Production of the triply heavy Ω_{ccc} and Ω_{bbb} baryons at e^+e^- colliders

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Nonrelativistic quantum chromodynamics (NRQCD) factorization formulism is an important approach to investigate the production of the heavy quarkonium. In this paper, we study the production of the Ω_{ccc} and Ω_{bbb} at the e^+e^- collider, using the NRQCD factorization formulism. We do the full calculation of the total and differential cross sections of the processes $e^+e^- \to \gamma^*/Z^* \to \Omega_{QQQ} + \bar{Q} + \bar{Q} + \bar{Q}$, in the leading order at the e^+e^- colliders with different energies. In this paper, Ω_{QQQ} means the triply heavy baryon Ω_{ccc} or Ω_{bbb}. And Q denotes the c or b quark. The results show that it is hard to observe them at the e^+e^- colliders directly.

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I. INTRODUCTION

The doubly charmed baryon Ξ_{cc} has been observed in both SELEX and LHC [1,2]. Then, different decay channels of the Ξ_{cc} were observed in LHC [3–5]. The lifetime of the Ξ_{cc} was measured in LHC also [6]. While, none of the triply heavy baryons has been observed in experiment. As the baryonic analogues of the heavy quarkonia, triply heavy baryons are helpful to understand the strong interaction between quarks, the hadron structures and weak decays of the heavy baryons. The triply heavy baryons have been studying in theory. About forty years ago, the heavy baryon spectroscopy was studied using the QCD bag model in [7], firstly. Then, the mass of the triply heavy baryons have been determined in kinds of theoretical models, such as, the QCD sum rules [8–14], the lattice QCD [15–20], the hypercentral quark model [21–24], the nonrelativistic quark model [25–30] and the relativistic quark model [31–34], the Faddeev equation [35–37], the symmetry-preserving Schwinger-Dyson equations [38,39], the variational method [40], the Regge trajectories [41–44], etc.

As that two pairs of heavy quarks need to be produced in the production of the doubly heavy baryon [45–51], three pairs of heavy quarks need to be produced in the production progress of the triply heavy baryon. This makes it difficult to produce and observe the triply heavy baryon in experiment. The production cross sections of the triply heavy baryons at the LHC have been evaluated via fragmentation of the heavy quark in Refs. [52–55]. In [56,57], the total and differential cross sections of the direct production processes of these baryons at the LHC have been calculated. And the results show that it is quite promising to discover the triply heavy baryons in LHC. The production of the Ω_{ccc} in heavy-ion collisions have been investigated in Refs. [58–60]. Compared with the hadron collider, the backgrounds at the e^+e^- collider are much less. So, it is necessary to study the production of the triply heavy baryons at the e^+e^- collider. The production of the triply charmed baryon in e^+e^- collisions has been estimated in Ref. [61]. To simply the calculation, the authors in Ref. [61] have taken an approximation ignoring the mass of the charm quark in the numerator of the quark propagator and in all traces. However, as pointed out in Ref. [62], the approximation will lead to quite large errors. In this work, we report the study on the production of the triply heavy baryons Ω_{ccc} and Ω_{bbb} at e^+e^- colliders. We calculate the total and differential cross sections of the direct production of the Ω_{ccc} and Ω_{bbb} exactly.

Because the constituent quarks of the triply heavy baryon are all heavy flavored, the NRQCD [63] can be used to describe the triply heavy baryon. The production of the Ω_{QQQ} can be factorized into two parts, the short-distance coefficient which describes the process of the production of the three heavy quark pairs, and the long-distance matrix element which describes the hadronization of three heavy quarks to the triply heavy baryon. The number of the Feynman diagrams corresponding to the short distance coefficient is large. To achieve the calculation, we utilize the characteristic of the constituent quarks as the authors have done in Refs. [56,57].

The present paper is organized as follows. In Sec. II, we will show the details on the calculation of the two parts,

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the short-distance coefficient and the long-distance matrix element mentioned above. In Sec. III, we will list the numerical results and conclusions. And in Sec. IV, we will give a brief summary.

II. PRODUCTION OF THE Ω_{ccc} AND Ω_{bbb} AT e^+e^- COLLIDERS

For the triply heavy baryons, the relative motions among the heavy valence quarks are nonrelativistic. The typical velocity v of the heavy quark in the rest frame of the Ω_{QQQ} is small. And there are three distinct energy scales in the triply heavy baryons, i.e., the mass of the heavy quark m, the three-momentum of the heavy quark mv, and the energy of the heavy quark mv^2. And the three energy scales have the relationship of $m \gg mv \gg mv^2$. The direct production of the triply heavy baryons can be described as follows. Three heavy quark pairs are produced firstly at energy scale m or higher followed by the formation of the triply heavy baryon at the energy scale mv. Namely, the production cross section of the triply heavy baryon can be factorized into the short-distance coefficient and the long-distance matrix element. The short-distance coefficient describes the process of the three heavy quark pairs production which can be expanded as a power series of α_s and α at the energy scale m or higher. The long-distance matrix element describes the hadronization of the triply heavy baryon from the pointlike three heavy quarks.

The triply heavy baryon must be in color singlet. In the leading order, the color wave function of the Ω_{QQQ} must be $\frac{1}{\sqrt{6}}\varepsilon^{\xi_1\xi_2\xi_3}Q_{1\xi_1}Q_{2\xi_2}Q_{3\xi_3}$ with ξ_i $(i=1,2,3)$ being the color index of the valence quark Q_i. As a ground state, the orbital angular momentum wave function is symmetrical. The exchange antisymmetry of the identical fermions implies that the Ω_{QQQ} must be the spin-symmetrical state and the spin of it must be $\frac{3}{2}$. As given in Ref. [56], in the leading Fock state description, the S-wave state of the nonrelativistic triply heavy baryon Ω_{QQQ} can be written as,

$$|\Omega_{QQQ},\frac{3}{2},S_Z\rangle = \sqrt{2M}\int\frac{d^3\vec{q}_1}{(2\pi)^3}\frac{d^3\vec{q}_2}{(2\pi)^3}\frac{1}{\sqrt{3!}} \times\sum_{\xi_1,\xi_2,\xi_3}\sum_{\eta_1,\eta_2,\eta_3}\frac{\varepsilon^{\xi_1\xi_2\xi_3}}{\sqrt{6}}\left\langle\frac{3}{2},S_Z\middle|\eta_1,\eta_2,\eta_3\right\rangle \times\frac{1}{\sqrt{2E_1 2E_2 2E_3}}\psi(\vec{q}_1,\vec{q}_2)|Q_1,\xi_1,\eta_1,\vec{q}_1\rangle \times|Q_2,\xi_2,\eta_2,\vec{q}_2\rangle|Q_3,\xi_3,\eta_3,\vec{q}_3\rangle, \qquad (1)$$

where, $\vec{q}_3=-\vec{q}_1-\vec{q}_2$, and

$$\langle Q_i,\xi_i,\eta_i,\vec{q}_i|Q_j,\xi_j,\eta_j,\vec{q}_j\rangle = \delta_{\eta_i\eta_j}\delta_{\xi_i\xi_j}(2\pi)^3 2E_f\delta^{(3)}(\vec{q}_i-\vec{q}_j),$$

with η_i and $(E_i,\vec{q}_i)$ $(i=1,2,3)$ being the spin and the four-momentum of the heavy quark Q_i; M being the mass of the baryon Ω_{QQQ}; $\langle\frac{3}{2},S_Z|\eta_1,\eta_2,\eta_3\rangle$ being the Clebsch-Gordan (C-G) coefficient; S_Z being the third component of the spin of the Ω_{ccc}, and $\psi(\vec{q}_1,\vec{q}_2)$ being the wave function in the momentum space which is normalized as follows

$$\int\frac{d^3\vec{q}_1}{(2\pi)^3}\frac{d^3\vec{q}_2}{(2\pi)^3}\psi^*(\vec{q}_1,\vec{q}_2)\psi(\vec{q}_1,\vec{q}_2)=1. \qquad (2)$$

In the heavy quark limit, the dependence of the short distance on the momenta, q_1 and q_2, can be neglected in the leading order. Namely, the momenta of the produced three identical heavy quarks can be treated as the same. As a result, the long-distance matrix is proportional to the wave function of the baryon at the origin,

$$\Psi(0,0)=\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\psi(q_1,q_2). \qquad (3)$$

In this paper, we consider only the contribution from the leading order short-distance coefficient, and the leading order long-distance matrix element. By the standard perturbation theory, the amplitude of the process $e^+e^-\to\Omega_{QQQ}+\bar{Q}+\bar{Q}+\bar{Q}$ can be written as,

$$A(e^+e^-\to\Omega_{QQQ}+\bar{Q}+\bar{Q}+\bar{Q}) =\frac{\sqrt{2M}}{\sqrt{(2m)^3}}\frac{\Psi(0,0)}{\sqrt{3!}} \times\mathcal{M}\Big(e^+e^-\to(QQQ)_1^{(\frac{3}{2},S_Z)}+\bar{Q}+\bar{Q}+\bar{Q}\Big), \qquad (4)$$

in which, $\mathcal{M}(e^+e^-\to(QQQ)_1^{(\frac{3}{2},S_Z)}+\bar{Q}+\bar{Q}+\bar{Q})$ is the matrix element in the leading order of the process $e^+e^-\to(QQQ)_1^{(\frac{3}{2},S_Z)}+\bar{Q}+\bar{Q}+\bar{Q}$.[1] It is the product of the short-distance coefficient with the color- and spin-wave functions in the long-distance matrix element.

Now, let us consider the short-distance coefficient. The contribution to the production of the three heavy quark pairs comes from the e^+e^- annihilation process mainly,

$$e^-(k_1)+e^+(k_2)\to Z^*/\gamma^* \to Q(p_1,\xi_1)+Q(p_2,\xi_2)+Q(p_3,\xi_3) +\bar{Q}(p_4,\chi_1)+\bar{Q}(p_5,\chi_2) +\bar{Q}(p_6,\chi_3), \qquad (5)$$

where, k_1 and k_2 are the 4-momenta of the electron and position; p_i $(i=1,6)$ are the 4-momenta of the produced Q

[1] $(QQQ)_1^{(S,S_Z)}$ means the total spin and the third component of the pointlike three Q-quarks are S and S_Z, respectively; the momenta of the three quarks are the same; and the three heavy quarks couple to a color singlet.

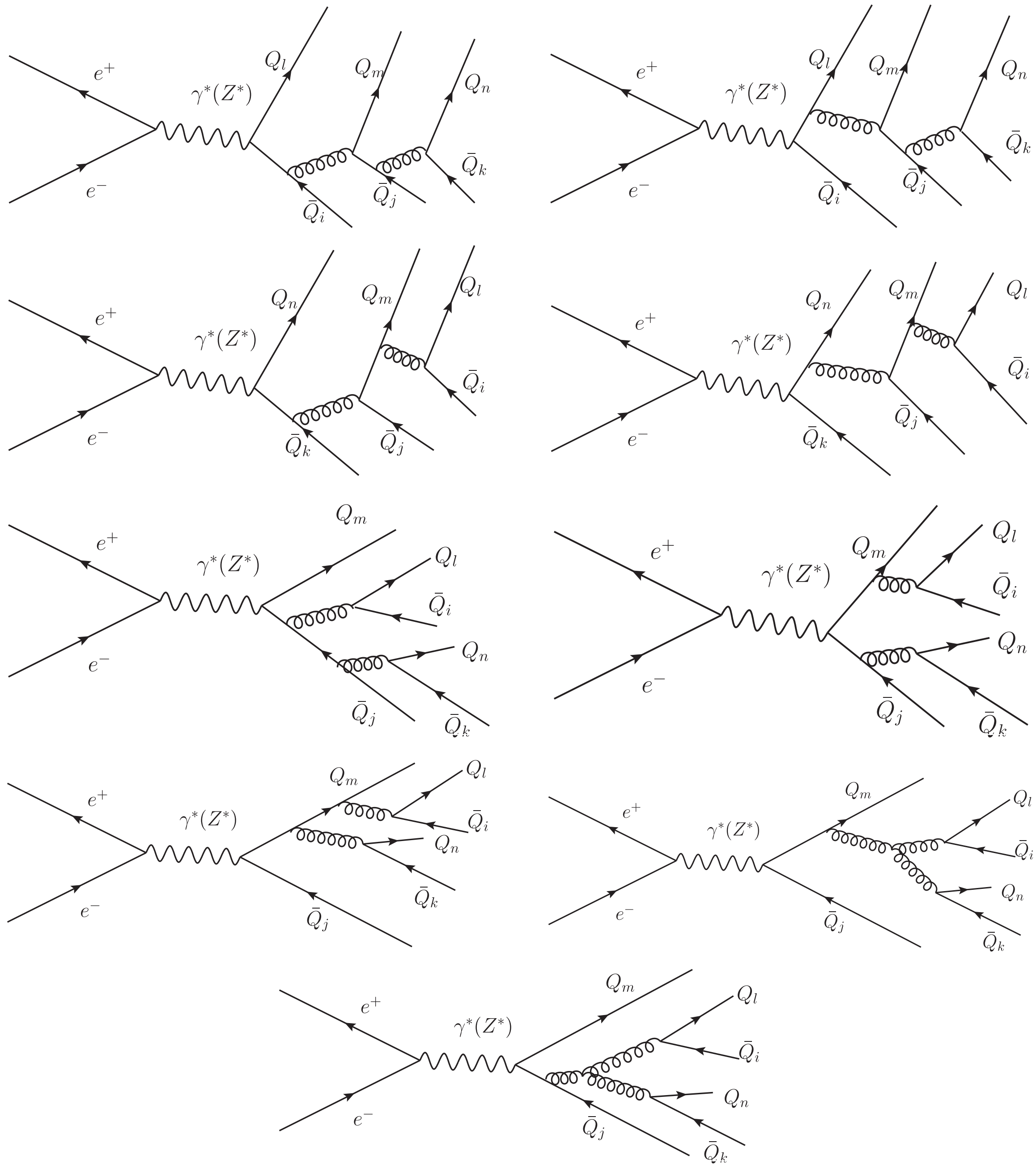

FIG. 1. Nine typical Feynman diagrams for the process (5). The indices $i, j, k, l, m, n = 1, 2, 3$ (with $i \neq j \neq k$ and $l \neq m \neq n$).

and $\bar{Q}$ quarks, with $p_1 = p_2 = p_3$; ξ_j and $\chi_j (j = 1, 2, 3)$ are the color indices of the Q_j-quarks and $\bar{Q}_j$-quarks, respectively. The produced three pairs of heavy quarks in process (5) have six permutations denoted as, $(Q_1\bar{Q}_iQ_2\bar{Q}_jQ_3\bar{Q}_k)$ with $i, j, k = 1, 2, 3$ and $i \neq j \neq k$.[2] In the calculation, we will disregard the contribution of the electroweak interaction between the heavy quarks for the process (5). As a result, there are seven inequivalent topology structures for each of the six permutations as the same as the one in [56]. So, there are 42 topology structures totally for the produced heavy quarks. Inserting the $e^+e^-\gamma^*$ vertex and $e^+e^-Z^*$ vertex into the 42 topology structures in all the allowed positions in the tree level, we get 576 Feynman diagrams for the process (5). And, we show nine typical ones in Fig. 1.

As pointed out above, the color configuration of the produced three heavy quarks is $\frac{1}{\sqrt{6}}\varepsilon^{\xi_1\xi_2\xi_3}Q_{1\xi_1}Q_{2\xi_2}Q_{3\xi_3}$. Setting $T^a = \frac{\lambda^a}{2}$ with λ^a $(a = 1, \ldots, 8)$ being the

[2] $(Q_1\bar{Q}_iQ_2\bar{Q}_jQ_3\bar{Q}_k)$ $(i, j, k = 1, 2, 3$ and $i \neq j \neq k)$ means the quark Q_1 in the same fermion line with the antiquark $\bar{Q}_i$, the quark Q_2 in the same fermion line with the antiquark $\bar{Q}_j$ and the quark Q_3 in the same fermion line with the antiquark $\bar{Q}_k$.

Gell-Mann matrices and using the color-flow method in [56], we can get the color factors of the 576 Feynman diagrams. The color factors of the last two diagrams in Fig. 1 are both,

$$\sum_{a,b,c}\sum_{\xi_1,\xi_2,\xi_3}\frac{1}{\sqrt{6}}\varepsilon^{\xi_1\xi_2\xi_3}f^{abc}(T^a)_{\xi_l\chi_i}(T^b)_{\xi_m\chi_j}(T^b)_{\xi_n\chi_k}=0.$$

in which, f^{abc} $[(a,b,c=1,\ldots,8)]$ is the structure constant of the $SU(3)$ group, and the index ξ_i $(i=1,2,3)$ appears twice, in $\varepsilon^{\xi_1\xi_2\xi_3}$ directly and as one of the indices ξ_l, ξ_m and ξ_n indirectly. And we see the result is independent onto the indices i, j, k, l, m, and n. As a result, we get the conclusion that the total contribution to the amplitude of the process (5) from the 72 Feynman diagrams involving the three-gluon vertex vanishes, because the color factors of these Feynman diagrams are all zero. The number of the Feynman diagrams which we need to consider reduces to 504. The color factors of the first seven Feynman diagrams in Fig. 1 are all the same,

$$\begin{aligned}\sum_{a,b}\sum_{\xi_1,\xi_2,\xi_3}&\frac{1}{\sqrt{6}}\varepsilon^{\xi_1\xi_2\xi_3}(T^a)_{\xi_l\chi_i}(T^aT^b)_{\xi_m\chi_j}(T^b)_{\xi_n\chi_k}\\&=(-1)^N\frac{4}{9}\frac{1}{\sqrt{6}}\varepsilon^{\chi_i\chi_j\chi_k},\end{aligned}$$

in which N is the number of permutations of transforming the set (ξ_l,ξ_m,ξ_n) to the set (ξ_1,ξ_2,ξ_3). For each of the Feynman diagrams, there is a Feynman factor $(-1)^{N^*}$, where N^* equals the total number of permutations of transforming the set (l,m,n) to the set (1,2,3) and transforming the set (i,j,k) to the set (1,2,3). Subsuming the Feynman factor into the corresponding color factor, we find that all the color factors of the remaining 504 Feynman diagrams are the same,

$$C_{\mathrm{col}}=(-1)^{N^*}(-1)^N\frac{4}{9}\frac{1}{\sqrt{6}}\varepsilon^{\chi_i\chi_j\chi_k}=\frac{4}{9}\frac{1}{\sqrt{6}}\varepsilon^{\chi_1\chi_2\chi_3}.$$

Let us consider the remained 504 Feynman diagrams. For each one of the six permutations given above, there are 84 Feynman diagrams, in which 42 ones correspond to the process $e^-+e^+\to Z^*\to Q_1+Q_2+Q_3+\bar{Q}_1+\bar{Q}_2+\bar{Q}_3$ and the other 42 ones correspond to the process $e^-+e^+\to\gamma^*\to Q_1+Q_2+Q_3+\bar{Q}_1+\bar{Q}_2+\bar{Q}_3$. In the leading order, the momenta of the three heavy quarks are considered to be equal. As a result, the contribution of every 84 Feynam diagrams corresponding to each of the six permutations are the same. The total amplitude for the process $e^+e^-\to(QQQ)_1^{(\frac{3}{2},S_Z)}+\bar{Q}+\bar{Q}+\bar{Q}$

$$\begin{aligned}\mathcal{M}(e^+e^-\to{}&(QQQ)_1^{(\frac{3}{2},S_Z)}+\bar{Q}+\bar{Q}+\bar{Q})\\&=\sum_{k=1}^{504}C_{\mathrm{col}}\Gamma_k=3!\sum_{k=1}^{84}C_{\mathrm{col}}\Gamma'_k,\end{aligned}\tag{6}$$

where Γ_k $(k=1,2\ldots,504)$ are the matrices without color factors of the Feynman diagrams, and Γ'_k $(k=1,2\ldots,84)$ are the matrices without color factors of the 84 Feynman diagrams corresponding to the permutation $(Q_1\bar{Q}_1Q_2\bar{Q}_2Q_3\bar{Q}_3)$.

III. NUMERICAL RESULTS AND DISCUSSIONS

Now, the cross section of the progress (5) can be written as

$$\begin{aligned}\sigma=\frac{1}{3!}\int\sum_{S_Z,\varsigma_i}&\frac{(2\pi)^4}{2\hat{s}}\delta^4(k_1+k_2-P-p_4-p_5-p_6)\\&\times d\Pi_4\frac{1}{4}\sum_{s_1,s_2,\chi_i}|\mathcal{A}(e^+e^-\to\Omega_{QQQ}\bar{Q}\,\bar{Q}\,\bar{Q})|^2,\end{aligned}\tag{7}$$

with

$$d\Pi_4=\frac{d^3P}{(2\pi)^32E}\frac{d^3p_4}{(2\pi)^32E_{p_4}}\frac{d^3p_5}{(2\pi)^32E_{p_5}}\frac{d^3p_6}{(2\pi)^32E_{p_6}},\tag{8}$$

where ς_i $(i=1,2,3)$, s_1 and s_2 are the spins of the antiquarks $\bar{Q}_i$, e^+ and e^-, respectively.

To do the numerical calculation, the parameters are taken as follows: $m_Z=91.18$ GeV, $\Gamma_Z=2.49$ GeV, $\alpha(m_Z)=1/127.95$, $\sin^2\theta_w=0.224$, $m_c=1.5$ GeV, $|\Psi_{\Omega_{ccc}}(0,0)|^2=0.36\times10^{-3}$ GeV^6 as in [61], and $m_b=4.9$ GeV, $|\Psi_{\Omega_{bbb}}(0,0)|^2=0.189$ GeV^6 as in [57].

For the electromagnetic coupling constant, we adopt

$$\alpha(q)=\frac{\alpha(m_Z)}{1-\frac{2\alpha(m_Z)}{3\pi}\log(\frac{q}{m_Z})},\tag{9}$$

where, q denotes the energy scale of the electromagnetic coupling constant, and we take $q=\frac{\sqrt{s}}{2}$ with $\sqrt{s}$ being the colliding energy in the center-of-mass frame. And the strong coupling constant, we adopt

$$\begin{aligned}\alpha_s(\mu)&=\frac{\alpha_s(m_Z)}{1+\frac{b_0}{2\pi}\alpha_s(m_Z)\log(\frac{\mu}{m_Z})},\quad\text{with}\\b_0&=11-\frac{2}{3}n_f,\end{aligned}\tag{10}$$

where μ is the energy scale and $\alpha_s(m_Z)=0.118$. For the production of the Ω_{ccc}, n_f is taken to be 4 when $\mu\le 2m_b$ and 5 when μ is larger than $2m_b$. And for the production of the Ω_{bbb}, n_f is taken to be 5. For comparison, we take two

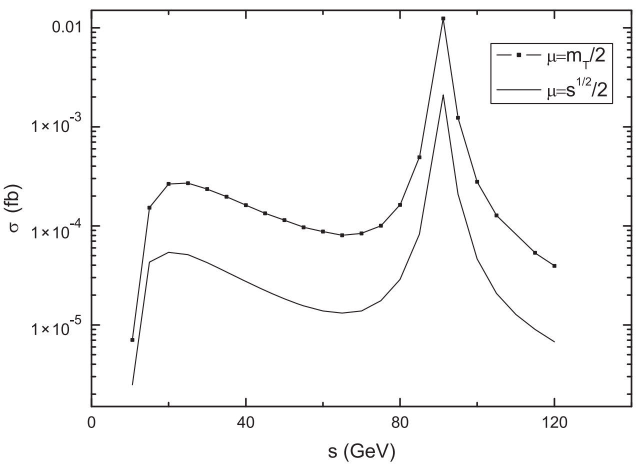

FIG. 2. The production cross section of the Ω_{ccc} produced by e^+e^- annihilation at different energies.

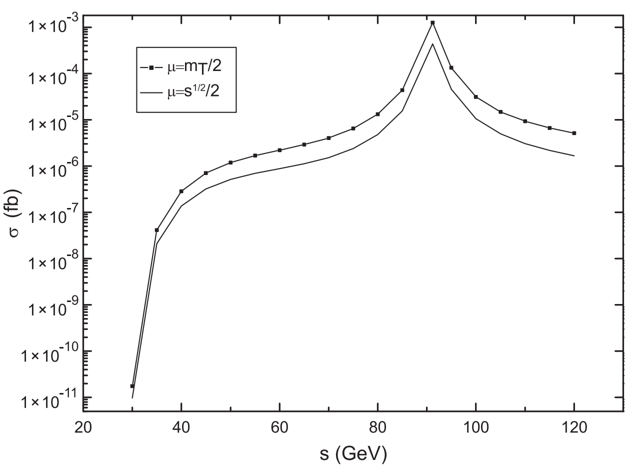

FIG. 3. The production cross section of the Ω_{bbb} produced by e^+e^- annihilation at different energies.

different values for μ, i.e., $\mu = \sqrt{s}/2$ and $\mu = m_T/2$, where m_T is the transverse mass of the produced baryon, namely $m_T^2 = p_T^2 + M^2$.[3] And in our numerical calculation, the integrals of the phase-pace are achieved by VEGAS [64].

SuperKEKB, the asymmetric beam energy e^-e^+ collider, is an upgrade of the KEKB accelerator facility. The target integrated luminosity of 50 ab^{-1} to be collected by the Belle II experiment. And a high luminosity e^+e^- collider named the Circular Electron-Positron Collider (CEPC) has been proposed by the Chinese particle physics community. The CEPC is designed to operate at three different modes, as a Higgs factory at $\sqrt{s} = 240$ GeV, as a Z factory at $\sqrt{s} = 91.2$ GeV and performing WW threshold scans around $\sqrt{s} = 160$ GeV [65]. We calculate the

[3]Here, p_T means the transverse momentum of the produced triply heavy baryon.

TABLE I. Production cross sections (in unit fb) of the Ω_{ccc} and Ω_{bbb} at the CEPC and Belle II.

	$\sqrt{s}$	μ	Ω_{ccc}	Ω_{bbb}
CEPC	91.2 GeV	$\sqrt{s}/2$	0.00204(5)	$0.437(6)\times 10^{-3}$
⋯	⋯	$m_T/2$	0.0124(3)	$1.25(2)\times 10^{-3}$
⋯	160 GeV	$\sqrt{s}/2$	$0.214(9)\times 10^{-5}$	$0.55(2)\times 10^{-6}$
⋯	⋯	$m_T/2$	$0.108(9)\times 10^{-4}$	$0.183(4)\times 10^{-5}$
Belle II	10.58 GeV	$\sqrt{s}/2$	$0.249(1)\times 10^{-5}$	
⋯	⋯	$m_T/2$	$0.707(3)\times 10^{-5}$	

production cross sections of the baryons Ω_{ccc} and Ω_{bbb} at CEPC with the energy $\sqrt{s} = 91.2$ GeV and $\sqrt{s} = 160$ GeV, and at Belle II with the center-of-mass energy $\sqrt{s} = 10.58$ GeV. The results are shown in Table I. And, we calculate the production of the Ω_{ccc} and Ω_{bbb} at the e^+e^- colliders with different colliding energies, which are shown in Figs. 2 and 3. From the Table I, Fig. 2 and Fig. 3, we see that the cross sections for the production of the Ω_{ccc} and Ω_{bbb} at $\sqrt{s} = 91.2$ GeV are larger than the corresponding ones at other colliding energies. This is because the cross sections contributed by the Z-boson exchange processes are proportional to a factor of $1/[(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2]$, which has a peak at the Z pole and decreases rapidly when the $\sqrt{s}$ deviates from m_Z, as pointed in Ref. [66].

In this paper, we also calculate the differential cross sections $d\sigma/dp_T$ of the production of the Ω_{ccc} and Ω_{bbb} at $\sqrt{s} = 91.2$ GeV, which are shown in Figs. 4 and 5.

As we see that both the integrated cross sections and differential cross sections are proportional to $|\Psi(0,0)|^2$, $\alpha^2(q)$ and $\alpha_s^4(\mu)$. So the numerical results can be changed by one even two orders using different the wave function at the origin of the triply heavy baryons, different running coupling constants and different energy scale choices. We can get the conclusion from the numerical results shown in Figs. 2 and 3, that the production cross sections of the Ω_{ccc}

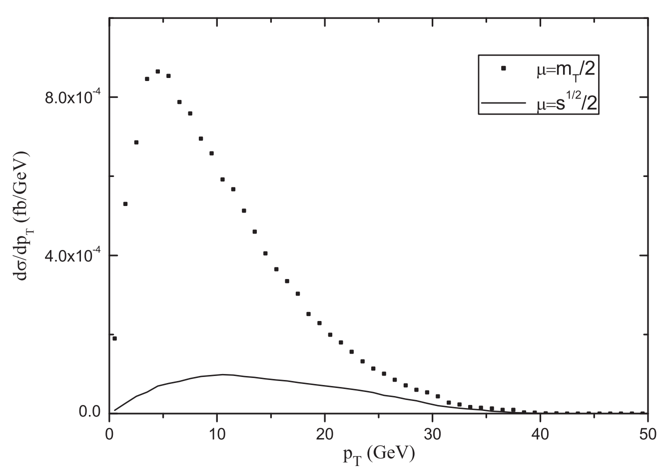

FIG. 4. The p_T distributions of the Ω_{ccc} produced at CEPC with $\sqrt{s} = 91.2$ GeV.

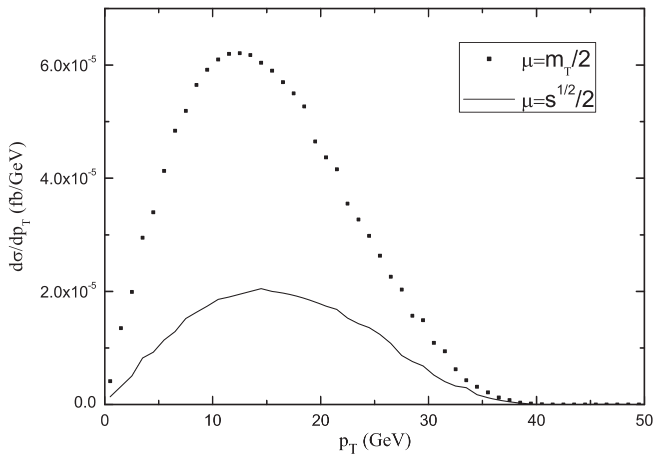

FIG. 5. The p_T distributions of the Ω_{bbb} produced at CEPC with $\sqrt{s} = 91.2$ GeV.

and Ω_{bbb} take their maximum values at $\sqrt{s} = m_Z$. From the numerical results in Table I, we know that it is impossible to observe Ω_{ccc} at SuperKEKB, if there is no other models to produce them at the e^+e^- colliders. Also, we see there are around 32–198 events for the production of Ω_{ccc} and 7–20 events for the production of Ω_{bbb}, at CEPC with 16 ab^{-1} integrated luminosity running at 91.2 GeV.

In this work, we also calculate the production cross section of the Ω_{ccc} using the same parameters as in Ref. [61]. And the numerical result is $0.20(2) \times 10^{-2}$ fb, which is about an order of magnitude smaller than the one 0.0404(4) fb in Ref. [61]. This huge difference may can be explained by the approximation adopted in Ref. [61].

IV. SUMMARY

We have studied the production of the Ω_{ccc} and Ω_{bbb} at the SuperKEKB and the CEPC. From the numerical results, we conclude that it is impossible to find the triply heavy baryons at SuperKEKB, and it is hard to find Ω_{ccc} and Ω_{bbb} at the CEPC because of the few events.

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