

Three-point functions of conserved supercurrents in 3D $\mathcal{N}=1$ SCFT: General formalism for arbitrary superspins

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We analyze the general structure of the three-point functions of conserved higher-spin supercurrents in 3D, $\mathcal{N}=1$ superconformal field theory. It is shown that supersymmetry imposes additional restrictions on correlation functions of conserved higher-spin currents. We develop a manifestly supersymmetric formalism to compute the three-point function $\langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle$, where $\mathbf{J}_{s_1}$, $\mathbf{J}'_{s_2}$, and $\mathbf{J}''_{s_3}$ are conserved higher-spin supercurrents with superspins s_1 , s_2 , and s_3 , respectively (integer or half-integer). Using a computational approach limited only by computer power, we analytically impose the constraints arising from the superfield conservation equations and symmetries under permutations of superspace points. Explicit solutions for three-point functions are presented, and we provide a complete classification of the results for $s_i \leq 20$; the pattern is very clear, and we propose that our classification holds for arbitrary superspins. We demonstrate that Grassmann-even three-point functions are fixed up to one parity-even structure and one parity-odd structure, while Grassmann-odd three-point functions are fixed up to a single parity-even structure. The existence of the parity-odd structure in the Grassmann-even correlation functions is subject to a set of triangle inequalities in the superspins. For completeness, we also analyze the structure of three-point functions involving conserved higher-spin supercurrents and scalar superfields.

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I. INTRODUCTION

A well-known implication of conformal symmetry [1–11] is that the general form of two- and three-point correlation functions of primary operators is fixed up to finitely many parameters. However, constructing explicit solutions for three-point functions of conserved current operators such as the energy-momentum tensor, vector currents, and more generally, higher-spin currents, remains an open problem. An interesting feature of three-dimensional (3D) conformal field theories is the existence of parity-odd structures in the three-point functions of conserved currents. These structures were overlooked in the seminal work by Osborn and Petkou [11] (see also [12]), which introduced the group-theoretic formalism to study the three-point functions of the energy-momentum tensor and vector currents. The parity-odd structures were discovered later using a polarization spinor approach in [13], where results for three-point functions of conserved (bosonic) higher-spin currents were obtained.

Soon after, it was proven by Maldacena and Zhiboedov in [14] that correlation functions involving the energy-momentum tensor and higher-spin currents are equal to those of free field theories.¹ This can be viewed as an extension of the Coleman-Mandula theorem [15] to conformal field theories; it was originally proven in three dimensions and was generalized to four- and higher-dimensional cases in [16–19] (see also [20,21]). In three-dimensional theories the general structure of the three-point function $\langle J_{s_1} J'_{s_2} J''_{s_3} \rangle$, where J_s denotes a conserved current of arbitrary spin- s , is fixed up to the following form [13,14]²:

$$\begin{aligned} \langle J_{s_1} J'_{s_2} J''_{s_3} \rangle &= a_1 \langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_{E_1} + a_2 \langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_{E_2} \\ &\quad + b \langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_O, \end{aligned} \quad (1.1)$$

¹An assumption of the Maldacena-Zhiboedov theorem is that the conformal theory under consideration possesses a unique spin-2 conserved current—the energy-momentum tensor. This assumption, however, does not hold in the presence of fermionic higher-spin currents. Hence, it also does not hold in superconformal theories possessing conserved higher-spin supercurrents.

²Recall: in a d -dimensional conformal field theory (CFT), a conserved current of spin- s is a totally symmetric and traceless tensor $J_{m_1 \dots m_s}$ of scale dimension $\Delta_J = s + d - 2$, satisfying the conservation equation $\partial^{m_1} J_{m_1 \dots m_s} = 0$.

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where $\langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_{E_1}$ and $\langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_{E_2}$ are parity-even solutions corresponding to free field theories, and $\langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_O$ is a parity-violating, or parity-odd, solution which is not generated by a free CFT. The existence of the parity-odd solution is subject to the following triangle inequalities on the spins:

$$s_1 \leq s_2 + s_3, \quad s_2 \leq s_1 + s_3, \quad s_3 \leq s_1 + s_2. \quad (1.2)$$

If any of the above inequalities are not satisfied, then the odd solution is incompatible with current conservation. Parity-odd solutions are unique to three dimensions and have been shown to arise in Chern-Simons theories interacting with parity-violating matter [22–32]. Existence and uniqueness of the odd solution has been proven in [28], while methods to obtain explicit solutions for arbitrary spin are contained in [16,17,33,34].

A natural follow-up question arises: in conformal field theories, what are the implications of supersymmetry on the general structure of three-point correlation functions? The study of correlation functions in superconformal theories has been carried out in diverse dimensions using the group-theoretic approach developed in the following publications [34–50]. It has been shown that superconformal symmetry imposes additional restrictions on the three-point functions of conserved currents compared to nonsupersymmetric theories. For example, it was pointed out in [44] that there is an apparent tension between supersymmetry and the existence of parity-violating structures. In contrast with the nonsupersymmetric case, parity-odd structures are not found in the three-point functions of the energy-momentum tensor and conserved vector currents [41–44]. For three-point functions of higher-spin currents the results are more unclear; however, it was shown in [47] that parity-odd structures can appear in the three-point functions of currents belonging to a superspin-2 current multiplet. Such a multiplet contains independent conserved currents of spin-2 and spin- $\frac{5}{2}$ (the spin-2 current is not equal to but possesses the same properties as the energy-momentum tensor). In general, for three-point functions involving conserved higher-spin currents, the conditions under which parity-violating structures can arise in supersymmetric theories are not well understood.

The intent of this paper is to address these concerns and provide a complete classification of conserved three-point functions in 3D $\mathcal{N} = 1$ superconformal field theory. To do this we develop a general formalism to study the three-point function

$$\langle J_{s_1}(z_1) J'_{s_2}(z_2) J''_{s_3}(z_3) \rangle, \quad (1.3)$$

where z_1, z_2, z_3 are points in 3D $\mathcal{N} = 1$ Minkowski superspace and the superfield $J_s(z)$ is a conserved higher-spin supercurrent of superspin- s (integer or half-integer). These currents are primary superfields transforming in an

irreducible representation of the 3D $\mathcal{N} = 1$ superconformal algebra, $\mathfrak{so}(3, 2|1) \cong \mathfrak{osp}(1|2; \mathbb{R})$. They are described by totally symmetric spin tensors of rank $2s$, $J_{\alpha_1 \dots \alpha_{2s}}(z) = J_{(\alpha_1 \dots \alpha_{2s})}(z)$, and satisfy the following superfield conservation equation:

$$D^{\alpha_1} J_{\alpha_1 \alpha_2 \dots \alpha_{2s}}(z) = 0, \quad (1.4)$$

where D^α is the conventional covariant spinor derivative in $\mathcal{N} = 1$ superspace. As a result of the superfield conservation equation (1.4), conserved supercurrents have scale dimension $\Delta_J = s + 1$ (saturating the unitary bound), and at the component level contain independent conserved currents of spin- s and $s + \frac{1}{2}$, respectively. The most important examples of conserved supercurrents in superconformal field theory are the supercurrent and flavor current multiplets, corresponding to the cases $s = \frac{3}{2}$ and $s = \frac{1}{2}$, respectively (for a review of the properties of supercurrent and flavor current multiplets in 3D theories, see [41,51] and the references therein). The supercurrent multiplet contains the energy-momentum tensor and the supersymmetry current.³ Likewise, the flavor current multiplet contains a conserved vector current. Three-point correlation functions of these currents contain important physical information about a given superconformal field theory and are highly constrained by superconformal symmetry.

The general structure of three-point functions of conserved (higher-spin) currents in 3D $\mathcal{N} = 1$ superconformal field theory was proposed in [40] to be fixed up to the following form:

$$\langle J_{s_1} J'_{s_2} J''_{s_3} \rangle = a \langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_E + b \langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_O, \quad (1.5)$$

where $\langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_E$ is a parity-even solution and $\langle J_{s_1} J'_{s_2} J''_{s_3} \rangle_O$ is a parity-odd solution. However, as was pointed out above there is a tension between supersymmetry and the existence of parity-odd structures, which means that the coefficient b in (1.5) vanishes in many correlators. In this paper we provide a complete classification for when the parity-odd structures are allowed and when they are not. In particular, we show that the odd solution does not appear in correlation functions that are overall Grassmann-odd (or fermionic). In the Grassmann-even (bosonic) three-point functions the existence of the parity-odd solution is subject to the following superspin triangle inequalities:

$$s_1 \leq s_2 + s_3, \quad s_2 \leq s_1 + s_3, \quad s_3 \leq s_1 + s_2. \quad (1.6)$$

When the triangle inequalities are simultaneously satisfied there is one even solution and one odd solution; however, if

³In $\mathcal{N}$ -extended superconformal theories, the supercurrent multiplet also contains the R -symmetry currents.

any of the above inequalities are not satisfied, then the odd solution is incompatible with the superfield conservation equations. Our classification is in perfect agreement with our previous results in [41,44] for the three-point functions of the energy-momentum tensor and conserved vector currents. They belong to the supermultiplets of superspins $s = \frac{3}{2}$ and $s = \frac{1}{2}$, respectively, and, hence, their three-point functions in superspace are Grassmann-odd. Based on our classification, it is implied that they do not possess parity-odd contributions, which is in agreement with the earlier results. Our classification is also in agreement with our previous result in [47] for the three-point function of the conserved supercurrent of superspin-2. This three-point function is Grassmann-even in superspace, and since the triangle inequalities (1.6) are satisfied, a parity-odd contribution is allowed.

Our method assumes only the constraints imposed by superconformal symmetry and superfield conservation equations; within the framework of our formalism we reproduce all known results concerning the structure of three-point functions of conserved supercurrents in 3D $\mathcal{N} = 1$ superconformal field theory (SCFT). We present new results for three-point functions involving higher-spin supercurrents, obtaining explicit and completely analytic results. We also analyze three-point functions involving scalar superfields, thus covering essentially all possible three-point functions in 3D $\mathcal{N} = 1$ superconformal field theory. Our method is based on a computational approach (by means of analytic/symbolic computer algebra in *Mathematica*) which constructs all possible structures for the correlation function for a given set of superspins s_1 , s_2 , and s_3 , consistent with its superconformal properties. Next, we extract the linearly independent structures by systematic application of linear dependence relations and then impose the superfield conservation equations and symmetries under permutations of superspace points. As a result we obtain the three-point function in a very explicit form which can be presented for relatively high superspins. The method can be applied for arbitrary superspins and is limited only by computer power. Due to these limitations we were able to carry out computations up to $s_i = 20$ (a “soft” limit, after which the calculations take many hours); however, with a sufficiently powerful computer one could extend this bound even further. The computational approach we have developed (based on the same method as in [34]) is completely algorithmic; one simply chooses the superspins of the fields and the solution for the three-point function consistent with conservation and point-switch symmetries is generated.

The analysis is computationally intensive for higher spins; to streamline the calculations we develop a hybrid, index-free formalism which combines the group-theoretic superspace formalism introduced by Osborn [35] and Park [37,38] and a method based on contraction of tensor indices with auxiliary spinors. This method is widely used throughout the literature to construct correlation functions of

higher-spin currents (see, e.g., [13,16,17,40,52,53]); however, this particular approach describes the correlation function completely in terms of a polynomial, $\mathcal{H}(X, \Theta; u, v, w)$, which is a function of two superconformally covariant three-point building blocks, X and Θ , and the auxiliary spinor variables u , v , and w . As a result one does not have to work with the superspace points explicitly when imposing the superfield conservation equations.

The results of this paper are organized as follows. In Sec. II we review the essentials of the group theoretic formalism used to construct correlation functions of primary superfields in 3D $\mathcal{N} = 1$ SCFT. In Sec. III we outline a method to impose all constraints arising from superfield conservation equations and point-switch symmetries on three-point functions of conserved higher-spin supercurrents. In particular, we introduce an index-free, auxiliary spinor formalism which allows us to construct a generating function for the three-point functions, and we outline the important aspects of our computational approach. Section IV is then devoted to the analysis of three-point functions involving conserved supercurrents. As a test of our approach, we present an explicit analysis for three-point correlation functions involving combinations of supercurrent and flavor current multiplets, reproducing the known results [41,44]. The results are then expanded to include conserved higher-spin supercurrents, for which we provide many examples and confirm the results of [47]. Here we also resolve a contradiction in the literature concerning the structure of the three-point function $\langle \mathbf{J}_{1/2} \mathbf{J}'_{1/2} \mathbf{J}''_2 \rangle$; it was found in [40] that this three-point function contains a parity-odd solution; however, it was shown later in [47] that parity-odd structures are inconsistent with conservation equations. In this paper we reexamine this three-point function and provide a straightforward explanation, based on the triangle inequalities (1.6), for why this structure cannot appear. In Sec. V, for completeness, we perform the analysis of correlation functions involving combinations of scalar superfields and conserved higher-spin supercurrents. Finally, in Sec. VI we comment on the general results in the context of superconformal field theories. The appendixes are devoted to mathematical conventions and various useful identities.

II. SUPERCONFORMAL SYMMETRY IN THREE DIMENSIONS

In this section we will review the pertinent aspects of the group-theoretic formalism used to compute three-point correlation functions of primary superfields in 3D $\mathcal{N} = 1$ superconformal field theories. For a more detailed review of the formalism the reader may consult [38,41].

A. Superconformal transformations and primary superfields

Let us begin by reviewing infinitesimal superconformal transformations and the transformation laws of primary

superfields. This section closely follows the notation of [54–56]. Now consider 3D, $\mathcal{N} = 1$ Minkowski superspace $\mathbb{M}^{3|2}$, parametrized by coordinates $z^A = (x^a, \theta^\alpha)$, where $a = 0, 1, 2$ and $\alpha = 1, 2$ are Lorentz and spinor indices, respectively. We consider infinitesimal superconformal transformations

$$\delta z^A = \xi z^A \Leftrightarrow \delta x^a = \xi^a(z) + i(\gamma^a)_{\alpha\beta} \xi^\alpha(z) \theta^\beta, \quad \delta \theta^\alpha = \xi^\alpha(z), \quad (2.1)$$

which are associated with the real first-order differential operator

$$\xi = \xi^A(z) \partial_A = \xi^a(z) \partial_a + \xi^\alpha(z) D_\alpha. \quad (2.2)$$

This operator satisfies the master equation $[\xi, D_\alpha] \propto D_\beta$, from which we obtain

$$\xi^\alpha = \frac{i}{6} D_\beta \xi^{\alpha\beta}. \quad (2.3)$$

As a consequence, the conformal Killing equation is satisfied,

$$\partial_a \xi_b + \partial_b \xi_a = \frac{2}{3} \eta_{ab} \partial_c \xi^c. \quad (2.4)$$

The solutions to the master equation are called the superconformal Killing vector fields of Minkowski superspace [55,57]. They span a Lie algebra isomorphic to the superconformal algebra $\mathfrak{osp}(1|2; \mathbb{R})$. The components of the operator ξ were calculated explicitly in [38,41], and are found to be

$$\xi^{\alpha\beta} = a^{\alpha\beta} - \lambda^\alpha_\gamma x^{\gamma\beta} - x^{\alpha\gamma} \lambda_\gamma^\beta + \sigma x^{\alpha\beta} + 4i\epsilon^{(\alpha} \theta^{\beta)} + x^{\alpha\gamma} x^{\beta\delta} b_{\gamma\delta} + ib_\delta^{(\alpha} x^{\beta)\delta} \theta^2 - 4i\eta_\gamma x^{\gamma(\alpha} \theta^{\beta)}, \quad (2.5a)$$

$$\xi^\alpha = \epsilon^\alpha - \lambda^\alpha_\beta \theta^\beta + \frac{1}{2} \sigma \theta^\alpha + b_{\beta\gamma} x^{\beta\alpha} \theta^\gamma + \eta_\beta (2i\theta^\beta \theta^\alpha - x^{\beta\alpha}), \quad (2.5b)$$

$$a_{\alpha\beta} = a_{\beta\alpha}, \quad \lambda_{\alpha\beta} = \lambda_{\beta\alpha}, \quad \lambda^\alpha_\alpha = 0, \quad b_{\alpha\beta} = b_{\beta\alpha}. \quad (2.5c)$$

The bosonic parameters $a_{\alpha\beta}$, $\lambda_{\alpha\beta}$, σ , $b_{\alpha\beta}$ correspond to infinitesimal translations, Lorentz transformations, scale transformations, and special conformal transformations, respectively, while the fermionic parameters ϵ^α and η^α correspond to Q -supersymmetry and S -supersymmetry transformations. Furthermore, the identity $D_{[\alpha} \xi_{\beta]} \propto \epsilon_{\alpha\beta}$ implies that

$$[\xi, D_\alpha] = -(D_\alpha \xi^\beta) D_\beta = \lambda^\beta_\alpha(z) D_\beta - \frac{1}{2} \sigma(z) D_\alpha, \quad (2.6)$$

$$\lambda_{\alpha\beta}(z) = -D_{(\alpha} \xi_{\beta)}, \quad \sigma(z) = D_\alpha \xi^\alpha. \quad (2.7)$$

The local parameters $\lambda^{\alpha\beta}(z)$ and $\sigma(z)$ are interpreted as being associated with combined special-conformal/Lorentz and scale transformations, respectively, and appear in the transformation laws for primary tensor superfields. For later use let us also introduce the z -dependent S -supersymmetry parameter

$$\eta_\alpha(z) = -\frac{i}{2} D_\alpha \sigma(z). \quad (2.8)$$

Explicit calculations of the local parameters give [38,41]

$$\lambda^{\alpha\beta}(z) = \lambda^{\alpha\beta} - x^{\gamma(\alpha} b_\gamma^{\beta)} + 2i\eta^{(\alpha} \theta^{\beta)} - \frac{i}{2} b^{\alpha\beta} \theta^2, \quad (2.9a)$$

$$\sigma(z) = \sigma + b_{\alpha\beta} x^{\alpha\beta} + 2i\theta^\alpha \eta_\alpha, \quad (2.9b)$$

$$\eta_\alpha(z) = \eta_\alpha - b_{\alpha\beta} \theta^\beta. \quad (2.9c)$$

Now consider a tensor superfield $\Phi_{\mathcal{A}}(z)$ transforming in an irreducible representation of the Lorentz group with respect to the index $\mathcal{A}$. Such a superfield is called primary with dimension Δ if it possesses the following superconformal transformation properties:

$$\delta \Phi_{\mathcal{A}} = -\xi \Phi_{\mathcal{A}} - \Delta \sigma(z) \Phi_{\mathcal{A}} + \lambda^{\alpha\beta}(z) (M_{\alpha\beta})_{\mathcal{A}}^{\mathcal{B}} \Phi_{\mathcal{B}}, \quad (2.10)$$

where ξ is the superconformal Killing vector, $\sigma(z)$ and $\lambda^{\alpha\beta}(z)$ are z -dependent parameters associated with ξ , and the matrix $M_{\alpha\beta}$ is a Lorentz generator.

1. Conserved supercurrents

In this paper we are primarily interested in the structure of three-point correlation functions involving conserved higher-spin supercurrents. In 3D, $\mathcal{N} = 1$ theories, a conserved higher-spin supercurrent of superspin- s (integer or half-integer) is defined as a totally symmetric spin tensor of rank $2s$, $\mathbf{J}_{\alpha_1 \dots \alpha_{2s}}(z) = \mathbf{J}_{(\alpha_1 \dots \alpha_{2s})}(z) = \mathbf{J}_{\alpha(2s)}(z)$, satisfying a conservation equation of the form

$$D^{\alpha_1} \mathbf{J}_{\alpha_1 \alpha_2 \dots \alpha_{2s}}(z) = 0, \quad (2.11)$$

where D^α is the conventional covariant spinor derivative (A11). Conserved currents are primary superfields as they possess the following infinitesimal superconformal transformation properties [38,41,57]:

$$\delta \mathbf{J}_{\alpha_1 \dots \alpha_{2s}}(z) = -\xi \mathbf{J}_{\alpha_1 \dots \alpha_{2s}}(z) - \Delta_{\mathbf{J}} \sigma(z) \mathbf{J}_{\alpha_1 \dots \alpha_{2s}}(z) + 2s \lambda_{(\alpha_1}^\delta(z) \mathbf{J}_{\alpha_2 \dots \alpha_{2s})\delta}(z). \quad (2.12)$$

The dimension $\Delta_{\mathbf{J}}$ is constrained by the conservation condition (2.11) to $\Delta_{\mathbf{J}} = s + 1$. Higher-spin supercurrents possess the following component structure:

$$\mathbf{J}_{\alpha(2s)}(z) = J_{\alpha(2s)}^{(0)}(x) + J_{\alpha(2s+1)}^{(1)}(x)\theta^{\alpha_{2s+1}} + \tilde{J}_{(\alpha_1 \dots \alpha_{2s-1}}^{(1)}(x)\theta_{\alpha_{2s}} + J_{\alpha(2s)}^{(2)}(x)\theta^2. \quad (2.13)$$

After imposing (2.11), a short calculation gives $\tilde{J}^{(1)} = 0$, while $J^{(2)}$ is a function of $J_{\alpha(2s)}^{(0)}$. On the other hand, the components $J^{(0)}$ and $J^{(1)}$ satisfy the following conservation equations:

$$\partial^{\alpha_1 \alpha_2} J_{\alpha_1 \alpha_2 \alpha(2s-2)}^{(0)}(x) = 0, \quad \partial^{\alpha_1 \alpha_2} J_{\alpha_1 \alpha_2 \alpha(2s-1)}^{(1)}(x) = 0. \quad (2.14)$$

Hence, at the component level, a higher-spin supercurrent of superspin- s contains conserved conformal currents of spin- s and spin- $(s + \frac{1}{2})$, respectively.

B. Two-point building blocks

Given two superspace points z_1 and z_2 , we define the two-point functions

$$\mathbf{x}_{12}^{\alpha\beta} = (x_1 - x_2)^{\alpha\beta} + 2i\theta_1^\alpha \theta_2^\beta - i\theta_{12}^\alpha \theta_{12}^\beta, \quad \theta_{12}^\alpha = \theta_1^\alpha - \theta_2^\alpha, \quad (2.15)$$

which transform under the superconformal group as follows:

$$\begin{aligned} \tilde{\delta}\mathbf{x}_{12}^{\alpha\beta} = & -\left(\lambda^\alpha{}_\gamma(z_1) - \frac{1}{2}\delta^\alpha{}_\gamma\sigma(z_1)\right)\mathbf{x}_{12}^{\gamma\beta} \\ & -\mathbf{x}_{12}^{\alpha\gamma}\left(\lambda_\gamma{}^\beta(z_2) - \frac{1}{2}\delta_\gamma{}^\beta\sigma(z_2)\right), \end{aligned} \quad (2.16a)$$

$$\tilde{\delta}\theta_{12}^\alpha = -\left(\lambda^\alpha{}_\beta(z_1) - \frac{1}{2}\delta^\alpha{}_\beta\sigma(z_1)\right)\theta_{12}^\beta - \mathbf{x}_{12}^{\alpha\beta}\eta_\beta(z_2). \quad (2.16b)$$

Here the total variation $\tilde{\delta}$ is defined by its action on an n -point function $\Phi(z_1, \dots, z_n)$ as

$$\tilde{\delta}\Phi(z_1, \dots, z_n) = \sum_{i=1}^n \xi_{z_i} \Phi(z_1, \dots, z_n). \quad (2.17)$$

Only (2.16a) transforms covariantly under superconformal transformations, as (2.16b) contains an inhomogeneous piece in its transformation law. Therefore, it will not appear as a building block in two- or three-point correlation functions. Due to the useful property, $\mathbf{x}_{21}^{\alpha\beta} = -\mathbf{x}_{12}^{\beta\alpha}$, the two-point function (2.15) can be split into symmetric and antisymmetric parts as follows:

$$\mathbf{x}_{12}^{\alpha\beta} = x_{12}^{\alpha\beta} + \frac{i}{2}\varepsilon^{\alpha\beta}\theta_{12}^2, \quad \theta_{12}^2 = \theta_{12}^\alpha \theta_{12\alpha}. \quad (2.18)$$

The symmetric component

$$x_{12}^{\alpha\beta} = (x_1 - x_2)^{\alpha\beta} + 2i\theta_1^\alpha \theta_2^\beta \quad (2.19)$$

is recognized as the bosonic part of the standard two-point superspace interval. The two-point functions possess the property:

$$\mathbf{x}_{12}^{\alpha\sigma}\mathbf{x}_{21\sigma\beta} = \mathbf{x}_{12}^2\delta_\beta^\alpha, \quad \mathbf{x}_{12}^2 = -\frac{1}{2}\mathbf{x}_{12}^{\alpha\beta}\mathbf{x}_{12\alpha\beta}. \quad (2.20)$$

Hence, we find

$$(\mathbf{x}_{12}^{-1})^{\alpha\beta} = -\frac{\mathbf{x}_{12}^{\beta\alpha}}{\mathbf{x}_{12}^2}. \quad (2.21)$$

It is now useful to introduce the normalized two-point functions, denoted by $\hat{\mathbf{x}}_{12}$,

$$\hat{\mathbf{x}}_{12\alpha\beta} = \frac{\mathbf{x}_{12\alpha\beta}}{(\mathbf{x}_{12}^2)^{1/2}}, \quad \hat{\mathbf{x}}_{12}^{\alpha\sigma}\hat{\mathbf{x}}_{21\sigma\beta} = \delta_\beta^\alpha. \quad (2.22)$$

Under superconformal transformations, $\mathbf{x}_{12}^2$ transforms with local scale parameters, while (2.22) transforms with local Lorentz parameters

$$\tilde{\delta}\mathbf{x}_{12}^2 = (\sigma(z_1) + \sigma(z_2))\mathbf{x}_{12}^2, \quad (2.23a)$$

$$\tilde{\delta}\hat{\mathbf{x}}_{12}^{\alpha\beta} = -\lambda^\alpha{}_\gamma(z_1)\hat{\mathbf{x}}_{12}^{\gamma\beta} - \hat{\mathbf{x}}_{12}^{\alpha\gamma}\lambda_\gamma{}^\beta(z_2). \quad (2.23b)$$

There are also the following differential identities for the action of covariant spinor derivatives on the two-point functions:

$$D_{(1)\gamma}\mathbf{x}_{12}^{\alpha\beta} = -2i\theta_{12}^\beta\delta_\gamma^\alpha, \quad D_{(1)\alpha}\mathbf{x}_{12}^{\alpha\beta} = -4i\theta_{12}^\beta, \quad (2.24)$$

where $D_{(i)\alpha}$ acts on the superspace point z_i . From here we can now construct an operator analogous to the conformal inversion tensor acting on the space of symmetric traceless spin tensors of arbitrary rank. Given a two-point function $\mathbf{x}$, we define the operator

$$\mathcal{I}_{\alpha(k)\beta(k)}(\mathbf{x}) = \hat{\mathbf{x}}_{(\alpha_1(\beta_1} \dots \hat{\mathbf{x}}_{\alpha_k)\beta_k)}, \quad (2.25)$$

along with its inverse

$$\mathcal{I}^{\alpha(k)\beta(k)}(\mathbf{x}) = \hat{\mathbf{x}}^{(\alpha_1(\beta_1} \dots \hat{\mathbf{x}}^{\alpha_k)\beta_k)}. \quad (2.26)$$

The spinor indices may be raised and lowered using the standard conventions as follows:

$$\mathcal{I}_{\alpha(k)}^{\beta(k)}(\mathbf{x}) = \varepsilon^{\beta_1\gamma_1} \dots \varepsilon^{\beta_k\gamma_k} \mathcal{I}_{\alpha(k)\gamma(k)}(\mathbf{x}). \quad (2.27)$$

Now due to the property

$$\mathcal{I}_{\alpha(k)\beta(k)}(-\mathbf{x}) = (-1)^k \mathcal{I}_{\alpha(k)\beta(k)}(\mathbf{x}), \quad (2.28)$$

the following identity holds for products of inversion tensors:

$$\mathcal{I}_{\alpha(k)\sigma(k)}(\mathbf{x}_{12})\mathcal{I}^{\sigma(k)\beta(k)}(\mathbf{x}_{21}) = \delta_{(\alpha_1}^{(\beta_1} \dots \delta_{\alpha_k)}^{\beta_k)}. \quad (2.29)$$

The objects (2.25) and (2.26) prove to be essential in the construction of correlation functions of primary operators with arbitrary spin. Indeed, the vector representation of the inversion tensor may be recovered in terms of the spinor two-point functions as follows:

$$I_{mn}(x) = -\frac{1}{2} \text{Tr}(\gamma_m \hat{x} \gamma_n \hat{x})|_{\theta=0}. \quad (2.30)$$

C. Three-point building blocks

Essential to the analysis of three-point correlation functions are three-point covariants/building blocks. Indeed, given three superspace points, z_1, z_2, z_3 , one can define the objects, $\mathcal{Z}_k = (X_{ij}, \Theta_{ij})$ as follows:

$$\begin{aligned} X_{ij\alpha\beta} &= -(\mathbf{x}_{ik}^{-1})_{\alpha\gamma} \mathbf{x}_{ij}^{\gamma\delta} (\mathbf{x}_{kj}^{-1})_{\delta\beta}, \\ \Theta_{ij\alpha} &= (\mathbf{x}_{ik}^{-1})_{\alpha\beta} \theta_{ki}^\beta - (\mathbf{x}_{jk}^{-1})_{\alpha\beta} \theta_{kj}^\beta, \end{aligned} \quad (2.31a)$$

where the labels (i, j, k) are a cyclic permutation of $(1, 2, 3)$. These objects possess the important property $X_{ij\alpha\beta} = -X_{ji\beta\alpha}$. As a consequence, the three-point building blocks (2.31) possess many properties similar to those of the two-point building blocks

$$X_{ij}^{\alpha\sigma} X_{ji\sigma\beta} = X_{ij}^2 \delta_\beta^\alpha, \quad X_{ij}^2 = -\frac{1}{2} X_{ij}^{\alpha\beta} X_{ij\alpha\beta}. \quad (2.32)$$

Hence, we find

$$(X_{ij}^{-1})^{\alpha\beta} = -\frac{X_{ij}^{\beta\alpha}}{X_{ij}^2}. \quad (2.33)$$

It is also useful to note that one may decompose X_{ij} into symmetric and antisymmetric parts similar to (2.18) as follows:

$$X_{ij\alpha\beta} = X_{ij\alpha\beta} - \frac{i}{2} \varepsilon_{\alpha\beta} \Theta_{ij}^2, \quad X_{ij\alpha\beta} = X_{ij\beta\alpha}, \quad (2.34)$$

where the symmetric spin tensor, $X_{ij\alpha\beta}$, can be equivalently represented by the three-vector $X_{ijm} = -\frac{1}{2} (\gamma_m)^{\alpha\beta} X_{ij\alpha\beta}$. Since the building blocks possess the same properties up to cyclic permutations of the points, we will only examine the properties of X_{12} and Θ_{12} , as these objects appear most frequently in our analysis of correlation functions. One can compute

$$X_{12}^2 = -\frac{1}{2} X_{12}^{\alpha\beta} X_{12\alpha\beta} = \frac{\mathbf{x}_{12}^2}{\mathbf{x}_{13}^2 \mathbf{x}_{23}^2}, \quad \Theta_{12}^2 = \Theta_{12}^\alpha \Theta_{12\alpha}. \quad (2.35)$$

The building block X_{12} also possesses the following superconformal transformation properties:

$$\tilde{\delta} X_{12\alpha\beta} = \lambda_\alpha^\gamma(z_3) X_{12\gamma\beta} + X_{12\alpha\gamma} \lambda_\beta^\gamma(z_3) - \sigma(z_3) X_{12\alpha\beta}, \quad (2.36a)$$

$$\tilde{\delta} \Theta_{12\alpha} = \left(\lambda_\alpha^\beta(z_3) - \frac{1}{2} \delta_\alpha^\beta \sigma(z_3) \right) \Theta_{12\beta}, \quad (2.36b)$$

and, therefore,

$$\tilde{\delta} X_{12}^2 = -2\sigma(z_3) X_{12}^2, \quad \tilde{\delta} \Theta_{12}^2 = -\sigma(z_3) \Theta_{12}^2, \quad (2.37)$$

i.e., (X_{12}, Θ_{12}) is superconformally covariant at z_3 . As a consequence, one can identify the three-point superconformal invariant

$$J = \frac{\Theta_{12}^2}{\sqrt{X_{12}^2}} \Rightarrow \tilde{\delta} J = 0, \quad (2.38)$$

which proves to be invariant under permutations of the superspace points, i.e.,

$$J = \frac{\Theta_{12}^2}{\sqrt{X_{12}^2}} = \frac{\Theta_{31}^2}{\sqrt{X_{31}^2}} = \frac{\Theta_{23}^2}{\sqrt{X_{23}^2}}. \quad (2.39)$$

Analogous to the two-point functions, it is also useful to introduce the normalized three-point building blocks, denoted by $\hat{X}_{ij}, \hat{\Theta}_{ij}$,

$$\hat{X}_{ij\alpha\beta} = \frac{X_{ij\alpha\beta}}{(X_{ij}^2)^{1/2}}, \quad \hat{\Theta}_{ij}^\alpha = \frac{\Theta_{ij}^\alpha}{(X_{ij}^2)^{1/4}}, \quad (2.40)$$

such that

$$\hat{X}_{ij}^{\alpha\sigma} \hat{X}_{ji\sigma\beta} = \delta_\beta^\alpha, \quad J = \hat{\Theta}_{ij}^2. \quad (2.41)$$

Compared with the standard three-point building blocks (2.31), the objects (2.40) transform only with local Lorentz parameters. Now given an arbitrary three-point building block, X , let us construct the following higher-spin inversion operator:

$$\mathcal{I}_{\alpha(k)\beta(k)}(X) = \hat{X}_{(\alpha_1\beta_1} \cdots \hat{X}_{\alpha_k\beta_k)}, \quad (2.42)$$

along with its inverse

$$\mathcal{I}^{\alpha(k)\beta(k)}(X) = \hat{X}^{(\alpha_1\beta_1} \cdots \hat{X}^{\alpha_k\beta_k)}. \quad (2.43)$$

These operators possess properties similar to the two-point higher-spin inversion operators (2.25), (2.26), and are essential to the analysis of three-point correlation functions involving higher-spin primary superfields. In particular, one can prove the following useful identities involving X_{ij} and Θ_{ij} at different superspace points:

$$\mathcal{I}_\alpha^\sigma(\mathbf{x}_{13})\mathcal{I}_{\beta'}^\gamma(\mathbf{x}_{13})\mathcal{I}_{\sigma\gamma}(\mathbf{X}_{12}) = \mathcal{I}_{\alpha\beta}(\mathbf{X}_{23}^I), \quad (2.44a)$$

$$\mathcal{I}_\alpha^\gamma(\mathbf{x}_{13})\hat{\Theta}_{12\gamma} = \hat{\Theta}_{23\alpha}^I, \quad (2.44b)$$

where we have defined

$$\Theta_{ij\alpha}^I = \mathcal{I}_{\alpha\beta}(-\mathbf{X}_{ij})\Theta_{ij}^\beta. \quad (2.45)$$

Note that $\mathbf{X}^I$ is defined in the same way, and using (2.42) we have $\mathbf{X}_{\alpha\beta}^I = \mathcal{I}_{(\alpha\alpha')(\beta\beta')}(-\mathbf{X})\mathbf{X}^{\alpha'\beta'} = -\mathbf{X}_{\alpha\beta}$, as expected. Using the inversion operators above, the identity (2.44a) (and cyclic permutations) admits the following generalization to higher spins:

$$\mathcal{I}_{\alpha(k)}^{\sigma(k)}(\mathbf{x}_{13})\mathcal{I}_{\beta(k)}^{\gamma(k)}(\mathbf{x}_{13})\mathcal{I}_{\sigma(k)\gamma(k)}(\mathbf{X}_{12}) = \mathcal{I}_{\alpha(k)\beta(k)}(\mathbf{X}_{23}). \quad (2.46)$$

Due to the transformation properties (2.36a) and (2.36b) it is often useful to make the identifications $(\mathbf{X}_1, \Theta_1) := (\mathbf{X}_{23}, \Theta_{23})$, $(\mathbf{X}_2, \Theta_2) := (\mathbf{X}_{31}, \Theta_{31})$, $(\mathbf{X}_3, \Theta_3) := (\mathbf{X}_{12}, \Theta_{12})$, in which case we have, e.g., $\mathbf{X}_{21} = -\mathbf{X}_3^T$; we will switch between these notations when convenient. Let us now introduce the following analogs of the covariant spinor derivative and supercharge operators involving the three-point objects:

$$\begin{aligned} \mathcal{D}_{(i)\alpha} &= \frac{\partial}{\partial \Theta_i^\alpha} + i(\gamma^m)_{\alpha\beta}\Theta_i^\beta \frac{\partial}{\partial X_i^m}, \\ \mathcal{Q}_{(i)\alpha} &= i\frac{\partial}{\partial \Theta_i^\alpha} + (\gamma^m)_{\alpha\beta}\Theta_i^\beta \frac{\partial}{\partial X_i^m}, \end{aligned} \quad (2.47)$$

which obey the standard commutation relations

$$\{\mathcal{D}_{(i)\alpha}, \mathcal{D}_{(i)\beta}\} = \{\mathcal{Q}_{(i)\alpha}, \mathcal{Q}_{(i)\beta}\} = 2i(\gamma^m)_{\alpha\beta} \frac{\partial}{\partial X_i^m}. \quad (2.48)$$

Some useful identities involving (2.47) are, e.g.,

$$\mathcal{D}_{(3)\gamma}\mathbf{X}_{3\alpha\beta} = -2i\varepsilon_{\gamma\beta}\Theta_{3\alpha}, \quad \mathcal{Q}_{(3)\gamma}\mathbf{X}_{3\alpha\beta} = -2\varepsilon_{\gamma\alpha}\Theta_{3\beta}. \quad (2.49)$$

We must also account for the fact that correlation functions of primary superfields obey differential constraints as a result of superfield conservation equations. Using (2.24) we obtain the following identities:

$$D_{(1)\gamma}\mathbf{X}_{3\alpha\beta} = 2i(\mathbf{x}_{13}^{-1})_{\alpha\gamma}\Theta_{3\beta}, \quad D_{(1)\alpha}\Theta_{3\beta} = -(\mathbf{x}_{13}^{-1})_{\beta\alpha}, \quad (2.50a)$$

$$D_{(2)\gamma}\mathbf{X}_{3\alpha\beta} = 2i(\mathbf{x}_{23}^{-1})_{\beta\gamma}\Theta_{3\alpha}, \quad D_{(2)\alpha}\Theta_{3\beta} = (\mathbf{x}_{23}^{-1})_{\beta\alpha}. \quad (2.50b)$$

Now given a function $f(\mathbf{X}_3, \Theta_3)$, there are the following differential identities which arise as a consequence of (2.49), (2.50a), and (2.50b):

$$D_{(1)\gamma}f(\mathbf{X}_3, \Theta_3) = (\mathbf{x}_{13}^{-1})_{\alpha\gamma}\mathcal{D}_{(3)}^\alpha f(\mathbf{X}_3, \Theta_3), \quad (2.51a)$$

$$D_{(2)\gamma}f(\mathbf{X}_3, \Theta_3) = i(\mathbf{x}_{23}^{-1})_{\alpha\gamma}\mathcal{Q}_{(3)}^\alpha f(\mathbf{X}_3, \Theta_3). \quad (2.51b)$$

These will prove to be essential for imposing differential constraints on three-point correlation functions of primary superfields.

III. GENERAL FORMALISM FOR CORRELATION FUNCTIONS OF PRIMARY SUPERFIELDS

In this section we develop a formalism to construct correlation functions of primary superfields in 3D superconformal field theories. We utilize a hybrid method which combines auxiliary spinors with the approach of [38,41].

A. Two-point functions

Let $\Phi_{\mathcal{A}}$ be a primary superfield with dimension Δ , where $\mathcal{A}$ denotes a collection of Lorentz spinor indices. The two-point correlation function of $\Phi_{\mathcal{A}}$ is fixed by superconformal symmetry to the form

$$\langle \Phi_{\mathcal{A}}(z_1)\Phi^{\mathcal{B}}(z_2) \rangle = c \frac{\mathcal{I}_{\mathcal{A}}^{\mathcal{B}}(\mathbf{x}_{12})}{(\mathbf{x}_{12}^2)^\Delta}, \quad (3.1)$$

where $\mathcal{I}$ is an appropriate representation of the inversion tensor and c is a constant real parameter. The denominator of the two-point function is determined by the conformal dimension of $\Phi_{\mathcal{A}}$, which guarantees that the correlation function transforms with the appropriate weight under scale transformations.

B. Three-point functions

In this subsection we will review the various properties of three-point correlation functions in 3D $\mathcal{N} = 1$ superconformal field theory. First we present the superfield ansatz introduced by Park in [38]. We then develop a new index-free formalism utilizing auxiliary spinors to simplify the overall form of the three-point function, with the ultimate aim of constructing a generating function for arbitrary spins.

1. Superfield ansatz

Concerning three-point correlation functions, let Φ, Ψ, Π be primary superfields with scale dimensions Δ_1, Δ_2 , and Δ_3 , respectively. The three-point function may be constructed using the general ansatz

$$\langle \Phi_{\mathcal{A}_1}(z_1) \Psi_{\mathcal{A}_2}(z_2) \Pi_{\mathcal{A}_3}(z_3) \rangle = \frac{\mathcal{I}^{(1)}_{\mathcal{A}_1} \mathcal{A}'_1(\mathbf{x}_{13}) \mathcal{I}^{(2)}_{\mathcal{A}_2} \mathcal{A}'_2(\mathbf{x}_{23})}{(\mathbf{x}_{13}^2)^{\Delta_1} (\mathbf{x}_{23}^2)^{\Delta_2}} \times \mathcal{H}_{\mathcal{A}'_1 \mathcal{A}'_2 \mathcal{A}_3}(\mathbf{X}_{12}, \Theta_{12}), \quad (3.2)$$

where the tensor $\mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}$ encodes all information about the correlation function and is related to the leading singular operator product expansion coefficient [11]. It is highly constrained by superconformal symmetry as follows:

- (i) Under scale transformations of $\mathbb{M}^{3|2}$, $z = (x, \theta) \mapsto z' = (\lambda^{-2}x, \lambda^{-1}\theta)$; hence, the three-point covariants transform as $(\mathbf{X}, \Theta) \mapsto (\mathbf{X}', \Theta') = (\lambda^2 \mathbf{X}, \lambda \Theta)$. As a consequence, the correlation function transforms as

$$\langle \Phi_{\mathcal{A}_1}(z'_1) \Psi_{\mathcal{A}_2}(z'_2) \Pi_{\mathcal{A}_3}(z'_3) \rangle = (\lambda^2)^{\Delta_1 + \Delta_2 + \Delta_3} \langle \Phi_{\mathcal{A}_1}(z_1) \Psi_{\mathcal{A}_2}(z_2) \Pi_{\mathcal{A}_3}(z_3) \rangle, \quad (3.3)$$

which implies that $\mathcal{H}$ obeys the scaling property

$$\mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\lambda^2 \mathbf{X}, \lambda \Theta) = (\lambda^2)^{\Delta_3 - \Delta_2 - \Delta_1} \mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta), \quad \forall \lambda \in \mathbb{R} \setminus \{0\}. \quad (3.4)$$

This guarantees that the correlation function transforms correctly under scale transformations.

- (ii) If any of the fields Φ , Ψ , Π obey differential equations, such as conservation laws in the case of conserved currents, then the tensor $\mathcal{H}$ is also constrained by differential equations which may be derived with the aid of identities (2.51a) and (2.51b).
- (iii) If any (or all) of the operators Φ , Ψ , Π coincide, the correlation function possesses symmetries under permutations of spacetime points, e.g.,

$$\langle \Phi_{\mathcal{A}_1}(z_1) \Phi_{\mathcal{A}_2}(z_2) \Pi_{\mathcal{A}_3}(z_3) \rangle = (-1)^{\epsilon(\Phi)} \langle \Phi_{\mathcal{A}_2}(z_2) \Phi_{\mathcal{A}_1}(z_1) \Pi_{\mathcal{A}_3}(z_3) \rangle, \quad (3.5)$$

where $\epsilon(\Phi)$ is the Grassmann parity of Φ . As a consequence, the tensor $\mathcal{H}$ obeys constraints which will be referred to as “point-switch identities.”

The constraints above fix the functional form of $\mathcal{H}$ (and therefore the correlation function) up to finitely many independent parameters. Hence, using the general formula (3.6), the problem of computing three-point correlation functions is reduced to deriving the general structure of the tensor $\mathcal{H}$ subject to the above constraints.

2. A note on conserved three-point functions

An important aspect of this construction is that depending on the way in which one constructs the general ansatz (3.6), it can be difficult to impose conservation equations on one of the three fields due to a lack of useful identities such as (2.50a) and (2.50b). For this reason it is useful to switch between the various representations of the three-point function. To illustrate this process

more clearly, consider the following example; suppose we have obtained a solution for the correlation function $\langle \Phi_{\mathcal{A}_1}(z_1) \Psi_{\mathcal{A}_2}(z_2) \Pi_{\mathcal{A}_3}(z_3) \rangle$, with the ansatz

$$\langle \Phi_{\mathcal{A}_1}(z_1) \Psi_{\mathcal{A}_2}(z_2) \Pi_{\mathcal{A}_3}(z_3) \rangle = \frac{\mathcal{I}^{(1)}_{\mathcal{A}_1} \mathcal{A}'_1(\mathbf{x}_{13}) \mathcal{I}^{(2)}_{\mathcal{A}_2} \mathcal{A}'_2(\mathbf{x}_{23})}{(\mathbf{x}_{13}^2)^{\Delta_1} (\mathbf{x}_{23}^2)^{\Delta_2}} \times \mathcal{H}_{\mathcal{A}'_1 \mathcal{A}'_2 \mathcal{A}_3}(\mathbf{X}_{12}, \Theta_{12}). \quad (3.6)$$

All information about this correlation function is encoded in the tensor $\mathcal{H}$, and one can impose conservation on z_1 and z_2 using the identities (2.50a), (2.50b), (2.51a), (2.51b). However, this particular formulation of the three-point function prevents us from imposing conservation on z_3 in a straightforward way. Let us now reformulate the ansatz with Π at the front as follows:

$$\langle \Pi_{\mathcal{A}_3}(z_3) \Psi_{\mathcal{A}_2}(z_2) \Phi_{\mathcal{A}_1}(z_1) \rangle = \frac{\mathcal{I}^{(3)}_{\mathcal{A}_3} \mathcal{A}'_3(\mathbf{x}_{31}) \mathcal{I}^{(2)}_{\mathcal{A}_2} \mathcal{A}'_2(\mathbf{x}_{21})}{(\mathbf{x}_{31}^2)^{\Delta_3} (\mathbf{x}_{21}^2)^{\Delta_2}} \times \tilde{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}'_2 \mathcal{A}'_3}(\mathbf{X}_{23}, \Theta_{23}). \quad (3.7)$$

In this case, all information about this correlation function is now encoded in the tensor $\tilde{\mathcal{H}}$, which has a completely different structure compared to $\mathcal{H}$. Conservation on Π can now be imposed by treating z_3 as the first point with the aid of identities analogous to (2.51a) and (2.51b). We now require an equation relating the tensors $\mathcal{H}$ and $\tilde{\mathcal{H}}$, which correspond to different representations of the same correlation function. Equating the two ansatz above, we obtain the following:

$$\begin{aligned} \tilde{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}_{23}, \Theta_{23}) &= (\mathbf{x}_{13}^2)^{\Delta_3 - \Delta_1} \left(\frac{\mathbf{x}_{21}^2}{\mathbf{x}_{23}^2} \right)^{\Delta_2} \mathcal{I}^{(1)}_{\mathcal{A}_1} \mathcal{A}'_1(\mathbf{x}_{13}) \\ &\times \mathcal{I}^{(2)}_{\mathcal{A}_2} \mathcal{B}_2(\mathbf{x}_{12}) \mathcal{I}^{(2)}_{\mathcal{B}_2} \mathcal{A}'_2(\mathbf{x}_{23}) \\ &\times \mathcal{I}^{(3)}_{\mathcal{A}_3} \mathcal{A}'_3(\mathbf{x}_{13}) \mathcal{H}_{\mathcal{A}'_1 \mathcal{A}'_2 \mathcal{A}_3}(\mathbf{X}_{12}, \Theta_{12}), \end{aligned} \quad (3.8)$$

where we have ignored any signs due to Grassmann parity. Before we can simplify the above equation, we must understand how the inversion tensor acts on $\mathcal{H}(\mathbf{X}, \Theta)$. Now let

$$\mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta) = \mathbf{X}^{\Delta_3 - \Delta_2 - \Delta_1} \hat{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta), \quad (3.9)$$

where $\hat{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta)$ is homogeneous degree 0 in $(\mathbf{X}, \Theta)$, i.e.,

$$\hat{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\lambda^2 \mathbf{X}, \lambda \Theta) = \hat{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta). \quad (3.10)$$

The tensor $\hat{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta)$ can be constructed from totally symmetric, homogeneous degree 0 combinations of ε , $\mathbf{X}$,

and Θ , compatible with the set of indices $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$; hence, we consider the following objects:

$$\varepsilon_{\alpha\beta}, \quad \hat{X}_{\alpha\beta}, \quad \hat{\Theta}_\alpha, \quad (\hat{X} \cdot \hat{\Theta})_\alpha = \hat{X}_{\alpha\beta} \hat{\Theta}^\beta, \quad J = \hat{\Theta}^2. \quad (3.11)$$

Now to simplify (3.8), consider

$$\begin{aligned} & \mathcal{I}^{(1)}_{\mathcal{A}_1} \mathcal{A}'_1(\mathbf{x}_{13}) \mathcal{I}^{(2)}_{\mathcal{A}_2} \mathcal{A}'_2(\mathbf{x}_{13}) \mathcal{I}^{(3)}_{\mathcal{A}_3} \mathcal{A}'_3(\mathbf{x}_{13}) \\ & \times \hat{\mathcal{H}}_{\mathcal{A}'_1 \mathcal{A}'_2 \mathcal{A}'_3}(\mathbf{X}_{12}, \Theta_{12}). \end{aligned} \quad (3.12)$$

Only combinations of the following fundamental products may appear in the result:

$$\mathcal{I}_{\alpha'}^{\alpha'}(\mathbf{x}_{13}) \mathcal{I}_{\beta'}^{\beta'}(\mathbf{x}_{13}) \varepsilon_{\alpha'\beta'} = -\varepsilon_{\alpha\beta}, \quad (3.13a)$$

$$\mathcal{I}_{\alpha'}^{\alpha'}(\mathbf{x}_{13}) \mathcal{I}_{\beta'}^{\beta'}(\mathbf{x}_{13}) \hat{X}_{12\alpha'\beta'} = \hat{X}_{23\alpha\beta}, \quad (3.13b)$$

$$\mathcal{I}_{\alpha'}^{\alpha'}(\mathbf{x}_{13}) \hat{\Theta}_{12\alpha'} = \hat{\Theta}_{23\alpha}^I, \quad (3.13c)$$

$$\mathcal{I}_{\alpha'}^{\alpha'}(\mathbf{x}_{13}) (\hat{X}_{12} \cdot \hat{\Theta}_{12})_{\alpha'} = -(\hat{X}_{23} \cdot \hat{\Theta}_{23})_{\alpha}, \quad (3.13d)$$

where $\hat{\Theta}_{ij}^I$ was defined in (2.45). For correlation functions involving the superconformal invariant, J , we must note that $J^I = (\hat{\Theta}^I)^2 = -J$. These identities are consequences of (2.44a) and (2.44b). If we now denote the above transformations by $\mathcal{I}_{13}$, it acts on $\hat{\mathcal{H}}(\mathbf{X}_{12}, \Theta_{12})$ as follows:

$$\hat{X}_{12} \xrightarrow{\mathcal{I}_{13}} \hat{X}_{23}, \quad \hat{\Theta}_{12} \xrightarrow{\mathcal{I}_{13}} \hat{\Theta}_{23}^I, \quad (3.14a)$$

$$\varepsilon \xrightarrow{\mathcal{I}_{13}} -\varepsilon, \quad \hat{X}_{12} \cdot \hat{\Theta}_{12} \xrightarrow{\mathcal{I}_{13}} -\hat{X}_{23} \cdot \hat{\Theta}_{23}^I, \quad J \xrightarrow{\mathcal{I}_{13}} -J^I. \quad (3.14b)$$

Hence, due to their transformation properties under $\mathcal{I}$, the objects (3.14a) are classified as “parity-even” as they are invariant under $\mathcal{I}$, while the objects (3.14b) are classified as “parity-odd,” as they are pseudo-invariant under $\mathcal{I}$. At this point it is convenient to partition our solution into “even” and “odd” sectors as follows:

$$\mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta) = \mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}^{(+)}(\mathbf{X}, \Theta) + \mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}^{(-)}(\mathbf{X}, \Theta), \quad (3.15)$$

where $\mathcal{H}^{(+)}$ contains all structures that are invariant under $\mathcal{I}$ and $\mathcal{H}^{(-)}$ contains all structures that are pseudo-invariant under $\mathcal{I}$. With this choice of convention, as a consequence of (2.44a) and (2.44b), the following relation holds:

$$\begin{aligned} \hat{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}^{I(\pm)}(\mathbf{X}_{23}, \Theta_{23}) &= \pm \mathcal{I}^{(1)}_{\mathcal{A}_1} \mathcal{A}'_1(\mathbf{x}_{13}) \mathcal{I}^{(2)}_{\mathcal{A}_2} \mathcal{A}'_2(\mathbf{x}_{13}) \\ &\times \mathcal{I}^{(3)}_{\mathcal{A}_3} \mathcal{A}'_3(\mathbf{x}_{13}) \hat{\mathcal{H}}_{\mathcal{A}'_1 \mathcal{A}'_2 \mathcal{A}'_3}^{(\pm)}(\mathbf{X}_{12}, \Theta_{12}), \end{aligned} \quad (3.16)$$

where $\hat{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}^{I(\pm)}(\mathbf{X}, \Theta) = \hat{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}^{(\pm)}(\mathbf{X}, \Theta^I)$. A result analogous to (3.14a) and (3.14b) that follows from the properties of the inversion tensor acting on $(\mathbf{X}, \Theta)$ is

$$\hat{X} \xrightarrow{\mathcal{I}_X} -\hat{X}, \quad \hat{\Theta} \xrightarrow{\mathcal{I}_X} \hat{\Theta}^I, \quad (3.17a)$$

$$\varepsilon \xrightarrow{\mathcal{I}_X} -\varepsilon, \quad \hat{X} \cdot \hat{\Theta} \xrightarrow{\mathcal{I}_X} \hat{X} \cdot \hat{\Theta}^I, \quad J \xrightarrow{\mathcal{I}_X} -J^I. \quad (3.17b)$$

Hence, to obtain the desired transformation properties as in (3.14a) and (3.14b), we consider $\mathcal{H}(-\mathbf{X}, \Theta)$ and obtain the formula

$$\begin{aligned} \mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}^{I(\pm)}(\mathbf{X}, \Theta) &= \pm \mathcal{I}^{(1)}_{\mathcal{A}_1} \mathcal{A}'_1(\mathbf{X}) \mathcal{I}^{(2)}_{\mathcal{A}_2} \mathcal{A}'_2(\mathbf{X}) \mathcal{I}^{(3)}_{\mathcal{A}_3} \mathcal{A}'_3(\mathbf{X}) \\ &\times \mathcal{H}_{\mathcal{A}'_1 \mathcal{A}'_2 \mathcal{A}'_3}^{(\pm)}(-\mathbf{X}, \Theta), \end{aligned} \quad (3.18)$$

which is generally more simple to compute. After substituting (3.16) into (3.8), we obtain the following relation between $\mathcal{H}$ and $\tilde{\mathcal{H}}$:

$$\tilde{\mathcal{H}}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}^{(\pm)}(\mathbf{X}, \Theta) = \pm (\mathbf{X}^2)^{\Delta_1 - \Delta_3} \mathcal{I}^{(2)}_{\mathcal{A}_2} \mathcal{A}'_2(\mathbf{X}) \mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}^{I(\pm)}(\mathbf{X}, \Theta). \quad (3.19)$$

It is now apparent that $\mathcal{I}$ acts as an intertwining operator between the various representations of the correlation function. Once $\tilde{\mathcal{H}}$ is obtained we can then impose conservation on Π as if it were located at the “first point,” using identities analogous to (2.51a) and (2.51b).

If we now consider the correlation function of three conserved primary superfields $\mathbf{J}_{\alpha(I)}$, $\mathbf{J}_{\beta(J)}'$, $\mathbf{J}_{\gamma(K)}''$, where $I = 2s_1$, $J = 2s_2$, $K = 2s_3$, then the general ansatz is

$$\begin{aligned} \langle \mathbf{J}_{\alpha(I)}(z_1) \mathbf{J}_{\beta(J)}'(z_2) \mathbf{J}_{\gamma(K)}''(z_3) \rangle &= \frac{\mathcal{I}_{\alpha(I)'}^{\alpha'}(\mathbf{x}_{13}) \mathcal{I}_{\beta(J)'}^{\beta'}(\mathbf{x}_{23})}{(\mathbf{x}_{13}^2)^{\Delta_1} (\mathbf{x}_{23}^2)^{\Delta_2}} \\ &\times \mathcal{H}_{\alpha'(I) \beta'(J) \gamma(K)}(\mathbf{X}_{12}, \Theta_{12}), \end{aligned} \quad (3.20)$$

where $\Delta_i = s_i + 1$. The constraints on $\mathcal{H}$ are then as follows:

(i) *Homogeneity*:

$$\begin{aligned} & \mathcal{H}_{\alpha(I) \beta(J) \gamma(K)}(\lambda^2 \mathbf{X}, \lambda \Theta) \\ &= (\lambda^2)^{\Delta_3 - \Delta_2 - \Delta_1} \mathcal{H}_{\alpha(I) \beta(J) \gamma(K)}(\mathbf{X}, \Theta). \end{aligned} \quad (3.21)$$

It is often convenient to introduce $\hat{\mathcal{H}}_{\alpha(I) \beta(J) \gamma(K)}(\mathbf{X}, \Theta)$, such that

$$\mathcal{H}_{\alpha(I) \beta(J) \gamma(K)}(\mathbf{X}, \Theta) = \mathbf{X}^{\Delta_3 - \Delta_2 - \Delta_1} \hat{\mathcal{H}}_{\alpha(I) \beta(J) \gamma(K)}(\mathbf{X}, \Theta), \quad (3.22)$$

where $\hat{\mathcal{H}}_{\alpha(I)\beta(J)\gamma(K)}(\mathbf{X}, \Theta)$ is homogeneous degree 0 in $(\mathbf{X}, \Theta)$, i.e.,

$$\hat{\mathcal{H}}_{\alpha(I)\beta(J)\gamma(K)}(\lambda^2 \mathbf{X}, \lambda \Theta) = \hat{\mathcal{H}}_{\alpha(I)\beta(J)\gamma(K)}(\mathbf{X}, \Theta). \quad (3.23)$$

- (ii) *Differential constraints:* After application of the identities (2.51a) and (2.51b) we obtain the following constraints:

$$\text{Conservation at } z_1: \mathcal{D}^\alpha \mathcal{H}_{\alpha\alpha(I-1)\beta(J)\gamma(K)}(\mathbf{X}, \Theta) = 0, \quad (3.24a)$$

$$\text{Conservation at } z_2: \mathcal{Q}^\beta \mathcal{H}_{\alpha(I)\beta\beta(J-1)\gamma(K)}(\mathbf{X}, \Theta) = 0, \quad (3.24b)$$

$$\text{Conservation at } z_3: \mathcal{Q}^\gamma \tilde{\mathcal{H}}_{\alpha(I)\beta(J)\gamma\gamma(K-1)}(\mathbf{X}, \Theta) = 0, \quad (3.24c)$$

where

$$\begin{aligned} \tilde{\mathcal{H}}_{\alpha(I)\beta(J)\gamma(K)}^{(\pm)}(\mathbf{X}, \Theta) \\ = (\mathbf{X}^2)^{\Delta_1 - \Delta_3} \mathcal{I}_{\beta(J)}^{\beta'(J)}(\mathbf{X}) \mathcal{H}_{\alpha(I)\beta'(J)\gamma(K)}^{I(\pm)}(\mathbf{X}, \Theta). \end{aligned} \quad (3.25)$$

- (iii) *Point-switch symmetries:* If the fields $\mathbf{J}$ and $\mathbf{J}'$ coincide, then we obtain the following point-switch identity:

$$\mathcal{H}_{\alpha(I)\beta(I)\gamma(K)}(\mathbf{X}, \Theta) = (-1)^{\epsilon(\mathbf{J})} \mathcal{H}_{\beta(I)\alpha(I)\gamma(K)}(-\mathbf{X}^T, -\Theta), \quad (3.26)$$

where $\epsilon(\mathbf{J})$ is the Grassmann parity of $\mathbf{J}$. Likewise, if the fields $\mathbf{J}$ and $\mathbf{J}''$ coincide, then we obtain the constraint

$$\tilde{\mathcal{H}}_{\alpha(I)\beta(J)\gamma(I)}(\mathbf{X}, \Theta) = (-1)^{\epsilon(\mathbf{J})} \mathcal{H}_{\gamma(I)\beta(J)\alpha(I)}(-\mathbf{X}^T, -\Theta). \quad (3.27)$$

In practice, imposing these constraints on correlation functions involving higher-spin supercurrents quickly becomes unwieldy using the tensor formalism, particularly due to the sheer number of possible tensor structures for a given set of superspins. Hence, in the next subsections we will develop an index-free formalism to handle the computations efficiently, using the same approach as [34].

3. Auxiliary spinor formalism

Suppose we must analyze the constraints on a general spin tensor $\mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta)$, where $\mathcal{A}_1 = \{\alpha_1, \dots, \alpha_I\}$, $\mathcal{A}_2 = \{\beta_1, \dots, \beta_J\}$, $\mathcal{A}_3 = \{\gamma_1, \dots, \gamma_K\}$ represent sets of totally symmetric spinor indices associated with the fields at points

z_1, z_2 , and z_3 , respectively. We introduce sets of commuting auxiliary spinors for each point; u at z_1 , v at z_2 , and w at z_3 , where the spinors satisfy

$$u^2 = \varepsilon_{\alpha\beta} u^\alpha u^\beta = 0, \quad v^2 = \varepsilon_{\alpha\beta} v^\alpha v^\beta = 0, \quad w^2 = \varepsilon_{\alpha\beta} w^\alpha w^\beta = 0. \quad (3.28)$$

Now if we define the objects

$$\mathbf{u}^{\mathcal{A}_1} \equiv \mathbf{u}^{\alpha(I)} = u^{\alpha_1} \dots u^{\alpha_I}, \quad (3.29a)$$

$$\mathbf{v}^{\mathcal{A}_2} \equiv \mathbf{v}^{\beta(J)} = v^{\beta_1} \dots v^{\beta_J}, \quad (3.29b)$$

$$\mathbf{w}^{\mathcal{A}_3} \equiv \mathbf{w}^{\gamma(K)} = w^{\gamma_1} \dots w^{\gamma_K}, \quad (3.29c)$$

then the generating polynomial for $\mathcal{H}$ is constructed as follows:

$$\mathcal{H}(\mathbf{X}, \Theta; u, v, w) = \mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta) \mathbf{u}^{\mathcal{A}_1} \mathbf{v}^{\mathcal{A}_2} \mathbf{w}^{\mathcal{A}_3}. \quad (3.30)$$

There is a one-to-one mapping between the space of symmetric traceless spin tensors and the polynomials constructed using the above method. Indeed, the tensor $\mathcal{H}$ is extracted from the polynomial by acting on it with the following partial derivative operators:

$$\frac{\partial}{\partial \mathbf{u}^{\mathcal{A}_1}} \equiv \frac{\partial}{\partial \mathbf{u}^{\alpha(I)}} = \frac{1}{I!} \frac{\partial}{\partial u^{\alpha_1}} \dots \frac{\partial}{\partial u^{\alpha_I}}, \quad (3.31a)$$

$$\frac{\partial}{\partial \mathbf{v}^{\mathcal{A}_2}} \equiv \frac{\partial}{\partial \mathbf{v}^{\beta(J)}} = \frac{1}{J!} \frac{\partial}{\partial v^{\beta_1}} \dots \frac{\partial}{\partial v^{\beta_J}}, \quad (3.31b)$$

$$\frac{\partial}{\partial \mathbf{w}^{\mathcal{A}_3}} \equiv \frac{\partial}{\partial \mathbf{w}^{\gamma(K)}} = \frac{1}{K!} \frac{\partial}{\partial w^{\gamma_1}} \dots \frac{\partial}{\partial w^{\gamma_K}}. \quad (3.31c)$$

The tensor $\mathcal{H}$ is then extracted from the polynomial as follows:

$$\mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta) = \frac{\partial}{\partial \mathbf{u}^{\mathcal{A}_1}} \frac{\partial}{\partial \mathbf{v}^{\mathcal{A}_2}} \frac{\partial}{\partial \mathbf{w}^{\mathcal{A}_3}} \mathcal{H}(\mathbf{X}, \Theta; u, v, w). \quad (3.32)$$

Auxiliary spinors are widely used in the construction of correlation functions throughout the literature (see, e.g., [13,16,17,40,52,53]); however, usually the entire correlator is contracted with auxiliary variables and as a result one produces a polynomial depending on all three superspace points and the auxiliary spinors. In contrast, this approach contracts the auxiliary spinors with the tensor $\mathcal{H}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3}(\mathbf{X}, \Theta)$, which depends only on $\mathbf{X}, \Theta$. As a result, it is straightforward to impose constraints on the correlation function as $\mathcal{H}$ does not depend on any of the superspace points explicitly.

The full three-point function may be translated into the auxiliary spinor formalism; recalling that $I = 2s_1$, $J = 2s_2$, $K = 2s_3$, first we define

$$\mathbf{J}_{s_1}(z_1; u) = \mathbf{J}_{\alpha(I)}(z_1) \mathbf{u}^{\alpha(I)}, \quad \mathbf{J}'_{s_2}(z_2; v) = \mathbf{J}_{\beta(J)}(z_2) \mathbf{v}^{\alpha(J)}, \quad (3.33a)$$

$$\mathbf{J}''_{s_3}(z_3; w) = \mathbf{J}_{\gamma(K)}(z_3) \mathbf{w}^{\gamma(K)}. \quad (3.33b)$$

The general ansatz for the three-point function is as follows:

$$\begin{aligned} & \langle \mathbf{J}_{s_1}(z_1; u) \mathbf{J}'_{s_2}(z_2; v) \mathbf{J}''_{s_3}(z_3; w) \rangle \\ &= \frac{\mathcal{I}^{(I)}(\mathbf{x}_{13}; u, \tilde{u}) \mathcal{I}^{(J)}(\mathbf{x}_{23}; v, \tilde{v})}{(\mathbf{x}_{13}^2)^{\Delta_1} (\mathbf{x}_{23}^2)^{\Delta_2}} \mathcal{H}(\mathbf{X}_{12}, \Theta_{12}; \tilde{u}, \tilde{v}, w), \end{aligned} \quad (3.34)$$

where

$$\mathcal{I}^{(s)}(\mathbf{x}; u, \tilde{u}) \equiv \mathcal{I}_x^{(s)}(u, \tilde{u}) = \mathbf{u}^{\alpha(s)} \mathcal{I}_{\alpha(s)}^{\alpha'(s)}(\mathbf{x}) \frac{\partial}{\partial \tilde{u}^{\alpha'(s)}} \quad (3.35)$$

is the inversion operator acting on polynomials degree s in $\tilde{u}$, and $\Delta_i = s_i + 1$. After converting the constraints summarized in the previous subsection into the auxiliary spinor formalism, we obtain the following:

(i) *Homogeneity*:

$$\begin{aligned} & \mathcal{H}(\lambda^2 \mathbf{X}, \lambda \Theta; u(I), v(J), w(K)) \\ &= (\lambda^2)^{\Delta_3 - \Delta_2 - \Delta_1} \mathcal{H}(\mathbf{X}, \Theta; u(I), v(J), w(K)), \end{aligned} \quad (3.36)$$

where we have used the notation $u(I)$, $v(J)$, and $w(K)$ to keep track of the homogeneity of the auxiliary spinors u , v , and w .

(ii) *Differential constraints*: First, define the following three differential operators:

$$D_1 = \mathcal{D}^\alpha \frac{\partial}{\partial u^\alpha}, \quad D_2 = \mathcal{Q}^\alpha \frac{\partial}{\partial v^\alpha}, \quad D_3 = \mathcal{Q}^\alpha \frac{\partial}{\partial w^\alpha}. \quad (3.37)$$

Conservation on all three points may be imposed using the following constraints:

$$\text{Conservation at } z_1: D_1 \mathcal{H}(\mathbf{X}, \Theta; u(I), v(J), w(K)) = 0, \quad (3.38a)$$

$$\text{Conservation at } z_2: D_2 \mathcal{H}(\mathbf{X}, \Theta; u(I), v(J), w(K)) = 0, \quad (3.38b)$$

$$\text{Conservation at } z_3: D_3 \tilde{\mathcal{H}}(\mathbf{X}, \Theta; u(I), v(J), w(K)) = 0, \quad (3.38c)$$

where, in the auxiliary spinor formalism, $\tilde{\mathcal{H}} = \tilde{\mathcal{H}}^{(+)} + \tilde{\mathcal{H}}^{(-)}$ is computed as follows:

$$\begin{aligned} & \tilde{\mathcal{H}}^{(\pm)}(\mathbf{X}, \Theta; u(I), v(J), w(K)) \\ &= \pm (\mathbf{X}^2)^{\Delta_1 - \Delta_3} \mathcal{I}_X^{(J)}(v, \tilde{v}) \\ &\quad \times \mathcal{H}^{I(\pm)}(\mathbf{X}, \Theta; u(I), \tilde{v}(J), w(K)), \end{aligned} \quad (3.39)$$

where $\mathcal{I}_X^{(s)}(v, \tilde{v}) \equiv \mathcal{I}^{(s)}(\mathbf{X}; v, \tilde{v})$.

(iii) *Point-switch symmetries*: If the fields $\mathbf{J}$ and $\mathbf{J}'$ coincide (hence $I = J$), then we obtain the following point-switch constraint

$$\begin{aligned} & \mathcal{H}(\mathbf{X}, \Theta; u(I), v(I), w(K)) \\ &= (-1)^{\epsilon(\mathbf{J})} \mathcal{H}(-\mathbf{X}^T, -\Theta; v(I), u(I), w(K)), \end{aligned} \quad (3.40)$$

where, again, $\epsilon(\mathbf{J})$ is the Grassmann parity of $\mathbf{J}$. Similarly, if the fields $\mathbf{J}$ and $\mathbf{J}''$ coincide (hence $I = K$), then we obtain the constraint

$$\begin{aligned} & \tilde{\mathcal{H}}(\mathbf{X}, \Theta; u(I), v(J), w(I)) \\ &= (-1)^{\epsilon(\mathbf{J})} \mathcal{H}(-\mathbf{X}^T, -\Theta; w(I), v(J), u(I)). \end{aligned} \quad (3.41)$$

To find an explicit solution for the polynomial (3.30), one must now consider all possible scalar combinations of $\mathbf{X}$, Θ , ε , u , v , and w with the appropriate homogeneity. Hence, let us introduce the following structures:

Bosonic:

$$P_1 = \varepsilon_{\alpha\beta} v^\alpha w^\beta, \quad P_2 = \varepsilon_{\alpha\beta} w^\alpha u^\beta, \quad P_3 = \varepsilon_{\alpha\beta} u^\alpha v^\beta, \quad (3.42a)$$

$$\mathbb{Q}_1 = \hat{\mathbf{X}}_{\alpha\beta} v^\alpha w^\beta, \quad \mathbb{Q}_2 = \hat{\mathbf{X}}_{\alpha\beta} w^\alpha u^\beta, \quad \mathbb{Q}_3 = \hat{\mathbf{X}}_{\alpha\beta} u^\alpha v^\beta, \quad (3.42b)$$

$$\mathbb{Z}_1 = \hat{\mathbf{X}}_{\alpha\beta} u^\alpha u^\beta, \quad \mathbb{Z}_2 = \hat{\mathbf{X}}_{\alpha\beta} v^\alpha v^\beta, \quad \mathbb{Z}_3 = \hat{\mathbf{X}}_{\alpha\beta} w^\alpha w^\beta. \quad (3.42c)$$

Fermionic:

$$R_1 = \varepsilon_{\alpha\beta} u^\alpha \hat{\Theta}^\beta, \quad R_2 = \varepsilon_{\alpha\beta} v^\alpha \hat{\Theta}^\beta, \quad R_3 = \varepsilon_{\alpha\beta} w^\alpha \hat{\Theta}^\beta, \quad (3.43a)$$

$$\mathbb{S}_1 = \hat{\mathbf{X}}_{\alpha\beta} u^\alpha \hat{\Theta}^\beta, \quad \mathbb{S}_2 = \hat{\mathbf{X}}_{\alpha\beta} v^\alpha \hat{\Theta}^\beta, \quad \mathbb{S}_3 = \hat{\mathbf{X}}_{\alpha\beta} w^\alpha \hat{\Theta}^\beta. \quad (3.43b)$$

A general solution for $\mathcal{H}(\mathbf{X}, \Theta)$ is composed of all possible combinations of P_i , $\mathbb{Q}_i$, $\mathbb{Z}_i$, R_i , $\mathbb{S}_i$, and $\mathbf{J}$ which possess the

correct homogeneity in u , v , and w . Comparing with (3.14a) and (3.14b), we can identify the objects P_i , S_i , and J as being parity-odd due to their transformation properties under inversions.

For the subsequent analysis of conserved three-point functions, due to the property (2.34), and the fact that in $\mathcal{N} = 1$ theories $\Theta^3 = 0 \Rightarrow X^2 = X^2$, it is generally more convenient to construct the polynomial in terms of the symmetric spin tensor, $X_{\alpha\beta}$, rather than $X_{\alpha\beta}$, resulting in the polynomial $\mathcal{H}(X, \Theta)$. Hence, we expand Q_i , Z_i , S_i as follows:

$$Q_i = Q_i - \frac{i}{2} P_i J, \quad Z_i = Z_i, \quad S_i = S_i, \quad (3.44)$$

where we have defined

$$Q_1 = \hat{X}_{\alpha\beta} v^\alpha w^\beta, \quad Q_2 = \hat{X}_{\alpha\beta} w^\alpha u^\beta, \quad Q_3 = \hat{X}_{\alpha\beta} u^\alpha v^\beta, \quad (3.45a)$$

$$Z_1 = \hat{X}_{\alpha\beta} u^\alpha u^\beta, \quad Z_2 = \hat{X}_{\alpha\beta} v^\alpha v^\beta, \quad Z_3 = \hat{X}_{\alpha\beta} w^\alpha w^\beta, \quad (3.45b)$$

$$S_1 = \hat{X}_{\alpha\beta} u^\alpha \hat{\Theta}^\beta, \quad S_2 = \hat{X}_{\alpha\beta} v^\alpha \hat{\Theta}^\beta, \quad S_3 = \hat{X}_{\alpha\beta} w^\alpha \hat{\Theta}^\beta. \quad (3.45c)$$

The polynomial $\mathcal{H}(X, \Theta)$ is now constructed from all possible combinations of P_i , Q_i , Z_i , R_i , S_i , and J . Once a general solution for $\mathcal{H}(X, \Theta)$ is obtained, one can convert back to ‘‘covariant form,’’ $\mathcal{H}(X, \Theta)$, by making the replacements

$$Q_i \rightarrow Q_i + \frac{i}{2} P_i J, \quad Z_i \rightarrow Z_i, \quad S_i \rightarrow S_i. \quad (3.46)$$

4. Generating function method

In general, it is a nontrivial technical problem to come up with an exhaustive list of possible solutions for $\mathcal{H}(X, \Theta; u, v, w)$ for a given set of superspins; however, this process can be simplified by introducing generating functions for the polynomial $\mathcal{H}(X, \Theta; u, v, w)$. First we introduce the function $\mathcal{F}(X)$, defined as follows:

$$\mathcal{F}(X) = X^\delta P_1^{k_1} P_2^{k_2} P_3^{k_3} Q_1^{l_1} Q_2^{l_2} Q_3^{l_3} Z_1^{m_1} Z_2^{m_2} Z_3^{m_3}, \quad (3.47)$$

where, typically, $\delta = \Delta_3 - \Delta_2 - \Delta_1$. The generating functions for Grassmann-even and Grassmann-odd correlators in $\mathcal{N} = 1$ theories are then defined as follows:

$$\mathcal{G}(X, \Theta | \Gamma) = \begin{cases} \mathcal{F}(X) J^\sigma, & \text{Bosonic,} \\ \mathcal{F}(X) R_1^{p_1} R_2^{p_2} R_3^{p_3} S_1^{q_1} S_2^{q_2} S_3^{q_3}, & \text{Fermionic.} \end{cases} \quad (3.48)$$

Here the non-negative integers, $\Gamma = \{k_i, l_i, m_i, p_i, q_i, \sigma\}$, $i = 1, 2, 3$, are constrained; for overall bosonic correlation functions they are solutions to the following linear system:

$$k_2 + k_3 + l_2 + l_3 + 2m_1 = I, \quad (3.49a)$$

$$k_1 + k_3 + l_1 + l_3 + 2m_2 = J, \quad (3.49b)$$

$$k_1 + k_2 + l_1 + l_2 + 2m_3 = K, \quad (3.49c)$$

with $\sigma = 0, 1$. Likewise, for overall fermionic correlation functions, the integers Γ are solutions to the following system:

$$k_2 + k_3 + l_2 + l_3 + 2m_1 + p_1 + q_1 = I, \quad (3.50a)$$

$$k_1 + k_3 + l_1 + l_3 + 2m_2 + p_2 + q_2 = J, \quad (3.50b)$$

$$k_1 + k_2 + l_1 + l_2 + 2m_3 + p_3 + q_3 = K, \quad (3.50c)$$

$$p_1 + p_2 + p_3 + q_1 + q_2 + q_3 = 1, \quad (3.50d)$$

where $I = 2s_1$, $J = 2s_2$, $K = 2s_3$ specify the spin structure of the correlation function. These equations are obtained by comparing the homogeneity of the auxiliary spinors u , v , w in the generating functions (3.48), against the index structure of the tensor $\mathcal{H}$. The solutions correspond to a linearly dependent basis of structures in which the polynomial $\mathcal{H}$ can be decomposed. Using *Mathematica* it is straightforward to generate all possible solutions to (3.49) and (3.50) for fixed values of the superspins.

Now let us assume there exists a finite number of solutions Γ_i , $i = 1, \dots, N$ to (3.49) and (3.50) for a given choice of I, J, K . The set of solutions $\Gamma = \{\Gamma_i\}$ may be partitioned into even and odd sets Γ^+ and Γ^- , respectively, by counting the number of pseudo-invariant basis structures present in a particular solution. Therefore we define

$$\begin{aligned} \Gamma^+ &= \Gamma |_{k_1+k_2+k_3+q_1+q_2+q_3+\sigma \pmod{2}=0}, \\ \Gamma^- &= \Gamma |_{k_1+k_2+k_3+q_1+q_2+q_3+\sigma \pmod{2}=1}. \end{aligned} \quad (3.51)$$

Hence, the even solutions are those such that $k_1 + k_2 + k_3 + q_1 + q_2 + q_3 + \sigma = \text{even}$ (i.e., contains an even number of parity-odd building blocks), while the odd solutions are those such that $k_1 + k_2 + k_3 + q_1 + q_2 + q_3 + \sigma = \text{odd}$ (contains an odd number of parity-odd building blocks). Let $|\Gamma^+| = N^+$ and $|\Gamma^-| = N^-$, with $N = N^+ + N^-$, and then the most general ansatz for the polynomial $\mathcal{H}$ in (3.30) is as follows:

$$\mathcal{H}(X, \Theta; u, v, w) = \mathcal{H}^{(+)}(X, \Theta; u, v, w) + \mathcal{H}^{(-)}(X, \Theta; u, v, w), \quad (3.52)$$

where

$$\mathcal{H}^{(+)}(X, \Theta; u, v, w) = \sum_{i=1}^{N^+} A_i \mathcal{G}(X, \Theta | \Gamma_i^+), \quad (3.53a)$$

$$\mathcal{H}^{(-)}(X, \Theta; u, v, w) = \sum_{i=1}^{N^-} B_i \mathcal{G}(X, \Theta | \Gamma_i^-), \quad (3.53b)$$

and A_i and B_i are real constants. Using this method one can generate all the possible structures for a given set of superspins (s_1, s_2, s_3) ; however, at this stage we must recall that the solutions generated using this approach are linearly dependent. To form a linearly independent set of solutions we must systematically take into account the following nonlinear relations between the primitive structures:

$$Z_2 Z_3 + P_1^2 - Q_1^2 = 0, \quad (3.54a)$$

$$Z_1 Z_3 + P_2^2 - Q_2^2 = 0, \quad (3.54b)$$

$$Z_1 Z_2 + P_3^2 - Q_3^2 = 0, \quad (3.54c)$$

$$P_1 Z_1 + P_2 Q_3 + P_3 Q_2 = 0, \quad Q_1 Z_1 - Q_2 Q_3 - P_2 P_3 = 0, \quad (3.55a)$$

$$P_2 Z_2 + P_1 Q_3 + P_3 Q_1 = 0, \quad Q_2 Z_2 - Q_1 Q_3 - P_1 P_3 = 0, \quad (3.55b)$$

$$P_3 Z_3 + P_1 Q_2 + P_2 Q_1 = 0, \quad Q_3 Z_3 - Q_1 Q_2 - P_1 P_2 = 0. \quad (3.55c)$$

These allow elimination of the combinations $Z_i Z_j$, $Z_i P_i$, $Z_i Q_i$. There is also another relation involving triple products:

$$P_1 P_2 P_3 + P_1 Q_2 Q_3 + P_2 Q_1 Q_3 + P_3 Q_1 Q_2 = 0, \quad (3.56)$$

which allows elimination of $P_1 P_2 P_3$. The relations above are identical to those appearing in the 3D CFT case [34]; however, they must be supplemented by relations involving the fermionic structures:

$$P_1 R_1 - Q_2 S_2 + Q_3 S_3 = 0, \quad P_1 S_1 - Q_2 R_2 + Q_3 R_3 = 0, \quad (3.57a)$$

$$P_2 R_2 - Q_3 S_3 + Q_1 S_1 = 0, \quad P_2 S_2 - Q_3 R_3 + Q_1 R_1 = 0, \quad (3.57b)$$

$$P_3 R_3 - Q_1 S_1 + Q_2 S_2 = 0, \quad P_3 S_3 - Q_1 R_1 + Q_2 R_2 = 0, \quad (3.57c)$$

$$Z_1 R_2 - Q_3 R_1 + P_3 S_1 = 0, \quad Z_2 R_1 - Q_3 R_2 - P_3 S_2 = 0, \quad (3.58a)$$

$$Z_2 R_3 - Q_1 R_2 + P_1 S_2 = 0, \quad Z_3 R_2 - Q_1 R_3 - P_1 S_3 = 0, \quad (3.58b)$$

$$Z_3 R_1 - Q_2 R_3 + P_2 S_3 = 0, \quad Z_1 R_3 - Q_2 R_1 - P_2 S_1 = 0, \quad (3.58c)$$

$$Z_1 S_2 - Q_3 S_1 + P_3 R_1 = 0, \quad Z_2 S_1 - Q_3 S_2 - P_3 R_2 = 0, \quad (3.59a)$$

$$Z_2 S_3 - Q_1 S_2 + P_1 R_2 = 0, \quad Z_3 S_2 - Q_1 S_3 - P_1 R_3 = 0, \quad (3.59b)$$

$$Z_3 S_1 - Q_2 S_3 + P_2 R_3 = 0, \quad Z_1 S_3 - Q_2 S_1 - P_2 R_1 = 0. \quad (3.59c)$$

These allow for elimination of the products $P_i R_i$, $P_i S_i$, $Z_i R_j$, $Z_i S_j$. As a consequence of (3.57a), the following also hold:

$$P_1 R_1 + P_2 R_2 + P_3 R_3 = 0, \quad (3.60a)$$

$$P_1 S_1 + P_2 S_2 + P_3 S_3 = 0. \quad (3.60b)$$

Applying the above relations to a set of linearly dependent polynomial structures significantly reduces the number of structures to consider for a given three-point function, since we are now restricted to linearly independent contributions. This process is relatively straightforward to implement using *Mathematica*'s pattern matching functions.

Now that we have taken care of linear dependence, it now remains to impose conservation on all three points in addition to the various point-switch symmetries; introducing the objects P_i , Q_i , Z_i , R_i , S_i proves to streamline this analysis significantly. First let us consider conservation; to impose conservation on z_1 (for either sector), we compute

$$D_1 \mathcal{H}(X, \Theta; u, v, w) = D_1 \left\{ \sum_{i=1}^N c_i \mathcal{G}(X, \Theta | \Gamma_i) \right\} = \sum_{i=1}^N c_i D_1 \mathcal{G}(X, \Theta | \Gamma_i). \quad (3.61)$$

We then solve for the coefficient c_i such that the result vanishes. To impose the superfield conservation equations, the identities (B1) are essential. The same approach applies for conservation on z_2 .

Next, to impose conservation on z_3 we must first obtain an explicit expression for $\tilde{\mathcal{H}}(X, \Theta)$ in terms of $\mathcal{H}(X, \Theta)$; that is, we must compute (e.g., for the even sector)

$$\begin{aligned} \tilde{\mathcal{H}}(X, \Theta; u(I), v(J), w(K)) \\ = (X^2)^{\Delta_1 - \Delta_3} \mathcal{I}_X^{(J)}(v, \tilde{v}) \mathcal{H}^I(X, \Theta; u(I), \tilde{v}(J), w(K)). \end{aligned} \quad (3.62)$$

Recall that any solution for $\mathcal{H}(X, \Theta)$ can be written in terms of the structures (3.42) and (3.43); given the transformation properties (3.16) and (3.19), the computation of $\mathcal{H}^I(X, \Theta)$ from $\mathcal{H}(X, \Theta)$ is equivalent to the following replacements:

$$P_1 \rightarrow -P_1, \quad P_2 \rightarrow -P_2, \quad P_3 \rightarrow -P_3, \quad (3.63a)$$

$$R_1 \rightarrow -S_1, \quad R_2 \rightarrow -S_2, \quad R_3 \rightarrow -S_3, \quad (3.63b)$$

$$S_1 \rightarrow R_1, \quad S_2 \rightarrow R_2, \quad S_3 \rightarrow R_3. \quad (3.63c)$$

Now to compute $\tilde{\mathcal{H}}(X, \Theta)$ from $\mathcal{H}^I(X, \Theta)$, we make use of the fact that $P_1, P_3, Q_1, Q_3, Z_2, R_2$, and S_2 are the only objects with $\tilde{v}$ dependence and apply the identities

$$\mathcal{I}_X(v, \tilde{v})P_1 = -Q_1, \quad \mathcal{I}_X(v, \tilde{v})P_3 = Q_3 + iP_3J, \quad (3.64a)$$

$$\mathcal{I}_X(v, \tilde{v})Q_1 = -P_1 + iQ_1J, \quad \mathcal{I}_X(v, \tilde{v})Q_3 = P_3, \quad (3.64b)$$

$$\mathcal{I}_X(v, \tilde{v})R_2 = -S_2, \quad \mathcal{I}_X(v, \tilde{v})S_2 = -R_2, \quad (3.64c)$$

$$\mathcal{I}_X^{(2)}(v, \tilde{v})Z_2 = -Z_2. \quad (3.64d)$$

Hence, given a solution for the polynomial $\mathcal{H}(X, \Theta)$, the computation of $\tilde{\mathcal{H}}(X, \Theta)$ is now equivalent to the following replacements of the basis structures (3.42) and (3.43):

$$P_1 \rightarrow Q_1, \quad P_2 \rightarrow -P_2, \quad P_3 \rightarrow -Q_3 - iP_3J, \quad (3.65a)$$

$$Q_1 \rightarrow -P_1 + iQ_1J, \quad Q_2 \rightarrow Q_2, \quad Q_3 \rightarrow P_3, \quad (3.65b)$$

$$Z_1 \rightarrow Z_1, \quad Z_2 \rightarrow -Z_2, \quad Z_3 \rightarrow Z_3, \quad (3.65c)$$

$$R_1 \rightarrow -S_1, \quad R_2 \rightarrow R_2, \quad R_3 \rightarrow -S_3, \quad (3.65d)$$

$$S_1 \rightarrow R_1, \quad S_2 \rightarrow -S_2, \quad S_3 \rightarrow R_3. \quad (3.65e)$$

These rules are obtained by combining (3.63) and (3.64). Conservation on z_3 can now be imposed using the operator D_3 .

It now remains to find out how point-switch symmetries act on the basis structures; this analysis is simpler when

working with $\mathcal{H}(X, \Theta)$, instead of $\mathcal{H}(X, \Theta)$. For permutation of superspace points z_1 and z_2 , we have $X \rightarrow -X$, $\Theta \rightarrow -\Theta$, $u \leftrightarrow v$. This results in the following replacement rules for the basis objects (3.42) and (3.43):

$$P_1 \rightarrow -P_2, \quad P_2 \rightarrow -P_1, \quad P_3 \rightarrow -P_3, \quad (3.66a)$$

$$Q_1 \rightarrow -Q_2, \quad Q_2 \rightarrow -Q_1, \quad Q_3 \rightarrow -Q_3, \quad (3.66b)$$

$$Z_1 \rightarrow -Z_2, \quad Z_2 \rightarrow -Z_1, \quad Z_3 \rightarrow -Z_3, \quad (3.66c)$$

$$R_1 \rightarrow -R_2, \quad R_2 \rightarrow -R_1, \quad R_3 \rightarrow -R_3, \quad (3.66d)$$

$$S_1 \rightarrow S_2, \quad S_2 \rightarrow S_1, \quad S_3 \rightarrow S_3. \quad (3.66e)$$

Likewise, for permutation of superspace points z_1 and z_3 we have $X \rightarrow -X$, $\Theta \rightarrow -\Theta$, $u \leftrightarrow w$, resulting in the following replacements:

$$P_1 \rightarrow -P_3, \quad P_2 \rightarrow -P_2, \quad P_3 \rightarrow -P_1, \quad (3.67a)$$

$$Q_1 \rightarrow -Q_3, \quad Q_2 \rightarrow -Q_2, \quad Q_3 \rightarrow -Q_1, \quad (3.67b)$$

$$Z_1 \rightarrow -Z_3, \quad Z_2 \rightarrow -Z_2, \quad Z_3 \rightarrow -Z_1, \quad (3.67c)$$

$$R_1 \rightarrow -R_3, \quad R_2 \rightarrow -R_2, \quad R_3 \rightarrow -R_1, \quad (3.67d)$$

$$S_1 \rightarrow S_3, \quad S_2 \rightarrow S_2, \quad S_3 \rightarrow S_1. \quad (3.67e)$$

We have now developed all the formalism necessary to analyze the structure of three-point correlation functions in 3D $\mathcal{N} = 1$ SCFT. To summarize, in the remaining sections of this paper we will analyze the three-point functions of conserved higher-spin supercurrents (for both integer and half-integer superspin) using the following method:

- (1) For a given set of superspins, we construct all possible (linearly dependent) structures for the polynomial $\mathcal{H}(X, \Theta; u, v, w)$, which is governed by the solutions to (3.49) and (3.50). The solutions are sorted into even and odd sectors.
- (2) We systematically apply the linear dependence relations (3.54a), (3.55a), (3.56), (3.57a), (3.58a), (3.59a) to the set of all polynomial structures. This is sufficient to form the most general linearly independent ansatz for the correlation function.
- (3) Using the method outlined in Sec. III B 3, we impose the superfield conservation equations on the correlation function, resulting in the differential constraints (3.38) on $\mathcal{H}$. The result of each computation is a large polynomial in the basis structures (3.42) and (3.43). The linear dependence relations are systematically applied to this polynomial again to ensure that it is composed of only linearly independent terms. The coefficients are read off the structures, resulting in algebraic constraint relations on the

coefficients A_i and B_i . This process significantly reduces the number of structures in the three-point function.

- (4) Once the general form of the polynomial $\mathcal{H}(X, \Theta; u, v, w)$ (associated with the conserved three-point function $\langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle$) is obtained for a given set of superspins (s_1, s_2, s_3) , we then impose any symmetries under permutation of superspace points, that is, (3.40) and (3.41) (if applicable). In certain cases, imposing these constraints can eliminate the remaining structures. The solution is then converted into covariant form $\mathcal{H}(X, \Theta; u, v, w)$.

The computations are done completely analytically with the use of *Mathematica* and the Grassmann package. By using pattern matching functions, the calculations are carried out purely among the basis structures (3.42) and (3.43); as a result we do not have to fix superspace points to certain values. The only chosen parameters are the spins. Due to computational limitations we could carry out the analysis up to $s_i = 20$ (some steps of the calculations involve millions of terms); however, with more optimization and sufficient computational resources this approach should hold for arbitrary superspins. Since there is an enormous number of possible three-point functions with $s_i \leq 20$, we present the final results (in the form of *Mathematica* outputs) for $\mathcal{H}(X, \Theta; u, v, w)$ for some particularly interesting examples, as the solutions and coefficient constraints become cumbersome to present beyond cases involving low superspins. We are primarily interested in counting the number of independent polynomial structures after imposing all the constraints.

IV. THREE-POINT FUNCTIONS OF CONSERVED SUPERCURRENTS

In the next subsections we analyze the structure of three-point correlation functions involving conserved higher-spin supercurrents. As a test of our approach we begin with an analysis of three-point functions involving currents with low superspins, such as the supercurrent and flavor current multiplets.

A. Supercurrent and flavor current correlators

The most important examples of conserved supercurrents in 3D $\mathcal{N} = 1$ superconformal field theories are the supercurrent and flavor current multiplets. The supercurrent multiplet is described by the spin tensor superfield, $J_{\alpha(3)}(z)$, with scale dimension $\Delta_J = 5/2$. It satisfies $D^{\alpha_1} J_{\alpha_1 \alpha_2 \alpha_3}(z) = 0$ and contains the energy-momentum tensor, $T_{\alpha(4)}(x) = D_{(\alpha_1} J_{\alpha_2 \alpha_3 \alpha_4)}(z)|_{\theta=0}$, and the supersymmetry current, $Q_{\alpha(3)}(x) = J_{\alpha(3)}(z)|_{\theta=0}$, as its independent component fields. Likewise, the flavor current multiplet is described by a spinor superfield, $L_\alpha(z)$, with scale dimension $\Delta_L = 3/2$. It satisfies the superfield conservation equation $D^\alpha L_\alpha(z) = 0$, and contains a conserved vector

current $V_{\alpha(2)}(x) = D_{(\alpha_1} L_{\alpha_2)}(z)|_{\theta=0}$. Three-point functions of these supercurrents were originally studied in [41,44] (for analysis of three-point functions of the component currents in 3D/4D CFT see [34,49]), and here we present the solutions for them using our formalism. The possible three-point functions involving the supercurrent and flavor current multiplets are

$$\langle L_\alpha(z_1) L_\beta(z_2) L_\gamma(z_3) \rangle, \quad \langle L_\alpha(z_1) L_\beta(z_2) J_{\gamma(3)}(z_3) \rangle, \quad (4.1)$$

$$\langle J_{\alpha(3)}(z_1) J_{\beta(3)}(z_2) L_\alpha(z_3) \rangle, \quad \langle J_{\alpha(3)}(z_1) J_{\beta(3)}(z_2) J_{\gamma(3)}(z_3) \rangle. \quad (4.2)$$

We note that in all cases the correlation functions are overall Grassmann-odd; hence, it is expected that each of them is fixed up to a single parity-even solution after imposing conservation on all three points. The analysis of these three-point functions is relatively straightforward using our computational approach.

Correlation function $\langle LLL \rangle$:

Let us first consider $\langle LLL \rangle$; within the framework of our formalism we study the three-point function $\langle \mathbf{J}_{1/2} \mathbf{J}'_{1/2} \mathbf{J}''_{1/2} \rangle$. The general ansatz for this correlation function, according to (3.20), is

$$\langle \mathbf{J}_\alpha(z_1) \mathbf{J}'_\beta(z_2) \mathbf{J}''_\gamma(z_3) \rangle = \frac{\mathcal{I}_\alpha^{\alpha'}(\mathbf{x}_{13}) \mathcal{I}_\beta^{\beta'}(\mathbf{x}_{23})}{(\mathbf{x}_{13}^2)^{3/2} (\mathbf{x}_{23}^2)^{3/2}} \mathcal{H}_{\alpha'\beta'\gamma}(X_{12}, \Theta_{12}). \quad (4.3)$$

Using the formalism outlined in Sec. III B, all information about this correlation function is encoded in the following polynomial:

$$\mathcal{H}(X, \Theta; u(1), v(1), w(1)) = \mathcal{H}_{\alpha\beta\gamma}(X, \Theta) u^\alpha v^\beta w^\gamma. \quad (4.4)$$

Using *Mathematica* we solve (3.50) for the chosen spins and substitute each solution into the generating function (3.48). This provides us with the following list of linearly dependent polynomial structures for the polynomial $\mathcal{H}(X, \Theta; u, v, w)$ in the even and odd sectors, respectively:

$$\text{Even: } \{Q_3 R_3, Q_2 R_2, Q_1 R_1, P_3 S_3, P_2 S_2, P_1 S_1\}, \quad (4.5a)$$

$$\text{Odd: } \{Q_3 S_3, Q_2 S_2, Q_1 S_1, P_3 R_3, P_2 R_2, P_1 R_1\}. \quad (4.5b)$$

After systematic application of the linear dependence relations (3.54a)–(3.59a) we obtain the following linearly independent sets:

$$\text{Even: } \{Q_3 R_3, Q_2 R_2, Q_1 R_1\}, \quad (4.6a)$$

$$\text{Odd: } \{Q_3 S_3, Q_2 S_2, Q_1 S_1\}. \quad (4.6b)$$

Next, we impose conservation on all three points, where we obtain the following constraints on the coefficients A_i and B_i :

$$\text{Even: } \left\{ A_1 \rightarrow A_1, A_2 \rightarrow -\frac{A_1}{3}, A_3 \rightarrow -\frac{A_1}{3} \right\}, \quad (4.7a)$$

$$\text{Odd: } \{B_1 \rightarrow 0, B_2 \rightarrow 0, B_3 \rightarrow 0\}, \quad (4.7b)$$

and the explicit solution for $\mathcal{H}(X, \Theta; u, v, w)$ is

$$\text{Even: } \frac{A_1}{X^{3/2}} \left(-\frac{1}{3} Q_1 R_1 - \frac{Q_2 R_2}{3} + Q_3 R_3 \right), \quad (4.8a)$$

$$\text{Odd: } 0. \quad (4.8b)$$

Hence, the three-point function is fixed up to a single parity-even polynomial structure. After imposing symmetries under permutation of spacetime points, e.g., $\mathbf{J} = \mathbf{J}' = \mathbf{J}''$, the remaining structure vanishes. This vanishing result is not surprising because it corresponds to the contribution proportional to the symmetric invariant tensor of the flavor symmetry group. In four dimensions this contribution is related to the chiral anomaly which does not exist in three dimensions. The correlator $\langle \mathbf{J}_{1/2} \mathbf{J}_{1/2} \mathbf{J}_{1/2} \rangle$ has, however, a nonvanishing contribution proportional to the totally antisymmetric structure constants. In our analysis in this paper any possible “antisymmetric” contributions are ignored when we impose the point-switch identities. The most general form of the three-point function of flavor current multiplets was found explicitly in [41,44], and we will not discuss it here.

Correlation function $\langle LLJ \rangle$:

The next example to consider is the mixed correlator $\langle LLJ \rangle$; to study this case we may examine the correlation function $\langle \mathbf{J}_{1/2} \mathbf{J}'_{1/2} \mathbf{J}''_{3/2} \rangle$. Using the general formula, the ansatz for this three-point function is

$$\begin{aligned} \langle \mathbf{J}_\alpha(z_1) \mathbf{J}'_\beta(z_2) \mathbf{J}''_{\gamma(3)}(z_3) \rangle &= \frac{\mathcal{I}_\alpha{}^{\alpha'}(\mathbf{x}_{13}) \mathcal{I}_{\beta'}{}^{\beta''}(\mathbf{x}_{23})}{(\mathbf{x}_{13}^2)^{3/2} (\mathbf{x}_{23}^2)^{3/2}} \\ &\times \mathcal{H}_{\alpha'\beta'\gamma(3)}(\mathbf{X}_{12}, \Theta_{12}). \end{aligned} \quad (4.9)$$

Using the formalism outlined in III B, all information about this correlation function is encoded in the following polynomial:

$$\mathcal{H}(X, \Theta; u(1), v(1), w(3)) = \mathcal{H}_{\alpha\beta\gamma(3)}(X, \Theta) u^\alpha v^\beta w^\gamma(3). \quad (4.10)$$

After solving (3.50), we obtain the following list of polynomial structures for $\mathcal{H}(X, \Theta; u, v, w)$ in the even and odd sectors, respectively:

$$\begin{aligned} \text{Even: } \{ &Q_3 R_3 Z_3, Q_2 R_2 Z_3, Q_1 R_1 Z_3, Q_1 Q_2 R_3, P_3 S_3 Z_3, \\ &P_2 S_2 Z_3, P_2 Q_1 S_3, P_1 S_1 Z_3, P_1 Q_2 S_3, P_1 P_2 R_3 \}, \end{aligned} \quad (4.11a)$$

$$\begin{aligned} \text{Odd: } \{ &Q_3 S_3 Z_3, Q_2 S_2 Z_3, Q_1 S_1 Z_3, Q_1 Q_2 S_3, P_3 R_3 Z_3, \\ &P_2 R_2 Z_3, P_2 Q_1 R_3, P_1 R_1 Z_3, P_1 Q_2 R_3, P_1 P_2 S_3 \}. \end{aligned} \quad (4.11b)$$

After systematic application of the linear dependence relations (3.54a)–(3.59a) we obtain the following linearly independent sets:

$$\text{Even: } \{P_1 P_2 R_3, Q_1 Q_2 R_3, P_1 Q_2 S_3, P_2 Q_1 S_3\}, \quad (4.12a)$$

$$\text{Odd: } \{P_1 P_2 S_3, Q_1 Q_2 S_3, P_1 Q_2 R_3, P_2 Q_1 R_3\}. \quad (4.12b)$$

Next, we impose conservation on all three points; we obtain the following constraints on the coefficients A_i and B_i :

$$\text{Even: } \left\{ A_1 \rightarrow A_1, A_2 \rightarrow \frac{A_1}{5}, A_3 \rightarrow -\frac{A_1}{5}, A_4 \rightarrow \frac{A_1}{5} \right\}, \quad (4.13a)$$

$$\text{Odd: } \{B_1 \rightarrow 0, B_2 \rightarrow 0, B_3 \rightarrow 0, B_4 \rightarrow 0\}, \quad (4.13b)$$

and the explicit solution for $\mathcal{H}(X, \Theta; u, v, w)$

$$\text{Even: } \frac{A_1}{\sqrt{X}} \left(\frac{1}{5} P_2 Q_1 S_3 - \frac{1}{5} P_1 Q_2 S_3 + P_1 P_2 R_3 + \frac{1}{5} Q_1 Q_2 R_3 \right), \quad (4.14a)$$

$$\text{Odd: } 0. \quad (4.14b)$$

Hence, after conservation, the three-point function is fixed up to a single even structure. This structure is also compatible with the symmetry $\mathbf{J} = \mathbf{J}'$; therefore $\langle LLJ \rangle$ is fixed up to a single structure.

Correlation function $\langle JJJ \rangle$:

The next example to consider is the mixed correlator $\langle JJJ \rangle$; to study this case we may examine the correlation function $\langle \mathbf{J}_{3/2} \mathbf{J}'_{3/2} \mathbf{J}''_{1/2} \rangle$. Using the general formula, the ansatz for this three-point function is

$$\begin{aligned} \langle \mathbf{J}_{\alpha(3)}(z_1) \mathbf{J}'_{\beta(3)}(z_2) \mathbf{J}''_{\gamma(3)}(z_3) \rangle &= \frac{\mathcal{I}_{\alpha(3)}{}^{\alpha'(3)}(\mathbf{x}_{13}) \mathcal{I}_{\beta(3)}{}^{\beta'(3)}(\mathbf{x}_{23})}{(\mathbf{x}_{13}^2)^{5/2} (\mathbf{x}_{23}^2)^{5/2}} \\ &\times \mathcal{H}_{\alpha'(3)\beta'(3)\gamma(3)}(\mathbf{X}_{12}, \Theta_{12}). \end{aligned} \quad (4.15)$$

Using the formalism outlined in Sec. III B, all information about this correlation function is encoded in the following polynomial:

$$\mathcal{H}(X, \Theta; u(3), v(3), w(1)) = \mathcal{H}_{\alpha(3)\beta(3)\gamma}(X, \Theta) \mathbf{u}^{\alpha(3)} \mathbf{v}^{\beta(3)} \mathbf{w}^{\gamma}. \quad (4.16)$$

After solving (3.50), we obtain the following list of (linearly dependent) polynomial structures in the even and odd sectors, respectively:

$$\begin{aligned} \textbf{Even: } & \{Q_3 R_3 Z_1 Z_2, Q_3^3 R_3, Q_2 R_2 Z_1 Z_2, Q_2 Q_3 R_1 Z_2, Q_2 Q_3^2 R_2, Q_1 R_1 Z_1 Z_2, \\ & Q_1 Q_3 R_2 Z_1, Q_1 Q_3^2 R_1, P_3 S_3 Z_1 Z_2, P_3 Q_3^2 S_3, P_3 Q_2 S_1 Z_2, P_3 Q_2 Q_3 S_2, \\ & P_3 Q_1 S_2 Z_1, P_3 Q_1 Q_3 S_1, P_3^2 Q_3 R_3, P_3^2 Q_2 R_2, P_3^2 Q_1 R_1, P_3^3 S_3, P_2 S_2 Z_1 Z_2, \\ & P_2 Q_3 S_1 Z_2, P_2 Q_3^2 S_2, P_2 P_3 R_1 Z_2, P_2 P_3 Q_3 R_2, P_2 P_3^2 S_2, P_1 S_1 Z_1 Z_2, \\ & P_1 Q_3 S_2 Z_1, P_1 Q_3^2 S_1, P_1 P_3 R_2 Z_1, P_1 P_3 Q_3 R_1, P_1 P_3^2 S_1\}, \end{aligned} \quad (4.17a)$$

$$\begin{aligned} \textbf{Odd: } & \{Q_3 S_3 Z_1 Z_2, Q_3^3 S_3, Q_2 S_2 Z_1 Z_2, Q_2 Q_3 S_1 Z_2, Q_2 Q_3^2 S_2, Q_1 S_1 Z_1 Z_2, \\ & Q_1 Q_3 S_2 Z_1, Q_1 Q_3^2 S_1, P_3 R_3 Z_1 Z_2, P_3 Q_3^2 R_3, P_3 Q_2 R_1 Z_2, P_3 Q_2 Q_3 R_2, \\ & P_3 Q_1 R_2 Z_1, P_3 Q_1 Q_3 R_1, P_3^2 Q_3 S_3, P_3^2 Q_2 S_2, P_3^2 Q_1 S_1, P_3^3 R_3, P_2 R_2 Z_1 Z_2, \\ & P_2 Q_3 R_1 Z_2, P_2 Q_3^2 R_2, P_2 P_3 S_1 Z_2, P_2 P_3 Q_3 S_2, P_2 P_3^2 R_2, P_1 R_1 Z_1 Z_2, \\ & P_1 Q_3 R_2 Z_1, P_1 Q_3^2 R_1, P_1 P_3 S_2 Z_1, P_1 P_3 Q_3 S_1, P_1 P_3^2 R_1\}. \end{aligned} \quad (4.17b)$$

After systematic application of the linear dependence relations (3.54a)–(3.59a) we obtain the following linearly independent sets:

$$\textbf{Even: } \{Q_2 Q_3^2 R_2, P_3 Q_2 Q_3 S_2, P_3 Q_1 Q_3 S_1, Q_1 Q_3^2 R_1, Q_3^3 R_3, P_3^2 Q_2 R_2, P_3^2 Q_1 R_1\}, \quad (4.18a)$$

$$\textbf{Odd: } \{Q_1 Q_3^2 S_1, Q_3^3 S_3, P_3 Q_1 Q_3 R_1, Q_2 Q_3^2 S_2, P_3 Q_2 Q_3 R_2, P_3^2 Q_2 S_2, P_3^2 Q_1 S_1\}. \quad (4.18b)$$

Next, we impose conservation on all three points; we obtain the following constraints on the coefficients A_i and B_i :

$$\textbf{Even: } \left\{ A_1 \rightarrow A_1, A_2 \rightarrow -A_1, A_3 \rightarrow A_1, A_4 \rightarrow A_1, \right. \\ \left. A_5 \rightarrow -\frac{7A_1}{3}, A_6 \rightarrow -\frac{A_1}{5}, A_7 \rightarrow -\frac{A_1}{5} \right\}, \quad (4.19a)$$

$$\textbf{Odd: } \{B_1 \rightarrow 0, B_2 \rightarrow 0, B_3 \rightarrow 0, B_4 \rightarrow 0, \\ B_5 \rightarrow 0, B_6 \rightarrow 0, B_7 \rightarrow 0\}, \quad (4.19b)$$

and the explicit solution for $\mathcal{H}(X, \Theta; u, v, w)$

$$\textbf{Even: } \frac{A_1}{X^{7/2}} \left(P_3 Q_1 Q_3 S_1 - P_3 Q_2 Q_3 S_2 - \frac{1}{5} P_3^2 Q_1 R_1 \right. \\ \left. - \frac{1}{5} P_3^2 Q_2 R_2 - \frac{7}{3} Q_3^3 R_3 + Q_1 Q_3^2 R_1 + Q_2 Q_3^2 R_2 \right), \quad (4.20a)$$

$$\textbf{Odd: } 0. \quad (4.20b)$$

Hence, after imposing conservation on all three points, the three-point function is fixed up to a single even structure. This structure is not compatible with the symmetry property $\mathbf{J} = \mathbf{J}'$; hence, $\langle JJJ \rangle = 0$.

Correlation function $\langle JJJ \rangle$:

The last example to consider is the three-point function of the supercurrent, $\langle JJJ \rangle$. To study it we may examine the correlation function $\langle \mathbf{J}_{3/2} \mathbf{J}'_{3/2} \mathbf{J}''_{3/2} \rangle$. Using the general formula, the ansatz for this three-point function is

$$\begin{aligned} \langle \mathbf{J}_{\alpha(3)}(z_1) \mathbf{J}'_{\beta(3)}(z_2) \mathbf{J}''_{\gamma(3)}(z_3) \rangle &= \frac{\mathcal{I}_{\alpha(3)}^{\alpha'(3)}(\mathbf{x}_{13}) \mathcal{I}_{\beta(3)}^{\beta'(3)}(\mathbf{x}_{23})}{(\mathbf{x}_{13}^2)^{5/2} (\mathbf{x}_{23}^2)^{5/2}} \\ &\times \mathcal{H}_{\alpha(3)\beta'(3)\gamma(3)}(X_{12}, \Theta_{12}). \end{aligned} \quad (4.21)$$

Using the formalism outlined in Sec. III B, all information about this correlation function is encoded in the following polynomial:

$$\begin{aligned} \mathcal{H}(X, \Theta; u(3), v(3), w(3)) \\ = \mathcal{H}_{\alpha(3)\beta(3)\gamma(3)}(X, \Theta) \mathbf{u}^{\alpha(3)} \mathbf{v}^{\beta(3)} \mathbf{w}^{\gamma(3)}. \end{aligned} \quad (4.22)$$

In this case there are a vast number of linearly dependent structures to consider, and the list is too large to present; however, after application of the linear dependence relations (3.54a)–(3.59a) we obtain the following linearly independent structures:

$$\begin{aligned} \text{Even: } \{ & Q_1 Q_2 Q_3^2 R_3, P_1 Q_2 Q_3^2 S_3, Q_1 Q_2^2 Q_3 R_2, P_1 Q_2^2 Q_3 S_2, Q_1^2 Q_2 Q_3 R_1, P_2 Q_1^2 Q_3 S_1, \\ & P_2 Q_1 Q_3^2 S_3, P_1 P_2 Q_3^2 R_3, P_3 Q_1 Q_2^2 S_2, P_3 Q_1^2 Q_2 S_1, P_1 P_3 Q_2^2 R_2, P_2 P_3 Q_1^2 R_1 \}, \end{aligned} \quad (4.23a)$$

$$\begin{aligned} \text{Odd: } \{ & P_2 Q_1 Q_3^2 R_3, P_1 P_2 Q_3^2 S_3, Q_1^2 Q_2 Q_3 S_1, Q_1 Q_2 Q_3^2 S_3, Q_1 Q_2^2 Q_3 S_2, P_1 Q_2 Q_3^2 R_3, \\ & P_2 Q_1^2 Q_3 R_1, P_1 Q_2^2 Q_3 R_2, P_1 P_3 Q_2^2 S_2, P_3 Q_1 Q_2^2 R_2, P_3 Q_1^2 Q_2 R_1, P_2 P_3 Q_1^2 S_1 \}. \end{aligned} \quad (4.23b)$$

Next, we impose conservation on all three points and obtain the following constraints on the coefficients A_i and B_i :

$$\begin{aligned} \text{Even: } \{ & A_1 \rightarrow A_1, A_2 \rightarrow \frac{A_1}{3}, A_3 \rightarrow -\frac{11A_1}{21}, A_4 \rightarrow \frac{A_1}{3}, \\ & A_5 \rightarrow -\frac{11A_1}{21}, A_6 \rightarrow -\frac{A_1}{3}, A_7 \rightarrow -\frac{A_1}{3}, A_8 \rightarrow -A_1, \\ & A_9 \rightarrow \frac{A_1}{7}, A_{10} \rightarrow -\frac{A_1}{7}, A_{11} \rightarrow \frac{A_1}{21}, A_{12} \rightarrow \frac{A_1}{21} \}, \end{aligned} \quad (4.24a)$$

$$\begin{aligned} \text{Odd: } \{ & B_1 \rightarrow 0, B_2 \rightarrow 0, B_3 \rightarrow 0, B_4 \rightarrow 0, B_5 \rightarrow 0, B_6 \rightarrow 0, \\ & B_7 \rightarrow 0, B_8 \rightarrow 0, B_9 \rightarrow 0, B_{10} \rightarrow 0, B_{11} \rightarrow 0, B_{12} \rightarrow 0 \}, \end{aligned} \quad (4.24b)$$

and the explicit solution for $\mathcal{H}(X, \Theta; u, v, w)$

$$\begin{aligned} \text{Even: } \frac{A_1}{X^{5/2}} \left(& -\frac{1}{7} P_3 Q_2 Q_1^2 S_1 - \frac{1}{3} P_2 Q_3 Q_1^2 S_1 + \frac{1}{7} P_3 Q_2^2 Q_1 S_2 - \frac{1}{3} P_2 Q_3^2 Q_1 S_3 \right. \\ & + \frac{1}{3} P_1 Q_2^2 Q_3 S_2 + \frac{1}{3} P_1 Q_2 Q_3^2 S_3 + \frac{1}{21} P_2 P_3 Q_1^2 R_1 + \frac{1}{21} P_1 P_3 Q_2^2 R_2 \\ & \left. - P_1 P_2 Q_3^2 R_3 - \frac{11}{21} Q_2 Q_3 Q_1^2 R_1 + Q_2 Q_3^2 Q_1 R_3 - \frac{11}{21} Q_2^2 Q_3 Q_1 R_2 \right), \end{aligned} \quad (4.25a)$$

$$\text{Odd: } 0. \quad (4.25b)$$

Hence the three-point function $\langle \mathbf{J}_{3/2} \mathbf{J}'_{3/2} \mathbf{J}''_{3/2} \rangle$ is fixed up to a single parity-even structure. The remaining polynomial structures are also compatible with the symmetry property $\mathbf{J} = \mathbf{J}' = \mathbf{J}''$; hence, the supercurrent three-point function $\langle JJJ \rangle$ is fixed up to a single parity-even structure. In terms of the number of independent structures, these results are consistent with [41].

B. General structure of $\langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle$

We performed a comprehensive analysis of the general structure of the three-point correlation function $\langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle$ using our computational approach. Due to computational limitations we were able to carry out this analysis for $s_i \leq 20$; however, the pattern in the solutions is very clear, and we propose that the results stated in this section hold for arbitrary superspins. We also want to emphasize that for given (s_1, s_2, s_3) our method produces a result which can be presented in an explicit form even for relatively high

superspins (see examples below). With a sufficiently powerful computer one can extend our results to larger values of s_i .

Based on our analysis we found that the general structure of the three-point correlation function $\langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle$ is constrained to the following form:

$$\langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle = a \langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle_E + b \langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle_O. \quad (4.26)$$

One of our main conclusions is that the odd structure, $\langle \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \mathbf{J}''_{s_3} \rangle_O$ does not appear in correlators that are overall Grassmann-odd (or fermionic). The existence of the odd solution in the Grassmann-even (bosonic) correlators depend on the following superspin triangle inequalities:

$$s_1 \leq s_2 + s_3, \quad s_2 \leq s_1 + s_3, \quad s_3 \leq s_1 + s_2. \quad (4.27)$$

When the triangle inequalities are simultaneously satisfied, there is one even solution and one odd solution; however, if any of the above inequalities are not satisfied, then the odd solution is incompatible with current conservation. Further, if any of the $\mathbf{J}$, $\mathbf{J}'$, $\mathbf{J}''$ coincide, then the resulting point-switch symmetries can kill off the remaining structures.

Before we discuss in more detail Grassmann-even and Grassmann-odd correlators and present explicit examples we would like to make some general comments. In particular, we observe that if the triangle inequalities are simultaneously satisfied, each polynomial structure in the three-point functions can be written as a product of at most 5 of the P_i , Q_i , with the Z_i completely eliminated. Another useful observation is that the triangle inequalities can be encoded in a discriminant, σ , which we define as follows:

$$\sigma(s_1, s_2, s_3) = q_1 q_2 q_3, \quad q_i = s_i - s_j - s_k - 1, \quad (4.28)$$

where (i, j, k) is a cyclic permutation of $(1, 2, 3)$. For $\sigma(s_1, s_2, s_3) < 0$, there is one even solution and one odd solution, while for $\sigma(s_1, s_2, s_3) \geq 0$ there is a single even solution. Also recall that the correlation function can be encoded in a tensor $\mathcal{H}$, which is a function of two three-point covariants, X and Θ . There are three different (equivalent) representations of a given three-point function, call them $\mathcal{H}^{(i)}$, where the superscript i denotes which point we set to act as the “third point” in the ansatz (3.6). As shown in Sec. III B 1, the representations are related by the

intertwining operator, $\mathcal{I}$. Since the dimensions of the conserved supercurrents Δ_i are related to the superspins as $\Delta_i = s_i + 1$, it follows that each $\mathcal{H}^{(i)}$ is homogeneous of degree q_i . Then it follows that the odd structure survives if and only if $\forall i, q_i < 0$. In other words, each $\mathcal{H}^{(i)}$ must be a rational function of X and Θ with homogeneity $q_i < 0$. The discriminant (4.28) simply encodes information about whether the $\mathcal{H}^{(i)}$ are simultaneously of negative homogeneity.

1. Grassmann-even correlators

The complete classification of results for Grassmann-even conserved three-point functions, including cases where there is a point-switch symmetry, is as follows:

- (i) In all the cases we have examined ($s_i \leq 20$) there is one even solution and one odd solution; however, the odd solution vanishes if the superspin triangle inequalities are not satisfied.
- (ii) $\langle \mathbf{J}_{s_1} \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \rangle$: Note that in this case s_2 must be an integer. For s_2 even, the solutions survive the point-switch symmetry for arbitrary s_1 (integer or half-integer). For s_2 odd the point-switch symmetry is not satisfied and the three-point function vanishes.
- (iii) $\langle \mathbf{J}_s \mathbf{J}_s \mathbf{J}_s \rangle$: In this case s is restricted to integer values. For s even the solutions are compatible with the point-switch symmetries.

The number of linearly independent structures grows rapidly with the superspins; therefore we only present results for some low superspin cases after imposing conservation on all three points.

Correlation function $\langle \mathbf{J}_1 \mathbf{J}'_1 \mathbf{J}''_1 \rangle$:

$$\text{Even: } \frac{A_1}{X^2} \left(\frac{11}{6} iJP_3 Q_1 Q_2 - \frac{1}{6} iJP_2 Q_1 Q_3 - \frac{1}{6} iJP_1 Q_2 Q_3 + P_1 P_3 Q_2 + \frac{5}{6} iJP_1 P_2 P_3 + P_2 P_3 Q_1 - \frac{1}{3} P_1 P_2 Q_3 + Q_1 Q_2 Q_3 \right), \quad (4.29a)$$

$$\text{Odd: } \frac{B_1}{X^2} (JQ_1 Q_3 Q_2 + iP_3 Q_1 Q_2). \quad (4.29b)$$

Correlation function $\langle \mathbf{J}_{1/2} \mathbf{J}'_{1/2} \mathbf{J}''_{1/2} \rangle$:

$$\text{Even: } A_1 \left(\frac{1}{2} iJP_2 Q_1 Z_3 + \frac{1}{2} iJP_1 Q_2 Z_3 + P_1 P_2 Z_3 - \frac{1}{3} Q_1 Q_2 Z_3 \right), \quad (4.30a)$$

$$\text{Odd: } 0. \quad (4.30b)$$

This three-point function was initially studied in [40], where it was shown that a parity-odd solution could arise. However, it was proven later in [47] that such a structure cannot be consistent with the superfield conservation equations. The approach we have developed also confirms that a parity-odd solution cannot exist; this is further supported by the fact that the superspin triangle inequalities are not satisfied for this three-point function.

Correlation function $\langle \mathbf{J}_{1/2} \mathbf{J}'_{1/2} \mathbf{J}''_3 \rangle$:

$$\text{Even: } XA_1 \left(\frac{1}{2} iJP_2 \mathbb{Q}_1 \mathbb{Z}_3^2 + \frac{1}{2} iJP_1 \mathbb{Q}_2 \mathbb{Z}_3^2 + P_1 P_2 \mathbb{Z}_3^2 - \frac{1}{5} \mathbb{Q}_1 \mathbb{Q}_2 \mathbb{Z}_3^2 \right), \quad (4.31a)$$

$$\text{Odd: } 0. \quad (4.31b)$$

This is another case where the superspin triangle inequalities are not satisfied; hence, the odd structure vanishes as expected.

Correlation function $\langle \mathbf{J}_{1/2} \mathbf{J}'_{3/2} \mathbf{J}''_2 \rangle$:

$$\text{Even: } \frac{A_1}{X} \left(\frac{5}{6} iJP_1^3 \mathbb{Q}_2 - \frac{7}{2} iJP_2 P_1^2 \mathbb{Q}_1 - \frac{9}{2} iJP_1 \mathbb{Q}_1^2 \mathbb{Q}_2 - \frac{3}{2} iJP_2 \mathbb{Q}_1^3 - 5P_1^2 \mathbb{Q}_1 \mathbb{Q}_2 - 3P_2 P_1 \mathbb{Q}_1^2 + \frac{5}{3} P_2 P_1^3 + \mathbb{Q}_1^3 \mathbb{Q}_2 \right), \quad (4.32a)$$

$$\text{Odd: } \frac{B_1}{X} \left(-\frac{3}{2} JP_1^2 \mathbb{Q}_1 \mathbb{Q}_2 - \frac{3}{2} JP_2 P_1^3 + J\mathbb{Q}_1^3 \mathbb{Q}_2 + \frac{3}{2} iP_1^3 \mathbb{Q}_2 + \frac{3}{2} iP_2 P_1^2 \mathbb{Q}_1 - \frac{3}{2} iP_1 \mathbb{Q}_1^2 \mathbb{Q}_2 - \frac{1}{2} iP_2 \mathbb{Q}_1^3 \right). \quad (4.32b)$$

Correlation function $\langle \mathbf{J}_{3/2} \mathbf{J}'_{3/2} \mathbf{J}''_2 \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_1}{X^2} & \left(\frac{1}{2} iJP_3 P_1^2 \mathbb{Q}_2^2 + \frac{11}{5} iJP_2 P_1^2 \mathbb{Q}_2 \mathbb{Q}_3 + \frac{44}{5} iJP_2 P_3 P_1 \mathbb{Q}_1 \mathbb{Q}_2 + \frac{11}{5} iJP_1 \mathbb{Q}_1 \mathbb{Q}_2^2 \mathbb{Q}_3 + \frac{11}{5} iJP_2^2 P_1 \mathbb{Q}_1 \mathbb{Q}_3 + \frac{1}{2} iJP_2^2 P_3 \mathbb{Q}_1^2 \right. \\ & + \frac{77}{10} iJP_3 \mathbb{Q}_1^2 \mathbb{Q}_2^2 + \frac{11}{5} iJP_2 \mathbb{Q}_1^2 \mathbb{Q}_2 \mathbb{Q}_3 + \frac{1}{10} iJP_2^2 P_3 P_1^2 + \frac{1}{5} P_2^2 P_1^2 \mathbb{Q}_3 + P_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3 + 4P_3 P_1 \mathbb{Q}_1 \mathbb{Q}_2^2 \\ & \left. + \frac{8}{5} P_2 P_1 \mathbb{Q}_1 \mathbb{Q}_2 \mathbb{Q}_3 + 4P_2 P_3 \mathbb{Q}_1^2 \mathbb{Q}_2 + P_2^2 \mathbb{Q}_1^2 \mathbb{Q}_3 + \mathbb{Q}_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3 \right), \quad (4.33a) \end{aligned}$$

$$\begin{aligned} \text{Odd: } \frac{B_1}{X^2} & \left(\frac{1}{2} JP_2 P_3 P_1^2 \mathbb{Q}_2 - \frac{1}{2} JP_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3 - \frac{1}{2} JP_3 P_1 \mathbb{Q}_1 \mathbb{Q}_2^2 + \frac{1}{2} JP_2^2 P_3 P_1 \mathbb{Q}_1 - 2JP_2 P_1 \mathbb{Q}_1 \mathbb{Q}_2 \mathbb{Q}_3 - \frac{1}{2} JP_2 P_3 \mathbb{Q}_1^2 \mathbb{Q}_2 \right. \\ & \left. - \frac{1}{2} JP_2^2 \mathbb{Q}_1^2 \mathbb{Q}_3 + J\mathbb{Q}_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3 - \frac{1}{2} iP_3 P_1^2 \mathbb{Q}_2^2 + iP_1 \mathbb{Q}_1 \mathbb{Q}_2^2 \mathbb{Q}_3 - \frac{1}{2} iP_2^2 P_3 \mathbb{Q}_1^2 + 2iP_3 \mathbb{Q}_1^2 \mathbb{Q}_2^2 + iP_2 \mathbb{Q}_1^2 \mathbb{Q}_2 \mathbb{Q}_3 \right). \quad (4.33b) \end{aligned}$$

Correlation function $\langle \mathbf{J}_2 \mathbf{J}'_2 \mathbf{J}''_2 \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_1}{X^3} & \left(\frac{23}{35} iJP_1 P_2^2 \mathbb{Q}_1 \mathbb{Q}_3^2 + iJP_1 P_3^2 P_2^2 \mathbb{Q}_1 - \frac{5}{7} iJP_3 P_2^2 \mathbb{Q}_1^2 \mathbb{Q}_3 + \frac{3}{35} iJP_1^2 P_3 P_2^2 \mathbb{Q}_3 + \frac{23}{35} iJP_1^2 P_2 \mathbb{Q}_2 \mathbb{Q}_3^2 + \frac{27}{35} iJP_2 \mathbb{Q}_1^2 \mathbb{Q}_2 \mathbb{Q}_3^2 \right. \\ & + iJP_1^2 P_3^2 P_2 \mathbb{Q}_2 + \frac{3}{7} iJP_3^2 P_2 \mathbb{Q}_1^2 \mathbb{Q}_2 + \frac{6}{7} iJP_1 P_3 P_2 \mathbb{Q}_1 \mathbb{Q}_2 \mathbb{Q}_3 + \frac{3}{7} iJP_1 P_3^2 \mathbb{Q}_1 \mathbb{Q}_2^2 + \frac{27}{35} iJP_1 \mathbb{Q}_1 \mathbb{Q}_2^2 \mathbb{Q}_3^2 \\ & + \frac{307}{35} iJP_3 \mathbb{Q}_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3 - \frac{5}{7} iJP_2^2 P_3 \mathbb{Q}_2^2 \mathbb{Q}_3 + P_3^2 P_2^2 \mathbb{Q}_1^2 + \frac{3}{35} P_1^2 P_2^2 \mathbb{Q}_3^2 + \frac{2}{7} P_2^2 \mathbb{Q}_1^2 \mathbb{Q}_3^2 + \frac{2}{7} P_1 P_2 \mathbb{Q}_1 \mathbb{Q}_2 \mathbb{Q}_3^2 + \frac{2}{7} P_3 P_2 \mathbb{Q}_1^2 \mathbb{Q}_2 \mathbb{Q}_3 \\ & \left. + P_1^2 P_3^2 \mathbb{Q}_2^2 - \frac{30}{7} P_3^2 \mathbb{Q}_1^2 \mathbb{Q}_2^2 + \frac{2}{7} P_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3^2 + \frac{2}{7} P_1 P_3 \mathbb{Q}_1 \mathbb{Q}_2^2 \mathbb{Q}_3 + \mathbb{Q}_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3^2 \right), \quad (4.34a) \end{aligned}$$

$$\text{Odd: } \frac{B_1}{X^3} \left(-\frac{5}{2} JP_3^2 \mathbb{Q}_1^2 \mathbb{Q}_2^2 - 2JP_1 P_2 P_3^2 \mathbb{Q}_1 \mathbb{Q}_2 + J\mathbb{Q}_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3^2 + iP_1 P_3^2 \mathbb{Q}_1 \mathbb{Q}_2^2 + iP_2 P_3^2 \mathbb{Q}_1^2 \mathbb{Q}_2 + iP_3 \mathbb{Q}_1^2 \mathbb{Q}_2^2 \mathbb{Q}_3 \right). \quad (4.34b)$$

This three-point function has been studied explicitly using a tensor formalism in [47], where it was shown that a parity-odd solution could arise in the three-point function. The approach we have developed can compute this correlator in seconds.

Correlation function $\langle \mathbf{J}_1 \mathbf{J}_2 \mathbf{J}_4'' \rangle$:

$$\begin{aligned} \text{Even: } A_5 & \left(-\frac{3}{8} iJP_2 P_1^4 Q_2 Z_3 - \frac{45}{28} iJP_1^3 Q_1 Q_2^2 Z_3 - \frac{25}{28} iJP_2^2 P_1^3 Q_1 Z_3 - \frac{51}{28} iJP_2 P_1^2 Q_1^2 Q_2 Z_3 - \frac{17}{28} iJP_2^2 P_1 Q_1^3 Z_3 \right. \\ & + \frac{5}{4} iJP_1 Q_1^3 Q_2^2 Z_3 + \frac{5}{8} iJP_2 Q_1^4 Q_2 Z_3 - \frac{33}{56} P_1^4 Q_2^2 Z_3 - \frac{15}{7} P_2 P_1^3 Q_1 Q_2 Z_3 - \frac{5}{4} P_2^2 P_1^2 Q_1^2 Z_3 \\ & \left. + \frac{27}{28} P_1^2 Q_1^2 Q_2^2 Z_3 + P_2 P_1 Q_1^3 Q_2 Z_3 + \frac{9}{56} P_2^2 Q_1^4 Z_3 + \frac{45}{56} P_2^2 P_1^4 Z_3 - \frac{5}{56} Q_1^4 Q_2^2 Z_3 \right), \end{aligned} \quad (4.35a)$$

$$\text{Odd: } 0. \quad (4.35b)$$

In this case we note that the superspin triangle inequalities are not satisfied, and therefore the odd solution vanishes after current conservation.

Correlation function $\langle \mathbf{J}_2 \mathbf{J}_2 \mathbf{J}_4'' \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_1}{X} & \left(\frac{18}{5} iJP_2 P_1^4 Q_2^3 - \frac{94}{7} iJP_2^3 P_1^4 Q_2 + \frac{90}{7} iJP_1^3 Q_1 Q_2^4 + \frac{180}{7} iJP_2^2 P_1^3 Q_1 Q_2^2 - \frac{94}{7} iJP_2^4 P_1^3 Q_1 + \frac{204}{7} iJP_2 P_1^2 Q_1^2 Q_2^3 \right. \\ & + \frac{180}{7} iJP_2^3 P_1^2 Q_1^2 Q_2 - 14iJP_1 Q_1^3 Q_2^4 + \frac{18}{5} iJP_2^4 P_1 Q_1^3 + \frac{204}{7} iJP_2^2 P_1 Q_1^3 Q_2^2 - 14iJP_2 Q_1^4 Q_2^3 + \frac{90}{7} iJP_2^3 Q_1^4 Q_2 \\ & + \frac{99}{35} P_1^4 Q_2^4 - \frac{54}{7} P_2^2 P_1^4 Q_2^2 + \frac{144}{7} P_2 P_1^3 Q_1 Q_2^3 - 16P_2^3 P_1^3 Q_1 Q_2 - \frac{54}{7} P_1^2 Q_1^2 Q_2^4 - \frac{54}{7} P_2^4 P_1^2 Q_1^2 \\ & \left. + 36P_2^2 P_1^2 Q_1^2 Q_2^2 - 16P_2 P_1 Q_1^3 Q_2^3 + \frac{144}{7} P_2^3 P_1 Q_1^3 Q_2 + \frac{99}{35} P_2^4 Q_1^4 - \frac{54}{7} P_2^2 Q_1^4 Q_2^2 + P_2^4 P_1^4 + Q_1^4 Q_2^4 \right), \end{aligned} \quad (4.36a)$$

$$\begin{aligned} \text{Odd: } \frac{B_1}{X} & \left(\frac{5}{6} JP_1^4 Q_2^4 + \frac{7}{2} JP_2^2 P_1^4 Q_2^2 + \frac{4}{3} JP_2 P_1^3 Q_1 Q_2^3 + 8JP_2^3 P_1^3 Q_1 Q_2 - \frac{9}{2} JP_1^2 Q_1^2 Q_2^4 + \frac{7}{2} JP_2^4 P_1^2 Q_1^2 - 8JP_2 P_1 Q_1^3 Q_2^3 \right. \\ & + \frac{4}{3} JP_2^3 P_1 Q_1^3 Q_2 + \frac{5}{6} JP_2^4 Q_1^4 - \frac{9}{2} JP_2^2 Q_1^4 Q_2^2 - JP_2^4 P_1^4 + JQ_1^4 Q_2^4 - \frac{5}{3} iP_2 P_1^4 Q_2^3 + iP_2^3 P_1^4 Q_2 + \frac{5}{3} iP_1^3 Q_1 Q_2^4 - 6iP_2^2 P_1^3 Q_1 Q_2^2 \\ & \left. + iP_2^4 P_1^3 Q_1 + 6iP_2 P_1^2 Q_1^2 Q_2^3 - 6iP_2^3 P_1^2 Q_1^2 Q_2 - iP_1 Q_1^3 Q_2^4 - \frac{5}{3} iP_2^4 P_1 Q_1^3 + 6iP_2^2 P_1 Q_1^3 Q_2^2 - iP_2 Q_1^4 Q_2^3 + \frac{5}{3} iP_2^3 Q_1^4 Q_2 \right). \end{aligned} \quad (4.36b)$$

Correlation function $\langle \mathbf{J}_4 \mathbf{J}_4 \mathbf{J}_4'' \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_1}{X^5} & \left(P_3^4 Q_2^4 P_1^4 + \frac{7P_2^4 Q_3^4 P_1^4}{1287} + \frac{6}{65} Q_2^4 Q_3^4 P_1^4 + \frac{284}{715} iJP_2 Q_2^3 Q_3^4 P_1^4 + \frac{4}{143} P_2^2 Q_2^2 Q_3^4 P_1^4 + \frac{86iJP_2^3 Q_2 Q_3^4 P_1^4}{1287} \right. \\ & + 2iJP_2 P_3^4 Q_2^3 P_1^4 - \frac{68}{195} iJP_3 Q_2^4 Q_3^3 P_1^4 + \frac{8}{143} iJP_2^2 P_3 Q_2^2 Q_3^3 P_1^4 + \frac{14iJP_2^4 P_3 Q_3^3 P_1^4}{1287} + \frac{4}{15} P_3^2 Q_2^4 Q_3^3 P_1^4 \\ & + \frac{8}{15} iJP_2 P_3^2 Q_2^3 Q_3^2 P_1^4 - \frac{26}{15} iJP_3^3 Q_2^4 Q_3 P_1^4 + \frac{2}{5} iJP_3^4 Q_1 Q_2^4 P_1^3 + \frac{44}{39} iJQ_1 Q_2^4 Q_3^4 P_1^3 + \frac{112}{715} P_2 Q_1 Q_2^3 Q_3^3 P_1^3 \\ & + \frac{4214iJP_2^2 Q_1 Q_2^2 Q_3^4 P_1^3}{6435} + \frac{86iJP_2^4 Q_1 Q_3^4 P_1^3}{1287} + \frac{4}{143} P_2^3 Q_1 Q_2 Q_3^4 P_1^3 - \frac{16}{195} P_3 Q_1 Q_2^4 Q_3^3 P_1^3 + \frac{64}{429} iJP_2 P_3 Q_1 Q_2^3 Q_3^3 P_1^3 \\ & + \frac{8}{143} iJP_2^3 P_3 Q_1 Q_2 Q_3^3 P_1^3 + \frac{62}{39} iJP_3^2 Q_1 Q_2^4 Q_3^3 P_1^3 + \frac{4}{15} P_3^3 Q_1 Q_2^4 Q_3 P_1^3 + \frac{8}{15} iJP_2 P_3^3 Q_1 Q_2^3 Q_3 P_1^3 - \frac{84}{5} P_3^4 Q_1^2 Q_2^4 P_1^2 \\ & + \frac{1748Q_1^2 Q_2^4 Q_3^4 P_1^2}{2145} + \frac{31006iJP_2 Q_1^2 Q_2^3 Q_3^4 P_1^2}{6435} + \frac{4}{143} P_2^4 Q_1^2 Q_3^4 P_1^2 + \frac{158}{715} P_2^2 Q_1^2 Q_2^2 Q_3^4 P_1^2 + \frac{4214iJP_2^3 Q_1^2 Q_2 Q_3^4 P_1^2}{6435} \\ & \left. - \frac{168}{5} iJP_2 P_3^4 Q_1^2 Q_2^3 P_1^2 - \frac{1322}{143} iJP_3 Q_1^2 Q_2^4 Q_3^3 P_1^2 + \frac{8}{143} iJP_2^4 P_3 Q_1^2 Q_3^3 P_1^2 + \frac{316}{715} iJP_2^2 P_3 Q_1^2 Q_2^2 Q_3^3 P_1^2 + \frac{974}{195} P_3^2 Q_1^2 Q_2^4 Q_3^3 P_1^2 \right) \end{aligned}$$

$$\begin{aligned}
& + \frac{1948}{195} iJP_2 P_3^2 Q_1^2 Q_2^2 Q_3^2 P_1^2 + \frac{1598}{39} iJP_3^3 Q_1^2 Q_2^4 Q_3 P_1^2 - \frac{588}{65} iJP_3^4 Q_1^3 Q_2^4 P_1 + \frac{5230iJP_1^3 Q_1^3 Q_2^4 Q_3 P_1}{1287} + \frac{284}{715} iJP_2^4 Q_1^3 Q_3^4 P_1 \\
& + \frac{2924P_2 P_3^3 Q_1^3 Q_2^4 P_1}{2145} + \frac{31006iJP_2^2 Q_1^3 Q_2^2 Q_3^4 P_1}{6435} + \frac{112}{715} P_2^3 Q_1^2 Q_2 Q_3^4 P_1 + 2iJP_2^4 P_3^4 Q_1^3 P_1 - \frac{7828P_3 Q_1^3 Q_2^4 Q_3^3 P_1}{2145} \\
& - \frac{8488}{715} iJP_2 P_3 Q_1^3 Q_2^2 Q_3^3 P_1 + \frac{64}{429} iJP_2^3 P_3 Q_1^3 Q_2 Q_3^3 P_1 - \frac{168}{5} iJP_2^2 P_3^4 Q_1^3 Q_2^2 P_1 - \frac{62}{429} iJP_3^3 Q_1^3 Q_2^4 Q_3^2 P_1 \\
& + \frac{8}{15} iJP_2^4 P_3^2 Q_1^3 Q_3^2 P_1 + \frac{1948}{195} iJP_2^2 P_3^2 Q_1^3 Q_2^2 Q_3^2 P_1 + \frac{336}{65} P_3^3 Q_1^3 Q_2^4 Q_3 P_1 + \frac{1344}{65} iJP_2 P_3^3 Q_1^3 Q_2^2 Q_3 P_1 \\
& + \frac{8}{15} iJP_2^3 P_3^3 Q_1^3 Q_2 Q_3 P_1 + P_2^4 P_3^4 Q_1^4 + \frac{462}{13} P_3^4 Q_1^4 Q_2^4 + \frac{6}{65} P_2^4 Q_1^4 Q_3^4 + Q_1^4 Q_2^4 Q_3^4 + \frac{5230iJP_2 Q_1^4 Q_2^3 Q_3^4}{1287} \\
& + \frac{1748P_2^2 Q_1^4 Q_2^2 Q_3^4}{2145} + \frac{44}{39} iJP_2^3 Q_1^4 Q_2 Q_3^4 - \frac{588}{65} iJP_2 P_3^4 Q_1^4 Q_2^3 - \frac{68}{195} iJP_2^4 P_3 Q_1^4 Q_3^3 + \frac{46414iJP_3 Q_1^4 Q_2^4 Q_3^3}{1287} \\
& - \frac{7828P_2 P_3 Q_1^4 Q_3^3 Q_3^3}{2145} - \frac{1322}{143} iJP_2^2 P_3 Q_1^4 Q_2^2 Q_3^3 - \frac{16}{195} P_2^3 P_3 Q_1^4 Q_2 Q_3^3 - \frac{84}{5} P_2^2 P_3^4 Q_1^4 Q_2^2 + \frac{4}{15} P_2^4 P_3^2 Q_1^4 Q_2^2 \\
& - \frac{15204}{715} P_3^2 Q_1^4 Q_2^2 Q_3^2 - \frac{62}{429} iJP_2 P_3^2 Q_1^4 Q_2^2 Q_3^2 + \frac{974}{195} P_2^2 P_3^2 Q_1^4 Q_2^2 Q_3^2 + \frac{62}{39} iJP_2^3 P_3^2 Q_1^4 Q_2 Q_3^2 + \frac{2}{5} iJP_2^3 P_3^4 Q_1^4 Q_2 \\
& - \frac{26}{15} iJP_2^4 P_3^3 Q_1^4 Q_3 - \frac{6636}{65} iJP_3^3 Q_1^4 Q_2^2 Q_3 + \frac{336}{65} P_2 P_3^3 Q_1^4 Q_2^2 Q_3 + \frac{1598}{39} iJP_2^2 P_3^3 Q_1^4 Q_2^2 Q_3 + \frac{4}{15} P_2^3 P_3^3 Q_1^4 Q_2 Q_3 \Big), \quad (4.37a)
\end{aligned}$$

$$\begin{aligned}
\text{Odd: } \frac{B_1}{X^5} & \left(\frac{35}{2} JP_3^4 Q_1^4 Q_2^4 - \frac{1}{2} JP_1^2 P_3^4 Q_1^2 Q_2^4 + 20JP_1 P_2 P_3^4 Q_1^3 Q_2^3 - 2JP_1^2 P_2 P_3^4 Q_1 Q_2^3 - \frac{1}{2} JP_2^2 P_3^4 Q_1^4 Q_2^2 - 2JP_1 P_2^3 P_3^4 Q_1^3 Q_2 \right. \\
& - \frac{16}{3} JP_1 P_3^3 Q_1^3 Q_2^4 Q_3 + 2JP_1^3 P_3^3 Q_1 Q_2^4 Q_3 - \frac{16}{3} JP_2 P_3^3 Q_1^4 Q_2^2 Q_3 + 2JP_2^3 P_3^3 Q_1^4 Q_2 Q_3 - \frac{27}{2} JP_3^2 Q_1^4 Q_2^4 Q_3^2 \\
& - 2JP_1^2 P_3^2 Q_1^2 Q_2^4 Q_3^2 - \frac{40}{3} JP_1 P_2 P_3^2 Q_1^3 Q_2^2 Q_3^2 - 2JP_2^2 P_3^2 Q_1^4 Q_2^2 Q_3^2 + JP_1^4 Q_1^4 Q_2^4 Q_3^4 - 5iP_1 P_3^4 Q_1^3 Q_2^4 + iP_1^3 P_3^4 Q_1 Q_2^4 \\
& \left. - 5iP_2 P_3^4 Q_1^4 Q_2^3 + iP_2^3 P_3^4 Q_1^4 Q_2 - \frac{5}{3} iP_3^3 Q_1^4 Q_2^4 Q_3 + \frac{10}{3} iP_1 P_3^2 Q_1^3 Q_2^4 Q_3^2 + \frac{10}{3} iP_2 P_3^2 Q_1^4 Q_2^3 Q_3^2 + iP_3 Q_1^4 Q_2^4 Q_3^3 \right). \quad (4.37b)
\end{aligned}$$

2. Grassmann-odd correlators

The classification of results for Grassmann-odd three-point functions, including cases where there is a point-switch symmetry, is as follows:

- (i) In all the cases we have examined ($s_i \leq 20$), the three-point functions are fixed up to a single parity-even solution after conservation on all three points. In general, any parity-odd structures are incompatible with conservation.
- (ii) $\langle \mathbf{J}_{s_1} \mathbf{J}_{s_1} \mathbf{J}'_{s_2} \rangle$: Note that in this case s_2 must be half-integer. For $s_1 \neq s_2$, the classification is as follows:
 Let $s_2 = 2k + \frac{1}{2}$, $k \in \mathbb{Z}_{\geq 0}$; for arbitrary s_1 (integer or half-integer) the point-switch symmetry is not satisfied, and therefore the three-point function vanishes in general.
 Let $s_2 = 2k + \frac{3}{2}$, $k \in \mathbb{Z}_{\geq 0}$; for arbitrary s_1 (integer or half-integer) the point-switch symmetry is satisfied, and therefore the three-point function is fixed up to a single parity-even structure.
- (iii) $\langle \mathbf{J}_s \mathbf{J}_s \mathbf{J}_s \rangle$: For $s = 2k + \frac{3}{2}$, $k \in \mathbb{Z}_{\geq 0}$ the solution is compatible with the point-switch symmetry. For $s = 2k + \frac{1}{2}$, $k \in \mathbb{Z}_{\geq 0}$ the three-point function vanishes.

We now present results after imposing conservation on all three points.

Correlation function $\langle \mathbf{J}_{1/2} \mathbf{J}'_{3/2} \mathbf{J}''_{5/2} \rangle$:

$$\text{Even: } \frac{A_1}{\sqrt{X}} \left(\frac{1}{3} P_1^3 Q_2 S_3 - P_2 P_1^2 Q_1 S_3 - \frac{3}{7} P_1 Q_1^2 Q_2 S_3 + \frac{1}{7} P_2 Q_1^3 S_3 - P_1^2 Q_1 Q_2 R_3 + P_2 P_1 Q_1^2 R_3 - 3P_2 P_1^3 R_3 + \frac{1}{7} Q_1^3 Q_2 R_3 \right), \quad (4.38a)$$

$$\text{Odd: } 0. \quad (4.38b)$$

Correlation function $\langle \mathbf{J}_2 \mathbf{J}_2' \mathbf{J}_{1/2}'' \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_1}{X^{9/2}} & \left(-\frac{2}{3} P_3 Q_1 Q_3^2 S_1 + \frac{2}{3} P_3 Q_2 Q_3^2 S_2 + \frac{1}{21} P_3^3 Q_1 S_1 - \frac{1}{21} P_3^3 Q_2 S_2 \right. \\ & \left. + \frac{4}{21} P_3^2 Q_1 Q_3 R_1 + \frac{4}{21} P_3^2 Q_2 Q_3 R_2 + Q_3^4 R_3 - \frac{4}{9} Q_1 Q_3^3 R_1 - \frac{4}{9} Q_2 Q_3^3 R_2 \right), \end{aligned} \quad (4.39a)$$

$$\text{Odd: } 0. \quad (4.39b)$$

In this instance we note that the superspin triangle inequalities are not satisfied, and therefore the odd solution vanishes after current conservation.

Correlation function $\langle \mathbf{J}_2 \mathbf{J}_2' \mathbf{J}_{3/2}'' \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_1}{X^{7/2}} & \left(-\frac{4}{17} P_2 Q_1 Q_3^3 S_3 + \frac{4}{17} P_1 Q_2 Q_3^3 S_3 - \frac{6}{17} P_2 Q_1^2 Q_3^2 S_1 + \frac{6}{17} P_1 Q_2^2 Q_3^2 S_2 - \frac{14}{51} P_3 Q_1^2 Q_2 Q_3 S_1 + \frac{14}{51} P_3 Q_1 Q_2^2 Q_3 S_2 \right. \\ & + \frac{1}{51} P_2 P_3^2 Q_1^2 S_1 - \frac{1}{51} P_1 P_3^2 Q_2^2 S_2 - \frac{11}{17} P_1 P_2 Q_3^3 R_3 + \frac{4}{51} P_2 P_3 Q_1^2 Q_3 R_1 + \frac{4}{51} P_1 P_3 Q_2^2 Q_3 R_2 + \frac{5}{119} P_3^2 Q_1 Q_2^2 R_2 \\ & \left. + \frac{5}{119} P_3^2 Q_1^2 Q_2 R_1 + Q_1 Q_2 Q_3^3 R_3 - \frac{26}{51} Q_1 Q_2^2 Q_3^2 R_2 - \frac{26}{51} Q_1^2 Q_2 Q_3^2 R_1 \right), \end{aligned} \quad (4.40a)$$

$$\text{Odd: } 0. \quad (4.40b)$$

Correlation function $\langle \mathbf{J}_{3/2} \mathbf{J}_{3/2}' \mathbf{J}_{5/2}'' \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_1}{X^{3/2}} & \left(-\frac{1}{3} P_2 Q_2 Q_1^3 S_1 + \frac{1}{3} P_1 Q_2^3 Q_1 S_2 + \frac{3}{5} P_1 P_2^2 Q_3 Q_1 S_3 - \frac{3}{5} P_1^2 P_2 Q_2 Q_3 S_3 + \frac{1}{15} P_2^2 Q_1^3 R_1 - \frac{3}{5} P_2^2 Q_3 Q_1^2 R_3 \right. \\ & \left. - \frac{2}{5} P_1 P_2 Q_2 Q_3 Q_1 R_3 + \frac{1}{15} P_1^2 Q_2^3 R_2 - \frac{3}{5} P_1^2 Q_2^2 Q_3 R_3 + \frac{11}{5} P_1^2 P_2^2 Q_3 R_3 - \frac{10}{21} Q_2^2 Q_1^3 R_1 - \frac{10}{21} Q_2^3 Q_1^2 R_2 + Q_2^2 Q_3 Q_1^2 R_3 \right), \end{aligned} \quad (4.41a)$$

$$\text{Odd: } 0. \quad (4.41b)$$

Correlation function $\langle \mathbf{J}_{3/2} \mathbf{J}_{3/2}' \mathbf{J}_{7/2}'' \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_2}{\sqrt{X}} & \left(\frac{1}{9} P_1^3 Q_2^3 S_3 - \frac{11}{9} P_2^2 P_1^3 Q_2 S_3 - P_2 P_1^2 Q_1 Q_2^2 S_3 + \frac{11}{9} P_2^2 P_1^2 Q_1 S_3 - \frac{5}{27} P_1 Q_1^2 Q_2^3 S_3 + P_2^2 P_1 Q_1^2 Q_2 S_3 \right. \\ & - \frac{1}{9} P_2^3 Q_1^3 S_3 + \frac{5}{27} P_2 Q_1^3 Q_2^2 S_3 - \frac{11}{9} P_2 P_1^3 Q_2^2 R_3 - \frac{1}{3} P_1^2 Q_1 Q_2^3 R_3 + \frac{11}{3} P_2^2 P_1^2 Q_1 Q_2 R_3 - \frac{11}{9} P_2^2 P_1 Q_1^2 R_3 + P_2 P_1 Q_1^2 Q_2^2 R_3 \\ & \left. - \frac{1}{3} P_2^2 Q_1^3 Q_2 R_3 + \frac{143}{27} P_2^3 P_1^3 R_3 + \frac{5}{81} Q_1^3 Q_2^3 R_3 \right), \end{aligned} \quad (4.42a)$$

$$\text{Odd: } 0. \quad (4.42b)$$

Correlation function $\langle \mathbf{J}_2 \mathbf{J}_2' \mathbf{J}_{7/2}'' \rangle$:

$$\begin{aligned} \text{Even: } \frac{A_1}{X^{3/2}} & \left(\frac{33}{805} P_2^3 Q_1^4 S_1 - \frac{9}{23} P_2 Q_2^2 Q_1^4 S_1 + \frac{132}{805} P_2^3 Q_3 Q_1^3 S_3 - \frac{4}{23} P_2 Q_2^2 Q_3 Q_1^3 S_3 + \frac{9}{23} P_1 Q_2^4 Q_1^2 S_2 + \frac{4}{23} P_1 Q_2^3 Q_3 Q_1^2 S_3 \right. \\ & \left. - \frac{44}{161} P_1 P_2^2 Q_2 Q_3 Q_1^2 S_3 - \frac{572}{805} P_1^2 P_2^3 Q_3 Q_1 S_3 + \frac{44}{161} P_1^2 P_2 Q_2^2 Q_3 Q_1 S_3 - \frac{33}{805} P_1^3 Q_2^4 S_2 - \frac{132}{805} P_1^3 Q_2^3 Q_3 S_3 \right) \end{aligned}$$

$$\begin{aligned}
& + \frac{572}{805} P_1^3 P_2^2 Q_2 Q_3 S_3 + \frac{3}{23} P_2^2 Q_2 Q_1^4 R_1 - \frac{66}{161} P_2^2 Q_2 Q_3 Q_1^3 R_3 - \frac{209}{161} P_1 P_2 Q_2^2 Q_3 Q_1^2 R_3 + \frac{858}{805} P_1 P_2^3 Q_3 Q_1^2 R_3 \\
& + \frac{3}{23} P_1^2 Q_2^4 Q_1 R_2 - \frac{66}{161} P_1^2 Q_2^3 Q_3 Q_1 R_3 - \frac{143}{805} P_1^2 P_2^2 Q_2 Q_3 Q_1 R_3 + \frac{858}{805} P_1^3 P_2 Q_2^2 Q_3 R_3 - \frac{429}{161} P_1^3 P_2^3 Q_3 R_3 - \frac{35}{69} Q_2^3 Q_1^4 R_1 \\
& - \frac{35}{69} Q_2^4 Q_1^3 R_2 + Q_2^2 Q_3 Q_1^3 R_3 \Big), \tag{4.43a}
\end{aligned}$$

$$\text{Odd: } 0. \tag{4.43b}$$

Correlation function $\langle \mathbf{J}_{5/2} \mathbf{J}'_{5/2} \mathbf{J}''_{5/2} \rangle$:

$$\begin{aligned}
\text{Even: } & \frac{A_1}{X^{7/2}} \left(-\frac{13}{33} P_2 Q_2 Q_3^2 Q_1^3 S_1 + \frac{5}{363} P_2 P_3^2 Q_2 Q_1^3 S_1 - \frac{65}{363} P_3 Q_2^2 Q_3 Q_1^3 S_1 + \frac{1}{33} P_2^2 P_3 Q_3 Q_1^3 S_1 + \frac{65}{363} P_3 Q_2^3 Q_3 Q_1^2 S_2 \right. \\
& - \frac{1}{11} P_2 Q_2 Q_3^3 Q_1^2 S_3 - \frac{5}{363} P_1 P_3^2 Q_2^3 Q_1 S_2 + \frac{13}{33} P_1 Q_2^3 Q_2^3 Q_1 S_2 + \frac{13}{99} P_1 P_2^2 Q_3^3 Q_1 S_3 + \frac{1}{11} P_1 Q_2^2 Q_3^3 Q_1 S_3 \\
& - \frac{1}{33} P_1^2 P_3 Q_2^3 Q_3 S_2 - \frac{13}{99} P_1^2 P_2 Q_2 Q_3^3 S_3 + \frac{70 P_3^2 Q_2^2 Q_1^3 R_1}{3267} + \frac{2}{33} P_2^2 Q_2^3 Q_1^3 R_1 - \frac{1}{363} P_2^2 P_3^3 Q_1^3 R_1 + \frac{28}{363} P_2 P_3 Q_2 Q_3 Q_1^3 R_1 \\
& + \frac{70 P_3^2 Q_2^3 Q_1^2 R_2}{3267} - \frac{26}{99} P_2^2 Q_3^3 Q_1^2 R_3 - \frac{52}{99} P_1 P_2 Q_2 Q_3^3 Q_1 R_3 + \frac{28}{363} P_1 P_3 Q_2^3 Q_3 Q_1 R_2 - \frac{1}{363} P_1^2 P_3^3 Q_2 R_2 \\
& \left. - \frac{26}{99} P_1^2 Q_2^2 Q_3^3 R_3 + \frac{13}{33} P_1^2 P_2^2 Q_3^3 R_3 + \frac{2}{33} P_1^2 Q_2^3 Q_2^3 R_2 - \frac{181}{363} Q_2^2 Q_3^3 Q_1^3 R_1 + Q_2^2 Q_3^3 Q_1^2 R_3 - \frac{181}{363} Q_2^3 Q_3^2 Q_1^2 R_2 \right), \tag{4.44a}
\end{aligned}$$

$$\text{Odd: } 0. \tag{4.44b}$$

Correlation function $\langle \mathbf{J}_{7/2} \mathbf{J}'_{7/2} \mathbf{J}''_{7/2} \rangle$:

$$\begin{aligned}
\text{Even: } & \frac{A_1}{X^{9/2}} \left(-\frac{3499 R_1 Q_2^3 Q_3^3 Q_1^4}{6997} + \frac{705 P_2^2 R_1 Q_2 Q_3^3 Q_1^4}{6997} - \frac{189 P_2 P_3^3 R_1 Q_2^2 Q_1^4}{90961} + \frac{647 P_2 P_3 R_1 Q_2^2 Q_3^2 Q_1^4}{6997} - \frac{45 P_2^3 P_3 R_1 Q_3^2 Q_1^4}{6997} \right. \\
& + \frac{P_2^3 P_3 R_1 Q_1^4}{6997} + \frac{2961 P_3^2 R_1 Q_2^3 Q_3 Q_1^4}{90961} - \frac{53 P_2^2 P_3^2 R_1 Q_2 Q_3 Q_1^4}{6997} + \frac{315 P_3^3 Q_2^3 S_1 Q_1^4}{90961} + \frac{255 P_2^3 Q_3^3 S_1 Q_1^4}{6997} \\
& - \frac{2985 P_2 Q_2^2 Q_3^3 S_1 Q_1^4}{6997} - \frac{1393 P_3 Q_2^3 Q_3^2 S_1 Q_1^4}{6997} + \frac{375 P_2^2 P_3 Q_2 Q_3^2 S_1 Q_1^4}{6997} - \frac{7 P_2^2 P_3^3 Q_2 S_1 Q_1^4}{6997} - \frac{15 P_2^3 P_3^2 Q_3 S_1 Q_1^4}{6997} \\
& + \frac{175 P_2 P_3^2 Q_2^3 Q_3 S_1 Q_1^4}{6997} + R_3 Q_2^3 Q_3^4 Q_1^3 - \frac{2091 P_2^2 R_3 Q_2 Q_3^4 Q_1^3}{6997} - \frac{3499 R_2 Q_2^4 Q_3^3 Q_1^3}{6997} + \frac{2961 P_3^2 R_2 Q_2^4 Q_3 Q_1^3}{90961} - \frac{315 P_3^3 Q_2^4 S_2 Q_1^3}{90961} \\
& + \frac{1393 P_3 Q_2^4 Q_3^2 S_2 Q_1^3}{6997} + \frac{255 P_2^3 Q_4^3 S_3 Q_1^3}{6997} - \frac{521 P_2 Q_2^2 Q_4^3 S_3 Q_1^3}{6997} - \frac{189 P_1 P_3^3 R_2 Q_2^4 Q_1^2}{90961} - \frac{3859 P_1 P_2 R_3 Q_2^2 Q_4^3 Q_1^2}{6997} \\
& + \frac{969 P_1 P_2^3 R_3 Q_3^4 Q_1^2}{6997} + \frac{647 P_1 P_3 R_2 Q_2^4 Q_3^2 Q_1^2}{6997} + \frac{2985 P_1 Q_2^4 Q_3^3 S_2 Q_1^2}{6997} - \frac{175 P_1 P_3^2 Q_2^4 Q_3 S_2 Q_1^2}{6997} + \frac{521 P_1 Q_2^3 Q_3^4 S_3 Q_1^2}{6997} \\
& + \frac{221 P_1 P_2^2 Q_2 Q_3^4 S_3 Q_1^2}{6997} - \frac{2091 P_1^2 R_3 Q_2^3 Q_3^4 Q_1}{6997} + \frac{1615 P_1^2 P_2^2 R_3 Q_2 Q_3^4 Q_1}{6997} + \frac{705 P_1^2 R_2 Q_2^4 Q_3^3 Q_1}{6997} - \frac{53 P_1^2 P_3^2 R_2 Q_2^4 Q_3 Q_1}{6997} \\
& + \frac{7 P_1^2 P_3^3 Q_2^4 S_2 Q_1}{6997} - \frac{375 P_1^2 P_3 Q_2^4 Q_3^2 S_2 Q_1}{6997} - \frac{323 P_1^2 P_3^2 Q_4^3 S_3 Q_1}{6997} - \frac{221 P_1^2 P_2 Q_2^2 Q_4^3 S_3 Q_1}{6997} + \frac{P_1^3 P_3^2 R_2 Q_2^4}{6997} \\
& + \frac{969 P_1^3 P_2 R_3 Q_2^2 Q_4^3}{6997} - \frac{969 P_1^3 P_3^2 R_3 Q_3^4}{6997} - \frac{45 P_1^3 P_3 R_2 Q_2^4 Q_3^2}{6997} - \frac{255 P_1^3 Q_2^4 Q_3^3 S_2}{6997} + \frac{15 P_1^3 P_2^2 Q_2^4 Q_3 S_2}{6997} \\
& \left. - \frac{255 P_1^3 Q_2^3 Q_4^3 S_3}{6997} + \frac{323 P_1^2 P_2^2 Q_2 Q_4^3 S_3}{6997} \right), \tag{4.45a}
\end{aligned}$$

$$\text{Odd: } 0. \tag{4.45b}$$

V. THREE-POINT FUNCTIONS OF SCALAR SUPERFIELDS

For completeness, in this section we analyze three-point correlation functions involving scalar superfields and conserved supercurrents. Some of the three-point functions contain parity-odd solutions, with their existence depending on both triangle inequalities and the weights of the scalars. We found that the following general results hold:

$$\langle \mathcal{O}\mathcal{O}'\mathbf{J}_s \rangle = a \langle \mathcal{O}\mathcal{O}'\mathbf{J}_s \rangle_E, \quad (5.1a)$$

$$\langle \mathbf{J}_{s_1}\mathbf{J}'_{s_2}\mathcal{O} \rangle = a \langle \mathbf{J}_{s_1}\mathbf{J}'_{s_2}\mathcal{O} \rangle_E + b \langle \mathbf{J}_{s_1}\mathbf{J}'_{s_2}\mathcal{O} \rangle_O. \quad (5.1b)$$

The correlation functions are analyzed using the same methods as in the previous sections; the full classification of results (for cases where there is a point-switch symmetry), is summarised below:

- (i) $\langle \mathcal{O}\mathcal{O}'\mathbf{J}_s \rangle$: In general there are solutions only for $\Delta_{\mathcal{O}} = \Delta_{\mathcal{O}'}$. For the Grassmann-even case the solution satisfies the point-switch symmetry $\mathcal{O} = \mathcal{O}'$ only for even s . For the Grassmann-odd case the solution satisfies the point-switch symmetry only for $s = 2k + \frac{3}{2}$, $k \in \mathbb{Z}_{\geq 0}$.
- (ii) $\langle \mathbf{J}_{s_1}\mathbf{J}'_{s_2}\mathcal{O} \rangle$: For $s_1 \neq s_2$, there is a single even solution for $\Delta_{\mathcal{O}} = 1$; otherwise the three-point function vanishes. For $s_1 = s_2$ there is one even and one odd solution and the point-switch symmetries are satisfied.

We now present explicit solutions for the above cases.

Correlation function $\langle \mathcal{O}\mathcal{O}'\mathbf{J}_{1/2} \rangle$:

For $\delta_1 = \delta_2 = \delta$, there is a single even solution compatible with conservation:

$$\text{Even: } X^{\frac{3}{2}-2\delta} A_1 R_3, \quad (5.2a)$$

$$\text{Odd: } 0. \quad (5.2b)$$

Correlation function $\langle \mathcal{O}\mathcal{O}'\mathbf{J}_1 \rangle$:

For $\delta_1 = \delta_2 = \delta$, there is a single even solution compatible with conservation:

$$\text{Even: } X^{2-2\delta} A_1 \mathbb{Z}_3, \quad (5.3a)$$

$$\text{Odd: } 0. \quad (5.3b)$$

Correlation function $\langle \mathcal{O}\mathcal{O}'\mathbf{J}_{3/2} \rangle$:

For $\delta_1 = \delta_2 = \delta$, there is a single even solution compatible with conservation:

$$\text{Even: } X^{\frac{5}{2}-2\delta} A_1 R_3 \mathbb{Z}_3, \quad (5.4a)$$

$$\text{Odd: } 0. \quad (5.4b)$$

Correlation function $\langle \mathcal{O}\mathcal{O}'\mathbf{J}_2 \rangle$:

For $\delta_1 = \delta_2 = \delta$, there is a single even solution compatible with conservation:

$$\text{Even: } X^{3-2\delta} A_1 \mathbb{Z}_3^2, \quad (5.5a)$$

$$\text{Odd: } 0. \quad (5.5b)$$

Correlation function $\langle \mathbf{J}_{1/2}\mathbf{J}'_{1/2}\mathcal{O} \rangle$:

In this case, the superspin triangle inequalities are satisfied, and there is one even and one odd solution for arbitrary δ :

$$\text{Even: } X^{-3+\delta} A_1 \left(\mathbb{Q}_3 + \frac{1}{2} i \delta J P_3 \right), \quad (5.6a)$$

$$\text{Odd: } X^{-3+\delta} B_2 \left(\frac{1}{2} i \delta J \mathbb{Q}_3 - \frac{3}{2} i J \mathbb{Q}_3 + P_3 \right). \quad (5.6b)$$

Correlation function $\langle \mathbf{J}_{1/2}\mathbf{J}'_{3/2}\mathcal{O} \rangle$:

In this case there is a solution only for $\delta = 1$:

$$\text{Even: } \frac{A_1}{X^3} \left(\mathbb{Q}_3 + \frac{1}{2} i J P_3 \right) \mathbb{Z}_2, \quad (5.7a)$$

$$\text{Odd: } 0. \quad (5.7b)$$

Correlation function $\langle \mathbf{J}_{3/2}\mathbf{J}'_{3/2}\mathcal{O} \rangle$:

In this case, the superspin triangle inequalities are satisfied and there is one even and one odd solution for arbitrary δ :

$$\begin{aligned} \text{Even: } X^{-5+\delta} A_1 \left(\frac{i \delta (\delta + 2) J P_3^3}{2(\delta - 6)} + \frac{3}{2} i \delta J P_3 \mathbb{Q}_3^2 \right. \\ \left. + \frac{3 \delta P_3^2 \mathbb{Q}_3}{\delta - 6} + \mathbb{Q}_3^3 \right), \end{aligned} \quad (5.8a)$$

$$\begin{aligned} \text{Odd: } \frac{1}{6} X^{-5+\delta} B_2 \left(3i(\delta + 1) J P_3^2 \mathbb{Q}_3 + i(\delta - 7) J \mathbb{Q}_3^3 \right. \\ \left. + \frac{2(\delta + 1) P_3^3}{\delta - 5} + 6 P_3 \mathbb{Q}_3^2 \right). \end{aligned} \quad (5.8b)$$

Correlation function $\langle \mathbf{J}_1\mathbf{J}'_2\mathcal{O} \rangle$:

In this case there is a solution only for $\delta = 1$:

$$\text{Even: } \frac{A_1}{X^4} (-5i J P_3 \mathbb{Q}_3 + P_3^2 - 5 \mathbb{Q}_3^2) \mathbb{Z}_2, \quad (5.9a)$$

$$\text{Odd: } 0. \quad (5.9b)$$

Correlation function $\langle \mathbf{J}_2\mathbf{J}'_2\mathcal{O} \rangle$:

In this case, the superspin triangle inequalities are satisfied, and there is one even and one odd solution for arbitrary δ :

$$\text{Even: } X^{-6+\delta} A_1 \left(\frac{2i\delta(\delta+2)JP_3^3Q_3}{\delta-8} + 2i\delta JP_3Q_3^3 + \frac{\delta(\delta+2)P_3^4}{(\delta-8)(\delta-6)} + \frac{6\delta P_3^2Q_3^2}{\delta-8} + Q_3^4 \right), \quad (5.10a)$$

$$\text{Odd: } X^{-6+\delta} B_2 \left(\frac{i(\delta+1)(\delta+3)JP_3^4}{8(\delta-7)} + \frac{3}{4}i(\delta+1)JP_3^2Q_3^2 + \frac{1}{8}i(\delta-9)JQ_3^4 + \frac{(\delta+1)P_3^3Q_3}{\delta-7} + P_3Q_3^3 \right). \quad (5.10b)$$

VI. CONCLUSION

The purpose of this paper was to develop a formalism to determine the general structure of three-point correlation functions of conserved supercurrents for arbitrary superspins in three-dimensional superconformal field theory. Our method produces explicit results up to $s_i = 20$ and is limited only by computer power. We found that the main difference in the general structure of the three-point function $\langle \mathbf{J}_{s_1} \mathbf{J}_{s_2} \mathbf{J}_{s_3}'' \rangle$ is whether it is Grassmann-odd or Grassmann-even in superspace. If $\langle \mathbf{J}_{s_1} \mathbf{J}_{s_2} \mathbf{J}_{s_3}'' \rangle$ is Grassmann-odd (that is, the sum of the superspins is half-integer), then the correlator is fixed up to a single parity-even contribution. If $\langle \mathbf{J}_{s_1} \mathbf{J}_{s_2} \mathbf{J}_{s_3}'' \rangle$ is Grassmann-even (that is, the sum of the superspins is an integer), then it is fixed up to one even solution and one odd solution; the existence of the latter, however, depends on whether the triangle inequalities are satisfied. The pattern of the number of independent structures is clear, and we have sufficient evidence to propose that our classification of results holds in general.

There are various possible directions to extend our results. An open question is whether it is possible to find generating functions for arbitrary superspins that encapsulate the results in this paper, similar to the ones found in nonsupersymmetric theories [16,17,58,59]. It would also be interesting to apply our methods to superconformal theories in higher dimensions (see [45,46,50] for recent progress) and to $\mathcal{N}$ -extended superconformal theories. Correlation functions of higher-spin currents in conformal theories with extended supersymmetry have practically not been studied; however, recent progress has been reported in [48]. An important difference compared to the $\mathcal{N} = 1$ case is that conserved currents can carry indices of the R -symmetry group. Concerning the study of three-point functions in four dimensions, in [50] a method was introduced to study three-point functions of conserved supercurrents $J_{\alpha(r)\dot{\alpha}(r)}$ for arbitrary superspins in 4D $\mathcal{N} = 1$ superconformal field theories. Explicit solutions were constructed for three-point functions involving higher-spin supercurrents and flavor current multiplets. The method of [50] was an extension of the one used in [46] where the classification problem was solved for generic three-point functions of conserved fermionic currents $S_{\alpha(k)}$ of arbitrary rank in 4D $\mathcal{N} = 1$ SCFT. We believe that the formalism developed in the present paper will generalize directly to 4D $\mathcal{N} = 1$ theories and will allow us

to extend the results of [50]. We leave these considerations for a future study.

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APPENDIX A: 3D CONVENTIONS AND NOTATION

For the Minkowski metric we use the “mostly plus” convention: $\eta_{mn} = \text{diag}(-1, 1, 1)$. Spinor indices are then raised and lowered with the $SL(2, \mathbb{R})$ invariant antisymmetric ε -tensor

$$\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon_{\alpha\gamma} \varepsilon^{\gamma\beta} = \delta_{\alpha}^{\beta}, \quad (A1a)$$

$$\phi_{\alpha} = \varepsilon_{\alpha\beta} \phi^{\beta}, \quad \phi^{\alpha} = \varepsilon^{\alpha\beta} \phi_{\beta}. \quad (A1b)$$

The γ -matrices are chosen to be real and are expressed in terms of the Pauli matrices, σ , as follows:

$$(\gamma_0)_{\alpha}^{\beta} = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad (\gamma_1)_{\alpha}^{\beta} = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (A2a)$$

$$(\gamma_2)_{\alpha}^{\beta} = -\sigma_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad (A2b)$$

$$(\gamma_m)_{\alpha\beta} = \varepsilon_{\beta\delta} (\gamma_m)_{\alpha}^{\delta}, \quad (\gamma_m)^{\alpha\beta} = \varepsilon^{\alpha\delta} (\gamma_m)_{\delta}^{\beta}. \quad (A2c)$$

The γ -matrices are traceless and symmetric,

$$(\gamma_m)^{\alpha}_{\alpha} = 0, \quad (\gamma_m)_{\alpha\beta} = (\gamma_m)_{\beta\alpha}, \quad (A3)$$

and also satisfy the Clifford algebra

$$\gamma_m \gamma_n + \gamma_n \gamma_m = 2\eta_{mn}. \quad (\text{A4})$$

For products of γ -matrices we make use of the identities

$$(\gamma_m)_\alpha{}^\rho (\gamma_n)_\rho{}^\beta = \eta_{mn} \delta_\alpha{}^\beta + \epsilon_{mnp} (\gamma^p)_\alpha{}^\beta, \quad (\text{A5a})$$

$$\begin{aligned} (\gamma_m)_\alpha{}^\rho (\gamma_n)_\rho{}^\sigma (\gamma_p)_\sigma{}^\beta &= \eta_{mn} (\gamma_p)_\alpha{}^\beta - \eta_{mp} (\gamma_n)_\alpha{}^\beta \\ &\quad + \eta_{np} (\gamma_m)_\alpha{}^\beta + \epsilon_{mnp} \delta_\alpha{}^\beta, \end{aligned} \quad (\text{A5b})$$

where we have introduced the 3D Levi-Civita tensor ϵ with $\epsilon^{012} = -\epsilon_{012} = 1$. We also have the orthogonality and completeness relations for the γ -matrices

$$(\gamma^m)_{\alpha\beta} (\gamma_m)^{\rho\sigma} = -\delta_\alpha{}^\rho \delta_\beta{}^\sigma - \delta_\alpha{}^\sigma \delta_\beta{}^\rho, \quad (\gamma_m)_{\alpha\beta} (\gamma_n)^{\alpha\beta} = -2\eta_{mn}. \quad (\text{A6})$$

The γ -matrices are used to swap from vector indices to spinor indices. For example, given some three-vector x_m , it may equivalently be expressed in terms of a symmetric second-rank spinor $x_{\alpha\beta}$ as follows:

$$x_{\alpha\beta} = (\gamma^m)_{\alpha\beta} x_m, \quad x_m = -\frac{1}{2} (\gamma_m)^{\alpha\beta} x_{\alpha\beta}, \quad (\text{A7a})$$

$$\det(x_{\alpha\beta}) = \frac{1}{2} x^{\alpha\beta} x_{\alpha\beta} = -x^m x_m = -x^2. \quad (\text{A7b})$$

The same conventions are also adopted for the spacetime partial derivatives ∂_m ,

$$\partial_{\alpha\beta} = (\gamma^m)_{\alpha\beta} \partial_m, \quad \partial_m = -\frac{1}{2} (\gamma_m)^{\alpha\beta} \partial_{\alpha\beta}, \quad (\text{A8a})$$

$$\partial_m x^n = \delta_m^n, \quad \partial_{\alpha\beta} x^{\rho\sigma} = -\delta_\alpha{}^\rho \delta_\beta{}^\sigma - \delta_\alpha{}^\sigma \delta_\beta{}^\rho, \quad (\text{A8b})$$

$$\xi^m \partial_m = -\frac{1}{2} \xi^{\alpha\beta} \partial_{\alpha\beta}. \quad (\text{A9})$$

We also define the supersymmetry generators Q_α ,

$$Q_\alpha = i \frac{\partial}{\partial \theta^\alpha} + (\gamma^m)_{\alpha\beta} \theta^\beta \frac{\partial}{\partial x^m}, \quad (\text{A10})$$

and the covariant spinor derivatives

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i (\gamma^m)_{\alpha\beta} \theta^\beta \frac{\partial}{\partial x^m}, \quad (\text{A11})$$

which anticommute with the supersymmetry generators, $\{Q_\alpha, D_\beta\} = 0$, and obey the standard anticommutation relations

$$\{D_\alpha, D_\beta\} = 2i (\gamma^m)_{\alpha\beta} \partial_m. \quad (\text{A12})$$

APPENDIX B: CONSERVATION IDENTITIES

For imposing superfield conservation equations on three-point correlation functions, the following identities are essential:

$$\mathcal{D}^\alpha Q_1 = \frac{i}{X^{1/2}} \left\{ v^\alpha R_3 + w^\alpha R_2 - Q_1 (\hat{X} \cdot \hat{\Theta})^\alpha \right\}, \quad (\text{B1a})$$

$$\mathcal{D}^\alpha Q_2 = \frac{i}{X^{1/2}} \left\{ u^\alpha R_3 + w^\alpha R_1 - Q_2 (\hat{X} \cdot \hat{\Theta})^\alpha \right\}, \quad (\text{B1b})$$

$$\mathcal{D}^\alpha Q_3 = \frac{i}{X^{1/2}} \left\{ u^\alpha R_2 + v^\alpha R_1 - Q_3 (\hat{X} \cdot \hat{\Theta})^\alpha \right\}, \quad (\text{B1c})$$

$$\mathcal{D}^\alpha Z_1 = \frac{i}{X^{1/2}} \left\{ 2u^\alpha R_1 - Z_1 (\hat{X} \cdot \hat{\Theta})^\alpha \right\}, \quad (\text{B1d})$$

$$\mathcal{D}^\alpha Z_2 = \frac{i}{X^{1/2}} \left\{ 2v^\alpha R_2 - Z_2 (\hat{X} \cdot \hat{\Theta})^\alpha \right\}, \quad (\text{B1e})$$

$$\mathcal{D}^\alpha Z_3 = \frac{i}{X^{1/2}} \left\{ 2w^\alpha R_3 - Z_3 (\hat{X} \cdot \hat{\Theta})^\alpha \right\}, \quad (\text{B1f})$$

$$\mathcal{D}^\alpha R_1 = \frac{1}{X^{1/2}} \left\{ -u^\alpha - \frac{i}{4} (\hat{X} \cdot u)^\alpha \mathbf{J} \right\}, \quad (\text{B1g})$$

$$\mathcal{D}^\alpha R_2 = \frac{1}{X^{1/2}} \left\{ -v^\alpha - \frac{i}{4} (\hat{X} \cdot v)^\alpha \mathbf{J} \right\}, \quad (\text{B1h})$$

$$\mathcal{D}^\alpha R_3 = \frac{1}{X^{1/2}} \left\{ -w^\alpha - \frac{i}{4} (\hat{X} \cdot w)^\alpha \mathbf{J} \right\}, \quad (\text{B1i})$$

$$\mathcal{D}^\alpha S_1 = \frac{1}{X^{1/2}} \left\{ (\hat{X} \cdot u)^\alpha - \frac{3i}{4} u^\alpha \mathbf{J} \right\}, \quad (\text{B1j})$$

$$\mathcal{D}^\alpha S_2 = \frac{1}{X^{1/2}} \left\{ (\hat{X} \cdot v)^\alpha - \frac{3i}{4} v^\alpha \mathbf{J} \right\}, \quad (\text{B1k})$$

$$\mathcal{D}^\alpha S_3 = \frac{1}{X^{1/2}} \left\{ (\hat{X} \cdot w)^\alpha - \frac{3i}{4} w^\alpha \mathbf{J} \right\}. \quad (\text{B1l})$$

Similar relations hold for the action of Q^α on the basis structures.

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