Primordial non-Gaussianity in the warm *k*-inflation

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This paper presents and investigates non-Gaussian perturbations for the warm *k*-inflation model that is driven by pure kinetic energy. The two complementary components of the overall non-Gaussianity are the three-point and four-point correlations. The intrinsic non-Gaussian component, denoted as the nonlinear parameter f_{NL}^{int}, is rooted in the three-point correlation for the inflaton field. Meanwhile, the δN part non-Gaussianity, denoted as $f_{NL}^{\delta N}$, is the contribution attributed to the four-point correlation function of the inflaton field. In this paper, the above two components in warm *k*-inflation are individually computed and analyzed under the condition that the dissipative coefficient in warm inflation is temperature independent. Then, comparisons and discussions between them are conducted, and the non-Gaussian theoretical results are compared with experimental observations to determine the range of model parameters within the allowable range of observation.

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I. INTRODUCTION

The inflationary model is the most appealing for explaining issues of the standard cosmology model such as horizon, flatness, and monopole puzzles. The dominant inflationary models have been categorized into two paradigms via the previous study. In the first paradigm, which is called cold inflation, the inflationary field diminishes potential and swiftly drives the universe toward the supercooling phase. To solve the "graceful exit" problem, there must be a reheating period to bring the universe back to a radiation-dominated phase at the end of inflation [1–3]. Warm inflation is the other paradigm, and in this paradigm, there is no reheating phase during warm inflation because radiation is continuously produced by interactions of the inflaton field with several other subdominant boson or fermion fields. The universal expansion exits gracefully, and the radiation energy density becomes dominant smoothly when the inflation ends [4–8].

The most general inflationary scenarios are based on the potential energy for the scalar field, in which the potential energy outweighs the dynamic energy and causes the universe to grow quasi-exponentially. However, Mukhanov first proposed the "*k*-inflation" model that is driven by kinetic energy terms for a scalar field ϕ [9]. In string theory, nonstandard kinetic components are studied based on the existence of higher-order corrections to the effective action of the scalar field. So, the *k*-inflation picture introduces novel mechanics to the inflation model. The *k*-inflation model has been generalized to warm inflation in our previous work [10,11]. The standard potential-driven warm inflation theory has been extended to a warm *k*-inflationary case including cosmological perturbations [11]. In addition, there are some kinds of new and more effective warm inflationary theories proposed recently [12,13]. A warm inflationary scenario can have interesting features to construct a unifying picture of very early inflation with dark matter or dark energy [14–16]. From many different warm inflationary cases, it can be concluded that there are large differences among different theories of warm inflation perturbations, which incorporate strong and weak regimes [17–19].

Calculating the two-point correlation function is a method to distinguish different inflation models, where two-point correlation statistical information is reflected by the power spectrum. However, the statistical information is limited in the power spectrum, so it cannot distinguish these models more effectively. Therefore, bispectral and non-Gaussian measurements that distinguish various inflation models become necessary and receive much attention [20,21]. When analyzing inflation models, non-Gaussianity is usually an important consideration. The Gaussian term is dominated in inflationary perturbations; i.e., the dominant

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term for slow roll inflation fluctuations deviates marginally from the pure Gaussian term. Gaussianity dominates the primordial curvature perturbations [22,23]. The three-point function and its Fourier transform, i.e., the bispectral representation, has the leading statistics ability to differentiate between non-Gaussian and Gaussian perturbations. The topic of non-Gaussianity in warm inflation has been studied in several scenarios, such as a general type of canonical and noncanonical warm inflation, and strong or weak dissipative regimes [24–28]. In recent years, numerous studies have investigated the primordial non-Gaussianity generated by warm inflationary models, but the research on non-Gaussianity in the warm k-inflationary case is still blank, which is the aim of this paper.

This paper provides the theory of non-Gaussianity in warm k-inflation under the condition that the dissipative coefficient Γ is phenomenologically independent of temperature. First, the dynamical equations of warm k-inflation for the flat Friedmann-Robertson-Walker (FRW) background are introduced. Typically, the observation limit of the nonlinear parameter f_{NL} is established to estimate the non-Gaussian level. Second, two complementary parts of the non-Gaussianity are discussed. The first one is the three-point correlation, which is presented from the field self-interaction and can be calculated by solving slow roll perturbation equations. The other one is the four-point correlation function, which can be derived using the δN formalism. Particularly, in the case of multiple inflation, the non-Gaussianity can be calculated conveniently using the δN form [29,30]. In this paper, the δN formalism is first introduced for calculating non-Gaussianity in warm k-inflation, and then the δN part non-Gaussianity is computed.

The rest of this paper is organized as follows: In Sec. II, the warm k-inflation model and its fundamental equations are introduced. In Sec. III, the δN formalism is applied to compute the non-Gaussianity produced by the four-point correlation, and the non-Gaussianity in the three-point correlation function is computed. Finally, in Sec. IV, the total results and discussions are obtained.

II. THE FRAMEWORK OF WARM *k*-INFLATION

Since the universe was built from multiple components during warm inflation, the total matter action can be represented as

$$S = \int d^4x\sqrt{-g}[\mathcal{L}(X,\phi) + \mathcal{L}_\gamma + \mathcal{L}_{\rm int}], \tag{1}$$

where g denotes the metric determinant, $X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, $\mathcal{L}(X,\phi)$ denotes the Lagrangian density in the inflaton field, $\mathcal{L}_\gamma$ denotes the Lagrangian density in the radiation field, and $\mathcal{L}_{\rm int}$ describes the inflaton field interacting with other fields.

The warm k-inflation model that was employed originally is [9]

$$\mathcal{L} = K(\phi)X + L(\phi)X^2 + \cdots. \tag{2}$$

By redefining

$$\phi_{\rm new} = \int d\phi_{\rm old}\mathcal{L}^{\frac{1}{4}}(\phi_{\rm old}), \tag{3}$$

where the $\phi_{\rm old}$ and $\mathcal{L}(\phi_{\rm old})$ mean the inflaton field and the Lagrangian density in Eq. (2). We can rewrite the Lagrangian density Eq. (2) in a simpler and more concise form as

$$\mathcal{L} = K_{\rm new}(\phi_{\rm new})X_{\rm new} + X^2_{\rm new}. \tag{4}$$

In the following, the new field is used without the subscript "new" for convenience. The $K(\phi)$ in the equation above is called the "kinetic function,", which is the function of the inflaton field ϕ.

Because of the redefined ϕ in our pure kinetic warm inflationary model, ϕ does not have the usual dimension of mass, and the major parameters in the model lack the traditional dimension of the canonical warm inflation. The inflaton field is dimensionless, and the dimensions of the corresponding major parameters are given as follows:

$$[\dot{\phi}] = [m], \qquad [X] = [m]^2, \qquad [K(\phi)] = [m]^2,$$
$$[\Gamma] = [m]^3, \qquad [r] = [m]^2. \tag{5}$$

The quantity Γ in the equation above is the dissipative coefficient in warm inflation which can describe the thermal dissipation of the inflaton field to radiation, and r is the dissipative strength parameter describing the strength of the thermal dissipative effect, defined as $r = \frac{\Gamma}{3H}$. Then, the Lagrangian density in the pure kinetic inflaton field can be simply expressed as in [9,10]

$$\mathcal{L}(X,\phi) = K(\phi)X + X^2. \tag{6}$$

A fluid with the energy-momentum tensor can appropriately characterize the inflaton field in an FRW universe with a flat spatial structure:

$$T^\phi_{\mu\nu} = (\rho_\phi + p_\phi)u_\mu u_\nu - p_\phi g_{\mu\nu}, \tag{7}$$

where ρ_ϕ, p_ϕ and u_μ, and u_ν denote the energy density, pressure, and four-velocity for the inflaton field, respectively. The energy momentum of the inflaton field is determined by varying the action for the inflaton field relative to the metric,

$$T^\phi_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \mathcal{L}_X\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L}(X,\phi), \tag{8}$$

where $\mathcal{L}_X$ represents a Lagrangian partial derivative of $\mathcal{L}(X,\phi)$ with respect to X. Then we have

$$\rho_\phi = 2X\mathcal{L}_X - \mathcal{L} = K(\phi)X + 3X^2, \tag{9}$$

$$p_\phi = \mathcal{L} = K(\phi)X + X^2, \tag{10}$$

and

$$u_\mu = \sigma\frac{\nabla_\mu\phi}{\sqrt{2X}}, \tag{11}$$

where σ denotes the sign of $\dot\phi$ (i.e., when $\dot\phi > 0$, $\sigma = 1$; otherwise, $\sigma = -1$).

In the warm k-inflation, the major dynamical equations are given by [10]

$$H^2 = \frac{1}{3M_P^2}(\rho_\phi + \rho_\gamma) = \frac{1}{3M_P^2}\rho, \tag{12}$$

$$\dot\rho_\phi + 3H(\rho_\phi + p_\phi) = -\Gamma\dot\phi^2, \tag{13}$$

$$\dot\rho_\gamma + 4H\rho_\gamma = \Gamma\dot\phi^2, \tag{14}$$

where $M_p^2 \equiv (8\pi G)^{-1}$, ρ is the total energy density of the universe, and ρ_γ denotes the energy density for radiation. The dissipative coefficient Γ phenomenologically describes the decay of the inflaton field ϕ via the interaction Lagrangian $\mathcal{L}_{\text{int}}$ during the inflationary phase. The dimension of Γ is different from that in canonical warm inflation as we stated in Eq. (5). The dissipative coefficient Γ in our manuscript thus has a different functional form from that in the canonical warm inflation. The specific functional form of Γ depends on the concrete microphysical interactions between inflaton and other subdominated fields in the warm k-inflationary scenario, which we will concentrate on our later research. In principle, $\Gamma\dot\phi^2$ is a proper approximation for the energy dissipated by the inflaton field ϕ into a thermalized radiation bath. And we have checked the assumption is self-consistent in our previous work [10].

Since the inflaton field and radiation are the dominant components of the universe during inflation, the total energy density ρ and pressure p are represented as

$$\rho = K(\phi)X + 3X^2 + \rho_\gamma \tag{15}$$

and

$$p = K(\phi)X + X^2 + \frac{1}{3}\rho_\gamma. \tag{16}$$

This paper considers a homogeneous background scalar field, so we have $X = \frac{1}{2}\dot\phi^2$. The motion equation for the inflaton field can be determined by varying the action function and considering the thermal damped effect in the warm inflationary scenario:

$$(3\dot\phi^2 + K)\ddot\phi + 3H(\dot\phi^2 + K + r)\dot\phi + \frac{1}{2}K_\phi\dot\phi^2 = 0, \tag{17}$$

where K_ϕ is a derivative of ϕ.

Because of the difficulty of solving the exact model through Eqs. (12), (14), and (17), a slow roll approximation is frequently used. A stability analysis was conducted to confirm the slow roll situation for the dynamical systems staying near the quasi-exponential inflationary attractor for a significant number of Hubble times [10]. The conditions of a slow roll are

$$\epsilon \ll 1, \qquad |\eta| \ll \frac{\mathcal{L}_X}{(\mathcal{L}_X + r)c_s^2}, \qquad |b| \ll 1,$$
$$|c| < 4, \qquad \frac{rc_s^2}{\mathcal{L}_X} \ll 1 - 2c_s^2, \tag{18}$$

where the parameters in the above equations are defined as

$$\epsilon = \frac{K_\phi\dot\phi}{HK}, \qquad \eta = \frac{K_{\phi\phi}\dot\phi}{HK_\phi}, \qquad b = \frac{\Gamma_\phi\dot\phi}{H\Gamma},$$
$$c = \frac{T\Gamma_T}{\Gamma}, \qquad c_s^2 \simeq \frac{\dot p}{\dot\rho} = \frac{\dot\phi^2 + K}{3\dot\phi^2 + K} < 1. \tag{19}$$

In the parameter definitions, the subscripts represent the partial derivative of the quantities for the inflaton field or temperature, while the dot represents the time derivative of quantities. Besides, quasi-exponential warm k-inflation proves that the term $\dot\phi^2 + K + r$ represents a small positive quantity, and $\dot\phi^2$ and $|K(\phi)|$ have the same order [10]. Then, the energy density of the inflaton field is on the order of $\frac{1}{4}|K|\dot\phi^2$, i.e., $\rho_\phi \sim \frac{1}{4}|K|\dot\phi^2$, and the inflationary period now enters the slow roll phase and has surpassed the radiation period, i.e., $\rho_\gamma < \rho_\phi$. Thus, the Friedmann equation (12) can be reduced to

$$H^2 \simeq \frac{1}{3M_P^2}\left(\frac{1}{4}|K|\dot\phi^2\right). \tag{20}$$

Then, based on the slow roll approximations and guaranteed by the slow roll conditions during the inflation [10,31], Eq. (17) can be rewritten as

$$6H(\dot\phi^2 + K + r) \simeq -K_\phi\dot\phi, \tag{21}$$

where the dissipative strength of the model is determined by the dissipative strength parameter r. In the weak dissipation regime ($r \ll \mathcal{L}_X$), the background dynamical evolution in the inflaton field is not affected by dissipation because it is too weak. However, the field fluctuations will be modified by the thermal variations in the radiation energy density, which will also affect the primordial spectrum of perturbations. In the strong dissipation regime ($r \gg \mathcal{L}_X$), the background dynamics and fluctuations will be dominated by thermal dissipation, making it simpler to satisfy slow roll conditions.

Generally, this paper considers radiation production to be quasistable, i.e., $\dot{\rho}_\gamma \ll 4H\rho_\gamma$. The density of radiation represented thus is

$$\rho_\gamma = \kappa T^4 \simeq \frac{3}{4} r\dot{\phi}^2, \tag{22}$$

where κ is the Stefan-Boltzmann constant and T is the temperature of the thermal bath. Based on Eqs. (19), (20), and (21), the relationship between ρ_γ and ρ_ϕ is obtained:

$$\rho_\gamma = \frac{r}{2(\mathcal{L}_X + r)} \epsilon \rho_\phi, \tag{23}$$

where $\mathcal{L}_X = \dot{\phi}^2 + K$. The following condition explains the epoch during which warm k-inflation takes place:

$$\rho_\phi \gg \frac{2(\mathcal{L}_X + r)}{r} \rho_\gamma. \tag{24}$$

In contrast, inflation ceases when the universe reaches a phase dominated by radiation, and this occurs when $\epsilon \simeq 1$, indicating $\rho_\phi \simeq \frac{2(\mathcal{L}_X + r)}{r}\rho_\gamma$ at the end of inflation. The number of e-folds of inflation is given by

$$N = \int_{t_i}^{t_e} H dt = \int_{\phi_i}^{\phi_e} \frac{H}{\dot{\phi}} d\phi \simeq \frac{\sigma}{2\sqrt{3}M_p} \int_{\phi_i}^{\phi_e} \sqrt{-K(\phi)} d\phi, \tag{25}$$

where ϕ_i is the initial value of the inflaton field and ϕ_e is the final value.

III. THE NON-GAUSSIANITY IN WARM k-INFLATION

Non-Gaussianity in warm k-inflation is comprised of two complementary elements: the δN component and the intrinsic component. These two components are now determined separately.

A. The δN part non-Gaussianity

The δN formalism is often used to compute the non-Gaussian property of multifield inflation, which can be found in numerous works [32–34]. According to cosmological observations, the primordial curvature perturbation, denoted as ζ, is a Gaussian dominated term with a nearly scale-invariant spectrum.

The expansion $N(t, \mathbf{x}) \equiv \ln[\frac{\tilde{a}(t)}{a(t_{\rm in})}]$ from any beginning flat slice at time $t_{\rm in}$ to a final slice can be described with uniform energy density, where $\tilde{a}(t, \mathbf{x})$ is the locally defined scale factor. As δN formalism suggests [32,33,35], and considering that the curvature perturbation ζ is almost Gaussian, we have

$$\zeta(t, \mathbf{x}) \simeq \delta N = N(t, t_{\rm i}, {\rm x}) - N(t, t_i). \tag{26}$$

For good accuracy, δN can perform series expansion of the initial scalar field,

$$\delta N = N_{,I}\delta\phi^I + \frac{1}{2} N_{,IJ}\delta\phi^I\delta\phi^J + \cdots, \tag{27}$$

where $N_{,I} \equiv \frac{\partial N}{\partial \phi^I}$ and $N_{,IJ} \equiv \frac{\partial^2 N}{\partial\phi^I \partial\phi^J}$. In the equation above, the items above the second order are omitted. Finally, the two-point correlation function and three-point correlation function could be stated in the form of δN:

$$\mathcal{P}_\zeta = \delta^{IJ} N_{,I} N_{,J} \mathcal{P}_{\phi *} \tag{28}$$

and

$$\begin{aligned}\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle &= N_{,I}N_{,J}N_{,K}\langle\delta\phi^I(\boldsymbol{k}_1)\delta\phi^J(\boldsymbol{k}_2)\delta\phi^K(\boldsymbol{k}_3)\rangle \\ &+ \frac{1}{2} N_{,I}N_{,J}N_{,KL}\langle\delta\phi^I(\boldsymbol{k}_1)\delta\phi^J(\boldsymbol{k}_2)(\delta\phi^K \star \delta\phi^L)(\boldsymbol{k}_3)\rangle \\ &+ \text{perms},\end{aligned} \tag{29}$$

where $\star$ represents convolution and the expanded high-order term is not written down. Now, this paper introduces the nonlinear parameter f_{NL} describing the non-Gaussian level, and they stand for observational limits. The power spectrum and bispectrum for curvature perturbation are defined as

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\rangle \equiv (2\pi)^3\delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2)\frac{2\pi^2}{k_1^3}\mathcal{P}_\zeta(k_1) \tag{30}$$

and

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_1)\rangle \equiv (2\pi)^3\delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)B_\zeta(k_1, k_2, k_3), \tag{31}$$

where $\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2}P_\zeta(k)$.

The bispectrum can be expressed as

$$B_\zeta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\frac{6}{5} f_{NL}[P_\zeta(\boldsymbol{k}_1)P_\zeta(\boldsymbol{k}_2) + \text{cyclic}]. \tag{32}$$

During our warm k-inflationary model, only one inflaton field is relevant, so the relation Eq. (27) is reduced to

$$\zeta(t, \boldsymbol{x}) = N_\phi\delta\phi + \frac{1}{2}N_{\phi\phi}(\delta\phi)^2. \tag{33}$$

Thus, the general δN part nonlinear parameter for our model can be described as

$$-\frac{3}{5}f_{NL}^{\delta N}=\frac{1}{2}\frac{N_{\phi\phi}}{N_{\phi}^{2}}. \tag{34}$$

The term $f_{NL}^{\delta N}$ is scale independent and can be obtained by Eq. (34). Inflation observations are computed at the time of the horizon crossing. Since the horizon crossing occurs within the region of slow roll inflation, it is appropriate to compute the δN part nonlinear parameter $f_{NL}^{\delta N}$ using slow roll approximations.

Considering the conditions of the slow roll, we have

$$N_{\phi}=\frac{\sigma}{2\sqrt{3}M_{p}}\sqrt{-K(\phi)}, \tag{35}$$

and from Eq. (35), we have

$$N_{\phi\phi}=-\frac{\sigma}{4\sqrt{3}M_{p}}[-K(\phi)]^{-\frac{1}{2}}K_{\phi}. \tag{36}$$

In terms of $-\frac{3}{5}f_{NL}^{\delta N}=\frac{1}{2}\frac{N_{\phi\phi}}{N_{\phi}^{2}}$, one can obtain

$$f_{NL}^{\delta N}=\frac{5}{6}\frac{\sigma\sqrt{3}M_{p}K_{\phi}}{(-K)^{\frac{3}{2}}}=-\frac{5}{12}\sigma\epsilon, \tag{37}$$

where $\epsilon=\frac{K_{\phi}\dot{\phi}}{HK}$ is a slow roll parameter. Thus, we have $f_{NL}^{\delta N}\ll 1$.

As suggested by the slow roll conditions in warm k-inflation, the amplitude of δN-form non-Gaussianity is not distinct in the slow roll regime. Then it can increase slightly accompanying the inflation for the universe. Given that the δN form non-Gaussianity is insufficiently large, using this part to show the whole non-Gaussianity caused by inflation is not enough and is not safe and complete, as some studies have shown [10,36]. In this case, calculating the intrinsic non-Gaussianity produced by the three-point correlation functions of the inflation field is essential.

B. The intrinsic part non-Gaussianity

Compared with cold inflation, warm inflation fluctuations are generated mainly by thermal fluctuations. In warm k-inflation, only one scalar field plays the role of inflaton. When small perturbations are considered, the full inflaton field can be extended as $\Phi(\boldsymbol{x},t)=\phi(t)+\delta\phi(\boldsymbol{x},t)$, where $\delta\phi$ is the usual perturbation field surrounding the homogeneous background field $\phi(t)$.

To get analytic results, we concentrate on the temperature independent case; i.e., Γ is a constant or $\Gamma=\Gamma(\phi)$. We treat the perturbations in spatially flat gauge $\zeta=\frac{H}{\dot{\phi}}\delta\phi$ and consider that the dissipative coefficient Γ is independent on temperature. Under these conditions, the inflaton fluctuations and radiation fluctuations are decoupled [19,37]. And considering that when the thermal dissipation effect is not strong (even a very weak dissipation effect can result in a warm inflation $T>H$), the radiation source term can be neglected and the comoving curvature perturbation $\mathcal{R}_k$ is adiabatic [11], so the analytic calculations of bispectrum can be performed in the following.

Horizon crossing occurs deeply inside the slow roll regime, and the observations of inflation are calculated at this time. In the warm k-inflation, due to the enhancement of the Hubble and thermal damped terms, the inflation evolution is overdamped inside the slow roll regime. The motion of the entire field perturbation can be explained by introducing random thermal noise ξ [11]:

$$\mathcal{L}_{X}c_{s}^{-2}\delta\ddot{\phi}_{k}(t)+3H(\mathcal{L}_{X}c_{s}^{-2}+r)\delta\dot{\phi}_{k}(t)+\mathcal{L}_{X}\frac{k_{c}^{2}}{a^{2}}\delta\phi_{k}(t)=\xi_{k}, \tag{38}$$

where ξ in the equation above is a white noise term in a thermal system with zero mean $\langle\xi\rangle=0$ and two-point correlation relation $\langle\xi(\mathbf{k},t)\xi(\mathbf{k}',t')\rangle=2\Gamma T(2\pi)^{3}\times\delta^{3}(\mathbf{k}-\mathbf{k}')\delta(t-t')$ with respect to the fluctuation-dissipation theorem [38]. The equation above is known as the Langevin equation, and it is used to describe the interaction between a scalar field and radiation. In the equation above, k_c is the comoving wave number. Guaranteed by the conditions of slow roll, the inertia term $\delta\ddot{\phi}_k$ is usually omitted to simplify the perturbation calculations [38,39].

To calculate $\delta\phi$, we expand $\delta\phi$ to second-order $\delta\phi=\delta\phi_1+\delta\phi_2$, where $\delta\phi_1=\mathcal{O}(\delta\phi)$ and $\delta\phi_2=\mathcal{O}(\delta\phi^2)$. Consequently, the evolution equation of the first- and second-order perturbation field in the Fourier space can be obtained:

$$\frac{d\delta\phi_{1}(\boldsymbol{k},t)}{dt}=\frac{1}{3H(6X+K+r)}[-\mathcal{L}_{X}k^{2}\delta\phi_{1}(\boldsymbol{k},t)+\xi(\boldsymbol{k},t)] \tag{39}$$

and

$$\begin{aligned}\frac{d\delta\phi_{2}(\boldsymbol{k},t)}{dt}=\frac{1}{3H(6X+K+r)}\Bigg[&-\mathcal{L}_{X}k^{2}\delta\phi_{2}(\boldsymbol{k},t)\\&-k^{2}\mathcal{L}_{X\phi}\int\frac{dp^{3}}{(2\pi)^{3}}\delta\phi_{1}(\boldsymbol{p},t)\delta\phi_{1}(\boldsymbol{k}-\boldsymbol{p},t)\\&-k^{2}\mathcal{L}_{XX}\int\frac{dp^{3}}{(2\pi)^{3}}\delta\phi_{1}(\boldsymbol{p},t)\delta X_{1}(\boldsymbol{k}-\boldsymbol{p},t)\Bigg].\end{aligned} \tag{40}$$

The quantity X_1 in the above equation can be obtained by

$$\delta X_{1}=\dot{\phi}\delta\dot{\phi}_{1}=\sqrt{2X}\frac{d}{dt}\delta\phi_{1}, \tag{41}$$

where k is the physical wave number, $\boldsymbol{k} \equiv \boldsymbol{k}_P = \frac{\boldsymbol{k}_c}{a}$ ($\boldsymbol{k}_c$ represents the comoving momentum, $\boldsymbol{k}_p$ represents the physical momentum, and $k = |\boldsymbol{k}|$).

Considering the slow variation of X, K, and r in the slow roll regime, we will ignore the time variation of them during the time interval $t_0 \sim t$ as in paper [38]. Then we can get the approximate analytic solutions by solving the evolution equations

$$\delta\phi_1(\boldsymbol{k},t) = \frac{1}{3H(6X+K+r)}\exp\left[-\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t-t_0)\right]\int_{t_0}^{t} dt' \exp\left[\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t'-t_0)\right]\xi(\boldsymbol{k},t') + \delta\phi_1(\boldsymbol{k},t_0)\exp\left[-\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t-t_0)\right] \tag{42}$$

and

$$\delta\phi_2(\boldsymbol{k},t) = \exp\left[-\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t-t_0)\right]\int_{t_0}^{t} dt' \exp\left[\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t'-t_0)\right] \times\left[A(k,t')\int\frac{dp^3}{(2\pi)^3}\delta\phi_1(\boldsymbol{p},t')\delta\phi_1(\boldsymbol{k}-\boldsymbol{p},t') + B(k,t')\int\frac{dp^3}{(2\pi)^3}\delta\phi_1(\boldsymbol{p},t')\xi(\boldsymbol{k}-\boldsymbol{p},t')\right] + \delta\phi_2(\boldsymbol{k},t_0)\exp\left[-\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t-t_0)\right]. \tag{43}$$

The parameters $A(\boldsymbol{k},t)$ and $B(\boldsymbol{k},t)$ appear in the equation above as

$$A(\boldsymbol{k},t) = -\frac{1}{3H(6X+K+r)}\left[k^2\mathcal{L}_{X\phi} + k^2\mathcal{L}_{XX}\frac{\sqrt{2X}k^2\mathcal{L}_X}{3H(6X+K+r)}\right] \tag{44}$$

and

$$B(\boldsymbol{k},t) = -\frac{\mathcal{L}_{XX}k^2\sqrt{2X}}{[3H(6X+K+r)]^2}. \tag{45}$$

Based on Eq. (42), a characteristic parameter $\tau(\phi) = \frac{3H(\mathcal{L}_X c_s^{-2}+r)}{\mathcal{L}_X k^2}$ can be defined to describe the efficiency of the thermalizing process. It is found that a larger k^2 indicates a faster relaxation rate. If the k^2 of one of the fields $\Phi(\mathbf{x},t)$ is large enough to relax within a Hubble time, the mode can be thermal. When the physical wave number k_p of the corresponding $\Phi(\mathbf{x},t)$ mode is smaller than the freeze-out physical wave number k_F, it is no longer affected by thermal noise ξ_k during a Hubble time. Based on the condition $\tau(\phi) = \frac{3H(\mathcal{L}_X c_s^{-2}+r)}{\mathcal{L}_X k^2} = \frac{1}{H}$, the freeze-out momentum k_F could be given by

$$k_F = \sqrt{\frac{3H^2}{c_s^2}\left(1+\frac{rc_s^2}{\mathcal{L}_X}\right)}. \tag{46}$$

As previously stated, the first-order inflaton perturbation $\delta\phi_1$ is a pure Gaussian field, and their bispectrum vanishes due to their statistical stochastic features. To calculate non-Gaussianity, the bispectrum resulting from two first-order and one second-order fluctuations should have the highest order. Then we have

$$\begin{aligned}\langle\delta\phi(\boldsymbol{k}_1,t)\delta\phi(\boldsymbol{k}_2,t)\delta\phi(\boldsymbol{k}_3,t)\rangle &= \exp\left[-\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t-t_0)\right]\int_{t_0}^{t} dt' \exp\left[\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t'-t_0)\right][A(k,t')\int\frac{dp^3}{(2\pi)^3} \\ &\times\langle\delta\phi_1(\boldsymbol{k}_1,t)\delta\phi_1(\boldsymbol{k}_2,t)\delta\phi_1(\boldsymbol{p},t')\delta\phi_1(\boldsymbol{k}_3-\boldsymbol{p},t')\rangle + B(k,t')\int\frac{dp^3}{(2\pi)^3}\langle\delta\phi_1(\boldsymbol{k}_1,t)\delta\phi_1(\boldsymbol{k}_2,t)\delta\phi_1(\boldsymbol{p},t')\xi(\boldsymbol{k}_3-\boldsymbol{p},t')\rangle] \\ &+ \exp\left[-\frac{\mathcal{L}_X k^2}{3H(6X+K+r)}(t-t_0)\right]\langle\delta\phi_1(\boldsymbol{k}_1,t)\delta\phi_1(\boldsymbol{k}_2,t)\delta\phi_2(\boldsymbol{k}_3,t_0)\rangle + (\boldsymbol{k}_1\leftrightarrow\boldsymbol{k}_3) + (\boldsymbol{k}_2\leftrightarrow\boldsymbol{k}_3).\end{aligned} \tag{47}$$

The bispectrum amplitude is determined when the cosmic scale departs the horizon. There are about 60 e-folds until the end of the inflation, and $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, and $\boldsymbol{k}_3$ are crossing the horizon all within a few e-folds. $k_F > H$ is deduced from the expression of k_F in the warm k-inflationary model. This indicates the correlations in our model, known as the thermalized correlations, which should be calculated at the crossing of the Hubble horizon $k = H$, are determined at an earlier freeze-out period $k = k_F$ [38,40,41]. Thus, the duration between corrections can be calculated by

$$\Delta t_F = t_H - t_F \simeq \frac{1}{H}\ln\left(\frac{k_F}{H}\right). \tag{48}$$

The bispectrum can then be reduced to

$$\begin{aligned}&\langle\delta\phi(\boldsymbol{k_1},t)\delta\phi(\boldsymbol{k}_2,t)\delta\phi(\boldsymbol{k}_3,t)\rangle\\&\quad\simeq 2A(k_F,t_F)\Delta t_F\left[\frac{dp^3}{(2\pi)^3}\langle\delta\phi_1(\boldsymbol{k}_1,t)\delta\phi_1(\boldsymbol{k}_1,p)\rangle\langle\delta\phi_1(\boldsymbol{k}_2,t)\delta\phi_1(\boldsymbol{k}_3-p,t)\rangle+(\boldsymbol{k}_1\leftrightarrow\boldsymbol{k}_3)+(\boldsymbol{k}_2\leftrightarrow\boldsymbol{k}_3)\right].\end{aligned} \tag{49}$$

According to Eqs. (31), (32), (49), and the relation $\zeta = \frac{H}{\dot\phi}\delta\phi$, the intrinsic non-Gaussian nonlinear parameter can be obtained,

$$\begin{aligned}f_{NL}^{\rm int} &= -\frac{5}{6}\frac{\dot\phi}{H}2A(k_F,t_F)\Delta t_F\\&= \frac{5}{3}\ln\sqrt{\frac{3}{c_s^2}\left(1+\frac{rc_s^2}{\mathcal{L}_X}\right)}\left[\frac{\sqrt{2X}k_F^2\mathcal{L}_{X\phi}}{3H^3(6X+K+r)}+\frac{2k_F^4\mathcal{L}_{XX}\mathcal{L}_X X}{9H^4(6X+K+r)^2}\right]\\&= \underbrace{\frac{5}{3}\ln\sqrt{\left(\frac{3}{c_s^2}+\frac{3r}{\mathcal{L}_X}\right)}\frac{K}{K+2X}\left(1+\frac{rc_s^2}{\mathcal{L}_X}\right)\epsilon}_{\text{term 1}}+\underbrace{\frac{5}{3}\ln\sqrt{\left(\frac{3}{c_s^2}+\frac{3r}{\mathcal{L}_X}\right)}\frac{4X(2X+K)}{(6X+K+r)^2}\left(\frac{1}{c_s^2}+\frac{r}{\mathcal{L}_X}\right)^2}_{\text{term 2}}.\end{aligned} \tag{50}$$

The intrinsic non-Gaussianity we obtained has a nearly equilateral shape. By exploiting conditions of slow roll in warm k-inflation, we have

$$\text{term 1} = \frac{5}{3}\ln\sqrt{\left(\frac{3}{c_s^2}+\frac{3r}{\mathcal{L}_X}\right)}\frac{K}{K+2X}\left(1+\frac{rc_s^2}{\mathcal{L}_X}\right)\epsilon \ll 1. \tag{51}$$

From this, it can be concluded that the second term dominates the intrinsic non-Gaussian nonlinear parameter, which can be obtained as follows:

$$f_{NL}^{\rm int} \simeq \frac{5}{3}\ln\sqrt{\left(\frac{3}{c_s^2}+\frac{3r}{\mathcal{L}_X}\right)}\frac{4X(2X+K)}{(6X+K+r)^2}\left(\frac{1}{c_s^2}+\frac{r}{\mathcal{L}_X}\right)^2. \tag{52}$$

From this equation, a small inflaton sound speed can significantly increase the amount of intrinsic non-Gaussianity, and strong thermal dissipation can also increase the proportion of intrinsic non-Gaussianity in warm k-inflation.

C. Discussions of result and parameters' restriction of the model

According to the above-mentioned field evolution equation, there are two dissipation terms, namely, Hubble dissipation $3H(\dot\phi^2 + K)\dot\phi$ and thermal dissipation $\Gamma\dot\phi$. Thus, we have $\mathcal{L}_X = K + 2X = K + \dot\phi^2$, and $r > \mathcal{L}_X$ indicates thermal effects dominate. This paper compares two portions of the non-Gaussianity based on the conclusions reached. $f_{NL}^{\delta N}$ is represented by the polymerization of the redefined slow roll parameters. So, $f_{NL}^{\delta N}$ should be far smaller than 1 in the slow roll inflationary regime, while the intrinsic part $f_{NL}^{\rm int}$ is much more than 1 if the sound speed of the inflaton field is small enough. Since the noncanonical effect is strong, it can be concluded that the intrinsic component of non-Gaussianity is the main component.

The entire nonlinear parameter could be estimated using the nonlinear parameter that we have calculated in two parts. That is,

$$\begin{aligned}f_{NL} &= -\frac{5}{12}\sigma\epsilon + \frac{5}{3}\ln\sqrt{\left(\frac{3}{c_s^2}+\frac{3r}{\mathcal{L}_X}\right)}\frac{K}{K+2X}\left(1+\frac{rc_s^2}{\mathcal{L}_X}\right)\epsilon\\&\quad+\frac{5}{3}\ln\sqrt{\left(\frac{3}{c_s^2}+\frac{3r}{\mathcal{L}_X}\right)}\frac{4X(2X+K)}{(6X+K+r)^2}\left(\frac{1}{c_s^2}+\frac{r}{\mathcal{L}_X}\right)^2\\&\simeq\left(\frac{1}{c_s^2}+\frac{r}{\mathcal{L}_X}\right)^2.\end{aligned} \tag{53}$$

The result obtained above indicates that in the thermal effect dominated regime $r \gg \mathcal{L}_X, f_{NL} \sim (\frac{r}{\mathcal{L}_X})^2$. Consequently, when thermal effects are strong in the universe, the non-Gaussianity is distinct. In the weak dissipative regime $r \ll \mathcal{L}_X$, $f_{NL} \sim c_s^{-4}$, as determined mainly by the sound speed. The result is different from the cold noncanonical inflationary cases, whose nonlinear parameter has a general relation $f_{NL} \sim c_s^{-2}$ [42,43]. That very small amounts of dissipation can result in warm inflation ($T > H$) was found in our previous work [11]. Hence, in the limit $r \to 0$, some warm inflationary quantities cannot reproduce the cold inflation results. In addition, the systemic calculations of cosmological perturbations for warm inflation and cold inflation are quite different; our non-Gaussian result on warm k-inflation thus cannot reduce to a cold inflation result when $r \ll 1$. Consequently, the non-Gaussianity in warm k-inflation depend on sound speed to a greater extent. If the universe is dominated by noncanonical effects, the non-Gaussianity is more obvious. We can see that both the thermal and noncanonical effects can enhance the magnitude of non-Gaussianity. However, noncanonical effects contribute more to the non-Gaussian magnitude than the thermal effect in warm k-inflation. Because of observational limitations $f_{NL} \sim \mathcal{O}(10^2)$ [44,45], the speed of sound c_s must not be too small ($c_s \gtrsim 0.31$), and the dissipative strength parameter r is required to be not extremely large; i.e., r should generally have the same order of $\mathcal{L}_X$. If the warm k-inflationary model can fit the observations well, neither the noncanonical effect nor the thermal effect should be too strong.

IV. CONCLUSIONS AND DISCUSSIONS

This paper investigates the entire primordial non-Gaussianity produced by warm k-inflation under the assumption of a temperature independent dissipative coefficient, which allows for an approximate analytical analysis of the problem. The essential equations of warm k-inflation are presented, such as the motion equation, e-folds, slow roll equations, as well as slow roll conditions. This paper emphasizes the key problem: non-Gaussianity resulted from warm k-inflation. The nonlinear parameter is usually used to quantify the degree of non-Gaussianity, and it consists of two components: the intrinsic part $f_{NL}^{\rm int}$ and the δN part $f_{NL}^{\delta N}$. The first component covers the impact of the three-point correlation, i.e., the intrinsic non-Gaussianity of the inflaton field. The second component is determined by a four-point correlation with inflaton perturbations. The original non-Gaussianity in warm k-inflation can be fully captured by these two components.

The formalism of δN is introduced and used to calculate the δN part non-Gaussianity. It is concluded from the obtained results that $f_{NL}^{\delta N}$ is defined as the linear combination of the redefined slow roll parameters. So, in slow roll inflation, $f_{NL}^{\delta N}$ is a first-order small quantity, and it indicates that the δN part non-Gaussianity for warm k-inflation is not significant. However, the situation is not the same for calculating intrinsic non-Gaussianity. To get the analytic result of intrinsic part non-Gaussianity, we restricted to the temperature independent case in our calculations. The intrinsic non-Gaussianity is principally driven by the sound speed and dissipation strength parameters, and it is produced by three-point correlations in the inflation. Throughout the entire non-Gaussianity in warm k-inflation, it is observed that $f_{NL}^{\rm int}$ dominates the $f_{NL}^{\delta N}$ part, sound speed plays the most important role in the non-Gaussianity of our model, and thermal dissipation effects also contribute to non-Gaussianity. A low sound speed and a large dissipative strength can much enhance the magnitude of non-Gaussianity, and both the parameters are constrained by the observations. Our result for the non-Gaussianity in the warm k-inflationary scenario departs from cold inflation, and it cannot reproduce cold inflationary results in the regime where $r \to 0$. The further research and comparison between non-Gaussianities in warm inflation and cold inflation deserve more attention, and this will be the focus of our future work.

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[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[4] A. Berera, Phys. Rev. Lett. **75**, 3218 (1995).
[5] A. Berera and L. Z. Fang, Phys. Rev. Lett. **74**, 1912 (1995).
[6] A. Berera, I. G. Moss, and R. O. Ramos, Rep. Prog. Phys. **72**, 026901 (2009).
[7] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).
[8] A. Berera, M. Gleiser, and R. O. Ramos, Phys. Rev. Lett. **83**, 264 (1999).

[9] C. Armendariz-Picon, T. Damour, and V. Mukhanov, Phys. Lett. B **458**, 209 (1999).
[10] Z. P. Peng, J. N. Yu, J. Y. Zhu, and X. M. Zhang, Phys. Rev. D **94**, 103531 (2016).
[11] Z. P. Peng, J. N. Yu, X. M. Zhang, and J. Y. Zhu, Phys. Rev. D **97**, 063523 (2018).
[12] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, Phys. Rev. Lett. **117**, 151301 (2016).
[13] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, Phys. Lett. B **813**, 136055 (2021).
[14] J. G. Rosa and L. B. Ventura, Phys. Lett. B **798**, 134984 (2019).
[15] J. G. Rosa and L. B. Ventura, Phys. Rev. Lett. **122**, 161301 (2019).
[16] X. M. Zhang, K. Li, Y. F. Guo, P. C. Chu, H. Liu, and J. Y. Zhu, Phys. Rev. D **104**, 103513 (2021).
[17] M. Bastero-Gil, A. Berera, I. G. Moss, and R. O. Ramos, J. Cosmol. Astropart. Phys. 12 (2014) 008.
[18] R. Herrera, S. del Campo, and C. Campuzano, J. Cosmol. Astropart. Phys. 10 (2006) 009.
[19] C. Graham and I. G. Moss, J. Cosmol. Astropart. Phys. 07 (2009) 013.
[20] M. Tegmark, M. A. Strauss, M. R. Blanton, K. Abazajian, S. Dodelson, H. Sandvik, X. M. Wang, D. H. Weinberg, I. Zehavi, N. A. Bahcall *et al.*, Phys. Rev. D **69**, 103501 (2004).
[21] D. H. Jeong and E. Komatsu, Astrophys. J. **703**, 1230 (2009).
[22] A. F. Heavens, Mon. Not. R. Astron. Soc. **299**, 805 (1998).
[23] P. A. R. Ade *et al.* (Planck Collabration), Astron. Astrophys. **594**, A17 (2016).
[24] I. G. Moss and C. Xiong, J. Cosmol. Astropart. Phys. 04 (2007) 007.
[25] S. Gupta, A. Berera, A. F. Heavens, and S. Matarrese, Phys. Rev. D **66**, 043510 (2002).
[26] X. M. Zhang, K. Li, H. Y. Ma, Q. Liu, and J. Y. Zhu, Phys. Rev. D **100**, 123521 (2019).
[27] M. Mirbabayi and A. Gruzinov, J. Cosmol. Astropart. Phys. 02 (2023) 012.
[28] I. G. Moss and T. Yeomans, J. Cosmol. Astropart. Phys. 08 (2011) 009.
[29] M. Sasaki, Prog. Theor. Phys. **120**, 159 (2008).
[30] L. E. Allen, S. Gupta, and D. Wands, J. Cosmol. Astropart. Phys. 01 (2006) 006.
[31] S. Weinberg, *Cosmology* (Oxford University Press, Oxford, England, 2008).
[32] D. H. Lyth and Y. Rodríguez, Phys. Rev. Lett. **95**, 121302 (2005).
[33] D. H. Lyth and Y. Rodríguez, Phys. Rev. D **71**, 123508 (2005).
[34] M. Sasaki and T. Tanaka, Prog. Theor. Phys. **99**, 763 (1998).
[35] L. Boubekeur and D. H. Lyth, Phys. Rev. D **73**, 021301(R) (2006).
[36] X. M. Zhang, H. Y. Ma, P. C. Chu, J. T. Liu, and J. Y. Zhu, J. Cosmol. Astropart. Phys. 03 (2016) 059.
[37] M. Bastero-Gil, A. Berera, and R. O. Ramos, J. Cosmol. Astropart. Phys. 07 (2011) 030.
[38] A. Berera, Nucl. Phys. **585B**, 666 (2000).
[39] A. Berera, Phys. Rev. D **54**, 2519 (1996).
[40] S. A. Bass, A. Dumitru, M. Bleicher, L. Bravina, E. Zabrodin, H. Stocker, and W. Greiner, Phys. Rev. C **60**, 021902(R) (1999).
[41] H. van Hecke, H. Sorge, and N. Xu, Phys. Rev. Lett. **81**, 5764 (1998).
[42] X. Chen, M. Huang, S. Kachru, and G. Shiu, J. Cosmol. Astropart. Phys. 01 (2007) 002.
[43] P. Nath and M. Piskunov, J. High Energy Phys. 02 (2019) 034.
[44] G. D'Amico, M. Lewandowski, L. Senatore, and P. Zhang, arXiv:2201.11518.
[45] Y. Akrami *et al.* (Planck Collaboration), Astron. Astrophys. **641**, A9 (2020).