Curvature-squared invariants of minimal five-dimensional supergravity from superspace

Gregory Gold,[*] Jessica Hutomo,[†] Saurish Khandelwal,[‡] and Gabriele Tartaglino-Mazzucchelli[§]
School of Mathematics and Physics, University of Queensland, St. Lucia, Brisbane, Queensland 4072, Australia

(Received 17 March 2023; accepted 13 April 2023; published 19 May 2023)

We elaborate on the off-shell superspace construction of curvature-squared invariants in minimal five-dimensional supergravity. This is described by the standard Weyl multiplet of conformal supergravity coupled to two compensators being a vector multiplet and a linear multiplet. In this setup, we review the definition of the off-shell two-derivative gauged supergravity together with the three independent four-derivative superspace invariants defined in Butter *et al.* [J. High Energy Phys. 02 (2015) 111]. We provide the explicit expression for the linear multiplet based on a prepotential given by the logarithm of the vector multiplet primary superfield. We then present for the first time the primary equations of motion for minimal gauged off-shell supergravity deformed by an arbitrary combination of these three four-derivative locally superconformal invariants. We also identify a four-derivative invariant based on the linear multiplet compensator and the kinetic superfield of a vector multiplet, which can be used to engineer an alternative supersymmetric completion of the scalar curvature squared.

DOI: 10.1103/PhysRevD.107.106013

I. INTRODUCTION

Almost five decades after the first (two-derivative) supergravity was constructed (for $\mathcal{N}=1$ supersymmetry in four dimensions), higher-order locally supersymmetric invariants are still largely unknown. Higher-order curvature terms play, however, a significant role in string theory, where quantum corrections take the form of an infinite series potentially constrained by supersymmetry order by order in the string tension α' and the string coupling g_s. Many open problems in string theory, for example, its vacua structure, are unresolved due to the lack of information about the full quantum corrected supergravity effective action. The complexity of such an effective theory is even made worse by the fact that the purely gravitational higher-curvature terms are related by supersymmetry to contributions depending on p-forms, which describe part of the string spectrum. These terms, which have not yet been systematically understood, play an important role in studying, for example, the moduli in compactified string theory and the low-energy description of string dualities; see, e.g., [1–3]. In the context of string-inspired holographic dualities, such as the AdS/CFT, higher-order $1/N$ corrections in quantum field theories translate into higher-curvature terms on the gravity side, making these contributions fundamental for precision tests in AdS/CFT. New interesting analyses on this topic have been performed in the last few years—see, for example, [4–12] and references therein.

One obstacle to constructing locally supersymmetric higher-order invariants is that often supersymmetry is only realized *on-shell*, meaning the symmetry algebra closes by using equations of motion. In on-shell approaches—which are, e.g., typically used in 10- and 11-dimensional theories—one needs to intertwine the construction of higher-order invariant terms in the Lagrangian of interest with a systematic and consistent deformation of the supersymmetry transformations, making the problem remarkably involved. This obstacle is simplified by using "off-shell supersymmetry," where one introduces extra (auxiliary) degrees of freedom to obtain supersymmetric multiplets possessing model-independent transformation rules. In a low number of space-time dimensions (D), in particular, $\mathrm{D}\leq 6$, off-shell techniques are by now well developed and understood for up to eight real supercharges—see [13–19] for reviews of off-shell approaches to supersymmetry and supergravity. In these cases, the construction of supergravity higher-derivative invariants can, in principle, be systematically approached. A restricted list of references using off-shell approaches to construct locally supersymmetric higher-derivative invariants includes [20–46].

[*]g.gold@uq.edu.au
[†]j.hutomo@uq.edu.au
[‡]s.khandelwal@uq.edu.au
[§]g.tartaglino-mazzucchelli@uq.edu.au

The scope of our paper is to enhance the classification of off-shell curvature-squared invariants of minimal five-dimensional (5D) supergravity. Minimal on-shell 5D supergravity was introduced four decades ago in [47,48], and the first off-shell description was given in [49] by the use of superspace techniques. Since then, 5D minimal supergravity and its matter couplings have been extensively studied at the component level, both in on- [50–53] and off-shell [54–63] settings. The superspace approach to general off-shell 5D $\mathcal{N}=1$ supergravity-matter systems has then been developed in [20,64–66]; see also [67] for a recent local supertwistor description of 5D conformal supergravity.

In our paper, we will specifically use the 5D $\mathcal{N}=1$ conformal superspace approach of [20].[1] This approach merges advantages of the 5D superconformal tensor calculus of [57–63] with the superspace approaches of [49,64–66]. In the superconformal setup (both in components and superspace), one enlarges the supergravity gauge group to be described by local superconformal transformations, plus potentially internal symmetries. Local Poincaré supersymmetry is then recovered by using an appropriate choice of compensating multiplets that are used to gauge fix extra nonphysical symmetries within the conformal algebra. For instance, in this setup, the off-shell formulation of minimal 5D supergravity is achieved by coupling the standard Weyl multiplet of 5D conformal supergravity to two off-shell conformal compensators: a vector multiplet and a hypermultiplet, the latter conveniently described by a linear multiplet. These will be the off-shell multiplets used in our paper. Within this setup, locally supersymmetric completions of the Weyl tensor squared and the scalar curvature squared were constructed for the first time, respectively, in [25] and [32] by using component fields techniques. Up to total derivatives, a generic combination of curvature-squared terms in five dimensions should include also a Ricci tensor-squared invariant. A third independent locally superconformal invariant that includes Ricci squared was indeed constructed in superspace in [20] by using a 5D analog of the "log multiplet" construction in 4D $\mathcal{N}=2$ supergravity of [33]. However, due to the computational complexity of the log multiplet in five dimensions, the component analysis of this invariant has not appeared so far—in a follow-up paper, we will report on the component structure of this invariant, which has been computed by making use of the computer algebra program Cadabra [72,73].

Note that the conformal approach described above is not unique. In five dimensions it is known that an efficient setup to describe general supergravity-matter couplings make use of a vector-dilaton Weyl multiplet as a multiplet of conformal supergravity in place of the standard Weyl one [59,61].[2] A remarkable property of systems based on the use of a 5D vector-dilaton Weyl multiplet, which is related to the Poincaré supergravity first introduced in [76], is the simplicity to define a third locally supersymmetric curvature-squared invariant. In fact, by employing a map between fields of the vector-dilaton Weyl multiplet and an off-shell vector multiplet, in [27] a locally supersymmetric extension of the Riemann tensor squared was constructed (a construction that, however, is not applicable for a standard Weyl multiplet). This, together with the Weyl-squared invariant of [25], was sufficient for Ozkan and Pang to construct in [30,32] a locally supersymmetric extension of the Gauss-Bonnet combination, which is expected to play a key role in the description of the first α' corrections to compactified string theory [77,78].

Despite the remarkable features mentioned above, two important disadvantages of the use of a vector-dilaton Weyl multiplet are that (i) the spectrum of the on-shell theory does not precisely match the one of minimal Poincaré supergravity as, in fact, it leads to an extra on-shell physical multiplet that includes a scalar (dilaton) field; and (ii) it is not possible to describe gauged supergravity and then anti–de Sitter (AdS) supergravity.[3] This second limitation has a clear impact if one is interested in using off-shell supergravity in the study of AdS/CFT. Indeed, the authors of [8,11,12] employed a formulation of minimal gauged supergravity in five dimensions based on the standard Weyl multiplet, for which, however, they could only use two of the three independent invariants, the ones of [25,30,32], explicitly known in terms of the component fields. To this regard, it is worth explaining that, as first discussed in [4], see also [5,8,11,12], the use of two invariants might suffice in five dimensions since a curvature-dependent redefinition of the metric can reabsorb one of the three curvature-squared terms. It remains, however, a nontrivial open problem to prove this statement for whole locally supersymmetric invariants (e.g., including fermions) and to have clear control of the supersymmetry transformations under

[1]Conformal superspace was originally introduced by D. Butter for 4D $\mathcal{N}=1$ supergravity in [68] and then extended to other space-time dimensions $2 \leq \mathrm{D} \leq 6$ for various amounts of supersymmetry in [20,38,41,69–71]—see also [18,19] for recent reviews.

[2]The vector-dilaton Weyl multiplet terminology is used here to stress that the variant multiplet of conformal supergravity in [59,61] is defined as an on-shell vector multiplet coupled to the standard Weyl multiplet. It was recently shown in [74,75] that an on-shell hypermultiplet in a standard Weyl multiplet background can be reinterpreted as yet another new variant Weyl multiplet of off-shell conformal supergravity, which was referred to as hyperdilaton Weyl.

[3]It has been proposed in [79], and successively described also in superspace in [20], how to gauge a system based on the vector-dilaton Weyl multiplet by appropriately deforming the constraint of the on-shell vector multiplet. However, so far, this construction has not been systematically studied as for gauged supergravities based on the standard Weyl multiplet, including curvature-squared invariants. Interestingly, the hyperdilaton Weyl multiplet of [74,75] has no apparent issues concerning gauging, at least for matter systems not including extra physical hypermultiplets.

this redefinition. All three invariants might also play a role to construct general higher-derivative invariants beyond four derivatives. It is also worth mentioning that the related recent analysis of [9] was based on the three independent curvature-squared invariants of [25,27,30,32] defined using a vector-dilaton Weyl multiplet. However, it remains unclear to us whether the analysis of [9] might have some issues with supersymmetry, due to the constraints in defining the gauging (or, equivalently, the cosmological constant term) in a vector-dilaton Weyl formulation.

Considering the potential subtleties in the recent studies in [5,8,9,11,12] it is natural to look back at [20] and elaborate on properties of the three independent curvature-squared invariants for minimal supergravity constructed in superspace. A fundamental property of these locally superconformal invariants is that they can all be constructed by using a standard Weyl multiplet, making straightforward their addition to the 5D minimal off-shell two-derivative gauged supergravity theory. In this paper, we then start to report on new results based on these invariants. More specifically, we will present here detailed expressions of all the composite primary superfields associated with each invariant—including a new expression for the log multiplet—which can be readily used for component analyses. We will then describe the primary equations of motion in superspace that describe minimal 5D gauged supergravity deformed by an arbitrary combination of three curvature-squared invariants. Our results are defined in superspace, but they can be straightforwardly translated in components by using the analysis of [20]. In particular, since all the expressions are fully covariant and described explicitly in terms of composites of descendants of the various multiplets, one could, for example, straightforwardly obtain the whole set of deformed supergravity equations of motion by the successive action of Q supersymmetry. This will then be a new step toward several applications of the three locally superconformal invariants of [20]. Moreover, we also introduce an alternative four-derivative invariant based on the linear multiplet compensator and the kinetic superfield of the vector multiplet compensator. This can be used to engineer a scalar curvature-squared invariant also in alternative off-shell supergravities as, for example, the formulation based on the recently introduced 5D hyperdilaton Weyl multiplet [75].

Our paper is organized as follows. In Sec. II, we describe the structure of 5D $\mathcal{N}=1$ superconformal multiplets that will be used in this work. In Sec. III we give the salient details of [20] concerning the superspace construction of various locally superconformal invariants (including curvature-squared ones), which will play the role of action principles. One of our new results in Sec. III includes the expression of a composite primary multiplet, which defines the log multiplet curvature-squared invariant. Section IV contains the main results of our paper: the superconformal primary equations of motion of all the curvature-squared terms for the minimal 5D gauged off-shell supergravity based on the standard Weyl multiplet. An alternative construction of a scalar curvature-squared invariant is presented in Sec. V. Our notation and conventions correspond to that of [20] (see also [75], where a handful of typos from [20] were fixed).

II. SUPERCONFORMAL MULTIPLETS IN 5D $\mathcal{N}=1$ SUPERSPACE

In this section, we review several superconformal multiplets that will serve as building blocks for the various curvature-squared invariants presented in this work. After describing the standard Weyl multiplet of conformal supergravity in 5D $\mathcal{N}=1$ conformal superspace, we move on to the discussion of the Abelian vector and off-shell linear multiplets. Here we make use of the approach and results given in [20]. We refer the reader to [49,64–66,80–83] for other works on flat and curved superspace and off-shell multiplets in five dimensions.

A. The standard Weyl multiplet

The standard Weyl multiplet of 5D $\mathcal{N}=1$ conformal supergravity [61] is associated with the gauging of the superconformal algebra $\mathrm{F}^2(4)$. The multiplet contains $32+32$ physical components described by a set of independent gauge fields: the vielbein $e_m{}^a$, the gravitino $\psi_m{}_\alpha^i$, the $\mathrm{SU}(2)_R$ gauge field $\phi_m{}^{ij}$, and a dilatation gauge field b_m. The other gauge fields associated with the remaining gauge symmetries—the spin connection $\omega_m{}^{ab}$, the S-supersymmetry connection $\phi_m{}_\alpha^i$, and the special conformal connection $\mathfrak{f}_m{}^a$—are composite fields, i.e., they are algebraically determined in terms of the other fields by imposing constraints on some of the curvature tensors. The standard Weyl multiplet also comprises a set of covariant auxiliary fields: a real antisymmetric tensor w_{ab}, a fermion χ^i_α, and a real auxiliary scalar D.

The 5D $\mathcal{N}=1$ conformal superspace is parametrized by local bosonic (x^m) and fermionic (θ_i) coordinates $z^M=(x^m,\theta^\mu_i)$, where $m=0,\ 1,\ 2,\ 3,\ 4,\ \mu=1,\ldots,4$, and $i=1,\ 2$. To perform the gauging of the superconformal algebra, one introduces covariant derivatives $\nabla_A=(\nabla_a,\nabla^i_\alpha)$, which have the form

$$\nabla_A=E_A-\omega_A{}^{\underline{b}}X_{\underline{b}}=E_A-\frac{1}{2}\Omega_A{}^{ab}M_{ab}-\Phi_A{}^{ij}J_{ij}-B_A\mathbb{D}-\mathfrak{F}_A{}^BK_B, \tag{2.1a}$$

$$=E_A-\frac{1}{2}\Omega_A{}^{ab}M_{ab}-\Phi_A{}^{ij}J_{ij}-B_A\mathbb{D}-\mathfrak{F}_A{}^{\alpha i}S_{\alpha i}-\mathfrak{F}_A{}^aK_a. \tag{2.1b}$$

Here $E_A=E_A{}^M\partial_M$ is the inverse supervielbein, M_{ab} are the Lorentz generators, J_{ij} are generators of the $\mathrm{SU}(2)_R$

R-symmetry group, $\mathbb{D}$ is the dilatation generator, and $K_A = (K_a, S_{\alpha i})$ are the special superconformal generators. The supervielbein 1-form is $E^A = \mathrm{d}z^M E_M{}^A$ with $E_M{}^A E_A{}^N = \delta_M^N$ and $E_A{}^M E_M{}^B = \delta_A^B$. We associate with each generator $X_{\underline{a}} = (M_{ab}, J_{ij}, \mathbb{D}, S_{\alpha i}, K_a)$ a connection super 1-form $\omega^{\underline{a}} = (\Omega^{ab}, \Phi^{ij}, B, \mathfrak{F}^{\alpha i}, \mathfrak{F}^a) = \mathrm{d}z^M \omega_M{}^{\underline{a}} = E^A \omega_A{}^{\underline{a}}$.

The algebra of covariant derivatives,

$$[\nabla_A, \nabla_B\} = -\mathcal{T}_{AB}{}^C \nabla_C - \frac{1}{2}\mathcal{R}(M)_{AB}{}^{cd} M_{cd} - \mathcal{R}(J)_{AB}{}^{kl} J_{kl} - \mathcal{R}(\mathbb{D})_{AB}\mathbb{D} - \mathcal{R}(S)_{AB}{}^{\gamma k} S_{\gamma k} - \mathcal{R}(K)_{AB}{}^c K_c, \tag{2.2}$$

is constrained to be expressed in terms of a single primary superfield, the super-Weyl tensor $W_{\alpha\beta}$.[4] It has the following properties:

$$W_{\alpha\beta} = W_{\beta\alpha}, \qquad K_A W_{\alpha\beta} = 0, \qquad \mathbb{D} W_{\alpha\beta} = W_{\alpha\beta}, \tag{2.3}$$

and satisfies the Bianchi identity

$$\nabla_\gamma^k W_{\alpha\beta} = \nabla_{(\alpha}^k W_{\beta\gamma)} + \frac{2}{5}\varepsilon_{\gamma(\alpha}\nabla^{\delta k} W_{\beta)\delta}. \tag{2.4}$$

In (2.2) $\mathcal{T}_{AB}{}^C$ is the torsion, and $\mathcal{R}(M)_{AB}{}^{cd}$, $\mathcal{R}(J)_{AB}{}^{kl}$, $\mathcal{R}(\mathbb{D})_{AB}$, $\mathcal{R}(S)_{AB}{}^{\gamma k}$, and $\mathcal{R}(K)_{AB}{}^c$ are the curvatures associated with Lorentz, $\mathrm{SU}(2)_R$, dilatation, S supersymmetry, and special conformal boosts, respectively.

The full algebra of covariant derivatives (2.2) (including the explicit expressions for the torsion and curvature components in terms of the descendant superfields) are given in Refs. [20,75]. To make use of the results of [20], it is important to note that in this paper we make use of the "traceless" frame conventional constraints for the conformal superspace algebra employed in Appendix C of [20], as well as in [75]. We also refer the reader to these papers for the description of how to reduce superspace results to standard component fields.

It is useful to introduce the dimension-3/2 superfields

$$W_{\alpha\beta\gamma}{}^k \coloneqq \nabla_{(\alpha}^k W_{\beta\gamma)}, \qquad X_\alpha^i \coloneqq \frac{2}{5}\nabla^{\beta i} W_{\beta\alpha}, \tag{2.5a}$$

and the dimension-2 descendant superfields constructed from spinor covariant derivatives of $W_{\alpha\beta}$,

$$W_{\alpha\beta\gamma\delta} \coloneqq \nabla_{(\alpha}^k W_{\beta\gamma\delta)k}, \qquad X_{\alpha\beta}{}^{ij} \coloneqq \nabla_{(\alpha}^{(i} X_{\beta)}^{j)}, \qquad Y \coloneqq \mathrm{i}\nabla^{\gamma k} X_{\gamma k}. \tag{2.5b}$$

It can be checked that only the superfields (2.5) and their vector derivatives appear upon taking successive spinor derivatives of $W_{\alpha\beta}$. The following relations define the tower of covariant fields in the standard Weyl multiplet and are particularly useful for analyzing the structure of curvature-squared invariants:

$$\nabla_\gamma^k W_{\alpha\beta} = W_{\alpha\beta\gamma}{}^k + \varepsilon_{\gamma(\alpha} X_{\beta)}^k, \tag{2.6a}$$

$$\nabla_\alpha^i X_\beta^j = X_{\alpha\beta}{}^{ij} + \frac{\mathrm{i}}{8}\varepsilon^{ij}\varepsilon_{\alpha\beta} Y - \frac{3\mathrm{i}}{2}\varepsilon^{ij}(\Gamma^a)_\alpha{}^\rho \nabla_a W_{\beta\rho} - 2\mathrm{i}\varepsilon^{ij} W_\alpha{}^\rho W_{\beta\rho} + \frac{\mathrm{i}}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta} W^{\gamma\delta} W_{\gamma\delta} - \frac{\mathrm{i}}{2}\varepsilon^{ij}(\Gamma^a)_\beta{}^\rho \nabla_a W_{\alpha\rho}, \tag{2.6b}$$

$$\nabla_\alpha^i W_{\beta\gamma\lambda}^j = -\frac{1}{2}\varepsilon^{ij}(W_{\alpha\beta\gamma\lambda} + 3\mathrm{i}(\Gamma_a)_{\alpha(\beta}\nabla^a W_{\gamma\lambda)} + 3\mathrm{i}\varepsilon_{\alpha(\beta}(\Gamma_a)_\gamma{}^\tau \nabla^a W_{\lambda)\tau}) - \frac{3}{2}\varepsilon_{\alpha(\beta} X_{\gamma\lambda)}{}^{ij}, \tag{2.6c}$$

$$\nabla_\alpha^i W_{\beta\gamma\lambda\rho} = -4\mathrm{i}(\Gamma_a)_{\alpha(\beta}\nabla^a W_{\gamma\lambda\rho)}{}^i - 6\mathrm{i} W_{\alpha(\beta\gamma}{}^i W_{\lambda\rho)} + 6\mathrm{i} W_{\alpha(\beta} W_{\gamma\lambda\rho)}{}^i + 6\mathrm{i}\varepsilon_{\alpha(\beta}(W_{\gamma\lambda} X_{\rho)}^i - 2(\Gamma_a)_\gamma{}^\tau \nabla^a W_{\lambda\rho)\tau}{}^i - W_\gamma{}^\tau W_{\lambda\rho)\tau}{}^i), \tag{2.6d}$$

$$\nabla_\alpha^i X_{\beta\gamma}{}^{jk} = \mathrm{i}\varepsilon^{i(j}(-3W_{(\beta}{}^\lambda W_{\gamma)\alpha\lambda}{}^{k)} - \varepsilon_{\alpha(\beta} W^{\rho\tau} W_{\gamma)\rho\tau}{}^{k)} - W_{\alpha\lambda} W_{\beta\gamma}{}^{\lambda k)} - \frac{3}{2}W_{\beta\gamma} X_\alpha^{k)} + \frac{1}{2}W_{\alpha(\beta} X_{\gamma)}^{k)} + \frac{3}{2}\varepsilon_{\alpha(\beta} W_{\gamma)\lambda} X^{k)\lambda} + 2(\Gamma^a)_\alpha{}^\rho \nabla_a W_{\beta\gamma\rho}{}^{k)} + 2(\Gamma^a)_{(\beta}{}^\rho \nabla_a W_{\gamma)\alpha\rho}{}^{k)} - (\Gamma^a)_{\alpha(\beta}\nabla_a X_{\gamma)}^{k)} + \varepsilon_{\alpha(\beta}(\Gamma^a)_{\gamma)\lambda}\nabla_a X^{k)\lambda}), \tag{2.6e}$$

$$\nabla_\alpha^i Y = 8(\Gamma^a)_\alpha{}^\beta \nabla_a X_\beta^i + 8W_\alpha{}^\beta X_\beta^i. \tag{2.6f}$$

Because of (2.4), the $X_{\alpha\beta}{}^{ij}$ and $W_{\alpha\beta\gamma\delta}$ dimension-2 superfields of the standard Weyl multiplet obey the following Bianchi identities:

$$\nabla_{(\alpha}{}^\gamma X_{\beta)\gamma}{}^{ij} = -\frac{1}{2}X^{\gamma(i} W_{\alpha\beta\gamma}{}^{j)}, \tag{2.7a}$$

$$\nabla_{(\alpha}{}^\lambda W_{\beta\gamma\tau)\lambda} = 3\mathrm{i}\nabla_{(\alpha}{}^\lambda(W_{\beta\gamma} W_{\tau)\lambda}). \tag{2.7b}$$

The independent descendant superfields of $W_{\alpha\beta}$ are all annihilated by K_a. However, under S supersymmetry, they transform as follows:

$$S_{\alpha i} W_{\beta\gamma\delta}{}^j = 6\delta_i^j \varepsilon_{\alpha(\beta} W_{\gamma\delta)}, \qquad S_{\alpha i} X_\beta^j = 4\delta_i^j W_{\alpha\beta}, \tag{2.8a}$$

$$S_{\alpha i} W_{\beta\gamma\delta\rho} = 24\varepsilon_{\alpha(\beta} W_{\gamma\delta\rho)}{}_i, \qquad S_{\alpha i} Y = 8\mathrm{i} X_{\alpha i}, \tag{2.8b}$$

$$S_{\alpha i} X_{\beta\gamma}{}^{jk} = -4\delta_i^{(j} W_{\alpha\beta\gamma}{}^{k)} + 4\delta_i^{(j}\varepsilon_{\alpha(\beta} X_{\gamma)}^{k)}. \tag{2.8c}$$

[4]Here and in what follows, an antisymmetric rank-2 tensor $T_{ab} = -T_{ba}$ can equivalently be written as $T_{ab} = (\Sigma_{ab})^{\alpha\beta} T_{\alpha\beta}$ and $T_{\alpha\beta} = 1/2(\Sigma^{ab})_{\alpha\beta} T_{ab}$.

The conformal supergravity gauge group $\mathcal{G}$ is generated by covariant general coordinate transformations $\delta_{\rm cgct}$, associated with a local superdiffeomorphism parameter ξ^A, and standard superconformal transformations $\delta_{\mathcal{H}}$, associated with the local superfield parameters: the dilatation σ, Lorentz $\Lambda^{ab} = -\Lambda^{ba}$, $\mathrm{SU}(2)_R$ $\Lambda^{ij} = \Lambda^{ji}$, and special conformal transformations $\Lambda^A = (\eta^{\alpha i}, \Lambda_K^a)$. The covariant derivatives transform as

$$\delta_{\mathcal{G}} \nabla_A = [\mathcal{K}, \nabla_A], \tag{2.9a}$$

where

$$\mathcal{K} = \xi^C \nabla_C + \frac{1}{2}\Lambda^{ab} M_{ab} + \Lambda^{ij} J_{ij} + \sigma \mathbb{D} + \Lambda^A K_A. \tag{2.9b}$$

A covariant (or tensor) superfield U transforms as

$$\delta_{\mathcal{G}} U = (\delta_{\rm cgct} + \delta_{\mathcal{H}}) U = \mathcal{K} U. \tag{2.10}$$

The superfield U is a "superconformal primary" of dimension Δ if $K_A U = 0$ (it suffices to require that $S_{\alpha i} U = 0$) and $\mathbb{D} U = \Delta U$.

B. The Abelian vector multiplet

In conformal superspace [20], a 5D $\mathcal{N} = 1$ Abelian vector multiplet [83,84] is described by a real primary superfield W of dimension 1,

$$(W)^* = W, \qquad K_A W = 0, \qquad \mathbb{D} W = W. \tag{2.11a}$$

The superfield W obeys the Bianchi identity

$$\nabla_\alpha^{(i} \nabla_\beta^{j)} W = \frac{1}{4} \varepsilon_{\alpha\beta} \nabla^{\gamma(i} \nabla_\gamma^{j)} W. \tag{2.11b}$$

Let us introduce the following descendants constructed from spinor derivatives of W:

$$\lambda_\alpha^i \coloneqq -\mathrm{i} \nabla_\alpha^i W, \qquad X^{ij} \coloneqq \frac{\mathrm{i}}{4} \nabla^{\alpha(i} \nabla_\alpha^{j)} W = -\frac{1}{4} \nabla^{\alpha(i} \lambda_\alpha^{j)}. \tag{2.12a}$$

These superfields, along with

$$F_{\alpha\beta} \coloneqq -\frac{\mathrm{i}}{4} \nabla_{(\alpha}^k \nabla_{\beta)k} W - W_{\alpha\beta} W = \frac{1}{4} \nabla_{(\alpha}^k \lambda_{\beta)k} - W_{\alpha\beta} W, \tag{2.12b}$$

satisfy the following identities:

$$\nabla_\alpha^i \lambda_\beta^j = -2\varepsilon^{ij} (F_{\alpha\beta} + W_{\alpha\beta} W) - \varepsilon_{\alpha\beta} X^{ij} - \varepsilon^{ij} \nabla_{\alpha\beta} W, \tag{2.13a}$$

$$\nabla_\alpha^i F_{\beta\gamma} = -\mathrm{i} \nabla_{\alpha(\beta} \lambda_{\gamma)}^i - \mathrm{i} \varepsilon_{\alpha(\beta} \nabla_{\gamma)}{}^\delta \lambda_\delta^i - \frac{3\mathrm{i}}{2} W_{\beta\gamma} \lambda_\alpha^i - W_{\alpha\beta\gamma}{}^i W + \frac{\mathrm{i}}{2} W_{\alpha(\beta} \lambda_{\gamma)}^i - \frac{3\mathrm{i}}{2} \varepsilon_{\alpha(\beta} W_{\gamma)}{}^\delta \lambda_\delta^i, \tag{2.13b}$$

$$\nabla_\alpha^i X^{jk} = 2\mathrm{i} \varepsilon^{i(j} \left(\nabla_\alpha{}^\beta \lambda_\beta^{k)} - \frac{1}{2} W_{\alpha\beta} \lambda^{\beta k)} + \frac{3\mathrm{i}}{4} X_\alpha^{k)} W \right). \tag{2.13c}$$

We also note that $F_{\alpha\beta} = \frac{1}{2} (\Sigma^{ab})_{\alpha\beta} F_{ab}$. Because of (2.11b), a dimension-2 superfield of a vector multiplet in the traceless frame obeys the following Bianchi identity:

$$\nabla_{(\alpha}{}^\gamma F_{\beta)\gamma} = \frac{1}{2} \lambda^{\gamma k} W_{\alpha\beta\gamma k}. \tag{2.14}$$

The actions of the S-supersymmetry generator on the descendants are given by

$$S_\alpha^i \lambda_\beta^j = -2\mathrm{i} \varepsilon_{\alpha\beta} \varepsilon^{ij} W, \qquad S_\alpha^i F_{\beta\gamma} = 4 \varepsilon_{\alpha(\beta} \lambda_{\gamma)}^i,$$
$$S_\alpha^i X^{jk} = -2 \varepsilon^{i(j} \lambda_\alpha^{k)}, \tag{2.15}$$

while all the fields are annihilated by the K_a generators.

In 5D $\mathcal{N} = 1$ conformal superspace, there exists a prepotential formulation for the Abelian vector multiplet, which was developed in [20], see also [64,66,80,81] for earlier related analyses in other superspaces. The authors of [20] introduced a real primary superfield V_{ij} of dimension -2, $\mathbb{D} V_{ij} = -2 V_{ij}$. It was also shown that V_{ij} transforms as an isovector under $\mathrm{SU}(2)_R$ transformations and is the 5D analog of Mezincescu's prepotential [85–87] for the 4D $\mathcal{N} = 2$ Abelian vector multiplet. This then allows us to represent the field strength W as

$$W = -\frac{3\mathrm{i}}{40} \nabla_{ij} \Delta^{ijkl} V_{kl}, \tag{2.16}$$

where we have defined the operators

$$\Delta^{ijkl} \coloneqq -\frac{1}{96} \varepsilon^{\alpha\beta\gamma\delta} \nabla_\alpha^{(i} \nabla_\beta^j \nabla_\gamma^k \nabla_\delta^{l)} = -\frac{1}{32} \nabla^{(ij} \nabla^{kl)} = \Delta^{(ijkl)}, \tag{2.17a}$$

$$\nabla^{ij} \coloneqq \nabla^{\alpha(i} \nabla_\alpha^{j)}. \tag{2.17b}$$

It should be noted that V_{ij} in (2.16) is defined modulo gauge transformations of the form

$$\delta V_{kl} = \nabla_\alpha^p \Lambda^\alpha{}_{klp}, \qquad \Lambda^\alpha{}_{klp} = \Lambda^\alpha{}_{(klp)}, \tag{2.18}$$

with the gauge parameter $\Lambda^\alpha{}_{klp}$ being a primary superfield,

$$S_\alpha^i \Lambda^\beta{}_{jkl} = 0, \qquad \mathbb{D} \Lambda^\beta{}_{jkl} = -\frac{5}{2} \Lambda^\beta{}_{jkl}. \tag{2.19}$$

C. The linear multiplet

The linear multiplet [88–98], or $\mathcal{O}(2)$ multiplet, can be described in terms of the primary superfield $G^{ij} = G^{ji}$, which is characterized by the properties

$$\nabla_\alpha^{(i} G^{jk)} = 0, \tag{2.20a}$$

$$K_A G^{ij} = 0, \qquad \mathbb{D} G^{ij} = 3 G^{ij}. \tag{2.20b}$$

We assume G^{ij} to be real, $(G^{ij})^* = \varepsilon_{ik}\varepsilon_{jl} G^{kl}$.

The component structure of G^{ij} is characterized by the following tower of identities:

$$\nabla_\alpha^i G^{jk} = 2\varepsilon^{i(j}\varphi_\alpha^{k)}, \tag{2.21a}$$

$$\nabla_\alpha^i \varphi_\beta^j = -\frac{\mathrm{i}}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta}F + \frac{\mathrm{i}}{2}\varepsilon^{ij}\mathcal{H}_{\alpha\beta} + \mathrm{i}\nabla_{\alpha\beta}G^{ij}, \tag{2.21b}$$

$$\nabla_\alpha^i F = -2\nabla_\alpha{}^\beta \varphi_\beta^i - 3W_{\alpha\beta}\varphi^{\beta i} - \frac{3}{2}X_{\alpha j}G^{ij}, \tag{2.21c}$$

$$\nabla_\alpha^i \mathcal{H}_a = 4(\Sigma_{ab})_\alpha{}^\beta \nabla^b \varphi_\beta^i - \frac{3}{2}(\Gamma_a)_\alpha{}^\beta W_{\beta\gamma}\varphi^{\gamma i} - \frac{1}{2}(\Gamma_a)_\gamma{}^\beta W_{\beta\alpha}\varphi^{\gamma i}, \tag{2.21d}$$

where we have defined the independent descendant superfields

$$\varphi_\alpha^i := \frac{1}{3}\nabla_{\alpha j} G^{ij}, \tag{2.22a}$$

$$F := \frac{\mathrm{i}}{12}\nabla^{\gamma i}\nabla_\gamma^j G_{ij} = -\frac{\mathrm{i}}{4}\nabla^{\gamma k}\varphi_{\gamma k}, \tag{2.22b}$$

$$\mathcal{H}_{abcd} := \frac{\mathrm{i}}{12}\varepsilon_{abcde}(\Gamma^e)^{\alpha\beta}\nabla_\alpha^i \nabla_\beta^j G_{ij} \equiv \varepsilon_{abcde}\mathcal{H}^e. \tag{2.22c}$$

Here $\mathcal{H}^a$ obeys the differential condition

$$\nabla_a \mathcal{H}^a = 0, \qquad \mathcal{H}^a := -\frac{1}{4!}\varepsilon^{abcde}\mathcal{H}_{bcde}. \tag{2.23}$$

The descendants (2.22) are all annihilated by K_a. Under the action of S supersymmetry, they transform as follows:

$$S_\alpha^i \varphi_\beta^j = -6\varepsilon_{\alpha\beta}G^{ij}, \quad S_\alpha^i F = 6\mathrm{i}\varphi_\alpha^i, \quad S_\alpha^i \mathcal{H}_b = -8\mathrm{i}(\Gamma_b)_\alpha{}^\beta \varphi_\beta^i. \tag{2.24}$$

We refer the reader to [20] for a superform description of the linear multiplet.

As described in [20], the linear multiplet constraints (2.20) may be solved in terms of an arbitrary primary real dimensionless scalar prepotential Ω,

$$S_\alpha^i \Omega = 0, \qquad \mathbb{D}\Omega = 0, \tag{2.25}$$

and the solution is

$$G^{ij} = -\frac{3\mathrm{i}}{40}\Delta^{ijkl}\nabla_{kl}\Omega. \tag{2.26}$$

A crucial property of G^{ij} defined by (2.26) is that it is invariant under gauge transformations of Ω of the form

$$\delta\Omega = -\frac{\mathrm{i}}{2}(\Gamma^a)^{\alpha\beta}\nabla_\alpha^i \nabla_\beta^j B_{aij}, \tag{2.27}$$

where the gauge parameter is assumed to have the properties

$$B_a{}^{ij} = B_a{}^{ji}, \qquad S_\alpha^i B_a{}^{jk} = 0, \qquad \mathbb{D}B_a{}^{ij} = -B_a{}^{ij}, \tag{2.28}$$

and is otherwise arbitrary.

To conclude this section, we introduce another result that will be used in the rest of the paper. Given a system of n Abelian vector multiplets W^I, with $I = 1, 2, \ldots n$, all satisfying (2.11), we can construct the following composite linear multiplet and its descendants [20]:

$$H^{ij} = C_{JK}\{2W^J X^{ijK} - \mathrm{i}\lambda^{\alpha J(i}\lambda_\alpha^{j)K}\}, \tag{2.29a}$$

$$\varphi_\alpha^i = C_{JK}\Big\{\mathrm{i}X^{ijJ}\lambda_{\alpha j}^K - 2\mathrm{i}F_{\alpha\beta}^J\lambda^{\beta iK} - \frac{3}{2}X_\alpha^i W^J W^K - 2\mathrm{i}W^J\nabla_{\alpha\beta}\lambda^{\beta iK} - \mathrm{i}(\nabla_{\alpha\beta}W^J)\lambda^{\beta iK} - 3\mathrm{i}W_{\alpha\beta}W^J\lambda^{\beta iK}\Big\}, \tag{2.29b}$$

$$F = C_{JK}\Big\{X^{ijJ}X_{ij}^K - F^{abJ}F_{ab}^K + 4W^J\Box W^K + 2(\nabla^a W^J)\nabla_a W^K + 2\mathrm{i}(\nabla_\alpha{}^\beta\lambda_\beta^{iJ})\lambda_i^{\alpha K} - 6W^{ab}F_{ab}^J W^K - \frac{39}{8}W^{ab}W_{ab}W^J W^K + \frac{3}{8}YW^J W^K + 6X^{\alpha i}\lambda_{\alpha i}^J W^K - 3\mathrm{i}W_{\alpha\beta}\lambda^{\alpha iJ}\lambda_i^{\beta K}\Big\}, \tag{2.29c}$$

$$\mathcal{H}_a = C_{JK}\Big\{-\frac{1}{2}\varepsilon_{abcde}F^{bcJ}F^{deK} + 4\nabla^b\Big(W^J F_{ba}^K + \frac{3}{2}W_{ba}W^J W^K\Big) + 2\mathrm{i}(\Sigma_{ba})^{\alpha\beta}\nabla^b(\lambda_\alpha^{iJ}\lambda_{\beta i}^K)\Big\}, \tag{2.29d}$$

where $\Box := \nabla^a\nabla_a$ and $C_{JK} = C_{(JK)}$ is a constant symmetric in J and K. Equation (2.29) is the superspace analog of the composite linear multiplet constructed in [61].

III. SUPERCONFORMAL ACTIONS

In this section, we review a main action principle that was used in [20] to construct various locally superconformally invariants (including curvature-squared ones) in superspace. A simple way to define it is based on a full superspace integral

$$S[\mathcal{L}] = \int \mathrm{d}^{5|8}z E\mathcal{L}, \qquad \mathrm{d}^{5|8}z := \mathrm{d}^5x\mathrm{d}^8\theta, \qquad E := \mathrm{Ber}(E_M{}^A), \tag{3.1}$$

where the Lagrangian $\mathcal{L}$ is a conformal primary superfield of dimension $+1$, $\mathbb{D}\mathcal{L} = \mathcal{L}$. This invariant can be proven to be locally superconformal invariant, that is, invariant under the supergravity gauge transformations (2.9).

A. BF action

The action involving the product of a linear multiplet with an Abelian vector multiplet is referred to as the BF action. Analogous to the component superconformal tensor calculus, this plays a fundamental role in the construction of general supergravity-matter couplings, see [57–63] for the 5D case, and it was a main building block for the invariants introduced in [20] that we focus on. In superspace, the BF action may be described by

$$S_{\mathrm{BF}} = \int \mathrm{d}^{5|8}z\, E\,\Omega W = \int \mathrm{d}^{5|8}z\, E\, G^{ij}V_{ij}. \tag{3.2a}$$

As implied by the equation above, the BF action can be written in different ways, see [20] for even more variants. In the first form in (3.2a), it involves the field strength of the vector multiplet W and the prepotential of the linear multiplet Ω. By using (2.26) and (2.16), and then integrating by parts, one may obtain the equivalent form of the BF action involving Mezincescu's prepotential V_{ij} and the field strength G^{ij} described by the right-hand side of (3.2a). One may also prove that the functionals $\int \mathrm{d}^{5|8}z\, E\,\Omega W$ and $\int \mathrm{d}^{5|8}z\, E\, G^{ij}V_{ij}$ are, respectively, invariant under the gauge transformations (2.27) and (2.18), thanks to the defining differential constraints satisfied by W and G^{ij}, Eqs. (2.11b) and (2.20a).

In components, and in our notation, the BF action takes the form [20]

$$\begin{aligned} S_{\mathrm{BF}} = -\int \mathrm{d}^5x e(& v_a\mathcal{H}^a + WF + X_{ij}G^{ij} + 2\lambda^{\alpha k}\varphi_{\alpha k} \\ & - \psi_a{}^\alpha_i(\Gamma^a)_\alpha{}^\beta\varphi^i_\beta W - \mathrm{i}\psi_a{}^\alpha_i(\Gamma^a)_\alpha{}^\beta\lambda_{\beta j}G^{ij} \\ & + \mathrm{i}\psi_a{}^\alpha_i(\Sigma^{ab})_\alpha{}^\beta\psi_{b\beta j}WG^{ij}). \end{aligned} \tag{3.2b}$$

In (3.2b), we have defined the usual component projection to $\theta = 0$, i.e., $U(z)| := U(z)|_{\theta=0}$. We associate the same symbol for the covariant component fields and the corresponding superfields, when the interpretation is clear from the context. Here $v_m := V_m|$ denotes a real Abelian gauge connection. Its real field strength is $f_{mn} := F_{mn}| = 2\partial_{[m}v_{n]}$. Note that the field strength f_{mn} may be expressed in terms of the bar-projected, covariant field strength $F_{ab} := F_{ab}|$ via the relation

$$\begin{aligned} F_{ab} &= f_{ab} + \mathrm{i}(\Gamma_{[a})_\alpha{}^\beta\psi_{b]}{}^\alpha_k\lambda^k_\beta + \frac{\mathrm{i}}{2}W\psi_{[a}{}^\gamma_k\psi_{b]}{}^k_\gamma, \\ f_{ab} &:= e_a{}^m e_b{}^n f_{mn}. \end{aligned} \tag{3.3}$$

When projected to components, the lowest component of the covariant superfield $\mathcal{H}_a$ satisfies the constraint $\nabla^a\mathcal{H}_a = 0$, where $\mathcal{H}_a := \mathcal{H}_a|$. It holds that

$$\mathcal{H}^a = h^a + 2(\Sigma^{ab})_\alpha{}^\beta\psi_b{}^\alpha_i\varphi^i_\beta - \frac{\mathrm{i}}{2}\varepsilon^{abcde}(\Sigma_{bc})_{\alpha\beta}\psi_d{}^\alpha_i\psi_e{}^\beta_j G^{ij}. \tag{3.4}$$

The constraint $\nabla^a\mathcal{H}_a = 0$ implies the existence of a gauge 3-form potential b_{mnp} and its exterior derivative $h_{mnpq} := 4\partial_{[m}b_{npq]}$. See [20,75] for more details.

B. Vector multiplet compensator

The two-derivative invariant for the vector multiplet compensator can be constructed using the above BF action principle (3.2a), but with the linear multiplet being a composite superfield. We denote by H^{ij}_{VM} the composite linear multiplet (2.29), which is built out of a single Abelian vector multiplet,

$$\begin{aligned} H^{ij}_{\mathrm{VM}} &= \mathrm{i}(\nabla^{\alpha(i}W)\nabla^{j)}_\alpha W + \frac{\mathrm{i}}{2}W\nabla^{\alpha(i}\nabla^{j)}_\alpha W \\ &= -\mathrm{i}\lambda^{\alpha i}\lambda^j_\alpha + 2WX^{ij}. \end{aligned} \tag{3.5}$$

One can check that H^{ij}_{VM} is a dimension-3 primary superfield, $S^k_\alpha H^{ij}_{\mathrm{VM}} = 0$. Thanks to the Bianchi identity (2.11b) obeyed by the field strength W, the composite superfield H^{ij}_{VM} satisfies the analyticity constraint

$$\nabla^{(i}_\alpha H^{jk)}_{\mathrm{VM}} = 0. \tag{3.6}$$

The vector multiplet action may then be rewritten as an integral over the full superspace,

$$S_{\mathrm{VM}} = \frac{1}{4}\int \mathrm{d}^{5|8}z\, E\, V_{ij}\, H^{ij}_{\mathrm{VM}}. \tag{3.7}$$

It is also possible to write the action as

$$S_{\mathrm{VM}} = \frac{1}{4}\int \mathrm{d}^{5|8}z\, E\,\mathbf{\Omega}_{\mathrm{VM}}W, \tag{3.8}$$

where we have introduced the primary superfield $\mathbf{\Omega}_{\rm VM}$ defined by [20]

$$\mathbf{\Omega}_{\rm VM} = \frac{\rm i}{4}(W\nabla^{ij}V_{ij} - 2(\nabla^{\alpha i}V_{ij})\nabla^j_\alpha W - 2V_{ij}\nabla^{ij}W). \quad (3.9)$$

This is a prepotential for $H^{ij}_{\rm VM}$ in the sense of (2.26).

The representations (3.7) and (3.8) allow us to compute the variation of $S_{\rm VM}$ with respect to the Mezincescu's prepotential,

$$\delta S_{\rm VM} = \frac{3}{4}\int {\rm d}^{5|8}z\, E\,\delta V_{ij}\, H^{ij}_{\rm VM}. \quad (3.10)$$

Note that the above variation vanishes when δV_{ij} is a gauge transformation (2.18). This implies that

$$\int {\rm d}^{5|8}z\, E\,\Lambda^\alpha{}_{ijk}\nabla^{(k}_\alpha H^{ij)}_{\rm VM} = 0, \quad (3.11)$$

that is, $\nabla^{(i}_\alpha H^{jk)}_{\rm VM} = 0$. This result is true for any dynamical system involving an Abelian vector multiplet [20]. The variation with respect to the prepotential V_{ij} couples to a composite linear multiplet that depends on the specific form of the associated action principle—let us call this, in general, $\mathbf{H}^{ij}$, which satisfies by construction the constraints (2.20). The equation of motion (EOM) for a vector multiplet is then $\mathbf{H}^{ij} = 0$. In the case of Eq. (3.7), the EOM for the vector multiplet compensator is $H^{ij}_{\rm VM} = 0$.

The superspace action $S_{\rm VM}$ can be reduced to components. The bosonic part of the component action reads [20]

$$\begin{aligned} S_{\rm VM} = \int {\rm d}^5x\,e\Big\{ &-\frac{1}{8}W^3\mathcal{R} + \frac{3}{2}W(\mathcal{D}^aW)\mathcal{D}_aW \\ &-\frac{3}{4}WX^{ij}X_{ij} + \frac{1}{8}\varepsilon_{abcde}v^af^{bc}f^{de} + \frac{3}{4}Wf^{ab}f_{ab} \\ &+\frac{9}{4}W^2W^{ab}f_{ab} + \frac{39}{32}W^3W^{ab}W_{ab} - \frac{3}{32}W^3Y\Big\}, \end{aligned} \quad (3.12)$$

where $\mathcal{R}$ denotes the scalar curvature. In the above, we have introduced the spin, dilatation, and ${\rm SU}(2)_R$ covariant derivative $\mathcal{D}_a$,

$$\mathcal{D}_a = e_a{}^m\mathcal{D}_m = e_a{}^m\left(\partial_m - \frac{1}{2}\omega_m{}^{bc}M_{bc} - b_m\mathbb{D} - \phi_m{}^{ij}J_{ij}\right). \quad (3.13)$$

The action is two-derivative and, upon gauge fixing dilatation by imposing $W = 1$, the first term gives a scalar curvature term $\mathcal{R}$. The gauge fixing $W = 1$ can be achieved by requiring $W \neq 0$, meaning that the vector multiplet is a conformal compensator.

C. Linear multiplet compensator

The action for the linear multiplet compensator can also be constructed using the BF action principle (3.2a). In this case, the dynamical part of the action is described by a vector multiplet built out of the linear multiplet. We denote by $\mathbf{W}$ the composite vector multiplet field strength,

$$\mathbf{W} := \frac{\rm i}{16}G\nabla^{\alpha i}\nabla^j_\alpha\left(\frac{G_{ij}}{G^2}\right) = \frac{1}{4}FG^{-1} - \frac{\rm i}{8}G_{ij}\varphi^{i\alpha}\varphi^j_\alpha G^{-3}, \quad (3.14)$$

with

$$G := \sqrt{\frac{1}{2}G^{ij}G_{ij}} \quad (3.15)$$

being nowhere vanishing, $G \neq 0$. At the component level, the vector multiplet (3.14) was first derived by Zucker [99] as a 5D analog of the improved 4D $\mathcal{N} = 2$ tensor multiplet [96]. The field strength $\mathbf{W}$ obeys the constraints (2.11).

The action for the linear multiplet compensator may then be rewritten as

$$S_{\rm L} = \int {\rm d}^{5|8}z\, E\,\Omega\mathbf{W}. \quad (3.16)$$

Varying the prepotential Ω leads to

$$\delta S_{\rm L} = \int {\rm d}^{5|8}z\, E\,\delta\Omega\mathbf{W}. \quad (3.17)$$

Similar to what we discussed for the vector multiplet case, the previous form holds for the first-order variation of a matter system that includes a linear multiplet with respect to its prepotential Ω. In particular, the variation must vanish if $\delta\Omega$ is the gauge transformation (2.27). This holds if $\mathbf{W}$ obeys the Bianchi identity (2.11b). In general, any dynamical system involving a linear multiplet then possesses a composite vector multiplet $\mathbf{W}$. The EOM for the linear multiplet is $\mathbf{W} = 0$ and for the specific case of the linear multiplet action of Eq. (3.16) this is given by $\mathbf{W}$ defined in (3.14).

The bosonic part of $S_{\rm L}$ is given by [20]

$$\begin{aligned} S_{\rm L} = \int {\rm d}^5x\,e\Big\{ &-\frac{3}{8}G\mathcal{R} + \frac{3}{32}GY - \frac{1}{8G}F^2 - \frac{3}{32}W^{ab}W_{ab}G \\ &+\frac{1}{4}G^{-1}(\mathcal{D}_aG^{ij})\mathcal{D}^aG_{ij} - \frac{1}{2}G^{-1}\mathcal{H}^a\mathcal{H}_a \\ &+\frac{1}{12}\varepsilon^{abcde}b_{cde}\left(\frac{1}{2}G^{-3}(\mathcal{D}_aG_{ik})(\mathcal{D}_bG_j{}^k)G^{ij} \right. \\ &\left. + G^{-1}R(J)_{ab}{}^{ij}G_{ij}\right)\Big\}. \end{aligned} \quad (3.18)$$

The action is two-derivative, and, with $G = 1$, the first term gives an $\mathcal{R}$ term.

For later use, it is useful to provide the explicit expressions of the composite descendant superfields of $\mathbf{W}$. These are given by

$$\begin{aligned}\boldsymbol{\lambda}_\alpha^i &= -\mathrm{i}\nabla_\alpha^i\mathbf{W}\\ &= G^{-1}\left\{-\frac{\mathrm{i}}{2}\nabla_{\alpha\beta}\varphi^{\beta i} + \frac{3\mathrm{i}}{4}W_{\alpha\beta}\varphi^{\beta i} + \frac{3\mathrm{i}}{8}G^{ij}X_{\alpha j}\right\}\\ &\quad + G^{-3}\left\{-\frac{\mathrm{i}}{8}FG^{ij}\varphi_{\alpha j} - \frac{\mathrm{i}}{8}G^{ij}\mathcal{H}_{\alpha\beta}\varphi_j^\beta + \frac{\mathrm{i}}{4}G_{jk}\varphi^{\beta k}\nabla_{\alpha\beta}G^{ij} + \frac{1}{4}\varphi^{\beta i}\varphi_\beta^j\varphi_{\alpha j}\right\}\\ &\quad + G^{-5}\left\{-\frac{3}{8}G^{ij}G_{kl}\varphi^{\beta k}\varphi_\beta^l\varphi_{\alpha j}\right\}, \end{aligned} \tag{3.19a}$$

$$\begin{aligned}\mathbf{X}^{ij} &= \frac{\mathrm{i}}{4}\nabla^{\alpha(i}\nabla_\alpha^{j)}\mathbf{W}\\ &= G^{-1}\left\{\frac{1}{2}\Box G^{ij} + \frac{3}{64}W^{ab}W_{ab}G^{ij} - \frac{3}{64}YG^{ij} + \frac{3\mathrm{i}}{4}X^{\alpha(i}\varphi_\alpha^{j)}\right\}\\ &\quad + G^{-3}\Big\{-\frac{1}{16}F^2G^{ij} - \frac{1}{16}\mathcal{H}^a\mathcal{H}_aG^{ij} + \frac{1}{4}\mathcal{H}^aG^{k(i}\nabla_aG_k{}^{j)} - \frac{1}{4}G_{kl}(\nabla^aG^{k(i})\nabla_aG^{j)l}\\ &\quad - \frac{3\mathrm{i}}{8}G^{ij}G_{kl}X^{\alpha k}\varphi_\alpha^l - \frac{\mathrm{i}}{8}F\varphi^{\alpha(i}\varphi_\alpha^{j)} + \frac{3\mathrm{i}}{8}W^{\alpha\beta}G^{ij}\varphi_\alpha^k\varphi_{\beta k}\\ &\quad + \frac{\mathrm{i}}{16}(\Gamma^a)^{\alpha\beta}(\mathcal{H}_a\varphi_\alpha^{(i}\varphi_\beta^{j)} + 8G^{k(i}(\nabla_a\varphi_\alpha^{j)})\varphi_{\beta k} + 2\varphi_\alpha^{(i}(\nabla_aG^{j)k})\varphi_{\beta k})\Big\}\\ &\quad + G^{-5}\Big\{\frac{3\mathrm{i}}{16}FG^{ij}G_{kl}\varphi^{\alpha k}\varphi_\alpha^l + \frac{3\mathrm{i}}{16}G^{k(i}G^{j)l}(\Gamma^a)_{\alpha\beta}\mathcal{H}_a\varphi_k^\alpha\varphi_l^\beta - \frac{3\mathrm{i}}{8}G^{mn}G^{k(i}(\nabla_{\alpha\beta}G_m{}^{j)})\varphi_k^\alpha\varphi_n^\beta\\ &\quad + \frac{3}{8}G^{k(i}\varphi_\alpha^{j)}\varphi^{\alpha l}\varphi_k^\beta\varphi_{\beta l} - \frac{3}{8}G^{kl}\varphi^{\alpha(i}\varphi_\alpha^{j)}\varphi_k^\beta\varphi_{\beta l}\Big\}\\ &\quad + G^{-7}\left\{\frac{15}{32}G^{ij}G_{kl}G_{mn}\varphi^{\alpha k}\varphi_\alpha^l\varphi^{\beta m}\varphi_\beta^n\right\}, \end{aligned} \tag{3.19b}$$

$$\begin{aligned}\mathbf{F}_{ab} &= \frac{1}{4}(\Sigma_{ab})^{\alpha\beta}\nabla_{(\alpha}^k\boldsymbol{\lambda}_{\beta)k} - W_{ab}\mathbf{W}\\ &= G^{-1}\left\{\frac{1}{2}\nabla_{[a}\mathcal{H}_{b]} - \frac{3\mathrm{i}}{8}G_{ij}X_{ab}{}^{ij} + \frac{\mathrm{i}}{4}W_{ab\alpha}{}^i\varphi_i^\alpha\right\}\\ &\quad + G^{-3}\Big\{\frac{1}{4}G_{ij}\mathcal{H}_{[a}\nabla_{b]}G^{ij} - \frac{1}{4}G_{ij}(\nabla_{[a}G^{ik})\nabla_{b]}G_k{}^j + \frac{\mathrm{i}}{2}G_{ij}(\Gamma_{[a})^{\alpha\beta}(\nabla_{b]}\varphi_\alpha^i)\varphi_\beta^j\\ &\quad - \frac{\mathrm{i}}{8}(\Gamma_{[a})^{\alpha\beta}(\nabla_{b]}G_{ij})\varphi_\alpha^i\varphi_\beta^j\Big\} + G^{-5}\left\{-\frac{3\mathrm{i}}{8}G^k{}_{(i}G^l{}_{j)}(\Gamma_{[a})^{\alpha\beta}(\nabla_{b]}G_{kl})\varphi_\alpha^i\varphi_\beta^j\right\}. \end{aligned} \tag{3.19c}$$

D. Gauged supergravity action

An off-shell formulation for 5D minimal supergravity can be obtained by coupling the standard Weyl multiplet to two off-shell compensators: vector and linear multiplets [20,54–63]. This is the 5D analog of the off-shell formulation for 4D $\mathcal{N} = 2$ supergravity [96,100]. The complete (gauged) supergravity action S_{gSG} is given by the following two-derivative action:

$$S_{\mathrm{gSG}} = S_{\mathrm{VM}} + S_{\mathrm{L}} + \kappa S_{\mathrm{BF}} = \int \mathrm{d}^{5|8}z\,E\left\{\frac{1}{4}V_{ij}H_{\mathrm{VM}}^{ij} + \boldsymbol{\Omega}\mathbf{W} + \kappa V_{ij}G^{ij}\right\} \tag{3.20a}$$

$$= \int \mathrm{d}^{5|8}z\,E\left\{\frac{1}{4}V_{ij}H_{\mathrm{VM}}^{ij} + \boldsymbol{\Omega}\mathbf{W} + \kappa\boldsymbol{\Omega}W\right\}. \tag{3.20b}$$

The BF action $\kappa S_{\rm BF}$ describes a supersymmetric cosmological term. The case $\kappa = 0$ case corresponds to Poincaré supergravity, while $\kappa \neq 0$ leads to gauged or anti–de Sitter supergravity.

Upon gauge fixing dilatation and superconformal symmetries (dilatation, S, and K) and integrating out the various auxiliary fields, one obtains the on-shell Poincaré supergravity action of [47,48]. The contributions from the scalar curvature terms in Eqs. (3.12) and (3.18) combine to give the normalized Einstein-Hilbert term $-\frac{1}{2}\mathcal{R}$ plus a cosmological constant, see, e.g., [20] for details.

In the remaining subsections, we elaborate on the structure of three independent curvature-squared invariants [20,25,27,30,32]. These invariants were constructed in superspace [20] in the standard Weyl multiplet background. In particular, we present the full expressions of all the composite primary multiplets that generate these invariants with the log multiplet appearing for the first time in its expanded form in terms of the descendants of W and $W_{\alpha\beta}$.

E. Weyl squared

We first consider a composite primary superfield that may be used to generate a supersymmetric completion of a Weyl-squared term. In superspace, it was described in [20] in terms of the super-Weyl tensor,

$$H^{ij}_{\rm Weyl} := -\frac{\rm i}{2}W^{\alpha\beta\gamma i}W_{\alpha\beta\gamma}{}^{j} + \frac{3{\rm i}}{2}W^{\alpha\beta}X_{\alpha\beta}{}^{ij} - \frac{3{\rm i}}{4}X^{\alpha i}X^{j}_{\alpha}. \quad (3.21)$$

It can be checked that $H^{ij}_{\rm Weyl}$ satisfies the constraints (2.20). The superfield $H^{ij}_{\rm Weyl}$ corresponds to the composite linear multiplet first constructed in components by Hanaki *et al.* in [25].

With the aid of the relations (2.6), the component fields of the composite linear multiplet are straightforward to compute. They include the $\theta = 0$ projection (or the "bar projection") of $H^{ij}_{\rm Weyl}$, together with the bar projection of the following descendant superfields of $H^{ij}_{\rm Weyl}$:

$$\varphi^{\alpha i}_{\rm Weyl} = \frac{1}{3}\nabla^{\alpha}_{j}H^{ij}_{\rm Weyl}, \quad (3.22a)$$

$$F_{\rm Weyl} = \frac{\rm i}{12}\nabla^{\alpha}_{i}\nabla_{\alpha j}H^{ij}_{\rm Weyl}, \quad (3.22b)$$

$$\mathcal{H}^{a}_{\rm Weyl} = \frac{\rm i}{12}(\Gamma^{a})^{\alpha\beta}\nabla_{\alpha i}\nabla_{\beta j}H^{ij}_{\rm Weyl}. \quad (3.22c)$$

Equation (3.22) play an important role in analyzing superconformal primary equations of motion in the next section. The resulting expression coincides, up to notations, to the results of [25]. We will give the full component expressions (3.22) in a follow-up paper.

By inserting the components of the composite linear multiplet (3.21) and (3.22) into the BF action (3.2), one may construct the following higher-derivative invariant in a standard Weyl multiplet background [20]:

$$S_{\rm Weyl} = \int {\rm d}^{5|8}z\, E\, V_{ij}H^{ij}_{\rm Weyl} \quad (3.23a)$$

$$= -\int {\rm d}^5x e(v_a\mathcal{H}^a_{\rm Weyl} + WF_{\rm Weyl} + X_{ij}H^{ij}_{\rm Weyl} + 2\lambda^k\varphi_{k{\rm Weyl}} - \psi_{ai}\Gamma^a\varphi^i_{\rm Weyl}W - {\rm i}\psi_{ai}\Gamma^a\lambda_j H^{ij}_{\rm Weyl} + {\rm i}\psi_{ai}\Sigma^{ab}\psi_{bj}WH^{ij}_{\rm Weyl}), \quad (3.23b)$$

where the spinor indices here are suppressed. This defines a locally supersymmetric extension of the Weyl-squared term [25,27,30,32].

F. log W

We now consider a composite linear superfield that includes a supersymmetric Ricci tensor-squared term. In superspace, it was described for the first time in [20] in analogy with the construction of a higher-derivative chiral invariant in 4D $\mathcal{N} = 2$ supergravity [33]. The composite superfield makes use of the standard Weyl multiplet coupled to the off-shell vector multiplet compensator. It takes the form[5]

$$H^{ij}_{\log W} = -\frac{3{\rm i}}{40}\Delta^{ijkl}\nabla_{kl}\log W = \frac{3{\rm i}}{1280}\nabla^{(ij}\nabla^{kl)}\nabla_{kl}\log W. \quad (3.24)$$

In general, such a linear multiplet could be defined by replacing W with any primary scalar superfield of weight q for which it is possible to prove that (3.24) satisfies all the linear multiplet constraints, Eq. (2.20), see [20]. However, for various applications, we choose to construct it in terms of the vector multiplet superfield strength W. Because of the complexity in computing the action of six spinor derivatives on the log multiplet, the component analysis of $H^{ij}_{\log W}$ has not appeared so far. This calculation can be performed with the aid of the Cadabra software. Here we find that the full expression of $H^{ij}_{\log W}$ in terms of the descendant superfields of the vector and standard Weyl multiplets is given by

[5]Note that there is an overall minus sign difference between the definition of the log W invariant in this paper and the one of [20].

$$
\begin{aligned}
H^{ij}_{\log W} = & -\frac{3\mathrm{i}}{8}W_{ab}X^{abij} + \frac{51\mathrm{i}}{64}X^{\alpha i}X^{j}_{\alpha} + W^{-1}\Big\{\frac{9}{64}X^{ij}Y - \frac{3\mathrm{i}}{8}F_{ab}X^{abij} - \frac{1}{2}\Box X^{ij} - \frac{9}{64}X^{ij}W^{ab}W_{ab} \\
& + \frac{1}{4}W^{ab}W_{ab}{}^{\alpha(i}\lambda^{j)}_{\alpha} - \frac{3}{4}(\Gamma^{a})^{\alpha\beta}X^{(i}_{\alpha}\nabla_{a}\lambda^{j)}_{\beta} + \frac{3}{4}(\Gamma^{a})^{\alpha\beta}\lambda^{(i}_{\alpha}\nabla_{a}X^{j)}_{\beta}\Big\} \\
& + W^{-2}\Big\{\frac{1}{2}X^{ij}\Box W + \frac{1}{2}(\nabla^{a}W)\nabla_{a}X^{ij} + \frac{1}{4}F^{ab}W_{ab}{}^{\alpha(i}\lambda^{j)}_{\alpha} - \frac{\mathrm{i}}{2}\lambda^{\alpha(j}\Box\lambda^{i)}_{\alpha} \\
& - \frac{\mathrm{i}}{4}(\nabla^{a}\lambda^{\alpha i})\nabla_{a}\lambda^{j}_{\alpha} - \frac{3\mathrm{i}}{16}\epsilon^{abcde}(\Sigma_{ab})^{\alpha\beta}W_{de}\lambda^{(i}_{\alpha}\nabla_{c}\lambda^{j)}_{\beta} - \frac{3\mathrm{i}}{8}(\Gamma^{a})^{\alpha\beta}\lambda^{i}_{\alpha}\lambda^{j}_{\beta}\nabla^{c}W_{ac} \\
& + \frac{3\mathrm{i}}{64}Y\lambda^{\alpha i}\lambda^{j}_{\alpha} + \frac{3}{8}(\Sigma_{ab})^{\alpha\beta}F^{ab}X^{(i}_{\alpha}\lambda^{j)}_{\beta} + \frac{3\mathrm{i}}{128}W^{ab}W_{ab}\lambda^{\alpha i}\lambda^{j}_{\alpha} + \frac{9\mathrm{i}}{256}\epsilon^{abcde}(\Gamma_{a})^{\alpha\beta}W_{bc}W_{de}\lambda^{i}_{\alpha}\lambda^{j}_{\beta} - \frac{3}{8}X^{ij}X^{\alpha k}\lambda_{\alpha k}\Big\} \\
& + W^{-3}\Big\{\frac{1}{8}X^{ij}F^{ab}F_{ab} - \frac{1}{8}X^{ij}X^{kl}X_{kl} - \frac{1}{4}X^{ij}(\nabla^{a}W)\nabla_{a}W - \frac{\mathrm{i}}{8}\epsilon^{abcde}(\Sigma_{ab})^{\alpha\beta}F_{de}\lambda^{(i}_{\alpha}\nabla_{c}\lambda^{j)}_{\beta} - \frac{\mathrm{i}}{4}(\Gamma^{a})^{\alpha\beta}F_{ab}\lambda^{(i}_{\alpha}\nabla^{b}\lambda^{j)}_{\beta} \\
& - \frac{\mathrm{i}}{4}(\Gamma^{a})^{\alpha\beta}\lambda^{i}_{\alpha}\lambda^{j}_{\beta}\nabla^{c}F_{ac} + \frac{\mathrm{i}}{4}(\Gamma^{a})^{\alpha\beta}X^{ij}\lambda^{k}_{\alpha}\nabla_{a}\lambda_{\beta k} + \frac{\mathrm{i}}{4}(\Gamma^{a})^{\alpha\beta}X^{k(i}\lambda^{j)}_{\alpha}\nabla_{a}\lambda_{\beta k} \\
& - \frac{\mathrm{i}}{4}(\Gamma^{a})^{\alpha\beta}\lambda^{(i}_{\alpha}(\nabla_{a}X^{j)l})\lambda_{\beta l} + \frac{\mathrm{i}}{4}\lambda^{\alpha i}\lambda^{j}_{\alpha}\Box W + \frac{3\mathrm{i}}{4}(\nabla^{a}W)\lambda^{\alpha(i}\nabla_{a}\lambda^{j)}_{\alpha} \\
& - \frac{\mathrm{i}}{2}(\Sigma_{ab})^{\alpha\beta}(\nabla^{a}W)\lambda^{(i}_{\alpha}\nabla^{b}\lambda^{j)}_{\beta} - \frac{3\mathrm{i}}{16}(\Gamma^{a})^{\alpha\beta}W_{ab}\lambda^{i}_{\alpha}\lambda^{j}_{\beta}\nabla^{b}W + \frac{3\mathrm{i}}{32}W_{ab}F^{ab}\lambda^{\alpha i}\lambda^{j}_{\alpha} \\
& + \frac{9\mathrm{i}}{64}\epsilon^{abcde}(\Gamma_{a})^{\alpha\beta}W_{bc}F_{de}\lambda^{i}_{\alpha}\lambda^{j}_{\beta} - \frac{3\mathrm{i}}{32}(\Sigma_{ab})^{\alpha\beta}X^{ij}W^{ab}\lambda^{k}_{\alpha}\lambda_{\beta k} - \frac{3\mathrm{i}}{8}X^{\alpha k}\lambda^{(i}_{\alpha}\lambda^{\beta j)}\lambda_{\beta k} - \frac{3\mathrm{i}}{8}X^{\alpha k}\lambda^{\beta i}\lambda^{j}_{\beta}\lambda_{\alpha k}\Big\} \\
& + W^{-4}\Big\{-\frac{3\mathrm{i}}{16}\lambda^{\alpha i}\lambda^{j}_{\alpha}(\nabla^{a}W)\nabla_{a}W - \frac{3\mathrm{i}}{8}(\Gamma^{a})^{\alpha\beta}X^{k(i}\lambda^{j)}_{\alpha}\lambda_{\beta k}\nabla_{a}W \\
& + \frac{3\mathrm{i}}{8}(\Gamma^{a})^{\alpha\beta}F_{ab}\lambda^{i}_{\alpha}\lambda^{j}_{\beta}\nabla^{b}W + \frac{3\mathrm{i}}{32}F^{ab}F_{ab}\lambda^{\alpha i}\lambda^{j}_{\alpha} + \frac{3\mathrm{i}}{64}\epsilon^{abcde}(\Gamma_{a})^{\alpha\beta}F_{bc}F_{de}\lambda^{i}_{\alpha}\lambda^{j}_{\beta} - \frac{3\mathrm{i}}{16}(\Sigma_{ab})^{\alpha\beta}X^{ij}F^{ab}\lambda^{k}_{\alpha}\lambda_{\beta k} \\
& - \frac{3\mathrm{i}}{16}X^{ij}X^{kl}\lambda^{\alpha}_{k}\lambda_{\alpha l} - \frac{3\mathrm{i}}{32}X^{kl}X_{kl}\lambda^{\alpha i}\lambda^{j}_{\alpha} - \frac{15}{64}(\Gamma^{a})^{\alpha\beta}\lambda^{i}_{\alpha}\lambda^{j}_{\beta}\lambda^{\gamma k}\nabla_{a}\lambda_{\gamma k} - \frac{9}{32}(\Gamma^{a})^{\alpha\beta}\lambda^{(i}_{\alpha}\lambda^{\rho j)}\lambda^{k}_{\beta}\nabla_{a}\lambda_{\rho k} \\
& - \frac{15}{32}(\Gamma^{a})^{\alpha\beta}\lambda^{(i}_{\alpha}\lambda^{\rho j)}\lambda^{k}_{\rho}\nabla_{a}\lambda_{\beta k} - \frac{15}{64}(\Gamma^{a})^{\alpha\beta}\lambda^{\rho i}\lambda^{j}_{\rho}\lambda^{k}_{\alpha}\nabla_{a}\lambda_{\beta k} \\
& + \frac{9}{32}(\Sigma_{ab})^{\alpha\beta}W^{ab}\lambda^{(i}_{\alpha}\lambda^{\rho j)}\lambda^{k}_{\beta}\lambda_{\rho k} + \frac{9}{64}(\Sigma_{ab})^{\alpha\beta}W^{ab}\lambda^{\rho i}\lambda^{j}_{\rho}\lambda^{k}_{\alpha}\lambda_{\beta k}\Big\} \\
& + W^{-5}\Big\{\frac{3}{8}(\Gamma^{a})^{\alpha\beta}\lambda^{(i}_{\alpha}\lambda^{\rho j)}\lambda^{k}_{\beta}\lambda_{\rho k}\nabla_{a}W + \frac{3}{8}(\Sigma_{ab})^{\alpha\beta}F^{ab}\lambda^{\rho i}\lambda^{j}_{\rho}\lambda^{k}_{\alpha}\lambda_{\beta k} + \frac{3}{8}X^{kl}\lambda^{\alpha i}\lambda^{j}_{\alpha}\lambda^{\beta}_{k}\lambda_{\beta l}\Big\} \\
& + W^{-6}\Big\{\frac{3\mathrm{i}}{32}\lambda^{\alpha i}\lambda^{j}_{\alpha}\lambda^{\beta k}\lambda^{l}_{\beta}\lambda^{\gamma}_{k}\lambda_{\gamma l} + \frac{3\mathrm{i}}{16}\lambda^{\alpha i}\lambda^{k}_{\alpha}\lambda^{\beta j}\lambda^{l}_{\beta}\lambda^{\gamma}_{k}\lambda_{\gamma l}\Big\}.
\end{aligned}
\tag{3.25}
$$

Using the explicit expression (3.25), together with the relations (2.6), (2.8c), (2.13), and (2.15), we have shown that $H^{ij}_{\log W}$ is indeed a primary and linear superfield satisfying (2.20). Furthermore, we have computed the descendants of the primary superfield $H^{ij}_{\log W}$ defined as

$$\varphi^{\alpha i}_{\log W} = \frac{1}{3}\nabla^{\alpha}_{j}H^{ij}_{\log W}, \tag{3.26a}$$

$$F_{\log W} = \frac{\mathrm{i}}{12}\nabla^{\alpha}_{i}\nabla_{\alpha j}H^{ij}_{\log \mathrm{W}}, \tag{3.26b}$$

$$\mathcal{H}^{a}_{\log W} = \frac{\mathrm{i}}{12}(\Gamma^{a})^{\alpha\beta}\nabla_{\alpha i}\nabla_{\beta j}H^{ij}_{\log \mathrm{W}}. \tag{3.26c}$$

By using (3.25) and the BF action, Eqs. (3.2a) and (3.2b), one may construct the following locally superconformal invariant in a standard Weyl multiplet background:

$$S_{\log W} = \int \mathrm{d}^{5|8}z\, E\, V_{ij}H^{ij}_{\log W} \tag{3.27a}$$

$$= -\int \mathrm{d}^5 x e (v_a \mathcal{H}^a_{\log W} + W F_{\log W} + X_{ij} H^{ij}_{\log W} + 2\lambda^k \varphi_{k \log W} - \psi_{ai}\Gamma^a \varphi^i_{\log W} W - \mathrm{i}\psi_{ai}\Gamma^a \lambda_j H^{ij}_{\log W} + \mathrm{i}\psi_{ai}\Sigma^{ab}\psi_{bj} W H^{ij}_{\log W}). \tag{3.27b}$$

The resulting component action includes, for example, a $(\Box\Box W)$ term which, upon gauge-fixing $W = 1$, includes a Ricci tensor-squared combination. A more detailed discussion of the (fairly involved) component structure of (3.26) will be given elsewhere.

G. Scalar curvature squared

Given a composite vector multiplet (3.14) and its corresponding descendants (3.19), we can then construct a composite linear multiplet defined by [20]

$$H^{ij}_{R^2} \coloneqq H^{ij}_{\mathrm{VM}}[\mathbf{W}] = \mathrm{i}(\nabla^{\alpha(i}\mathbf{W})\nabla^{j)}_{\alpha}\mathbf{W} + \frac{\mathrm{i}}{2}\mathbf{W}\nabla^{\alpha(i}\nabla^{j)}_{\alpha}\mathbf{W} = -\mathrm{i}\lambda^{\alpha i}\lambda^j_\alpha + 2\mathbf{W}\mathbf{X}^{ij}. \tag{3.28}$$

Inserting the composite field $H^{ij}_{R^2}$ and its independent descendants ($\varphi^{\alpha i}_{R^2}$, F_{R^2}, and $\mathcal{H}^a_{R^2}$) into the BF action principle (3.2) leads to the following supersymmetric invariant:

$$S_{R^2} = \int \mathrm{d}^{5|8} z\, E\, V_{ij} H^{ij}_{R^2} \tag{3.29a}$$

$$= -\int \mathrm{d}^5 x e \Big(v_a \mathcal{H}^a_{R^2} + W F_{R^2} + X_{ij} H^{ij}_{R^2} + 2\lambda^k \varphi_{kR^2} - \psi_{ai}\Gamma^a \varphi^i_{R^2} W - \mathrm{i}\psi_{ai}\Gamma^a \lambda_j H^{ij}_{R^2} + \mathrm{i}\psi_{ai}\Sigma^{ab}\psi_{bj} W H^{ij}_{R^2}\Big). \tag{3.29b}$$

At the component level, the above action generates the scalar curvature-squared invariant constructed in [30,32].

IV. SUPERCONFORMAL EQUATIONS OF MOTION

Let us now combine the gauged supergravity action S_{gSG} with the three independent curvature-squared invariants described by Eqs. (3.23), (3.27), and (3.29) to form a higher-derivative action

$$S_{\mathrm{HD}} = S_{\mathrm{gSG}} + \alpha S_{\mathrm{Weyl}} + \beta S_{\log W} + \gamma S_{R^2}. \tag{4.1}$$

The goal of this section is to obtain superconformal primary equations of motion in superspace that describe minimal 5D gauged supergravity deformed by an arbitrary combination of the three curvature-squared invariants described by the action above.

We can obtain these equations of motion by varying the superspace action (4.1) with respect to the superfield prepotentials of the standard Weyl multiplet ($\mathfrak{U}$), the vector multiplet compensator (V_{ij}), and the linear multiplet compensator (Ω). Such variations lead to the supercurrent superfield $\mathcal{J}$, the linear multiplet of the EOM of V_{ij}, and the vector multiplet of the EOM of Ω, respectively. Alternatively, we can reduce (4.1) to components and vary it with respect to the highest dimension independent fields (Y, X_{ij}, and F) of the corresponding multiplets. The resulting equations of motion then describe the primary fields, i.e., the bottom components, of the multiplets of the equations of motion that arise from the variation of the full superfields. It is then straightforward to reinterpret them as the primary superfields of the equations of motion. By making use of code developed in Cadabra, the full higher-derivative action in components has been obtained by substituting the explicit form of composite primary multiplets described in Secs. III E–III G together with their descendants. These results, and details of the derivation of the equations of motion that we derived by using a combination of both superspace and components arguments, will be presented in an upcoming paper. The important point to stress is that the equations of motion are fully locally superconformal covariant. From them, successively acting with spinor derivatives (which is equivalent to taking successive Q supersymmetry transformations), one can obtain the whole tower of independent equations of motion. Note that the component action computed from (4.1) includes thousands of terms when fermions are considered, and it is not manifestly covariant due to the presence of naked gravitini and Chern-Simons terms. These would become covariant only after taking field variations and several integration by parts. An efficient alternative, and algorithmic, way to attack this problem is then by analyzing the multiplets of the equations of motion starting from their primaries. Moreover, one could extract as much information as possible about the structure of the on-shell action (including all fermionic contributions) by directly working with the equations of motion in superspace.

In the next subsections, we will simply state the final results and show that the three primary equations of motion satisfy all necessary consistency checks dictated by their general structures. From this point of view, the results of this section stand on their own.

A. Vector multiplet

The vector multiplet equation of motion is obtained by varying (4.1) with respect to the superfield V_{ij} or, equivalently, the field X_{ij}. The resulting EOM is

$$0 = \frac{3}{4} H^{ij}_{\mathrm{VM}} + \kappa G^{ij} + \alpha H^{ij}_{\mathrm{Weyl}} + \beta H^{ij}_{\log W} + \gamma H^{ij}_{R^2}. \tag{4.2}$$

Note that the first two terms correspond to the EOM for the vector multiplet in the two-derivative supergravity theory S_{gSG}, while the remaining three terms describe the contribution coming from the three curvature-squared invariants. It is clear that, as expected, the right-hand side of (4.2) is a linear multiplet satisfying (2.20).

B. Linear multiplet

The linear multiplet equation of motion is obtained by varying (4.1) with respect to the superfield Ω or, equivalently, the auxiliary field F. The resulting EOM is

$$0 = \mathbf{W} + \kappa W + \gamma W_{R^2}, \tag{4.3}$$

with

$$\begin{aligned} W_{R^2} = G^{-1}\Big[&\frac{1}{2}X^{ij}\mathbf{X}_{ij} - \frac{1}{2}F^{ab}\mathbf{F}_{ab} + W\Box\mathbf{W} + \mathbf{W}\Box W + (\nabla^a W)\nabla_a\mathbf{W} \\ &+ \frac{3}{16}YW\mathbf{W} - \frac{3}{2}W^{ab}(F_{ab}\mathbf{W} + \mathbf{F}_{ab}W) - \frac{39}{16}W^{ab}W_{ab}W\mathbf{W} \\ &+ \frac{\mathrm{i}}{2}\lambda^{\alpha i}\nabla_\alpha{}^\beta\boldsymbol{\lambda}_{\beta i} + \frac{\mathrm{i}}{2}\boldsymbol{\lambda}^{\alpha i}\nabla_\alpha{}^\beta\lambda_{\beta i} + \frac{3}{2}X^{\alpha i}(W\boldsymbol{\lambda}_{\alpha i} + \mathbf{W}\lambda_{\alpha i}) - \frac{3\mathrm{i}}{2}W^{\alpha\beta}\lambda^i_\alpha\boldsymbol{\lambda}_{\beta i}\Big] \\ &+ G^{-3}\Big[\frac{1}{4}G^{ij}\varphi_{\beta i}(\lambda_{\alpha j}\nabla^{\alpha\beta}\mathbf{W} + \boldsymbol{\lambda}_{\alpha j}\nabla^{\alpha\beta}W) + \frac{1}{2}G^{ij}\varphi_{\beta i}(W\nabla^{\alpha\beta}\boldsymbol{\lambda}_{\alpha j} + \mathbf{W}\nabla^{\alpha\beta}\lambda_{\alpha j}) \\ &- \frac{1}{2}G^{ij}\varphi_{\alpha i}(F^{\alpha\beta}\boldsymbol{\lambda}_{\beta j} + \mathbf{F}^{\alpha\beta}\lambda_{\beta j}) - \frac{1}{4}G^{ij}F(W\mathbf{X}_{ij} + \mathbf{W}X_{ij}) \\ &- \frac{3}{4}G^{ij}W^{\alpha\beta}\varphi_{\alpha i}(\mathbf{W}\lambda_{\beta j} + W\boldsymbol{\lambda}_{\beta j}) + \frac{\mathrm{i}}{4}FG^{ij}\lambda^\alpha_i\boldsymbol{\lambda}_{\alpha j} + \frac{3\mathrm{i}}{4}G_{ij}X^{\alpha i}\varphi^j_\alpha W\mathbf{W} \\ &+ \frac{1}{4}G_{ij}\varphi^{\alpha i}(X^{jk}\boldsymbol{\lambda}_{\alpha k} + \mathbf{X}^{jk}\lambda_{\alpha k}) - \frac{\mathrm{i}}{4}\varphi^{\alpha i}\varphi^j_\alpha(X_{ij}\mathbf{W} + \mathbf{X}_{ij}W) - \frac{1}{4}\varphi^{\alpha i}\varphi^j_\alpha\lambda^\beta_i\boldsymbol{\lambda}_{\beta j}\Big] \\ &+ G^{-5}\Big[\frac{3\mathrm{i}}{8}G^{ij}G^{kl}\varphi^\alpha_k\varphi_{\alpha l}(X_{ij}\mathbf{W} + \mathbf{X}_{ij}W - \mathrm{i}\lambda^\beta_i\boldsymbol{\lambda}_{\beta j})\Big]. \end{aligned} \tag{4.4}$$

It is possible to check explicitly that W_{R^2} is primary, $S^i_\alpha W_{R^2} = 0$. Moreover, we find that W_{R^2} can be expressed as

$$W_{R^2} = \frac{\mathrm{i}}{32}G\nabla_{ij}\mathcal{R}^{ij}_1, \tag{4.5a}$$

where

$$\mathcal{R}^{ij}_1 = G^{-2}\left(\delta^i_k\delta^j_l - \frac{1}{2G^2}G^{ij}G_{kl}\right)H^{kl}_{\text{bilinear}}, \tag{4.5b}$$

and

$$H^{kl}_{\text{bilinear}} = 2W\mathbf{X}^{kl} + 2\mathbf{W}X^{kl} - 2\mathrm{i}\lambda^{\alpha(k}\boldsymbol{\lambda}^{\,l)}_{\alpha}. \tag{4.5c}$$

This is exactly the structure of the composite vector multiplets $\mathbb{W}_n$ in (5.1) with $n = 1$ and with a precise choice of composite linear multiplet $H^{kl} := H^{kl}_{\text{bilinear}}$. See Sec. V for more detail on these composite vector multiplets. In addition to the remarkably simple form of (4.5), this result guarantees that the right-hand side of Eq. (4.3), and in particular (4.4), is a primary superfield satisfying the vector multiplet constraints (2.11), as expected. This is a very nontrivial consistency check of Eq. (4.4).

C. Standard Weyl multiplet

The conformal supergravity equation of motion is obtained by varying (4.1) with respect to the standard Weyl multiplet prepotential superfield $\mathfrak{U}$ or, equivalently, the field Y. The resulting EOM is

$$0 = \mathcal{J} = J_{\text{EH}} + \alpha J_{\text{Weyl}} + \beta J_{\log W} + \gamma J_{R^2}, \tag{4.6a}$$

with

$$J_{\text{EH}} = \frac{3}{32}(G - W^3), \tag{4.6b}$$

$$J_{\text{Weyl}} = -\frac{3}{64}WY + \frac{3}{16}WW^{ab}W_{ab} + \frac{3}{32}F_{ab}W^{ab} - \frac{3}{16}\lambda^\alpha_i X^i_\alpha, \tag{4.6c}$$

$$J_{\log W} = -\frac{3}{1024}WY - \frac{69}{1024}W^{ab}W_{ab}W + \frac{3}{32}\Box W - \frac{3}{64}F_{ab}W^{ab} - \frac{3}{256}\lambda^\alpha_j X^j_\alpha + \frac{3}{128}F^{ab}F_{ab}W^{-1} - \frac{9}{128}X^{ij}X_{ij}W^{-1} + \frac{3\mathrm{i}}{32}(\Gamma^a)^{\alpha\beta}W^{-1}\lambda^i_\alpha\nabla_a\lambda_{\beta i} + \frac{3}{64}W^{-1}(\nabla^a W)\nabla_a W - \frac{3\mathrm{i}}{128}(\Sigma^{ab})^{\alpha\beta}F_{ab}\lambda^i_\alpha\lambda_{\beta i}W^{-2} - \frac{3\mathrm{i}}{64}X^{ij}\lambda^\alpha_i\lambda_{\alpha j}W^{-2} - \frac{3\mathrm{i}}{32}(\Gamma_b)^{\alpha\beta}\lambda^j_\alpha\lambda_{\beta j}W^{-2}\nabla^b W - \frac{3}{256}\lambda^{\alpha i}\lambda^\beta_i\lambda^j_\alpha\lambda_{\beta j}W^{-3}, \tag{4.6d}$$

$$J_{R^2} = -\frac{3}{8}W\mathbf{W}^2 + \frac{3}{32}G^{-1}(WG^{ij}\mathbf{X}_{ij} + G^{ij}X_{ij}\mathbf{W} - \mathrm{i}G^{ij}\lambda^\alpha_i\boldsymbol{\lambda}_{\alpha j}). \tag{4.6e}$$

Here J_{EH} is the EOM from the gauged supergravity action S_{gSG}, which does not have any contribution from the cosmological constant term κ.

Analogous to the case of 4D $\mathcal{N} = 2$ conformal supergravity [101,102], the 5D Weyl multiplet may be described by a single unconstrained real prepotential $\mathfrak{U}$ [20]. Given a system of matter superfields φ^i, one can construct a Noether coupling between $\mathfrak{U}$ and the matter supercurrent $\mathcal{J}$ of the form

$$S[\varphi^i] = \int \mathrm{d}^{5|8}z\, E\mathfrak{U}\mathcal{J} = \int \mathrm{d}^5 x e(YJ + \cdots), \tag{4.7}$$

where $J = \mathcal{J}|$. The supercurrent $\mathcal{J}$ is a dimension-3 primary real scalar superfield. The conformal supergravity EOM (4.6) is obtained by varying the supergravity action with respect to $\mathfrak{U}$,

$$\frac{\delta S[\varphi^i]}{\delta\mathfrak{U}} = \mathcal{J} = 0. \tag{4.8}$$

The supercurrent multiplet in five dimensions was constructed by Howe and Lindström [83]. It satisfies the conservation equation

$$\nabla^{ij}\mathcal{J} = 0, \tag{4.9}$$

when all matter superfields equations of motion are satisfied. Thus, as a consistency check, we shall prove that the expression $\mathcal{J}$ in (4.6) satisfies the conservation constraint (4.9). It has been shown in [20] that this constraint holds for J_{EH}. For each invariant, we have indeed verified that the corresponding J is a primary superfield of dimension 3. It also satisfies $\nabla^{ij}J = 0$ provided the vector and linear multiplets equations of motion of Eqs. (4.2) and (4.3), respectively, are imposed. Using Cadabra, an explicit calculation shows that, off-shell, it holds

$$\nabla^{ij}J_{\mathrm{Weyl}} = \frac{3\mathrm{i}}{4}WH^{ij}_{\mathrm{Weyl}}, \tag{4.10a}$$

$$\nabla^{ij}J_{\log W} = \frac{3\mathrm{i}}{4}WH^{ij}_{\log W}, \tag{4.10b}$$

$$\nabla^{ij}J_{R^2} = \frac{3\mathrm{i}}{4}WH^{ij}_{R^2} - \frac{3\mathrm{i}}{4}G^{ij}W_{R^2}. \tag{4.10c}$$

It is then clear that the right-hand sides of (4.10) are proportional to the composite vector and linear multiplets appearing in (4.2) and (4.3). Consequently, the supercurrent conservation equation (4.9) is satisfied once the equations of motion for the compensators are used. This represents a very nontrivial consistency check of (4.6b)–(4.6e).

V. AN ALTERNATIVE SCALAR CURVATURE-SQUARED INVARIANT

Recall the action defined in terms of a composite vector multiplet superfield $\mathbf{W}$ written in Eqs. (3.16) and (3.14), respectively. There also exists an infinite number of alternative vector multiplets composite of a linear multiplet compensating superfield G_{ij} and a superfield associated with a primary real $\mathcal{O}^{(2n)}$ multiplet $H^{i_1\cdots i_{2n}} = H^{(i_1\cdots i_{2n})}$, such that $\nabla^{(j}_\alpha H^{i_1\cdots i_{2n})} = 0$. In five dimensions, this was constructed in [20] by extending the 4D $\mathcal{N} = 2$ analysis of [87]. We refer to [20] for details, including the precise definition and literature on $\mathcal{O}^{(2n)}$ multiplets, and simply state the final result here. The following superfields

$$\mathbb{W}_n = \frac{\mathrm{i}(2n)!}{2^{2n+3}(n+1)!(n-1)!}G\nabla_{ij}\mathcal{R}^{ij}_n, \tag{5.1}$$

where

$$\mathcal{R}^{ij}_n = G^{-2n}\left(\delta^i_k\delta^j_l - \frac{1}{2G^2}G^{ij}G_{kl}\right) \times H^{kli_1\cdots i_{2n-2}}G_{(i_1 i_2}\cdots G_{i_{2n-3}i_{2n-2})}, \tag{5.2}$$

all satisfy the constraints (2.11) for any positive integer n. In fact, $\mathbb{W}_1$ is precisely the structure seen in W_{R^2} of Eq. (4.5a). By considering $n = 2$ and choosing H^{ijkl} to be the square of a linear multiplet H^{ij} (distinguished from G^{ij}), $H^{ijkl} = H^{(ij}H^{kl)}$, we can engineer an alternative scalar curvature-squared invariant. The result is in spirit similar to the scalar curvature-squared invariant engineered for 4D $\mathcal{N}=2$ in [24] and directly related to 5D $\mathcal{N}=1$ results in [39].[6] Let us show how this works.

Consider the $n=2$ composite superfield,

$$\mathbb{W}_2 = \frac{\mathrm{i}}{32} G\nabla_{ij}\mathcal{R}_2^{ij}, \tag{5.3}$$

where

$$\mathcal{R}_2^{ij} = G^{-4}\left(\delta_k^i\delta_l^j - \frac{1}{2G^2}G^{ij}G_{kl}\right)H^{(kl}H^{mn)}G_{mn}. \tag{5.4}$$

By explicitly computing (5.3), we may define $\mathbb{W}_2$ as a linear combination of real functions, $\mathcal{P}_A$ and $\mathcal{P}_{AB}{}^{ij}$, which are themselves composed of descendants of the linear multiplets,

$$\mathbb{W}_2 = 2\mathcal{P}_A F^A + 2\mathrm{i}\mathcal{P}_{AB}{}^{ij}\varphi_i^{\alpha A}\varphi_{\alpha j}^B. \tag{5.5}$$

Here the index $A=1$, 2 indicates the two linear superfields, $G_{ij}^1 = G_{ij}$ and $G_{ij}^2 = H_{ij}$. Note that this is analogous to Eq. (2.5) in [39] with the first A index fixed so that $\mathcal{F}_{AB}\to\mathcal{P}_B$ and with an appropriate normalization factor added to the second term. All functions, $\mathcal{P}_A$ and $\mathcal{P}_{AB}{}^{ij}$, are defined as follows:

$$\mathcal{P}_1 = \frac{1}{8}H^2G^{-3} - \frac{3}{32}(G_{kl}H^{kl})^2G^{-5}, \tag{5.6a}$$

$$\mathcal{P}_2 = \frac{1}{8}(G_{kl}H^{kl})G^{-3}, \tag{5.6b}$$

$$\mathcal{P}_{11}{}^{ij} = -\frac{3}{16}G^{ij}H^2G^{-5} - \frac{3}{16}(G_{kl}H^{kl})H^{ij}G^{-5} + \frac{15}{64}(G_{kl}H^{kl})^2G^{ij}G^{-7}, \tag{5.6c}$$

$$\mathcal{P}_{12}{}^{ij} = \mathcal{P}_{21}{}^{ij} = \frac{1}{8}H^{ij}G^{-3} - \frac{3}{16}(G_{kl}H^{kl})G^{ij}G^{-5}, \tag{5.6d}$$

$$\mathcal{P}_{22}{}^{ij} = \frac{1}{8}G^{ij}G^{-3}. \tag{5.6e}$$

It is then a straightforward exercise to show that functions with two A indices are derivatives of functions with one, that is,

$$\mathcal{P}_{AB}{}^{ij} = \frac{\partial\mathcal{P}_A}{\partial G_{ij}^B}. \tag{5.7}$$

They also satisfy the following constraints:

$$\mathcal{P}_{AB}{}^{ij} = \mathcal{P}_{(AB)}{}^{ij}, \qquad \mathcal{P}_{AB}{}^{ij}G_{jk}^B = -\frac{1}{2}\delta_k^i\mathcal{P}_A. \tag{5.8}$$

Last, we may define functions of two derivatives on $\mathcal{P}_A$,

$$\mathcal{P}_{ABC}{}^{ijkl} := \frac{\partial\mathcal{P}_{AB}{}^{ij}}{\partial G_{kl}^C} = \frac{\partial^2\mathcal{P}_A}{\partial G_{ij}^B\partial G_{kl}^C}, \tag{5.9}$$

which satisfy

$$\mathcal{P}_{ABC}{}^{ijkl} = \mathcal{P}_{(ABC)}{}^{ijkl}, \qquad \mathcal{P}_{ABC}{}^{ijkl}\epsilon_{jk} = 0. \tag{5.10}$$

These are the constraints needed to ensure that $\mathbb{W}_2$ in (5.5) satisfies (2.11), which in our case are satisfied by construction.

To engineer the alternative scalar curvature-squared invariant, consider G_{ij} to be a compensator and H_{ij} to be composite of a vector multiplet, which is built out of a single Abelian vector multiplet, as in Eq. (3.5),

$$H^{ij} := H_{\mathrm{VM}}^{ij}. \tag{5.11}$$

In finding the $\mathbb{X}_2^{ij}$ descendant of $\mathbb{W}_2$, we are interested in squared contributions of F_{VM} being a descendant field of H_{VM}^{ij}. This is apparent from the fact that F_{VM} satisfies

$$F_{\mathrm{VM}} = 4W\Box W + \cdots, \tag{5.12}$$

where $\Box W$ gives rise to an $\mathcal{R}$ contribution. Roughly speaking, by considering (5.5)–(5.6e) with the choice (5.11), we are squaring the kinetic term of the vector multiplet compensator which, in turn, leads to a scalar curvature-squared invariant. In fact, if we look at the dimension-2 scalar descendant of $\mathbb{W}_2$, we obtain

$$\mathbb{X}_2^{ij} := \frac{\mathrm{i}}{4}\nabla^{\alpha(i}\nabla_\alpha^{j)}\mathbb{W}_2 = \frac{1}{8}G^{ij}G^{-3}F_{\mathrm{VM}}^2 + \cdots = \mathcal{P}_{22}{}^{ij}F_{\mathrm{VM}}^2 + \cdots, \tag{5.13a}$$

$$G_{ij}\mathbb{X}_2^{ij} = 4G^{-1}W^2(\Box W)^2 + \cdots. \tag{5.13b}$$

Specifically, Eq. (5.13b) is one term in the component action given by the BF action principle. If one proceeds in setting to constants G and W, by gauge fixing dilatation and using (two-derivative) equations of motion, we are left with an $\mathcal{R}^2$ contribution to the four-derivative component action. Although we have not yet analyzed in detail the equations of motion and the component structure of this invariant, we expect it might play a role in studying higher-derivative invariants in alternative off-shell superspace settings, as, for example, the recent off-shell supergravity constructed in [75] by using the variant hyperdilaton Weyl multiplet of conformal supergravity. We leave for the future more investigations along this line.

[6] G. T.-M. is grateful for discussions with M. Ozkan on scalar curvature-squared invariants.

ACKNOWLEDGMENTS

We are grateful to M. Ozkan and Y. Pang for discussions and collaboration related to this work. This work is supported by the Australian Research Council (ARC) Future Fellowship FT180100353 and by the Capacity Building Package of the University of Queensland. G. G. and S. K. are supported by the postgraduate scholarships at the University of Queensland.

[1] I. Antoniadis, S. Ferrara, R. Minasian, and K. S. Narain, R^4 couplings in M and type II theories on Calabi-Yau spaces, Nucl. Phys. **B507**, 571 (1997).

[2] I. Antoniadis, R. Minasian, S. Theisen, and P. Vanhove, String loop corrections to the universal hypermultiplet, Classical Quantum Gravity **20**, 5079 (2003).

[3] J. T. Liu and R. Minasian, Higher-derivative couplings in string theory: Dualities and the B-field, Nucl. Phys. **B874**, 413 (2013).

[4] M. Baggio, N. Halmagyi, D. R. Mayerson, D. Robbins, and B. Wecht, Higher derivative corrections and central charges from wrapped M5-branes, J. High Energy Phys. 12 (2014) 042.

[5] N. Bobev, A. M. Charles, D. Gang, K. Hristov, and V. Reys, Higher-derivative supergravity, wrapped M5-branes, and theories of class $\mathcal{R}$, J. High Energy Phys. 04 (2021) 058.

[6] N. Bobev, A. M. Charles, K. Hristov, and V. Reys, The Unreasonable Effectiveness of Higher-Derivative Supergravity in AdS_4 Holography, Phys. Rev. Lett. **125**, 131601 (2020).

[7] N. Bobev, A. M. Charles, K. Hristov, and V. Reys, Higher-derivative supergravity, AdS_4 holography, and black holes, J. High Energy Phys. 08 (2021) 173.

[8] N. Bobev, K. Hristov, and V. Reys, AdS_5 holography and higher-derivative supergravity, J. High Energy Phys. 04 (2022) 088.

[9] J. T. Liu and R. J. Saskowski, Four-derivative corrections to minimal gauged supergravity in five dimensions, J. High Energy Phys. 05 (2022) 171.

[10] K. Hristov, ABJM at finite N via 4d supergravity, J. High Energy Phys. 10 (2022) 190.

[11] N. Bobev, V. Dimitrov, V. Reys, and A. Vekemans, Higher-derivative corrections and AdS_5 black holes, Phys. Rev. D **106**, L121903 (2022).

[12] D. Cassani, A. Ruipérez, and E. Turetta, Corrections to AdS_5 black hole thermodynamics from higher-derivative supergravity, J. High Energy Phys. 11 (2022) 059.

[13] S. J. Gates, M. T. Grisaru, M. Rocek, and W. Siegel, Superspace or one thousand and one lessons in supersymmetry, Front. Phys. **58**, 1 (1983).

[14] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, NJ, 1992).

[15] I. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity: Or a Walk through Superspace* (IOP, Bristol, 1998).

[16] D. Z. Freedman and A. Van Proeyen, *Supergravity* (Cambridge University Press, Cambridge, England, 2012).

[17] E. Lauria and A. Van Proeyen, $\mathcal{N} = 2$ supergravity in $D = 4, 5, 6$ dimensions, Lect. Notes Phys. **966**, XII (2020).

[18] S. M. Kuzenko, E. S. N. Raptakis, and G. Tartaglino-Mazzucchelli, Superspace approaches to $\mathcal{N} = 1$ supergravity, arXiv:2210.17088.

[19] S. M. Kuzenko, E. S. N. Raptakis, and G. Tartaglino-Mazzucchelli, Covariant superspace approaches to $\mathcal{N} = 2$ supergravity, arXiv:2211.11162.

[20] D. Butter, S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Conformal supergravity in five dimensions: New approach and applications, J. High Energy Phys. 02 (2015) 111.

[21] E. Bergshoeff, A. Salam, and E. Sezgin, A supersymmetric R^2-action in six dimensions and torsion, Phys. Lett. B **173**, 73 (1986).

[22] G. Lopes Cardoso, B. de Wit, and T. Mohaupt, Corrections to macroscopic supersymmetric black hole entropy, Phys. Lett. B **451**, 309 (1999).

[23] T. Mohaupt, Black hole entropy, special geometry and strings, Fortschr. Phys. **49**, 3 (2001).

[24] B. de Wit and F. Saueressig, Off-shell $N = 2$ tensor supermultiplets, J. High Energy Phys. 09 (2006) 062.

[25] K. Hanaki, K. Ohashi, and Y. Tachikawa, Supersymmetric completion of an R^2 term in five-dimensional supergravity, Prog. Theor. Phys. **117**, 533 (2007).

[26] B. de Wit, S. Katmadas, and M. van Zalk, New supersymmetric higher-derivative couplings: Full $N = 2$ superspace does not count!, J. High Energy Phys. 01 (2011) 007.

[27] E. A. Bergshoeff, J. Rosseel, and E. Sezgin, Off-shell $D = 5$, $N = 2$ Riemann squared supergravity, Classical Quantum Gravity **28**, 225016 (2011).

[28] F. Coomans and A. Van Proeyen, Off-shell $N = (1, 0)$, $D = 6$ supergravity from superconformal methods, J. High Energy Phys. 1102 (2011) 049; 01 (2012) 119(E).

[29] E. Bergshoeff, F. Coomans, E. Sezgin, and A. Van Proeyen, Higher derivative extension of 6D chiral gauged supergravity, J. High Energy Phys. 07 (2012) 011.

[30] M. Ozkan and Y. Pang, Supersymmetric completion of Gauss-Bonnet combination in five dimensions, J. High Energy Phys. 03 (2013) 158; 07 (2013) 152(E).

[31] D. Butter, S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Conformal supergravity in three dimensions: Off-shell actions, J. High Energy Phys. 10 (2013) 073.

[32] M. Ozkan and Y. Pang, All off-shell R^2 invariants in five dimensional $\mathcal{N} = 2$ supergravity, J. High Energy Phys. 08 (2013) 042.

[33] D. Butter, B. de Wit, S. M. Kuzenko, and I. Lodato, New higher-derivative invariants in $N = 2$ supergravity and the Gauss-Bonnet term, J. High Energy Phys. 12 (2013) 062.

[34] S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, $N = 6$ superconformal gravity in three dimensions from superspace, J. High Energy Phys. 01 (2014) 121.

[35] M. Ozkan, Supersymmetric curvature-squared invariants in five and six dimensions, Ph.D. thesis, Texas A&M University, 2013.

[36] D. Butter, B. de Wit, and I. Lodato, Non-renormalization theorems and $N = 2$ supersymmetric backgrounds, J. High Energy Phys. 03 (2014) 131.

[37] S. M. Kuzenko and J. Novak, On curvature-squared terms in $N = 2$ supergravity, Phys. Rev. D **92**, 085033 (2015).

[38] D. Butter, S. M. Kuzenko, J. Novak, and S. Theisen, Invariants for minimal conformal supergravity in six dimensions, J. High Energy Phys. 12 (2016) 072.

[39] M. Ozkan, Off-shell $\mathcal{N} = 2$ linear multiplets in five dimensions, J. High Energy Phys. 11 (2016) 157.

[40] D. Butter, F. Ciceri, B. de Wit, and B. Sahoo, Construction of All $N = 4$ Conformal Supergravities, Phys. Rev. Lett. **118**, 081602 (2017).

[41] D. Butter, J. Novak, and G. Tartaglino-Mazzucchelli, The component structure of conformal supergravity invariants in six dimensions, J. High Energy Phys. 05 (2017) 133.

[42] J. Novak, M. Ozkan, Y. Pang, and G. Tartaglino-Mazzucchelli, Gauss-Bonnet Supergravity in Six Dimensions, Phys. Rev. Lett. **119**, 111602 (2017).

[43] D. Butter, J. Novak, M. Ozkan, Y. Pang, and G. Tartaglino-Mazzucchelli, Curvature-squared invariants in six-dimensional $\mathcal{N} = (1, 0)$ supergravity, J. High Energy Phys. 04 (2019) 013.

[44] D. Butter, F. Ciceri, and B. Sahoo, $N = 4$ conformal supergravity: The complete actions, J. High Energy Phys. 01 (2020) 029.

[45] S. Hegde and B. Sahoo, New higher derivative action for tensor multiplet in $\mathcal{N} = 2$ conformal supergravity in four dimensions, J. High Energy Phys. 01 (2020) 070.

[46] M. Mishra and B. Sahoo, Curvature squared action in four dimensional $N = 2$ supergravity using the dilaton Weyl multiplet, J. High Energy Phys. 04 (2021) 027.

[47] E. Cremmer, Supergravities in 5 dimensions, in *Supergravity and Superspace*, edited by S. W. Hawking and M. Roček (Cambridge University Press, Cambridge, England, 1981), pp. 267–282.

[48] A. H. Chamseddine and H. Nicolai, Coupling the SO(2) supergravity through dimensional reduction, Phys. Lett. **96B**, 89 (1980).

[49] P. S. Howe, Off-shell $N = 2$ and $N = 4$ supergravity in five-dimensions, in *Quantum Structure of Space and Time*, edited by M. J. Duff and C. J. Isham (Cambridge University Press, Cambridge, England, 1982), pp. 239–253.

[50] M. Günaydin, G. Sierra, and P. K. Townsend, The geometry of $N = 2$ Maxwell-Einstein supergravity and Jordan algebras, Nucl. Phys. **B242**, 244 (1984).

[51] M. Günaydin, G. Sierra, and P. K. Townsend, Gauging the $D = 5$ Maxwell-Einstein supergravity theories: More on Jordan algebras, Nucl. Phys. **B253**, 573 (1985).

[52] M. Günaydin and M. Zagermann, The gauging of five-dimensional, $N = 2$ Maxwell-Einstein supergravity theories coupled to tensor multiplets, Nucl. Phys. **B572**, 131 (2000).

[53] A. Ceresole and G. Dall'Agata, General matter coupled $N = 2$, $D = 5$ gauged supergravity, Nucl. Phys. **B585**, 143 (2000).

[54] M. Zucker, Minimal off-shell supergravity in five dimensions, Nucl. Phys. **B570**, 267 (2000).

[55] M. Zucker, Gauged $N = 2$ off-shell supergravity in five dimensions, J. High Energy Phys. 08 (2000) 016.

[56] M. Zucker, Off-shell supergravity in five-dimensions and supersymmetric brane world scenarios, Fortschr. Phys. **51**, 899 (2003).

[57] T. Kugo and K. Ohashi, Supergravity tensor calculus in 5D from 6D, Prog. Theor. Phys. **104**, 835 (2000).

[58] T. Kugo and K. Ohashi, Off-shell $d = 5$ supergravity coupled to matter-Yang-Mills system, Prog. Theor. Phys. **105**, 323 (2001).

[59] T. Fujita and K. Ohashi, Superconformal tensor calculus in five dimensions, Prog. Theor. Phys. **106**, 221 (2001).

[60] T. Kugo and K. Ohashi, Gauge and non-gauge tensor multiplets in 5D conformal supergravity, Prog. Theor. Phys. **108**, 1143 (2003).

[61] E. Bergshoeff, S. Cucu, M. Derix, T. de Wit, R. Halbersma, and A. Van Proeyen, Weyl multiplets of $N = 2$ conformal supergravity in five-dimensions, J. High Energy Phys. 06 (2001) 051.

[62] E. Bergshoeff, S. Cucu, T. de Wit, J. Gheerardyn, R. Halbersma, S. Vandoren, and A. Van Proeyen, Superconformal $N = 2$, $D = 5$ matter with and without actions, J. High Energy Phys. 10 (2002) 045.

[63] E. Bergshoeff, S. Cucu, T. de Wit, J. Gheerardyn, S. Vandoren, and A. Van Proeyen, $N = 2$ supergravity in five dimensions revisited, Classical Quantum Gravity **21**, 3015 (2004).

[64] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Super-Weyl invariance in 5D supergravity, J. High Energy Phys. 04 (2008) 032.

[65] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Five-dimensional superfield supergravity, Phys. Lett. B **661**, 42 (2008).

[66] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, 5D supergravity and projective superspace, J. High Energy Phys. 02 (2008) 004.

[67] P. S. Howe and U. Lindström, Superconformal geometries and local twistors, J. High Energy Phys. 04 (2021) 140.

[68] D. Butter, $N = 1$ conformal superspace in four dimensions, Ann. Phys. (Amsterdam) **325**, 1026 (2010).

[69] D. Butter, $N = 2$ conformal superspace in four dimensions, J. High Energy Phys. 10 (2011) 030.

[70] D. Butter, S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Conformal supergravity in three dimensions: New off-shell formulation, J. High Energy Phys. 09 (2013) 072.

[71] S. M. Kuzenko and E. S. N. Raptakis, Conformal (p, q) supergeometries in two dimensions, J. High Energy Phys. 02 (2023) 166.

[72] K. Peeters, A field-theory motivated approach to symbolic computer algebra, Comput. Phys. Commun. **176**, 550 (2007).

[73] K. Peeters, Introducing cadabra: A symbolic computer algebra system for field theory problems, arXiv:hep-th/0701238.

[74] G. Gold, S. Khandelwal, W. Kitchin, and G. Tartaglino-Mazzucchelli, Hyper-dilaton Weyl multiplet of 4D, $\mathcal{N} = 2$ conformal supergravity, J. High Energy Phys. 09 (2022) 016.

[75] J. Hutomo, S. Khandelwal, G. Tartaglino-Mazzucchelli, and J. Woods, Hyperdilaton Weyl multiplets of 5D and 6D minimal conformal supergravity, Phys. Rev. D **107**, 046009 (2023).

[76] H. Nishino and S. Rajpoot, Alternative $N = 2$ supergravity in five dimensions with singularities, Phys. Lett. B **502**, 246 (2001).

[77] B. Zwiebach, Curvature squared terms and string theories, Phys. Lett. **156B**, 315 (1985).

[78] S. Deser and A. N. Redlich, String induced gravity and ghost freedom, Phys. Lett. B **176**, 350 (1986); **186**, 461(E) (1987).

[79] F. Coomans and M. Ozkan, An off-shell formulation for internally gauged $D = 5$, $N = 2$ supergravity from superconformal methods, J. High Energy Phys. 01 (2013) 099.

[80] S. M. Kuzenko, On compactified harmonic/projective superspace, 5D superconformal theories, and all that, Nucl. Phys. **B745**, 176 (2006).

[81] S. M. Kuzenko and W. D. Linch III, On five-dimensional superspaces, J. High Energy Phys. 02 (2006) 038.

[82] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Five-dimensional $N = 1$ AdS superspace: Geometry, off-shell multiplets and dynamics, Nucl. Phys. **B785**, 34 (2007).

[83] P. S. Howe and U. Lindström, The supercurrent in five dimensions, Phys. Lett. **103B**, 422 (1981).

[84] B. Zupnik, Harmonic superpotentials and symmetries in gauge theories with eight supercharges, Nucl. Phys. **B554**, 365 (1999); **B644**, 405 (2002).

[85] L. Mezincescu, On the superfield formulation of O(2) supersymmetry, Dubna Report No. JINR-P2-12572, 1979.

[86] P. S. Howe, K. S. Stelle, and P. K. Townsend, Supercurrents, Nucl. Phys. **B192**, 332 (1981).

[87] D. Butter and S. M. Kuzenko, New higher-derivative couplings in 4D $N = 2$ supergravity, J. High Energy Phys. 03 (2011) 047.

[88] M. F. Sohnius, Supersymmetry and central charges, Nucl. Phys. **B138**, 109 (1978).

[89] B. de Wit, J. W. van Holten, and A. Van Proeyen, Central charges and conformal supergravity, Phys. Lett. **95B**, 51 (1980).

[90] B. de Wit, J. W. van Holten, and A. Van Proeyen, Structure of $N = 2$ supergravity, Nucl. Phys. **B184**, 77 (1981); **B222**, 516(E) (1983).

[91] B. de Wit, P. G. Lauwers, R. Philippe, S. Q. Su, and A. Van Proeyen, Gauge and matter fields coupled to $N = 2$ supergravity, Phys. Lett. **134B**, 37 (1984).

[92] J. Wess, Supersymmetry and internal symmetry, Acta Phys. Austriaca **41**, 409 (1975).

[93] W. Siegel, Superfields in higher dimensional space-time, Phys. Lett. **80B**, 220 (1979).

[94] W. Siegel, Off-shell central charges, Nucl. Phys. **B173**, 51 (1980).

[95] M. F. Sohnius, K. S. Stelle, and P. C. West, Representations of extended supersymmetry, in *Superspace and Supergravity*, edited by S. W. Hawking and M. Roček (Cambridge University Press, Cambridge, England, 1981), p. 283.

[96] B. de Wit, R. Philippe, and A. Van Proeyen, The improved tensor multiplet in $N = 2$ supergravity, Nucl. Phys. **B219**, 143 (1983).

[97] A. Karlhede, U. Lindström, and M. Roček, Self-interacting tensor multiplets in $N = 2$ superspace, Phys. Lett. **147B**, 297 (1984).

[98] U. Lindström and M. Roček, New hyperkähler metrics and new supermultiplets, Commun. Math. Phys. **115**, 21 (1988).

[99] M. Zucker, Supersymmetric brane world scenarios from off-shell supergravity, Phys. Rev. D **64**, 024024 (2001).

[100] S. M. Kuzenko, On $N = 2$ supergravity and projective superspace: Dual formulations, Nucl. Phys. **B810**, 135 (2009).

[101] S. M. Kuzenko and S. Theisen, Correlation functions of conserved currents in $N = 2$ superconformal theory, Classical Quantum Gravity **17**, 665 (2000).

[102] D. Butter and S. M. Kuzenko, $N = 2$ supergravity and supercurrents, J. High Energy Phys. 12 (2010) 080.