

Introducing tools to test Higgs boson interactions via WW scattering. II. The coupled-channel formalism and scalar resonances

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In this work, we explore in detail the presence of scalar resonances in the WW fusion process in the context of the LHC experiments working in the theoretical framework provided by Higgs effective field theories (HEFTs). While the phenomenology of vector resonances is reasonably understood in the framework of Weinberg sum-rules and unitarization studies, scalar resonances are a lot less constrained and, more importantly, do depend on HEFT low-energy effective couplings different from the ones of vector resonances that are difficult to constrain experimentally. More specifically, unitarization techniques combined with the requirement of causality allows us to set nontrivial bounds on Higgs self-interactions. This is due to the need for considering coupled channels in the scalar case along the unitarization process. As a byproduct, we can gain some relevant information on the Higgs sector from $WW \rightarrow WW$ elastic processes without needing to consider two-Higgs production.

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I. INTRODUCTION

In a companion paper [1], we highlighted the importance of the $W_L W_L$ scattering in investigating the Higgs self-couplings, and therefore the Higgs potential, at the LHC in the framework of the Higgs effective field theory (HEFT). We argued that unitarization of the amplitudes was a convenient—sometimes even necessary—ingredient in this analysis. Summarizing, there are two reasons for this. On the one hand, departures from the minimal Standard Model (SM) typically lead to violations of unitarity at large energies with fast rising amplitudes. Taking into account that the fundamental theory has presumably to be renormalizable and unitary, this may lead to hypersensitivity to deviations of the effective theory coefficients with respect to their SM values. Even if this is not the case (for instance, because deviations with respect to the SM are tiny), it is known that lack of unitarity typically generates resonances in various channels, which in a sense is the way the effective theory has to remember that it derives from a *bona fide* microscopic theory. The properties of these resonances are typically very sensitive even to some

small deviations with respect the SM and thus worth investigating.

In Ref. [1], we listed and renormalized all the suitable on-shell local operators of the vector and scalar sector of the HEFT describing at low energies an extended electroweak symmetry breaking sector contributing to $2 \rightarrow 2$ processes. Similar results were also reported in Ref. [2] in the off-shell case, but without consideration of the Higgs self-coupling. In Ref. [3], the complete process $W_L^+ W_L^- \rightarrow hh$ where the triple Higgs coupling contributes at tree level and its renormalization plays a role, was in turn considered, confirming the results in Ref. [1]. We also studied the presence of vector resonances ($IJ = 11$) in the spectrum and their characteristics. These resonances appeared after unitarization of the WW elastic partial waves that would otherwise grow uncontrolled with the center-of-mass energy within the HEFT framework.

Unitarization of the amplitudes was carried out, making use of the Inverse Amplitude Method (IAM) [4–13], where the appearance of resonances can be understood after the resummation of an infinite chain of bubble diagrams, hence dynamically. In the vector case, the only such resummation possible is with $I = 1$ intermediate states, $WW \rightarrow ZZ \rightarrow WW \rightarrow \dots \rightarrow ZZ$, but for the scalar amplitude, $I = 0$ double-Higgs state insertions are permitted, leading to a chains of bubbles of the form $WW \rightarrow hh \rightarrow ZZ \rightarrow \dots \rightarrow ZZ$. The details of how to build the multichannel version of the IAM will be specified in the forthcoming sections, and the interested reader may find more information in Refs. [9,14].

It should be clear that unitarization in the $IJ = 00$ channel and the ensuing possible resonances are a very

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promising tool to study and eventually set relevant constraints on the Higgs self-couplings and all other parameters in an effective theory. As stated, a given set of parameters of the effective theory typically leads to the appearance of resonances, required by unitarity. If these resonances happen to have a low mass and should have already been experimentally seen, the absence of detection should translate into bounds on the effective parameters. On the other hand, it may happen (and it does happen) that a given effective theory gives rise to unphysical resonances, located in the first Riemann sheet. The corresponding set of parameters can also be excluded as no fundamental microscopic theory should give rise to acausal behavior. Thus, unitarization and resonances are important not only to reconcile effective theories with experiment but also to set bounds and exclude regions of parameter space. This is one of the purposes of the present work.

When compared to works that made use of the Equivalence Theorem [15–22] (ET) in its extreme version, where no transverse modes of the electroweak gauge bosons were allowed inside the loops [23,24], it was found that including the complete $\mathcal{O}(g)$ calculation did not noticeably modify masses and widths of the vector resonances. Consequently, we found a clear hierarchy among the HEFT coefficients as far as the properties of the resonances is concerned: the positions of the vector poles are mainly controlled by the parameters surviving in the extreme $g = 0$ ET limit, namely, a_4 and a_5 , with only small variations when the new $\mathcal{O}(g)$ operators are introduced via a_3 and ζ parameters [1] (see below for the proper definition of these parameters). This is important because the number of free parameters explodes in effective theories and it is relevant to know beforehand those that may be more relevant for phenomenology.

In this work, we begin by briefly reviewing the basic setup and notation of the HEFT. Then, we proceed to examining the existing bounds on the various parameters involved in the HEFT, paying special attention to those involving the Higgs. In Sec. III, the isospin projections and several technical aspects of the calculations are reviewed in a cursory manner (including several comments on the approximations made). Section IV is devoted to a

discussion, in the present context, on the IAM when several coupled channels are present. A comparison and a discussion on how the presence of coupled channels alters previously existing results is included there, too. The next section is devoted to a in-depth analysis of the various resonances that appear for a range of parameters. It is seen that for certain values of the Higgs potential causality violations appears. This implies that there cannot be a consistent microscopic, i.e., fundamental, theory whose low-energy realization—the HEFT—is described by such values. Other sets of parameters can be excluded because they would lead to resonances that should have been observed.

II. EFFECTIVE LAGRANGIAN

The HEFT is a nonlinear gauge effective field theory (EFT) that includes a light (with respect to the scale of new physics) Higgs-like field, and it is a natural extension of the Electroweak Chiral Lagrangian [25–29]. All the details about how to build this effective theory that only draws from the local and global properties of the electroweak sector can be found in the references listed in Ref. [1]. We will work under the approximation that the custodial symmetry remains exact and consequently the electromagnetism is removed from our theory along with other possible sources of custodial breaking. In practice, it will suffice to set $g' = 0$ and leave custodial breaking operators aside.

The low-energy degrees of freedom of this EFT are the electroweak gauge bosons W^\pm, Z ; their associated Goldstone bosons arising from the spontaneous symmetry breaking ω^\pm, z ; and the light Higgs, which in the nonlinear realization of the chiral symmetry remains a singlet in clear contrast to the $SU(2)_L$ doublet of the linear case. The theory is built as an expansion of powers of the momenta of the external Goldstones that quickly leads to a violation of unitarity of the amplitudes even within the range of convergence of the EFT.

The relevant pieces for the calculations of on-shell $2 \rightarrow 2$ processes in the scalar and vector sector of the HEFT up to next-to-leading order (NLO) and under the assumptions mentioned before are listed hereunder,

$$\mathcal{L}_2 = -\frac{1}{2g^2} \text{Tr}(\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}) - \frac{1}{2g^2} \text{Tr}(\hat{B}_{\mu\nu} \hat{B}^{\mu\nu}) + \frac{v^2}{4} \mathcal{F}(h) \text{Tr}(D^\mu U^\dagger D_\mu U) + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h) \quad (1)$$

$$\begin{aligned} \mathcal{L}_4 = & -ia_3 \text{Tr}(\hat{W}_{\mu\nu} [V^\mu, V^\nu]) + a_4 (\text{Tr}(V_\mu V_\nu))^2 + a_5 (\text{Tr}(V_\mu V^\mu))^2 + \frac{\gamma}{v^4} (\partial_\mu h \partial^\mu h)^2 \\ & + \frac{\delta}{v^2} (\partial_\mu h \partial^\mu h) \text{Tr}(D_\mu U^\dagger D^\mu U) + \frac{\eta}{v^2} (\partial_\mu h \partial_\nu h) \text{Tr}(D^\mu U^\dagger D^\nu U) + i \frac{\zeta}{v} \text{Tr}(\hat{W}_{\mu\nu} V^\mu) \partial^\nu h, \end{aligned} \quad (2)$$

with the building blocks

$$\begin{aligned}
U &= \exp\left(\frac{i\omega^a \sigma^a}{v}\right) \in SU(2)_V, & V_\mu &= D_\mu U^\dagger U, & \mathcal{F}(h) &= 1 + 2a\left(\frac{h}{v}\right) + b\left(\frac{h}{v}\right)^2 + \dots, \\
D_\mu U &= \partial_\mu U + i\hat{W}_\mu U, & \hat{W}_\mu &= g\frac{\vec{W}_\mu \cdot \vec{\sigma}}{2}, & \hat{W}_{\mu\nu} &= \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\mu + i[\hat{W}_\mu, \hat{W}_\nu], \\
V(h) &= \frac{1}{2}M_h^2 h^2 + \lambda_3 v h^3 + \frac{\lambda_4}{4}h^4 + \dots
\end{aligned} \tag{3}$$

The so-called *anomalous*, i.e., not present in the SM, chiral couplings follow the notation in Ref. [1]. The ω^a fields are the Goldstone degrees of freedom that are gathered in the unitary U matrix included in the custodial group $SU(2)_L \times SU(2)_R/SU(2)_V$.

We will use the parametrization $\lambda_{3,4} = d_{3,4}\lambda_{\text{SM}}$, where λ_{SM} is the SM Higgs self-interaction that defines its mass $M_h^2 = 2\lambda_{\text{SM}}v^2$. In our beyond-SM (BSM) theory, the departures from the SM Higgs potential are carried out by the dimensionless couplings d_3 and d_4 . These last parameters will play a key role in the present study since they enter at tree level in the two-Higgs production unlike in the $I = 1$ case.

In our previous work [1], the complete list of counterterms associated to the electroweak and chiral parameters of the custodial-preserving Lagrangian was presented up to order s^2 . In the Landau gauge and by making use of the Equivalence Theorem, we performed the one-loop on-shell renormalization of the three processes involved. We cross-checked our results with previous work that assumed a massless scenario [24] and also with results in an off-shell basis [2,30].

Finally,

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \tag{4}$$

where the last two pieces correspond to the gauge fixing and Faddeev-Popov terms, respectively, essential for adding the quantum corrections. Regarding this matter, we will be working in the Landau gauge ($\xi = 0$) with massless Goldstones and no interactions among the Goldstone sector and the Faddeev-Popov ghosts since they are proportional to the gauge parameter. There is no fundamental reason to do so, but this approach makes the calculations simpler. In the HEFT, there are no interactions between the Higgs and the ghost sector either, the former being a singlet in the nonlinear realization of the chiral symmetry and thereupon absent in both the gauge fixing and Faddeev-Popov terms.

III. RELEVANT AMPLITUDES

Let us begin by reviewing the experimental situation regarding the existing bounds on the couplings of the HEFT. Not all low-energy constants are constrained. At present, there are no bounds available on various $\mathcal{O}(p^4)$ parameters. It is important to note that the Higgs potential

parameters λ_3 and λ_4 (i.e., d_3 and d_4) are poorly constrained and not constrained at all, respectively, from an experimental point of view. See the Table I for existing information of the parameters of the HEFT. However, it should be mentioned that from the results of Ref. [1] it becomes clear that some of the parameters in the $\mathcal{O}(p^4)$ HEFT are not very relevant in determining the properties of the dynamical resonances that appear and consequently they only play a marginal role in the restoration of unitarity. Indeed, of all the effective couplings contributing to the $I = 1$ channel, a_3 and particularly ζ are not that important in determining the mass and width of the resonances. In the $I = 0$ case studied here, various other couplings enter, and it is *per se* interesting to assess the relevance of each of them, as they are in principle unknown except for general order-of-magnitude estimates. Needless to say, it is particularly interesting to assess the relevance of the Higgs scalar couplings for the reasons already mentioned.

In the $I = 0$ channel, there are three $2 \rightarrow 2$ processes that need to be taken into account: $WW \rightarrow WW$, where by WW we refer generically to any initial state with two W or two Z that is compatible with the prescribed isospin projection (we refer to this process as the elastic one); $WW \rightarrow hh$; and $hh \rightarrow hh$. Along the unitarization process, all three become coupled and need to be considered.

TABLE I. Current experimental constraints on bosonic HEFT anomalous couplings at 95% C.L. See our work [1] about the issue to extract the a_4 bound from the CMS analysis of Ref. [31]. Note that d_4 is not constrained at all at present from the experimental point of view.

Couplings	References	Experiments
$0.89 < a < 1.13$	[32]	LHC
$-0.76 < b < 2.56$	[33]	ATLAS
$-3.3 < d_3 < 8.5$	[34]	CMS
d_4
$ a_1 < 0.004$	[35]	Large Electron Positron Collider (LEP) (S parameter)
$-0.06 < a_2 - a_3 < 0.20$	[36]	LEP & LHC
$-0.0061 < a_4 < 0.0063$	[37]	The Compact Muon Solenoid Collaboration (CMS) (from $WZ \rightarrow 4l$)
$ a_5 < 0.0008$	[31]	CMS (from $WZ/WW \rightarrow 2l2j$)

The first step is to consider the tree plus one-loop perturbative contribution to the three subprocesses, including the necessary counterterms [1]. The relevant amplitudes are described below.

A. Tree-level amplitudes and counterterms

In this section, we show the tree-level amplitudes of the processes relevant for the study. The level of precision that we aim for requires having physical gauge bosons in the external states, as said before in the Introduction. For reasons that will be clear immediately below, we just need to compute the three processes presented here to get the suitable scalar-isospin projection. Because of the lengthy

expressions that we get, we give the results split in the different channels participating in the process using the following notation that was used in Ref. [1]: a super-index indicates the different processes labeled as WW for $W^+W^- \rightarrow ZZ$, Wh for $W^+W^- \rightarrow hh$, and hh for $hh \rightarrow hh$. Also, each amplitude carries a subindex xy that represents a process with a particle y propagating in the x channel. In the case with $x = c$ and no y , \mathcal{A}_c represents the contact interaction of the four external particles. For instance, the amplitude \mathcal{A}_{sh}^{WW} represents a Higgs exchanged in the s channel of $W^+W^- \rightarrow ZZ$ scattering. The tree-level amplitudes corresponding to the processes under discussion are as follows:

(1) $W^+W^- \rightarrow ZZ$:

$$\begin{aligned}
 \mathcal{A}_c^{WW} &= g^2(((-2a_3 + a_4)g^2 + 1)((\varepsilon_1\varepsilon_4)(\varepsilon_2\varepsilon_3) + (\varepsilon_1\varepsilon_3)(\varepsilon_2\varepsilon_4)) \\
 &\quad + 2((2a_3 + a_5)g^2 - 1)(\varepsilon_1\varepsilon_2)(\varepsilon_3\varepsilon_4)) \\
 \mathcal{A}_{sh}^{WW} &= -\frac{a^2g^2M_W^2(\varepsilon_1\varepsilon_2)(\varepsilon_3\varepsilon_4)}{(p_1 + p_2)^2 - M_H^2} + \frac{ag^4\zeta}{4((p_1 + p_2)^2 - M_H^2)} [2(\varepsilon_3\varepsilon_4)((p_1\varepsilon_2)(p_2\varepsilon_1) \\
 &\quad - (\varepsilon_1\varepsilon_2)(p_1 + p_2)^2) + 2(\varepsilon_1\varepsilon_2)(p_3\varepsilon_4)(p_4\varepsilon_3)] \\
 \mathcal{A}_{tW}^{WW} &= -\frac{(1 - 2a_3g^2)g^2}{(p_1 - p_3)^2 - M_W^2} [-4((\varepsilon_1\varepsilon_2)(p_1\varepsilon_3)(p_2\varepsilon_4) + (\varepsilon_1\varepsilon_4)(p_1\varepsilon_3)(p_4\varepsilon_2) \\
 &\quad + (\varepsilon_2\varepsilon_3)(p_3\varepsilon_1)(p_2\varepsilon_4) + (\varepsilon_3\varepsilon_4)(p_3\varepsilon_1)(p_4\varepsilon_2)) + 2((\varepsilon_2\varepsilon_4)((p_1\varepsilon_3)(p_2 + p_4)\varepsilon_1 + (p_3\varepsilon_1)(p_2 + p_4)\varepsilon_3) \\
 &\quad + (\varepsilon_1\varepsilon_3)((p_2\varepsilon_4)(p_1 + p_3)\varepsilon_2 + (p_4\varepsilon_2)(p_1 + p_3)\varepsilon_4) - (\varepsilon_1\varepsilon_3)(\varepsilon_2\varepsilon_4)((p_1 + p_3)p_2 + (p_2 + p_4)p_1)] \\
 \mathcal{A}_{uW}^{WW} &= \mathcal{A}_{tW}(p_3 \leftrightarrow p_4, \varepsilon_3 \leftrightarrow \varepsilon_4),
 \end{aligned} \tag{5}$$

where ε_i is the abbreviation for $\varepsilon_L(p_i)$.

(2) $WW \rightarrow hh$:

$$\begin{aligned}
 \mathcal{A}_c^{Wh} &= \frac{g^2b}{2}(\varepsilon_1\varepsilon_2) - \frac{g^2\eta}{v^2}((\varepsilon_1p_4)(\varepsilon_2p_3) + (p_3\varepsilon_1)(\varepsilon_2p_4)) - \frac{2g^2\delta}{v^2}(p_3p_4)(\varepsilon_1\varepsilon_2) \\
 &\quad + \frac{g^2\zeta}{v^2}((\varepsilon_1\varepsilon_2)(p_1 + p_2)^2 - 2(p_1\varepsilon_2)(p_2\varepsilon_1)) \\
 \mathcal{A}_{sh}^{Wh} &= \frac{3g^2M_h^2}{2((p_1 + p_2)^2 - M_h^2)}(a(\varepsilon_1\varepsilon_2) + \frac{\zeta}{v^2}((\varepsilon_1\varepsilon_2)(p_1 + p_2)^2 - 2(p_1\varepsilon_2)(p_2\varepsilon_1))) \\
 \mathcal{A}_{t\omega}^{Wh} &= \frac{2a^2g^2 + a\zeta g^4}{2(p_1 - p_3)^2}((p_3\varepsilon_1)(p_4\varepsilon_2)) \\
 \mathcal{A}_{tW} &= \frac{a^2g^2M_W^2}{((p_1 - p_3)^2 - M_W^2)}\left(\varepsilon_1\varepsilon_2 + \frac{(p_4\varepsilon_2)(\varepsilon_1p_3)}{(p_1 - p_3)^2}\right) + \frac{ag^4\zeta}{2((p_1 - p_3)^2 - M_W^2)}(2M_h^2(\varepsilon_1\varepsilon_2) \\
 &\quad - (p_4\varepsilon_2)(p_2\varepsilon_1) - (\varepsilon_1p_3)(\varepsilon_2p_3) + M_W^2\frac{(p_4\varepsilon_2)(\varepsilon_1p_3)}{(p_1 - p_3)^2}) \\
 \mathcal{A}_{u\omega}^{Wh} &= \mathcal{A}_{t\omega}^{Wh}(p_3 \leftrightarrow p_4) \\
 \mathcal{A}_{uW}^{Wh} &= \mathcal{A}_{tW}^{Wh}(p_3 \leftrightarrow p_4).
 \end{aligned} \tag{6}$$

(3) $hh \rightarrow hh$:

$$\begin{aligned}
\mathcal{A}_c^{hh} &= \frac{8\gamma}{v^4} ((p_1 p_4)(p_2 p_3) + (p_1 p_3)(p_2 p_4) + (p_1 p_2)(p_3 p_4)) - 6\lambda_4 \\
\mathcal{A}_{sh}^{hh} &= -\frac{36\lambda_3^2 v^2}{(p_1 + p_2)^2 - M_h^2} \\
\mathcal{A}_{th}^{hh} &= \mathcal{A}_{sh}^{hh}(p_2 \leftrightarrow -p_3) \\
\mathcal{A}_{uh}^{hh} &= \mathcal{A}_{sh}^{hh}(p_2 \leftrightarrow -p_4).
\end{aligned} \tag{7}$$

In all the previous expressions, the various couplings and parameters v, a, b, a_3, a_4, \dots contain the corresponding counterterms needed for the one-loop renormalization: $v \rightarrow v + \delta v, a \rightarrow a + \delta a, b \rightarrow b + \delta b, a_3 \rightarrow a_3 + \delta a_3, \dots$. The set of counterterms required to render finite the physical amplitudes is provided in the Appendix.

In Ref. [1], the interested reader can find a more detailed construction of the isospin projections (isoscalar, isovector, and isotensor) of the $2 \rightarrow 2$ processes that concern us here. In this section, we will only summarize the main points relevant for the $I = 0$ case. We emphasize that in the scalar case the two-Higgs final state is also present and that through the coupled channel the unitarization mechanism contributes to the elastic $WW \rightarrow WW$ channel, and this is a relevant fact because this turns out to be easier to handle experimentally than processes involving two-Higgses final states, as we will see in the coming sections. This opens an interesting window: the dynamical resonances potentially present in elastic WW scattering carry information from Higgs production. We will exploit this potential below.

The generic process $W_L^a W_L^b \rightarrow W_L^c W_L^d$, assuming an exactly preserved custodial symmetry and using Bose symmetry, can be written

$$\begin{aligned}
\mathcal{A}^{abcd} &= \delta^{ab} \delta^{cd} \mathcal{A}(p^a, p^b, p^c, p^d) \\
&+ \delta^{ac} \delta^{bd} \mathcal{A}(p^a, -p^c, -p^b, p^d) \\
&+ \delta^{ad} \delta^{bc} \mathcal{A}(p^a, -p^d, p^c, -p^b),
\end{aligned} \tag{8}$$

which allows us to write the following amplitudes in the more familiar charge basis:

$$\begin{aligned}
\mathcal{A}^{+-00} &= \mathcal{A}(p^a, p^b, p^c, p^d) \\
\mathcal{A}^{+--+} &= \mathcal{A}(p^a, p^b, p^c, p^d) + \mathcal{A}(p^a, -p^c, -p^b, p^d) \\
\mathcal{A}^{++++} &= \mathcal{A}(p^a, -p^c, -p^b, p^d) + \mathcal{A}(p^a, -p^d, p^c, -p^b).
\end{aligned} \tag{9}$$

Given this, one can see that once the ‘‘fundamental’’ amplitude $W_L^+ W_L^- \rightarrow Z_L Z_L$ has been computed the other ones are obtained simply by crossing symmetry.

Since we are interested in a partial-wave analysis of unitarity, it is suitable to build the fixed-isospin amplitudes of the process that read

$$\begin{aligned}
T_0 &= 3\mathcal{A}^{+-00} + \mathcal{A}^{++++} \\
T_1 &= 2\mathcal{A}^{+--+} - 2\mathcal{A}^{+-00} - \mathcal{A}^{++++} \\
T_2 &= \mathcal{A}^{++++}.
\end{aligned} \tag{10}$$

In contrast to our previous work, we will now be interested in the scalar projection, T_0 .

Within our theoretical framework, the Higgs is a weak isospin singlet ($I = 0$), so it is straightforward to write the following relations for the $W_L^a W_L^b \rightarrow hh$ and $hh \rightarrow hh$ processes:

$$\begin{aligned}
\mathcal{M}(W_L^a W_L^b \rightarrow hh) &= \mathcal{M}^{ab}(p^a, p^b, p_{h,1}, p_{h,2}), \\
T_{Wh,0} &= \sqrt{3}\mathcal{M}^{+-},
\end{aligned} \tag{11}$$

$$\mathcal{T}(hh \rightarrow hh) = \mathcal{T}(p_{h,1}, p_{h,2}, p_{h,3}, p_{h,4}) = \mathcal{T}_{hh,0}. \tag{12}$$

B. One-loop real part: The Equivalence Theorem

Taking into account that we will be eventually interested in exploring a large set of parameters, it is important to be equipped with computational algorithms that run fast. This is one of the reasons that make convenient, in order to determine the real part of the one-loop amplitude, to appeal to the so-called Equivalence Theorem. This allows us to replace the scattering of physical W 's by the one of the corresponding Goldstone bosons. The details concerning this approximation can be found in Ref. [38] and were also briefly reviewed in Ref. [1].

Another reason why it may be convenient to use the ET has to do with the convenience to make direct contact with some existing analytical results that are only available in the ET limit and for $g = 0$, where the expressions become analytically tractable, because a full one-loop calculation in the $g \neq 0$ limit involves expressions that do not have a simple analytical continuation to the complex s plane.

We chose to work in the 't Hooft-Landau gauge that is particularly simple from the renormalization point of view. As discussed in Ref. [1], the results we obtain should be gauge invariant at the order where we are working, even if, in principle, the leading order of the ET approximation is by itself not gauge invariant. Recall that the ET relies on the splitting of the polarization vector $\epsilon_L^\mu = k^\mu/M_W + v^\mu$, with v^μ being of order M_W/\sqrt{s} . We note that we use the ET for the one-loop correction only, not for the tree-level contribution that is calculated using physical W 's. The one-loop correction to the partial wave is of $O(s^2)$, and the (potentially gauge-dependent) corrections to the ET might change the $O(s)$ contribution, but the latter—tree level—is calculated exactly without appealing to the ET. Therefore, gauge invariance is respected even if the splitting is itself not gauge invariant. As a sanity check, where a comparison can be made, all counterterms agree with those computed in a general gauge.

A further check is provided by comparing the imaginary part obtained in this way with the one computed exactly via the optical theorem.

Even using the ET, the one-loop real part cannot be expressed when masses are not neglected in terms of simple functions and the results are derived numerically. Fortunately, most of the effective coefficients enter only at order s^2 , and then they are formally tree level from a computational point of view, even though they give contributions of the same order as the one loop. This makes exploring the parameter landscape simpler. An exception to this rule are the parameters a , b , d_3 , and d_4 that enter at order s . Accordingly, every modification of any of those requires a new one-loop calculation of the real part.

For further reference, we give below the expression for the tree and one-loop results in the limit $M_W = M_H = 0$ [24]. They are useful to identify physical poles in the full-fledged calculation that, as said, is not amenable to analytical continuation to the appropriate Riemann sheet. In the limit where all the particles are massless and hence the SM values g and λ_{SM} are set to zero, the amplitudes with W s in the external states vanish, and if we want to have some analytical expressions for the tree level for our unitarity study, we are forced to go to the ET and place Goldstone bosons in the external legs.

The authors in Ref. [24] worked with the chiral parameters α and β , instead of a and b , since they introduced a vacuum-tilt extra free parameter, $\xi = \sqrt{\frac{v}{f}}$, interpolating between composite models ($v = f$) and the SM limit ($f \rightarrow \infty$) where the new resonant states completely decouple from the theory (this vacuum-tilt parameter should not be confused with the gauge parameter). In fact, with the massless and naive custodial limit $g = g' = 0$ in Ref. [24] [also known as *naive Equivalence Theorem* (nET)], all the gauge dependence disappears, and all the amplitudes are trivially gauge invariant. This parametrization makes contact with ours with the redefinitions $a = \alpha\sqrt{\xi}$ and $b = \beta\xi$. In our framework, only the electroweak scale is used for the Higgs mechanism and weighs both Goldstone and Higgs fields, so we rewrite their amplitudes in the particular case: $\xi = 1$, $a = \alpha$, and $b = \beta$.

The expressions are shown below, and, in contrast to the full calculation previously described, due to the simplicity of the formulas, we do not split the full amplitude in the different channels:

(1) $\omega\omega \rightarrow \omega\omega$ —Massless limit:

$$\mathcal{A}^{\text{tree}} = (1 - a^2) \frac{s}{v^2} + \frac{4}{v^4} (2a_5 s^2 + a_4 (t^2 + u^2)) \quad (13)$$

$$\mathcal{A}^{\text{loop}} = \frac{1}{576\pi^2 v^4} [f^W(s, t, u) s^2 + (1 - a^2)^2 (g(s, t, u) t^2 + g(s, u, t) u^2)] \quad (14)$$

with the definitions

$$f^W(s, t, u) = 20 - 40a^2 + 56a^4 - 72a^2b + 36b^2 + \Delta(12 - 24a^2 + 30a^4 - 36a^2b + 18b^2) \\ + (-18 + 36a^2 - 36a^4 + 36a^2b - 18b^2) \log\left(\frac{-s}{\mu^2}\right) + 3(1 - a^2)^2 \left[\log\left(\frac{-t}{\mu^2}\right) + \log\left(\frac{-u}{\mu^2}\right) \right] \quad (15)$$

$$g(s, t, u) = 26 + 12\Delta - 9 \log\left(\frac{-t}{\mu^2}\right) - 3 \log\left(\frac{-u}{\mu^2}\right).$$

(2) $\omega\omega \rightarrow hh$ —Massless limit:

$$\mathcal{M}^{\text{tree}} = (a^2 - b) \frac{s}{v^2} + \frac{2\delta}{v^4} s^2 + \frac{\eta}{v^4} (t^2 + u^2) \quad (16)$$

$$\mathcal{M}^{\text{loop}} = \frac{a^2 - b}{576\pi^2 v^2} \left[f^{\text{WH}}(s, t, u) \frac{s^2}{v^2} + \frac{a^2 - b}{v^2} (g(s, t, u)t^2 + g(s, u, t)u^2) \right], \quad (17)$$

where

$$\begin{aligned} f^{\text{WH}}(s, t, u) = & -8(-9 + 11a^2 - 2b) - 6\Delta(-6 + 7a^2 - b) - 36(1 - a^2) \log\left(\frac{-s}{\mu^2}\right) \\ & + 3(a^2 - b) \left(\log\left(\frac{-t}{\mu^2}\right) + \log\left(\frac{-u}{\mu^2}\right) \right) \end{aligned} \quad (18)$$

and the function $g(s, t, u)$ is the same as in the elastic case.

(3) $hh \rightarrow hh$ —Massless limit:

$$\mathcal{T}^{\text{tree}} = \frac{2\gamma}{v^4} (s^2 + t^2 + u^2). \quad (19)$$

Notice that this process has no $\mathcal{O}(p^2)$ contribution since in the massless Higgs limit the triple self-coupling of the Higgs vanishes and there is no diagram contributing to the process,

$$\mathcal{T}^{\text{loop}} = \frac{3(a^2 - b)^2}{32\pi^2 v^4} (f^H(s)s^2 + f^H(t)t^2 + f^H(u)u^2), \quad (20)$$

with

$$f^H(s) = 2 + \Delta - \log\left(\frac{-s}{\mu^2}\right). \quad (21)$$

C. One-loop imaginary part: The optical theorem

With respect to the imaginary part of the one loop calculation, it is most easily determined exactly by using

$$\mathcal{A}_{\text{loop}}^{(4)} = \text{Re}[\mathcal{A}_{\text{loop}}^{(4)}(\omega^+\omega^- \rightarrow zz)] + i(\sigma_W(s)|\mathcal{A}_{\text{tree}}^{(2)}|^2 + \sigma_H(s)|\mathcal{A}_{\text{tree}}^{(2)}(W_L^+W_L^- \rightarrow hh)|^2). \quad (24)$$

$\mathcal{A}_{\text{tree}}^{(2)}(W_L^+W_L^- \rightarrow hh)$ in the imaginary part is the tree level amplitude of the VBS two-Higgs production, Eq. (6), and

$$\sigma_{W,H} = \sqrt{1 - \frac{4M_{W,H}^2}{s}}.$$

Can we safely compute the real part of the one-loop amplitude within the ET? The technical issue of gauge invariance was already discussed in the previous subsection. Now, we can ask ourselves what the precision of such

the optical theorem. The details about our calculation of the imaginary part using this theorem are gathered in Ref. [1], but for the scalar case, we find one difference with respect to our previous study: an intermediate double-Higgs $I = 0$ state is now permitted.

Once we know the discontinuity of a complex amplitude, $\mathcal{A}(s)$, across the physical cut, we find

$$\text{Im}\mathcal{A}(s) = \sum_{|\psi(I=0)\rangle} \sigma(s)|\mathcal{A}(s)|^2, \quad (22)$$

where $\sigma(s) = \sqrt{1 - \frac{(M_1+M_2)^2}{s}}$ is the two-body phase space. This allows us to compute the imaginary part of any amplitude at the one-loop level from the tree-level result.

As an example, we show the different contributions in the full $I = 0$ isospin amplitude in the process $W_L^+W_L^- \rightarrow Z_L Z_L$:

$$\mathcal{A}(W_L^+W_L^- \rightarrow Z_L Z_L) = \mathcal{A}_{\text{tree}}^{(2)} + \mathcal{A}_{\text{tree}}^{(4)} + \mathcal{A}_{\text{loop}}^{(4)}. \quad (23)$$

$\mathcal{A}_{\text{tree}}^{(2)} + \mathcal{A}_{\text{tree}}^{(4)}$ is the full tree-level contribution (5) and $\mathcal{A}_{\text{loop}}^{(4)}$ is the one-loop amplitude

an approximation is. In the context of partial wave analysis, *perturbative unitarity* relates the imaginary part of the NLO wave and its leading-order (LO) modulus

$$\text{Im}t_{00}^{\text{WW},(4)} = \sigma_W |t_{00}^{\text{WW},(2)}|^2 + \sigma_H |t_{00}^{\text{Wh},(2)}|^2. \quad (25)$$

These partial waves are defined in detail in the next section.

Our test consists in computing the left-hand side of (25) within the ET, via a one-loop calculation, and checks whether the relation (25), with the rhs determined using physical W 's, stands and what level of agreement is obtained. For this, let us define the error, in a percentage, assumed by the ET with the quantity

$$\Delta_{\text{Full-ET}} = \frac{|Im t_{00}^{\omega\omega,(4)} - (\sigma_W |t_{00}^{WW,(2)}|^2 + \sigma_H |t_{00}^{WH,(2)}|^2)|}{\sigma_W |t_{00}^{WW,(2)}|^2 + \sigma_H |t_{00}^{WH,(2)}|^2} \cdot 100. \quad (26)$$

This quantity includes, by construction, couplings from the leading-order Lagrangian (1), and it is completely independent of any $\mathcal{O}(p^4)$ parameter since our calculation is made up to $\mathcal{O}(p^4)$ and they just enter at tree level and consequently do not produce any imaginary part for the left-hand side of Eq. (25).

Figure 1 shows $\Delta_{\text{Full-ET}}$ for the BSM-HVV interaction $a = 0.9$, the self-couplings of the Higgs set to their standard values, and various b values. The behavior is that expected for the ET: the longitudinal components of the electroweak gauge bosons are well represented by the associated Goldstone boson for high energies compared to the gauge boson masses. This is independent of the value of b in the plot.

What we also observe in Fig. 1 is that for values of b close to $a^2 = 0.9^2 = 0.81$ the error grows. This “failure” of the ET can be understood by going to the completely

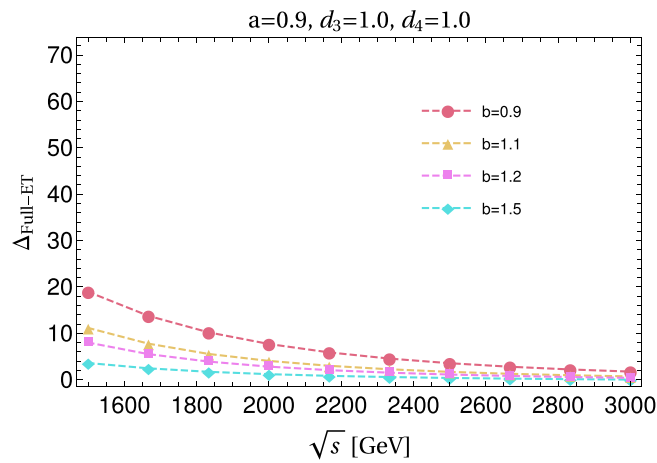


FIG. 1. In this plot, we show in a percentage the quantity $\Delta_{\text{Full-ET}}$ defined in (26) plotted with respect to the center-of-mass energy. In this figure, we set $a = 0.9$, $d_3 = 1.0$, and $d_4 = 1.0$ and show different values of b . This figure is independent of $\mathcal{O}(p^4)$ parameters. This shows that for large values of s the imaginary part computed via the ET agrees at the 10% level with the one determined (exactly) via the optical theorem. As explained in the text, although the discrepancy may look large at low values of s , the amplitude is very small there and does not contribute significantly to the position and width of possible resonances. See also Fig. 2.

massless limit, useful for $\sqrt{s} \gg M_{\omega,h}$, where the leading-order $\omega\omega$ and ωh amplitudes are proportional to $(1 - a^2)$ and $(a^2 - b)$ [Eqs. (13) and (16)], respectively. For a fixed value of a , close to the SM which cancels the $\omega\omega$ massless amplitude, the closer b is to 0.81 , the worse the comparison with the full calculation is. This happens because the right-hand side of (25) approaches zero, being proportional to $b - a^2$, and cancels the leading part of the denominator of $\Delta_{\text{Full-ET}}$. Generally speaking, if one considers the SM parameters $b = a^2 = 1$, we do not find good agreement, and more terms in the ET expansion, such as $\mathcal{O}(g^2)W_L\omega \rightarrow \omega\omega$, would be needed at low s values as explained in Ref. [38].

However, the apparent failure just described is actually a mirage because we are dealing with partial waves that are very small numerically. To see this, we show in Fig. 2 a check of perturbative unitarity for a benchmark point away from the SM by a 10%, except for the Higgs self-couplings which remain set in their SM values.

It could seem that Fig. 2 enters into contradiction with the previous Fig. 1 that shows a worse agreement for the same benchmark point, while it is almost unnoticeable for the low-energy points in the former. This situation, that is reproduced with any choice of parameters, is explained by the fact that, by construction, the chiral amplitudes are much smaller at low energies than in the high-energy regime where this uncontrolled growth leads precisely to the violation of unitarity. Having said that, we can conclude that it is safe for us to make use of the ET for the one-loop level along the whole range of energies since the differences with the full calculation are on one hand negligible in the high-energy regime and, on the other

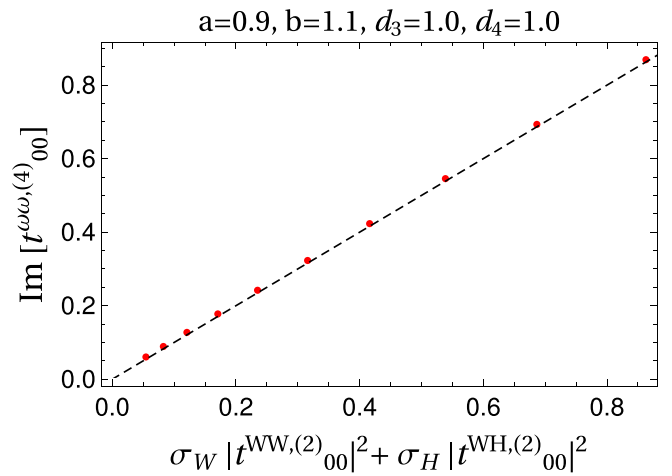


FIG. 2. Figure showing perturbative unitarity (25) for the chiral couplings specified in the title. The data shown here are ten red points equally spaced along the energy range 1500–3000 GeV, within the validity region of the theory. The dashed line is the bisector of the first quadrant, and the points that lie over it satisfy perturbative unitarity exactly.

hand, the low-energy contributions are much smaller when one is very close to the SM values. Away from the SM limit (which is of course our main focus), the agreement is good everywhere.

IV. UNITARIZATION

The resonances that we are seeking cannot be described by a series expansions in the momenta since they should arise as poles in S -matrix elements and that is why we need nonperturbative methods to extend the predictivity of the low-energy theory, that eventually will lose unitarity, to the strongly interacting regime at higher energies.

The loss of unitarity requires unitarization methods, and there is a variety of such methods (K-matrix, N/D , IAM, etc.; see Ref. [12] for a complete summary), but they have been proven to show the same qualitative results. They are based on partial wave analysis and make use of amplitudes with fixed spin (J) and isospin (I) after the projection

$$t_{IJ}(s) = \frac{1}{32K\pi} \int_{-1}^1 d(\cos\theta) P_J(\cos\theta) T_I(s, \cos\theta), \quad (27)$$

where K is a constant whose value is $K = 2$ or 1 depending on whether the particles participating in the process are identical or not and T_I are the fixed isospin amplitudes that are built from Feynman diagrams and weak isospin relations. These are Eqs. (9)–(12) of Sec. III A.

As we did in the vector case, we will assume that the scalar wave admits an expansion in powers of the momenta

$$t_{00} = t_{00}^{(2)} + t_{00}^{(4)} + \dots, \quad (28)$$

restricting ourselves to the lowest order ($I, J = 0, 0$) for the study. Hence, with this analysis, we will just be looking for scalar-isoscalar resonances.

A. Coupled channel formalism

In our preceding study [1], we focused on the case of vector-isovector resonances present in the $W_L W_L$ elastic scattering, so the single-channel formalism of the IAM was the way to go. As already anticipated in the Introduction, for the scalar case, we must take into account the presence of scalar waves coming from double-Higgs configurations (in addition to the $I = 0$ projection of $\omega\omega$) in the intermediate states of the resummation of bubble diagrams. This mixing of different possible intermediate states is represented in the matrix form of the scalar-isoscalar partial wave

$$t_{00} = \begin{pmatrix} t_{00}^{WW} & t_{00}^{Wh} \\ t_{00}^{Wh} & t_{00}^{hh} \end{pmatrix} \quad (29)$$

that is the fundamental structure that will be rendered unitary.

For the case $b = a^2$ and in the high-energy limit where the mass of the Higgs can be neglected, the off-diagonal elements (what we call the crossed channel) of (29) vanish in the ET limit when we set $g = 0$, i.e., in the nET framework. This actually leads to the decoupling limit: there is no mixing among the different scalar channels whatsoever. However, this is not true as soon as we set $g \neq 0$, even if $b = a^2$, and the full coupling matrix needs to be considered.

It can be found, for example in Ref. [9], that when cutting the expansion of the scalar wave at NLO [$\mathcal{O}(p^4)$] the multichannel IAM amplitude is just the generalization of the elastic case in matrix form

$$t_{00}^{IAM} = t_{00}^{(2)} \cdot (t_{00}^{(2)} - t_{00}^{(4)})^{-1} \cdot t_{00}^{(2)}. \quad (30)$$

The elements of the IAM matrix are all the unitary scalar waves participating in the process up to NLO: unitary WW in the first diagonal entry, Wh in the off-diagonal and hh in the second diagonal element.

This IAM matrix, besides keeping the analytical properties on the right cut required for partial wave analysis, has a low-energy expansion that coincides with (28) and fulfills the exact unitarity condition

$$Im t_{00}^{IAM} = \sigma (t_{00}^{IAM})^\dagger t_{00}^{IAM}, \quad (31)$$

where σ is the two-body phase space. At this point, we find an ambiguity in the crossed channel of this expression: we have two kinds of particles with different masses, the gauge bosons and the Higgs, but yet we only include a unique phase space, that we choose to be the one with the W boson mass, $\sigma = \sqrt{1 - \frac{4M_W^2}{s}}$. This choice, of course, will be of no relevance at the high-energy regime that we want to explore where $M_W^2 \approx M_H^2 \ll s$ and $\sigma \approx 1$.

From (30), it can be seen how the scalar resonances, if present, are located at the zeros of the determinant

$$\Delta_{00}(s_R) \equiv \det(t_{00}^{(2)}(s_R) - t_{00}^{(4)}(s_R)) = 0, \quad (32)$$

where the Breit-Wigner resonances occur at $s_R = (M_R - \frac{i}{2}\Gamma_R)^2$ in the s -complex plane.

With this coupled-channel formalism, more channels are available for the resummation of the intermediate and in the final states, making the resonances appearing in the scattering characteristically broader. They are short lived, compared to those found in single-channel, massive states. If these poles in the zeros of the determinant (32) are to be interpreted as Breit-Wigner-like resonant states, we will be applying the broadly used criterion that the width satisfies $\Gamma < \frac{M}{4}$, meaning this that the pole is located near the real axis as one can see from the definition of s_R above. Otherwise, we would have found a simple enhancement of the scalar amplitude not to be interpreted as a physical

pole with such an enhancement produced by the presence of a pole far from the real axis.

As in the vector case, we shall be looking for poles appearing in the second Riemann sheet where the Breit-Wigner interpretation leads to positive widths, required by causality arguments. If some pole appears in the first Riemann sheet, where imaginary parts are positive, it would be associated with a spurious resonance with negative width that cannot be present in a physical theory. Thus, we find here an empirical approach in order to discriminate *a priori* plausible sets of parameters in the HEFT.

B. Scalar resonances

Now that we have presented the coupled-channel formalism for the scalar waves, we are prepared to search for scalar resonances in the chiral parameter space.

Following the way of building the fixed $I = 0$ isospin amplitude in Ref. [1], the scalar partial wave is obtained from (27)

$$t_{00} = \frac{1}{64\pi} \int_{-1}^1 d(\cos\theta) T_0(s, \cos\theta), \quad (33)$$

where we have used that the Legendre polynomial $P_0(\cos\theta) = 1$.

Previous works such as Refs. [39,40] already searched for scalar resonances in WW scattering following the procedure developed in the preceding section. These works relied on the ET (even at tree level) in the naive custodial limit, $g = g' = 0$, and the former assumed a completely massless scenario, so both of them could get exact analytical continuations of the partial waves to the second Riemann sheet to look for the resonant states. This is a step that we are not able to perform in our calculation, as the resulting expressions do not have an analytic treatment.

The first task for our numerical analysis, following the ideas in Ref. [1], is to find modifications in the properties of the scalar resonances studied, e.g., in Ref. [40] once one relaxes the $g = 0$ approximation and includes gauge bosons in the external states at tree level and in internal lines of the one-loop calculation. In that study, the authors considered the relevant chiral parameter space giving scalar resonance masses in the range $1.8 \text{ TeV} < M_S < 2.2 \text{ TeV}$. No coupled-channel formalism was used, and instead they assumed the decoupled-channel limit within the nET by setting $b = a^2$ for the particular case $b = a = 1$. For some benchmark points in the mentioned region, we get the modifications on the location of the scalar points after allowing for transverse gauge propagation (see Table II).

From this table, we extract similar conclusions as in the vector case. On one hand, the masses of the scalar resonances are pushed up by 2%–3% once the $SU(2)_L$ coupling is set to its SM value, the very same behavior as for $I = 1$. On the other hand, we observe variations in the

TABLE II. Values for the location of the scalar poles $\sqrt{s_S} = M_S - \frac{i}{2}\Gamma_S$ for $g = 0$ and $g \neq 0$ for some points in the $a_4 - a_5$ plane and in the decoupling limit $b = a^2$ within the nET with $a = b = 1$. The self-interactions of the Higgs are set to the SM values. Note that the coupling to other $I = 0$ channels is ignored here for the purpose of assessing the effect of switching on the transverse modes.

$\sqrt{s_S}$ (GeV)	$a_4 \times 10^4$	$a_5 \times 10^4$	$g = 0$	$g \neq 0$
1	1	-0.2	$1805 - \frac{i}{2}130$	$1856 - \frac{i}{2}125$
2	2	-1	$2065 - \frac{i}{2}160$	$2119 - \frac{i}{2}150$
3.5	3.5	-2	$2175 - \frac{i}{2}170$	$2231 - \frac{i}{2}163$

widths¹ of around 4%–6%, values much greater than in the vector case where the differences were almost unnoticeable. This gives us an idea of the significance of the propagation of transverse modes.

However, the above results for M_S, Γ_S are only tentative because when one wants to make a full calculation beyond the nET and consider physical vector bosons in the external states, even in the case where $b = a^2$, there is no decoupling and one needs the coupled-channel formalism to get a proper description of the dynamics of the system in the $IJ = 00$ channel. Let us now proceed to study how coupling the various relevant channels affects the results. The modifications will be substantial in fact.

All the $\mathcal{O}(p^2)$ parameters are included in all the amplitudes of (29), but as one can see from the \mathcal{L}_4 Lagrangian (2), not every $\mathcal{O}(p^4)$ coupling affects all the channels. In particular, WW depends on a_4, a_5, a_3 , and ζ ; Wh depends on δ, η , and ζ ; and the hh elastic process depends only on γ . The operators accompanying these couplings could eventually dominate the corresponding amplitudes at high energies due to the presence of the four derivatives. However, not all these couplings contribute to the NLO scalar amplitude with the same strength. The aforementioned contribution is represented in Fig. 3 for values of the parameters of the expected (absolute) size of 10^{-3} .

From Fig. 3, we see an evident hierarchy among the different couplings: a_4 and a_5 contributions are much more relevant than those of a_3 and ζ in the elastic WW . For the crossed scattering, Wh, δ , and η contributions are much more important than the one of ζ . This picture reinforces the conclusion that those operators surviving in the $g = 0$ limit (the nET limit) are more relevant than the other ones. The reason why this happens lies in the fundamental

¹In this work the widths are determined by measuring the difference in energy between the values where the resonance curve intercepts a straight line drawn at half the value of the maximum of the resonance curve. This is a simple method that should not differ significantly from an exact determination of the pole position in the complex energy plane, which is not accessible to us.

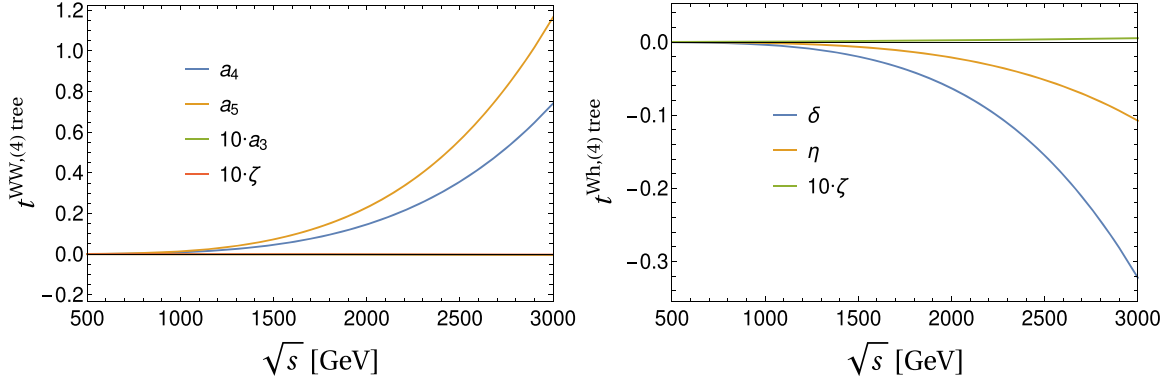


FIG. 3. Plot of the NLO tree-level scalar wave separated in the different chiral coupling contributions for (left axis) the elastic WW and (right axis) the crossed channel Wh . All the values are chosen to be of the maximum expected size of 10^{-3} .

structure of the HEFT. To be consistent in the chiral counting, both Higgs mass ($\sim\sqrt{\lambda_{\text{SM}}}v$) and EW gauge boson mass ($\sim gv$) must be understood as $\mathcal{O}(p)$ soft scales; therefore, a local operator with one gauge coupling plus three derivatives (like those accompanying a_3 and ζ) is of chiral order 4, just like one with four derivatives (a_4, a_5, \dots), but the latter dominates by far at high energies.

The behavior presented above agrees with what we found from vector-isovector resonances [1]: the pole position was almost completely determined by a_4 and a_5 with subleading effects after adding a_3 and ζ , at least for values of a, b, d_3 , and d_4 close to the SM values. This is why in the forthcoming analysis, in order to keep it as simple as possible, we will only consider the influence of a_4 and a_5 in determining the properties of resonances in the $IJ = 11$ channel and neglect the role of a_3 and ζ .

The space of parameters to analyze in the $IJ = 00$ case is considerably larger than in the vector case, and some sort of hierarchy is needed in order to proceed. One point to check is whether in the scalar case a_4 and a_5 dictates to a very good approximation the structure of resonances as it happens in the vector case (assuming for the time being that we stay close to the SM values $a = b = d_3 = d_4 = 1$). To study this, we will focus first on the benchmark points (BPs) in the $a_4 - a_5$ plane defined in Table III. Other works have studied the spectrum of resonances in WW scattering,

in particular the group of Ref. [41] that made use of Weinberg sum rules [42] derived from the W^3B correlator, to set minimal bounds for the masses of vector resonances allowed by experimental constraints of the chiral parameters. For the region in the $a_4 - a_5$ plane that we are interested in, they found that, for any scenario where an axial state is decoupled, the minimal mass for an experimentally allowed vector resonance is around 2 TeV. We slightly relax that condition and require a parameter space where, if present, the vector resonances satisfy $M_V \gtrsim 1.8$ TeV. We choose the minimal mass for any observable scalar resonance to be the same value of $M_S \gtrsim 1.8$ TeV and assume that any lighter state should have already been seen in the experiment.

At this point, one should recall that only particular combinations of a_4 and a_5 appear in the various channels, namely, $5a_4 + 8a_5$ for $IJ = 00$, $a_4 - 2a_5$ for $IJ = 11$, and $2a_4 + a_5$ for $IJ = 20$ [43]. In previous studies, it was found that isotensor resonances are always acausal and the corresponding region $2a_4 + a_5 < 0$ is to be excluded from our considerations. Thus, we select BPs outside the region excluded by isotensor acausal resonances [44] and within the vector-isovector and scalar-isoscalar space. In particular, we select one BP (BP1) that belongs to the region where both isovector and isoscalar resonances appear, satisfying the condition commented above regarding the vector resonance mass. The other two BPs (BP2 and BP3)

TABLE III. Properties of the scalar resonances for the selected benchmark points in the $a_4 - a_5$ plane, with the $\mathcal{O}(p^2)$ parameters set to their standard values, in both single-channel (S.C.) and coupled-channel (C.C.) formalism. We also include the values of the properties of vector resonances if present. The dash symbol $-$ indicates the absence of a zero in the determinant, Eq. (32). The $\mathcal{O}(p^2)$ chiral parameters are set to their SM values. We see that coupling channels modifies very substantially masses and widths. Those poles not fulfilling the resonance condition are in boldface.

	$a_4 \times 10^4$	$a_5 \times 10^4$	S.C.	C.C.	$M_V - \frac{i}{2}\Gamma_V$
BP1	3.5	1	1044 - $\frac{i}{2}$ 50	1844 - $\frac{i}{2}$487	2540 - $\frac{i}{2}$ 27
BP2	-1	2.5	1219 - $\frac{i}{2}$ 75	2156 - $\frac{i}{2}$637	...
BP3	1	1	1269 - $\frac{i}{2}$ 75	2244 - $\frac{i}{2}$675	...

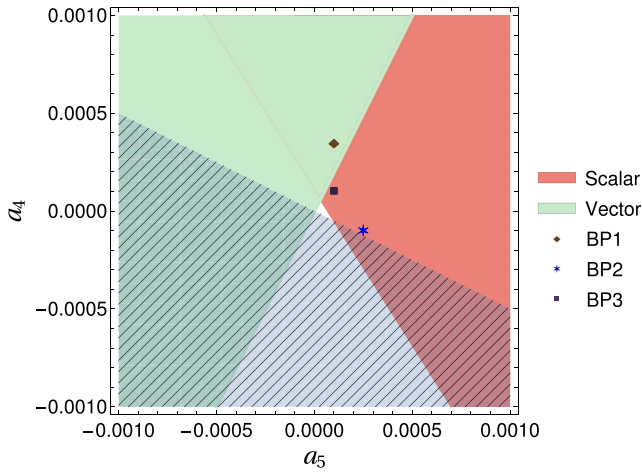


FIG. 4. Regions in the $a_4 - a_5$ plane where scalar (red) and vector (green) resonant states appear. The striped area represents excluded parameter space by the presence of acausal isotensor resonances. The benchmark points used in this study are marked in the plot. One of them, BP1, lies in the region where both isoscalar and isovector states show up, and the other two, BP2 and BP3, lie in the purely isoscalar sector.

lie in the purely scalar-isoscalar region. In Fig. 4, we see the position of the BPs that we have just described within the $a_4 - a_5$ plane where resonances are present when the coupled-channel formalism is applied.

These BPs are gathered in Table III where we also include, even if they do not have a physical relevance, the values after applying the single channel formalism to the WW scalar wave, obviating the crossed channel and the elastic hh scattering. Both values are obtained with $g \neq 0$.

The first thing that one notices is that when (correctly) considering coupled channels the results differ considerably

from the ones one would obtain in the single channel and the resonance masses and widths visibly increase. Recall that here we are assuming $a = b = 1$ where naively one would expect to have decoupling (this is the case in the nET). This is not so because we are setting $g \neq 0$. In fact some of the would-be resonances even disappear as such by just becoming broad enhancements. Recall that conventionally a physical resonance must satisfy $\Gamma < M/4$ and this is not the case in many cases when applying the coupled-channel formalism. Obviously, coupled channels matter.

Finally, let us mention that in pp collisions the scattering of vector bosons (VBS) is a subdominant process, but the production of hh pairs via VBS is further suppressed with respect to the elastic channel $WW \rightarrow WW$. A relevant issue that will be studied below is the intensity of the coupling of the dynamical resonances to hh final states. As we will see, they would be more visible in the elastic $WW \rightarrow WW$ channel and tend to couple weakly to final hh pairs. To what extent this depends on the various couplings is an interesting question, too.

C. Checking unitarity

The bad high-energy behavior, manifest in the amplitudes of the effective theory even, must be avoided in order to give reasonable predictions that do not overestimate the number of predicted events in WW fusion subprocesses. Here, we provide evidence that this is the case when the partial scalar waves are unitarized.

Indeed, the IAM amplitude (30) is built to keep the desired unitarity property (31) as long as there is a good description of the amplitudes across the physical cut. One can easily show from the exact unitarity condition that the unitarized partial waves for the elastic processes (WW and

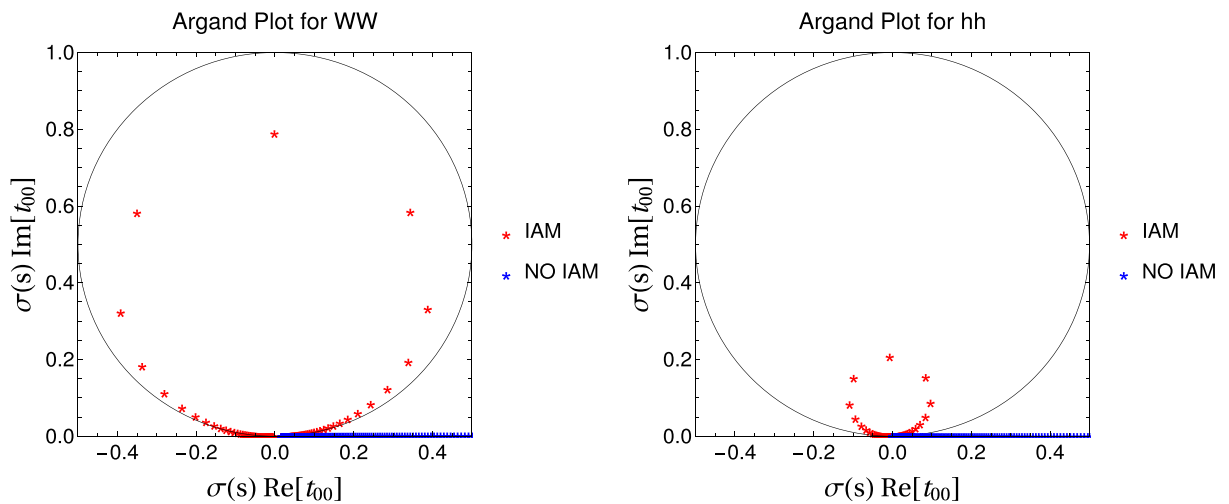


FIG. 5. Argand plot for the scalar wave of the (left) elastic WW and (right) elastic hh scattering for BP3 and the chiral coupling $\gamma = 10^{-3}$. The rest of the parameters are set to their SM values. In red dots, the unitarized amplitude satisfying the unitarity condition; in blue dots, the nonunitary chiral amplitude from the Lagrangians (1) and (2). The Wh crossed channel alone does not need to satisfy this condition of lying on or inside the circumference.

hh) must lie on or within the circumference of radius $1/2$ centered at $(0, 1/2)$ when plotting the imaginary versus the real part of the unitarized amplitude. This is not the case for the crossed-channel Wh . As a demonstration of the good behavior of our scalar waves, we show in Fig. 5 the Argand plot for the elastic processes described with the chiral parameters of BP3 plus $\gamma = 10^{-3}$. The rest of the chiral couplings keep their SM values.

As we can see in Fig. 5, no matter the point (every one of them corresponding to different energies), they all fall within the unitary circle. The fact that they do not lie exactly over the circumference is because, for the selection of parameters chosen for illustration, there is a big component of inelasticity in the process, i.e., the crossed channel cannot be neglected.

D. Some theoretical insight

Physical resonances should always be located in the second Riemann sheet. It is sometimes not fully appreciated that the presence of unphysical singularities, e.g., in the first Riemann sheet, is useful to restrict the space of parameters of an effective field theory. In hadron physics, for instance, it is widely known that a broad range of parameters in the pion Effective Chiral Lagrangian is excluded because it leads to acausal resonances in isotensor resonances. This result automatically translates into the HEFT because the expressions are very similar. An upper bound on the combination $a_5 + 2a_4$ (see Ref. [43]) emerges. This restricts the range of parameters that can be considered for an effective theory. In other words, no UV completion may exist that leads to such low-energy constant.

This lack of causality can also be understood directly on Lagrangian terms in some cases. In Ref. [45], it was seen in a general setting how such restrictions may arise.

In the context of HEFT, in Ref. [46], the following sum rule was derived

$$\frac{1-a^2}{v^2} = \frac{1}{6\pi} \int_0^\infty \frac{ds}{s} (2\sigma_{I=0}(s)^{\text{tot}} + 3\sigma_{I=1}(s)^{\text{tot}} - 5\sigma_{I=2}(s)^{\text{tot}}), \quad (34)$$

where σ_I^{tot} is the total cross section in the isospin channel I . This interesting result was derived making full use of the ET and setting $g = 0$. Taking into account that, unless there is an unlikely strong enhancement of the $I = 2$ isospin channel, the rhs is positive definite, this would exclude values of the effective coupling a greater than 1. Note that we just saw that there were no physical resonances in the isotensor channel. And, indeed, no satisfactory microscopic model has been constructed with $a > 1$ to our knowledge.

As we have seen, there are some deviations with respect to the ET predictions when using the proper longitudinal vector boson amplitudes and they affect the analytic properties of the amplitude. In Ref. [44], it was seen that

a complete calculation (as opposed to the simpler nET treatment) changes the previous result in several ways. For instance, it is not true that a given order in the chiral expansion corresponds to a definite power of s —a property that is used in order to derive (34). Therefore, when gauge transverse propagation q is included, the order s contribution will have corrections from all orders in perturbation theory. The contribution to the left-hand side of the integral obtained will then be of the form

$$\frac{3 - a^2 + \mathcal{O}(g^2)}{v^2}. \quad (35)$$

However, the right cut changes, too, when g is taken to be nonzero due to W propagation in the t channel, and this could compensate the modification on the lhs. Finally, as we have seen, crossing symmetry is not manifest in the Mandelstam variables when one moves away from the nET. This is again a necessary ingredient to derive (34).

These subtleties, however, do not mean that the $a > 1$ forbidden region is not present; it just means that proving this when the propagation of transverse modes is taken into account is not so easy. Indeed, in Ref. [44], it was seen that for $a > 1$ the IAM led to pathologies in resonances appearing in various channels, including acausal resonances—poles in the first Riemann sheet.

Therefore, it seems that an efficient way of setting bounds on the low-energy constants is provided by discarding those regions of parameter space in the effective theory, i.e., in the infrared, where resonances are acausal. The regions described by these effective theories do not have an ultraviolet completion.

V. RESULTS

As previously mentioned, bounds on the parameters of the HEFT from the study of unitarity and resonances can come in two ways. One is simply by experimentally falsifying a given set of parameters because they should give rise to resonances that are not seen in experiment. The other is giving rise to unphysical acausal resonances that lay on the wrong Riemann sheet.

To exploit all the potential of the analysis, we will group the parameters of the HEFT into two different sets. One of them contains all parameters that enter the $\mathcal{O}(p^2)$ Lagrangian; namely a, b, d_3 , and d_4 . In the preliminary analysis previously presented, these values were all set to their SM values $a = b = d_3 = d_4 = 1$. The other set contains all the $\mathcal{O}(p^4)$ parameters: a_4, a_5, γ, η , and δ . We shall assume that none of the parameters in the second group exceeds in absolute value 10^{-3} . We will not include the chiral parameters a_3 and ζ because in Ref. [1] it was demonstrated that they play only a marginal role in the determination of vector resonances.

In what follows, we will first study the influence of the relevant $\mathcal{O}(p^4)$ parameters while keeping the first set to

their SM values. Later, we will repeat the analysis for values of a , b that slightly differ from the SM, but still keeping $d_3 = d_4 = 1$. Finally, we will study the influence of d_3 and d_4 , but keeping the SM values $a = b = 1$ to test the sensitivity of scalar resonances to the parameters in the Higgs potential.

For a given set of $\mathcal{O}(p^2)$ parameters (a, b, d_3 , and d_4), once one fixes a_4 and a_5 , the pole position in the elastic WW channel is pretty much determined (up to small a_3 and ζ corrections that we neglect). To the extent that the elastic channel may dominate resonance production, we can treat the effect of the rest of parameters that participate in the mixing among the scalar channels as a perturbation. We have, for instance, searched for resonances in the case $a_4 = a_5 = 0$ while varying the remaining $\mathcal{O}(p^4)$ terms with a negative result. The presence of resonances (both in the vector and scalar channels) is largely triggered by nonzero values of the chiral couplings a_4 and a_5 .

However, not every set of low-energy parameters may correspond to an effective description of a strongly interacting theory. Therefore, we have to be able to discriminate which of the zeroes of Eq. (32) is a physical and which is not and also which resonances should have also been observed.

On one hand, we will be looking for resonant states that satisfy the condition $\Gamma < M/4$. If this is not fulfilled, we will be talking of an enhancement of the unitarized amplitude but never to be interpreted as a resonant state. In that case, even if Eq. (32) has a zero, the parameters M and Γ are not directly related the properties of a Breit-Wigner resonance. On the other hand, there are zeros that, even satisfying the aforementioned condition, cannot be taken as physical states since they have negative Breit-Wigner widths. These spurious states cannot be present in any physical theory. Analytically speaking, these zeros are found in the first Riemann sheet, above the physical cut in the complex s plane.

Unlike in the case of the nET, due to the complicated structure of the one-loop amplitude, we cannot perform analytical continuation to the second Riemann-sheet and find poles analytically. To contour that difficulty, we have three tools at our disposal: (1) comparison with the nET in order to see if a pole represents a modification of a pole previously known to exist in the simplified model; (2) fitting the partial wave to a two Breit-Wigner resonances, leaving the sign of the width as a free variable; and (3) checking the behavior of the phase shift across the resonance. Of all three possibilities, tool 1 is not very informative because, as seen, the modifications with respect the nET are large when coupled channels play a role, tool 2 is quite useful, but tool 3 is the method of choice (particularly when combined with tool 2).

The phase of an amplitude that contains a physical resonance presents a shift from $\pi/2$ to $-\pi/2$ in the pole position. The derivative of the phase should always be positive as the expression $\Gamma \sim \left(\frac{\partial\delta(s)}{\delta\sqrt{s}}\right)^{-1}$, where $\delta(s)$ represents the phase, can be derived analytically. In this way, we study the causal character of all resonances found. All cases are met: one resonance, two physical resonances, and also two resonances where only one of them happens to be physical.

Let us now proceed with the study in the case $a = b = d_3 = d_4 = 1$.

To study the impact of the new parameters, we focus on the three BP points defined by specific values of a_4 and a_5 previously used. The new parameters are: γ , that enters in elastic hh , and δ and η that carry out the mixing between the two elastic processes as one can see in (29).

The effect of each $\mathcal{O}(p^4)$ anomalous coupling is reflected in the Tables IV–VI below where we study the separate influence of each one for the above benchmark points. In the following analysis, we keep the SM values for a, b, d_3 , and d_4 .

TABLE IV. Pole position for the benchmark points in Table III varying the $\mathcal{O}(p^4)$ parameter γ . The rest of the parameters are set to their SM values. Values in boldface indicate broad resonances that do not satisfy $\Gamma < M/4$.

$M_S - \frac{i}{2}\Gamma_S$	$\gamma = 0$	$\gamma = 0.5 \times 10^{-4}$	$\gamma = 1 \times 10^{-4}$	$\gamma = -0.5 \times 10^{-4}$	$\gamma = -1 \times 10^{-4}$	$\gamma = 1 \times 10^{-2}$
BP1	1844 $- \frac{i}{2}$ 487	1668 $- \frac{i}{2}$ 212	1594 $- \frac{i}{2}$ 162	1119 $- \frac{i}{2}$ 50
BP2	2156 $- \frac{i}{2}$ 637	1881 $- \frac{i}{2}$ 212	1781 $- \frac{i}{2}$ 162	1269 $- \frac{i}{2}$ 62
BP3	2244 $- \frac{i}{2}$ 675	1931 $- \frac{i}{2}$ 200	1831 $- \frac{i}{2}$ 162	1319 $- \frac{i}{2}$ 75

TABLE V. Pole position for the benchmark points in Table III varying the $\mathcal{O}(p^4)$ parameter δ . The rest of the parameters are set to their SM values. Values in boldface indicate broad resonances that do not satisfy $\Gamma < M/4$.

$M_S - \frac{i}{2}\Gamma_S$	$\delta = 0$	$\delta = 0.5 \times 10^{-4}$	$\delta = 1 \times 10^{-4}$	$\delta = -0.5 \times 10^{-4}$	$\delta = -1 \times 10^{-4}$
BP1	1844 $- \frac{i}{2}$ 487	1744 $- \frac{i}{2}$ 362	1669 $- \frac{i}{2}$ 300	1994 $- \frac{i}{2}$ 1100	⊗
BP2	2156 $- \frac{i}{2}$ 637	1981 $- \frac{i}{2}$ 387	1869 $- \frac{i}{2}$ 300	2644 $- \frac{i}{2}$ Γ	...
BP3	2244 $- \frac{i}{2}$ 675	2031 $- \frac{i}{2}$ 400	1906 $- \frac{i}{2}$ 287

TABLE VI. Pole position for the benchmark points in Table III varying the $\mathcal{O}(p^4)$ parameter η . The rest of the parameters are set to their SM values. Values in boldface indicate broad resonances that do not satisfy $\Gamma < M/4$.

$M_S - \frac{i}{2}\Gamma_S$	$\eta = 0$	$\eta = 0.5 \times 10^{-4}$	$\eta = 1 \times 10^{-4}$	$\eta = -0.5 \times 10^{-4}$	$\eta = -1 \times 10^{-4}$
BP1	1844 - $\frac{i}{2}$ 487	1806 - $\frac{i}{2}$ 437	1769 - $\frac{i}{2}$ 387	1881 - $\frac{i}{2}$ 575	1931 - $\frac{i}{2}$ 712
BP2	2156 - $\frac{i}{2}$ 637	2094 - $\frac{i}{2}$ 512	2031 - $\frac{i}{2}$ 437	2256 - $\frac{i}{2}$ 887	2394 - $\frac{i}{2}$ Γ
BP3	2244 - $\frac{i}{2}$ 675	2156 - $\frac{i}{2}$ 537	2094 - $\frac{i}{2}$ 450	2356 - $\frac{i}{2}$ 925	2544 - $\frac{i}{2}$ Γ

From the previous table, we can see that the appearance of a nonzero γ makes the profile of the zero narrower in such a way that we can even talk of a Breit-Wigner resonance with $\Gamma < M/4$. The values in boldface for $\gamma = 0$ do not satisfy this condition. In the case of an extreme value of γ , a value we would not expect for naturalness reasons, we recover the single-channel approximation, and the coupled-channel formalism is not necessary anymore. This is shown in the last column for a value of $\gamma = 10^{-2}$, where very narrow resonances appear.

The symbol $-$ represents the absence of a zero in the determinant of the IAM matrix. We have also introduced the symbol \otimes to indicate the situation where there are two poles in the unitarized amplitude but one is unphysical following the phase-shift criteria; analytically, it corresponds to a pole in the first Riemann sheet, which leads to a violation of causality with a negative width. Also, whenever our code is not able to calculate the width over the profile of the “resonance” because it is too wide and the half maximum surpasses the HEFT validity, we include the symbol Γ , knowing that such a BP can never represent a physical resonance.

From the Tables IV–VI above we can see a really different scenario from the one in the vector-isovector case. The location of the pole changes 15%–20% when we use reasonable values of γ and δ ($\sim 10^{-4}$) and softer variations of around 4%–8% for values of η of the same order. The lesson thus is clear: we cannot give a good description of the resonant scalar states from WW scattering without paying attention to the coupled channels. In the first table, we have also included a big γ value ($\sim 10^{-2}$) to make evident that the pole position in that case is very similar to that obtained using the single-channel formalism neglecting the extra $I = 0$ intermediate states. In fact, for very non-natural values of γ (~ 1), the single-channel resonance is reproduced exactly.

The importance of the mixing parameters in determining the properties of the scalar resonances is now evident.

In the Tables IV–VI, we have studied the effect in the resonance properties of the different couplings separately. However, this may not be the general case since they are all independent and they are not strongly constrained (or even constrained at all) by the experiment, especially the ones belonging to the Higgs sector, so they could all differ from zero. Hence, it is not the individual effects but the simultaneous contribution of them all that we are interested

in. In Fig. 6, we show for the BPs in Table III the space parameter in the $\delta - \eta$ plane where physical resonances with scalar masses heavier than 1.8 TeV are allowed for different values of γ .

No matter the value of γ or the benchmark point selected, the presence of an unphysical pole appearing in the first Riemann sheet leads us to exclude the parameter space above the bands. This whole range of parameters cannot describe any physical extension of the SM. We also find that the greater the value of γ is, the more restriction we find (there are more excluded space above the band), especially for BP1.

Below the bands, we find a nonresonant scenario: we do not find any zero in the determinant of the unitarized amplitude.

Because the above benchmark points correspond to relatively large masses, the amplitudes are to a large extent dominated by the NLO [i.e., $\mathcal{O}(p^4)$] contributions. Those appearing in the Wh mixed channel vanish when $\delta = \eta = 0$, so the decoupling limit results should be retrieved then.

The question of whether these resonances could be visible in the experiment requires a much more detailed study with Monte Carlo techniques that is beyond the scope of this first study of scalar resonances. However, from the parton level processes studied here, and by looking at the relative size of the residues of the corresponding poles in every channel, we can say whether it is more likely to be a bound system of two W 's or a hh composite state. Once the pole structure is factorized from the unitarized amplitude, we are left with function which is a mixture of the other dynamical variables of the system, i.e., momentum structures and couplings of the Lagrangian.

As an example, we show in Fig. 7 the amplitude of two unitarized amplitudes that show a broad (left panel) and a narrow (right panel) resonances. In both cases, they correspond to zeros of the determinant of the IAM amplitude. We observe that the bigger the γ parameter, the stronger the coupling to a hh final state is, although the WW channel is strongly favored always. In any case, even if the dynamical resonances have a strong admixture of Higgs, they will be easier to spot in the WW elastic channel. This is a very clear prediction.

To conclude this section, let us consider the case where some of the $\mathcal{O}(p^2)$ parameters differ from the SM limit. We shall still keep $d_3 = d_4 = 1$, but let us take $a = 0.95$ and $b = 0.805$. These values correspond to a minimal

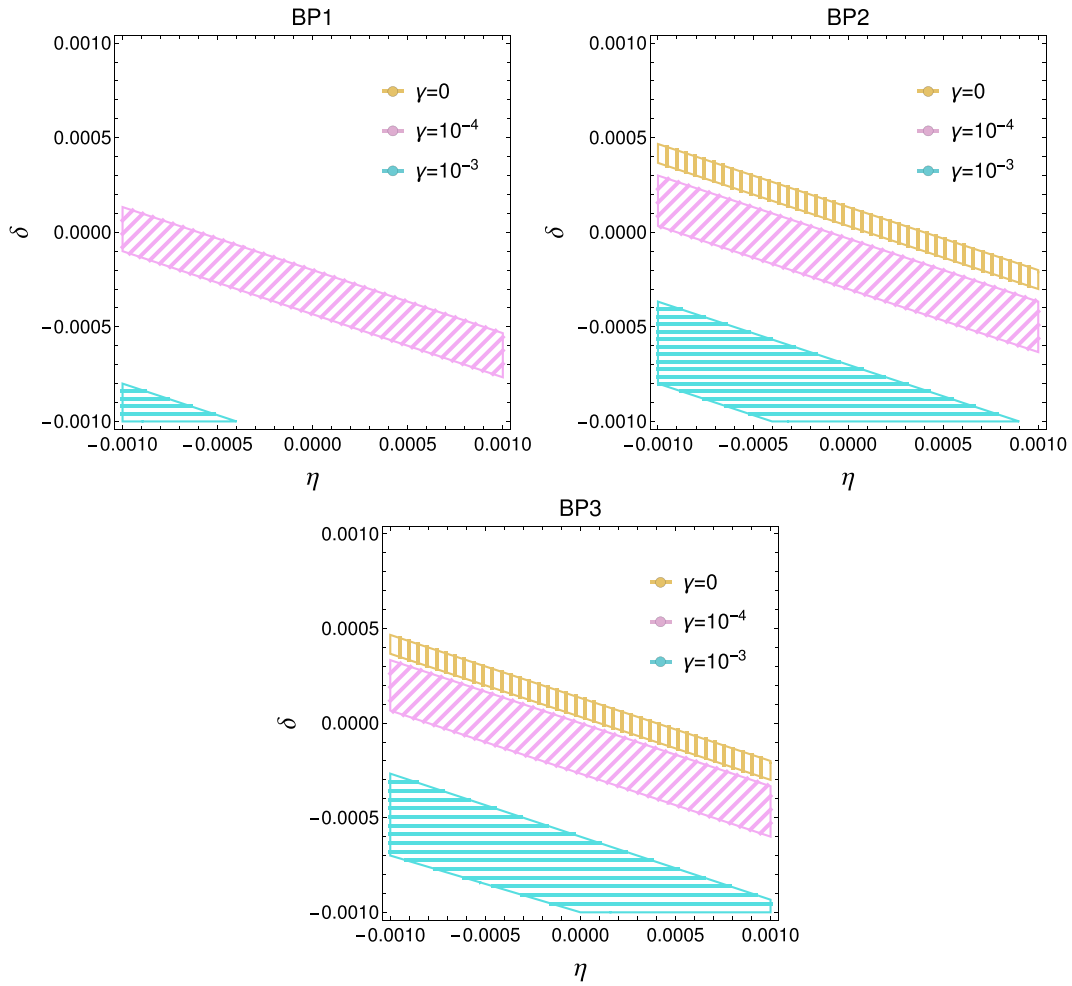


FIG. 6. Regions in the $\delta - \eta$ plane where physical resonances satisfying $M_S > 1.8$ TeV and for the benchmark points in Table III appear for different values of γ : $\gamma = 0$ (golden vertical lines), $\gamma = 10^{-4}$ (pink tilted lines), and $\gamma = 10^{-3}$ (blue horizontal lines). For all the values of γ , the region above the bands is excluded by the presence of a nonphysical pole. Below the bands, we find a nonresonant scenario.

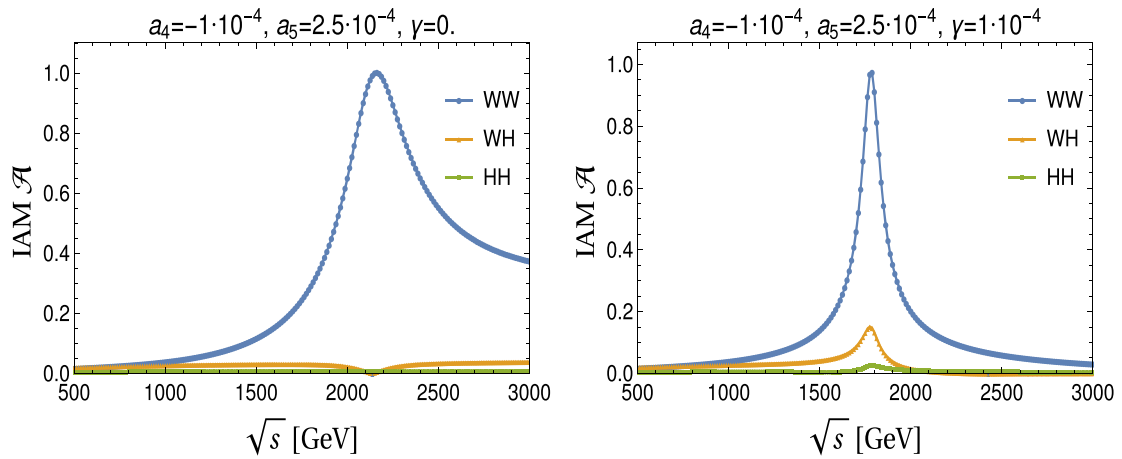


FIG. 7. Profile of the unitarized amplitude showing a zero in the determinant for the chiral couplings specified in the title and with the rest of the parameters set to the corresponding SM values.

composite Higgs model living in the subgroup $H = SO(5)/SO(4)$, which presents a symmetry in the Higgs function $\mathcal{F}(h)$ with the relation $b = 2a^2 - 1$. The interested reader may find in Ref. [25] a complete review for the different realizations of the HEFT, including this minimal extension.

In this case, and with the already mentioned expected maximum size of the $\mathcal{O}(p^4)$ of $\sim 10^{-3}$, we have not found any resonant state that fulfills all the requirements of this study, even though some scenarios present resonant profiles that are too wide. All the physical resonances for this choice of a and b appear for values of $a_{4,5}$ of order 10^{-2} .

Not much can be concluded in this case.

VI. BOUNDS ON d_3 AND d_4 FROM RESONANCES

In this section, we will take the $\mathcal{O}(p^2)$ couplings a, b to be equal to their SM value $a = b = 1$ and explore how the resonance scene depends on the triple and quartic Higgs couplings.

A. d_3

The issue of determining the triple Higgs self-coupling is of utmost importance because it would help us to explore the properties of the Higgs potential, crucial to understanding the nature of the Higgs boson itself. However, such a measurement is quite involved at the LCH because it relies on the ability of the experiment to find a double

Higgs final state (through its decay products) coming from the fusion of two radiated (off-shell) electroweak gauge bosons or, alternatively, from top pairs. Up to now, not enough statistics have been collected from the experiment, which translates into a very wide range in the experimental bound for this coupling: $-3.3 < d_3 < 8.5$. The upper limit of this interval would make the interaction of $\mathcal{O}(1)$ since the BSM self-interaction is described by $\lambda_3 = d_3 \lambda_{SM}$ with $\lambda_{SM} \sim 0.13$.

The fact that this coupling d_3 enters now at tree level in the calculation of the $I = 0$ processes Wh and hh makes the resonant scalar states in the spectrum of WW scattering more sensitive to it and, hence, a good approach to the problem of investigating the Higgs potential.

We start by analyzing the effect of this coupling separately, when the rest of the chiral parameters are set to their SM values, and for the benchmark points in Table III. The results are gathered in Table VII.

We find that for $d_3 \gtrsim 2.5$ a second pole clearly appears (notation pole1 over pole2) in the low-energy region around ~ 1 TeV, and it is also physical because it is found in the second Riemann sheet of the complex s plane. However, one of the physical poles is located at energy scales much lower than our preestablished bound of 1.8 TeV, so, in principle, the corresponding set of parameters should be discarded. The results are shown in Table VII. In fact, there are already hints of this first resonance at $d_3 = 1.7$.

TABLE VII. Values of the pole position of the benchmark points in Table III changing d_3 . The rest of the parameters are set to their SM values. The cells with two complex numbers indicate the pole position of the two physical Breit-Wigner poles in the denominator of the unitarized amplitude.

$M_S - \frac{i}{2}\Gamma_S$	$d_3 = 0.5$	$d_3 = 1$	$d_3 = 2$	$d_3 = 3$	$d_3 = 4$	$d_3 = 5$
BP1	2006 $- \frac{i}{2}\Gamma$	1884 $- \frac{i}{2}\mathbf{487}$	1681 $- \frac{i}{2}187$	994 $- \frac{i}{2}25$ 1756 $- \frac{i}{2}65$	1044 $- \frac{i}{2}38$ 2069 $- \frac{i}{2}26$	993 $- \frac{i}{2}23$ 2444 $- \frac{i}{2}25$
BP2	2369 $- \frac{i}{2}\Gamma$	2156 $- \frac{i}{2}\mathbf{637}$	1906 $- \frac{i}{2}237$	1119 $- \frac{i}{2}27$ 1869 $- \frac{i}{2}75$	1219 $- \frac{i}{2}37$ 2094 $- \frac{i}{2}31$	1181 $- \frac{i}{2}21$ 2444 $- \frac{i}{2}25$
BP3	2468 $- \frac{i}{2}\Gamma$	2244 $- \frac{i}{2}\mathbf{675}$	1969 $- \frac{i}{2}250$	1131 $- \frac{i}{2}19$ 1894 $- \frac{i}{2}75$	1269 $- \frac{i}{2}37$ 2094 $- \frac{i}{2}20$	1231 $- \frac{i}{2}23$ 2444 $- \frac{i}{2}25$

TABLE VIII. Values of the pole position of the benchmark points in Table III with $\gamma = 0.5 \times 10^{-4}$ changing d_3 . The rest of the parameters are set to their SM values. The cells with two complex numbers indicate the pole position of the two physical Breit-Wigner poles in the denominator of the unitarized amplitude.

$M_S - \frac{i}{2}\Gamma_S$	$d_3 = 0.5$	$d_3 = 1$	$d_3 = 2$	$d_3 = 3$	$d_3 = 4$	$d_3 = 5$
BP1	1769 $- \frac{i}{2}275$	1668 $- \frac{i}{2}212$	1544 $- \frac{i}{2}112$	994 $- \frac{i}{2}23$ 1569 $- \frac{i}{2}25$	1044 $- \frac{i}{2}37$ 1769 $- \frac{i}{2}34$	994 $- \frac{i}{2}27$ 1994 $- \frac{i}{2}54$
BP2	1981 $- \frac{i}{2}262$	1881 $- \frac{i}{2}212$	1719 $- \frac{i}{2}125$	1106 $- \frac{i}{2}27$ 1656 $- \frac{i}{2}50$	1219 $- \frac{i}{2}37$ 1781 $- \frac{i}{2}34$	1118 $- \frac{i}{2}26$ 1994 $- \frac{i}{2}50$
BP3	2031 $- \frac{i}{2}250$	1931 $- \frac{i}{2}200$	1769 $- \frac{i}{2}125$	1131 $- \frac{i}{2}37$ 1681 $- \frac{i}{2}38$	1269 $- \frac{i}{2}37$ 1781 $- \frac{i}{2}27$	1231 $- \frac{i}{2}23$ 1994 $- \frac{i}{2}53$

TABLE IX. Values of the pole position of the benchmark points in Table III changing d_4 with $\gamma = 0.5 \times 10^{-4}$. The rest of the parameters are set to their SM values.

$M_S - \frac{i}{2}\Gamma_S$	$d_4 = 0.5$	$d_4 = 1$	$d_4 = 2$	$d_4 = 3$	$d_4 = 4$	$d_4 = 5$	$d_4 = 8$
BP1	$1794 - \frac{i}{2}250$	$1668 - \frac{i}{2}212$	$1494 - \frac{i}{2}137$	$1381 - \frac{i}{2}112$	$1306 - \frac{i}{2}87$	$1256 - \frac{i}{2}75$	$1169 - \frac{i}{2}50$
BP2	$1981 - \frac{i}{2}225$	$1881 - \frac{i}{2}212$	$1719 - \frac{i}{2}175$	$1606 - \frac{i}{2}125$	$1531 - \frac{i}{2}112$	$1481 - \frac{i}{2}87$	$1381 - \frac{i}{2}75$
BP3	$2031 - \frac{i}{2}225$	$1931 - \frac{i}{2}200$	$1781 - \frac{i}{2}162$	$1669 - \frac{i}{2}137$	$1594 - \frac{i}{2}112$	$1544 - \frac{i}{2}100$	$1444 - \frac{i}{2}75$

Of course, the possibility of a light scalar resonance ($\lesssim 1.8$ TeV) being very weakly coupled to WW channel and, hence, viable but hard to detect yet due to limited statistics remains a logical possibility to be further studied. However, if we discard such possibility, the bound on d_3 becomes very stringent.

We have checked that the inclusion of a natural value of γ does not alter the fact that one of the states is too light, making the restriction on d_3 not significantly modified as

can be seen in Table VIII. In this table, we reproduce the same analysis that we have just presented but set the value $\gamma = 0.5 \times 10^{-4}$.

The next step is to check the impact of the crossed channels by varying η and δ in the phenomenological constraint found.

By doing so, we have not found any resonant state fulfilling all the criteria that we have imposed. For the three selected benchmark points in Table III, the behavior is quite

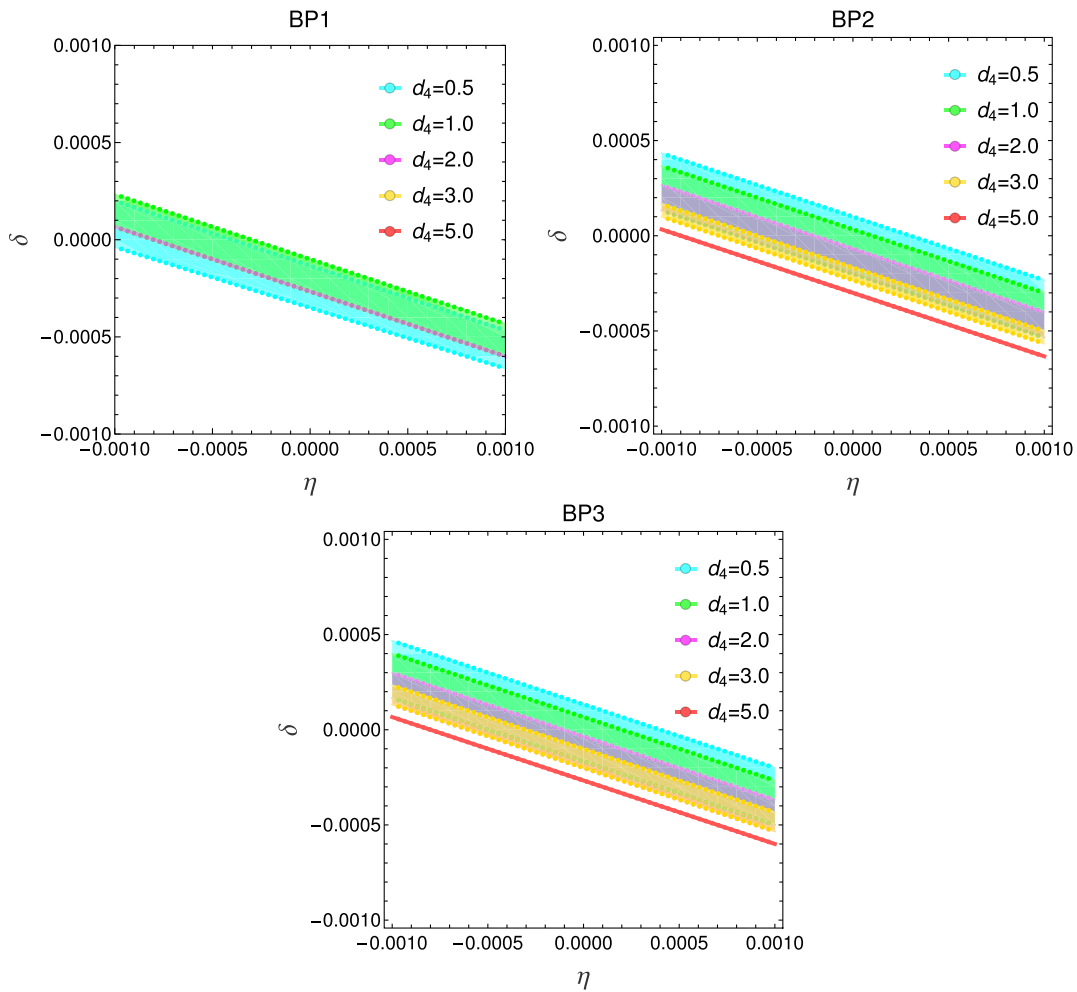


FIG. 8. Regions in the $\delta - \eta$ plane where physical resonances satisfying $M_S > 1.8$ TeV appear for different values of d_4 and setting $\gamma = 0.5 \times 10^{-4}$ for specific values of a_4 and a_5 corresponding to (top left) BP1, (top right) BP2, and (bottom) BP3 in Table III. For all the values of $d_4 \lesssim 6$ and for all the benchmark points, the region above the bands is excluded by the presence of a nonphysical pole. Below the bands, we find a nonresonant scenario.

similar and can be summed up in the following three situations depending on the region in the $\eta - \delta$ plane: the first scenario (1) with a single light resonance (~ 1 TeV), another scenario (2) where two physical resonances appear but one is too light and a third new scenario (3) with a chain of three resonances but the more massive one is classified as unphysical by the phase shift criteria. With all this, the bound $d_3 \lesssim 2.5$ is not modified.

B. d_4

The coupling d_4 parametrizes the strength of the self-interaction of four Higgses, and as it happens with d_3 , it enters now at the lowest order in chiral perturbation theory and contributes at tree level in the hh process. From experiment, it is extremely poorly constrained because of the difficulty of measuring the pointlike coupling of four Higgses. For this study and in the absence of any relevant experimental bounds up to date, we will be considering values up to $d_4 \lesssim 10$, which would make the interaction of order $\mathcal{O}(1)$. Negative values of d_4 are to be excluded outright due to vacuum-stability reasons.

To start the analysis, we select the benchmark points from the tables above and see how the value of d_4 affects the properties of the poles. In particular, we focus on the case where $\gamma = 0.5 \times 10^{-4}$, which for all scenarios allowed the presence of resonances satisfying $\Gamma < M/4$.

From Table IX, we can say that, if all the rest of parameters are set to their SM values, we could exclude values of $d_4 \gtrsim 2$ for BP2 and BP3 and BP1 would be excluded since these parameters lead to light resonances that should have already been seen. As always, we assume (rightly or wrongly) that any scalar resonance above 1.8 TeV should have been observed. And as always, we also force the vector resonances, if present, to be heavier than that scale.

The question whether the crossed channel (with the parameters δ and η leading at high energies) could affect this result is depicted in the following graphs, where, for different values of d_4 , we show the regions in the $\delta - \eta$ plane where resonances $M_S > 1.8$ TeV can appear.

In Fig. 8, we see how, in fact, some regions that were nonresonant show resonances after activating the crossed-channels parameters from the values in Table IX. The more we depart from the SM value $d_4 = 1$, the more restriction we get. In fact, for $d_4 = 5$ and the values of Table IX, we only find resonances in the lines $\delta = -\frac{1}{3000} - \frac{\eta}{3}$ for BP2 and $\delta = -\frac{8}{3000} - \frac{\eta}{3}$ for BP3.

We do not find any physical resonant state with $M_S \gtrsim 1.8$ TeV and $d_4 \gtrsim 6$.

In this case varying d_4 , the same behavior that has been observed varying γ (Fig. 6) is reproduced: above the color bands, we get excluded regions by the appearance of a second nonphysical pole (again using the phase-shift criteria), and below the bands, we get a nonresonant scenario with an absence of any zeros in the determinant of the unitarized amplitude.

VII. CONCLUSIONS

Resonances are a characteristic feature in a HEFT as soon as one departs from the minimal Standard Model. Their role is to restore unitarity, and their properties, mass, and width and also their coupling to the various initial and final states are in close relation with the low-energy constants present in the HEFT. Detecting one of such a resonance would undoubtedly signal the existence of additional microscopic degrees of physics but also point to particular regions in the space of effective theories, suggesting fundamental physics of a certain kind.

Studying the properties of the possible resonances provides precious information to experimentalist as to what type of signal is to be expected in extensions of the Standard Model. It was seen in previous studies that the resonances appearing in WW elastic scattering are typically narrow and not very pronounced, indicating that, while the HEFT may be perturbatively nonunitary, it is, in a sense, close to unitarity because the departures from the Standard Model are not large from a numerical point of view. While this picture remains true in the vector resonance case, we have seen that when considering the $IJ = 00$ case, where the formalism of coupled channels is unavoidable when transverse gauge degrees of freedom are included, scalar resonances become substantially broader.

Assuming that no resonances exist below the scales that have already been experimentally probed, the next-to-leading $\mathcal{O}(p^4)$ coefficients in the HEFT should be at most of order 10^{-3} and probably of order 10^{-4} . In this work, we have assumed that no resonance, vector or scalar, exists below 1.8 TeV, and from that, we derive bounds on the HEFT couplings. But there is another way of restricting the HEFT; namely, if in the unitarization process one encounters acausal or unphysical resonances, the corresponding set of parameters in the effective theory can be ruled out.

We assume three different conditions to characterize a resonance as physical: (1) $\Gamma < M/4$; (2) the set of parameters in the HEFT must not produce vector or scalar resonances below 1.8 TeV; and (3) all resonances (usually one, but sometimes two) must lie in the second Riemann sheet. We also assume, as said, that any resonance above 1.8 TeV should have been observed by now.

The space of parameters is fairly large, so the present study has to be understood only as a first exploration of this landscape that surely merits an more systematic analysis. Let us nevertheless summarize the more relevant constraints.

First, we have verified that not all $\mathcal{O}(p^4)$ coefficients are equally important. Those determining the appearance of resonances correspond to operators that survive in the nET limit. This, which is in agreement with previous studies in the vector case, and it is quite useful as it tells us where to look for resonances in the vast space of HEFT. Note that the inclusion of transverse modes becomes relevant (unlike in the vector case) in the scalar case. Second, besides a_4 and a_5 , three new parameters appear at next-to-leading order.

Taking all the $\mathcal{O}(p^2)$ couplings to be identical to the SM, we have found that, for specific values of a_4 and a_5 , the resonance spectrum that could be observed is more restricted in the $\delta - \eta$ plane the lower the value of γ is. For $\gamma = 0$, resonances live in a narrow band of values of δ and η , and the greater the value of γ , the broader the band is. The region above these narrow bands can be excluded on causality grounds. So, this places a very strong restriction in parameter space. It is also seen almost immediately that if a resonance is present in one channel it is present in all, but they always couple more strongly to WW final states.

In this study, and making use of the arguments explained above, we have also set encouraging theoretical bounds on the self-interactions of the Higgs, especially in the case of the triple self-coupling whose BSM deviations are parametrized via d_3 (in units of λ_{SM}). We have found for this coupling that whenever it exceeds $d_3 \sim 2.5$ a second very light pole appears, which we assume would have already been detected in the experiment. The emergence of this light pole becomes noticeable even before, from $d_3 \sim 1.7$. The absence of such a resonance make us exclude all the values above this threshold. This behavior is not significantly modified when considering nonzero values for the parameters δ , γ , and η , so from this study, we could set a bound $d_3 \lesssim 2$, much more restrictive than current experimental bounds (assuming, of course, that indeed no scalar resonance exists below 1.8 TeV). This is an important prediction; even if a resonance is more likely to be observed in the WW elastic channel, the Higgs self-coupling enters in the determination of its properties via the coupled-channel formalism, and it should not be dramatically different from its SM value.

For the case of the four-Higgs coupling, parametrized by d_4 (again in units of λ_{SM}), there are no experimental bounds in the literature to our knowledge. From this study of the single resonance spectrum, we have set an overall phenomenological bound $d_4 \lesssim 6$ with regions of the parameter

space where it could be more restrictive, in particular for the point with both scalar and vector contribution (our BP1).

In conclusion, somewhat unexpectedly, the study of possible scalar resonances in WW fusion places very interesting restrictions on the space of Higgs couplings, a region that is hard to experimentally study. We have presented here some, we believe, relevant results, but certainly this line of research deserves further more systematic studies.

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APPENDIX: RELEVANT COUNTERTERMS

In this appendix, we present the complete list of counterterms needed to absorb the one-loop divergences of the three relevant processes of this study. They have been obtained in the on-shell (OS) [47] scheme and with the approximations mentioned in the previous sections of this piece of work; they are valid in the custodial limit, $g' = 0$, and within the Landau gauge ($\xi = 0$) where the Goldstones are massless. In this scheme, the physical mass is placed in the pole of the renormalized denominator with residue 1. With all this, we just need to redefine two of the bare masses of the electroweak sector, M_h and M_W , and no gauge parameter whatsoever due to its multiplicative renormalization,

$$\begin{aligned}
 \delta M_{h,div}^2 &= \frac{\Delta}{32\pi^2 v^2} (3[6(2a^2 + b)M_W^4 - 6a^2 M_W^2 M_h^2 + (3d_3^2 + d_4 + a^2)M_h^4]), \\
 \delta M_{W,div}^2 &= \frac{\Delta}{48\pi^2 v^2} (M_W^2 [3(b - a^2)M_h^2 + (-69 + 10a^2)M_W^2]), \\
 \delta Z_{h,div} &= \frac{\Delta}{16\pi^2 v^2} (3a^2(3M_W^2 - M_h^2)), \\
 \delta Z_{w,div} &= \frac{\Delta}{16\pi^2 v^2} ((b - a^2)M_h^2 + 3(a^2 + 2)M_W^2),
 \end{aligned} \tag{A1}$$

where $\Delta = \frac{1}{\epsilon} + \log(4\pi) + \gamma_E$ represents the divergence.

In our setting, we let all the mass of the Higgs, the vacuum expectation value (vev), and λ_{SM} to get radiative corrections at NLO so the relation $M_H = 2\lambda_{\text{SM}}v$ is just

valid at tree level. On the contrary, the relation $M_W = \frac{1}{2}gv$ is kept at all orders. This is why, in our case, it is meaningless to add a counterterm δg since it is a derived quantity:

$$\frac{\delta g^2}{g^2} = \frac{\delta M_W^2}{M_W^2} + \frac{\delta v^2}{v^2}. \quad (\text{A2})$$

In what concerns the chiral parameters, we just need to renormalize those couplings accompanying local operators of the scalar sector with no custodial-breaking pieces in them; these are $a, b, a_3, a_4, a_5, \gamma, \delta, \eta$, and ζ :

$$\begin{aligned} \delta v_{\text{div}}^2 &= \frac{\Delta}{16\pi^2} ((b - a^2)M_h^2 + 3(a^2 + 2)M_W^2), & \delta T_{\text{div}} &= -\frac{\Delta}{32\pi^2 v} 3(d_3 M_h^4 + 6a M_W^4), \\ \delta a &= \frac{\Delta}{32\pi^2 v^2} (6a(-2a^2 + b + 1)M_W^2 + (5a^3 - a(2 + 3b) - 3d_3(a^2 - b))M_h^2), \\ \delta b &= \frac{\Delta}{32\pi^2 v^2} (6(3a^4 - 6a^2b + b(b + 2))M_W^2 \\ &\quad - (21a^4 - a^2(8 + 19b) + b(4 + 2b) + 6ad_3(1 + 2b - 3a^2) - 3d_4(b - a^2))M_h^2), \\ \delta \lambda_{\text{div}} &= \frac{\Delta}{64\pi^2 v^4} ((5a^2 - 2b + 3(d_3(3d_3 - 1) + d_4))M_h^4 - 12(2a^2 + 1)M_W^2 M_h^2 + 18(a(2a - 1) + b)M_W^4), \\ \delta \lambda_3 &= \frac{\Delta}{64\pi^2 v^4} (36abM_W^4 + 6(3a^3 - 3ab - d_3(5a^2 + 1))M_W^2 M_h^2 + (-9a^3 + 3ab + d_3(10a^2 - b) + 9d_3d_4)M_h^4), \\ \delta \lambda_4 &= \frac{\Delta}{64\pi^2 v^4} (36b^2M_W^4 - 12(a^2 - b)(8a^2 - 2b - 9ad_3)M_W^2 M_h^2 \\ &\quad + (96a^4 + 4b^2 - d_3(114a^3 - 42ab) + 9d_4^2 + a^2(-64b + 27d_3^2 + 12d_4))M_h^4), \\ \delta a_3 &= -\frac{\Delta}{384\pi^2} (1 - a^2), & \delta a_4 &= -\frac{\Delta}{192\pi^2} (1 - a^2)^2, \\ \delta a_5 &= -\frac{\Delta}{768\pi^2} (5a^4 - 2a^2(3b + 2) + 3b^2 + 2), \\ \delta \gamma &= -\frac{\Delta}{64\pi^2} 3(b - a^2)^2, & \delta \delta &= -\frac{\Delta}{192\pi^2} (b - a^2)(7a^2 - b - 6), & \delta \eta &= -\frac{\Delta}{48\pi^2} (b - a^2)^2, \\ \delta \zeta &= \frac{\Delta}{96\pi^2} a(b - a^2). \end{aligned} \quad (\text{A3})$$

All these counterterms have been proven to have the good SM behavior and to be consistent with the existing literature in the appropriate limits.

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