Toward minimal composite Higgs models from regular geometries in bottom-up holography

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We study a bottom-up, holographic description of a field theory yielding the spontaneous breaking of an approximate $SO(5)$ global symmetry to its $SO(4)$ subgroup. The weakly coupled, six-dimensional gravity dual has regular geometry. One of the dimensions is compactified on a circle that shrinks smoothly to zero size at a finite value of the holographic direction, hence introducing a physical scale in a way that mimics the effect of confinement in the dual four-dimensional field theory. We study the spectrum of small fluctuations of the bulk fields carrying $SO(5)$ quantum numbers, which can be interpreted as spin-0 and spin-1 bound states in the dual field theory. This work supplements an earlier publication focused only on the $SO(5)$ singlet states. We explore the parameter space of the theory, paying particular attention to composite states that have the right quantum numbers to be identified as pseudo-Nambu-Goldstone bosons (PNGBs). We find that in this model the PNGBs are generally heavy, with masses of the same order as other bound states, indicating the presence of a sizeable amount of explicit symmetry breaking in the field-theory side. Nevertheless, we also find a qualitatively new, unexpected result. When the dimension of the field-theory operator inducing $SO(5)$ breaking is close to half the space-time dimensionality, there exists a region of parameter space in which the PNGBs and the lightest scalar are both parametrically light in comparison to all other bound states of the field theory. Although this region is known to yield metastable classical backgrounds, this finding might be relevant to model building in the composite Higgs context.

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I. INTRODUCTION

It has been just over ten years since the discovery of the Higgs boson [1,2], and in the intervening time composite Higgs models (CHMs) [3–5], in which the Higgs fields originate as composite, pseudo-Nambu-Goldstone bosons (PNGBs) in a more fundamental theory have gained much attention in the literature; see Refs. [6–8] and the summary tables in Refs. [9–11]. Phenomenological and model-building studies of realizations of this idea [12–58] have been complemented by growing literature of dedicated lattice calculations that analyze strongly coupled field theories providing, at least partial, short-distance completions [59–78]. But providing a compelling microscopic origin for CHMs with minimal $SO(5)/SO(4)$ coset is nontrivial; see, for instance, Ref. [79].

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Within string theory and supergravity, it has been discovered that gauge-gravity dualities, or holography [80–83], provide an alternative way to study special field theories in their nonperturbative regime. Applications include the holographic description of confinement [84–87], the study of the composite (glueball) mass spectra [88–99], chiral symmetry breaking [100,101], and masses of mesons [102–106]. But it is very challenging to embed within string theory and supergravity fully realistic dynamical models yielding the low-energy theories relevant to CHMs. A few steps toward a top-down construction for CHMs with the $SO(5)/SO(4)$ coset have been taken recently [107]. A more general, pragmatic, bottom-up approach to holography exists, in which the gravity dual is constructed classically on the basis of *ad hoc* simplifying assumptions. Indeed, much work on the minimal $SO(5)/SO(4)$ coset has been developed in this context and makes it a quite compelling scenario [108–115]. Other CHMs suitable for lattice explorations have been the subject of recent bottom-up holographic studies [116–119].

In this paper, we outline a new bottom-up holographic realization of the $SO(5)/SO(4)$ paradigm needed for minimal CHMs. The model is an extended version of the very simple one that was studied in Ref. [120], admitting the same class of background geometries. While Ref. [120] focuses on identifying regions of parameter space relevant to understanding the physics of the dilaton, the approximate Goldstone boson associated with scale invariance, along the programmatic lines developed in Refs. [121–126],[1] in this publication, and in future ones [131,132], we are interested in investigating whether the PNGB states associated with the $SO(5)/SO(4)$ coset have model-building potential in the CHM context.

Let us first summarize the salient features of the model, borrowing results from Ref. [120]. A single scalar field is coupled to gravity in six dimensions, and its dynamics is governed by a polynomial potential [133–139]. A free parameter in the potential determines the dimensionality of the operator (deformation) in the dual five-dimensional field-theory interpretation of the gravity model. Furthermore, one of the spacelike dimensions is a circle, its size shrinking along the holographic direction, which introduces a smooth end of space in the regular geometry. As proposed in Ref. [84], in the dual field-theory interpretation this amounts to introducing a mass gap, in a way that mimics what happens for confining theories. Notice that this is the main difference with respect to Refs. [108–115], namely, the existence of a lift to a completely smooth geometry in six dimensions, which imposes constraints on the bulk profiles of the scalar fields of the five-dimensional gravity theory, obtained after reduction on the circle.

It has been shown in Ref. [120] that a phase transition occurs in the theory. This result was obtained by applying holographic renormalization [140–142] to compute the free energy. Whether the regular backgrounds are stable depends on the magnitude of the field-theory deformation. As explained in the body of the paper, the size of such a deformation, and its associated condensate, are both extracted from the profile of the bulk fields, but are not independent of one another, as they are constrained by the aforementioned regularity conditions.

The calculation of the spectrum of fluctuations of the bulk fields has a natural holographic interpretation in terms of bound states of the dual theory. Reference [120] reports this spectrum for states that carry no $SO(4)$ quantum numbers, computed by exploiting the powerful algorithmic process developed in Refs. [91–98,143,144]. Depending on the region of parameter space of interest, the lightest scalar fluctuation is found to be either a tachyon, or a generic, massive spin-0 state, or a parametrically light dilaton only in rather special cases. In particular, the light dilaton emerges only in regions of parameter space for which the regular backgrounds are metastable, while, in the region of parameter space in which the regular geometry is stable, the lightest scalar may show some suppression of its mass, but this is quantitatively a modest effect and never a parametric one. In the body of this paper, we provide some technical details that are necessary to the exposition, in the interest of making the presentation self-contained, and of fixing the notation, while referring instead to Ref. [120] for extensive details and numerical results.

In this paper, we replace the singlet scalar with an $SO(5)$ vector multiplet, gauge the $SO(5)$ symmetry in the six-dimensional gravity geometry compactified on a circle, adopt the R_ξ gauge as in Ref. [143], and compute the mass spectrum of new states carrying $SO(4)$ quantum numbers. In doing so, we make essential use of the fact that we identify the single scalar field of Ref. [120] with the absolute value of the $SO(5)$ multiplet field, in such a way that the latter obeys the same equations of motion as the former, and hence we consider identical classical background solutions. The approximate, global symmetry-breaking pattern $SO(5) \to SO(4)$ emerges in the dual field theory. Despite our interest in CHMs, here we describe the theory in isolation, and we do not couple it to external, weakly coupled, elementary fields, deferring the actual construction of CHMs to future publications [131,132]. While in the stable region of parameter space, none of the composite states can be made parametrically light, interestingly we find that there exists a metastable region in which the spectrum contains parametrically light PNGBs accompanied by a light pseudo-dilaton. This is suggestive, as it indicates the need to include a dilaton in the low-energy effective theory [145–147]. Even taken in isolation, the emergence of a dilaton has striking, potential phenomenological implications, and it is the subject of vast literature; see, for example, Refs. [148–159] and references therein. If one extends the chiral Lagrangian to the dilaton effective field theory [160–173], then the dilaton field might have an important role to play also in the construction of a viable CHM; see, for instance, Refs. [53,56].

The paper is organized as follows. We present the model in Sec. II, and describe the classical solutions of interest in Sec. III, borrowing relevant results from Ref. [120], but dispensing with repeating technical details and intermediate results. We then compute the mass spectrum of the fluctuations of the system, focusing on the states carrying $SO(4)$ quantum numbers. We compare the results to those for the singlets [120] by exploring the three-dimensional parameter space. We summarize the main results and outline future research directions in Sec. V. We relegate to the appendices the technical details that are useful to reproduce our main original results.

[1]The common theme to this sequence of papers is that a light dilaton might emerge in the spectrum of strongly coupled models if the dynamics brings them in proximity of (tachyonic) instabilities, the simplest holographic realization of which is related to the Breitenlohner-Freedman unitarity bound [127]. See also the critical discussion in Ref. [128] that proposes a bottom-up model in which the dilaton is not parametrically light, as well as the models of the earlier Refs. [129,130] that do not yield a light dilaton at all.

II. THE MODEL

In this section, we provide the weakly coupled gravity description of the model we want to analyze, which is closely related to the one studied in Ref. [120]. The two-derivative bulk action describes gravity in $D = 6$ dimensions, coupled to a real scalar field $\mathcal{X}$ transforming in the fundamental representation of a gauged $SO(5)$ symmetry. We add two boundaries in the radial direction at $\rho = \rho_1$ and $\rho = \rho_2$, respectively, and hence the action includes appropriate boundary-localized terms. The boundaries have the only purpose of acting as regulators: Physical results can be recovered by extrapolating to the limit in which the boundaries are removed.

The bulk, gauged $SO(5)$ is broken to $SO(4)$ by a nontrivial vacuum expectation value (VEV) of the combination $\phi \equiv \sqrt{\mathcal{X}^T\mathcal{X}}$; the field ϕ can be identified with the one appearing in Ref. [120]. The (putative) dual four-dimensional field theory has a global $SO(5)$ symmetry inherited from the bulk $SO(5)$. Its breaking is generically interpreted as an admixture of spontaneous and explicit breaking effects, due to the coupling and VEV of the operator dual to the bulk field ϕ. In the treatment of the bulk theory, we adopt the R_ξ gauge, for which purpose we follow the procedure (and notation) in Ref. [143], which requires us to add both bulk and boundary terms, but we do not report them in this section.

A. The six-dimensional action

We first write the model in $D = 6$ dimensions. The field content consists of gravity, scalar fields $\mathcal{X}_\alpha$ transforming in the 5 of the gauge group $SO(5)$, and $SO(5)$ gauge fields $\mathcal{A}_{\hat{M}\alpha}{}^{\beta}$. The six-dimensional space-time indexes are denoted by $\hat{M} = 0, 1, 2, 3, 5, 6$, while the components of the fundamental representation of $SO(5)$ are denoted by greek indexes $\alpha = 1, \ldots, 5$. The generators t^A $(A = 1, \ldots, 10)$ of $SO(5)$ are normalized so that $\mathrm{Tr}(t^A t^B) = \frac{1}{2}\delta^{AB}$. The action is

$$\mathcal{S}_6 = \mathcal{S}_6^{(\mathrm{bulk})} + \sum_{i=1,2} \mathcal{S}_{5,i}, \tag{1}$$

$$\mathcal{S}_6^{(\mathrm{bulk})} = \int \mathrm{d}^6 x \sqrt{-\hat{g}_6}\Bigg\{\frac{\mathcal{R}_6}{4} - \frac{1}{2}\hat{g}^{\hat{M}\hat{N}}(D_{\hat{M}}\mathcal{X})^T D_{\hat{N}}\mathcal{X} - \mathcal{V}_6(\mathcal{X}) - \frac{1}{2}\mathrm{Tr}\left[\hat{g}^{\hat{M}\hat{P}}\hat{g}^{\hat{N}\hat{Q}}\mathcal{F}_{\hat{M}\hat{N}}\mathcal{F}_{\hat{P}\hat{Q}}\right]\Bigg\}, \tag{2}$$

$$\mathcal{S}_{5,i} = (-)^i \int \mathrm{d}^5 x \sqrt{-\tilde{\hat{g}}}\left\{\frac{\mathcal{K}}{2} + \lambda_i(\mathcal{X}) + f_i(\tilde{\hat{g}}_{\hat{M}\hat{N}})\right\}\Bigg|_{\rho=\rho_i}, \tag{3}$$

where the bulk part is $\mathcal{S}_6^{(\mathrm{bulk})}$, and the two boundary actions $\mathcal{S}_{5,i}$, with $i = 1, 2$, are localized at the values $\rho = \rho_{1,2}$ of the radial coordinate. Our conventions are such that the six-dimensional metric $\hat{g}_{\hat{M}\hat{N}}$ has determinant $\hat{g}_6$ and signature mostly $+$. The six-dimensional Ricci scalar is $\mathcal{R}_6$. The induced metric on the boundaries is denoted as $\tilde{\hat{g}}_{\hat{M}\hat{N}}$, the extrinsic curvature is $\mathcal{K}$, and it appears in the Gibbons-Hawking-York term of the boundary actions. The terms denoted with f_i depend explicitly on the induced metric on the boundary, as in Ref. [120].

The covariant derivatives are defined as follows:

$$(D_{\hat{M}}\mathcal{X})_\alpha \equiv \partial_{\hat{M}}\mathcal{X}_\alpha + ig\mathcal{A}_{\hat{M}\alpha}{}^{\beta}\mathcal{X}_\beta, \tag{4}$$

and the field-strength tensors are

$$\mathcal{F}_{\hat{M}\hat{N}\alpha}{}^{\beta} \equiv 2\Big(\partial_{[\hat{M}}\mathcal{A}_{\hat{N}]\alpha}{}^{\beta} + ig\mathcal{A}_{[\hat{M}\alpha}{}^{\gamma}\mathcal{A}_{\hat{N}]\gamma}{}^{\beta}\Big), \tag{5}$$

where antisymmetrization is defined as $[n_1 n_2] \equiv \frac{1}{2}(n_1 n_2 - n_2 n_1)$. The coupling g is a free parameter.

Both the boundary potentials $\lambda_i(\mathcal{X})$, as well as the bulk scalar potential $\mathcal{V}_6(\mathcal{X})$, are taken to be $SO(5)$ invariant, and hence, functions of the single variable $\phi \equiv \sqrt{\mathcal{X}^T\mathcal{X}}$. We adopt the explicit form of $\mathcal{V}_6(\phi)$ following Ref. [120] by expressing it in terms of a superpotential $\mathcal{W}_6$ that satisfies the relation

$$\mathcal{V}_6 = \frac{1}{2}\sum_\alpha \left(\frac{\partial \mathcal{W}_6}{\partial \mathcal{X}_\alpha}\right)^2 - \frac{5}{4}\mathcal{W}_6^2, \tag{6}$$

where the superpotential is given by

$$\mathcal{W}_6 \equiv -2 - \frac{\Delta}{2}\mathcal{X}^T\mathcal{X} = -2 - \frac{\Delta}{2}\phi^2, \tag{7}$$

and hence, one finds that

$$\mathcal{V}_6 = -5 - \frac{\Delta(5-\Delta)}{2}\phi^2 - \frac{5\Delta^2}{16}\phi^4. \tag{8}$$

We retain this elegant formulation only for convenience, even though the model itself is not supersymmetric (there are no fermionic fields), nor do the backgrounds discussed in this paper originate from solving first-order equations derivable from the superpotential $\mathcal{W}_6$.

B. Dimensional reduction

One of the dimensions is a circle parametrized by the angular variable $0 \leq \eta < 2\pi$. We adopt the (soliton) ansatz

$$\mathrm{d}s_6^2 = e^{-2\chi}\mathrm{d}x_5^2 + e^{6\chi}(\mathrm{d}\eta + \chi_M \mathrm{d}x^M)^2, \tag{9}$$

where the space-time index $M = 0, 1, 2, 3, 5$. The five-dimensional metric has the domain-wall form

$$\mathrm{d}s_5^2 = \mathrm{d}r^2 + e^{2A(r)}\mathrm{d}x_{1,3}^2 = e^{2\chi(\rho)}\mathrm{d}\rho^2 + e^{2A(\rho)}\mathrm{d}x_{1,3}^2, \tag{10}$$

and we dimensionally reduce the theory, so that the reduced action is then

$$\mathcal{S}_5 = \mathcal{S}_5^{(\mathrm{bulk})} + \sum_{i=1,2}\mathcal{S}_{4,i}, \tag{11}$$

$$\begin{aligned}\mathcal{S}_5^{(\mathrm{bulk})} = \int \mathrm{d}^5x\sqrt{-g_5}\Bigg\{&\frac{R}{4}-\frac{1}{2}g^{MN}\left[6\partial_M\chi\partial_N\chi+\sum_{\alpha=1}^{5}(D_M\mathcal{X})_\alpha(D_N\mathcal{X})_\alpha+e^{-6\chi}\sum_{A=1}^{10}(D_M\mathcal{A}_6)^A(D_N\mathcal{A}_6)^A\right]\\ &-e^{-2\chi}\mathcal{V}_6-\frac{1}{2}g^2e^{-8\chi}\mathcal{X}^T\mathcal{A}_6^2\mathcal{X}-\frac{1}{16}e^{8\chi}g^{MP}g^{NQ}F^{(\chi)}_{MN}F^{(\chi)}_{PQ}-\frac{1}{2}e^{2\chi}\mathrm{Tr}[g^{MP}g^{NQ}\mathcal{F}_{MN}\mathcal{F}_{PQ}]\\ &-g^{MN}(ig)\chi_M\mathcal{X}^T\mathcal{A}_6D_N\mathcal{X}-2e^{2\chi}g^{MN}g^{OP}\chi_M\mathrm{Tr}(\mathcal{F}_{NO}D_P\mathcal{A}_6)\\ &-\frac{1}{2}g^2g^{MN}\chi_M\chi_N\mathcal{X}^T\mathcal{A}_6^2\mathcal{X}+e^{2\chi}g^{MP}g^{NQ}\chi_M\chi_N\mathrm{Tr}(D_P\mathcal{A}_6D_Q\mathcal{A}_6)-e^{2\chi}g^{MN}g^{PQ}\chi_M\chi_N\mathrm{Tr}(D_P\mathcal{A}_6D_Q\mathcal{A}_6)\Bigg\}, \end{aligned} \tag{12}$$

$$\mathcal{S}_{4,i} = (-)^i\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left\{\frac{K}{2}+e^{-\chi}\lambda_i(\mathcal{X})+e^{-\chi}f_i(\chi)\right\}\Bigg|_{\rho=\rho_i}. \tag{13}$$

The five-dimensional metric g_{MN} has determinant g_5, the induced metric on the boundaries is $\tilde{g}_{MN}$, the five-dimensional Ricci scalar is R, and K is the extrinsic curvature. The field strength for the vector χ_M is given by $F^{(\chi)}_{MN}\equiv\partial_M\chi_N-\partial_N\chi_M$. We define $\mathcal{A}_6\equiv\mathcal{A}_6^At^A$, where $\mathcal{A}_6^A$ is a scalar that transforms in the adjoint representation of $SO(5)$ and originates from the sixth component of the gauge field in six dimensions.

We are interested in background solutions in which $\mathcal{A}_6=0$, $\mathcal{A}_M=0$, $\chi_M=0$, while the metric and the scalars $\mathcal{X}_\alpha$ and χ depend on the radial coordinate ρ only. The background fields satisfy the equations of motion

$$\partial_\rho^2\mathcal{X}_\alpha+(4\partial_\rho A-\partial_\rho\chi)\partial_\rho\mathcal{X}_\alpha=\frac{\partial\mathcal{V}_6}{\partial\mathcal{X}_\alpha}, \tag{14}$$

$$\partial_\rho^2\chi+(4\partial_\rho A-\partial_\rho\chi)\partial_\rho\chi=-\frac{\mathcal{V}_6}{3}, \tag{15}$$

$$3(\partial_\rho A)^2-\frac{1}{2}\partial_\rho\mathcal{X}_\alpha\partial_\rho\mathcal{X}_\alpha-3(\partial_\rho\chi)^2=-\mathcal{V}_6, \tag{16}$$

with boundary conditions given by

$$\left(\partial_\rho\mathcal{X}_\alpha-\frac{\partial\lambda_i}{\partial\mathcal{X}_\alpha}\right)\Bigg|_{\rho_i}=0,\quad \left(6\partial_\rho\chi+\lambda_i+f_i-\frac{\partial f_i}{\partial\chi}\right)\Bigg|_{\rho_i}=0,\quad \left(\frac{3}{2}\partial_\rho A+\lambda_i+f_i\right)\Bigg|_{\rho_i}=0. \tag{17}$$

For vanishing $f_i=0$, one obtains solutions that lift to domain walls in $D=6$ dimensions, for which

$$\mathcal{A}=A-\chi=3\chi. \tag{18}$$

The solutions which we will be interested in break the $SO(5)$ symmetry to $SO(4)$ due to a nontrivial background profile of $\phi(\rho)\neq0$. It is hence convenient to decompose $\mathcal{X}_\alpha$ as $5=1\oplus4$ in terms of irreducible representations of $SO(4)$, which we denote as ϕ and $\pi^{\hat{A}}$, respectively. We use the parametrization

$$\mathcal{X}=\exp\left[2i\pi^{\hat{A}}t^{\hat{A}}\right]\phi\mathcal{X}_0,\qquad \mathcal{X}_0=(0,0,0,0,1)^T, \tag{19}$$

and adopt the decomposition

$$\mathcal{A}_{\hat{M}\alpha}{}^{\beta}=\mathcal{A}_{\hat{M}}{}^{\bar{A}}(t^{\bar{A}})_\alpha{}^\beta+\mathcal{A}_{\hat{M}}{}^{\hat{A}}(t^{\hat{A}})_\alpha{}^\beta, \tag{20}$$

where $t^{\hat{A}}$ $(\hat{A}=1,\ldots,4)$ and $t^{\bar{A}}$ $(\bar{A}=5,\ldots,10)$ are, respectively, the broken and unbroken generators of $SO(5)$ with respect to $\mathcal{X}_0$. An example of such a basis of generators is given in Appendix A. The generators obey the normalization conditions $\mathrm{Tr}(t^{\bar{A}}t^{\hat{B}})=0$, $\mathrm{Tr}(t^{\hat{A}}t^{\hat{B}})=\frac{1}{2}\delta^{\hat{A}\hat{B}}$, and $\mathrm{Tr}(t^{\bar{A}}t^{\bar{B}})=\frac{1}{2}\delta^{\bar{A}\bar{B}}$.

As the boundary potentials $\lambda_i(\phi)$ are $SO(5)$ invariant, the boundary conditions for $\mathcal{X}_\alpha$ given in Eq. (17) become

$$0=\left(\left[\partial_\rho\phi-\frac{\partial\lambda_i}{\partial\phi}\right]\frac{\mathcal{X}_\alpha}{\phi}+2i\partial_\rho\pi^{\hat{A}}(t^{\hat{A}})_\alpha{}^\beta\mathcal{X}_\beta\right)\Bigg|_{\rho_i}, \tag{21}$$

which are solved by imposing

$$\partial_\rho\phi|_{\rho_i}=\frac{\partial\lambda_i}{\partial\phi}\Bigg|_{\rho_i},\qquad \partial_\rho\pi^{\hat{A}}|_{\rho_i}=0. \tag{22}$$

These boundary conditions select background solutions in which, without loss of generality, we choose $\pi^{\hat{A}}=0$. Hence, the only background functions that are nonzero are A, ϕ, and χ.

C. Truncation to quadratic order

As ϕ, A, and χ are the only functions that are nontrivial in the background, it is convenient to simplify the reduced action further by power expanding the other scalar and gauge fields and truncating the expansion at the quadratic order. The resulting action admits the same classical solutions and still contains enough information to compute the linearized equations of motion for the small fluctuations of all the fields.

By treating the remaining degrees of freedom (other than ϕ, χ, and g_{MN}) as perturbations, at quadratic order the five-dimensional action can then be written as

$$\mathcal{S}_5^{(2)} = \mathcal{S}_5^{(\mathrm{bulk},2)} + \sum_{i=1,2} \mathcal{S}_{4,i}, \tag{23}$$

where the bulk action is

$$\begin{aligned}\mathcal{S}_5^{(\mathrm{bulk},2)} = \int \mathrm{d}^5x\sqrt{-g_5}\Big\{&\frac{R}{4} - \frac{1}{2}g^{MN}G_{ab}\partial_M\Phi^a\partial_N\Phi^b - \mathcal{V}_5(\Phi^a)\\ &-\frac{1}{2}g^{MN}G^{(0)}_{ab}\partial_M\Phi^{(0)a}\partial_N\Phi^{(0)b} - \frac{1}{2}m^{(0)2}_{ab}\Phi^{(0)a}\Phi^{(0)b}\\ &-\frac{1}{2}g^{MN}G^{(1)}_{AB}\mathcal{H}^{(1)\,A}_M\mathcal{H}^{(1)\,B}_N - \frac{1}{4}g^{MO}g^{NP}H^{(1)}_{AB}F_{MN}{}^AF_{OP}{}^B\Big\},\end{aligned} \tag{24}$$

and the boundary actions are

$$\mathcal{S}_{4,i} = (-)^i\int \mathrm{d}^4x\sqrt{-\tilde{g}}\Big\{\frac{K}{2} + e^{-\chi}\lambda_i(\phi) + e^{-\chi}f_i(\chi)\Big\}\Big|_{\rho=\rho_i}. \tag{25}$$

The sigma-model metric for the active scalars $\Phi^a = \{\phi,\chi\}$ is $G_{ab} = \mathrm{diag}(1,6)$, and the potential is $\mathcal{V}_5(\phi,\chi) = e^{-2\chi}\mathcal{V}_6(\phi)$. The scalars $\Phi^{(0)a} = \{\mathcal{A}_6^{\bar{A}},\mathcal{A}_6^{\hat{A}}\}$ have sigma-model metric

$$G^{(0)} = \left(\begin{array}{c|c} e^{-6\chi}\mathbb{1}_{6\times6} & \\ \hline & e^{-6\chi}\mathbb{1}_{4\times4}\end{array}\right) \tag{26}$$

and mass matrix

$$\frac{m^{(0)2}}{g^2} = \left(\begin{array}{c|c} \mathbb{0}_{6\times6} & \\ \hline & \frac{1}{4}\phi^2e^{-8\chi}\mathbb{1}_{4\times4}\end{array}\right). \tag{27}$$

The 1-forms $V_M{}^A = \{\chi_M,\mathcal{A}_M{}^{\bar{A}},\mathcal{A}_M{}^{\hat{A}}\}$ have field strengths $F_{MN}{}^A = 2\partial_{[M}V_{N]}{}^A$, and

$$H^{(1)} = \left(\begin{array}{c|c|c} \frac{1}{4}e^{8\chi} & & \\ \hline & e^{2\chi}\mathbb{1}_{6\times6} & \\ \hline & & e^{2\chi}\mathbb{1}_{4\times4}\end{array}\right), \tag{28}$$

while the gauge-invariant combinations of derivatives of the pseudo-scalars and 1-forms given by $\mathcal{H}^{(1)\,A}_M = \{0,0,\partial_M\pi^{\hat{A}} + \frac{g}{2}\mathcal{A}_M{}^{\hat{A}}\}$ have

$$G^{(1)} = \left(\begin{array}{c|c|c} 0 & & \\ \hline & \mathbb{0}_{6\times6} & \\ \hline & & \phi^2\mathbb{1}_{4\times4}\end{array}\right). \tag{29}$$

III. BACKGROUND SOLUTIONS

All calculations presented in this paper make use of regular background solutions in which the size of the circle shrinks to zero size. We refer to such solutions as confining, with abuse of language, and borrow their characterization from Ref. [120], to which we refer the reader for technical details and expanded discussions. The space of the solutions of interest depends on two parameters. The parameter Δ is related to the dimension of the deforming parameter, or operator, in the five-dimensional theory. An additional parameter ϕ_I controls the behavior of the active scalars in proximity of the end of space and ultimately controls the size of the deformation.

The solutions of interest are not known in closed form, but only numerically, and can be obtained starting from the (IR) expansion of the background functions [120]. Assuming the space ends at some value ρ_o of the radial direction, we can write the regular solutions as a power expansion in the small difference $(\rho-\rho_o)$:

$$\begin{aligned}\phi(\rho) = \phi_I - \frac{1}{16}\Delta\phi_I(20+\Delta(5\phi_I^2-4))(\rho-\rho_o)^2\\ + \mathcal{O}((\rho-\rho_o)^4),\end{aligned} \tag{30}$$

$$\begin{aligned}\chi(\rho) = \chi_I + \frac{1}{3}\log(\rho-\rho_o) + \frac{1}{288}(-80+8(\Delta-5)\\ \times\Delta\phi_I^2 - 5\Delta^2\phi_I^4)(\rho-\rho_o)^2 + O((\rho-\rho_o)^4),\end{aligned} \tag{31}$$

$$\begin{aligned}A(\rho) = A_I + \frac{1}{3}\log(\rho-\rho_o) + \frac{7}{576}(80+\Delta\phi_I^2(40\\ + \Delta(5\phi_I^2-8)))(\rho-\rho_o)^2 + \mathcal{O}((\rho-\rho_o)^4),\end{aligned} \tag{32}$$

where χ_I, A_I are additional integration constants, besides the aforementioned ρ_o and ϕ_I. In order to avoid a conical singularity in the plane described by ρ and η, we set $\chi_I = 0$, and it is shown explicitly in Ref. [120] that the curvature invariants up to quadratic order (in six dimensions) are finite for these solutions.

For asymptotically large values of the radial coordinate ρ, all the backgrounds of interest approach the geometry of AdS_6. One can hence expand the functional form of the background functions in powers of the small parameter $z \equiv e^{-\rho}$. The detailed form of such UV expansions depends nontrivially on the value of Δ, and a (incomplete) catalog of examples can be found in the Appendix of Ref. [120], which we do not reproduce here. We denote the five integration constants as ϕ_J, ϕ_V, χ_U, χ_5, and A_U. They appear in the background functions in the following general way:

$$\phi(z) = \phi_J z^{\Delta_J} + \cdots + \phi_V z^{\Delta_V} + \cdots, \tag{33}$$

$$\chi(z) = \chi_U - \frac{1}{3}\log(z) + \cdots + (\chi_5 + \cdots)z^5 + \cdots, \tag{34}$$

$$A(z) = A_U - \frac{4}{3}\log(z) + \cdots. \tag{35}$$

In these expressions, A_U and χ_U can be set to zero without loss of generality by trivial redefinitions of the metric and coordinates in six dimensions, for example, following the procedure adopted in Ref. [124]. As we are interested only in computing the spectrum of fluctuations in units of the mass of the lightest tensorial glueball, our results are not affected by these two parameters, and we will not discuss them any further. In the expansions in Ref. [120], χ_5 is conventionally defined in such a way that if $\chi_5 = 0$, then $A = 4\chi$. Finally, the two parameters ϕ_J and ϕ_V appear in $\phi(z)$, as the coefficients of the z^{Δ_J} and z^{Δ_V} terms of the expansion.[2] We adopt the convention that $\Delta_J = \min(\Delta, 5 - \Delta)$ and $\Delta_V = 5 - \Delta_J$, hence, always interpreting Δ_V as the dimension of the operator in the dual field theory corresponding to ϕ, and Δ_J as the dimension of its coupling. We refer the reader to Ref. [120] for more details and for the calculation of the free energy for a number of choices of Δ and ϕ_I.

IV. MASS SPECTRUM OF FLUCTUATIONS

Reference [120] reports the spectrum of fluctuations of the $SO(5)$ singlets computed using the gauge-invariant formalism of Refs. [91–98], which allows one to resolve the mixing between fluctuations of the fields ϕ, χ, and the metric. The resulting variables are denoted, respectively, as $\mathfrak{a}^1$, $\mathfrak{a}^2$, and $\mathfrak{e}$. We denote as $\mathfrak{v}^1$ the fluctuations associated with χ_M. For the same background solutions, we now consider the $SO(5)$ multiplets $\mathcal{A}_6^{\bar{A}}$, $\mathcal{A}_6^{\hat{A}}$, $\mathcal{A}_M^{\bar{A}}$, $\mathcal{A}_M^{\hat{A}}$, and $\pi^{\hat{A}}$. None of these additional fields develop VEVs; hence, they do not mix with the components of the background metric. Yet, because of the presence of a bulk $SO(5)$ gauge symmetry, to compute the spectrum of their fluctuations we elect to introduce the R_ξ gauge and to identify gauge-invariant physical combinations, borrowing the formalism developed in Ref. [143] (see also Ref. [107]). The resulting gauge-invariant fluctuations are denoted as $\mathfrak{a}^3$, $\mathfrak{a}^4$, $\mathfrak{v}^2$, $\mathfrak{v}^3$, and $\mathfrak{p}$, respectively.

[2] The limiting case $\Delta = 5/2$ requires a generalization. The expansion in this case is written explicitly in Ref. [120].

We restrict the discussion to the $SO(5)$ multiplets, and the equations they obey, rather than repeating details that can be found in Ref. [120]. The equations of motion are the following:

$$0 = \left[\partial_\rho^2 + (4\partial_\rho A - 7\partial_\rho\chi)\partial_\rho - e^{2\chi-2A}q^2\right]\mathfrak{a}^3, \tag{36}$$

$$0 = \left[\partial_\rho^2 + (4\partial_\rho A - 7\partial_\rho\chi)\partial_\rho - \frac{g^2\phi^2}{4} - e^{2\chi-2A}q^2\right]\mathfrak{a}^4, \tag{37}$$

$$0 = \left[\partial_\rho^2 + (2\partial_\rho A + \partial_\rho\chi)\partial_\rho - e^{2\chi-2A}q^2\right]\mathfrak{v}^2, \tag{38}$$

$$0 = \left[\partial_\rho^2 + (2\partial_\rho A + \partial_\rho\chi)\partial_\rho - \frac{g^2\phi^2}{4} - e^{2\chi-2A}q^2\right]\mathfrak{v}^3, \tag{39}$$

$$0 = \left[\partial_\rho^2 - \left(2\partial_\rho A + \partial_\rho\chi + \frac{2\partial_\rho\phi}{\phi}\right)\partial_\rho - \frac{g^2\phi^2}{4} - e^{2\chi-2A}q^2\right]\mathfrak{p}, \tag{40}$$

where $q^2 \equiv \eta_{\mu\nu}q^\mu q^\nu$, and q^μ is the four-momentum.

We study numerically the solutions of these linearized fluctuations in the range $\rho_1 \leq \rho \leq \rho_2$, with $\rho_1 > \rho_o$. In principle, in order to recover the physical results, we should apply boundary conditions at $\rho = \rho_1$ and $\rho = \rho_2$, and then repeat the process by taking the $\rho_1 \to \rho_o$ and $\rho_2 \to +\infty$ limits separately. To be more specific, for the scalars $\mathfrak{a}^3$ and $\mathfrak{a}^4$, we impose Dirichlet boundary conditions at $\rho = \rho_1$ and $\rho = \rho_2$: $\mathfrak{a}^{3,4}|_{\rho_i} = 0$. For the vectors $\mathfrak{v}^2$ and $\mathfrak{v}^3$, we impose Neumann boundary conditions at $\rho = \rho_1$ and $\rho = \rho_2$: $\partial_\rho\mathfrak{v}^{2,3}|_{\rho_i} = 0$. Conversely, the pseudo-scalar $\mathfrak{p}$ obeys Dirichlet boundary conditions for $\rho = \rho_1$ and Neumann for $\rho = \rho_2$ [107,118]. In practice, in order to improve the numerical convergence of this process, we make use of the asymptotic expansions, both in the IR and UV, of the general solutions of the fluctuation equations; see an example in Appendix B. We impose upon them the aforementioned boundary conditions and require continuity of the functions and their derivatives with the expansions thus constrained. The system is overconstrained, yielding a discrete spectrum of values of $M^2 = -q^2$. We will discuss in a future publication how these conclusions are modified in the presence of nontrivial boundary terms [131], which nontrivially parametrize the effect of coupling the theory to external fields.

In order to discuss our new results and provide an interpretation for them, we must first pause and explain the physical meaning of the parameters ϕ_I and g. The former is the parameter controlling the size of $SO(5)$ symmetry-breaking effects. Interestingly, despite the fact that ϕ obeys a second-order, nonlinear differential equation, the requirement of regularity of the geometry at the end of the space imposes a nontrivial relation between the two free parameters that appear in a generic confining solution. For concreteness, we can think of them as the coefficients ϕ_J and ϕ_V appearing in the UV expansions related to the explicit and spontaneous breaking of the symmetry on the field-theory side. The numerical study in Ref. [120] demonstrates that for $0 < \Delta < 5$, there is a critical value of ϕ_I denoted $\phi_I(c)$, such that when $\phi_I > \phi_I(c)$ there exists an alternative classical solution that has a lower value of the free energy for the same value of the source. This result demonstrates the existence of a phase transition. We choose to display $\phi_I = \phi_I(c)$ in the plots because this choice minimizes the mass of the lightest spin-0 state.

The parameter g controls the self-coupling of the bulk gauge fields. It is related to the coupling (and decay) of the composite vector mesons to two PNGBs in the effective description of the dual field theory. But these statements require some more qualification, in view of the notational conventions we adopted. In the action $\mathcal{S}_6$, we ignored a multiplicative factor of $2/\kappa^2$. In this paper, we are only solving classical background equations and linearized equations for the fluctuations around the chosen background solutions. In this process, $2/\kappa^2$ is just an overall factor that disappears from the final results. These classical results are exact if one takes the limit $\kappa \to 0$ while holding fixed g and ϕ_I. Close to the classical regime, perturbative corrections can be organized in loop diagrams, provided the coupling g is not too strong. A naive estimate of the upper bound yields $\frac{3g^2\kappa^2}{256\pi^3} \ll 1$ [174].[3]

We show in Figs. 1 and 2 examples of mass spectra for selected choices of $\Delta < 5/2$ and $\Delta \geq 5/2$, respectively. Hence, for each representative choice of Δ, we produce one plot in which we fix $g = 5$ and vary ϕ_I, and a second plot in which we fix $\phi_I = \phi_I(c)$ and vary g. We show all the states of the system, differentiating them by color, and the shape of the markers [for different $SO(4)$ representations]. We also reproduce the results for the $SO(5)$ singlet for completeness of the presentation, but also to set up their physics implication. More examples of the numerical results are presented in Appendix C.

For any value of Δ, we find that the mass of the axial-vector states, transforming as 4 of $SO(4)$, is larger than that of the vectors, and the difference grows with g. Also, the mass of the lightest PNGBs grows with g. When varying ϕ_I for $\Delta \lesssim 2$ and fixed g, the mass of the spin-0 states transforming as 4 of $SO(4)$ grows with ϕ_I. In field-theory terms, in this regime we are enhancing the effect of explicit symmetry breaking compared to the spontaneous breaking, and there is no real sense in which these states are genuine PNGBs, despite having the right quantum numbers. But for $\Delta \geq 2.5$, we see that the mass of the lightest spin-0 states transforming as 4 of $SO(4)$ can be made arbitrarily light by choosing large values of ϕ_I. Unfortunately though, the critical values of ϕ_I are rather small, and such large choices fall into the tachyonic part of the spectrum. The general conclusion of this exercise is that for all choices of Δ and g, if we restrict attention to the stable region of parameter space with $\phi_I \leq \phi_I(c)$, then the mass of the PNGBs shows no indications of being suppressed.

Interestingly, we find something new when we focus our attention on the case where $2 \lesssim \Delta < 2.5$. Reference [120] found the existence of a metastable region of parameter space with large ϕ_I, in which the lightest scalar is a dilaton. Here, we find that also the PNGBs transforming as a 4 of $SO(4)$ are light in this region of parameter space, their masses being suppressed with respect to the scale of all other bound states. This can be seen in the bottom-left panel of Fig. 1.

To demonstrate that the mass of these two states can be dialed to be arbitrarily small, compared to the typical mass scale of all other bound states represented by the mass of the spin-2 particles, in Fig. 3, we display some more information about the choices $\Delta = 2.35$, $\Delta = 2.40$, and $\Delta = 2.45$. We show in the left panels of the figure the dependence on g of the spectrum, for a choice of $\phi_I = 3$, large enough to fall in the portion of parameter space that contains a light dilaton together with a light set of PNGBs transforming as the 4 of $SO(4)$.

While Ref. [120], for such large values of ϕ_I, found that the confining background solutions are metastable, we produce here three expanded and detailed plots showing that the free energy is almost degenerate with another branch of solutions. The plots on the right panels of Fig. 3 show the free energy $\hat{\mathcal{F}}$ computed using holographic renormalization, as a function of the source $\hat{\phi}_J$, and normalized appropriately. The plots are expanded versions of those in Ref. [120], and for these choices, are obtained with the following relations:

$$\mathcal{F} = -\frac{1}{40} e^{4A_U - \chi_U} \left(16\Delta\left(\frac{5}{2} - \Delta\right)\phi_J\phi_V - 75\chi_5\right), \tag{41}$$

$$\Lambda^{-1} \equiv \int_{\rho_o}^{\infty} \mathrm{d}\rho\, e^{\chi(\rho) - A(\rho)}, \tag{42}$$

and the rescaling $\hat{\mathcal{F}} \equiv \mathcal{F}/\Lambda^5$, $\hat{\phi}_J \equiv \phi_J/\Lambda^{\Delta}$. We do not repeat the details here, except for specifying that in the plots the choice $\phi_I = 3$ is equivalent to confining solutions with $\hat{\phi}_J = 6.78$ for $\Delta = 2.35$, $\hat{\phi}_J = 6.83$ for $\Delta = 2.40$, and $\hat{\phi}_J = 6.88$ for $\Delta = 2.45$. The reason why we show these

[3]The factor of 3 in this expression is the second Casimir of the adjoint $C_2(\mathrm{Adj}) = 3$ for $SO(5)$; it would be $C_2(\mathrm{Adj}) = N_c$ for $SU(N_c)$.

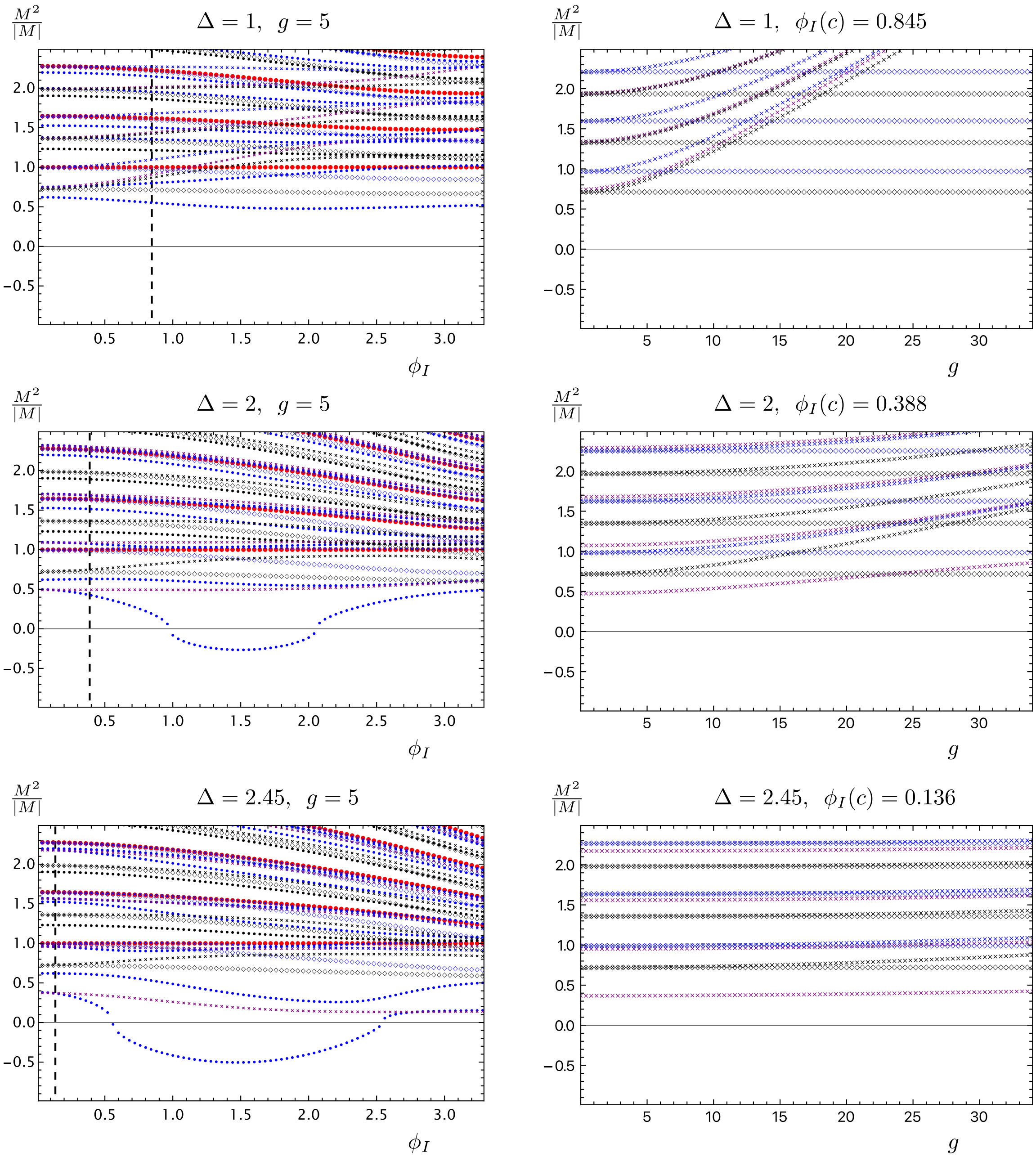

FIG. 1. Mass spectrum $\frac{M^2}{|M|}$ of fluctuations computed for confining backgrounds, with various choices of Δ, as a function of the IR parameter ϕ_I for $g = 5$ (left) and as a function of g for $\phi_I = \phi_I(c)$ (right). For each Δ, we show the spectrum of scalar (blue), pseudoscalar (purple), vector (black), and tensor (red) states. The values of the IR and UV cutoffs in the calculations are, respectively, given by $\rho_1 - \rho_o = 10^{-9}$ and $\rho_2 - \rho_o = 5$ in all of the cases. The different symbols refer to the quantum numbers with respect to the unbroken $SO(4)$ symmetry: Disks are used for singlets and have already been reported in Ref. [120]; diamonds represent the 6 of $SO(4)$, and crosses the 4 of $SO(4)$. All masses are normalized to the mass of the lightest spin-2 state. Because the masses of the $SO(5)$ singlets do not depend on g, we do not repeat them in the right panels, which display only nontrivial $SO(5)$ multiplets.

plots is that, besides the confining solutions, the analysis of the free energy carries over also for (singular) solutions respecting five-dimensional Poincaré invariance, and we can show a graphical comparison. In particular, one can explicitly see that for $\phi_I = 3$, in all three examples reported here, the confining solutions do not minimize the free energy. In the limit of large ϕ_I, the metastable solutions might be long-lived. Whether there are regions of parameter space that allow for the construction of a viable CHM relying on the existence of a long-lived metastable vacuum is an important question that would require a dedicated study.

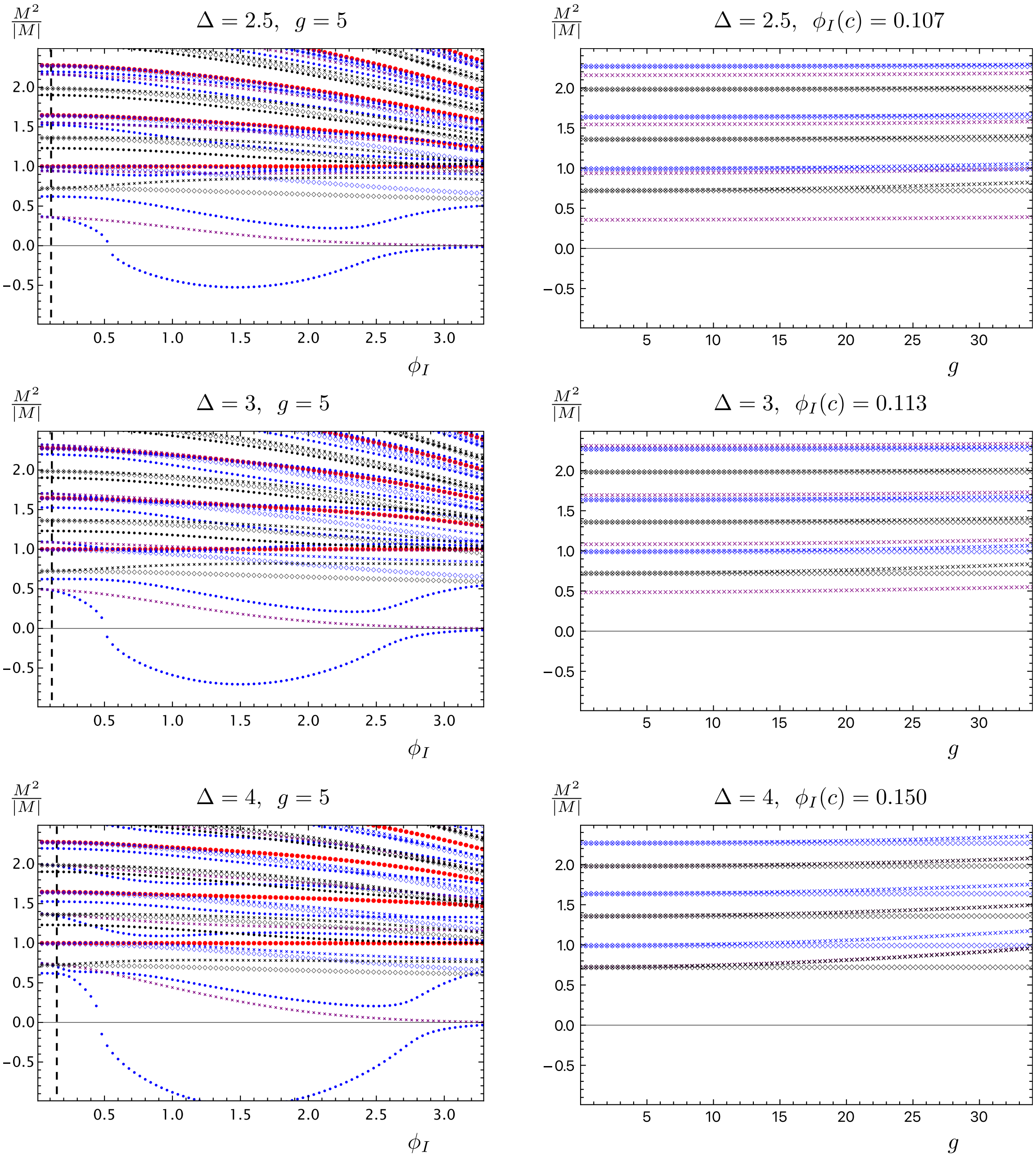

FIG. 2. Mass spectrum $\frac{M^2}{|M|}$ of fluctuations computed for confining backgrounds, with various choices of Δ, as a function of the IR parameter ϕ_I for $g = 5$ (left) and as a function of g for $\phi_I = \phi_I(c)$ (right). For each Δ, we show the spectrum of scalar (blue), pseudoscalar (purple), vector (black), and tensor (red) states. The values of the IR and UV cutoffs in the calculations are, respectively, given by $\rho_1 - \rho_o = 10^{-9}$ and $\rho_2 - \rho_o = 5$ in all of the cases. The different symbols refer to the quantum numbers with respect to the unbroken $SO(4)$ symmetry: Disks are used for singlets and have already been reported in Ref. [120]; diamonds represent the 6 of $SO(4)$, and crosses the 4 of $SO(4)$. All masses are normalized to the mass of the lightest spin-2 state. Because the masses of the $SO(5)$ singlets do not depend on g, we do not repeat them in the right panels, which display only nontrivial $SO(5)$ multiplets.

While for small ϕ_I one could argue that the choice of potential adopted in this paper is as good as any, because it can be obtained as a power expansion of more complicated potentials in the regime of small ϕ, one expects model dependence to affect the large ϕ_I region. Top-down models are not affected by this limitation. For example, Fig. 7 of Ref. [107] shows the spectrum of a top-down model with $SO(5)$ symmetry breaking to $SO(4)$. The results at large ϕ_I qualitatively resemble, for large values of the VEV, the case $\Delta \geq 5/2$ of this paper, as for arbitrarily large values of ϕ_I the PNGBs become arbitrarily light, but also other states, including a tachyon, persist and their masses appear to be suppressed compared to the typical scale of the other bound state masses. We do not know if top-down models showing

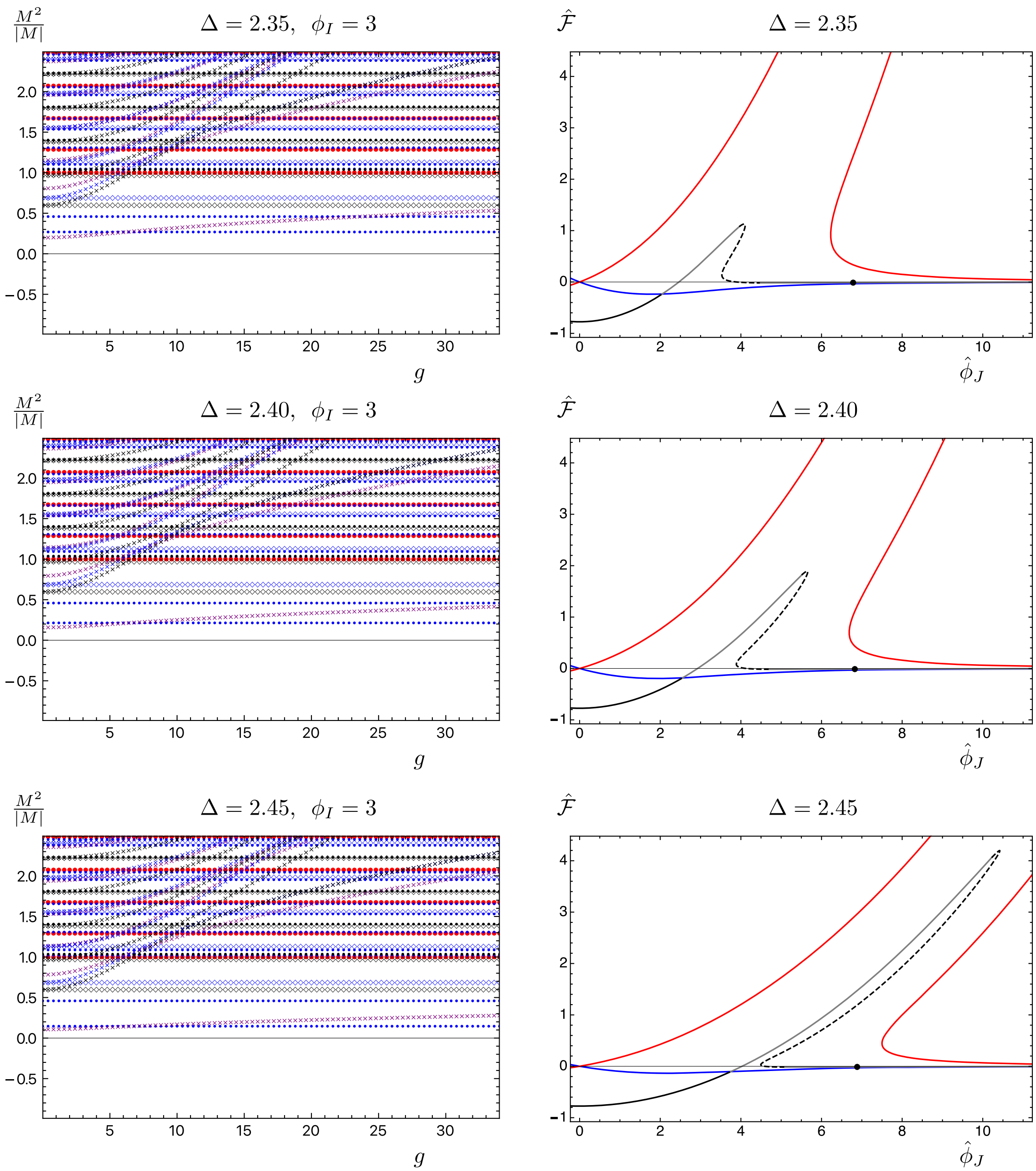

FIG. 3. Mass spectrum $\frac{M^2}{|M|}$ of fluctuations computed for confining backgrounds for $\Delta = 2.35$ (top), $\Delta = 2.40$ (middle), and $\Delta = 2.45$ (bottom), and $\phi_I = 3$ as a function of g (left), and free energy $\hat{\mathcal{F}}$ a function of the normalized source $\hat{\phi}_J$ (right). For the spectrum, scalar (blue), pseudo-scalar (purple), vector (black), and tensor (red) states are displayed as disks for $SO(4)$ singlets, as diamonds to represent the 6 of $SO(4)$, and crosses for the 4 of $SO(4)$. The values of the IR and UV cutoffs in the calculations are, respectively, given by $\rho_1 - \rho_o = 10^{-9}$ and $\rho_2 - \rho_o = 5$. All masses are normalized to the mass of the lightest spin-2 state. The definition of (normalized) free energy $\hat{\mathcal{F}}$ and (normalized) source $\hat{\phi}_J$ can be found in Ref. [120]. The black (stable), gray (metastable), and dashed black (tachyonic) regions of the curve refer to the confining solutions of interest, while red and blue curves refer to singular solutions. The black dots on the right panels denote the solutions with $\phi_I = 3$.

the feature we uncovered here exist, namely, in which both dilaton and PNGBs are parametrically light, but there is no tachyon. Nevertheless, this is a new, unexpected result, which might have important phenomenological implications that are worth studying in the future.

V. OUTLOOK

In this paper, we studied the spectrum of bound states carrying $SO(5)$ quantum numbers in a strongly coupled, confining field theory modeled by its higher-dimensional, weakly coupled gravity dual. We paid particular attention to the states that have the correct quantum numbers to be identified as PNGBs, as the $SO(5)$ global symmetry of the field theory is broken both explicitly and spontaneously to its $SO(4)$ subgroup. We studied the spectrum as a function of three parameters. Δ is the parameter that, for $\Delta > 5/2$, is interpreted in the field theory as the dimension of the scalar operator controlling $SO(5)$ breaking, and for $\Delta < 5/2$ as the dimension of the coupling of the operator. ϕ_I is the parameter controlling the size of the symmetry breaking, and g controls the self-coupling of vector fields, as well as their coupling to the PNGBs.

The main results of our analysis are twofold. First, we showed that if we restrict our attention to the region of parameter space in which the confining solutions are stable, as identified in Ref. [120], then neither the scalar $SO(5)$ singlet nor the $SO(4)$ multiplets are parametrically light for any value of $\phi_I \leq \phi_I(c)$, Δ, and g that we considered. Second, we identified a metastable region of parameter space with $2 \lesssim \Delta < 2.5$ and large ϕ_I, for which both the scalar $SO(5)$ singlet and the lightest spin-0 states transforming as 4 of $SO(4)$ become arbitrarily light when approaching $\Delta \to 5/2$. In this case, the former is a dilaton, and the latter is a multiplet of PNGBs. The existence of this region of parameter space, and the fact that both types of particles are parametrically light, are both new and unexpected results, deserving further investigation.

This is the first step toward the construction of a composite Higgs model, in which the Higgs fields emerge as the PNGBs of a new strongly coupled theory. The next step requires one to couple the system to the standard model gauge fields and to study vacuum alignment in the theory as a function of the strength of additional symmetry-breaking parameters, which in the gravity theory correspond to boundary-localized terms. Whether or not this will allow one to explore other, enlarged regions of parameter space as viable for CHM model building is not known, as is not known whether the presence of the dilaton in the metastable region of parameter space has phenomenologically relevant implications. These interesting questions will be addressed in future research.

The data generated for this manuscript can be downloaded from the Zenodo repository [175].

ACKNOWLEDGMENTS

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APPENDIX A: BASIS OF $SO(5)$ GENERATORS

For concreteness, we present here an example of a basis of $SO(5)$ generators, which we chose so that the first four generators $t^{\hat{A}}$, with $\hat{A} = 1, \ldots, 4$, span the coset $SO(5)/SO(4)$, with the conventions in Eq. (19), while the unbroken $SO(4)$ is generated by $t^{\bar{A}}$, with $\bar{A} = 5, \ldots, 10$,

$$t^1 = \frac{i}{2}\begin{pmatrix} 0&0&0&0&-1\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0 \end{pmatrix}, \quad t^2 = \frac{i}{2}\begin{pmatrix} 0&0&0&0&0\\0&0&0&0&-1\\0&0&0&0&0\\0&0&0&0&0\\0&1&0&0&0 \end{pmatrix}, \quad t^3 = \frac{i}{2}\begin{pmatrix} 0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-1\\0&0&0&0&0\\0&0&1&0&0 \end{pmatrix}, \quad t^4 = \frac{i}{2}\begin{pmatrix} 0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-1\\0&0&0&1&0 \end{pmatrix},$$

$$t^5 = \frac{i}{2}\begin{pmatrix} 0&0&0&-1&0\\0&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&0&0&0&0 \end{pmatrix}, \quad t^6 = \frac{i}{2}\begin{pmatrix} 0&0&0&0&0\\0&0&0&-1&0\\0&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0 \end{pmatrix}, \quad t^7 = \frac{i}{2}\begin{pmatrix} 0&0&0&0&0\\0&0&0&0&0\\0&0&0&-1&0\\0&0&1&0&0\\0&0&0&0&0 \end{pmatrix},$$

$$t^8 = \frac{i}{2}\begin{pmatrix} 0&0&-1&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0 \end{pmatrix}, \quad t^9 = \frac{i}{2}\begin{pmatrix} 0&0&0&0&0\\0&0&-1&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&0&0 \end{pmatrix}, \quad t^{10} = \frac{i}{2}\begin{pmatrix} 0&-1&0&0&0\\1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0 \end{pmatrix}. \tag{A1}$$

APPENDIX B: ASYMPTOTIC EXPANSIONS OF THE FLUCTUATIONS

The linearized equations governing the dynamics of the small fluctuations around the classical solutions are subject to boundary conditions that, as explained in the body of the paper, can be implemented by matching to the asymptotic expansion of the solutions in a way that resembles the process of improvement in lattice field theory. It is hence useful to report here such asymptotic expansions. We find it convenient to include also the $SO(5)$ singlets, together with the $SO(4)$ multiplets.

1. IR expansions

We start from the IR expansion of the fluctuations. For convenience, we put $\rho_o = 0$ and $A_I = 0$ in this subsection,[4] while setting $\chi_I = 0$ in order to avoid a conical singularity. We then expand the solutions of the linearized equations in powers of small ρ. We can write the expansion for a general value of Δ.

For the scalar fluctuations, we find

$$\begin{aligned}\mathfrak{a}^1 &= \mathfrak{a}^1_{I,0} + \mathfrak{a}^1_{I,l}\log(\rho) + \frac{1}{4}\rho^2\bigg[-\frac{1}{4}\Delta(\mathfrak{a}^1_{I,0}(\Delta(15\phi_I^2-4)+20) + 6\phi_I(\mathfrak{a}^2_{I,0}-\mathfrak{a}^2_{I,l})(\Delta(5\phi_I^2-4)+20))\\ &\quad + q^2(\mathfrak{a}^1_{I,0}-\mathfrak{a}^1_{I,l}) - \frac{1}{48}\mathfrak{a}^1_{I,l}(\Delta(25\Delta\phi_I^4 + 20(10-11\Delta)\phi_I^2 + 48(\Delta-5)) + 400)\\ &\quad + \log(\rho)\left(\mathfrak{a}^1_{I,l}\left(-\frac{15\Delta^2\phi_I^2}{4} + (\Delta-5)\Delta + q^2\right) - \frac{3}{2}\mathfrak{a}^2_{I,l}\Delta\phi_I(\Delta(5\phi_I^2-4)+20)\right)\bigg] + \mathcal{O}(\rho^4),\end{aligned} \tag{B1}$$

$$\begin{aligned}\mathfrak{a}^2 &= \mathfrak{a}^2_{I,0} + \mathfrak{a}^2_{I,l}\log(\rho) + \frac{1}{4}\rho^2\bigg[-\frac{1}{4}\Delta\phi_I(\mathfrak{a}^1_{I,0}-\mathfrak{a}^1_{I,l})(\Delta(5\phi_I^2-4)+20) + q^2(\mathfrak{a}^2_{I,0}-\mathfrak{a}^2_{I,l})\\ &\quad -\frac{3}{8}\mathfrak{a}^2_{I,0}(\Delta\phi_I^2(\Delta(5\phi_I^2-8)+40)+80) + \frac{13}{48}\mathfrak{a}^2_{I,l}(\Delta\phi_I^2(\Delta(5\phi_I^2-8)+40)+80)\\ &\quad + \log(\rho)\left(-\frac{5}{4}\mathfrak{a}^1_{I,l}\Delta^2\phi_I^3 + \mathfrak{a}^1_{I,l}(\Delta-5)\Delta\phi_I + \mathfrak{a}^2_{I,l}\left(-\frac{15}{8}\Delta^2\phi_I^4 + 3(\Delta-5)\Delta\phi_I^2 + q^2 - 30\right)\right)\bigg] + \mathcal{O}(\rho^4),\end{aligned} \tag{B2}$$

$$\mathfrak{a}^3 = \mathfrak{a}^3_{I,0} + \rho^2\left(\frac{1}{2}\mathfrak{a}^3_{I,0}q^2\log(\rho) + \mathfrak{a}^3_{I,2}\right) + \mathcal{O}(\rho^4), \tag{B3}$$

$$\mathfrak{a}^4 = \mathfrak{a}^4_{I,0} + \rho^2\left(\frac{1}{2}\mathfrak{a}^4_{I,0}\left(q^2 + \frac{g^2\phi_I^2}{4}\right)\log(\rho) + \mathfrak{a}^4_{I,2}\right) + \mathcal{O}(\rho^4). \tag{B4}$$

For the pseudo-scalar fluctuations, we find

$$\mathfrak{p} = \mathfrak{p}_{I,0} + \rho^2\left[\mathfrak{p}_{I,2} + \frac{1}{2}\mathfrak{p}_{I,0}\left(q^2 + \frac{g^2}{4}\phi_I^2\right)\log(\rho)\right] + \mathcal{O}(\rho^4). \tag{B5}$$

For the vector fluctuations, we find

$$\begin{aligned}\mathfrak{v}^1 &= \mathfrak{v}^1_{I,-2}\rho^{-2} + \frac{1}{2}q^2\mathfrak{v}^1_{I,-2}\log(\rho) + \mathfrak{v}^1_{I,0} + \frac{1}{12288}\rho^2[1536q^2\mathfrak{v}^1_{I,0} + 80\Delta^2\mathfrak{v}^1_{I,-2}\phi_I^4(2(8\Delta^2-50\Delta+75)-3q^2)\\ &\quad + 128(\Delta-5)\Delta\mathfrak{v}^1_{I,-2}\phi_I^2(-3(\Delta-5)\Delta+3q^2-50) - 64(9q^4+60q^2-500)\mathfrak{v}^1_{I,-2} + 125\Delta^4\mathfrak{v}^1_{I,-2}\phi_I^8\\ &\quad - 1000(\Delta-2)\Delta^3\mathfrak{v}^1_{I,-2}\phi_I^6 + 768q^4\mathfrak{v}^1_{I,-2}\log(\rho)] + \mathcal{O}(\rho^4),\end{aligned} \tag{B6}$$

$$\begin{aligned}\mathfrak{v}^2 &= \mathfrak{v}^2_{I,0} + \mathfrak{v}^2_{I,l}\log(\rho) + \frac{1}{96}\rho^2[24q^2(\mathfrak{v}^2_{I,0}-\mathfrak{v}^2_{I,l}) + \mathfrak{v}^2_{I,l}(-5\Delta^2\phi_I^4 + 8(\Delta-5)\Delta\phi_I^2 - 80)\\ &\quad + 24q^2\mathfrak{v}^2_{I,l}\log(\rho)] + \mathcal{O}(\rho^4),\end{aligned} \tag{B7}$$

[4]The dependence on ρ_o and A_I can be reinstated by making the substitutions $\rho \to \rho - \rho_o$ and $q^2 \to e^{-2A_I}q^2$ in the expressions.

$$\mathfrak{v}^3 = \mathfrak{v}^3_{I,0} + \mathfrak{v}^3_{I,l}\log(\rho) + \frac{1}{96}\rho^2[(24q^2 + 6g^2\phi_I^2)\mathfrak{v}^3_{I,0} + (-80 - 24q^2 - 6g^2\phi_I^2 - 40\Delta\phi_I^2 + \Delta^2(8\phi_I^2 - 5\phi_I^4))\mathfrak{v}^3_{I,l} + (24q^2 + 6g^2\phi_I^2)\log(\rho)\mathfrak{v}^3_{I,l}] + \mathcal{O}(\rho^4). \tag{B8}$$

For the tensor fluctuations, we find

$$\mathfrak{e} = \mathfrak{e}_{I,0} + \mathfrak{e}_{I,l}\log(\rho) + \frac{1}{192}\rho^2[48q^2(\mathfrak{e}_{I,0} - \mathfrak{e}_{I,l}) - 25\Delta^2\mathfrak{e}_{I,l}\phi_I^4 + 40(\Delta - 5)\Delta\mathfrak{e}_{I,l}\phi_I^2 - 400\mathfrak{e}_{I,l} + 48\mathfrak{e}_{I,l}q^2\log(\rho)] + \mathcal{O}(\rho^4). \tag{B9}$$

2. UV expansions

The expansions for large ρ in the UV regime of the dual field-theory interpretation depend nontrivially on the parameter Δ. For illustration purposes, in this subsection we set $\Delta = 3$, and $A_U = 0 = \chi_U$.[5] We write the expansions in terms of $z \equiv e^{-\rho}$.

For the scalar fluctuations, we find

$$\mathfrak{a}^1 = \mathfrak{a}^1_2 z^2 + \mathfrak{a}^1_3 z^3 + \frac{1}{2}\mathfrak{a}^1_2 q^2 z^4 + \frac{1}{6}\mathfrak{a}^1_3 q^2 z^5 + \frac{1}{48}\mathfrak{a}^1_2(2q^4 - 99\phi_J^2)z^6 + \mathcal{O}(z^7), \tag{B10}$$

$$\mathfrak{a}^2 = \mathfrak{a}^2_0 - \frac{1}{6}\mathfrak{a}^2_0 q^2 z^2 + \frac{1}{24}\mathfrak{a}^2_0 q^4 z^4 + \mathfrak{a}^2_5 z^5 + \frac{1}{144}\mathfrak{a}^2_0 q^2(q^4 - 14\phi_J^2)z^6 + \mathcal{O}(z^7), \tag{B11}$$

$$\mathfrak{a}^3 = \mathfrak{a}^3_0 - \frac{1}{2}\mathfrak{a}^3_0 q^2 z^2 + \mathfrak{a}^3_3 z^3 - \frac{1}{8}\mathfrak{a}^3_0 q^4 z^4 + \frac{1}{10}\mathfrak{a}^3_3 q^2 z^5 - \frac{1}{144}\mathfrak{a}^3_0 q^2(q^4 + 10\phi_J^2)z^6 + \mathcal{O}(z^7), \tag{B12}$$

$$\mathfrak{a}^4 = \mathfrak{a}^4_0 - \frac{1}{2}\mathfrak{a}^4_0 q^2 z^2 + \mathfrak{a}^4_3 z^3 + \frac{1}{16}\mathfrak{a}^4_0(g^2\phi_J^2 - 2q^4)z^4 + \frac{1}{20}(\mathfrak{a}^4_0 g^2\phi_J\phi_V + 2\mathfrak{a}^4_3 q^2)z^5 - \frac{1}{288}\mathfrak{a}^4_0(2q^6 + (20 + g^2)q^2\phi_J^2 - 4g^2\phi_V^2)z^6 + \mathcal{O}(z^7). \tag{B13}$$

For the pseudo-scalar fluctuations, we find

$$\mathfrak{p} = \mathfrak{p}_0 + \mathfrak{p}_1 z + \left(\frac{\mathfrak{p}_0 q^2}{2} + \frac{\mathfrak{p}_1\phi_V}{\phi_J}\right)z^2 + \frac{2\mathfrak{p}_0 q^2\phi_J\phi_V + \mathfrak{p}_1 q^2\phi_J^2 + 2\mathfrak{p}_1\phi_V^2}{6\phi_J^2}z^3 + \frac{\mathfrak{p}_0\left(q^4\phi_J + \frac{g^2\phi_J^3}{2}\right) + 4\mathfrak{p}_1 q^2\phi_V}{24\phi_J}z^4 + \mathcal{O}(z^5). \tag{B14}$$

For the vector fluctuations, we find

$$\mathfrak{v}^1 = \mathfrak{v}^1_0 - \frac{1}{6}q^2\mathfrak{v}^1_0 z^2 + \frac{1}{24}q^4\mathfrak{v}^1_0 z^4 + \mathfrak{v}^1_5 z^5 + \frac{1}{144}q^2\mathfrak{v}^1_0(q^4 - 14\phi_J^2)z^6 + \frac{1}{70}q^2(70\mathfrak{v}^1_0\chi_5 - 2\mathfrak{v}^1_0\phi_J\phi_V + 5\mathfrak{v}^1_5)z^7 + \mathcal{O}(z^8), \tag{B15}$$

$$\mathfrak{v}^2 = \mathfrak{v}^2_0 - \frac{1}{2}q^2\mathfrak{v}^2_0 z^2 + \mathfrak{v}^2_3 z^3 - \frac{1}{8}q^4\mathfrak{v}^2_0 z^4 + \frac{1}{10}q^2\mathfrak{v}^2_3 z^5 - \frac{1}{144}q^2\mathfrak{v}^2_0(q^4 + 10\phi_J^2)z^6 + \frac{1}{1400}(5(q^4 + 45\phi_J^2)\mathfrak{v}^2_3 + 6q^2\mathfrak{v}^2_0(75\chi_5 - 26\phi_J\phi_V))z^7 + \mathcal{O}(z^8), \tag{B16}$$

$$\mathfrak{v}^3 = \mathfrak{v}^3_0 - \frac{1}{2}q^2\mathfrak{v}^3_0 z^2 + \mathfrak{v}^3_3 z^3 - \frac{1}{8}\mathfrak{v}^3_0\left(q^4 - \frac{g^2}{2}\phi_J^2\right)z^4 + \frac{1}{10}\left(q^2\mathfrak{v}^3_3 + \frac{g^2}{2}\mathfrak{v}^3_0\phi_J\phi_V\right)z^5 + \mathcal{O}\left(z^6\right). \tag{B17}$$

[5]The dependence on χ_U and A_U can be reinstated by making the substitution $q^2 \to e^{2\chi_U - 2A_U}q^2$ in the expressions.

For the tensor fluctuations, we find

$$\mathfrak{e} = \mathfrak{e}_0 - \frac{1}{6}\mathfrak{e}_0 q^2 z^2 + \frac{1}{24}\mathfrak{e}_0 q^4 z^4 + \mathfrak{e}_5 z^5 + \mathcal{O}(z^6). \quad \text{(B18)}$$

The choice $\Delta = 3$ yields a particularly simple expansion in powers of z. In the process of carrying out the numerical calculations for this paper, we computed the UV expansions for all values of Δ for which we plot the spectrum. We do not report all of these expansions here, but we notice that for special choices of Δ the formal expansion changes to include also logarithmic terms in the form $z^n \log^m(z)$.

APPENDIX C: MORE MASS SPECTRA

In this appendix, we report a few additional examples of spectra in Figs. 4–6. The choices of Δ are such as to

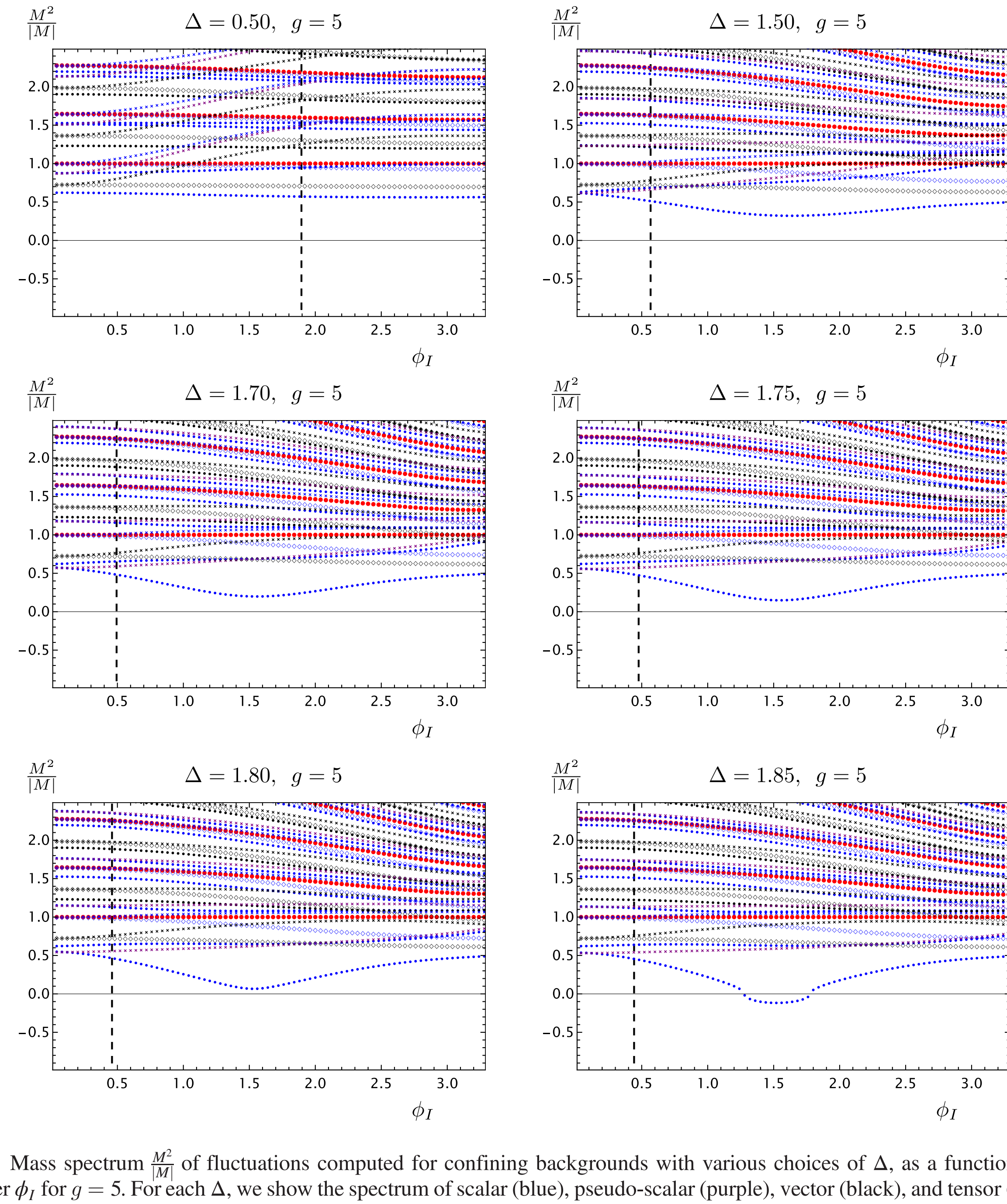

FIG. 4. Mass spectrum $\frac{M^2}{|M|}$ of fluctuations computed for confining backgrounds with various choices of Δ, as a function of the IR parameter ϕ_I for $g = 5$. For each Δ, we show the spectrum of scalar (blue), pseudo-scalar (purple), vector (black), and tensor (red) states. The values of the IR and UV cutoffs in the calculations are, respectively, given by $\rho_1 - \rho_o = 10^{-9}$ and $\rho_2 - \rho_o = 5$ in all of the cases. The different symbols refer to the quantum numbers with respect to the unbroken $SO(4)$ symmetry: Disks are used for singlets and have already been reported in Ref. [120], diamonds represent the 6 of $SO(4)$, and crosses the 4 of $SO(4)$. All masses are normalized to the mass of the lightest spin-2 state.

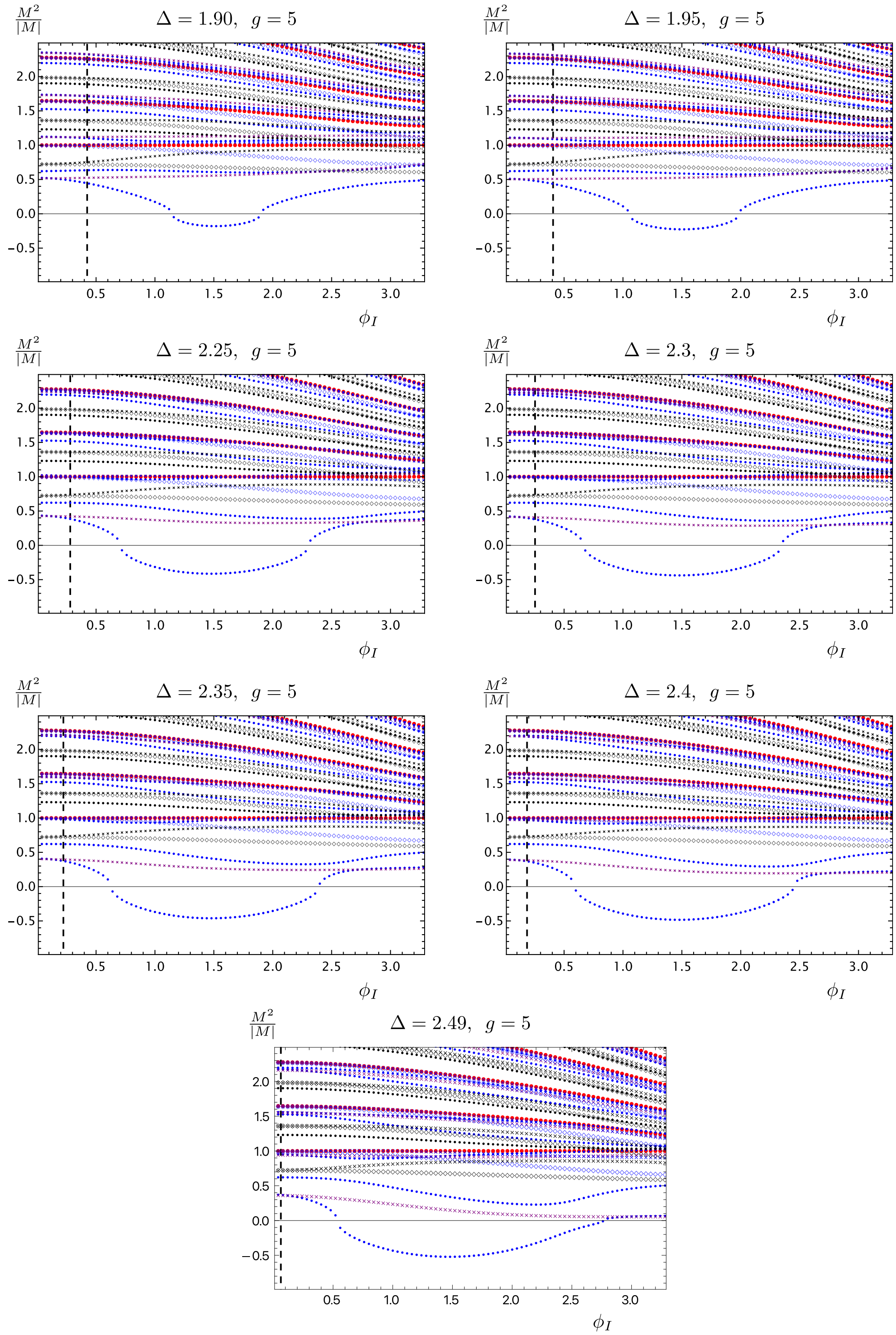

FIG. 5. Mass spectrum $\frac{M^2}{|M|}$ of fluctuations computed for confining backgrounds with various choices of Δ, as a function of the IR parameter ϕ_I for $g = 5$. For each Δ, we show the spectrum of scalar (blue), pseudo-scalar (purple), vector (black), and tensor (red) states. The values of the IR and UV cutoffs in the calculations are, respectively, given by $\rho_1 - \rho_o = 10^{-9}$ and $\rho_2 - \rho_o = 5$ in all of the cases. The different symbols refer to the quantum numbers with respect to the unbroken $SO(4)$ symmetry: Disks are used for singlets and have already been reported in Ref. [120], diamonds represent the 6 of $SO(4)$, and crosses the 4 of $SO(4)$. All masses are normalized to the mass of the lightest spin-2 state.

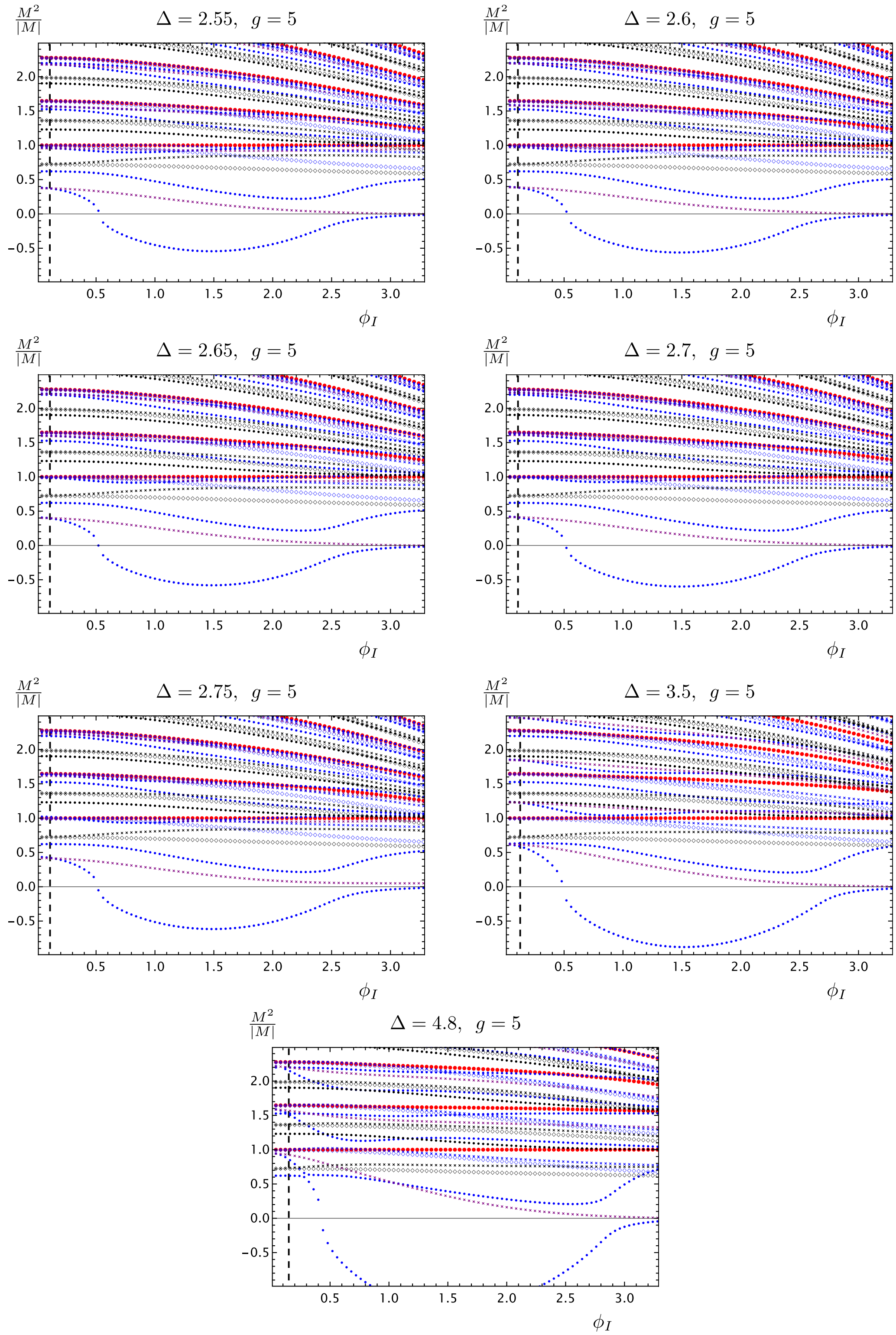

FIG. 6. Mass spectrum $\frac{M^2}{|M|}$ of fluctuations computed for confining backgrounds with various choices of Δ, as a function of the IR parameter ϕ_I for $g = 5$. For each Δ, we show the spectrum of scalar (blue), pseudo-scalar (purple), vector (black), and tensor (red) states. The values of the IR and UV cutoffs in the calculations are, respectively, given by $\rho_1 - \rho_o = 10^{-9}$ and $\rho_2 - \rho_o = 5$ in all of the cases. The different symbols refer to the quantum numbers with respect to the unbroken $SO(4)$ symmetry: Disks are used for singlets and have already been reported in Ref. [120], diamonds represent the 6 of $SO(4)$, and crosses the 4 of $SO(4)$. All masses are normalized to the mass of the lightest spin-2 state.

include the entirety of the catalog in Ref. [120]. We fix the indicative value $g = 5$ in all plots. Qualitatively, all these plots resemble at least one of those in the main body of the paper, though quantitative features may be amplified or suppressed by changes in Δ.

[1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B **716**, 1 (2012).

[2] S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B **716**, 30 (2012).

[3] D. B. Kaplan and H. Georgi, $SU(2) \times U(1)$ breaking by vacuum misalignment, Phys. Lett. **136B**, 183 (1984).

[4] H. Georgi and D. B. Kaplan, Composite Higgs and custodial SU(2), Phys. Lett. **145B**, 216 (1984).

[5] M. J. Dugan, H. Georgi, and D. B. Kaplan, Anatomy of a composite Higgs model, Nucl. Phys. **B254**, 299 (1985).

[6] G. Panico and A. Wulzer, The composite Nambu-Goldstone Higgs, Lect. Notes Phys. **913**, 1 (2016).

[7] O. Witzel, Review on composite Higgs models, Proc. Sci. LATTICE2018 (**2019**) 006 [arXiv:1901.08216].

[8] G. Cacciapaglia, C. Pica, and F. Sannino, Fundamental composite dynamics: A review, Phys. Rep. **877**, 1 (2020).

[9] G. Ferretti and D. Karateev, Fermionic UV completions of composite Higgs models, J. High Energy Phys. 03 (2014) 077.

[10] G. Ferretti, Gauge theories of partial compositeness: Scenarios for Run-II of the LHC, J. High Energy Phys. 06 (2016) 107.

[11] G. Cacciapaglia, G. Ferretti, T. Flacke, and H. Serôdio, Light scalars in composite Higgs models, Front. Phys. **7**, 22 (2019).

[12] E. Katz, A. E. Nelson, and D. G. E. Walker, The intermediate Higgs, J. High Energy Phys. 08 (2005) 074.

[13] R. Barbieri, B. Bellazzini, V. S. Rychkov, and A. Varagnolo, The Higgs boson from an extended symmetry, Phys. Rev. D **76**, 115008 (2007).

[14] P. Lodone, Vector-like quarks in a composite Higgs model, J. High Energy Phys. 12 (2008) 029.

[15] B. Gripaios, A. Pomarol, F. Riva, and J. Serra, Beyond the minimal composite Higgs model, J. High Energy Phys. 04 (2009) 070.

[16] J. Mrazek, A. Pomarol, R. Rattazzi, M. Redi, J. Serra, and A. Wulzer, The other natural two Higgs doublet model, Nucl. Phys. **B853**, 1 (2011).

[17] D. Marzocca, M. Serone, and J. Shu, General composite Higgs models, J. High Energy Phys. 08 (2012) 013.

[18] C. Grojean, O. Matsedonskyi, and G. Panico, Light top partners and precision physics, J. High Energy Phys. 10 (2013) 160.

[19] J. Barnard, T. Gherghetta, and T. S. Ray, UV descriptions of composite Higgs models without elementary scalars, J. High Energy Phys. 02 (2014) 002.

[20] G. Cacciapaglia and F. Sannino, Fundamental composite (Goldstone) Higgs dynamics, J. High Energy Phys. 04 (2014) 111.

[21] G. Ferretti, UV completions of partial compositeness: The case for a SU(4) gauge group, J. High Energy Phys. 06 (2014) 142.

[22] A. Arbey, G. Cacciapaglia, H. Cai, A. Deandrea, S. Le Corre, and F. Sannino, Fundamental composite electroweak dynamics: Status at the LHC, Phys. Rev. D **95**, 015028 (2017).

[23] G. von Gersdorff, E. Pontón, and R. Rosenfeld, The dynamical composite Higgs, J. High Energy Phys. 06 (2015) 119.

[24] L. Vecchi, A dangerous irrelevant UV-completion of the composite Higgs, J. High Energy Phys. 02 (2017) 094.

[25] G. Cacciapaglia, H. Cai, A. Deandrea, T. Flacke, S. J. Lee, and A. Parolini, Composite scalars at the LHC: The Higgs, the sextet and the octet, J. High Energy Phys. 11 (2015) 201.

[26] T. Ma and G. Cacciapaglia, Fundamental composite 2HDM: SU(N) with 4 flavours, J. High Energy Phys. 03 (2016) 211.

[27] F. Feruglio, B. Gavela, K. Kanshin, P. A. N. Machado, S. Rigolin, and S. Saa, The minimal linear sigma model for the Goldstone Higgs, J. High Energy Phys. 06 (2016) 038.

[28] T. DeGrand, M. Golterman, E. T. Neil, and Y. Shamir, One-loop chiral perturbation theory with two fermion representations, Phys. Rev. D **94**, 025020 (2016).

[29] S. Fichet, G. von Gersdorff, E. Pontòn, and R. Rosenfeld, The excitation of the global symmetry-breaking vacuum in composite Higgs models, J. High Energy Phys. 09 (2016) 158.

[30] J. Galloway, A. L. Kagan, and A. Martin, A UV complete partially composite-pNGB Higgs, Phys. Rev. D **95**, 035038 (2017).

[31] A. Agugliaro, O. Antipin, D. Becciolini, S. De Curtis, and M. Redi, UV complete composite Higgs models, Phys. Rev. D **95**, 035019 (2017).

[32] A. Belyaev, G. Cacciapaglia, H. Cai, G. Ferretti, T. Flacke, A. Parolini, and H. Serodio, Di-boson signatures as standard candles for partial compositeness, J. High Energy Phys. 01 (2017) 094.

[33] N. Bizot, M. Frigerio, M. Knecht, and J. L. Kneur, Nonperturbative analysis of the spectrum of meson resonances in an ultraviolet-complete composite-Higgs model, Phys. Rev. D **95**, 075006 (2017).

[34] C. Csaki, T. Ma, and J. Shu, Maximally Symmetric Composite Higgs Models, Phys. Rev. Lett. **119**, 131803 (2017).

[35] M. Chala, G. Durieux, C. Grojean, L. de Lima, and O. Matsedonskyi, Minimally extended SILH, J. High Energy Phys. 06 (2017) 088.
[36] M. Golterman and Y. Shamir, Effective potential in ultraviolet completions for composite Higgs models, Phys. Rev. D **97**, 095005 (2018).
[37] C. Csaki, T. Ma, and J. Shu, Trigonometric Parity for Composite Higgs Models, Phys. Rev. Lett. **121**, 231801 (2018).
[38] T. Alanne, D. Buarque Franzosi, and M. T. Frandsen, A partially composite Goldstone Higgs, Phys. Rev. D **96**, 095012 (2017).
[39] T. Alanne, D. Buarque Franzosi, M. T. Frandsen, M. L. A. Kristensen, A. Meroni, and M. Rosenlyst, Partially composite Higgs models: Phenomenology and RG analysis, J. High Energy Phys. 01 (2018) 051.
[40] F. Sannino, P. Stangl, D. M. Straub, and A. E. Thomsen, Flavor physics and flavor anomalies in minimal fundamental partial compositeness, Phys. Rev. D **97**, 115046 (2018).
[41] T. Alanne, N. Bizot, G. Cacciapaglia, and F. Sannino, Classification of NLO operators for composite Higgs models, Phys. Rev. D **97**, 075028 (2018).
[42] N. Bizot, G. Cacciapaglia, and T. Flacke, Common exotic decays of top partners, J. High Energy Phys. 06 (2018) 065.
[43] C. Cai, G. Cacciapaglia, and H. H. Zhang, Vacuum alignment in a composite 2HDM, J. High Energy Phys. 01 (2019) 130.
[44] A. Agugliaro, G. Cacciapaglia, A. Deandrea, and S. De Curtis, Vacuum misalignment and pattern of scalar masses in the SU(5)/SO(5) composite Higgs model, J. High Energy Phys. 02 (2019) 089.
[45] D. Buarque Franzosi, G. Cacciapaglia, and A. Deandrea, Sigma-assisted low scale composite Goldstone–Higgs, Eur. Phys. J. C **80**, 28 (2020).
[46] G. Cacciapaglia, T. Ma, S. Vatani, and Y. Wu, Towards a fundamental safe theory of composite Higgs and dark matter, Eur. Phys. J. C **80**, 1088 (2020).
[47] H. Gertov, A. E. Nelson, A. Perko, and D. G. E. Walker, Lattice-friendly gauge completion of a composite Higgs with top partners, J. High Energy Phys. 02 (2019) 181.
[48] V. Ayyar, M. F. Golterman, D. C. Hackett, W. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, Radiative contribution to the composite-Higgs potential in a two-representation lattice model, Phys. Rev. D **99**, 094504 (2019).
[49] G. Cacciapaglia, H. Cai, A. Deandrea, and A. Kushwaha, Composite Higgs and dark matter model in SU(6)/SO(6), J. High Energy Phys. 10 (2019) 035.
[50] D. Buarque Franzosi and G. Ferretti, Anomalous dimensions of potential top-partners, SciPost Phys. **7**, 027 (2019).
[51] G. Cacciapaglia, S. Vatani, and C. Zhang, Composite Higgs meets Planck scale: Partial compositeness from partial unification, Phys. Lett. B **815**, 136177 (2021).
[52] G. Cacciapaglia, A. Deandrea, T. Flacke, and A. M. Iyer, Gluon-photon signatures for color octet at the LHC (and beyond), J. High Energy Phys. 05 (2020) 027.
[53] T. Appelquist, J. Ingoldby, and M. Piai, Nearly Conformal Composite Higgs Model, Phys. Rev. Lett. **126**, 191804 (2021).
[54] G. Cacciapaglia, T. Flacke, M. Kunkel, and W. Porod, Phenomenology of unusual top partners in composite Higgs models, J. High Energy Phys. 02 (2022) 208.
[55] A. Banerjee, D. B. Franzosi, and G. Ferretti, Modelling vector-like quarks in partial compositeness framework, J. High Energy Phys. 03 (2022) 200.
[56] T. Appelquist, J. Ingoldby, and M. Piai, Composite two-Higgs doublet model from dilaton effective field theory, Nucl. Phys. **B983**, 115930 (2022).
[57] G. Ferretti, Compositeness above the electroweak scale and a proposed test at LHCb, EPJ Web Conf. **258**, 08002 (2022).
[58] A. Banerjee and G. Ferretti, Vacuum misalignment in presence of four-Fermi operators, arXiv:2302.11598.
[59] A. Hietanen, R. Lewis, C. Pica, and F. Sannino, Fundamental composite Higgs dynamics on the lattice: SU(2) with two flavors, J. High Energy Phys. 07 (2014) 116.
[60] W. Detmold, M. McCullough, and A. Pochinsky, Dark nuclei. II. Nuclear spectroscopy in two-color QCD, Phys. Rev. D **90**, 114506 (2014).
[61] R. Arthur, V. Drach, M. Hansen, A. Hietanen, C. Pica, and F. Sannino, SU(2) gauge theory with two fundamental flavors: A minimal template for model building, Phys. Rev. D **94**, 094507 (2016).
[62] R. Arthur, V. Drach, A. Hietanen, C. Pica, and F. Sannino, $SU(2)$ gauge theory with two fundamental flavours: Scalar and pseudoscalar spectrum, arXiv:1607.06654.
[63] C. Pica, V. Drach, M. Hansen, and F. Sannino, Composite Higgs dynamics on the lattice, EPJ Web Conf. **137**, 10005 (2017).
[64] J. W. Lee, B. Lucini, and M. Piai, Symmetry restoration at high-temperature in two-color and two-flavor lattice gauge theories, J. High Energy Phys. 04 (2017) 036.
[65] V. Drach, T. Janowski, and C. Pica, Update on SU(2) gauge theory with NF = 2 fundamental flavours, EPJ Web Conf. **175**, 08020 (2018).
[66] V. Drach, T. Janowski, C. Pica, and S. Prelovsek, Scattering of Goldstone bosons and resonance production in a composite Higgs model on the lattice, J. High Energy Phys. 04 (2021) 117.
[67] V. Drach, P. Fritzsch, A. Rago, and F. Romero-López, Singlet channel scattering in a composite Higgs model on the lattice, Eur. Phys. J. C **82**, 47 (2022).
[68] E. Bennett, D. K. Hong, J. W. Lee, C.-J. D. Lin, B. Lucini, M. Piai, and D. Vadacchino, $Sp(4)$ gauge theory on the lattice: Towards SU(4)/Sp(4) composite Higgs (and beyond), J. High Energy Phys. 03 (2018) 185.
[69] E. Bennett, D. K. Hong, J. W. Lee, C.-J. D. Lin, B. Lucini, M. Piai, and D. Vadacchino, $Sp(4)$ gauge theories on the lattice: $N_f = 2$ dynamical fundamental fermions, J. High Energy Phys. 12 (2019) 053.
[70] E. Bennett, D. K. Hong, J. W. Lee, C. J. D. Lin, B. Lucini, M. Mesiti, M. Piai, J. Rantaharju, and D. Vadacchino, $Sp(4)$ gauge theories on the lattice: Quenched fundamental and antisymmetric fermions, Phys. Rev. D **101**, 074516 (2020).

[71] E. Bennett, D. K. Hong, H. Hsiao, J. W. Lee, C. J. D. Lin, B. Lucini, M. Mesiti, M. Piai, and D. Vadacchino, Lattice studies of the $Sp(4)$ gauge theory with two fundamental and three antisymmetric Dirac fermions, Phys. Rev. D **106**, 014501 (2022).
[72] V. Ayyar, T. DeGrand, M. Golterman, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, Spectroscopy of SU(4) composite Higgs theory with two distinct fermion representations, Phys. Rev. D **97**, 074505 (2018).
[73] V. Ayyar, T. Degrand, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, Baryon spectrum of SU(4) composite Higgs theory with two distinct fermion representations, Phys. Rev. D **97**, 114505 (2018).
[74] V. Ayyar, T. DeGrand, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, Finite-temperature phase structure of SU(4) gauge theory with multiple fermion representations, Phys. Rev. D **97**, 114502 (2018).
[75] V. Ayyar, T. DeGrand, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, Partial compositeness and baryon matrix elements on the lattice, Phys. Rev. D **99**, 094502 (2019).
[76] G. Cossu, L. Del Debbio, M. Panero, and D. Preti, Strong dynamics with matter in multiple representations: SU(4) gauge theory with fundamental and sextet fermions, Eur. Phys. J. C **79**, 638 (2019).
[77] Y. Shamir, M. Golterman, W. I. Jay, E. T. Neil, and B. Svetitsky, S parameter from a prototype composite-Higgs model, Proc. Sci. LATTICE2021 (**2022**) 611 [arXiv:2110.05198].
[78] L. Del Debbio, A. Lupo, M. Panero, and N. Tantalo, Spectral reconstruction in SU(4) gauge theory with fermions in multiple representations, Proc. Sci. LATTICE2021 (**2022**) 092 [arXiv:2112.01158].
[79] F. Caracciolo, A. Parolini, and M. Serone, UV completions of composite Higgs models with partial compositeness, J. High Energy Phys. 02 (2013) 066.
[80] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. **38**, 1113 (1999); Adv. Theor. Math. Phys. **2**, 231 (1998).
[81] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B **428**, 105 (1998).
[82] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. **2**, 253 (1998).
[83] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, Large N field theories, string theory and gravity, Phys. Rep. **323**, 183 (2000).
[84] E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories, Adv. Theor. Math. Phys. **2**, 505 (1998)
[85] I. R. Klebanov and M. J. Strassler, Supergravity and a confining gauge theory: Duality cascades and chi SB resolution of naked singularities, J. High Energy Phys. 08 (2000) 052.
[86] J. M. Maldacena and C. Nunez, Towards the Large N Limit of Pure $N = 1$ SuperYang-Mills, Phys. Rev. Lett. **86**, 588 (2001).
[87] A. Butti, M. Grana, R. Minasian, M. Petrini, and A. Zaffaroni, The baryonic branch of Klebanov-Strassler solution: A supersymmetric family of SU(3) structure backgrounds, J. High Energy Phys. 03 (2005) 069.
[88] R. C. Brower, S. D. Mathur, and C. I. Tan, Glueball spectrum for QCD from AdS supergravity duality, Nucl. Phys. **B587**, 249 (2000).
[89] C. K. Wen and H. X. Yang, QCD_4 glueball masses from AdS_6 black hole description, Mod. Phys. Lett. A **20**, 997 (2005).
[90] S. Kuperstein and J. Sonnenschein, Non-critical, near extremal AdS_6 background as a holographic laboratory of four dimensional YM theory, J. High Energy Phys. 11 (2004) 026.
[91] M. Bianchi, M. Prisco, and W. Mueck, New results on holographic three point functions, J. High Energy Phys. 11 (2003) 052.
[92] M. Berg, M. Haack, and W. Mueck, Bulk dynamics in confining gauge theories, Nucl. Phys. **B736**, 82 (2006).
[93] M. Berg, M. Haack, and W. Mueck, Glueballs vs. gluinoballs: Fluctuation spectra in non-AdS/non-CFT, Nucl. Phys. **B789**, 1 (2008).
[94] D. Elander, Glueball spectra of SQCD-like theories, J. High Energy Phys. 03 (2010) 114.
[95] D. Elander and M. Piai, Light scalars from a compact fifth dimension, J. High Energy Phys. 01 (2011) 026.
[96] D. Elander, Light scalar from deformations of the Klebanov-Strassler background, Phys. Rev. D **91**, 126012 (2015).
[97] D. Elander and M. Piai, Calculable mass hierarchies and a light dilaton from gravity duals, Phys. Lett. B **772**, 110 (2017).
[98] D. Elander and M. Piai, Glueballs on the baryonic branch of Klebanov-Strassler: Dimensional deconstruction and a light scalar particle, J. High Energy Phys. 06 (2017) 003.
[99] D. Elander, A. F. Faedo, C. Hoyos, D. Mateos, and M. Piai, Multiscale confining dynamics from holographic RG flows, J. High Energy Phys. 05 (2014) 003.
[100] A. Karch and E. Katz, Adding flavor to AdS/CFT, J. High Energy Phys. 06 (2002) 043.
[101] J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik, and I. Kirsch, Chiral symmetry breaking and pions in nonsupersymmetric gauge/gravity duals, Phys. Rev. D **69**, 066007 (2004).
[102] T. Sakai and S. Sugimoto, Low energy hadron physics in holographic QCD, Prog. Theor. Phys. **113**, 843 (2005).
[103] T. Sakai and S. Sugimoto, More on a holographic dual of QCD, Prog. Theor. Phys. **114**, 1083 (2005).
[104] M. Kruczenski, D. Mateos, R. C. Myers, and D. J. Winters, Meson spectroscopy in AdS/CFT with flavor, J. High Energy Phys. 07 (2003) 049.
[105] C. Nunez, A. Paredes, and A. V. Ramallo, Flavoring the gravity dual of $N = 1$ Yang-Mills with probes, J. High Energy Phys. 12 (2003) 024.
[106] J. Erdmenger, N. Evans, I. Kirsch, and E. J. Threlfall, Mesons in gauge/gravity duals—A review, Eur. Phys. J. A **35**, 81 (2008).
[107] D. Elander and M. Piai, Towards top-down holographic composite Higgs: Minimal coset from maximal supergravity, J. High Energy Phys. 03 (2022) 049.

[108] R. Contino, Y. Nomura, and A. Pomarol, Higgs as a holographic pseudo-Goldstone boson, Nucl. Phys. **B671**, 148 (2003).
[109] K. Agashe, R. Contino, and A. Pomarol, The minimal composite Higgs model, Nucl. Phys. **B719**, 165 (2005).
[110] K. Agashe and R. Contino, The minimal composite Higgs model and electroweak precision tests, Nucl. Phys. **B742**, 59 (2006).
[111] K. Agashe, R. Contino, L. Da Rold, and A. Pomarol, A custodial symmetry for $Zb\bar{b}$, Phys. Lett. B **641**, 62 (2006).
[112] R. Contino, L. Da Rold, and A. Pomarol, Light custodians in natural composite Higgs models, Phys. Rev. D **75**, 055014 (2007).
[113] A. Falkowski and M. Perez-Victoria, Electroweak breaking on a soft wall, J. High Energy Phys. 12 (2008) 107.
[114] R. Contino, The Higgs as a composite Nambu-Goldstone boson, in *Physics of the Large and the Small* (2011), pp. 235–306, 10.1142/9789814327183_0005.
[115] R. Contino, D. Marzocca, D. Pappadopulo, and R. Rattazzi, On the effect of resonances in composite Higgs phenomenology, J. High Energy Phys. 10 (2011) 081.
[116] J. Erdmenger, N. Evans, W. Porod, and K. S. Rigatos, Gauge/Gravity Dynamics for Composite Higgs Models and the Top Mass, Phys. Rev. Lett. **126**, 071602 (2021).
[117] J. Erdmenger, N. Evans, W. Porod, and K. S. Rigatos, Gauge/gravity dual dynamics for the strongly coupled sector of composite Higgs models, J. High Energy Phys. 02 (2021) 058.
[118] D. Elander, M. Frigerio, M. Knecht, and J. L. Kneur, Holographic models of composite Higgs in the Veneziano limit. Part I. Bosonic sector, J. High Energy Phys. 03 (2021) 182.
[119] D. Elander, M. Frigerio, M. Knecht, and J. L. Kneur, Holographic models of composite Higgs in the Veneziano limit. Part II. Fermionic sector, J. High Energy Phys. 05 (2022) 066.
[120] D. Elander, A. Fatemiabhari, and M. Piai, Phase transitions and light scalars in bottom-up holography, arXiv:2212.07954.
[121] D. B. Kaplan, J. W. Lee, D. T. Son, and M. A. Stephanov, Conformality lost, Phys. Rev. D **80**, 125005 (2009).
[122] V. Gorbenko, S. Rychkov, and B. Zan, Walking, weak first-order transitions, and complex CFTs, J. High Energy Phys. 10 (2018) 108.
[123] V. Gorbenko, S. Rychkov, and B. Zan, Walking, weak first-order transitions, and complex CFTs II. Two-dimensional Potts model at $Q > 4$, SciPost Phys. **5**, 050 (2018).
[124] D. Elander, M. Piai, and J. Roughley, Dilatonic states near holographic phase transitions, Phys. Rev. D **103**, 106018 (2021).
[125] D. Elander, M. Piai, and J. Roughley, Light dilaton in a metastable vacuum, Phys. Rev. D **103**, 046009 (2021).
[126] D. Elander, M. Piai, and J. Roughley, Coulomb branch of $N = 4$ SYM and dilatonic scions in supergravity, Phys. Rev. D **104**, 046003 (2021).
[127] P. Breitenlohner and D. Z. Freedman, Stability in gauged extended supergravity, Ann. Phys. (N.Y.) **144**, 249 (1982).
[128] A. Pomarol, O. Pujolas, and L. Salas, Holographic conformal transition and light scalars, J. High Energy Phys. 10 (2019) 202.
[129] D. Arean, I. Iatrakis, M. Järvinen, and E. Kiritsis, V-QCD: Spectra, the dilaton and the *S*-parameter, Phys. Lett. B **720**, 219 (2013).
[130] D. Areán, I. Iatrakis, M. Järvinen, and E. Kiritsis, The discontinuities of conformal transitions and mass spectra of V-QCD, J. High Energy Phys. 11 (2013) 068.
[131] D. Elander, A. Fatemiabhari, and M. Piai, A note on holographic vacuum alignment (to be published).
[132] D. Elander, A. Fatemiabhari, and M. Piai, A new bottom-up holographic composite Higgs model (to be published).
[133] W. D. Goldberger and M. B. Wise, Modulus Stabilization with Bulk Fields, Phys. Rev. Lett. **83**, 4922 (1999).
[134] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, Modeling the fifth-dimension with scalars and gravity, Phys. Rev. D **62**, 046008 (2000).
[135] W. D. Goldberger and M. B. Wise, Phenomenology of a stabilized modulus, Phys. Lett. B **475**, 275 (2000).
[136] C. Csaki, M. L. Graesser, and G. D. Kribs, Radion dynamics and electroweak physics, Phys. Rev. D **63**, 065002 (2001).
[137] N. Arkani-Hamed, M. Porrati, and L. Randall, Holography and phenomenology, J. High Energy Phys. 08 (2001) 017.
[138] R. Rattazzi and A. Zaffaroni, Comments on the holographic picture of the Randall-Sundrum model, J. High Energy Phys. 04 (2001) 021.
[139] L. Kofman, J. Martin, and M. Peloso, Exact identification of the radion and its coupling to the observable sector, Phys. Rev. D **70**, 085015 (2004).
[140] M. Bianchi, D. Z. Freedman, and K. Skenderis, Holographic renormalization, Nucl. Phys. **B631**, 159 (2002).
[141] K. Skenderis, Lecture notes on holographic renormalization, Classical Quantum Gravity **19**, 5849 (2002).
[142] I. Papadimitriou and K. Skenderis, AdS/CFT correspondence and geometry, IRMA Lect. Math. Theor. Phys. **8**, 73 (2005).
[143] D. Elander, M. Piai, and J. Roughley, Holographic glueballs from the circle reduction of Romans supergravity, J. High Energy Phys. 02 (2019) 101.
[144] D. Elander, M. Piai, and J. Roughley, Probing the holographic dilaton, J. High Energy Phys. 06 (2020) 177.
[145] A. A. Migdal and M. A. Shifman, Dilaton effective Lagrangian in gluodynamics, Phys. Lett. **114B**, 445 (1982).
[146] S. Coleman, *Aspects of Symmetry: Selected Erice Lectures* (Cambridge University Press, Cambridge, England, 1985).
[147] W. D. Goldberger, B. Grinstein, and W. Skiba, Distinguishing the Higgs Boson from the Dilaton at the Large Hadron Collider, Phys. Rev. Lett. **100**, 111802 (2008).
[148] D. K. Hong, S. D. H. Hsu, and F. Sannino, Composite Higgs from higher representations, Phys. Lett. B **597**, 89 (2004).
[149] D. D. Dietrich, F. Sannino, and K. Tuominen, Light composite Higgs from higher representations versus electroweak precision measurements: Predictions for CERN LHC, Phys. Rev. D **72**, 055001 (2005).
[150] M. Hashimoto and K. Yamawaki, Techni-dilaton at conformal edge, Phys. Rev. D **83**, 015008 (2011).
[151] T. Appelquist and Y. Bai, A light dilaton in walking gauge theories, Phys. Rev. D **82**, 071701 (2010).
[152] L. Vecchi, Phenomenology of a light scalar: The dilaton, Phys. Rev. D **82**, 076009 (2010).

[153] Z. Chacko and R. K. Mishra, Effective theory of a light dilaton, Phys. Rev. D **87**, 115006 (2013).

[154] B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, and J. Terning, A Higgslike dilaton, Eur. Phys. J. C **73**, 2333 (2013).

[155] B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, and J. Terning, A naturally light dilaton and a small cosmological constant, Eur. Phys. J. C **74**, 2790 (2014).

[156] T. Abe, R. Kitano, Y. Konishi, K. y. Oda, J. Sato, and S. Sugiyama, Minimal dilaton model, Phys. Rev. D **86**, 115016 (2012).

[157] E. Eichten, K. Lane, and A. Martin, A Higgs impostor in low-scale technicolor, arXiv:1210.5462.

[158] P. Hernandez-Leon and L. Merlo, Distinguishing a Higgs-like dilaton scenario with a complete bosonic effective field theory basis, Phys. Rev. D **96**, 075008 (2017).

[159] J. Cruz Rojas, D. K. Hong, S. H. Im, and M. Järvinen, Holographic light dilaton at the conformal edge, arXiv:2302.08112.

[160] S. Matsuzaki and K. Yamawaki, Dilaton Chiral Perturbation Theory: Determining the Mass and Decay Constant of the Technidilaton on the Lattice, Phys. Rev. Lett. **113**, 082002 (2014).

[161] M. Golterman and Y. Shamir, Low-energy effective action for pions and a dilatonic meson, Phys. Rev. D **94**, 054502 (2016).

[162] A. Kasai, K. i. Okumura, and H. Suzuki, A dilaton-pion mass relation, arXiv:1609.02264.

[163] M. Hansen, K. Langaeble, and F. Sannino, Extending chiral perturbation theory with an isosinglet scalar, Phys. Rev. D **95**, 036005 (2017).

[164] M. Golterman and Y. Shamir, Effective pion mass term and the trace anomaly, Phys. Rev. D **95**, 016003 (2017).

[165] T. Appelquist, J. Ingoldby, and M. Piai, Dilaton EFT framework for lattice data, J. High Energy Phys. 07 (2017) 035.

[166] T. Appelquist, J. Ingoldby, and M. Piai, Analysis of a dilaton EFT for lattice data, J. High Energy Phys. 03 (2018) 039.

[167] M. Golterman and Y. Shamir, Large-mass regime of the dilaton-pion low-energy effective theory, Phys. Rev. D **98**, 056025 (2018).

[168] O. Cata and C. Muller, Chiral effective theories with a light scalar at one loop, Nucl. Phys. **B952**, 114938 (2020).

[169] O. Catà, R. J. Crewther, and L. C. Tunstall, Crawling technicolor, Phys. Rev. D **100**, 095007 (2019).

[170] T. Appelquist, J. Ingoldby, and M. Piai, Dilaton potential and lattice data, Phys. Rev. D **101**, 075025 (2020).

[171] M. Golterman, E. T. Neil, and Y. Shamir, Application of dilaton chiral perturbation theory to $N_f = 8$, SU(3) spectral data, Phys. Rev. D **102**, 034515 (2020).

[172] M. Golterman and Y. Shamir, Explorations beyond dilaton chiral perturbation theory in the eight-flavor SU(3) gauge theory, Phys. Rev. D **102**, 114507 (2020).

[173] T. Appelquist, J. Ingoldby, and M. Piai, Dilaton effective field theory, Universe **9**, 10 (2023).

[174] E. Ponton, Four lectures on TeV scale extra dimensions, in *The Dark Secrets of the Terascale* (2013), pp. 283–374, 10.1142/9789814390163_0007.

[175] D. Elander, A. Fatemiabhari, and M. Piai, Toward minimal composite Higgs models from regular geometries in bottom-up holography—data release, Zenodo, 10.5281/zenodo.7705408 (2023).