Resonant production of dark photons from axions without a large coupling

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Dark photons could be produced resonantly by the oscillating axion field in the early universe. This resonant production mechanism has been used in various contexts, including dark photon dark matter and primordial magnetic field production. However, for this resonant production to work in an expanding universe, a large axion-dark photon coupling is required, which is not easy to realize in terms of model building and requires the introduction of many charged fermions and/or the complex clockwork mechanism. In this paper, we present a new scenario that efficiently produces dark photons from the axion with a much smaller coupling. This is possible by modifying the dynamics of axion and significantly delaying the onset of oscillations, as in the so-called trapped misalignment mechanism. As a specific example, we consider models in which dark photon production occurs efficiently despite the small axion-dark photon coupling by temporally trapping an axion in a wrong minimum and releasing it after the Hubble parameter becomes much smaller than the axion mass. In this scenario, it is expected that the polarization asymmetry of dark photons and gravitational waves generated from dark photons will be significantly reduced.

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I. INTRODUCTION

Dark photons are known as a plausible candidate for physics beyond the Standard Model [1]. Recently they have attracted much attention as a candidate for dark matter, which can be probed by various experiments/observations via the kinetic mixing with the standard model photons. For reviews, see, e.g., Refs. [2,3].

The production mechanism of dark photon dark matter has been studied in the literature, such as the production from the inflationary fluctuations [4,5], the resonant production from the axion oscillation [6–8], gravitational production during reheating [9,10], misalignment production of coherent oscillation [11–13], production from cosmic strings [14,15], resonant production from dark Higgs [16] and production from spectator scalar field during inflation [17]. In this paper, we focus on the resonant production of dark photons from the axion condensate.

Dark photons can be produced resonantly through the coherent oscillation of the axion field [18]. The axion field begins to oscillate when the Hubble parameter becomes comparable to the axion mass. If the coupling between the axion and dark photon is sufficiently strong, dark photon production occurs efficiently until saturation is reached. This production then significantly affects the axion dynamics through backreaction. For example, when applied to the QCD axion, the axion abundance can be reduced to several percent as a result of dark photon production [19] (see also Ref. [20]). When dark photons are produced resonantly, they often exhibit a large circular polarization asymmetry. This is because one of the circular polarization modes is amplified, depending on the axion's velocity sign, which is typically biased either positively or negatively. The dark photons also generate stochastic gravitational wave background, which inherit the circular polarization asymmetry, providing a unique signal for this scenario [21–28]. In addition, if the dark photon mass is generated by the Higgs mechanism, the dark photons produced by the process can effectively stabilize the dark Higgs field at its origin until late times, where the hidden U(1) symmetry is restored. Then, the dark Higgs can lead to an additional short-term inflation phase [29] or the early dark energy [30]. Focusing on the coupling with the standard model photons instead of dark photons, the primordial large-scale magnetic field may also be produced by the same mechanism (e.g., [31]).[1]

[1]When electrons or other charged particles are present, they can hamper the growth of the gauge fields. This makes it harder to generate magnetic fields using the resonant production method.

While the resonant production of dark photons is a very interesting phenomenon and is being investigated in a variety of contexts, it necessitates a large axion-dark photon coupling, which, in turn, requires a nontrivial UV completion [6]. The situation is exacerbated when the dark gauge coupling is small, because the anomalous axion-dark photon coupling is considered to be proportional to the gauge coupling squared. For example, one way to realize such a large axion-dark photon coupling is to introduce many symmetry-breaking scalar fields as in the clockwork mechanism [32] (see also Refs. [33–40]). Alternatively, one may introduce a large number of charged fermions that run in the loop diagram.

In this paper, we show a novel possibility to efficiently produce dark photons from coherent oscillations of the axion for a much smaller coupling. This is due to a modification of the axion dynamics. Specifically, we consider an additional potential which traps the axion in early times. Such a potential can significantly delay the commencement of axion oscillations compared to the conventional scenario where the axion starts to oscillate when the Hubble parameter becomes comparable to the axion mass. The trapping effect on the axion has been investigated in the context of the QCD axion with an extra Peccei-Quinn symmetry breaking term [32,41–46], a lighter QCD axion [47], and the axion or axionlike particle undergoing a first-order phase transition [48]. It was shown that the trapped misalignment mechanism [32,44,46] can significantly increase or decrease the axion abundance, depending on the minimum in which the axion is trapped, as compared to the conventional misalignment mechanism [49–51]. Here we show that, by temporally trapping the axion in a wrong minimum and delaying the onset of oscillations as in the trapped misalignment mechanism, dark photons are resonantly produced even for the axion-dark photon couplings that are comparable to or smaller than $\mathcal{O}(1)$.

Finally, let us comment on related literature [27], which pointed out that a large initial velocity of axion due to large Peccei-Quinn symmetry breaking and the time-dependent decay constant could result in tachyonic dark photon production, even if the coupling between the axion and dark photon is weak. In such a case, a large circular polarization asymmetry is expected for the produced dark photons as well as gravitational waves. On the other hand, as we will discuss later, in our scenario with the trapped misalignment mechanism, dark photon production efficiently occurs via narrow resonance while the decay constant is not dynamical. Also, the circular polarization asymmetry is suppressed, which is confirmed by numerical calculations.

II. MODEL

Let us consider the system of axion (ϕ) and dark photon (A_μ). The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A_\mu A^\mu - \frac{\beta}{4f_a}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor, $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}/2\sqrt{-g}$ with $\epsilon^{0123} = 1$ is its dual, m_A is the mass of the dark photon, f_a is the axion decay constant and β is the axion-dark photon coupling. Here we consider the following axion potential,

$$V(\phi) = m_a^2 f_a^2\left[1 - \cos\left(\frac{\phi}{f_a}\right)\right] + V_{\rm trap}(\phi), \tag{2}$$

with $V_{\rm trap}$ being the potential which traps the axion in early times. For instance, one can consider the following trapping potential as a toy model,

$$V_{\rm trap}(\phi) = \frac{1}{2}m_*^2(\phi - \phi_*)^2\theta(t_* - t), \tag{3}$$

where $\theta(t)$ is the step function with respect to the cosmic time t, t_* is the time when the trapping ends, and $m_*\,(> m_a)$ is the mass stabilizing the axion around $\phi = \phi_*$.[2] We also consider the model motivated by the trapped QCD axion model [46] with the potential given by

$$V(\phi) = m_a(t)^2 f_a^2\left[1 - \cos\left(\frac{\phi}{f_a}\right)\right] + V_{\rm trap}(\phi), \tag{4}$$

where the time-dependent mass is given by

$$m_a(t) = \begin{cases} m_{a0}\left(\frac{t}{t_*}\right)^{b/2} & \text{for } t < t_* \\ m_{a0} & \text{otherwise} \end{cases}. \tag{5}$$

The trapping potential is given by

$$V_{\rm trap}(\phi) = \Lambda_H^4\left[1 - \cos\left(\frac{N_H\phi}{f_a}\right)\right], \tag{6}$$

where N_H is an integer. In case of the QCD axion, the time dependent mass is expressed as $m_{a0}(\Lambda/T)^b$ with $\Lambda \sim O(0.1)$ GeV being the QCD scale and $b \simeq 4$. In this model, first, the axion falls into one of the minima of the trapping potential (6). Figure 1 shows the schematic illustration of the potential with the above two cases.

[2]This potential is introduced as a toy model just for the illustration of the trapping behavior. As a realistic model, the Witten effect [52] can naturally realizes a similar situation. It induces a quadratic potential having a periodic multibranch structure and the monopole annihilation triggers the sudden disappearance of the potential. See [41,42,53] for the cosmological applications.

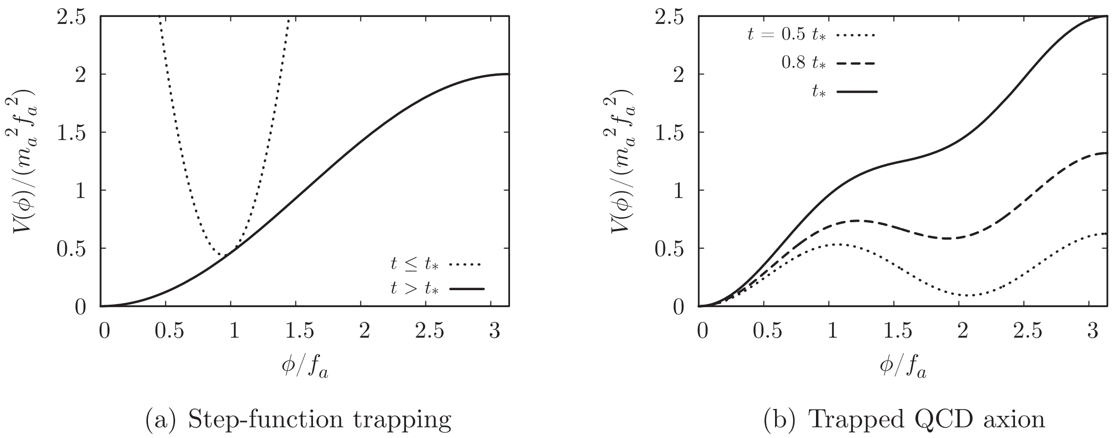


FIG. 1. Schematic illustration of the two potentials, a trapping by a step function of time with $m_* = 4m_a$ and $\phi_* = f_a$ (left) and the trapped QCD axion with $N_H = 3$ and $\Lambda_H^2 = 0.5 m_{a0} f_a$ (right).

From the Lagrangian, one obtains the following equations of motion,

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + V'(\phi) = -\frac{\beta}{4f_a} F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{7}$$

$$\ddot{\boldsymbol{A}} + H\dot{\boldsymbol{A}} - \frac{\nabla^2 \boldsymbol{A}}{a^2} + m_A^2 \boldsymbol{A} - \frac{\beta}{f_a a}(\dot{\phi}\nabla\times\boldsymbol{A} - \nabla\phi\times(\dot{\boldsymbol{A}} - \nabla A_0)) = 0, \tag{8}$$

where a is the scale factor, $H = \dot{a}/a$ is the Hubble parameter, with the overdot denoting the derivative with respect to the cosmic time. In this paper, we only focus on the dark photon production from the homogeneous axion oscillations and neglect the backreaction from produced dark photons. In this case, since the homogeneous axion field only couples to the transverse mode, only the transverse mode can be exited and we can neglect the longitudinal mode. Then, one can decompose the gauge field by circular polarization states,

$$\boldsymbol{A}(t,\boldsymbol{x}) = \sum_{\lambda=\pm}\int \frac{d^3k}{(2\pi)^3} \boldsymbol{e}_\lambda(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} A_\lambda(t,\boldsymbol{k}), \tag{9}$$

where $\boldsymbol{e}_\pm(\boldsymbol{k})$ are circular polarization vectors satisfying $\boldsymbol{k}\cdot\boldsymbol{e}_\pm(\boldsymbol{k}) = 0$ and $\boldsymbol{k}\times\boldsymbol{e}_\pm(\boldsymbol{k}) = \mp i|\boldsymbol{k}|\boldsymbol{e}_\pm(\boldsymbol{k})$. Then, assuming the homogeneous background including the axion field, the evolution equation for each mode is simplified as

$$\ddot{A}_\pm + H\dot{A}_\pm + \left(m_A^2 + \frac{k^2}{a^2} \mp \frac{k}{a}\frac{\beta\dot{\phi}_0}{f_a}\right)A_\pm = 0, \tag{10}$$

where ϕ_0 is the background value of the axion coherent oscillation. Note that it shows that the dark photon experiences tachyonic instabilities depending on the coupling β and the sign of $\dot{\phi}_0$. Soon after the beginning of the axion oscillation, the dark photon field can be produced exponentially. The equation of motion for the axion coherent oscillation is

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V'(\phi_0) = \frac{\beta}{f_a}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle, \tag{11}$$

where the right-hand side represents the backreaction by the produced dark photons and the angular bracket represents an average over volume. It can be calculated in terms of the Fourier modes of the circular polarization states as,

$$\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle = -\frac{1}{a^3}\int \frac{d^3k}{(2\pi)^3}\frac{k}{2}\frac{d}{dt}(|A_+|^2 - |A_-|^2). \tag{12}$$

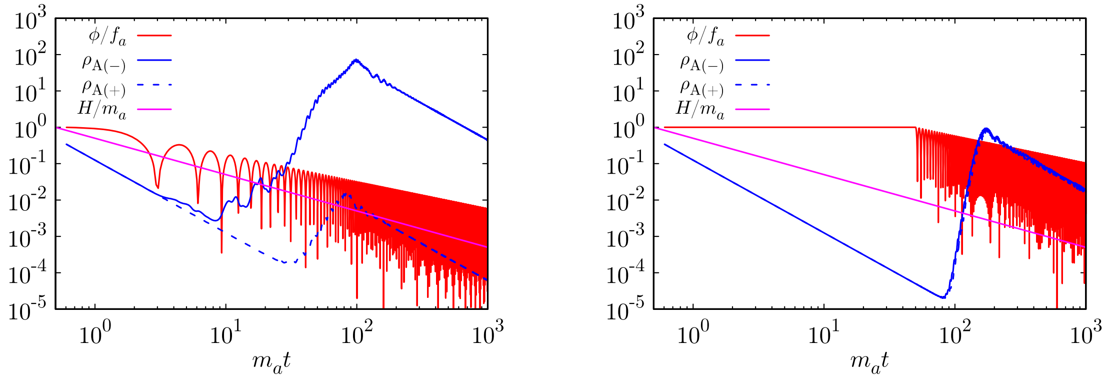
The electric and magnetic fields can be represented respectively by $E_i = -\dot{A}_i/a$, $B_i = \epsilon_{ijk}\partial_j A_k/a^2$ under the temporal gauge ($A_0 = 0$). The energy density of the dark photon can be calculated as follows

$$\rho_A^{(\pm)} = \frac{1}{2}\int \frac{dk}{k}\left[\mathcal{P}_E^{(\pm)}(k) + \mathcal{P}_B^{(\pm)}(k)\right], \tag{13}$$

with the power spectra given by

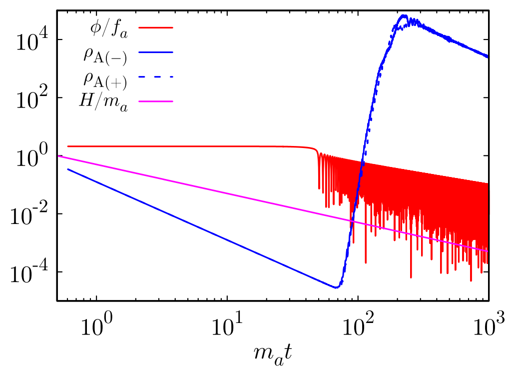
$$\mathcal{P}_E^{(\pm)}(k) = \frac{k^3|\dot{A}_\pm|^2}{2\pi^2 a^2}, \qquad \mathcal{P}_B^{(\pm)}(k) = \frac{k^5|A_\pm|^2}{2\pi^2 a^4}. \tag{14}$$

In the case with small β, and a sufficient long trapping of the axion before the onset of oscillations, the gauge field is amplified by the narrow resonance rather than the tachyonic instability. When it begins to oscillate,



(a) $\beta = 10$ without trap

(b) Step-function trapping, $\beta = 1$, $m_a t_* = 50$



(c) Trapped QCD axion, $\beta = 1$, $m_a t_* = 50$

FIG. 2. Time evolution of the energy density of the dark photon (blue), the axion field value (red) and the Hubble parameter (magenta). Solid and dashed blue lines corresponds respectively to the minus and plus circular polarization modes. We have taken $m_A = 0$ and $\beta = 10$ (top-left), $\beta = 1$ and $m_a t_* = 50$ (top-right and bottom), $m_* = 4m_a$ (top-right), $N_H = 3$ and $\Lambda_H^2 = 0.5 m_a f_a$ (bottom).

the homogeneous axion field evolves as $\phi(t) = \theta_i f_a (a_{\rm osc}/a)^{3/2}\cos(m_a t)$. Then Eq. (10) can be expressed as the following Mathieu equation,

$$\frac{\partial^2 \mathcal{A}_\pm}{\partial z^2} + [p - 2q\cos(2z)]\mathcal{A}_\pm = 0, \tag{15}$$

where we have defined $z \equiv m_a t/2 + \pi/4$, $\mathcal{A}_\pm \equiv a^{1/2} A_\pm$, $p = 4k^2/(am_a)^2 + 4m_A^2/(m_a)^2$, and $q = \mp 2k\beta\theta_i (a_{\rm osc}/a)^{3/2}/(am_a)$, and we have assumed $H \ll m_a$. Here, we are interested in the case of the narrow resonance, $|q| < 1$, with $\beta\theta_i \lesssim 1$. Then, the parametric amplification occurs in the first resonance band satisfying $1 - |q| < p < 1 + |q|$ [54,55]. Neglecting the dark photon mass for simplicity, the resonance condition reads

$$1 - \frac{\beta\theta_i}{2}\left(\frac{a_{\rm osc} m_a}{2k}\right)^{3/2} \lesssim \frac{am_a}{2k} \lesssim 1 + \frac{\beta\theta_i}{2}\left(\frac{a_{\rm osc} m_a}{2k}\right)^{3/2}, \tag{16}$$

where we dropped higher order terms of q since $|q| \ll 1$. Note that the center of the instability band is at $k/a = m_a/2$ and the band width is proportional to $\beta\theta_i$. Therefore the time interval during the resonant amplification can be estimated as

$$z_{\rm res} = \frac{1}{2} m_a (t_{\rm end} - t_{\rm osc}) \approx \frac{\beta\theta_i m_a}{H_{\rm osc}}\left(\frac{2k}{a_{\rm osc} m_a}\right)^{-3/2}, \tag{17}$$

where $t_{\rm end}$ is the time at the end of the resonance. Neglecting the backreaction to the axion, the gauge field is maximally amplified as $\mathcal{A}_\pm \propto \exp(\mu z_{\rm res})$ with $\mu \simeq |q|/2$ being the growth rate. Thus, the total growth exponent can be computed as follows

$$\mu z_{\rm res} \simeq \frac{1}{4}|q| m_a (t_{\rm end} - t_{\rm osc}) \simeq \frac{1}{2\sqrt{2}}\left(\frac{k}{a_{\rm osc} m_a}\right)^{-1/2} \frac{(\beta\theta_i)^2 m_a}{H_{\rm osc}}. \tag{18}$$

Note that the wave number giving the maximum enhancement is $k \approx a_{\rm osc} m_a/2$ since the momentum of produced dark photons should kinematically satisfy $k/a_{\rm osc} \gtrsim m_a/2$.

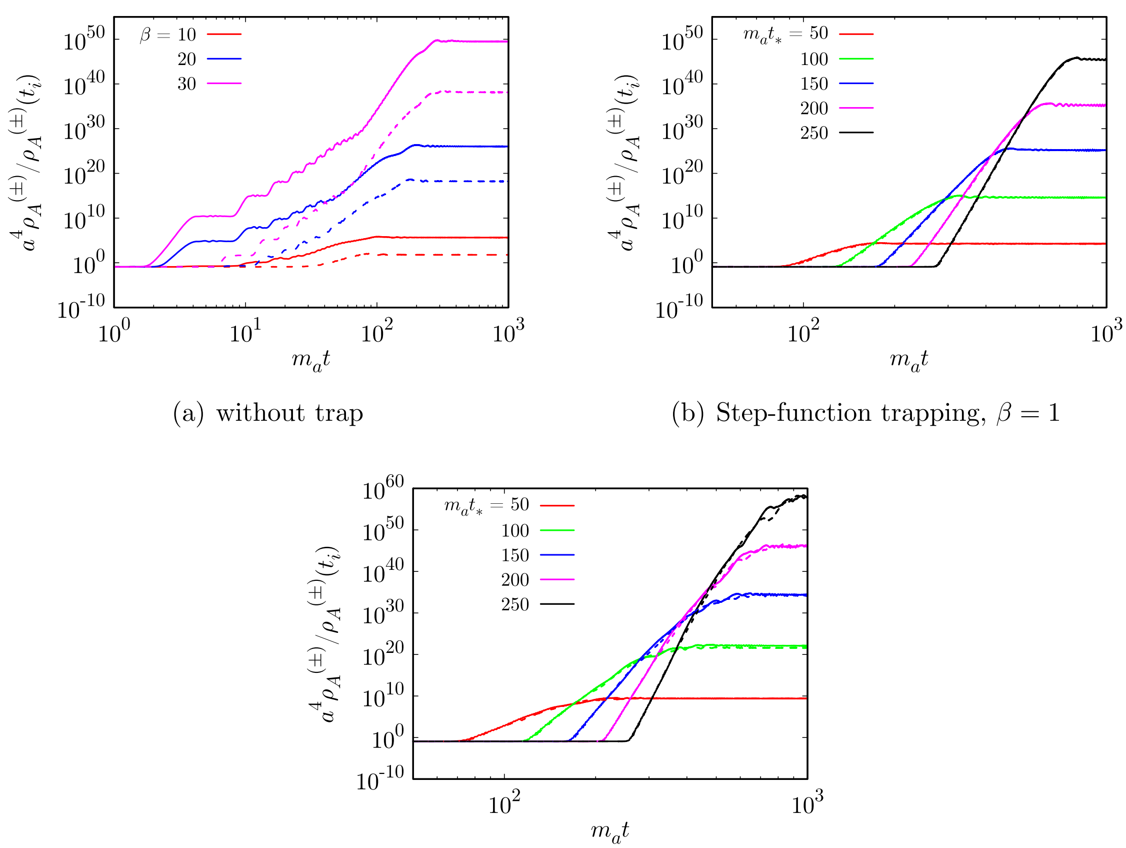
In the conventional scenario with $H_{\rm osc} \sim m_a$, the growth exponent is $(\beta\theta_i)^2$. Then the significant dark photon production never occurs for a small coupling ($\beta \ll 1$). In the trapped axion scenario, on the other hand, the growth exponent can be enhanced by a factor of $m_a/H_{\rm osc}$. Thus, the explosive dark photon production is possible even for a small coupling if the onset of the axion oscillation is delayed significantly.

In the next section, we will numerically solve the equations of motion for the axion and dark photon fields and examine whether the trapping effect of the axion can induce resonant production of dark photons even for small β. Although we could perform lattice simulations in principle, our goal only requires us to estimate the growth rate of dark photon fields. Therefore, we will restrict ourselves to the linear analysis and ignore the feedback of the produced dark photons on the axion dynamics.

III. NUMERICAL RESULTS

We solve the equation of motion (10) for the dark photon numerically, assuming a homogeneous background that includes the axion field. Since our interest is in the growth of dark photon fields for various axion potentials, we neglect the backreaction and set the rhs of Eq. (11) to zero. We consider three scenarios of axion dynamics: one without the trapping potential $V_{\rm trap}$, one with a step function of time, and one with a trapped QCD axion model.

In Fig. 2, we show the time evolution of the axion field value and the energy density of each circular polarization mode of the dark photon field for the three cases mentioned above. In the case without trapping shown in panel (a), it can be seen that a large asymmetry in the circular polarization is generated in the dark photons produced in the first few oscillations, due to the cosmic expansion. Although the back reaction to the axion is not included, the production of dark photons stops at some point because instability band becomes smaller due to cosmic expansion. On the other hand, in the two trapped cases shown in panels (b) and (c), the axion oscillation period is consistently much



(a) without trap

(b) Step-function trapping, $\beta = 1$

(c) Trapped QCD axion, $\beta = 1$

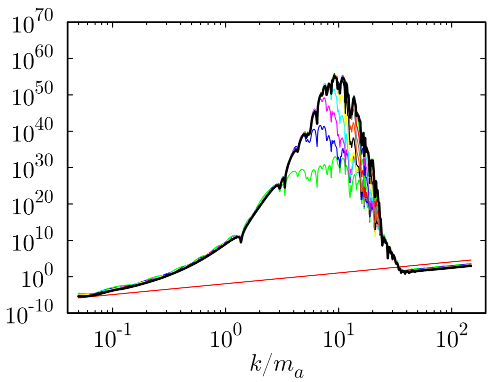
FIG. 3. Time evolution of the comoving energy density of the dark photon normalized by the initial value. The thick and dashed line corresponds respectively to the minus and the plus circular polarization modes. We set $m_A = 0$, $m_* = 4m_a$ (top-right), $N_H = 3$ and $\Lambda_H^2 = 0.5 m_a f_a$ (bottom) and neglected the backreaction to the axion.

shorter than Hubble time from right after the trapping ends, and therefore both plus and minus modes are equally generated, resulting in no circular polarization asymmetry. This is in contrast to the conventional case (a) where there is a large difference between two modes of generated dark photons. In both cases (b) and (c), we set $\beta = 1$, but it can be seen that resonant production of dark photons occurs efficiently.

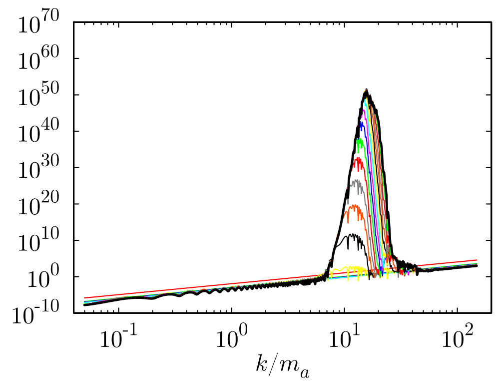
In Fig. 3, we show how much the energy density of dark photons increased compared to the initial value in the three cases, by changing β and the end time of trapping t_*. In the case without trapping, it can be seen that dark photons are generated more efficiently as β increases. On the other hand, in two trapped cases with $\beta = 1$, it can be seen that dark photons are generated more efficiently as the end time of trapping becomes later. As before the circular polarization asymmetry is suppressed in the trapped cases.

We also show the time evolution of the spectrum of dark photons for each case in Fig. 4. Compared to when there is no trap, we can see that the spectrum becomes sharper. This is what is expected from the previous discussion that resonance is narrow. Since the horizontal axis is the comoving wave number, we can also see that the fastest growing mode moves to a larger comoving wave number as times goes.

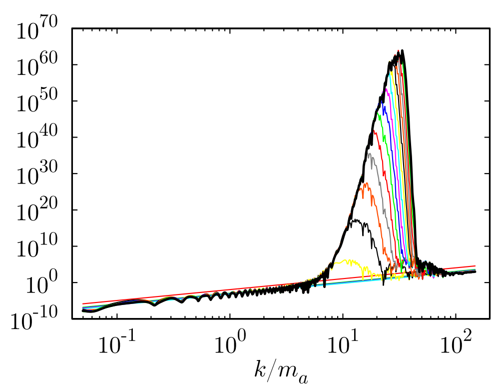
Figure 5 shows the total growth factor of the energy density of dark photons as a function of the growth exponent $\mu z_{\rm res}$ defined by Eq. (18). The growth can be well-fitted by the analytic formula (dotted lines) especially in the case of trapped QCD axion model. The deviation from the fitting can be understood by the effect of the cosmic expansion. In particular, when the Hubble parameter is comparable to the mass, the growth shows a step-like behavior due to the tachyonic instability as one can see in Fig. 3(a). In this case, the growth is clearly delayed compared with the monochromatic exponential growth which we assumed in the derivation of the total growth exponent in Eq. (18). If the onset of the oscillation is delayed due to the trapping and the cosmic expansion becomes negligible in one oscillation period, the growth can be regarded as a monochromatic exponential function, which explains why it can be well approximated by the fitting function.



(a) $\beta = 30$ without trap

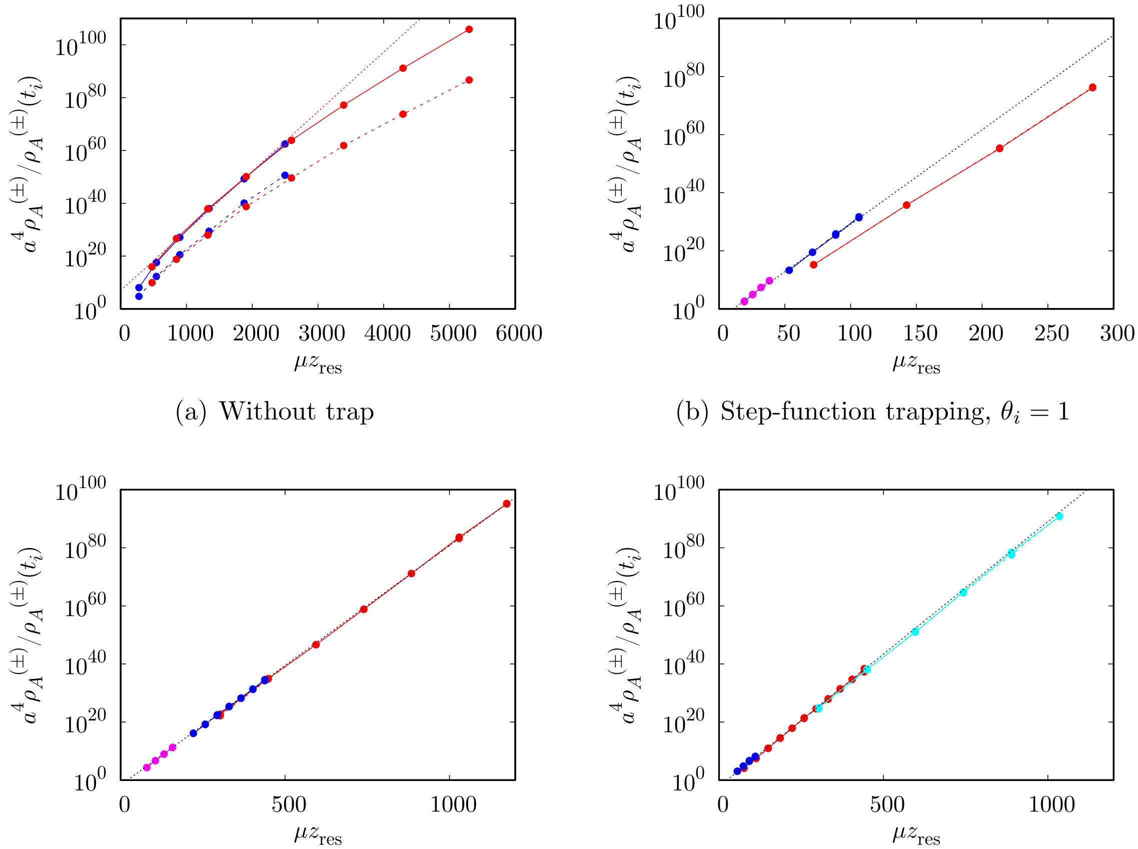


(b) Step-function trapping, $\beta = 1$, $m_a t_* = 250$



(c) Trapped QCD axion, $\beta = 1$, $m_a t_* = 250$

FIG. 4. Time evolution of the spectrum of comoving number density of the dark photon. We set $m_A = 0$, $m_* = 4m_a$ (top-right), $N_H = 3$ and $\Lambda_H^2 = 0.5 m_a f_a$ (bottom) and neglected the backreaction to the axion. The time evolves from bottom to top and the thick black line corresponds to the final time $m_a t = 1000$.



(a) Without trap

(b) Step-function trapping, $\theta_i = 1$

(c) Trapped QCD axion, $N_H = 3$

(d) Trapped QCD axion, $N_H = 6$

FIG. 5. Total growth factor of comoving energy density of dark photons as a function of $\mu z_{\rm res}$ with $k = a_{\rm osc}m_a$ [see Eq. (18)]. Top left: the case without trapping with $\theta_i = 2\pi/3$. Dotted line is a fitting function, $\exp(0.052\mu z_{\rm res})$. Top right: step-function trapping case with $\theta_i = 1$. Dotted line is a fitting function, $\exp(0.75\mu z_{\rm res})$ Bottom panels: trapped QCD axion model with $N_H = 3$ (left), 6 (right). We have taken $\beta = 1$ (red), 0.5 (blue), 0.3 (magenta), and 2 (cyan). Dotted lines are fitting formula, $\exp(0.19\mu z_{\rm res})$ (left) and $\exp(0.21\mu z_{\rm res})$ (right).

IV. DISCUSSION AND CONCLUSIONS

We have proposed a modification to the axion dynamics in this paper to demonstrate that resonant dark photon production can occur efficiently, even for a small axion-dark photon coupling. This relies on a later onset of the axion oscillations than is typical. We have studied the time evolution of dark photons by varying the trapping duration for various coupling constants. Our results have shown that the required axion-dark photon coupling constant becomes small in proportion to the square root of the Hubble parameter at the onset of axion oscillations, as expected from the analytical estimate of the growth rate. In other words, the total growth exponent is proportional to $\beta^2 m_a/H_{\rm osc}$ [see Eq. (18)]. This implies that the resonant production of dark photons can occur efficiently by trapping axions for an arbitrarily small coupling constant, eliminating the need for a complicated UV completion to obtain the large axion-dark photon coupling. While the axion dynamics are more complex, only two potentials are needed, and the efficiency of dark photon production depends on when the trapping ends, which typically happens when the two potential heights become comparable.

Our mechanism is applicable to a variety of dark photon production scenarios. For example, it could be applied to scenarios where the abundance of the QCD axion with a large decay constant is suppressed by producing dark photons. In the conventional scenario where the axion starts to oscillate when the Hubble parameter is equal to the axion mass, the axion abundance is reduced to about 1/10 [19], but we need to run lattice numerical simulations to determine how much the axion abundance is reduced in our mechanism. In particular, because of the relative weakness of the cosmic expansion effect, the interactions between axions and dark photons may continue to be effective for a longer time after the back reaction becomes important, and may significantly change the final ratio of the axion and dark photon energy abundances. This is left for a further study.

One may apply this mechanism to the early dark energy models using an axion field [30,56] where the axion-dark photon coupling is used either for an efficient reheating to avoid the use of contrived axion potential of the original model [57] or for generating a heavy mass for dark Higgs. The source of the trapping potential could behave as dark radiation or hot dark matter.

Another application may be the production of the primordial magnetic field [31]. However, it relies on the inverse cascade decay of helical magnetic fields. On the other hand, in our mechanism using the trapped axion, the circular polarization asymmetry of the produced photons is significantly suppressed. Therefore, it is difficult to generate the primordial magnetic field by our mechanism.

Similarly, for gravitational waves produced by dark photons, circular polarization asymmetry is expected to be significantly reduced compared to the conventional case due to the weak asymmetry of the first few oscillations of the axion, but we need to run lattice numerical simulations to obtain a quantitative evaluation.

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