi^\dagger\sigma^i\partial_i\phi + \phi^\dagger\sigma^i\partial_i\chi - g\varphi(\chi^\dagger\phi + \phi^\dagger\chi)\right). \quad (30)$$

Clearly, we can see that under the action of Galilean conformal symmetry generators, the variation in Lagrangian either changes by a total time derivative term or vanishes trivially. This assures that Galilean conformal symmetries are preserved at the level of *action* in $d = 4$ dimensions. The invariance of the Lagrangian under J_{ij} is trivially satisfied (i.e., $\delta_J\mathbb{L} = 0$). We can then conclude that the Galilean Yukawa theory (16) is invariant under infinite Galilean conformal symmetry generators.

2. Conserved charges

The Noether theorem suggests that, for every continuous symmetry of the Lagrangian, there exists a corresponding global conserved charge. Owing to the existence of an infinite number of symmetries, we can deduce that the Galilean Yukawa theory admits an infinite tower of conserved charges. The aim of this subsection is to construct those charges. We begin by briefly outlining the systematic procedure we would employ to construct the charges. Let us consider a generic Lagrangian $\mathbb{L}$ in $(d+1)$ spacetime which depends upon the general field Φ, i.e.,

$$\mathbb{L} \equiv \mathbb{L}(\Phi, \partial_t\Phi, \partial_i\Phi). \quad (31)$$

Now consider the transformation of the field $\Phi \to \Phi + \delta_1\Phi$. If we invoke the Euler Lagrange equations of motion, the Lagrangian can at most change by a total time derivative; i.e., we are studying the variation of the Lagrangian on-shell,

$$\delta\mathbb{L}|_{\text{on-shell}} = \int d^dx\partial_t\Theta_t(\Phi, \partial_t\Phi, \partial_i\Phi, \delta_1\Phi). \quad (32)$$

Now if we consider the infinitesimal symmetry transformation instead, i.e., $\Phi \to \Phi + \delta_2\Phi$, then the Lagrangian should differ only by a total derivative, i.e.,

$$\delta\mathbb{L}|_{\text{off-shell}} = \int d^dx\partial_t\alpha_t(\Phi, \partial_t\Phi, \partial_i\Phi, \delta_2\Phi). \quad (33)$$

Since the symmetry transformations leave the Lagrangian invariant, i.e., choosing $\delta_1 = \delta_2$ forces the off-shell variation to be equal to the on-shell variation,

$$\delta\mathbb{L}|_{\text{on-shell}} = \delta\mathbb{L}|_{\text{off-shell}}.$$

Thus we can deduce

$$\partial_t(\Theta_t - \alpha_t) = 0.$$

Hence, the corresponding global conserved charge is given by

$$Q = \int d^dx(\Theta_t - \alpha_t). \quad (34)$$

Noether's procedure described above allows one to deduce the conserved charges associated with the Galilean conformal symmetry generators $\{L^{(n)}, M_i^{(n)}, J_{ij}\}$ in $(3+1)$ dimensions. The on-shell variation of the Lagrangian (17) leads to

$$\Theta_t = -\sqrt{2}i\phi^\dagger\delta_1\phi.$$

Now, the off-shell variation of (17) under $L^{(n)}$ and $M_i^{(n)}$ leads to

$$\alpha_t|_{L^{(n)}} = f_n(t)\left(\frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) - \sqrt{2}i\phi^\dagger\partial_t\phi - \chi^\dagger\sigma^i\partial_i\phi + \phi^\dagger\sigma^i\partial_i\chi - g\varphi(\chi^\dagger\phi + \phi^\dagger\chi)\right),$$
$$\alpha_t|_{M_i^{(n)}} = 0,$$

where $f_n(t) = t^{n+1}$ is a Laurent polynomial in t. The conserved charges for the Galilean Yukawa theory becomes

$$Q_{L^{(n)}} = \int d^3x\left\{-\sqrt{2}i\dot{f}_n\phi^\dagger\left(x^j\partial_j\phi + \frac{3}{2}\phi\right) - f\left(\frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) - \chi^\dagger\sigma^i\partial_i\phi + \phi^\dagger\sigma^i\partial_i\chi - g\varphi(\chi^\dagger\phi + \phi^\dagger\chi)\right)\right\}, \quad (35)$$

$$Q_{M^{(n)}} = -\int d^3x\sqrt{2}i\phi^\dagger(\xi^i\partial_i\phi), \quad (36)$$

where $\xi = \xi^j\partial_j$ is a spatially constant vector in time with $\xi^j = t^{(n+1)}(1,1,1)$. The finite conserved charges can be deduced from (35) and (36) by appropriately restricting n to $(-1,0,1)$. For example, restricting $n = -1$ in (35) leads to the Noether charge corresponding to the time translations (Hamiltonian), i.e.,

$$Q_{L^{(-1)}} = -\int d^3x\left(\frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) - \chi^\dagger\sigma^i\partial_i\phi + \phi^\dagger\sigma^i\partial_i\chi - g\varphi(\chi^\dagger\phi + \phi^\dagger\chi)\right), \quad (37)$$

which correctly reproduces the canonical Hamiltonian (21). This example also serves as a verification check for the conserved charges obtained for the Galilean Yukawa theory. In a similar fashion, we can construct other finite charges for the Galilean Yukawa theory by appropriately restricting n to -1, 0, or 1 in (35) and (36). A similar analysis yields the Noether charge for rotations (J), i.e.,

$$Q_{\omega} = \int d^3x \left(-2\sqrt{2} i\omega^{ij} x_i \phi^{\dagger} \partial_j \phi + \frac{1}{2} \omega^{ij} \epsilon_{ijk} \sigma_k \phi^{\dagger} \phi \right), \quad (38)$$

where ω^{ij} is a constant antisymmetric matrix, i.e., $\omega^{ij} = -\omega^{ji}$.

III. QUANTUM FIELD DESCRIPTION OF GALILEAN YUKAWA THEORY

In the last section, we provided the classical field description of the Galilean Yukawa theory. In this section we delve further into the theory by exploring the quantum field description of the Galilean Yukawa theory. Our motivation to study the quantum properties stems from the realization that the theory admits an infinite number of conserved charges [(35) and (36)] at the classical level. It shall be interesting to understand how the symmetries behave at the quantum level. To this end, our primary goal is to construct a quantum field description of the Galilean Yukawa theory. Our analysis relies upon the functional techniques. For the sake of brevity, we shall revert to (14) to exploit the quantum nature of the theory. The *action* S for the theory reads

$$S = \int dt\, d^3x \left(\frac{1}{2} (\partial_i \varphi)(\partial_i \varphi) + i\bar{\psi}(\gamma^t \partial_t + \gamma^i \partial_i)\psi - g\bar{\psi}\psi\varphi \right). \quad (39)$$

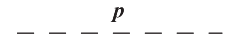
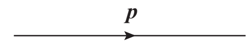
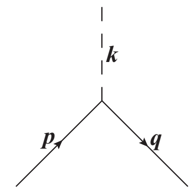
The fermionic field ψ carries a mass dimension $[\psi] = [\bar{\psi}] = 3/2$, and the scalar field φ admits $[\varphi] = 1$. This restricts the coupling strength g to dimensionless, i.e., $[g] = 0$, which makes for the case of a marginally renormalizable theory. For the rest of this paper, our focus will be on the one-loop renormalization of th Galilean Yukawa theory in $(3+1)$ dimensions.

A. Feynman rules

For notational convenience, we introduce $\boldsymbol{p} = (\omega, p_i)$, where ω is the energy associated with the field and p_i is the spatial momentum of the field. The Feynman rules for the Galilean Yukawa theory (39) are

1. For scalar propagator: $G(p, \omega) = \dfrac{i}{p^2}$.
2. For fermion propagator: $D(p, \omega) = \dfrac{i}{\gamma^t \omega + \gamma^i p_i}$.
3. For vertex: $V = -ig$.
4. Overall multiplicative factor of -1 for each internal fermion loop.

TABLE I. Feynman rules for the Galilean Yukawa theory.

1.	$G(\omega, p_i)$	
2.	$D(\omega, p_i)$	
3.	V	

Diagrammatic representation of Feynman rules is given in Table I. Note that if we suppress the scalar degree of freedom, we end up with a pure Galilean fermion theory whose quantization is described by the fermion propagator. Tree level quantization (from both canonical and functional methods) of free Galilean fermions has been studied in [38,39]. Also, if we suppress the fermionic part of the Lagrangian, we end up with a Galilean scalar field. An uninteresting feature of Galilean scalar field theory is that it does not admit any kinetic terms. Because of the lack of scalar dynamical degrees of freedom, we have not considered any self-interaction term such as φ^4. Our point of interest lies in incorporating the matter degrees of freedom that happen to be fermionic fields ψ in our case. In the next section, we study the one-loop corrections to the propagators and vertex.

B. Regularization

Owing to the existence of a three point vertex in the Feynman rules, the theory admits a correction to the scalar propagator, fermion propagator, and vertex. In the general treatment of quantum field theory, the loop corrections are generally UV divergent quantities that must be regularized by restricting the infinite modes in momentum and energy integrals up to a UV cutoff regulator. In the context of Galilean field theories, the unequal footing of space and time forces one to consider the two cutoff regulators viz. Ω in the energy sector and Λ in the momentum sector. Following along the lines of [14], we define the superficial degree of divergence by a set of two numbers $(\mathbb{D}, \mathbb{F})$, i.e.,

$$\mathbb{D} = \begin{pmatrix} \text{Powers of } \omega \\ \text{in the numerator} \end{pmatrix} - \begin{pmatrix} \text{Powers of } \omega \\ \text{in the denominator} \end{pmatrix}, \quad (40)$$

$$\mathbb{F} = \begin{pmatrix} \text{Powers of } \vec{p} \\ \text{in the numerator} \end{pmatrix} - \begin{pmatrix} \text{Powers of } \vec{p} \\ \text{in the denominator} \end{pmatrix}. \quad (41)$$

The knowledge of the superficial degree of divergence is helpful in understanding the extent to which the divergences can appear in the theory. However, as is often the case, the actual degree of divergence can be softer than the predicted superficial degree of divergence [40,41]. In what follows, we shall put (40) and (41) into use whenever required. We begin by evaluating the correction to the fermion propagator.

1. Correction to the fermion propagator

The Feynman diagram for the same is given by Fig. 1. The loop integral (Σ) corresponding to Fig. 1 can be evaluated by integrating along the unconstrained variables $(\omega_q, \vec{q})$, i.e.,

$$\Sigma(\omega_p,\vec{p}) = \frac{i(-ig)^2}{(2\pi)^4}\int d\omega_q d^3q\left(\frac{i}{\gamma^t(\omega_p+\omega_q)+\gamma^i(p_i+q_i)}\right) \times \frac{i}{q^j q_j}, \tag{42}$$

which can be rearranged to

$$\Sigma(\omega_p,\vec{p}) = \frac{i(-ig)^2}{(2\pi)^4}\int d\omega_q d^3q \frac{1}{q^2(p+q)^2} \times (\gamma^t(\omega_q+\omega_p)+(\gamma^i(p_i+q_i))). \tag{43}$$

The superficial degree of divergence is given by (2,0), suggesting a quadratic divergence in the energy sector and a logarithmic divergence in the momentum sector. We introduce the cutoffs Ω in the energy sector and Λ in the momentum sector. The integral evaluates to take the following value:

$$\Sigma(\omega_p,\vec{p}) = -i\frac{g^2}{8\pi|\vec{p}|}(\gamma^t\omega_p+\gamma^i p_i)\Omega. \tag{44}$$

Note that the loop integral above does not exhibit any logarithmic divergence offered due to the infinite modes of the momentum as predicted by the superficial degree of divergence. This is because the integral linear in $\vec{q}$ vanishes due to the antisymmetricity of the integral. Also, the actual degree of divergence offered due to the energy sector does not agree with the superficial degree of divergence. This behavior is not surprising since we know that the superficial degree of divergence renders only a naive idea about the extent of divergences in the theory. We note that in the cutoff limit Ω, the integral $\int d\omega_q \gamma^t \omega_q$ vanishes owing to the fact that the integrand is an odd function of ω_q. Hence, the true degree of divergence of the integral turns out to be linear.

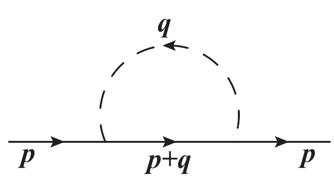


FIG. 1. Correction to the fermion propagator.

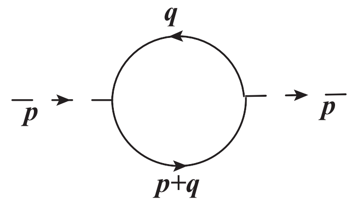


FIG. 2. Correction to the scalar propagator.

2. Correction to the scalar propagator

The Feynman diagram for the correction to the scalar propagator is given in Fig. 2. The loop integral (Π) in Fig. 2 takes on the following form:

$$\Pi(\omega_p,\vec{p}) = \frac{-i}{(2\pi)^4}\int d\omega_q d^3q(-ig)^2 \mathrm{Tr}\left(\left(\frac{i}{\gamma^t\omega_q+\gamma^i q_i}\right) \times \left(\frac{i}{\gamma^t(\omega_q+\omega_p)+\gamma^j(q_j+p_j)}\right)\right). \tag{45}$$

The superficial degree of divergence in this case can be evaluated to (1,1). This suggests that the loop integral diverges linearly with the momentum cutoff and energy cutoff, respectively. As it turns out, the integral can be evaluated to the following value:

$$\Pi(\omega_p,\vec{p}) = \frac{3ig^2}{2\pi^3}\Omega\Lambda. \tag{46}$$

We note that the one-loop correction to the scalar propagator diverges linearly with both energy as well as momentum cutoff, which is in agreement with the superficial degree of divergence.

3. Correction to the vertex

The Feynman diagram for the correction to the vertex is given in Fig. 3. The loop integral (Γ) takes the following form:

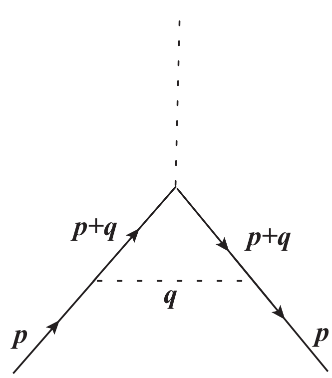


FIG. 3. Correction to the vertex.

$$\Gamma(\omega_p, \vec{p}) = \frac{i}{(2\pi)^4} \int d\omega_q d^3 q (-ig)^3 \frac{i}{q^j q_j} \times \left(\frac{i}{\gamma^t(\omega_p + \omega_q) + \gamma^i (p_i + q_i)} \right)^2. \quad (47)$$

As before, we note that the superficial degree of divergence turns out to be $(1, -1)$. This is suggestive of linear divergence offered due to the infinite energy modes. Interestingly, the vertex correction in the momentum sector does not offer any divergence superficially. The integral can be evaluated to

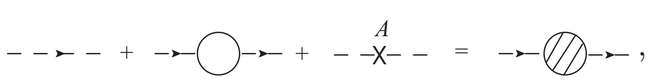
$$\Gamma(\omega_p, \vec{p}) = -\frac{ig^3}{8\pi|\vec{p}|} \Omega. \quad (48)$$

Clearly, the integral diverges linearly. We note that in all three corrections, the integral necessarily diverges due to the cutoff offered at large energy values. This shall not be surprising at all given that the scalar propagator does not admit any kinetic term which means that we always have to integrate over infinite energy modes, rendering us a linear factor of Ω in each of the corrections.

C. Renormalization and beta function

Having evaluated all one-loop corrections in the previous subsection, the stage is set to address the question of renormalization of the theory. As already mentioned, the dimensionless nature of the coupling strength g makes the Galilean Yukawa theory a reasonable candidate for a renormalizable theory. By introducing counterterms to the theory (i.e., subtracting the divergent pieces in the various cutoffs), we shall be able to absorb the divergent integrals (44), (46), and (48) in the field and coupling redefinitions.

We begin our analysis with the correction to the scalar propagator. The correction to the scalar propagator (46) suggests that we should add the following counterterm:



where the coefficient A can be chosen to render a finite propagator at one-loop. The diagrammatic representation takes down the following mathematical expression:

$$\frac{i}{p^2} + \frac{i}{p^2}\left[\frac{3ig^2\Omega\Lambda}{2\pi^3}\right]\frac{i}{p^2} + \frac{i}{p^2} A \frac{i}{p^2} \equiv \text{finite}$$
$$\Rightarrow \frac{i}{p^2}\left\{1 + \frac{i}{p^2}\left[\frac{3ig^2\Omega\Lambda}{2\pi^3} + A\right]\right\} \equiv \text{finite.}$$

After a bit of simple algebra, one can reduce the above expression to

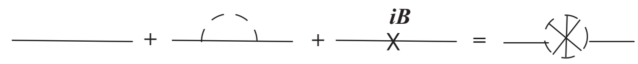


FIG. 4. Renormalization of the fermion propagator.

$$\frac{i}{p^2 - i\left[\frac{3ig^2\Omega\Lambda}{2\pi^3} + A\right]} \equiv \text{finite.}$$

Clearly, for the above expression to yield a finite value, we must have the quantity in the bracket to vanish identically, i.e.,

$$\frac{3ig^2\Omega\Lambda}{2\pi^3} + A = 0,$$

which restricts A to[5]

$$A = -\frac{3ig^2\Omega\Lambda}{2\pi^3} = -im^2, \quad (49)$$

where $m^2 = \frac{3g^2\Omega\Lambda}{2\pi^3}$ is the *mass parameter*. Evidently, the propagator renormalization of the scalar field has introduced a *mass* scale in our theory. The corresponding counterterm in the Lagrangian is

$$(\mathcal{L}_1)_{ct} = -\frac{1}{2} m^2 \varphi^2. \quad (50)$$

Remarkably, the scalar field has acquired the *mass* under renormalization. The emergence of the *mass* term signals the breaking of the conformal feature of the Galilean Yukawa theory. However, the interesting thing to note is that φ is a nondynamical field whose renormalization results in the *mass* scale at the quantum level. The emergence of a mass term for a nondynamical field is something that has never been seen in Galilean field theories. Note that the appearance of the mass term under renormalization is not surprising. In fact, there is nothing sacrosanct about the emergence of mass in a renormalized theory. We shall recall that in the Lorentzian massless φ^4 theory, the propagator correction leads to the mass term [41].

Now let us turn our attention toward the renormalization of the fermion propagator. Diagrammatically, we can represent this in Fig. 4. Following along the lines of scalar propagator renormalization, we can deduce that the counterterm takes the following form:

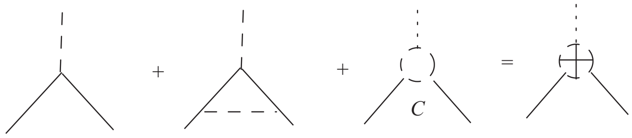
[5]Note that subtracting only the divergent pieces is one possible renormalization scheme out of many. We could also add to A a finite term, without spoiling the finiteness of $A + 3ig^2\Omega\Lambda/(2\pi^3)$. This ambiguity is fixed by measurement, according to the standard renormalization procedure. For more details on renormalization see [41].

$$(\mathcal{L}_2)_{ct} = iB\bar{\psi}(\gamma^t\partial_t + \gamma^i\partial_i)\psi, \tag{51}$$

where the coefficient B should be fixed to absorb the divergences in the theory. Using (44), we can fix B as

$$B = \frac{g^2}{8\pi|p|}\Omega. \tag{52}$$

Our last hunt is to renormalize the vertex term. The counterterm required to absorb the divergences in the vertex can be shown to take the following form:



where the coefficient C can be fixed using (48) to

$$C = \frac{g^2}{8\pi|p|}\Omega. \tag{53}$$

The counterterm for the vertex in the Lagrangian takes the following form then:

$$(\mathcal{L}_3)_{ct} = gC\bar{\psi}\psi\varphi. \tag{54}$$

Now that we have added the counterterms to our theory, we can define the bare Lagrangian $\mathcal{L}_b$ as

$$\mathcal{L}_b = \mathcal{L} + (\mathcal{L}_1)_{ct} + (\mathcal{L}_2)_{ct} + (\mathcal{L}_3)_{ct},$$

where $\mathcal{L}$ is given by (14). The bare Lagrangian takes the following form:

$$\mathcal{L}_b = \frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) + i\bar{\psi}(\gamma^t\partial_t + \gamma^i\partial_i)\psi - g\bar{\psi}\psi\varphi - \frac{1}{2}m^2\varphi^2 + iB\bar{\psi}(\gamma^t\partial_t + \gamma^i\partial_i)\psi + Cg\bar{\psi}\psi\varphi.$$

We can collect the coefficients to write

$$\mathcal{L}_b = \frac{1}{2}(\partial_i\varphi)(\partial_i\varphi) - \frac{1}{2}m^2\varphi^2 + (1+B)i\bar{\psi}(\gamma^t\partial_t + \gamma^i\partial_i)\psi - (1-C)g\bar{\psi}\psi\varphi. \tag{55}$$

We can then make the following redefinition for the fields:

$$\varphi_{(b)} = \varphi, \quad \psi_{(b)} = \sqrt{1+B}\psi. \tag{56}$$

The index b, appearing on the left-hand side of the fields, represents the bare field. Note that under renormalization, the scalar field does not get renormalized but instead leads to the *mass* term in the theory. Using (56), we can write the bare Lagrangian as

$$\mathcal{L}_b = \frac{1}{2}(\partial_i\varphi_{(b)})(\partial_i\varphi_{(b)}) - \frac{1}{2}m^2\varphi_{(b)}^2 + i\bar{\psi}_{(b)}(\gamma^t\partial_t + \gamma^i\partial_i)\psi_{(b)} - \frac{g(1-C)}{(1+B)}\bar{\psi}_{(b)}\psi_{(b)}\varphi_{(b)}. \tag{57}$$

We can clearly see that with the choice (56), one must redefine the coupling as well. Define

$$g_{(b)} = g\frac{(1-C)}{(1+B)}. \tag{58}$$

Thus, the bare Lagrangian (57) can be written down in terms of bare variables $\psi_{(b)}, \varphi_{(b)}$, and $g_{(b)}$ as

$$\mathcal{L}_b = \frac{1}{2}(\partial_i\varphi_{(b)})(\partial_i\varphi_{(b)}) - \frac{1}{2}m^2\varphi_{(b)}^2 + i\bar{\psi}_{(b)}(\gamma^t\partial_t + \gamma^i\partial_i)\psi_{(b)} - g_{(b)}\bar{\psi}_{(b)}\psi_{(b)}\varphi_{(b)}. \tag{59}$$

We note that even though, the theory can be made renormalizable, the bare Lagrangian does not share the same form as the starting Lagrangian. The emergence of the mass term for the scalar field is the captivating feature of the Galilean Yukawa theory. As already explained, the emergence of the mass term is the signature of anomalous breaking of conformal symmetry. To see this explicitly, we shall construct the beta function ($\beta(g)$) for the theory. The significance of the beta function is not just limited to conformal breaking of symmetry but, in fact, it also helps us to understand the validity of the theory with the cutoff scale involved. The beta function is defined as

$$\beta = \Omega\frac{\partial g}{\partial\Omega}. \tag{60}$$

We shall see from (58) that

$$g_{(b)} = g\frac{(1 - \frac{g^2\Omega}{8\pi|p|})}{(1 + \frac{g^2\Omega}{8\pi|p|})}. \tag{61}$$

Since the running coupling g is generally assumed to be small, we can condense the above expression to a simpler form by retaining the terms only up to $\mathcal{O}(g^3)$, i.e.,

$$g_{(b)} = \left(1 - \frac{2g^2\Omega}{8\pi|p|}\right)g. \tag{62}$$

Since the bare coupling $g_{(b)}$ has to be taken independent of the cutoff, we can differentiate the above expression to arrive at

$$\frac{\partial g}{\partial \Omega} = \frac{2g^3}{8\pi|p|}\left(1 + \frac{6g^2\Omega}{8\pi|p|}\right).$$

Retaining the terms only up to $\mathcal{O}(g^3)$, we can write

$$\frac{\partial g}{\partial \Omega} = \frac{2g^3}{8\pi|p|}. \tag{63}$$

Now note that in quantum field theories, the cutoff is often taken to be of the order of incoming momentum (or energy). Thus we can always define $\Lambda = b|p|$, where $b > 0$ is a constant parameter. Also, the two cutoffs Ω and Λ can be algebraically related by $\Omega = a\Lambda$, where $a \neq 1$ is a positive constant that parametrizes the discrepancy in the two cutoffs. These two conditions allow us to relate Ω with the momentum $|p|$, i.e., in the limit $\Omega \to \infty$, $\frac{\Omega}{|p|} = ab$. We can then write (63) as

$$\beta(g) = \Omega \frac{\partial g}{\partial \Omega} = g^3\left(\frac{ab}{4\pi}\right). \tag{64}$$

Since we know that both a and b are strictly positive, this suggests that $\beta(g)$ is always positive, i.e., $\beta(g) > 0$. This again confirms the presence of conformal anomalies in the theory and is in agreement with [28,29]. We shall also make note of the fact that the theory is devoid of asymptotic freedom; i.e., the Galilean Yukawa theory becomes strongly coupled at large momentum (or energies). This becomes evident if we integrate (64) between a reference scale Ω_0 and Ω; i.e., we get

$$g^2(\Omega) = \frac{g^2(\Omega_0)}{1 - \frac{ab}{2\pi} g^2(\Omega_0)\ln(\frac{\Omega}{\Omega_0})}. \tag{65}$$

It is straightforward to see from (65) that at small values of momentum, i.e., $\Omega \sim \Omega_0$, we have $g(\Omega) \sim g(\Omega_0)$. However, at large momentum values, i.e., $\Omega \gg \Omega_0$, the running coupling increases with the cutoff Ω confirming the invalidity of the theory at large energies. Another interesting thing to note here is the existence of the Landau pole in the theory. We can check that (65) shoots up at $\Omega = \Omega_0 \exp(\frac{2\pi}{abg^2(\Omega_0)})$. The existence of the Landau pole is a feature often observed in quantum field theories that are not asymptotically free.

IV. SUMMARY AND OUTLOOK

Let us summarize what we have accomplished in this paper. We have presented the classical and quantum field descriptions of an interacting Galilean conformal field theory. We have taken the case of massless Galilean fermions coupled to a massless Galilean scalar field. The introduction of scalar-fermionic interaction incorporates the dynamical degrees of freedom into the free Galilean scalar field theory, which otherwise is an example of a nonrelativistic conformal field theory with a nondynamical degree of freedom. At the classical level, the Lagrangian for the theory is obtained by null reducing the Lagrangian for the relativistic Yukawa theory in one higher dimension. The resulting theory is found to be invariant under the full Galilean conformal algebra, hence the name Galilean Yukawa theory. We further exploit the presence of infinite symmetries in the theory by constructing the conserved charges (35) and (36) for the theory. The coupling strength in the theory is observed to be dimensionless, which makes for the case of a marginally renormalizable theory; i.e., the theory may or may not be renormalizable. Interestingly, what we have found is that the theory is renormalizable at least to one-loop. Our prescription for quantization of the Galilean Yukawa theory relies on path integral techniques. We regularize the UV divergences in the theory by setting the energy and momentum cutoff (Ω, Λ). An interesting feature that emerges at quantum level is the entry of *mass* in the scalar sector of the theory. The admission of the *mass* term in the Lagrangian suggests that the conformal invariance of the theory is broken at the quantum level. This is captured by the behavior of the beta function which increases monotonically (and grows cubically) with the coupling. This suggests that the theory is not asymptotically free. The lack of asymptotic freedom is also captured by the Landau pole in the theory. The Galilean Yukawa theory shares this interesting feature of anomalous breaking of conformal symmetry with Galilean quantum electrodynamics [14]. However, an underlying difference between the two theories is the *mass* term. It must be noted that Galilean quantum electrodynamics does not lead to any mass term. This is because the Galilean quantum electrodynamics is obtained as a null reduction of Lorentzian quantum electrodynamics in one higher dimension. It is well known that in a relativistic setting, the photon does not acquire mass under renormalization, courtesy of Ward identities. Thus the renormalization of gauge fields in the Galilean limit is modeled in such a way that the gauge fields do not acquire mass. The Galilean Yukawa theory is the first example of a Galilean field theory where *mass* crops up.

Note that the scalar field in this theory exhibits nondynamical degrees of freedom. The emergence of the *mass* term at the quantum level calls for further investigations since there is no precise notion of mass in the Galilean setting. However, the theory shares similar features with Galilean quantum electrodynamics. It must be noted that further studies on global conformal anomalies, especially regarding anomalous Ward identities in the context of Galilean field theories, might be more tractable with the Galilean Yukawa theory than the gauge theory such as Galilean quantum electrodynamics. Also, the quantum field description presented in this paper is only valid up to one-loop in the perturbation. It shall also be very interesting to establish the renormalizability at all orders of the perturbations.

Note that many of the recent studies [14,22,23] on the quantization program of interacting Galilean field theories deal with matter-induced degrees of freedom. One of our future goals is to extend the quantization program developed in this paper to non-Abelian gauge theories such as Galilean Yang Mills (GYM) [11]. GYM is an example of a self-interacting theory; hence it shall be interesting to explore the quantization of pure GYM in this setting. Also, studying the quantum properties of Galilean QCD will be an avenue of future research.

We also wish to extend the quantization program for Carrollian field theories ($c \to 0$, a degenerate twin of Galilean field theories) by developing a prescription similar to the one described in this paper. Carrollian physics has recently gained attention much to the fact that it plays an essential role in understanding gravity in asymptotically flat spacetime [42–44]. Carrollian theories are promising candidates to study flat space holography. Also, recent study carried out with Carroll fluid allows one to model Carroll fluid as a possible dark energy candidate [45]. It shall be interesting if we can manage to uncover some interesting physics by probing the quantum properties of Carrollian field theories developed (see, for example, [46–49] and references therein) in recent years. Our recent work on the renormalization of scalar Carrollian electrodynamics [50] is a step in this direction.

ACKNOWLEDGMENTS

A. S. thanks Kinjal Banerjee and Peter Horvathy for useful discussion on Galilean fermions. A. S. also thanks Stefano Baiguera, Lorenzo Cederle, and Silvia Penati for several discussions on renormalization of nonrelativistic field theories. A note of thanks goes to JAXODRAW for developing their free JAVA program [51] for drawing Feynman diagrams.

[1] Yusuke Nishida and Dam Thanh Son, Unitary Fermi gas, epsilon expansion, and nonrelativistic conformal field theories, Lect. Notes Phys. **836**, 233 (2012).
[2] Paulo F. Bedaque, H. W. Hammer, and U. van Kolck, The three boson system with short range interactions, Nucl. Phys. **A646**, 444 (1999).
[3] Sean A. Hartnoll, Joseph Polchinski, Eva Silverstein, and David Tong, Towards strange metallic holography, J. High Energy Phys. 04 (2010) 120.
[4] Dam Thanh Son, Newton-Cartan geometry and the quantum Hall effect, arXiv:1306.0638.
[5] R. Jackiw, A particle field theorist's lectures on supersymmetric, non-Abelian fluid mechanics and d-branes, arXiv:physics/0010042.
[6] R. Jackiw and A. P. Polychronakos, Dynamical poincare symmetry realized by field-dependent diffeomorphisms, arXiv:hep-th/9809123.
[7] Arjun Bagchi, Joydeep Chakrabortty, and Aditya Mehra, Galilean field theories and conformal structure, J. High Energy Phys. 04 (2018) 144.
[8] Arjun Bagchi, Rudranil Basu, and Aditya Mehra, Galilean conformal electrodynamics, J. High Energy Phys. 11 (2014) 061.
[9] C. Duval and P. A. Horvthy, Non-relativistic conformal symmetries and newtoncartan structures, J. Phys. A **42**, 465206 (2009).
[10] C. Duval, On Galilean isometries, Classical Quantum Gravity **10**, 2217 (1993).
[11] Arjun Bagchi,, Rudranil Basu, Ashish Kakkar, and Aditya Mehra, Galilean Yang-Mills theory, J. High Energy Phys. 04 (2016) 051.
[12] Arjun Bagchi, Rudranil Basu, Minhajul Islam, Kedar S. Kolekar, and Aditya Mehra, Galilean gauge theories from null reductions, J. High Energy Phys. 04 (2022) 176.
[13] Rabin Banerjee and Soumya Bhattacharya, A new formulation of Galilean electrodynamics, arXiv:2211.12023.
[14] Kinjal Banerjee and Aditya Sharma, Quantization of interacting Galilean field theories, J. High Energy Phys. 08 (2022) 066.
[15] J. Lukierski, P. C. Stichel, and W. J. Zakrzewski, Exotic Galilean conformal symmetry and its dynamical realisations, Phys. Lett. A **357**, 1 (2006).
[16] D. T. Son and M. Wingate, General coordinate invariance and conformal invariance in nonrelativistic physics: Unitary Fermi gas, Ann. Phys. (Amsterdam) **321**, 197 (2006).
[17] R. Jackiw and V. P. Nair, Anyon spin and the exotic central extension of the planar Galilei group, Phys. Lett. B **480**, 237 (2000).
[18] Jean-Marc Lévy-Leblond, Nonrelativistic particles and wave equations, Commun. Math. Phys. **6**, 286 (1967).
[19] C. Duval, P. A. Horvathy, and L. Palla, Spinors in nonrelativistic Chern-Simons electrodynamics, Ann. Phys. (N.Y.) **249**, 265 (1996).
[20] Peng-Xiang Hao, Wei Song, Zehua Xiao, and Xianjin Xie, A BMS-invariant free fermion model, arXiv:2211.06927.
[21] Zhe-fei Yu and Bin Chen, Free field realization of the BMS Ising model, arXiv:2211.06926.
[22] Shira Chapman, Lorenzo Di Pietro, Kevin T. Grosvenor, and Ziqi Yan, Renormalization of Galilean electrodynamics, J. High Energy Phys. 10 (2020) 195.
[23] Stefano Baiguera, Lorenzo Cederle, and Silvia Penati, Supersymmetric Galilean electrodynamics, J. High Energy Phys. 09 (2022) 237.
[24] B. Julia and H. Nicolai, Null-killing vector dimensional reduction and Galilean geometrodynamics, Nucl. Phys. **B439**, 291 (1995).

[25] E. S. Santos, M. de Montigny, F. C. Khanna, and A. E. Santana, Galilean covariant Lagrangian models, J. Phys. A **37**, 9771 (2004).
[26] Eric Bergshoeff, Jan Rosseel, and Thomas Zojer, Non-relativistic fields from arbitrary contracting backgrounds, Classical Quantum Gravity **33**, 175010 (2016).
[27] Dennis Hansen, Beyond Lorentzian Physics, Ph.D. thesis, ETH Zurich, Zurich, 2021.
[28] Kristan Jensen, Anomalies for Galilean fields, SciPost Phys. **5**, 005 (2018).
[29] Akash Jain, Galilean anomalies and their effect on hydrodynamics, Phys. Rev. D **93**, 065007 (2016).
[30] C. Duval, G. Burdet, H. P. Künzle, and M. Perrin, Bargmann structures and Newton-Cartan theory, Phys. Rev. D **31**, 1841 (1985).
[31] C. Duval, G. W. Gibbons, P. A. Horvathy, and P. M. Zhang, Carroll versus Newton and Galilei: Two dual non-Einsteinian concepts of time, Classical Quantum Gravity **31**, 085016 (2014).
[32] K. N. Lian, Gravity theories via algebra gauging, arXiv:2112.13403.
[33] C. Duval, G. W. Gibbons, and P. A. Horvathy, Conformal Carroll groups, J. Phys. A **47**, 335204 (2014).
[34] Arjun Bagchi, Joydeep Chakrabortty, and Aditya Mehra, Galilean field theories and conformal structure, J. High Energy Phys. 04 (2018) 144.
[35] Arjun Bagchi and Ipsita Mandal, On representations and correlation functions of Galilean conformal algebras, Phys. Lett. B **675**, 393 (2009).
[36] Arjun Bagchi, Rajesh Gopakumar, Ipsita Mandal, and Akitsugu Miwa, GCA in 2d, J. High Energy Phys. 08 (2010) 004.
[37] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems*, Princeton paperbacks (Princeton University Press, Princeton, NJ, 1992).
[38] E. S. Santos, M. de Montigny, and F. C. Khanna, Canonical quantization of Galilean covariant field theories, Ann. Phys. (Amsterdam) **320**, 21 (2005).
[39] Marc de Montigny, Faqir C. Khanna, and Fuad M. Saradzhev, Path-integral quantization of Galilean Fermi fields, Ann. Phys. (Amsterdam) **323**, 1191 (2008).
[40] Michael Edward Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Westview Press, Reading, USA, 1995), p. 842.
[41] Lewis H. Ryder, *Quantum Field Theory*, 2nd ed. (Cambridge University Press, Cambridge, England, 1996).
[42] C. Duval, G. W. Gibbons, and P. A. Horvathy, Conformal Carroll groups and BMS symmetry, Classical Quantum Gravity **31**, 092001 (2014).
[43] Jelle Hartong, Holographic reconstruction of 3D flat spacetime, J. High Energy Phys. 10 (2016) 104.
[44] Biel Cardona, Joaquim Gomis, and Josep M Pons, Dynamics of Carroll strings, J. High Energy Phys. 07 (2016) 050.
[45] Jan de Boer, Jelle Hartong, Niels A. Obers, Watse Sybesma, and Stefan Vandoren, Carroll symmetry, dark energy and inflation, Front. Phys. **10**, 810405 (2022).
[46] Kinjal Banerjee, Rudranil Basu, Aditya Mehra, Akhila Mohan, and Aditya Sharma, Interacting conformal Carrollian theories: Cues from electrodynamics, Phys. Rev. D **103**, 105001 (2021).
[47] Arjun Bagchi, Aditya Mehra, and Poulami Nandi, Field theories with conformal Carrollian symmetry, J. High Energy Phys. 05 (2019) 108.
[48] Arjun Bagchi, Daniel Grumiller, and Poulami Nandi, Carrollian superconformal theories and super BMS, J. High Energy Phys. 05 (2022) 044.
[49] Minhajul Islam, Carrollian Yang-Mills Theory, arXiv:2301.00953.
[50] Aditya Mehra and Aditya Sharma, Towards Carrollian quantization: The case of carrollian electrodynamics, arXiv:2302.13257.
[51] D. Binosi and L. Theussl, JaxoDraw: A graphical user interface for drawing Feynman diagrams, Comput. Phys. Commun. **161**, 76 (2004).