

Radio excess from stimulated dark matter decay

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Despite an intense theoretical and experimental effort over the past decade, observations of the extragalactic radio background at multiple frequencies below 10 GHz are not understood in terms of known radio sources and may represent a sign of new physics. In this paper, we identify a new class of dark sector models with feebly interacting particles, where dark photons oscillate into ordinary photons that contribute to the radio background. Our scenario can explain both the magnitude and the spectral index of the radio background, while being consistent with other cosmological and astrophysical constraints. These models predict new relativistic degrees of freedom and spectral distortions of the cosmic microwave background, which could be detected in the next generation of experiments.

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I. INTRODUCTION

The cosmic microwave background (CMB) is one of the most well-studied electromagnetic signals in science, particularly in the frequency range of 60–600 GHz. In this range, the CMB monopole dominates over astrophysical backgrounds and is consistent with a blackbody distribution, with distortions limited to less than 1 part in 10^4 [1]. Spatial fluctuations in the brightness of the CMB are on the level of 1 part in 10^5 , and the power spectrum of these fluctuations forms a key pillar of modern cosmology [2]. These facts constitute strong evidence for the primordial origins of the CMB.

Much less is known about the extragalactic radio background (ERB) at frequencies $\nu \lesssim 10$ GHz. The ARCADE 2 collaboration combined their own observations [3] with measurements from other radio telescopes [4–7], spanning a frequency range of 22 MHz–90 GHz, and found that the ERB can be modeled as

$$T(\nu) = T_0 + T_R \left(\frac{\nu}{310 \text{ MHz}} \right)^\beta, \quad (1)$$

where $T(\nu)$ denotes the brightness temperature. This is not a thermodynamic temperature and does not assume thermal equilibrium; $T(\nu)$ indicates an intensity at frequency ν that is equal to the intensity of a blackbody at frequency ν , with thermodynamic temperature T . $T(\nu)$ is related to the photon spectrum via $T(\omega) = (\pi^2/\omega) dn_\gamma/d\omega$, where $dn_\gamma/d\omega$ is the number density of photons per unit energy (taking $\hbar = c = k_B = 1$), and $\omega \equiv 2\pi\nu$. The power-law fit in Ref. [3] found $T_0 = 2.725(1)$ K, $T_R = 24.1(21)$ K, and $\beta = -2.599(36)$. For $\nu \gtrsim 100$ GHz, this is consistent with the CMB temperature of $T_0^{\text{FIRAS}} = 2.7255(85)$ K measured by FIRAS [1,8]. At lower frequencies, however, the power law is an excellent fit to $T_{\text{exc}} \equiv T - T_0$. A more recent

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reanalysis including all-sky maps from the LWA1 Low Frequency Sky Survey (LFSS) reached a similar conclusion [9].

Explaining T_{exc} with known sources of radio emission has thus far been surprisingly difficult [10]. Almost all attempts to do so have relied on synchrotron emission, since synchrotron sources typically produce $-3 \lesssim \beta \lesssim -2.5$ [11,12]. Extragalactic radio synchrotron sources such as active galactic nuclei and star-forming galaxies can produce $\beta \approx -2.7$ [13–17], and the source counts distribution is thought to be relatively well understood. However, numerous studies have estimated the emission from extragalactic radio sources to be three to 10 times smaller than T_{exc} [16,18–22] between 100 MHz–10 GHz. Electrons that are reaccelerated during cluster mergers have been proposed as a significant contribution to T_{exc} [23], albeit under optimistic assumptions; this mechanism also produces a softer power law than is required.

An alternative explanation of T_{exc} is contamination from galactic synchrotron emission, which has a spectral index, and needs to be subtracted from radio data to obtain the extragalactic component [24–26]. However, additional sources of synchrotron are strongly constrained by the lack of accompanying inverse Compton emission in x-rays and the fact that such sources are not typical of Milky Way-like galaxies [26–28]. More sophisticated modeling using cosmic ray propagation models disfavors the possibility of significant galactic contamination [20].

A more exotic class of solutions involves radio emission from hypothetical early structures [29], including black holes [30–33] and star-forming galaxies [34] at high redshifts. These solutions typically require large, persistent magnetic fields [32], whose origin and impact on inverse Compton cooling are debated [35].

The ERB has also been studied in conjunction with 21-cm experiments, including EDGES [36], LEDA [37], LOFAR [38], and HERA [39]. Some of these measurements are in tension with a T_{exc} that is fully produced at high redshifts. In particular, the LEDA and LOFAR results constrain any cosmological radio background with $T_{\text{exc}} \propto \omega^{-2.6}$ at $\sim 10\%$ ($13.2 < z < 27.4$) [29] and $\sim 46\%$ ($z = 9.1$) [40] of the present-day radio excess, respectively.

Another aspect of the ERB that makes astrophysical explanations difficult is the spatial smoothness of the emission [41], deduced from measurements of the anisotropy of the radio sky [41–46]. These measurements find that fluctuations in T_{exc} , $\Delta T/T_{\text{exc}}$, are $\sim 10^{-2}$ across a range of angular scales and frequencies; this is smoother than expected if radio emission is correlated with the present-day dark matter distribution [41]. Note that the ERB could not have been generated significantly before recombination, since such a large nonthermal distribution would have been erased by scattering with baryons. The LEDA and LOFAR results discussed above also favor production at $z \lesssim 10$.

New physics explanations proposed thus far have primarily focused on synchrotron emission from DM annihilation and decay [47–50]. However, these models also run into similar issues: they may result in nonsmooth emission, underproduce T_{exc} , unless the magnetic fields responsible for synchrotron production have unusual or unlikely properties [49,50], require a new, large population of faint sources [21], or require a large portion of the isotropic, extragalactic gamma-ray background to come from DM annihilation [48], which is disfavored [51,52].

In this paper, we show that a simple class of experimentally viable new-physics models can explain the amplitude, power-law dependence and smoothness of T_{exc} . These models rely on three basic ingredients: (1) a particle decaying into dark photons A' ; (2) the presence of a thermal bath of A' , which stimulates this decay; and (3) A' resonantly oscillating into radio photons. This class of models leads to $T_{\text{exc}} \propto \omega^{-5/2}$, close to the observed power-law dependence. Relatively low- z resonant oscillations as well as $dn_{A'}/d\omega \propto \omega^{-1/2}$ prior to oscillations are both crucial elements of such models; we have not been able to identify viable alternatives. One possibility is the decay of DM particles to final states charged under A' , and A' appearing as “dark bremsstrahlung.” While having a spectrum similarly enhanced in the infrared, the DM has to be light and millicharged, with an effective electromagnetic charge exceeding current experimental bounds [53]. Another alternative is DM decay into a dark photon A'' which then resonantly converts twice via $A'' \rightarrow A' \rightarrow \gamma$, with some dark states charged under a dark photon A'' facilitating the $A'' \rightarrow A'$ transition. However, two resonant conversions make it difficult to produce a sufficiently large T_{exc} while ensuring that $T_{\text{exc}} \propto \omega^{-5/2}$ over nearly three decades of frequency.

The remainder of this paper is organized as follows. We begin by introducing the class of models and deriving the expected T_{exc} from them. Next, we discuss our fit of a particular example model to the radio data and relevant experimental constraints. We then move on to discuss the anisotropy of the ERB produced by the model. We conclude with future prospects for confirmation of this model. We adopt *Planck* 2018 cosmology throughout [2].

II. MODEL

A particle physics model that has the following three features can generate an ERB with $T_{\text{exc}} \propto \omega^{-5/2}$:

- (1) A cold component of DM a that is stable on cosmological timescales but undergoes a two-body decay with lifetime τ_{vac} in vacuum (for simplicity, we take a to be all of DM);
- (2) One of the daughter particles of the decay is a dark photon, A' , with an existing blackbody distribution, characterized by a temperature $T'(z) = T'_0(1+z)$, where T'_0 is its temperature today; and

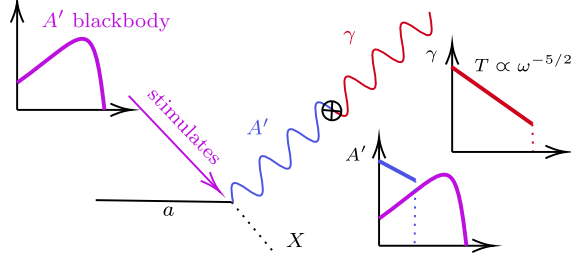


FIG. 1. A schematic representation of the key aspects of the proposed class of models.

- (3) The A' has mass 10^{-15} eV $\lesssim m_{A'} \lesssim 10^{-9}$ eV and is emitted relativistically with energy $\omega_{A'}$. A' mixes with the Standard Model photon γ with kinetic mixing parameter ϵ .

These three features are summarized in Fig. 1. The existence of the thermal population of A' enhances the decay rate of a due to Bose enhancement [54,55], leading to a redshift-dependent effective decay lifetime $\tau(z)$, where

$$\tau(z) = \tau_{\text{vac}} [1 + n f_{A'}^{\text{BB}}(z)]^{-1}, \quad (2)$$

with $f_{A'}^{\text{BB}} = (e^{\omega_{A'}/T} - 1)^{-1}$ being the blackbody occupation number of A' with energy $\omega_{A'}$. Note that $\omega_{A'} \gg m_{A'}$ so that the mass of A' can be neglected in the thermal distribution. $f_{A'}^{\text{BB}}$ can be easily generated if A' was produced in thermal equilibrium with the Standard Model and subsequently fell out of equilibrium and free streamed; A' can also stay in a thermal distribution after decoupling if there are other particles in the dark sector scattering rapidly with it.

There are several parameters that depend on the specifics of the model. The value of n depends on the occupation number of the other daughter particle. In addition, a can decay into $\alpha = 1$ or 2 dark photons. Finally, $\omega_{A'}$ depends on the kinematics of the decay of a , with $\omega_{A'} = m_a/2$ if it decays into a pair of relativistic final states. In Appendix B, we describe a concrete particle physics model that realizes all three features. To guarantee $T_{\text{exc}} \propto \omega^{-5/2}$, processes such as inverse decays of A' into a cannot significantly distort either the power-law index of the A' spectrum produced by the decay or the blackbody distribution of A' across all relevant frequencies. In accordance with this fiducial model, we fix $n = 2$, $\alpha = 1$ and $\omega_{A'} = m_a/2$ for all results that are specific to it, although we emphasize that this is only one example, out of potentially many, that we have studied in detail.

The small kinetic mixing between A' and γ enables resonant conversion between the two particles whenever $m_{A'}^2 = m_\gamma^2$ [56,57], where m_γ^2 is the effective photon plasma mass squared. This quantity is proportional to the free electron number density, n_e . The converted photons ultimately form the present-day T_{exc} . We calculate the

sky-averaged conversion probability per redshift $d\langle P_{A' \rightarrow \gamma} \rangle / dz$ taking into account inhomogeneities using the formalism developed in Refs. [57,58] (see also Refs. [59–61]).

III. RADIO BACKGROUND PRODUCTION

The particle a decays throughout cosmic history, producing a number density of A' per unit energy given by (see Appendix A for a complete derivation, which follows from Ref. [62])

$$\frac{dn_{A'}}{d\omega}(\omega, z) = \frac{\alpha \rho_a(z)}{m_a \omega \tau(z_\star) H(z_\star)} \Theta(z_\star - z), \quad (3)$$

where $1 + z_\star \equiv \omega_{A'}(1 + z)/\omega$ is the redshift at which a daughter A' with frequency ω at redshift z was produced, H is the Hubble parameter, $\rho_a(z)$ is the DM energy density, and Θ is the Heaviside step function. The total spectrum of A' is then a sum of this decay spectrum and the blackbody spectrum of the thermal A' distribution, which is subdominant in the energy range of interest.

In comoving coordinates, the spectrum of photons produced is obtained by integrating the spectrum of dark photons times $d\langle P_{A' \rightarrow \gamma} \rangle / dz$; i.e.,

$$\frac{1}{(1+z)^3} \frac{dn_\gamma}{dx} = \int_z^\infty dz' \frac{d\langle P_{A' \rightarrow \gamma} \rangle}{dz'} \frac{1}{(1+z')^3} \frac{dn_{A'}}{dx}(x, z'), \quad (4)$$

where $x \equiv \omega/[T_0(1+z)]$. For the range of $m_{A'}$ that is of interest, resonant conversions occur only after recombination. In the range $3 \times 10^{-4} \lesssim x \lesssim 0.2$, which are relevant for T_{exc} , these photons only evolve via redshifting, with the baryonic fluid being essentially transparent to them [55].

Substituting Eq. (3) into Eq. (4), we find

$$\frac{dn_\gamma}{dx}(x, z) = \frac{\rho_a(z) \alpha}{m_a} \frac{1}{x} \underbrace{\frac{1}{\tau(z_\star)}}_{\propto x^{-1}} \underbrace{\frac{1}{H(z_\star)}}_{\propto x^{3/2}} \int_z^{z_\star} dz' \underbrace{\frac{d\langle P_{A' \rightarrow \gamma} \rangle}{dz'}}_{\propto x^{-1}}. \quad (5)$$

This gives $dn_\gamma/dx \propto x^{-3/2}$, or $T_{\text{exc}} \propto \omega^{-5/2}$, the desired frequency dependence. Note that z_\star must occur during matter domination in order for $H(z_\star) \propto x^{-3/2}$, while $d\langle P_{A' \rightarrow \gamma} \rangle / dz \propto x^{-1}$ is derived in Refs. [56,57]. $\tau(z_\star) \propto x$ follows from Eq. (2) when $T' \gg \omega_{A'}$.

Equation (5) is the main result of this paper. For our fiducial model, we find the following approximate parametric dependence of T_{exc} at $z = 0$:

$$T_{\text{exc}}(\omega) \approx 10 \text{ K} \left(\frac{\nu}{310 \text{ MHz}} \right)^{-5/2} \left(\frac{T'_0}{0.2 T_0} \right) \left(\frac{10^{21} \text{ s}}{\tau_{\text{vac}}} \right) \times \left(\frac{10^{-4} \text{ eV}}{\omega_{A'}} \right)^{3/2} \left(\frac{2 \times 10^{-4} \text{ eV}}{m_a} \right) \left(\frac{P_{\text{GHz}}}{10^{-5}} \right), \quad (6)$$

where P_{GHz} is $\langle P_{A' \rightarrow \gamma} \rangle$ for $\nu = 1$ GHz today, which the FIRAS measurement of the CMB energy spectrum limits to be $P_{\text{GHz}} \lesssim 10^{-2}$ [56,58].

IV. FIT AND CONSTRAINTS

We can now perform a fit of T_{exc} predicted by our model to the measured data from Ref. [3]. There are six parameters in our fit: five parameters $\{m_a, m_{A'}, \tau_{\text{vac}}, T'_0, \epsilon\}$ from our new physics model and T_0 . Fitting the radio data places some requirements on the model parameters. For T_{exc} to be present across all data points, we require $m_a/2 \gtrsim 2\pi \times 10$ GHz. On the other hand, photons at 22 MHz must originate from decays of a during matter domination to satisfy $T_{\text{exc}} \propto \omega^{-5/2}$, leading to the requirement that $m_a/2 \lesssim 2\pi(1 + z_{\text{eq}}) \times 22$ MHz, where z_{eq} is the redshift of matter-radiation equality. Together, this means 8×10^{-5} eV $\lesssim m_a \lesssim 6 \times 10^{-4}$ eV. Resonant conversion must predominantly occur after z_* for 10 GHz photons so that the full power law extends to at least that frequency; the requirements on m_a show that $z_* \lesssim 6$, which favors 10^{-14} eV $\lesssim m_{A'} \lesssim 3 \times 10^{-13}$ eV. Finally, T' must be sufficiently large for $f_{A'}^{\text{BB}} \gg 1$ and $\tau(z_*) \propto x$ for the 10 GHz data points; this imposes $T'_0/T_0 \gtrsim 0.06$.

Several important experimental constraints exist on the class of models under consideration. First, $T'_0 \lesssim 0.4T_0$, in order to avoid introducing excessive effective relativistic degrees of freedom [2]. The kinetic mixing between A' and γ also leads to spectral distortions due to $\gamma \rightarrow A'$ resonant conversions; the FIRAS measurement of the CMB spectrum [1] leads to constraints on the order of $\epsilon < 10^{-7} - 10^{-5}$ in the $m_{A'}$ range of interest [57,58].

DM decaying into radiation is constrained by the CMB power spectrum, large scale structure, and the Milky Way satellite population [63–68]; however, these results assume a constant decay width. Conservatively requiring the total energy density of a that has decayed away with stimulated decay from the A' thermal population by the present day to be less than 2.2%, we find $\tau_{\text{vac}} > 10^{19}$ s $\times \max[2.1, 6.3(10^{-4} \text{ eV}/\omega_{A'}) (T'_0/0.2T_0)]$ (see Appendix C 1 for details). We also note that the decaying particle a may be a subcomponent f_{DM} of DM, which evades the lifetime bound altogether if $f_{\text{DM}} \lesssim 2.2\%$ [67].

21-cm power spectrum measurements from both LEDA and LOFAR are both in tension with T_{exc} being fully produced at high redshifts ($z \gtrsim 9.1$), placing strong constraints on models that produce T_{exc} primordially. In our model, however, $A' \rightarrow \gamma$ resonant conversions occur predominantly at $z \lesssim 6$.

We explore the posterior on the model parameters using nested sampling [69–71] implemented in DYNESTY [72]. Our priors, described in detail in Appendix E, are constructed such that they have zero probability density in parameter regions incompatible with (1) the FIRAS spectral distortion limits of Refs. [57,58] and (2) the DM lifetime

limit discussed above. Priors on T'_0 and T_0 are also chosen to account for the N_{eff} and FIRAS constraints on these parameters, respectively. The priors are otherwise broad and not biased toward the data points. The posterior on the excess temperature is shown on the left in Fig. 2. In computing T_{exc} from our model, we include an irreducible contribution from unresolved extragalactic radio sources of $T_{\text{eg}} = 0.23 \text{ K}(\nu/\text{GHz})^{-2.7}$ [16,41], which is at least three times smaller than the measured brightness temperature. This parametrization of T_{eg} is in excellent agreement with other independent estimates [73]. Although this power-law expression strictly applies in the range 100 MHz–10 GHz, we estimate the contribution outside of this frequency range by extrapolation. The marginal and joint posterior distributions are shown on the right in Fig. 2. The marginal posterior medians correspond to $m_a \simeq 2 \times 10^{-4}$ eV, $m_{A'} \simeq 2.5 \times 10^{-14}$ eV, $\tau_{\text{vac}} \simeq 1.3 \times 10^{21}$ s, $T'_0 \simeq 0.22T_0$, and $\epsilon \simeq 10^{-7}$. At these median values, roughly 0.1% of the dark matter decays away by the present day. We find an excellent fit to the data over a wide range of allowed model parameters. The posterior broadens at $\nu \gtrsim 10$ GHz where there are no data points and correspond to different choices of m_a and cutoffs in T_{exc} at $2\pi\nu = m_a/2$. In Appendix E, we show the extended corner plot for posterior distributions of all parameters of interest, along with other systematic variations; these do not qualitatively change our main result.

V. SMOOTHNESS

Upper limits on the anisotropy of the ERB have been reported for $4 \times 10^3 \lesssim \ell \lesssim 6 \times 10^4$ in the 4–8 GHz range by VLA [42,43] and ATCA [44], while actual measurements of the power spectrum have been made by LOFAR [46] and TGSS [45] at ~ 140 MHz for $10^2 \lesssim \ell \lesssim 10^4$. Such measurements can be challenging: Observations at ~ 140 MHz disagree with each other by a factor of 3 and face issues such as galactic foreground contamination and calibration errors [46]. To assess the smoothness of T_{exc} in our model, we take the 4–8 GHz upper limits and results from LOFAR—which reports a lower power—as approximate upper limits, noting that astrophysical sources can contribute more power [46] and that these observations are expected to improve.

There are two possible contributions to the ERB anisotropy produced by our model that are essentially independent: (1) decay anisotropy due to DM density correlations from the point at which a decays and (2) conversion anisotropy due to correlations in free electron density fluctuations δ_e during $A' \rightarrow \gamma$ conversions, since $m_\gamma^2 \propto n_e$. The decay anisotropy was found to exceed the radio anisotropy power spectrum in the 4–8 GHz range, unless the decay that produces these photons happens at $z_* \gtrsim 5$ [41,49]. This is easily satisfied in the range of parameters providing a good fit.

We compute the conversion anisotropy power spectrum by first writing down the two-point correlation function of the

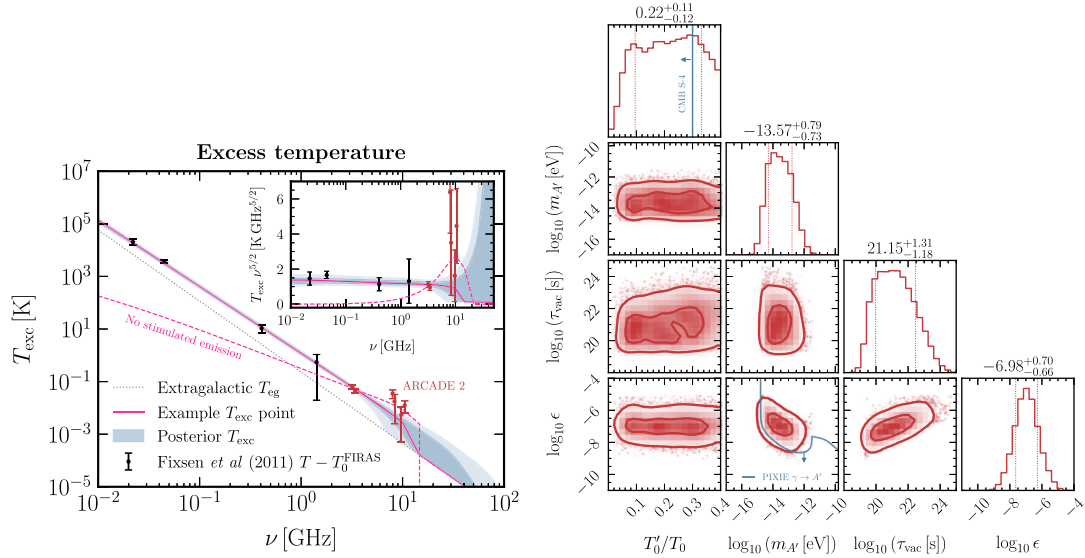


FIG. 2. Left: Pointwise posterior for T_{exc} within our proposed model, showing the middle 68% and 95% regions (dark and light blue regions, respectively). We include a subdominant but irreducible contribution from unresolved extragalactic sources T_{eg} [16] (dashed gray) for completeness. The spectrum for a single point in parameter space is shown in pink. Radio data, plotted as $T - T_0^{\text{FIRAS}}$, include measurements from ARCADE 2 and are shown in red [3], with results from other telescopes compiled in the same reference shown in black. A fit to the ARCADE 2 data points assuming no stimulated emission and $T_{\text{eg}} = 0$ is shown by the pink dashed line. The inset shows the same quantities scaled by a factor of $\nu^{5/2}$ in order to more clearly expose the posterior in relation to the measured data. Right: The inferred marginal and joint posterior distributions over a subset of parameters— T'_0/T_0 , $m_{A'}$, τ_{vac} , and ϵ —in our fiducial model. Median and middle 68% containment values are indicated in the subtitles, while for $\log_{10}(m_a/\text{eV})$, we obtain $-3.66^{+0.31}_{-0.29}$ (not shown). The potential reaches of PIXIE [58,74] and CMB-S4 [75] in our parameter space are shown in blue, with projected unconstrained regions indicated by the arrow.

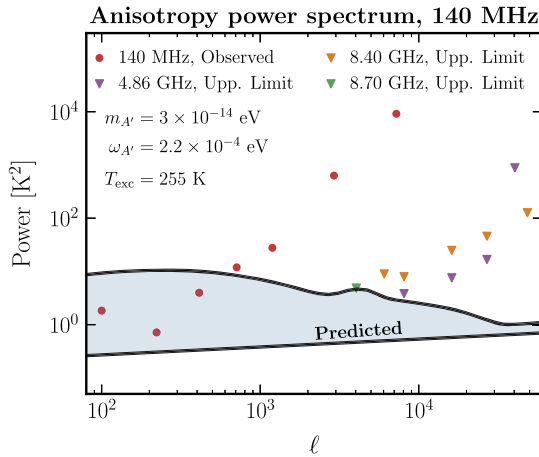


FIG. 3. Predicted anisotropy power spectrum with normally (solid black line, below) and log-normally (solid black line, above) distributed baryon fluctuations. The model parameters are $m_{A'} = 3 \times 10^{-14}$ eV and $\omega_{A'} = 2.2 \times 10^{-4}$ eV. Upper limits from VLA at 4.86 GHz [42] (purple triangles), 8.4 GHz [43] (orange triangles), and ATCA at 8.7 GHz [44] (green triangle) are shown and have been rescaled to $T_{\text{exc}} = 255$ K, the expected value at 140 MHz. Representative data points of the power spectrum measured by LOFAR are shown in red [46].

conversion probability of $A' \rightarrow \gamma$ in two different directions in the sky. As such, it depends on the two-point function of δ_e , which we model as either a Gaussian or a log-normal random field. The anisotropy power spectrum can then be obtained by performing a spherical harmonics decomposition and using the Limber approximation [76–78] following the method outlined in Ref. [79]. Further details of our calculation can be found in Appendix D.

Figure 3 shows the predicted anisotropy power spectrum in units of K^2 , defined using the same conventions as in the CMB power spectrum [2], for normally and log-normally distributed δ_e , with the result between these two choices shaded in blue. We have chosen $m_{A'} = 3 \times 10^{-14}$ eV and $\omega_{A'} = 2.2 \times 10^{-4}$ eV; this value of $\omega_{A'}$ lies just outside the 68% highest posterior density interval for $\omega_{A'}$ in order to guarantee $z_\star > 5$ for the ATCA data at 8.7 GHz but still provides a good fit to the brightness data. This can be taken as an estimate for the uncertainty associated with these distributions. This power should be compared to the LOFAR data, as well as the upper limits from VLA and ATCA at their respective frequencies ν , which have been rescaled by $(140 \text{ MHz}/\nu)^{2\beta}$, with $\beta = -2.6$; this assumes that $\Delta T/T_{\text{exc}}$ is independent of frequency. Our calculated

anisotropy power spectrum for a fiducial choice of $m_{A'} = 3 \times 10^{-14}$ eV and $\omega_{A'} = 2.2 \times 10^{-4}$ eV lies below the 4–8 GHz measurements for both choices of the two-point PDF of δ_e , while only the log-normal two-point PDF exceeds the low- ℓ measurements by LOFAR. However, significant scatter exists between adjacent frequency bands in the data for $\ell \lesssim 10^3$ [46]. Producing a sufficiently smooth radio background is thus highly plausible in our model. We leave a study of whether the sum of anisotropies due to our model and to unresolved extragalactic sources can reproduce all observations to future work. Currently, the VLA and ATCA data appear to be in tension with the LOFAR result—under our assumption that $\Delta T/T_{\text{exc}}$ is independent of frequency—while the anisotropies due to unresolved extragalactic sources are still not well understood (Ref. [46], for example, models them as being uniformly distributed in the sky and subsequently adding some *ad hoc* sinusoidal clustering).

VI. CONCLUSION

In this paper, we have provided a new physics explanation for the ERB, which is brighter than expected from known astrophysical sources. Our model consists of a DM candidate, which decays into dark photons in the presence of a thermal bath of the latter; the dark photons then resonantly convert to ordinary photons, producing the ERB. The anisotropy of the signal is expected to be relatively smooth, consistent with measurements of the anisotropy of the radio background [41,46]. Future experiments may corroborate the predictions of our model. PIXIE [74] may be sensitive to spectral distortions expected from our model and is expected to almost fully cover the 95% region of our posterior distribution in the ϵ - $m_{A'}$ plane. The thermal population of A' may also lead to a value of N_{eff} that is detectable in future CMB experiments such as CMB-S4 [75]. The potential reach of both experiments are shown in Fig. 2. If A' thermally decoupled from the Standard Model at a temperature just above the top quark mass, we obtain $T'_0/T_0 = 0.33$, well within reach of CMB-S4.

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Note added.—Reference [97] appeared several months after our work was posted on the arXiv, studying the model proposed here in detail, and pointing out some potential limitations of the model. First, we emphasize that all of the numerical results shown here use the full numerical result shown in Eq. (5), with the frequency dependence shown in braces below being approximate, leading to the approximate relation $T_{\text{exc}} \propto \omega^{-5/2}$. All of the effects that go beyond this approximate relation, which are discussed in detail in Ref. [97], are already included in Eq. (5).

There were two limitations pointed out in Ref. [97]. First, the median parameter values of the posterior distribution, $m_{A'} = 3 \times 10^{-14}$ eV and $\omega_{A'} = 1.1 \times 10^{-4}$ eV, lead to $z_{\star} < 5$ for radio photons at 8.7 GHz, leading to a potentially large decay anisotropy. While this is true, choosing, e.g., $\omega_{A'} = 2.2 \times 10^{-4}$ eV, which lies just outside the 68% highest posterior density interval for $\omega_{A'}$, leads to $z_{\star} \approx 5.2$, while maintaining a good fit to the data points. This is the parameter point that we show in Fig. 3. Second, Ref. [97] suggests that self-stimulation of the decay of a may be important, unless the thermal width of a is as small as $\delta E/E \sim 10^{-29}$. Although this may appear to be a small thermal width, such a cold population of axionlike particles is easily achieved in the context of the misalignment mechanism (see, for example, Ref. [98]). Self-stimulation is also heavily suppressed by the large coherence length of a as well as gravitational gradients [99].

APPENDIX A: DARK PHOTON SPECTRUM

The appendixes below are organized as follows. In App. A, we derive in detail the dark photon spectrum as a function of frequency and redshift in the class of models considered in the paper. Appendix B gives an extended description of a particular model, which presents all the ingredients necessary to explain the extragalactic radio background (ERB). Appendix C presents some of the important experimental limits on our model, including a derivation of the cosmological limits on the dark matter lifetime in the presence of stimulated decay, as well as limits on the dark sector temperature due to the presence of extra relativistic degrees of freedom. Appendix D presents the calculation of the anisotropy of the ERB due to photon-dark photon resonant conversions. In App. E, we describe our data analysis procedure, before presenting in App. F our extended results, including a complete corner plot of our parameter space and other systematics checks such as the use of different datasets and fits using the ARCADE 2 data points only. Finally, in App. G, we give a brief summary of the spectrum of radiation expected from synchrotron sources.

In this section, we provide a detailed derivation of the dark photon spectrum as a function of frequency and redshift, in the presence of stimulated emission.

Consider a two-body decay of a heavy particle a into two daughter particles, at least one of which is a dark photon A' , emitted relativistically with energy $\omega_{A'}$. In the presence of a blackbody distribution of A' with temperature T' , a undergoes stimulated decay, with a decay width $\Gamma(z)$ that is related to the usual vacuum decay rate Γ_{vac} via

$$\Gamma(z) = (1 + n f_{A'}^{\text{BB}}) \Gamma_{\text{vac}},$$

where $f_{A'}^{\text{BB}} \equiv [\exp(\omega_{A'}/T') - 1]^{-1}$ is the blackbody occupation number at energy $\omega_{A'}$. n is an integer that depends on the details of the model; $n = 2$ in the fiducial particle physics model that we describe below. Note that for $\omega_{A'} \ll T'$, we obtain $\Gamma(z) \simeq (nT'/\omega_{A'}) \Gamma_{\text{vac}}$, an approximation that we will use frequently to obtain parametric estimates.

Given $\Gamma(z')$, the number density of A' produced per unit time at redshift z' is

$$\frac{dn_{A'}}{dt} = \alpha \Gamma(z') \frac{\rho_a(z')}{m_a},$$

where $\alpha = 2$ if both daughter particles are A' and $\alpha = 1$ otherwise, $\rho_a(z)$ is the energy density of a at redshift z , and m_a is the mass of a . With this, we can find the contribution to the dark photon number density at redshift z per unit frequency ω , due to a decay occurring at redshift z' :

$$\frac{dn_{A'}(z)}{d\omega dz'} = \frac{dn_{A'}(z')}{dt} \frac{dt}{dz'} \frac{(1+z)^3}{(1+z')^3} \delta_{\text{D}} \left(\omega \frac{1+z'}{1+z} - \omega_{A'} \right),$$

where the redshift factors rescale the number density of particles produced at redshift z' down to redshift z , and the Dirac delta function enforces the fact that the decay produces dark photons with energy $\omega_{A'}$. Integrating over z' gives

$$\frac{dn_{A'}}{d\omega} = \int_z^\infty dz' \frac{\rho_a(z')}{m_a} \frac{\alpha \Gamma(z')}{H(z)(1+z)} \frac{(1+z)^3}{(1+z')^3} \delta_{\text{D}} \left(\omega \frac{1+z'}{1+z} - \omega_{A'} \right) = \frac{\alpha \rho_a(z)}{m_a} \frac{\Gamma(z_\star)}{\omega H(z_\star)} \Theta(z_\star - z), \quad (\text{A1})$$

where

$$1 + z_\star \equiv \frac{\omega_{A'}}{\omega} (1 + z) \quad (\text{A2})$$

is the redshift at which the decay of a produced the dark photon at frequency ω , with $\omega \leq \omega_{A'}$. We have also assumed that $\rho_a \propto (1+z)^3$. This expression agrees with Ref. [62] if we simply set $\Gamma(z_\star) \rightarrow \Gamma_{\text{vac}}$. In terms of comoving number density and $x \equiv \omega/T_0$ where T_0 is the CMB temperature today, we find

$$\frac{1}{(1+z)^3} \frac{dn_{A'}}{dx} = \frac{\alpha \rho_{a,0}}{m_a} \frac{\Gamma(z_\star)}{x H(z_\star)} \Theta(z_\star - z), \quad (\text{A3})$$

which is almost constant in redshift, aside from the step function. $\rho_{a,0}$ is the dark matter energy density today. Equation (A1) is the expression we use in the paper.

APPENDIX B: PARTICLE PHYSICS MODEL

1. Model building considerations

As we explained in the paper, the particle physics model we need to explain the radio background requires three features that we repeat here:

- (1) A cold component of DM a that is stable on cosmological timescales but undergoes a two-body decay with lifetime τ_{vac} in vacuum. For simplicity, we take a to be all of DM;
- (2) One of the daughter particles is a dark photon A' , which has an existing blackbody distribution of A' characterized by a temperature $T'(z) = T'_0(1+z)$, where T'_0 is its temperature today; and
- (3) A' has a mass $10^{-15} \text{ eV} \lesssim m_{A'} \lesssim 10^{-9} \text{ eV}$ and is emitted relativistically with energy $\omega_{A'}$. A' mixes with the Standard Model (SM) photon γ with kinetic mixing parameter ϵ .

These conditions appear simple to meet; for example, the model proposed in Ref. [62] where the DM is a dark axion decaying into two identical dark photons, which oscillates into SM photons, could be supplemented by an additional thermal population of dark photons. However, there are several challenges to building a successful model. In order to produce the right power law in the ERB, the A' power-law spectrum must maintain $dn_{A'}/d\omega \propto \omega^{-1/2}$ between $3 \times 10^{-4} \lesssim x \lesssim 0.2$, where $x \equiv \omega/T_{\text{CMB}}$, from the redshift of production $z_*(x)$ until resonant conversions $A' \rightarrow \gamma$ are mostly complete. Furthermore, the A' blackbody spectrum also cannot be significantly distorted; specifically, A' particles from the bath at each value of x must be described by a blackbody spectrum at $z_*(x)$. In the model of Ref. [62], both of these requirements can be violated by inverse decays $A'A' \rightarrow a$ or other A' scattering processes. Moreover, it is difficult to maintain thermal equilibrium in the blackbody spectrum through scattering with another particle in the dark sector bath, since such scattering processes likely distort the power-law spectrum significantly. For typical parameters required to produce the full T_{exc} , scattering between a low-energy A' in the power-law spectrum and a blackbody A' is too rapid to guarantee that the A' blackbody spectrum remains undistorted at all relevant times. While it is possible that even with significant distortion to the blackbody spectrum, a reasonable fit to the ERB can still result, checking this possibility would require us to integrate the Boltzmann equation over a wide range of A' frequencies. In this paper, we avoid this computational challenge by building a slightly different fiducial model without significant distortion.

Before discussing the details of our fiducial model, we will first discuss how to check the size of any A' spectral distortion due to a single process. We begin by writing down the Boltzmann equation with the relevant process contributing to the collision term. Neglecting for simplicity the existence of entropy dumps in the Standard Model so that x for any A' particle is a constant, the Boltzmann equation governing the occupation number $f_{A'}$ can be written

$$\frac{df_{A'}(x, t)}{dt} = \frac{C[f_{A'}]}{\omega}, \quad (\text{B1})$$

where $C[f_{A'}]$ is the collision integral, which we will define for particular processes below. Here, we adopt the convention that $f_{A'}$ is related to the number density via

$$n_{A'} = 3 \int \frac{d^3\vec{p}}{(2\pi)^3} f_{A'},$$

where the factor of 3 accounts for the degeneracy in the spin state of A' . To determine if any process causes a significant distortion to the spectrum of blackbody A' or low-energy A' from the decay of a , we divide Eq. (B1) by $f_{A'}$ and integrate with respect to time or, equivalently, redshift, to obtain the

change in $\log f_{A'}$, which gives a measure of the fraction of A' at that value of x that undergoes a scattering process. Our criterion for a small distortion to the A' spectra is therefore

$$|\Delta \log f_{A'}(x)| = \int_{z_{\min}}^{z_{\max}} \frac{dz}{H(z)(1+z)} \frac{|C[f_{A'}]|}{\omega f_{A'}(x, z)} \ll 1, \quad (\text{B2})$$

where z_{\min} and z_{\max} are the lowest and highest redshifts, respectively, for which the collision term is important.

Since we are only interested in producing the radio excess over a finite frequency range, we only need to check that distortions are small over a limited range of x . The radio frequency data points with an excess temperature over the CMB temperature span the frequency range 22 MHz–10 GHz, which we can cover by considering $3 \times 10^{-4} < x < 0.2$.

The power-law component at fixed x is produced by decays of a at a redshift $1 + z_* \equiv \omega_{A'}/(xT_0)$, where $\omega_{A'}$ is the energy of the emitted A' at the point of decay (for a decaying into two massless particles, this is simply $\omega_{A'} = m_a/2$), and T_0 is the blackbody CMB temperature today. After it is produced, it must stay in this power law without undergoing significant distortions until the present day.¹ For the blackbody component to effectively produce the stimulated emission that we need, on the other hand, the blackbody spectrum must not be significantly distorted prior to redshift $z_*(x)$; distortions after this redshift are unimportant. This means that for the power-law spectrum, for each value of x , we need to consider $z_{\min} = 0$ and $z_{\max} = z_*(x)$, while for the blackbody spectrum, $z_{\min} = z_*(x)$.

2. Fiducial model

We will now describe our fiducial particle physics model, which has all three properties required to produce the radio background laid out in the previous section. We will show that both the low-energy and blackbody distribution of the dark photons do not undergo any significant distortion.

Our fiducial model is a modified version of the model proposed in Ref. [62]. The important particles in this model are: (i) an axionlike dark matter a with mass m_a and decay constant f_a , and (ii) two dark photons, which we label A_ψ and A_{osc} . The three requirements for producing the radio excess as discussed in the main body of the paper are satisfied as follows:

- (1) The DM decays through the process $a \rightarrow A_\psi A_{\text{osc}}$;
- (2) A blackbody distribution of A_{osc} described by a temperature T' is also present, leading to stimulated decay of a , and

¹Technically, the dark photons can become significantly distorted after all resonant oscillations into photons have been completed, but for simplicity, we set this more stringent requirement.

- (3) The dark photon A_{osc} possesses a small kinetic mixing term with the SM photon and has a mass $10^{-15} \text{ eV} \lesssim m_{\text{osc}} \lesssim 10^{-9} \text{ eV}$ so that it undergoes resonant conversions into the SM photon after recombination.

The terms in the dark sector Lagrangian that are relevant to us are therefore

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 - \frac{a}{4f_a} F_{\psi,\mu\nu} \tilde{F}_{\text{osc}}^{\mu\nu} - \frac{\epsilon}{2} F_{\text{osc}}^{\mu\nu} F_{\mu\nu} + \mathcal{L}_K, \quad (\text{B3})$$

where \mathcal{L}_K contains the other kinetic terms of the dark photons, including a mass of m_{osc} for A_{osc} . Note that a discrete symmetry under which $A_\psi \rightarrow -A_\psi$ and $a \rightarrow -a$ explicitly forbids decays of a to a pair of dark photons of the same species, as well as mixing between A_ψ and the SM photon, naturally leading to the Lagrangian shown above. By allowing a to decay into two different dark photons, we can now introduce a fermion ψ that scatters rapidly with A_ψ to ensure that A_ψ always equilibrates into a thermal distribution with temperature T' without destroying the low-energy spectrum of A_{osc} .² With this modification, we overcome the main difficulty faced by the model in Ref. [62]: there are now no low-energy A_ψ to interact with the blackbody distribution of A_{osc} , which was the main source of distortion for the blackbody spectrum.³

We are now ready to check that distortions to both the low-energy A_{osc} power-law spectrum and the A_{osc} blackbody are small. The most important process, which enters at order $1/f_a^2$, is inverse decays, $A_\psi A_{\text{osc}} \rightarrow a$, which leads to the largest distortion. The collision integral is given by

²For simplicity, we assume that the entire dark sector is described by a common temperature T' .

³While this completes the basic description of our model, there are some requirements on ψ to keep this model tractable and avoid tracking the spectra of all of these particles as a function of time. First, Compton $\psi A_\psi \rightarrow \psi A_\psi$ and double-Compton $\psi A_\psi A_\psi \leftrightarrow \psi A_\psi$ scattering must be sufficiently rapid to ensure that A_ψ is always described by a simple, blackbody distribution. This is easily satisfied, as long as ψ has a large coupling to A_ψ and is sufficiently abundant. Second, the process $a A_{\text{osc}} \leftrightarrow \psi \bar{\psi}$ must not lead to significant distortion of A_{osc} . This is hard to determine without tracking the full evolution of the spectrum of A_{osc} , but it is possible to avoid this entirely by choosing a sufficiently massive ψ to kinematically forbid $a A_{\text{osc}} \rightarrow \psi \bar{\psi}$ and making ψ asymmetric. We find that $m_\psi \sim 30 \text{ eV}$ with a coupling to A_ψ of $\alpha_D = 1$, with an asymmetric number density today of $n_{\psi,0} = 2.5 \times 10^{-3} \text{ cm}^{-3}$ can provide sufficiently efficient scattering with A_ψ . This value of m_ψ is large enough to kinematically forbid $a A_{\text{osc}} \rightarrow \psi \bar{\psi}$. Introducing a similar fermion χ with $m_\chi \sim m_\psi$ that couples to A_{osc} instead guarantees that no distortion can occur when $T' > m_\psi, m_\chi$, while allowing distortions to build up once $T' < m_\psi, m_\chi$ and ψ, χ has frozen out. This also prevents $\psi \bar{\psi} \rightarrow a A_{\text{osc}}$ from being significant at $T' < m_\psi, m_\chi$, since $\bar{\psi}$ annihilates away at $T' \sim m_\psi$.

$$C_{A_{\text{osc}} A_\psi \rightarrow a}[f_{\text{osc}}] = -\frac{f_{\text{osc}}}{6} \int \frac{d^3 \vec{k}_{A_\psi}}{(2\pi)^3 2\omega_{A_\psi}} \int \frac{d^3 \vec{p}_a}{(2\pi)^3 2E_a} \times (2\pi)^4 \delta_D^4(k_{\text{osc}} + k_{A_\psi} - p_a) \times |\mathcal{M}|_{A_{\text{osc}} A_\psi \rightarrow a}^2 f_{A_\psi}(\vec{k}_{A_\psi}).$$

We use k, \vec{k} , and ω for incoming four-momentum, three-momentum, and energy; likewise, we have p, \vec{p} , and E for outgoing states. The subscript ‘‘osc’’ represents quantities associated with A_{osc} . There is no Bose enhancement in the outgoing state, since we can treat the entire population of a as having zero momentum. This also allows us to neglect the contribution from the backward reaction to the collision integral. The squared matrix element for this process (summed over initial and final states) is

$$|\mathcal{M}|_{A_{\text{osc}} A_\psi \rightarrow a}^2 = \frac{m_a^4}{2f_a^2},$$

while the occupation number of A_ψ is given by the blackbody occupation number, $f_{A_\psi} = [\exp(\omega_{A_\psi}/T') - 1]^{-1}$. These simple expressions allow us to perform the collision integral relatively easily, to obtain

$$C_{A_{\text{osc}} A_\psi \rightarrow a}[f_{\text{osc}}] = -\frac{f_{\text{osc}} m_a^4 T'}{96\pi \omega_{\text{osc}} f_a^2} \times \log \left[1 - \exp \left(-x \frac{(1+z_\star)^2}{(1+z)^2} \right) \right], \quad (\text{B4})$$

where $\omega_{\text{osc}} = x T_{\text{CMB}}$. With this expression, we can now assess the total distortion to the blackbody and power-law distributions of A_{osc} and check that for typical model parameters, these distortions are small, based on the criterion set out in Eq. (B2). For the blackbody distribution, we want to perform the integral in Eq. (B2) starting from $z_{\text{min}} = z_\star$, allowing us to approximate the collision term as

$$C_{A_{\text{osc}} A_\psi \rightarrow a}[f_{\text{osc}}] = -\frac{f_{\text{osc}} m_a^4 T'}{96\pi \omega_{\text{osc}} f_a^2} \log \left(x \frac{(1+z_\star)^2}{(1+z)^2} \right). \quad (\text{B5})$$

Integrating this gives

$$|\Delta \log f_{\text{osc}}| \sim \frac{2^{5/2} m_a^{3/2} \gamma x^{1/2} T_0^{3/2}}{96\pi H_0 \sqrt{\Omega_m} f_a^2} \sim 1.8 \times 10^{-4} \left(\frac{\gamma}{0.2} \right) \left(\frac{2 \times 10^{-4} \text{ eV}}{m_a} \right)^{3/2} \times \left(\frac{10^{21} \text{ s}}{\tau_{\text{vac}}} \right) \left(\frac{x}{0.2} \right)^{1/2}, \quad (\text{B6})$$

where $\gamma \equiv T'_0/T_0$, a redshift invariant quantity. This indicates that the total distortion to the blackbody distribution is small.

For the low-energy spectrum, we integrate Eq. (B2) from $z_{\min} = 0$ and $z_{\max} = z_*$. The integral can be approximately done in two parts: (i) $0 < 1 + z < (1 + z_*)/\sqrt{x}$, where we can expand the expression in Eq. (B4) using $\log(1 - \alpha) \simeq -\alpha$, and (ii) $(1 + z_*)/\sqrt{x} < 1 + z < 1 + z_*$, where we can use the approximation in Eq. (B5). The second part of the integral dominates, but the total contribution to the distortion is given numerically by

$$|\Delta \log f_{\text{osc}}| \sim 0.21 \left(\frac{\gamma}{0.2} \right) \left(\frac{2 \times 10^{-4} \text{ eV}}{m_a} \right)^{3/2} \times \left(\frac{10^{21} \text{ s}}{\tau_{\text{vac}}} \right) \left(\frac{3 \times 10^{-4}}{x} \right)^{3/4}. \quad (\text{B7})$$

This distortion appears to be somewhat large for the fiducial values shown here, but the relative uncertainty on the data point at $x = 3.8 \times 10^{-4}$ or 22 MHz is $\sim 25\%$ and so is enough to absorb the distortion obtained here. Moreover, the posterior distribution from our fits include larger values of m_a and τ_{vac} , which decreases the size of the distortion. We therefore conclude that inverse decays do not significantly distort either component of the A_{osc} spectrum.⁴

Our proposed model therefore satisfies the requirement that all distortions to the A'_{osc} power law and blackbody spectrum are small, and it is a viable model for explaining the ERB. We stress that many of the required conditions are invoked in order to simplify the analysis and to allow an unambiguous prediction of the ERB in this model. Some of the conditions on the distortions, for example, may be relaxed under a complete analysis, which would include numerically solving the Boltzmann equations for each mode.

APPENDIX C: EXPERIMENTAL CONSTRAINTS

1. Dark matter lifetime constraint with stimulated decay

Cosmological constraints on the decay lifetime of dark matter have been obtained for decays without stimulated emission [63–68]. These results show that if a subcomponent f_{dcdm} decays between recombination and today, *Planck* 2018 and baryon acoustic oscillation (BAO) data

⁴Although elastic scattering processes should be subleading at order $1/f_a^4$, they can, in fact, be very rapid for two reasons. First, processes like $aA_{\text{osc}} \rightarrow aA_{\text{osc}}$ receive a large Bose enhancement from the existing population of A_{osc} , which significantly increases the scattering rate. Secondly, the matrix element of $aA_{\text{osc}} \rightarrow aA_{\text{osc}}$ contains a divergence. Such divergences are commonplace even in Standard Model processes, such as $e^-Z \rightarrow e^-Z$ [100]. In the context of cosmological fluids, these divergences are regulated by thermal corrections to the self-energy of A_ψ , which impart an imaginary contribution to the mass of A_ψ [100]. We have checked that even including Bose enhancement and regulating this divergence with loop contributions from a thermal distribution of ψ [101], all elastic scattering processes are indeed subdominant to the leading inverse decay process discussed here.

limit $f_{\text{dcdm}} < 0.0216$ [67] at the 95% confidence level. This result is consistent with the uncertainty in the dark matter energy density reported by *Planck* 2018 [2]. From this, we can deduce a limit on the decay lifetime with stimulated emission, by adopting a limit of $f_{\text{lim}} = 0.0216$ for the proportion of dark matter that decays away by the present day. For a stimulated emission lifetime given by $\tau(z) = \tau_{\text{vac}}(1 + 2f_{A'}^{\text{BB}})^{-1}$, which is the expression we obtain with our fiducial model, we require⁵

$$\int_0^t \frac{dt}{\tau_{\text{vac}}} \left(1 + \frac{2}{e^{\omega_{A'}/T} - 1} \right) < f_{\text{lim}}.$$

Numerically, we find

$$\int_0^t \frac{dt}{\tau_{\text{vac}}} \left(1 + \frac{2}{e^{\omega_{A'}/T} - 1} \right) \approx \frac{1}{\tau_{\text{vac}}} \times \begin{cases} 1.5 \times 10^{18} \text{ s} \left(\frac{2T_0}{\omega_{A'}} \right), & \frac{\omega_{A'}}{2T_0} < 3.2, \\ H_0^{-1}, & \frac{\omega_{A'}}{2T_0} \geq 3.2, \end{cases}$$

which leads to a limit on the vacuum decay lifetime of

$$\tau_{\text{vac}} > 10^{19} \text{ s} \times \max \left[2.1, 6.3 \left(\frac{10^{-4} \text{ eV}}{\omega_{A'}} \right) \left(\frac{\gamma}{0.2} \right) \right]. \quad (\text{C1})$$

This limit is roughly an order of magnitude stronger than the lifetime limits on DM decays without stimulated emission for our fiducial choice of parameters [67].

2. Constraints on relativistic degrees of freedom

We complete our discussion of the particle physics model by discussing its effect on N_{eff} , and relevant constraints. A_{osc} , A_ψ contribute to N_{eff} as relativistic degrees of freedom. In addition, other fermions that couple to these bosons can contribute as well if they are sufficiently light. Assuming the existence of two such Dirac fermions on top of A_{osc} and A_ψ before recombination, the total energy density of these particles divided by the energy density of a neutrino is

$$\Delta N_{\text{eff}} \approx \frac{2 \times 3 + 2 \times 4 \times \frac{7}{8}}{2 \times \frac{7}{8}} \frac{\gamma^4}{\left(\frac{4}{11} \right)^{4/3}} = 0.04 \left(\frac{\gamma}{0.2} \right)^4. \quad (\text{C2})$$

This is to be compared with the *Planck* measurement of $N_{\text{eff}} = 2.99 \pm 0.17$ [2], which limits $\Delta N_{\text{eff}} < 0.34$ at the 95% confidence level. Including the irreducible contribution from the thermal distribution of just one dark photon, which must be present in the class of models considered in this work, gives $\Delta N_{\text{eff}} \approx 0.01(\gamma/0.2)^4$.

⁵This corresponds to an approximation of the effect of stimulation; we leave a more complete treatment for future work.

APPENDIX D: ANISOTROPY DUE TO DARK PHOTON CONVERSIONS

In this appendix, we obtain the conversion anisotropy power spectrum, which originates from variations in electron density fluctuations along two lines of sight separated in the sky by some angle. Our discussion follows Ref. [79] closely, but with some novel and peculiar features originating from the resonance conversion process; for clarity, we repeat many of the same calculations presented in that reference.

For dark photon conversions, the observed temperature in any direction in the sky \hat{n} at a fixed frequency is directly proportional to the total probability of conversion $P(\hat{n})$ of dark photons into photons. We begin by defining the fluctuation of the conversion probability in some particular direction in the sky \hat{n} , defined explicitly by $\delta P(\hat{n}) \equiv P(\hat{n}) - \langle P \rangle$, where $\langle P \rangle$ is the sky-averaged conversion probability. In any particular direction \hat{n} , we have [57,58]

$$P(\hat{n}) = \frac{\pi\epsilon^2 m_{A'}^4}{\omega_0} \int_0^{z_\star} dz \frac{\delta_D(m_\gamma^2(\vec{r}, z) - m_{A'}^2)}{H(z)(1+z)^2}, \quad (\text{D1})$$

where $m_\gamma^2(\vec{r}, z)$ is the effective plasma mass of photons along the particular line of sight, and ω_0 is the present-day, observed angular frequency of the photons. $\vec{r} \equiv \hat{n}\chi(z)$, where $\chi(z)$ is the comoving distance traveled by light between z and the present day. $\omega_0(1+z_\star)$ is the energy of the daughter particle from the DM decay; photons that have energy ω_0 today were produced by decays at z_\star , which is the maximum redshift we integrate up to.

We can compute the sky-averaged conversion probability by integrating over the one-point probability density function (PDF) $f_1(m_\gamma^2; z)$ of m_γ^2 to obtain [57,58]

$$\begin{aligned} \langle P \rangle &= \frac{\pi\epsilon^2 m_{A'}^4}{\omega_0} \int_0^{z_\star} dz \int dm_\gamma^2 f_1(m_\gamma^2; z) \frac{\delta_D(m_\gamma^2 - m_{A'}^2)}{H(z)(1+z)^2} \\ &= \frac{\pi\epsilon^2 m_{A'}^4}{\omega_0} \int_0^{z_\star} dz \frac{f_1(m_\gamma^2 = m_{A'}^2; z)}{H(z)(1+z)^2}, \end{aligned} \quad (\text{D2})$$

so that

$$\begin{aligned} \delta P(\hat{n}) &= \frac{\pi\epsilon^2 m_{A'}^4}{\omega_0} \int_0^{z_\star} dz \frac{1}{H(z)(1+z)^2} [\delta_D(m_\gamma^2(\vec{r}, z) - m_{A'}^2) \\ &\quad - f_1(m_\gamma^2 = m_{A'}^2; z)]. \end{aligned} \quad (\text{D3})$$

We now decompose the observed conversion probability over the whole sky into spherical harmonics $Y_{\ell m}(\hat{n})$, with coefficients $a_{\ell m}$ given by

$$a_{\ell m} = \frac{1}{\langle P \rangle} \int d\hat{n} \delta P(\hat{n}) Y_{\ell m}^*(\hat{n}).$$

The anisotropy power spectrum C_ℓ for the conversion probability, defined as $C_\ell \equiv \langle a_{\ell m}^* a_{\ell m} \rangle$, can then be computed as

$$C_\ell = \frac{1}{\langle P \rangle^2} \int d\hat{n} \int d\hat{n}' \langle \delta P(\hat{n}) \delta P(\hat{n}') \rangle Y_{\ell m}^*(\hat{n}) Y_{\ell m}(\hat{n}'),$$

where $\langle \dots \rangle$ in the integral should be interpreted as an all-sky average. Note that defined in the following manner, C_ℓ is dimensionless, since it describes fluctuations in conversion probability.

To make further progress, we define the quantity

$$Q(\vec{r}, z; \vec{r}', z') \equiv \langle [\delta_D(m_\gamma^2(\vec{r}, z) - m_{A'}^2) - f_1(m_\gamma^2 = m_{A'}^2; z)] [\delta_D(m_\gamma^2(\vec{r}', z') - m_{A'}^2) - f_1(m_\gamma^2 = m_{A'}^2; z')] \rangle, \quad (\text{D4})$$

a two-point correlation function along two different lines of sight, described by comoving coordinates \vec{r} and \vec{r}' , with photons passing through each point at redshifts z and z' , respectively. In fact, homogeneity and isotropy guarantee that this function does not depend on \vec{r} and \vec{r}' separately, but only on $|\vec{r} - \vec{r}'|$. Inserting this into the expression above for C_ℓ , we find

$$C_\ell = \frac{1}{\langle P \rangle^2} \left[\frac{\pi\epsilon^2 m_{A'}^4}{\omega_0} \right]^2 \int_0^{z_\star} dz W(z) \int_0^{z_\star} dz' W(z') \int d\hat{n} \int d\hat{n}' Q(|\vec{r} - \vec{r}'|, z, z') Y_{\ell m}^*(\hat{n}) Y_{\ell m}(\hat{n}'), \quad (\text{D5})$$

where $W(z) \equiv [H(z)(1+z)^2]^{-1}$. We can now write Q in terms of its Fourier transform over $\vec{r} - \vec{r}'$, \tilde{Q} , giving

$$\begin{aligned} Q(|\vec{r} - \vec{r}'|, z, z') &= \int \frac{d^3 \vec{k}}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{r} - \vec{r}')} \tilde{Q}(k, z, z') \\ &= (4\pi)^2 \sum_{p,s=0}^{\infty} \sum_{q=-p}^p \sum_{t=-s}^s \int \frac{d^3 \vec{k}}{(2\pi)^3} \tilde{Q}(\vec{k}, z, z') i^{p-s} j_p(kr) j_s(kr') Y_{pq}^*(\hat{k}) Y_{pq}(\hat{n}) Y_{st}(\hat{k}) Y_{st}^*(\hat{n}') \\ &= \frac{2}{\pi} \sum_{p=0}^{\infty} \sum_{q=-p}^p \int dk k^2 \tilde{Q}(k, z, z') j_p(kr) j_p(kr') Y_{pq}(\hat{n}) Y_{pq}^*(\hat{n}'). \end{aligned}$$

In the second line, we have used the Rayleigh expansion for plane waves, and j_p is the spherical Bessel function of order p . In the last line, we use the orthogonality of spherical harmonics to integrate over the solid angle. Substituting this expression into Eq. (D5) and integrating over \hat{n} and \hat{n}' , once again exploiting the orthogonality of spherical harmonics, gives

$$C_\ell = \frac{1}{\langle P \rangle^2} \left[\frac{\pi \epsilon^2 m_{A'}^4}{\omega_0} \right]^2 \frac{2}{\pi} \int_0^{z^*} dz W(z) \int_0^{z^*} dz' W(z') \int dk k^2 \tilde{Q}(k, z, z') j_\ell(kr) j_\ell(kr').$$

Finally, we can simplify this integral further assuming the Limber approximation [76–78], which is a high- ℓ expansion that works particularly well for the multipoles in which we are interested [102] and allows us to approximate $k^2 j_\ell(kr) j_\ell(kr') \approx (\pi/2) \delta_D(k - \ell/r) \delta_D(r - r')/r^2$. This finally gives

$$C_\ell = \frac{1}{\langle P \rangle^2} \left[\frac{\pi \epsilon^2 m_{A'}^4}{\omega_0} \right]^2 \int_0^{z^*} \frac{dz}{r^2(z)} W^2(z) H(z) \tilde{Q}(k = \ell/r, z, z). \quad (\text{D6})$$

This compact expression makes the calculation of the conversion anisotropy numerically tractable; in particular, the correlation function Q defined in Eq. (D4) need only be evaluated at points along two different lines of sight that have equal redshifts.

We now need an expression for $\tilde{Q}(k = \ell/r, z, z)$. First, we define $f_2(\rho, m_\gamma^2(\vec{r}), m_\gamma^2(\vec{r}'); z)$ as the two-point PDF for m_γ^2 at two different points, \vec{r} and \vec{r}' , at redshift z , with $\rho \equiv |\vec{r} - \vec{r}'|$. Note that homogeneity and isotropy guarantee that f_2 only depends on ρ . With this, the averaging in Eq. (D4) is performed by integrating the quantity with respect to $m_\gamma^2(\vec{r})$ and $m_\gamma^2(\vec{r}')$, weighted by f_2 , which gives

$$\begin{aligned} Q(|\vec{r} - \vec{r}'|, z, z) &= f_2(|\vec{r} - \vec{r}'|, m_\gamma^2(\vec{r}) = m_{A'}^2, m_\gamma^2(\vec{r}')) \\ &= m_{A'}^2; z - [f_1(m_\gamma^2 = m_{A'}^2; z)]^2, \end{aligned} \quad (\text{D7})$$

with

$$\tilde{Q}(k, z, z) = 4\pi \int d\rho \rho^2 j_0(k\rho) Q(\rho, z, z).$$

Given both the one- and two-point PDF of m_γ^2 , we can now proceed to calculate the anisotropy power spectrum.

We demonstrate that the conversion anisotropy signal can satisfy existing experimental bounds by assuming that 1) the free-electron number density is exactly equal to the baryon number density so that the one- and two-point PDFs of m_γ^2 are given by the one- and two-point PDFs of baryonic fluctuations δ_b divided by one and two powers of $\overline{m_\gamma^2}(z)$, the mean value of m_γ^2 at redshift z , respectively. This is a good assumption for $z \lesssim 6$, where resonant conversions generally take place, since reionization has been completed, and all baryons are ionized; and (2) the baryonic fluctuations follow analytic one- and two-point PDFs. We consider two analytic forms for the one- and two-points PDFs: normally distributed and log-normally distributed fluctuations, to give an indication of the dependence on the PDF. In both cases, the PDFs are fully specified by the correlation function $\xi_b(|\vec{r} - \vec{r}'|; z)$, which is related to the power spectrum of baryons P_b via

$$1\xi_b(\rho; z) = \int \frac{d^3\vec{k}}{(2\pi)^3} j_0(k\rho) P_b(k; z),$$

with $\sigma_b^2(z) = \xi_b(0; z)$ being the variance of baryonic fluctuations. For normally distributed fluctuations,

$$\begin{aligned} f_1(m_\gamma^2 = m_{A'}^2; z) &= \frac{1}{m_\gamma^2(z) \sqrt{2\pi\sigma_b^2(z)}} \exp\left[-\frac{g(m_{A'}^2; z)^2}{2\sigma_b^2(z)}\right], \\ f_2(\rho, m_\gamma^2(\vec{r}) = m_{A'}^2, m_\gamma^2(\vec{r}') = m_{A'}^2; z) &= \frac{1}{2\pi m_\gamma^2(z) \sqrt{\sigma_b^4(z) - \xi_b^2(\rho; z)}} \exp\left(-\frac{g(m_{A'}^2; z)^2}{\sigma_b^2(z) + \xi_b(\rho; z)}\right), \end{aligned} \quad (\text{D8})$$

where $g(m_{A'}^2; z) \equiv m_{A'}^2/\overline{m_\gamma^2}(z) - 1$, while for log-normally distributed fluctuations [57,103],

$$\begin{aligned} f_1(m_\gamma^2 = m_{A'}^2; z) &= \frac{1}{m_\gamma^2(z) \sqrt{2\pi\Sigma^2(z)}} \exp\left[-\frac{L^2(m_{A'}^2; z)}{2\Sigma^2(z)}\right] \frac{1}{1 + g(m_{A'}^2; z)}, \\ f_2(\rho, m_\gamma^2(\vec{r}) = m_{A'}^2, m_\gamma^2(\vec{r}') = m_{A'}^2; z) &= \frac{1}{2\pi m_\gamma^2(z) \sqrt{\Sigma^4(z) - X^2(\rho; z)}} \exp\left(-\frac{L^2(m_{A'}^2; z)}{\Sigma^2(z) + X(\rho; z)}\right) \frac{1}{[1 + g(m_{A'}^2; z)]^2}, \end{aligned} \quad (\text{D9})$$

where $\Sigma^2(z) = \log[1 + \sigma_b^2(z)]$, $X(\rho; z) = \log[1 + \xi_b(\rho; z)]$, and $L(m_{A'}^2; z) = \log[1 + g(m_{A'}^2; z)] + \Sigma^2(z)/2$.

In the limit that $\rho \rightarrow \infty$, we obtain $\xi_b \rightarrow 0$ and $f_2 \rightarrow f_1^2$ for both distributions, indicating that the two-point PDF factorizes into the product of two, independent one-point PDFs at sufficiently large separations, when correlations are unimportant.

Equations (D7) and (D6) give us the following general expression for C_ℓ , which is the main result of this section:

$$C_\ell = \frac{1}{\langle P \rangle^2} \int_0^{z_*} \frac{dz}{r^2(z)} H(z) \left[\frac{d\langle P \rangle}{dz} \right]^2 \int d\rho 4\pi\rho^2 j_0(\ell\rho/r) \times \left[\frac{f_2(\rho, m_\gamma^2(\vec{r}) = m_{A'}^2, m_\gamma^2(\vec{r}') = m_{A'}^2; z)}{f_1^2(m_\gamma^2 = m_{A'}^2; z)} - 1 \right], \quad (\text{D10})$$

an expression that we can evaluate numerically for either choice of baryon fluctuation PDFs. Note that the integrand over ρ is finite as $\rho \rightarrow 0$. Following the convention used in CMB anisotropy power spectrum analyses as well as Ref. [46], the temperature anisotropy power spectrum is defined to be $\ell(\ell+1)C_\ell \langle T \rangle^2$, where $\langle T \rangle$ is the sky-averaged brightness temperature.

For small angular scales ($\ell \gtrsim 3000$), upper limits on the anisotropy of the ERB have been obtained by the Very Large Array (VLA) at 4.86 GHz [42] and 8.4 GHz [43], as well as the Australia Telescope Compact Array (ATCA) at 8.7 GHz [44]. These measurements were made with the intention of looking for CMB anisotropies at the arcminute scales, and all set an approximate upper limit of $\Delta T/T_{\text{exc}} \lesssim 10^{-2}$ for the ERB [41]. More recently, LOFAR and TGSS observations have been used to measure the anisotropy power spectrum at ~ 140 MHz [45,46]. This latest result confirms the fact that a currently unknown population of dim but numerous synchrotron sources is required to explain the ERB; they also find that these sources must exhibit some nontrivial clustering to produce the right power spectrum.

Figure 4 shows the predicted anisotropy power spectrum, $[\ell(\ell+1)/(2\pi)]C_\ell \langle T \rangle^2$, as a function of ℓ , obtained by integrating Eq. (D10) numerically for both normally (lower line) and log-normally (upper line) distributed baryon fluctuations.⁶ The range between the two lines, shaded in blue, gives some indication of the uncertainty of our prediction. Representative values of $m_{A'} = 3 \times 10^{-14}$ eV and $\omega_{A'} = 2 \times 10^{-4}$ eV have been chosen, but the results are qualitatively similar for other values of these parameters. This result should be compared with existing measurements of the anisotropy power, including (1) observed upper limits of the CMB anisotropy power spectrum at small scales between 4–9 GHz by the VLA [42,43] and the

ATCA, subsequently reinterpreted in Ref. [41] as an upper limit on the anisotropy power of the excess radio power above the CMB and known point sources and (2) the LOFAR anisotropy power spectrum of the radio background at 140 MHz [46]. We take $\langle T \rangle = 255$ K, the expected radio temperature at 140 MHz based on the power-law fit in Eq. (1) of the paper. The anisotropy power reported by the high frequency measurements at frequency ν_{high} has been rescaled by $(140 \text{ MHz}/\nu_{\text{high}})^{2\beta}$, where $\beta = 2.6$; this assumes that the relative size of fluctuations $\delta T/T$ is independent of frequency and can be rescaled by $\langle T \rangle^2$ at each frequency. Note that the observed upper limits in the 4–9 GHz range are not necessarily in tension with the $\ell \gtrsim 10^3$ results at 140 MHz, since the anisotropy can in fact be frequency dependent.

The experimental results for the anisotropy power spectrum shown in Fig. 4 cover $10^2 \lesssim \ell \lesssim 6 \times 10^4$, finding a power of $1\text{--}10^2 \text{ K}^2$ across this range; this represents a temperature fluctuation of $0.004 \lesssim \Delta T/T_{\text{exc}} \lesssim 0.04$, a result that seems unusually smooth if radio emission were correlated with large scale structure at low redshifts [41]. Our predicted conversion anisotropy power spectrum for the fiducial values shown here lies mostly below the measured anisotropy power and is therefore consistent with these experimental results, except for $\ell \lesssim 800$ at 140 MHz. However, as mentioned in the paper, lower- ℓ measurements of the anisotropy are difficult, and within the LOFAR dataset, significant scatter can be observed in the anisotropy power between adjacent frequency bands, which are subsequently combined to produce the final

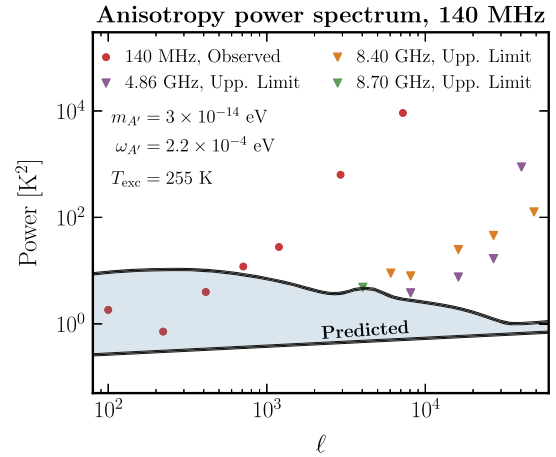


FIG. 4. Predicted anisotropy power spectrum with normally (solid black line, below) and log-normally (solid black line, above) distributed baryon fluctuations. The model parameters are $m_{A'} = 3 \times 10^{-14}$ eV and $\omega_{A'} = 2 \times 10^{-4}$ eV. Upper limits from VLA at 4.86 GHz [42] (purple triangles), 8.4 GHz [43] (orange triangles), and ATCA at 8.7 GHz [44] (green triangle) are shown and have been rescaled to $T_{\text{exc}} = 255$ K, the expected value at 140 MHz. Representative data points of the power spectrum measured by LOFAR are shown in red [46].

⁶The theoretical prediction from our model only has a weak dependence on the observational frequency, coming from the integration limit z_* ; we have only included the largest fluctuations here.

result shown here [46]. We have thus demonstrated that our model can, in principle, produce a sufficiently smooth conversion anisotropy power spectrum; more detailed studies involving more realistic one- and two-point PDFs, as well as potentially a more careful analysis of the 4–9 GHz radio data, may be of interest in future work.

APPENDIX E: DATA ANALYSIS AND STATISTICAL METHODOLOGY

1. Likelihood and posterior sampling

We assume the likelihood of the observed brightness temperatures $T_{\text{obs},i}(\nu_i)$ given a model for the emission $T(\nu; \theta)$ to be Gaussian, where i indexes individual data points, and $\theta = \{m_a, m_{A'}, \tau_{\text{vac}}, \epsilon, T'_0, T_0\}$ are the parameters of interest describing the fiducial new-physics model. Assuming uncorrelated measurements, the total likelihood for a dataset is given by $p(T_{\text{obs}}|\theta) = \prod_i N(T_{\text{obs},i}(\nu_i)|T(\nu_i; \theta), \sigma_{T,i})$, where $\sigma_{T,i}$ are the corresponding standard deviation uncertainties describing the noise model.

Given a prior $p(\theta)$ as defined in Sec. E 2 below, we infer an approximation for the posterior distribution $p(\theta|T_{\text{obs}}) = p(T_{\text{obs}}|\theta)p(\theta)/\mathcal{Z}$, where \mathcal{Z} is the marginal evidence $Z = \int d\theta p(T_{\text{obs}}|\theta)p(\theta)$. This is done through Monte Carlo nested sampling implemented in DYNESTY [72]. One thousand live points are used to model the posterior, and sampling is performed until the estimated expected contribution to the log evidence is less than $\Delta \log \mathcal{Z} = 0.05$, with an otherwise default configuration of the static nested sampler.

2. Constrained prior definition

A baseline prior is assumed such that $\log_{10} m_a \sim \mathcal{U}(-7, -1)$, $\log_{10} m_{A'} \sim \mathcal{U}(-17, -10)$, $T'_0/T_0 \sim \mathcal{U}(0, 0.4)$, $\log_{10} \tau_{\text{vac}} \sim \mathcal{U}(16, 25)$, $\log_{10} \epsilon \sim \mathcal{U}(-11, -4)$, and $T_0 \sim \mathcal{N}(2.7255, 0.00086)$, with the prior on T_0 motivated by an analysis of COBE/FIRAS CMB data in the 2.27–21.33 cm^{-1} frequency range [1,8], and the prior on T'_0/T_0 motivated by the constraints derived in Sec. C 2. Furthermore, for our fiducial analysis, we wish to incorporate external constraints on the considered parameter space as mentioned in the main body of this paper. Practical implementations of nested sampling require that the prior distribution be provided as a transformation from the unit hypercube to the target prior density. Although such a transformation is readily available for commonly used prior specifications, e.g., uniform- or Gaussian-distributed, transformations from an arbitrary prior such as that respecting the constraints laid out in the main body through combinations of multiple parameters are not always readily available or analytically tractable.

In order to construct an implicit prior for our purposes, we follow a procedure similar to that outlined in Ref. [104] and use normalizing flows [105,106], which use a series of

bijections parametrized by neural networks with tractable Jacobians in order to define transformations from a simple base distribution to complex, expressive target distributions. Specifically, we generate a large number 10^6 of samples from the baseline prior, remove the samples incompatible with the external constraints, and use a normalizing flow consisting of eight neural spline flows [107] to learn a transformation from a standard Gaussian $N(0, \mathbb{1})$ into the constrained target prior density. Samples u from the unit hypercube proposal can then be transformed to those corresponding to a standard Gaussian through an affine transformation $u \rightarrow 0 + \mathbb{1} \cdot F_{\mathcal{N}}^{-1}(u)$, where $F_{\mathcal{N}}^{-1}$ is the inverse cumulative distribution function of the Gaussian distribution. These samples can then be further transformed to those on the constrained prior density by passing them through the learned normalizing flow.

3. Prior-predictive check

Generally, in Bayesian analyses and, in particular, when nontrivial priors are utilized such as in this work, it is useful to conduct prior-predictive checks in order to instill confidence that the prior does not bias the analysis towards “favorable” regions of the parameter space. In that case, formally low-likelihood parameter points could still correspond to high posterior density regions.

Figure 5 shows the 68 and 95% contours of the prior induced on the excess temperature above the modeled extragalactic background, defined as $T_{\text{eg}} = 0.23 \text{ K}(\nu/\text{GHz})^{-2.7}$ [16,41]. It can be seen that the induced prior covers a large range of possible excess temperatures and does not bias the current analysis in favor of the observed data points (shown in black and red).

APPENDIX F: EXTENDED RESULTS

1. Parameter posteriors

Figure 6 shows the individual and pairwise marginal posteriors on all the modeled parameters obtained for the baseline analysis.

2. Systematic variations on the analysis

In this section, we consider various systematic variations of our baseline analysis. In particular, we consider:

- (1) Results obtained using the data from Ref. [9] instead of those from Ref. [3]. Reference [9] includes recent data points from the LWA1 Low Frequency Sky Survey (LLFSS), which spans a frequency range of 40–80 MHz. These new data points are consistent with the power-law fit in Ref. [3]. Reference [9] also independently reanalyzed the data from ARCADE 2 and the other experiments used in Ref. [3], finding broad agreement, but with small changes to the central values and more significant changes to the error bars, despite using a similar data analysis method. Reference [9] therefore provides a useful

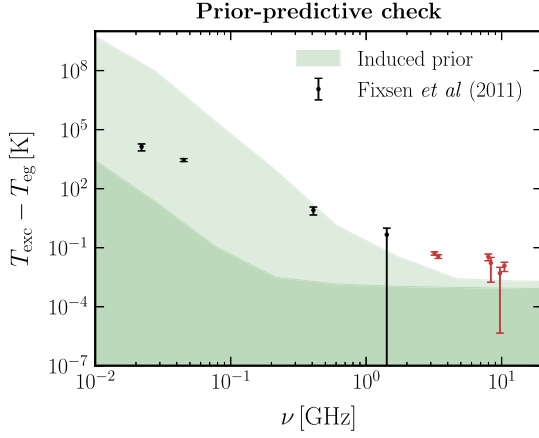


FIG. 5. 68% (dark green) and 95% (light green) containment of the induced prior on the excess temperature above the modeled extragalactic background, $T_{\text{eg}} = 0.23 \text{ K}(\nu/\text{GHz})^{-2.7}$ [16]. Measured data points from Ref. [3] are shown for comparison. The induced prior covers a large range of possible excess temperatures and does not bias the analysis in favor of the observed data points.

systematics check on how the ERB spectrum is extracted;

- (2) Results obtained by fitting only the ARCADE 2 data $\gtrsim 3$ GHz and ignoring the lower-frequency radio measurements. In this case, we neglect stimulated emission, since this is not necessary to produce a good fit, simplifying the model and reducing to that first introduced in Ref. [62]. ARCADE 2 is the only experiment in our list that is designed primarily with absolute zero-level calibration in mind, in order to produce an accurate determination of the sky brightness [10], which warrants a separate investigation; and
- (3) Results obtained without the modeled contribution from unresolved extragalactic radio sources [16,41]. This should concretely demonstrate that our model does not rely on unresolved extragalactic radio sources for a good fit, which can be anticipated from the fact that these sources are at least three times less bright than the measured T_{exc} .

Figure 7 shows the excess temperature posterior for all these systematic variations, while Table I shows the posterior summaries for each case considered. The parameters ranges compatible with observations show minor variations from case to case, while providing a visually good fit to the data in all cases.

We summarize the inferred posteriors in Table I through the median and 68% containment of the individual marginal parameter posteriors (labeled “marginal”) and the 68% highest posterior density intervals, describing the shortest interval in the higher-dimensional joint parameter

space where 68% of the posterior mass is contained (labeled “HDPI”).

3. Power-law fit and model comparison

As a point of comparison, we also show results using a variant of the commonly used power-law ansatz [equivalent to Eq. (1) in the paper] for the excess temperature over T_0 ,

$$T(\nu) = T_0 \left[1 + A_{\text{GHz}} \left(\frac{\nu}{1 \text{ GHz}} \right)^\beta \right] \quad (\text{F1})$$

which is parametrized through the CMB black-body temperature T_0 , excess temperature at 1 GHz A_{GHz} , and spectral index β . The results of these fits for the Fixsen *et al.* and Dowell & Taylor datasets are shown in the left and right columns of Fig. 8, respectively. As previously reported in the literature, these parametrizations provide a formally good fit to the low-frequency radio data [9,40]. Posterior summaries for these fits are given in Table II in the same format as Table I.

We show a quantitative comparison between the power law and new physics models as follows. For a given model \mathcal{M} (in our case, corresponding to either the new-physics model or the power-law ansatz), the model evidence, also known as the marginal likelihood, is defined for data x and parameters θ as $\mathcal{Z}_{\mathcal{M}} \equiv \int d\theta p_{\mathcal{M}}(x|\theta) p_{\mathcal{M}}(\theta)$, where $p_{\mathcal{M}}(x|\theta)$ and $p_{\mathcal{M}}(\theta)$ are the model likelihood and prior, respectively. The model evidence is a measure of compatibility between the model and data and implicitly penalizes excessive model complexity—this is relevant in our case, since the new physics and power-law models have different number of parameters (3 and 6, respectively). The ratio of evidence between two models \mathcal{M}_1 and \mathcal{M}_2 is known as the Bayes factor and can be formally used in a model comparison setting to quantify how much better of a fit \mathcal{M}_1 is compared to \mathcal{M}_2 .

We show the log-Bayes factors in favor of the power-law model in the last column of Table II. The power-law model tends to provide a formally better fit using both the Fixsen *et al.* and Dowell & Taylor datasets, corresponding to log-Bayes factors in favor of the power-law model of $\Delta \log \mathcal{Z} = 1.89$ and $\Delta \log \mathcal{Z} = 0.76$, respectively. This reflects the fact that a power law with $\beta \approx -2.6$ is a slightly better fit to the data points, rather than $\beta \approx -5/2$ as predicted by our model. We note however that given significant differences between models considered, a principled model comparison is difficult. Specifically, $\mathcal{O}(1)$ Bayes factors should not be taken at face value, since they can depend sensitively on chosen prior specifications and ranges; see Ref. [108] for a detailed discussion of this point. Finally, we note that even though synchrotron emission from unknown sources can plausibly produce a power law close to $\beta \approx -2.6$ (see App. G), this is highly dependent on

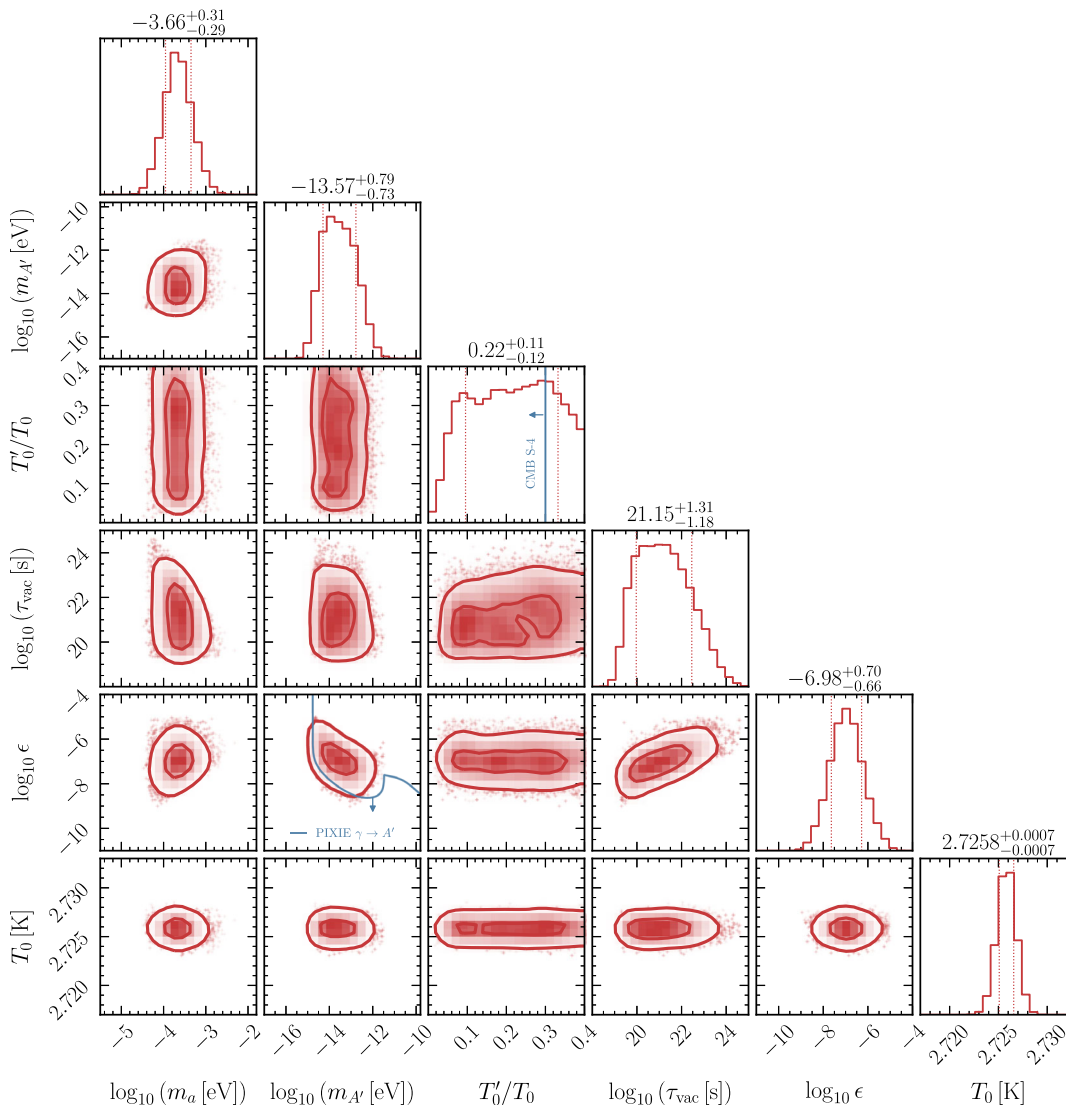


FIG. 6. Joint and individual marginal posterior obtained in the fiducial model, same format as Fig. 2 in the paper.

the electron distribution within these individual sources as a function of source brightness and counts; it is therefore not immediately obvious that any future model ascribing T_{exc} to currently unknown synchrotron sources would provide a better fit than our model does.

APPENDIX G: A RECAP OF SYNCHROTRON RADIATION

As extensively discussed in the paper, many astrophysical solutions to the ERB advocate for the presence of synchrotron radiation from unresolved astrophysical sources. In this appendix, we briefly review synchrotron emission with a particular focus on its spectral features. The basic physics behind synchrotron radiation is straightforward: a charged particle accelerated in a magnetic field will radiate. Nonrelativistic particles emit cyclotron radiation, with a frequency given by the cyclotron frequency

$\omega_{\text{cyc}} = qB/m$, where q is the particle charge, B the magnetic field in which it moves, and m its mass. Relativistic particles emit what we call synchrotron radiation: The frequency spectrum is far richer, covering a range well away from the cyclotron frequency.

Taking into account strong relativistic effects (such as forward beaming), one finds that the characteristic frequency of the synchrotron radiation of a *single* charged particle is $\omega_{\text{syn}} \sim \gamma^2 \omega_{\text{cyc}}$ [109]. The spectrum of the radiation is broad: For $\omega < \omega_{\text{syn}}$, the spectrum falls off like a power law $\propto \omega^{1/3}$, while for $\omega > \omega_{\text{syn}}$, the spectrum falls off exponentially.

This is, however, very different from what we observe in astrophysical systems. The reason is that we need to consider what happens to the total synchrotron radiation emitted by a *population* of charged particles, with a given energy distribution. Let us consider an ensemble of charged

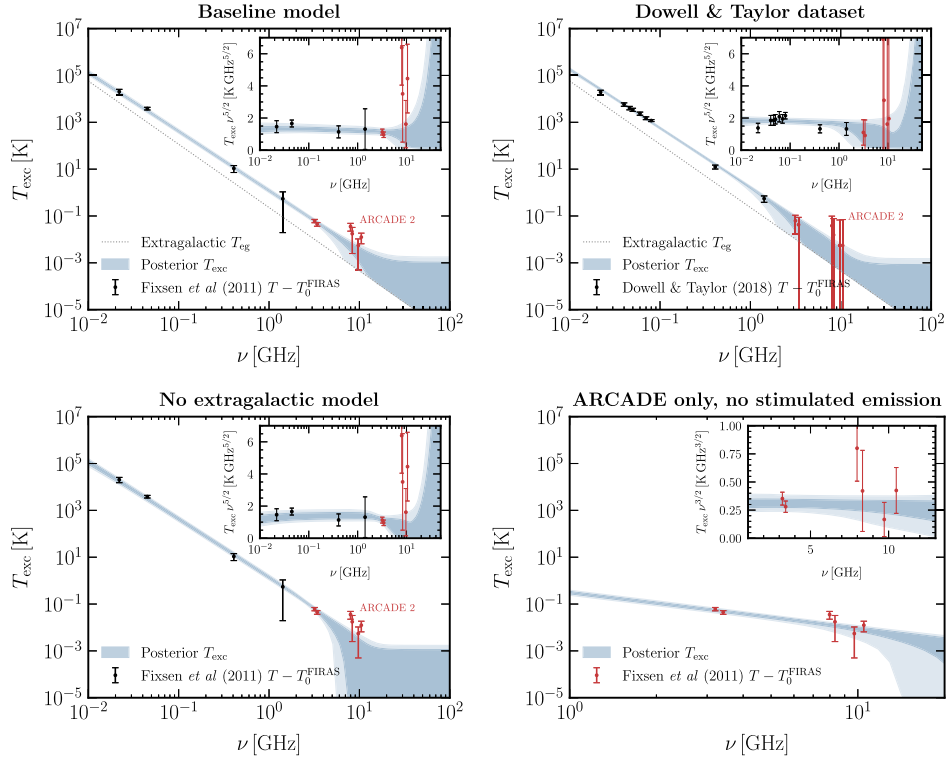


FIG. 7. Systematic variations on the analysis, same format as Fig. 2 in the paper.

particles with number density distribution per unit energy following a power law $dN/dE \propto E^p$. The power per unit energy emitted by this ensemble of charges reads

$$\frac{dP}{dE} \propto \frac{dN}{dE} P_E \propto E^p E^2 B^2,$$

where P_E is the power emitted by a single particle, given by the Larmor formula $P_E \propto B^2 \gamma^2 \propto B^2 E^2$ (we drop here all the irrelevant constants given that we are only interested in the final spectral shape of the synchrotron radiation (single power). Let us now assume that the *single-charge* synchrotron spectrum is peaked enough to assume that all the power is emitted at the synchrotron frequency, ω_{syn} .

This implies a one-to-one relation between the observed frequency and the energy of the single particle $\nu \simeq \omega_{\text{syn}}/(2\pi) \propto E^2 B$. It then follows

$$\frac{dP}{d\nu} = \frac{dP}{dE} \frac{dE}{d\nu} \propto E^{p+2} B \times \nu^{-1/2} B^{-1/2} \propto B \left(\frac{\nu}{B}\right)^{\frac{1+p}{2}},$$

and therefore, the spectral index reads $\alpha = (1+p)/2$. In terms of the spectral index of brightness temperature, β , this corresponds to $\beta = (p-3)/2$, since the brightness temperature is related to the intensity via $T_\nu \propto I_\nu/\nu^2$. Therefore, the spectral index for synchrotron radiation is intrinsically related to the energy distribution of the charged particles that generate

TABLE I. Posterior summaries for all the systematic variations as compared to our baseline analysis.

Config.	Summary	$\log_{10} m_a$	$\log_{10} m_{A'}$	T'_0/T_0	$\log_{10} \tau_a$	$\log_{10} \epsilon$	T_0
		$\log_{10}[\text{eV}]$	$\log_{10}[\text{eV}]$		$\log_{10}[\text{s}]$		[K]
Fixsen (Baseline)	Marginals	$-3.66^{+0.10}_{-0.26}$	$-13.57^{+0.32}_{-0.68}$	$0.22^{+0.05}_{-0.12}$	$21.15^{+0.47}_{-1.10}$	$-6.98^{+0.25}_{-0.60}$	$2.7258^{+0.0003}_{-0.0007}$
	HPDI	$[-3.97, -3.37]$	$[-14.52, -13.04]$	$[0.08, 0.32]$	$[19.58, 21.98]$	$[-7.65, -6.30]$	$[2.7251, 2.7266]$
No EG model	Marginals	$-3.96^{+0.11}_{-0.26}$	$-13.22^{+0.28}_{-0.75}$	$0.22^{+0.05}_{-0.11}$	$21.48^{+0.58}_{-1.27}$	$-7.23^{+0.27}_{-0.71}$	$2.7259^{+0.0003}_{-0.0007}$
	HPDI	$[-4.27, -3.75]$	$[-13.85, -12.47]$	$[0.09, 0.33]$	$[19.96, 22.71]$	$[-7.87, -6.43]$	$[2.7252, 2.7267]$
Dowell	Marginals	$-4.12^{+0.15}_{-0.33}$	$-13.28^{+0.30}_{-0.74}$	$0.22^{+0.05}_{-0.11}$	$21.61^{+0.60}_{-1.26}$	$-7.25^{+0.27}_{-0.78}$	$2.7256^{+0.0003}_{-0.0007}$
	HPDI	$[-4.54, -3.76]$	$[-14.01, -12.50]$	$[0.15, 0.40]$	$[20.10, 22.98]$	$[-8.06, -6.50]$	$[2.7249, 2.7264]$
Fixsen, no stim. ARCADE only	Marginals	$-3.38^{+0.13}_{-0.30}$	$-13.59^{+0.32}_{-0.64}$		$20.33^{+0.34}_{-0.65}$	$-6.61^{+0.20}_{-0.45}$	$2.7256^{+0.0002}_{-0.0007}$
	HPDI	$[-3.78, -3.10]$	$[-14.54, -13.06]$		$[19.36, 20.80]$	$[-7.15, -6.08]$	$[2.7249, 2.7264]$

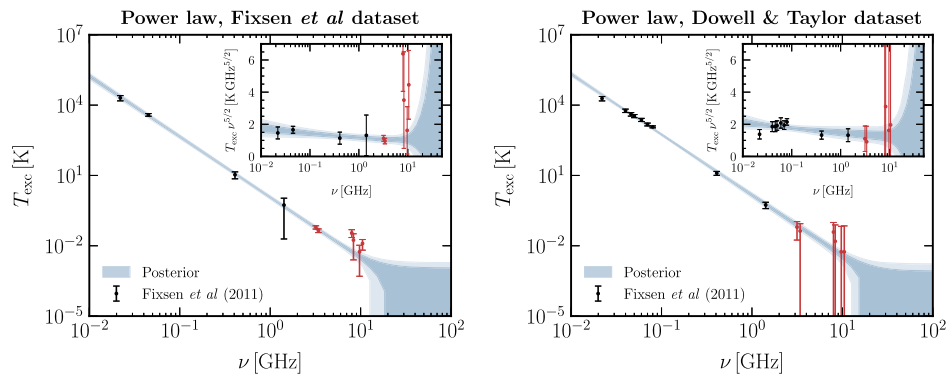


FIG. 8. Excess temperature for the power-law fit, same format as Fig. 2 in the paper.

the radiation. In particular, a power law is expected if the energy distribution is also a power law as assumed above.

One well-known example for which a power-law distribution is expected comes from particle acceleration by astrophysical shocks [110–114]. For example, if one considers shock waves that propagate in the interstellar medium outside a supernova remnant, it can be shown that at linear level, $p = -2$ [110,111], which implies $\alpha = -0.5$ and $\beta = -2.5$. The spectral index predicted by the simplest

scenario of shock acceleration coincides with the spectral index expected in our model, and is therefore slightly different than the power-law best-fit obtained in Ref. [3]. Nevertheless, steeper spectra are possible taking into account nonlinear effects in the theory of diffusive shock acceleration [115]. As a matter of fact, these effects are crucial to match observations of the supernova remnant Cassiopeia A, which exhibits a steeper spectral index, $\alpha \simeq -0.77$ or $\beta \simeq -2.77$ [116].

TABLE II. Posterior summaries for the power-law fits. The last column additionally shows the log-Bayes factor $\Delta \log \mathcal{Z}$ in favor of the power-law fit, with positive values corresponding to a preference for the power-law model.

Config.	Summary	A_{GHz}	β	T_0	$\Delta \log \mathcal{Z}$
				[K]	
Fixsen Power law	Marginals	$0.43^{+0.01}_{-0.04}$	$-2.58^{+0.01}_{-0.04}$	$2.7258^{+0.0003}_{-0.0008}$	1.89
	HPDI	[0.40, 0.47]	[-2.62, -2.55]	[2.7250, 2.7266]	
Dowell Power law	Marginals	$0.54^{+0.03}_{-0.06}$	$-2.58^{+0.02}_{-0.05}$	$2.7255^{+0.0003}_{-0.0008}$	0.76
	HPDI	[0.47, 0.61]	[-2.63, -2.53]	[2.7247, 2.7264]	

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