Perturbative aspects of *CPT*-even Lorentz-violating scalar chromodynamics

B. Altschul,[1,*] L. C. T. Brito,[2,†] J. C. C. Felipe,[3,‡] S. Karki,[1,§] A. C. Lehum,[4,‖] and A. Yu. Petrov[5,¶]

[1]*Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208*
[2]*Departamento de Física, Instituto de Ciências Naturais Universidade Federal de Lavras, Caixa Postal 3037, 37200-900, Lavras, MG, Brazil*
[3]*Instituto de Engenharia, Ciência e Tecnologia, Universidade Federal dos Vales do Jequitinhonha e Mucuri, Avenida Um 4050, 39447-790, Cidade Universitária, Janaúba, MG, Brazil*
[4]*Faculdade de Física, Universidade Federal do Pará, 66075-110, Belém, PA, Brazil*
[5]*Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970 João Pessoa, PB, Brazil*

(Received 11 April 2023; accepted 19 May 2023; published 1 June 2023)

In this work, we formulate the theory of Lorentz-violating scalar quantum chromodynamics with an arbitrary non-Abelian gauge group. This theory belongs to the class of models encompassed by the standard model extension framework. At the lowest order in the theory's Lorentz violation parameters, we calculate the divergent quantum corrections, including the renormalization group β-functions of the theory. The Lorentz-violating sector is shown to be scale invariant if there is a particular relation between the couplings.

DOI: 10.1103/PhysRevD.107.115002

I. INTRODUCTION

The physics of elementary particles rest on well-tested principles of symmetry. The presumptions that there are certain exact spacetime symmetries (described by the Lorentz group) and internal symmetries (underlying strong and electroweak physics and described by an overall non-Abelian gauge group) in the current standard model (SM) are key examples. However, there is also a common understanding that the SM as we observe it is really just an effective theory, describing low-energy elementary particle interactions using a renormalizable quantum field theory. Thus, any symmetry that is apparent at observable scales may actually be just a low-energy approximation, with those symmetries being violated at more fundamental levels. One natural way to obtain an extension of the currently understood SM is, therefore, by relaxing at least one of the fundamental symmetries imposed on the theory. In this paper, we are specifically concerned with the case of explicit breaking of some of the spacetime symmetries—specifically the isotropy and Lorentz boosts symmetries, which together generate the Lorentz group.

One of the most important directions in the study of Lorentz symmetry breaking consists of formulating and studying the possible Lorentz-breaking extensions of various field theoretic models. The most important advancement in this area was the formulation of the Lorentz-violating (LV) standard model extension (SME) [1,2]. The SME is an effective theory framework in which additional operators are added to the action of the SM; these operators are structurally similar to the usual SM operators, but unlike the terms in the usual SM Lagrange density (which are taken to be scalars under proper, orthochronous Lorentz transformations), the SME operators may have free Lorentz indices. Since the foundational work near the end of the last century, a large number of studies of classical and quantum aspects of various LV theories—most commonly LV extensions of spinor quantum electrodynamics (QED)—have been completed. (See, for example [3,4].) In addition, the SME approach has also been generalized to the include the presence of classical gravitation [5]. In this context, the perturbative studies of LV non-Abelian gauge theories are extremely natural.

There have already been some interesting results obtained using perturbative analyses of LV non-Abelian gauge theories coupled to spinor matter—notably including SME generalizations of the SM's quantum chromodynamics (QCD) sector with quarks and gluons: first, the one-loop renormalization of LV non-Abelian gauge theories with fermions completed (including chiral fermions) in

[*]altschul@mailbox.sc.edu
[†]lcbrito@ufla.br
[‡]jean.cfelipe@ufvjm.edu.br
[§]karkis@email.sc.edu
[‖]lehum@ufpa.br
[¶]petrov@fisica.ufpb.br

Refs. [6–8]; second, perturbative generation of the non-Abelian generalization of the Carroll-Field-Jackiw term [9]; third, perturbative generation of a non-Abelian aether-like term [10]. However, coupling of non-Abelian LV theories to scalar matter has not been explored at all, leaving this sector as a very natural area to study. Explicitly, our aim is to extend the calculations of two-, three- and four-point correlation functions, previously obtained in LV scalar QED in [11,12], to the non-Abelian case. That is, we will be formulating and studying scalar QCD, and thereby obtaining the one-loop divergent quantum corrections for the theory, from which its renormalization group behavior may be determined. More specifically, in this paper we are focusing on the analysis of the gluon-scalar interaction, since the gluon self-interaction and gluon-ghost interaction were studied in Refs. [6,7]. Throughout this paper, we use standard particle physics conventions [natural units $c = \hbar = 1$, and $(+---)$ as the spacetime signature].

The structure of the paper is as follows. In Sec. II, we introduce the action for our theory, including gauge fixing and ghost contributions. In Sec. III, we discuss the generation of the non-Abelian aether term. In Sec. IV, we obtain the scalar-vector vertex functions, and in Sec. V we study the β-functions that describe the resulting renormalization group (RG) behavior. Our conclusions are presented in Sec. VI. There are also two appendices. Appendix A collects the Passarino-Veltman basis integrals used throughout our calculations, and in Appendix B, the calculation of the gluon self-energy in the presence of scalar matter is presented in a little more detail.

II. *CPT*-EVEN LV SCALAR CHROMODYNAMICS

Let us consider the non-Abelian generalization of the model studied in [11,12], described by the Lagrange density

$$\mathcal{L} = (D_\mu\phi_i)^\dagger(\eta^{\mu\nu} + c^{\mu\nu})D_\nu\phi_i - m^2\phi_i^\dagger\phi_i - \frac{\lambda}{4}(\phi_i^\dagger\phi_i)^2 - \frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu} + \frac{1}{4}\kappa^{\mu\nu\alpha\beta}F_{a\mu\nu}F_{a\alpha\beta} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{ghost}}, \quad (1)$$

where a is the non-Abelian gauge group index [we may sometimes specialize to the gauge group $SU(N)$, or even to the physical $SU(3)$ of QCD, for definiteness]; the scalar fields ϕ_i are in the adjoint representation (meaning the octet in QCD); $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + gf_{abc}A_b^\mu A_c^\nu$ is the gluon field strength; $D^\mu = \partial^\mu - ieA_a^\mu T_a$ is the covariant derivative, with T_a being the generators $[T_a]_{bc} = if_{abc}$ of the gauge group in the adjoint; and $c^{\mu\nu}$ and $\kappa^{\mu\nu\alpha\beta}$ are dimensionless constant tensors that describe the CPT-even but LV operators in the scalar and vector sectors. All the scalars have the same mass m, which we take to be real, so that gauge symmetry is not spontaneously broken—meaning that $m^2 > 0$. (Studies of theories with both Lorentz symmetry breaking and spontaneous gauge symmetry breaking will be undertaken in the future.) Note that a term like $F_a^{\mu\nu}F_{a\mu\nu}$ with a sum over group indices may also be written as trace over $F^{\mu\nu}F_{\mu\nu}$, in terms of the group-valued field strengths $F^{\mu\nu}$.

Prior to the inclusion of the gauge-fixing and ghost terms, the Lagrange density (1) contains two tensors that describe the Lorentz-violating backgrounds through which the scalar and vector fields propagate. However, the number of physically meaningful parameters is actually fewer than one might expect, based just on counting the number of parameters in the SME tenors. Physically observable quantities cannot actually depend on $c^{\mu\nu}$ without also depending on $\kappa^{\mu\nu\alpha\beta}$, through the specific linear combination $c^{\mu\nu} + \kappa_\alpha{}^{\mu\alpha\nu}$ [13]. This quantity measures the mismatch between the effective metric appearing in the kinetic terms for different sectors of the theory. If $c^{\mu\nu} + \kappa_\alpha{}^{\mu\alpha\nu} = 0$, then the whole theory is actually nothing more than standard scalar QCD, written in skewed coordinates. Whenever possible, it is desirable to have the triviality of the Lorentz violation in this case be evident in the description of the theory.

For simplicity—especially for when we shall be looking at the RG β-functions—we shall assume that $c^{\mu\nu}$ takes an (aetherlike) traceless ($c^\mu{}_\mu = 0$) form $c^{\mu\nu} = Q_1u^\mu u^\nu$, dependent on a single preferred null vector u^μ with $u^2 = 0$. The Lorentz violation coefficient in the gauge sector will also depend solely on u^μ, taking the form

$$\kappa^{\mu\nu\alpha\beta} = \frac{Q_2}{Q_1}(c^{\mu\alpha}\eta^{\nu\beta} - c^{\mu\beta}\eta^{\nu\alpha} + \eta^{\mu\alpha}c^{\nu\beta} - \eta^{\mu\beta}c^{\nu\alpha}) \quad (2)$$

in terms of $c^{\mu\nu}$. (In the limit of vanishing coupling, $g = 0$, a $\kappa^{\mu\nu\alpha\beta}$ of this form is indicative of birefringence-free gauge boson propagation.) The case mentioned above—in which the apparent Lorentz violation is actually fictitious—corresponds to $Q_1 + 2Q_2 = 0$. When this relation is satisfied, the theory is actually just Lorentz-invariant scalar QCD, but expressed in a coordinate system in which the distance along the light-front axis direction u^μ is measured on a different scale than distances along other four-vector directions. General linear transformations of the global coordinates may be used to change u^μ, Q_1, and Q_2, but the quantity $Q_1 + 2Q_2$ remains invariant under such transformations, meaning that it may be measured independently of the choice of coordinate system.

With the simplified SME tensors, the Lagrange density (1) becomes

$$\mathcal{L} = (D_\mu\phi_i)^\dagger(\eta^{\mu\nu} + Q_1u^\mu u^\nu)D_\nu\phi_i - m^2\phi_i^\dagger\phi_i - \frac{\lambda}{4}(\phi_i^\dagger\phi_i)^2 - \frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu} + Q_2u^\mu u^\nu F_{a\mu}{}^\alpha F_{a\nu\alpha} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{ghost}}. \quad (3)$$

We must further include in the Lagrange density (1) or (3) a gauge-fixing term, together with the corresponding Faddeev-Popov ghost contributions. The gauge-fixing term

$\mathcal{L}_{\rm GF}$ is a further LV generalization of the usual Lorenz-like gauge condition used for non-Abelian gauge theories

$$\mathcal{L}_{\rm GF} = \frac{1}{2\xi}\left(\partial^\mu A_{a\mu} + \frac{1}{2}\kappa_G^{\mu\nu}\partial_\mu A_{a\nu}\right)^2. \quad (4)$$

In general, there is freedom to choose any $\kappa_G^{\mu\nu}$; however, the goal of selecting this generalized gauge-fixing term is to have the simplest possible propagator for the gauge sector, with the same $\kappa^{\mu\nu\alpha\beta}$ in the physical and pure gauge components of the propagator tensor. It is fairly clear that to make this possible, there must be a specific relationship between the physical $\kappa^{\mu\nu\alpha\beta}$ and the $\kappa_G^{\mu\nu}$ assigned to the gauge-fixing term, with $\kappa_G^{\mu\nu} = 4\lambda Q_2 u^\mu u^\nu$ for some constant λ. If $Q_1 + 2Q_2 = 0$, so that the apparent Lorentz violation in (1) is actually fictitious (being just a coordinate artifact), having a gauge-fixing term of this form should make it possible to eliminate an appropriately chosen $\kappa_G^{\mu\nu}$ tensor via the same coordinate redefinition that sets $Q_1 = Q_2 = 0$.

The structure of the $\mathcal{L}_{\rm GF}$ term necessitates that there should also be a ghost term, which takes the form

$$\mathcal{L}_{\rm ghost} = -\bar{C}_a\left(\partial^\mu D_{ab\mu} + \frac{1}{2}\kappa_G^{\mu\nu}\partial_\mu D_{ab\nu}\right)C_b, \quad (5)$$

where C_a and $\bar{C}_a$ are the Faddeev-Popov ghost and antighost field, respectively. Once again, if the Lorentz violation is unphysical, we hope, with the right coordinate transformation, to be able to eliminate the Lorentz violation in $\mathcal{L}_{\rm ghost}$ along with the other terms.

Since we shall be evaluating loop integrals only to first order in the Lorentz violation coefficients, we may neglect anything with more than one power of Q_1 or Q_2 when determining the gauge field propagator. To get this propagator, we look at the gauge part of the action in momentum space, at leading order,

$$S_A = -\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}A_{a\mu}\left[p^2\eta^{\mu\nu} - \left(1-\frac{1}{\xi}\right)p^\mu p^\nu + 2Q_2u^\mu u^\nu p^2 - 4\frac{(\xi-\lambda)}{\xi}Q_2u^\nu u^\alpha p_\alpha p^\mu + 2Q_2u^\alpha u^\beta p_\alpha p_\beta\eta^{\mu\nu}\right]A_{a\nu}. \quad (6)$$

We can read off from the gauge field action S_A what the corresponding equation of motion will be. To determine the propagator, we shall consider a numerator with the possible Lorentz structures

$$\Delta_{\mu\delta} = \eta_{\mu\delta} - \zeta\frac{p_\mu p_\delta}{p^2} + \beta Q_2 u_\mu u_\delta + \gamma Q_2 u^\alpha u_\delta\frac{p_\alpha p^\mu}{p^2} + \sigma Q_2 u^\alpha u^\beta\frac{p_\alpha p_\beta}{p^2}\eta_{\mu\delta} \quad (7)$$

and insist that it produce a Kronecker δ-function upon contraction with the integrand in (6). Of course, (7) is not the most general p^μ-dependent matrix that might be used to invert the bilinear structure in (6). In fact, this $\Delta_{\mu\delta}$ is restricted to have no poles higher than the ones that appear in the normal gauge boson propagator. The motivation for this is that Lorentz violation in the pure gauge sector may be eliminated by a linear coordinate transformation. (However, only if the SME coefficients satisfy $Q_1 + 2Q_2 = 0$ will the same transformation that sets $Q_2 = 0$ also eliminate the apparent Lorentz violation in the scalar sector, by also setting $Q_1 = 0$.) Therefore, we would like to find a propagator for the gauge field that resembles the propagator for a Lorentz-invariant non-Abelian gauge field, but expressed in skewed coordinates—if this is possible.

The first thing to notice is that $\zeta = (1-\xi)$ has to be satisfied, confirming the usual behavior in the absence of the Lorentz violation. Moreover, upon contracting and keeping the terms which are first order in Q_2, we find a total of twelve terms, which, taken together, should vanish. They are

$$\begin{aligned}0 = {}&\beta u^\nu u_\delta p^2 + \gamma u_\alpha u_\delta p^\alpha p^\nu + \sigma u^\alpha u^\beta p_\alpha p_\beta \delta^\nu_\delta \\ &- \beta\left(1-\frac{1}{\xi}\right)u_\alpha u_\delta p^\alpha p^\nu - \gamma\left(1-\frac{1}{\xi}\right)u_\alpha u_\delta p^\alpha p^\nu - \sigma\left(1-\frac{1}{\xi}\right)u^\alpha u^\beta p_\alpha p_\beta\frac{p_\delta p^\nu}{p^2} \\ &+ 2u^\nu u_\delta p^2 - 4\frac{\xi-\lambda}{\xi}u^\alpha u^\nu p_\alpha p_\delta + 2u^\alpha u^\beta p_\alpha p_\beta\delta^\nu_\delta \\ &- 2(1-\xi)u^\alpha u^\nu p_\alpha p_\delta + 4(1-\xi)\frac{\xi-\lambda}{\xi}u^\alpha u^\nu p_\alpha p_\delta - 2(1-\xi)u^\alpha u^\beta p_\alpha p_\beta\frac{p_\delta p^\nu}{p^2}.\end{aligned} \quad (8)$$

For this to be zero, the set of terms with each different contraction structure of u^μ and p^μ must vanish. For this to happen, the first and seventh terms together require $\beta = -2$. Similarly, the third and ninth terms require $\sigma = -2$. However, with this σ, we see that we cannot generally cancel the sixth and the twelfth term.

If this complication is temporarily ignored, we can also compute $\gamma = 2(1-\xi)$ and $\lambda = \frac{1+\xi}{2}$, which depend on the gauge parameter ξ. The latter expression may appear particularly problematic, as it seems peculiar that the quantity $\kappa_G^{\mu\nu}$ describing the extent of the Lorentz violation in the ghost loops should depend explicitly on ξ. A further problem may crop up with a more general $\kappa^{\mu\nu\alpha\beta}$ than (2), for which the propagator $\Delta_{\mu\nu}$ may not be manifestly symmetric in its Lorentz indices μ and ν. However, in the Feynman gauge, $\xi = 1$, all of these problems are circumvented.

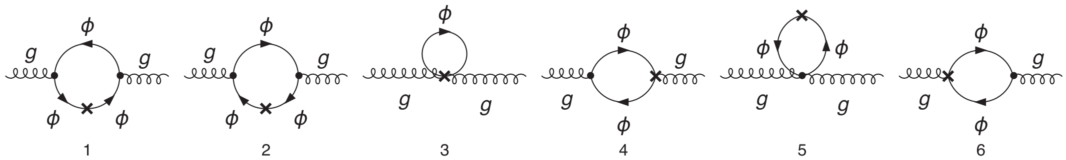

FIG. 1. Feynman diagrams for the LV corrections to the gluon field self-energy. Continuous and curly lines represent scalar and gluon propagators, respectively. The crossed vertices correspond to LV vertex insertions.

Moreover, this fixes the value $\lambda = 1$, so that the Lorentz violation in the kinetic terms for the gauge and ghost fields enter with the same physical magnitudes.

If we insisted on using a different gauge, then the simple $\Delta_{\mu\delta}$ from (7) would not be sufficient to describe the gauge boson propagator. A more complicated structure, with higher poles, would be required [14]. However, the analysis in Ref. [14] is also simplified by the fact that it only considers a massive Abelian gauge theory. In a theory with a vector mass (whether of generalized Proca or Stueckelberg form), there is a consistency condition on the gauge field; this is a Lorentz-violating generalization of the Lorenz condition obeyed by the Abelian vector potential in the presence of a Proca mass. This generalized consistency condition may be used to simplify the action and the propagator for the physical vector boson modes, but this useful simplifying condition unfortunately becomes trivial when the physical photon mass vanishes.

Fortunately, by using the Feynman gauge—which is generally the most convenient gauge to use anyway—all the complications and caveats are avoided. With the propagators established, in the next section we shall compute the one-loop radiative corrections to the $\kappa^{\mu\nu\alpha\beta}$ term in the gauge field action, as generated by the coupling of the gauge field to virtual scalar matter loops. To evaluate the relevant correlation functions, we shall use adapted versions of a set of *Mathematica* packages [15–17]. Note, however, that the contributions coming solely from gluon and ghost propagators (both of which are manifestations of the non-Abelian gauge sector) have already been calculated [6], although not directly using the gauge fixing prescription described in this section.

III. CORRECTIONS TO THE NON-ABELIAN AETHERLIKE TERM

Let us start with evaluating the matter loop corrections to the non-Abelian aether term $Q_2 u_\mu u_\nu F_a^{\mu\alpha} F^\nu_{a\alpha}$ appearing in (1). The diagrams we must compute are depicted in Figs. 1–3.

First, we calculate the gluon self-energy given by the Feynman diagrams in Fig. 1. The general structure of the aetherlike LV gluon self-energy has the form

$$\langle \mathrm{T}\, A_a^\mu(p_1) A_b^\nu(p_2)\rangle = \left(-\frac{g^2 C_A Q_1}{48\pi^2\epsilon} + \text{finite}\right)\Pi^{\mu\nu}(p_1)\mathrm{tr}(T_a T_b)(2\pi)^4\delta^{(4)}(p_1+p_2), \tag{9}$$

where the representation constant $C_A = N$ for the for the adjoint representation of the $SU(N)$ gauge group, and the Lorentz structure is

$$\Pi^{\mu\nu}(p_1) = \eta^{\mu\nu}(p_1\cdot u)^2 - (p_1^\mu u^\nu + p_1^\nu u^\mu)(p_1\cdot u) + p_1^2 u^\mu u^\nu; \tag{10}$$

here we have used the conservation law $p_2 = -p_1$ for the external momentum. In this and the following calculations we define, as usual, the dimensional extension $\epsilon = \frac{D-4}{2}$.

The matter corrections to the aetherlike LV three-gluon vertex, Fig. 2, can similarly be cast as

$$\langle \mathrm{T} A_a^\alpha(p_1) A_b^\mu(p_2) A_c^\nu(p_3)\rangle = \left(-\frac{g^2 C_A Q_1}{48\pi^2\epsilon} + \text{finite}\right)\Pi^{\alpha\mu\nu}(p_1,p_2,p_3)tr([T_a,T_b]T_c)(2\pi)^4\delta^{(4)}(p_1+p_2+p_3), \tag{11}$$

where

$$\Pi^{\alpha\mu\nu}(p_1,p_2,p_3) = (p_1-p_3)^\alpha u^\mu u^\nu + (p_3-p_2)^\nu u^\alpha u^\mu + (p_2-p_1)^\mu u^\nu u^\alpha + u^\alpha\eta^{\mu\nu}(p_1-p_3)\cdot u + u^\nu\eta^{\mu\alpha}(p_3-p_2)\cdot u + u^\mu\eta^{\alpha\nu}(p_2-p_1)\cdot u. \tag{12}$$

Naturally, the symbol $[T_a, T_b]$ stands for the commutators between the normalized group generators, $[T_a, T_b] = i f_{abc} T_c$.

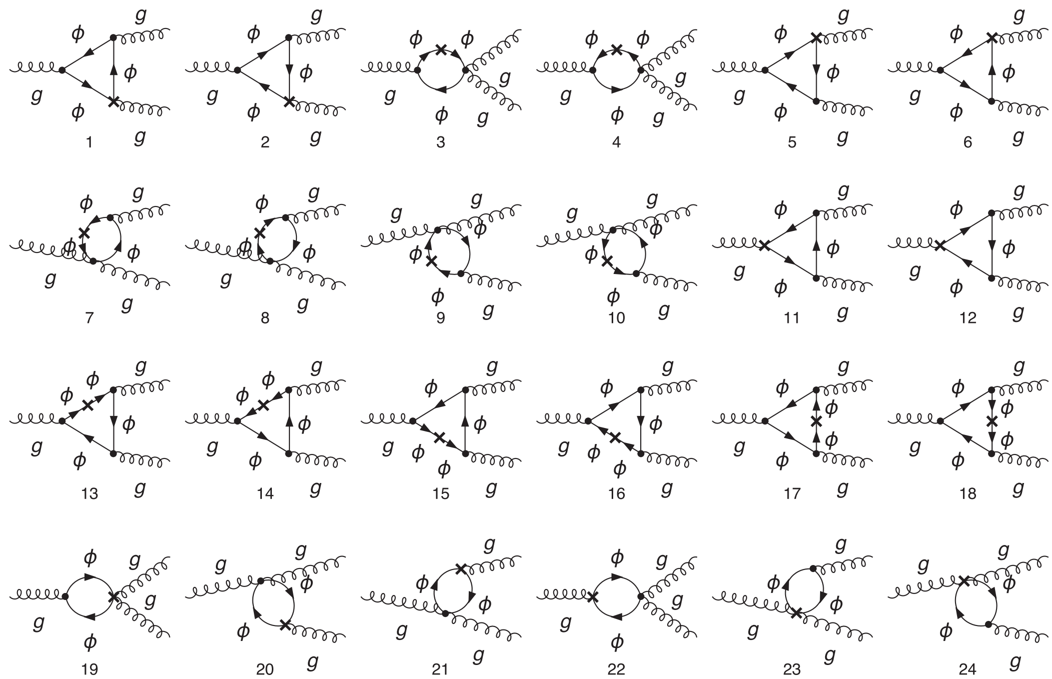

FIG. 2. Feynman diagrams for the LV corrections to the three-gluon vertex interaction.

Finally, matter loop corrections to the aetherlike LV four-gluon vertex, shown in Fig. 3, are given by

$$\langle \mathrm{T}\, A^{\alpha}_{a}(p_1)A^{\beta}_{b}(p_2)A^{\mu}_{c}(p_3)A^{\nu}_{d}(p_4)\rangle = \left(-\frac{g^2 C_A Q_1}{48\pi^2\epsilon} + \text{finite}\right)\Pi^{\alpha\beta\mu\nu}(p_1,p_2,p_3,p_4)\mathrm{tr}([T^a,T^d][T^b,T^c] + [T^a,T^c][T^b,T^d])(2\pi)^4\delta^{(4)}(p_1+p_2+p_3+p_4), \tag{13}$$

where in this case

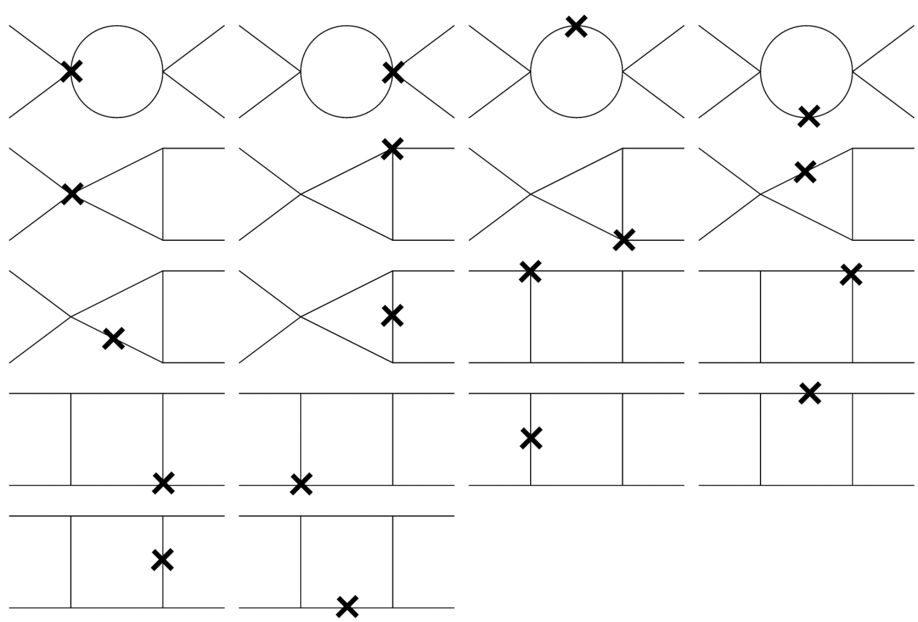

FIG. 3. Topologies for the four-point function Feynman diagrams with single LV vertex insertions. Both gauge and scalar internal lines are implicitly included.

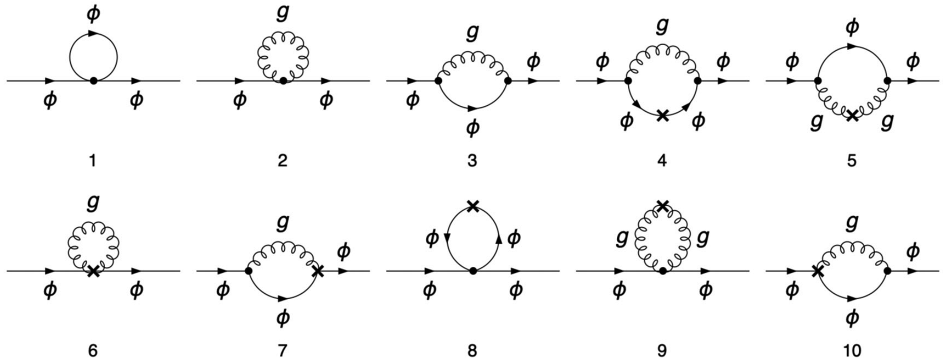

FIG. 4. Feynman diagrams for the scalar field self-energy.

$$\Pi^{\alpha\beta\mu\nu}(p_1,p_2,p_3,p_4) = u^\alpha u^\beta \eta^{\mu\nu} + u^\mu u^\nu \eta^{\alpha\beta} + u^\alpha u^\nu \eta^{\beta\mu} + u^\beta u^\mu \eta^{\alpha\nu} - 2(u^\alpha u^\mu \eta^{\beta\nu} + u^\beta u^\mu \eta^{\beta\nu}). \quad (14)$$

Using the Lie algebra of the generators T_a of the $SU(N)$ gauge group and combining the results (9), (11), and (13), we obtain the following effective Lagrange density

$$\mathcal{L}_{\rm eff} = {\rm tr}\left[\left(Z_{Q_2}Q_2 - \frac{g^2 C_A Q_1}{96\pi^2\epsilon} + {\rm finite}\right) u_\mu u_\nu F^{\mu\alpha} F^\nu{}_\alpha\right], \quad (15)$$

where $Z_{Q_2}Q_2 = (Q_2 + \delta_{Q_2})$, with δ_{Q_2} being the appropriate counterterm, which in the minimal subtraction (MS) renormalization scheme is evidently given by

$$\delta_{Q_2} = \frac{g^2 C_A Q_1}{96\pi^2\epsilon}. \quad (16)$$

This is one of the pieces needed to determine how the RG behavior of the theory is affected by the charged scalars. In the next section we compute the scalar self-energy contribution, as well as the gluon-matter contribution, in order to complete the computation the β functions for the model.

IV. SCALAR SELF-ENERGY AND THE GLUON-MATTER VERTEX

Let us now compute the one-loop scalar self-energy (Fig. 4), $\Sigma(p) = \langle {\rm T}\phi_a^i(p)\phi_b^{j*}(-p)\rangle$. The expression corresponding to Fig. 4's diagram 1 (or "diagram 4–1") is given by

$$\Sigma_1(p) = -i\int \frac{d^4k}{(2\pi)^4}\frac{2\lambda N_s}{(k^2-m^2)}\delta_{ij}\delta_{ab} \quad (17)$$

$$= \frac{\lambda N_s}{4\pi^2} A_0(m^2)\delta_{ij}\delta_{ab}, \quad (18)$$

where $A_0(m^2)$ is one of the Passarino-Veltman basis integrals (see Appendix A), and N_s is the number of scalar fields [which must be a multiple of (N^2-1) for $SU(N)$ adjoint scalars].

The expression for the tadpole diagram 4–2 is proportional to $A_0(0)$—that is, vanishing (up to irrelevant, regulator-dependent finite terms). However, the corresponding expression for diagram 4–3 is

$$\Sigma_3(p) = i\int \frac{d^4k}{(2\pi)^4}\frac{g^2\eta_{\mu\nu}(k+p)^\mu (k+p)^\nu f^{acd} f^{bcd}}{(k^2-m^2)(k-p)^2}\delta_{ij} \quad (19)$$

$$= \frac{g^2 C_A}{16\pi^2}[A_0(m^2) - 2(p^2+m^2)B_0(p^2,0,m^2)]\delta_{ij}\delta_{ab}, \quad (20)$$

where $B_0(p^2,0,m^2)$ is another integral from the Passarino-Veltman basis.

The expression for the first diagram with a LV vertex insertion, diagram 4–4, is given by

$$\Sigma_4(p) = -i\int \frac{d^4k}{(2\pi)^4}\frac{g^2 Q_1 (k+p)^2 (k\cdot u)^2 f^{acd} f^{bcd}}{(k^2-m^2)^2 (k-p)^2}\delta_{ij} \quad (21)$$

$$= -\frac{g^2 Q_1 C_A}{24\pi^2}\frac{(p\cdot u)^2}{p^4}\Big[2(p^2+m^2)A_0(m^2) - (2p^4 + 4m^2p^2 + 3m^4)B_0(p^2,0,m^2) + (p^2+m^2)^2 B_0(0,m^2,m^2) - (p^6 + 2m^2p^4 + 2m^4p^2 + m^6)C_0(0,p^2,p^2,m^2,m^2,0)\Big]\delta_{ij}\delta_{ab}. \quad (22)$$

Here we have encountered the last of the Passarino-Veltman basis integrals that we need; like the others, $C_0(0,p^2,p^2,m^2,m^2,0)$ is given in Appendix A.

Continuing, the expression for diagram 4–5 is

$$\Sigma_5(p) = -i\int \frac{d^4k}{(2\pi)^4}\frac{g^2Q_2[p^2(k\cdot u)^2 + k^2(p\cdot u)^2 - 2(k\cdot p)(k\cdot u)(p\cdot u)]f^{acd}f^{bcd}}{2(k^2)^2[(k-p)^2-m^2]}\delta_{ij} \tag{23}$$

$$= -\frac{g^2Q_2C_A}{192\pi^2}\frac{(p\cdot u)^2}{p^2}\Big[A_0(m^2) - 2(p^2+m^2)B_0(p^2,0,m^2) - (p^2-m^2)B_0(0,0,0) + (p^2-m^2)^2C_0(0,p^2,p^2,0,0,m^2)\Big]\delta_{ij}\delta_{ab}. \tag{24}$$

Diagram 4–6 is once again a tadpole proportional to $A_0(0)=0$. The following diagram, 4–7, can be expressed as

$$\Sigma_7(p) = i\int\frac{d^4k}{(2\pi)^4}\frac{g^2Q_1[(k+p)\cdot u]^2f^{acd}f^{bcd}}{(k^2-m^2)^2(k-p)^2}\delta_{ij} \tag{25}$$

$$= \frac{g^2Q_2C_A}{48\pi^2}\frac{(p\cdot u)^2}{p^4}\Big[(4p^2+m^2)A_0(m^2) - (7p^4+4m^2p^2+m^4)B_0(p^2,0,m^2)\Big]\delta_{ij}\delta_{ab}. \tag{26}$$

The expression corresponding to diagram 4–8 is proportional to the trace of the LV tensor $c^{\mu\nu}$—that is to $u^2=0$. Since diagram 4–9 is tadpole, it can also be expressed as

$$\Sigma_9(p) = i\int\frac{d^4k}{(2\pi)^4}\frac{g^2Q_2(k\cdot u)^2f^{acd}f^{bcd}}{4(k^2)^2}\delta_{ij} \propto A_0(0) = 0. \tag{27}$$

Finally, the expression for diagram 4–10 is

$$\Sigma_{10}(p) = i\int\frac{d^4k}{(2\pi)^4}\frac{g^2Q_1[(k-p)\cdot u]^2f^{acd}f^{bcd}}{(k^2-m^2)(k+p)^2}\delta_{ij} = \Sigma_7(p). \tag{28}$$

Adding all the above expressions, along with the counterterm diagrams, we find

$$\Sigma(p) = \Bigg[\frac{4\lambda N_s m^2 - g^2C_A(2p^2+m^2)}{16\pi^2\epsilon} - \frac{g^2(Q_1+Q_2)C_A(p\cdot u)^2}{4\pi^2\epsilon}(\delta_2p^2 - \delta_{m^2}m^2) + \delta_{Q_1}(p\cdot u)^2 + \text{finite}\Bigg]\delta_{ij}\delta_{ab}, \tag{29}$$

where δ_2, δ_{m^2}, and δ_{Q_1} are the counterterms in question, with the forms: $\delta_2 = Z_2 - 1$, $\delta_{m^2} = (m_0^2Z_{m^2} - m^2)/m^2$, and $\delta_{Q_1} = Q_1(Z_{Q_1}-1)$. Here, Z_2, Z_{m^2}, and Z_{Q_1} are the corresponding renormalization constants (in particular, Z_2 is the renormalization constant for the scalar field redefinition $\phi_a \to \sqrt{Z_2}\phi_a$), and m_0^2 is the bare scalar mass squared. Imposing finiteness of (29) through the MS scheme, we find

$$\delta_2 = \frac{g^2C_A}{8\pi^2\epsilon}, \tag{30a}$$

$$\delta_{m^2} = \frac{(\lambda N_s - g^2C_A)}{16\pi^2\epsilon}, \tag{30b}$$

$$\delta_{Q_1} = \frac{g^2(Q_1+Q_2)C_A}{4\pi^2\epsilon}. \tag{30c}$$

For completeness, we shall also evaluate the radiative corrections to the gluon-matter-current interaction vertex. The Feynman diagrams of this process are depicted in Fig. 5. The general form taken by the vertex function is

$$\Gamma = i\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}f_{abc}\phi_a(p_1)\phi_b^*(p_2) \times \Gamma_\mu A_c^\mu(p_3)(2\pi)^4\delta(p_1+p_2+p_3). \tag{31}$$

The corresponding ultraviolet-divergent expression may be written as

$$\Gamma^\mu(p_1,p_2,p_3) = \frac{g^3C_A(p_2-p_1)^\mu}{16\pi^2\epsilon} + \frac{3g^3(Q_1+Q_2)C_A[(p_2-p_1)\cdot u]u^\mu}{16\pi^2\epsilon} + \delta_1 g(p_2-p_1)^\mu - \tilde{\delta}_{Q_1}[(p_2-p_1)\cdot u]u^\mu, \tag{32}$$

where δ_1 and $\tilde{\delta}_{Q_1}$ are the appropriate counterterms, and we have once again used momentum conservation to simplify the final expression. Setting (32) to be vanishing, we find the counterterm values

$$\delta_1 = \frac{g_0Z_2\sqrt{Z_3} - g}{g} = -\frac{g^2C_A}{16\pi^2\epsilon}, \tag{33a}$$

$$\tilde{\delta}_{Q_1} = \frac{g_0(Q_1)_0Z_2\sqrt{Z_3} - Q_1}{Q_1} = \frac{3g^2(Q_1+Q_2)C_A}{16\pi^2\epsilon}, \tag{33b}$$

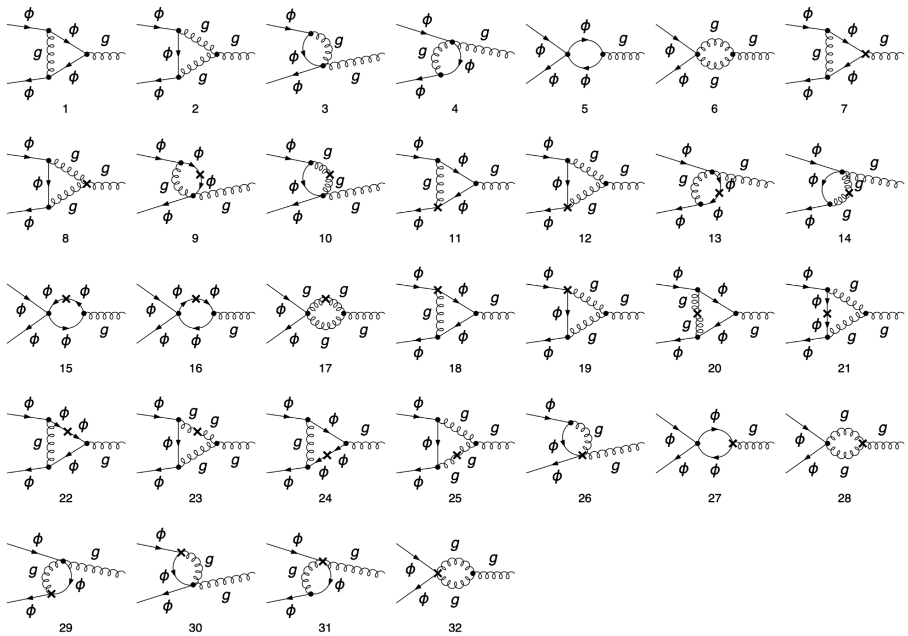

FIG. 5. Feynman diagrams for the gluon-matter vertex.

where Z_3 is the renormalization constant for the gluon field redefinition $A_\mu \to \sqrt{Z_3} A_\mu$. The counterterm $\delta_3 = Z_3 - 1$ is computed in Appendix B.

It is important to mention that the renormalization of the $\lambda\phi^4$ operator is just like in the ordinary theory, since the LV corrections are necessarily proportional to $u^2 = 0$. Any LV corrections to the scalar vertex must involve contractions of u^μ with derivatives of the field, making them manifestly finite by power counting.

V. β-FUNCTIONS

We now proceed to compute the scalar-matter coupling corrections to the one-loop β-functions of the Lorentz violation parameters Q_1 and Q_2. In the previous sections we have found the one-loop counterterms for the scalar and gluon fields and the gluon-matter vertex in our model. In order to evaluate the relevant Z factors to compute the β-functions of Q_1 and Q_2, we expand each renormalization constant Z_i as power series in the coupling constants, which can be determined order by order using perturbation theory,

$$Z_i = 1 + \delta_i^{(1)} + \delta_i^{(2)} + \cdots. \tag{34}$$

The relation between the bare and renormalized coupling constants may be cast as

$$Z_{Q_1} Q_1 = \mu^{-2\epsilon} (Q_1)_0 Z_2 = Q_1 + \delta_{Q_1}, \tag{35a}$$

$$Z_{Q_2} Q_2 = \mu^{-2\epsilon} (Q_2)_0 Z_3 = Q_2 + \delta_{Q_2}, \tag{35b}$$

where, as noted above, Z_2 ($\phi_a \to \sqrt{Z_2}\phi_a$) and Z_3 ($A_a^\mu \to \sqrt{Z_3} A_a^\mu$) are the field strength renormalization constants for the scalar and gauge boson fields, respectively.

Taking the scaling relations (35) between the bare and renormalized couplings, we can compute β_{Q_1} to be

$$\beta_{Q_1} = \lim_{\epsilon \to 0} \left[-2\epsilon Q_1 \left(1 + \delta_2 - \frac{\delta_{Q_1}}{Q_1} \right) \right] = \frac{g^2 (Q_1 + 2Q_2) C_A}{4\pi^2}. \tag{36}$$

When $Q_1 + 2Q_2 = 0$, there is no RG flow for this SME parameter. Similarly, the scalar matter-loop fluctuations result in the following corrections to the β_{Q_2} function:

$$\beta_{Q_2} = \lim_{\epsilon \to 0} \left[-2\epsilon Q_2 \left(1 + \delta_3 - \frac{\delta_{Q_2}}{Q_2} \right) \right] = \frac{g^2 N_s (Q_1 + 2Q_2)}{48\pi^2} + \cdots, \tag{37}$$

where the dots mean that we have included only the matter-loop corrections to β_{Q_2}.

VI. FINAL REMARKS

Renormalization is an essential part of the work to understand a quantum field theory. The regularization and renormalization processes allow us to verify the consistency of a specific field-theoretic model at the quantum level and to understand its underlying behavior at high energy scales. Even in theories, such as the SME, in which some of the core principles of relativistic field theory may have been discarded, it is still crucial to understand the theory's renormalization properties—either by generalizing and reformulating formal renormalization theorems or by explicit perturbative calculations. It is, in fact, one of the greatest strengths of the SME approach that, since it is an effective field theory, it is amenable to calculation of radiative corrections in essentially just the same way as the usual SM.

This paper has addressed the case where Lorentz symmetry is broken, but non-Abelian gauge symmetries are still preserved. More specifically, we formulated the LV extension of a non-Abelian gauge theory coupled to a scalar matter. We calculated the contributions to scalar and gluon field strength renormalization, as well as to the vector-scalar interaction vertex, at the lowest orders in the gauge couplings and the Lorentz violation parameters. Using these results, we computed the one-loop β-functions for the model. From the forms of β_{Q_1} and β_{Q_2}, it is evident that there is no running of these coupling constants when there is a particular relationship between the SME coefficients in the gauge and matter sectors, specifically when $Q_1 + 2Q_2 = 0$. When this relation is satisfied, the LV couplings are scale independent at this order; this corresponds to a theory in which the Lorentz violation may be eliminated from both sectors by a transformation to skewed four-dimensional Cartesian coordinates. The inclusion of quantum corrections should not change the fact that the theory is actually Lorentz invariant, but merely expressed in oblique coordinates.

The theory we have considered is, more generally, part of the non-Abelian gauge and scalar sectors of the LV SME. As we have already noted, there is still a significant amount of further work to be done to understand these types of theories, with couplings between of non-Abelian gauge fields and charged scalar matter—in particular, concerning the explicit calculation of radiative corrections. The present contribution gives a first step in this direction, but there are important topics that have still not been touched, such as the effects of spontaneous gauge symmetry breaking. Since the electroweak sector of the SM includes a multiplet of scalar Higgs fields that break $SU(2)_L$ gauge invariance, this is an important area in which further research will need to be undertaken in the future.

ACKNOWLEDGMENTS

The authors are grateful to J. R. Nascimento for important discussions. The work of A. Yu. P. has been partially supported by the CNPq project No. 301562/2019-9.

APPENDIX A: PASSARINO-VELTMAN BASIS INTEGRALS

Many of the integrals appearing in Sec. IV were reduced to linear combinations of the following standard integrals, which are components of the Passarino-Veltman basis:

$$A_0(m^2) = \int d^D k \frac{1}{k^2 - m^2} = \frac{m^2}{\epsilon} + \text{finite}; \tag{A1}$$

$$B_0(p^2, m_1^2, m_2^2) = \int d^D k \frac{1}{(k^2 - m_1^2)((k+p)^2 - m_2^2)} = \frac{1}{\epsilon} + \text{finite}; \tag{A2}$$

$$C_0(p_1^2, (p_1 - p_2)^2, p_2^2, m_1^2, m_2^2, m_3^2) = \int d^D k \frac{1}{(k^2 - m_1^2)((k+p_1)^2 - m_2^2)((k+p_2)^2 - m_3^2)} = \text{finite}. \tag{A3}$$

APPENDIX B: GLUON SELF-ENERGY

The Feynman diagrams for the ordinary one-loop gluon self-energy are depicted in Fig. 6. The general structure of the gluon two-point function has the form

$$\Gamma_A = \int \frac{d^4 p}{(2\pi)^4} A_a^\mu(p) \Pi_{\mu\nu}(p) A_a^\nu(-p). \tag{B1}$$

The expressions corresponding to the individual diagrams shown in Fig. 6 are

$$\Pi_1^{\mu\nu}(p) = i \int \frac{d^4 k}{(2\pi)^4} \frac{2g^2 N_s \eta^{\mu\nu}}{(k^2 - m^2)} = -\frac{g^2 N_s}{8\pi^2} \eta^{\mu\nu} A_0(m^2), \tag{B2}$$

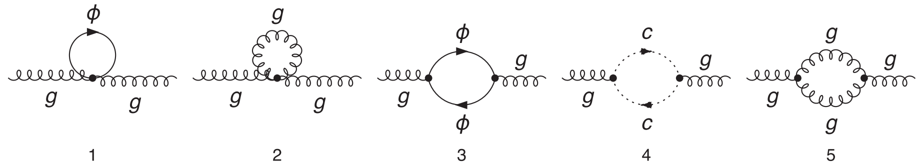

FIG. 6. Feynman diagrams for the ordinary gluon self-energy. Dashed lines represent Faddeev-Popov ghost propagators.

$$\Pi_3^{\mu\nu}(p) = -i\int \frac{d^4k}{(2\pi)^4}\frac{g^2N_s(2k-p)^\mu(2k-p)^\nu}{(k^2-m^2)((k-p)^2-m^2)} \tag{B3}$$

$$= \frac{g^2N_s}{48\pi^2}\frac{1}{p^2}[(p^2\eta^{\mu\nu}+2p^\mu p^\nu)A_0(m^2)+(4m^2-p^2)(p^2\eta^{\mu\nu}-p^\mu p^\nu)B_0(p^2,m^2,m^2)],$$

$$\Pi_4^{\mu\nu}(p) = i\int \frac{d^4k}{(2\pi)^4}\frac{g^2C_Ak^\mu(k-p)^\nu}{k^2(k-p)^2} = \frac{g^2C_A}{192\pi^2}(p^2\eta^{\mu\nu}+2p^\mu p^\nu)B_0(p^2,0,0), \tag{B4}$$

$$\Pi_5^{\mu\nu}(p) = -i\int \frac{d^4k}{(2\pi)^4}\frac{g^2C_A[\eta^{\mu\nu}(5p^2+2k^2-2(k\cdot p))-p^\mu(5k^\nu+2p^\nu)+5k^\mu(2k^\nu-p^\nu)]}{2k^2(k-p)^2}$$

$$= \frac{g^2C_A}{192\pi^2}(p^2\eta^{\mu\nu}+2p^\mu p^\nu)B_0(p^2,0,0), \tag{B5}$$

while diagram 6–2, being a tadpole, is proportional to $A_0(0) = 0$.

Adding these contributions, substituting the integrals (see again Appendix A), and keeping only the ultraviolet-divergent terms, we have

$$\Pi^{\mu\nu}(p) = (p^2\eta^{\mu\nu}-p^\mu p^\nu)\left[\frac{g^2(5C_A-N_s)}{12\pi^2\epsilon}-\delta_3\right]. \tag{B6}$$

Imposing finiteness, we immediately find

$$\delta_3 = \frac{g^2(5C_A-N_s)}{12\pi^2\epsilon}. \tag{B7}$$

Notice that if we have only one octet of QCD scalar fields, we will have $N_s = N^2-1 = 8$ and $C_A = N = 3$.

[1] D. Colladay and V. A. Kostelecký, *CPT* violation and the Standard Model, Phys. Rev. D **55**, 6760 (1997).

[2] D. Colladay and V. A. Kostelecký, Lorentz violating extension of the Standard Model, Phys. Rev. D **58**, 116002 (1998).

[3] V. A. Kostelecký, C. D. Lane, and A. G. M. Pickering, One loop renormalization of Lorentz violating electrodynamics, Phys. Rev. D **65**, 056006 (2002).

[4] A. F. Ferrari, J. R. Nascimento, and A. Yu. Petrov, Radiative corrections and Lorentz violation, Eur. Phys. J. C **80**, 459 (2020).

[5] V. A. Kostelecký, Gravity, Lorentz violation, and the Standard Model, Phys. Rev. D **69**, 105009 (2004).

[6] D. Colladay and P. McDonald, One-loop renormalization of pure Yang-Mills with Lorentz violation, Phys. Rev. D **75**, 105002 (2007).

[7] D. Colladay and P. McDonald, One-loop renormalization of QCD with Lorentz violation, Phys. Rev. D **77**, 085006 (2008).

[8] D. Colladay and P. McDonald, One-loop renormalization of the electroweak sector with Lorentz violation, Phys. Rev. D **79**, 125019 (2009).

[9] M. Gomes, J. R. Nascimento, E. Passos, A. Yu. Petrov, and A. J. da Silva, On the induction of the four-dimensional Lorentz-breaking non-Abelian Chern-Simons action, Phys. Rev. D **76**, 047701 (2007).

[10] A. J. G. Carvalho, D. R. Granado, J. R. Nascimento, and A. Yu. Petrov, Non-Abelian aether-like term in four dimensions, Eur. Phys. J. C **79**, 817 (2019).

[11] A. P. Baêta Scarpelli, J. C. C. Felipe, L. C. T. Brito, and A. Yu. Petrov, One-loop calculations in *CPT*-even Lorentz-breaking scalar QED, Mod. Phys. Lett. A **37**, 2250100 (2022).

[12] B. Altschul, L. C. T. Brito, J. C. C. Felipe, S. Karki, A. C. Lehum, and A. Yu. Petrov, Three- and four-point functions in *CPT*-even Lorentz-violating scalar QED, Phys. Rev. D **107**, 045005 (2023).

[13] B. Altschul, Astrophysical limits on Lorentz violation for all charged species, Astropart. Phys. **28**, 380 (2007).

[14] M. Cambiaso, R. Lehnert, and R. Potting, Massive photons and Lorentz violation, Phys. Rev. D **85**, 085023 (2012).

[15] R. Mertig, M. Bahm, and A. Denne, Feyn Calc—computer-algebraic calculation of Feynman amplitudes, Comput. Phys. Commun. **64**, 345 (1991).

[16] T. Hahn, Generating Feynman diagrams and amplitudes with FeynArts 3, Comput. Phys. Commun. **140**, 418 (2001).

[17] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr, and B. Fuks, FeynRules 2.0—a complete toolbox for tree-level phenomenology, Comput. Phys. Commun. **185**, 2250 (2014).