

k-essence induced by derivative couplings of the inflaton

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We consider two models which couple derivatives of the inflaton to ordinary matter, both to fermions and to scalars. Such couplings induce changes to the inflaton kinetic energy, analogous to the cosmological Coleman-Weinberg potentials which come from nonderivative couplings. Our purpose is to investigate whether these quantum-induced *k*-essence models can provide efficient reheating without affecting the observational constraints on primordial inflation. Our numerical studies show that it is difficult to preserve both properties.

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I. INTRODUCTION

Single scalar inflation is the simplest model consistent with current data:

$$\mathcal{L} = \frac{R\sqrt{-g}}{16\pi G} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}. \quad (1)$$

Given a desired expansion history $a(t)$, one can construct a scalar potential and an initial condition which will support it [1–3]. This is important because the observational constraints on inflation can be phrased in terms of the expansion history $a(t)$ and its derivatives:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x} \Rightarrow H(t) \equiv \frac{\dot{a}}{a}, \quad \epsilon(t) \equiv -\frac{\dot{H}}{H^2}. \quad (2)$$

These constraints are, first, that the number of *e*-foldings from the start of inflation at t_i to its end [when $\epsilon(t_e) = 1$] should be large enough to explain the horizon problem:

$$N \equiv \ln \left[\frac{a(t_e)}{a(t_i)} \right] \gtrsim 60. \quad (3)$$

Additional constraints come from the slow-roll approximations for the scalar and tensor power spectra,

$$\Delta_{\mathcal{R}}^2(k) \simeq \frac{GH^2(t_k)}{\pi\epsilon(t_k)}, \quad \Delta_h^2(k) \simeq \frac{16}{\pi}GH^2(t_k), \quad (4)$$

where t_k is the time of the first horizon crossing at which $k = a(t_k)H(t_k)$. The observed scalar perturbations experience first crossing over a period of about ten *e*-foldings,

starting about 50 *e*-foldings before the end of inflation. Near the beginning of this period, the value of $\Delta_{\mathcal{R}}^2(k)$ must be about 2×10^{-9} , and the scalar spectral index n_s must obey [4]

$$1 - n_s \equiv -\frac{\partial \ln(\Delta_{\mathcal{R}}^2)}{\partial \ln(k)} \simeq 2\epsilon + \frac{\dot{\epsilon}}{H\epsilon} \simeq 0.035. \quad (5)$$

Finally, the nondetection of primordial tensors implies [4]

$$r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} \simeq 16\epsilon \lesssim 0.036. \quad (6)$$

The observational constraints of Eqs. (3)–(6) result in very flat potentials, whose minimum is infinitesimally close to zero, and with initial conditions which seem unnaturally fine-tuned to some people. However, our concern here is the *additional* constraints which arise from coupling the inflaton to ordinary matter in order to facilitate reheating. The simplest couplings involve the undifferentiated inflaton—for example, a Yukawa coupling $\varphi\bar{\Psi}\Psi$ to fermions. What happens then is that vacuum fluctuations of ordinary matter induce Coleman-Weinberg potentials that are neither Planck-suppressed nor limited to local functionals of metric which could be completely eliminated by local counterterms [5]. Because they are not Planck-suppressed, these cosmological Coleman-Weinberg potentials typically cause dramatic changes in the inflationary expansion history, which endangers the observational constraints [Eqs. (3)–(6)].

Although cosmological Coleman-Weinberg potentials cannot be completely eliminated by allowed counterterms, two partial subtraction schemes are possible:

- (1) *Hubble subtraction*, in which a local function of the inflaton is used to nullify quantum effects at the onset of inflation [6].

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- (2) *Ricci subtraction*, in which a local function of the inflaton and the Ricci scalar are used to nullify quantum effects for $\epsilon = 0$ [7].

Neither technique gives good results. Employing Hubble subtraction [6] shows that, with a moderate coupling constant, inflation never ends for fermionic couplings of effective potential, and it ends too soon for vector boson couplings. Making the coupling constants very small results in acceptable inflation at the price of inefficient reheating. Ricci subtraction gives even worse results [7]. With this scheme, neither model experiences more than a single e -folding of inflation, no matter how small the coupling constant. This is because Ricci subtraction introduces an extra, unsuppressed degree of freedom, which makes a fatal change in the first Friedmann equation.

Because nonderivative couplings are so problematic, we have decided here to explore the consequences of derivative couplings.¹ Because the inflaton oscillates during reheating, its derivative should be just about as effective as the undifferentiated field at communicating kinetic energy to ordinary matter. Of course, a derivative coupling will not induce a Coleman-Weinberg potential; it will instead generate a non-linear function of the inflaton kinetic energy, which is a kind of the quantum-induced k -essence model [8–10]. Our goal is to check whether the resulting models, with Hubble subtraction, can resolve the inconsistency between the efficiency of the reheating and the observational constraints on inflation.

This paper consists of six sections, of which the first is nearly done. In Sec. II, we consider a model with derivative couplings of the inflaton to fermions, and work out the induced k -essence. Section III does the same for couplings to scalars. The two modified Friedmann equations and the scalar evolution equation are derived in Sec. IV. Section V investigates the effective kinetic energy induced by fermions and scalars. Our conclusions are presented in Sec. VI.

II. MODEL WITH DERIVATIVE COUPLING OF THE INFLATON TO FERMIONS

The inflaton could be derivative-coupled to a massless fermion $\Psi(x)$:

$$\frac{\partial_\mu}{m_c^3} \left\{ \sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \times \left[\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(1 - \frac{D}{2}\right) [4M + 4\frac{M^3}{H^2}] + \frac{H^2}{16\pi^2} \{8M + [4M + 4\frac{M^3}{H^2}] [\psi(1 + i\frac{M}{H}) + \psi(1 - i\frac{M}{H})]\} + \mathcal{O}(D-4) \right] \right\}. \quad (10)$$

Note that ψ in the second line is not a fermion field, but rather the digamma function $\psi(z) \equiv \frac{d}{dz} \ln[\Gamma(z)]$.

One can see that two counterterms are needed to renormalize the first line of Eq. (10):

$$\begin{aligned} \mathcal{L}_{\text{ct}} &= -\frac{1}{4} \delta z_1 (\partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu})^2 \sqrt{-g} - \frac{1}{8} \delta z_2 (\partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu})^4 \sqrt{-g}, \\ \frac{\delta S_{\text{ct}}}{\delta \varphi} &\Rightarrow \delta z_1 \partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi (\partial_\rho \varphi \partial_\sigma \varphi g^{\rho\sigma})] + \delta z_2 \partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi (\partial_\rho \varphi \partial_\sigma \varphi g^{\rho\sigma})^3]. \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{L}_{\text{fermion}} &= \bar{\Psi} \gamma^b e^\mu{}_b \left(\partial_\mu + \frac{i}{2} A_{\mu cd} J^{cd} \right) \Psi \sqrt{-g} \\ &+ \frac{1}{2m_c^3} \partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu} \bar{\Psi} \Psi \sqrt{-g}. \end{aligned} \quad (7)$$

Here, $e^\mu{}_b(x)$ is the vierbein field with $g^{\mu\nu}(x) = e^\mu{}_b(x) e^\nu{}_c(x) \eta^{bc}$, and $A_{\mu cd}(x) = e^\nu{}_c [e_{\nu d, \mu} - \Gamma^\rho{}_{\mu\nu} e_{\rho d}]$ is the spin connection. The symbol γ_{ij}^b represents the 4×4 gamma matrices which obey $\{\gamma^b, \gamma^c\} = -2\eta^{bc} I$, and $J^{cd} \equiv \frac{i}{4} [\gamma^c, \gamma^d]$ are the Lorentz representation matrices for Dirac fermions. The final term is the coupling between the derivative inflaton and fermions, and $\frac{1}{m_c^3}$ is the strength of the dimensionful coupling.

Taking the functional derivative of the sum of Eqs. (1) and (7), and then replacing $\bar{\Psi} \Psi$ with the coincident fermion propagator gives the effective field equation,

$$\begin{aligned} \partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi] - V'(\varphi) \sqrt{-g} \\ + \frac{1}{m_c^3} \partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \times \text{Tr}\{iS[M](x, x)\}] = 0. \end{aligned} \quad (8)$$

The sign flip in the final term is due to the definition of the fermion propagator, $i[S_j](x; x') \equiv \langle \Psi_i(x) \bar{\Psi}_j(x') \rangle$. To compute the final term of the effective equation, we employ the coincidence limit of the massive fermion propagator on de Sitter background ($\epsilon = 0$) [11,12],

$$iS[M](x; x) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(1 - \frac{D}{2}\right) \frac{\Gamma(\frac{D}{2} + i\frac{M}{H}) \Gamma(\frac{D}{2} - i\frac{M}{H})}{\Gamma(1 + i\frac{M}{H}) \Gamma(1 - i\frac{M}{H})} \times MI. \quad (9)$$

Here, $M \equiv \frac{-1}{2m_c^3} \partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu}$ is the fermion mass, and I stands for the identity matrix. Expanding Eq. (9) around $D = 4$ and substituting into Eq. (8), the unregulated limit of the quantum-induced term can be obtained:

¹This idea was suggested by the unnamed referee of Ref. [6], to whom we are grateful.

We make the following choice of δz_1 and δz_2 :

$$\begin{aligned}\delta z_1 &= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(1 - \frac{D}{2}\right) \frac{2}{m_c^6} + \frac{H^2(1-\gamma)}{4\pi^2 m_c^6}, \\ \delta z_2 &= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(1 - \frac{D}{2})}{2m_c^{12}H^2} + \frac{H^2}{16\pi^2} \frac{[-\gamma + \zeta(3)]}{m_c^{12}H^2},\end{aligned}\tag{12}$$

in order to absorb the divergences and to eliminate the two lowest-order terms in the small-field expansion of Eq. (10). After combining Eqs. (12) and (11) with Eq. (10), the renormalized result is

$$\partial_\mu \left\{ \sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \times \frac{H^4}{16\pi^2} \left[\frac{8\gamma}{m_c^3 H} \left(\frac{M}{H}\right) + \frac{8\gamma - 8\zeta(3)}{m_c^3 H} \left(\frac{M}{H}\right)^3 + \left[\frac{4}{m_c^3 H} \left(\frac{M}{H}\right) + \frac{4}{m_c^3 H} \left(\frac{M}{H}\right)^3 \right] [\psi(1 + i\frac{M}{H}) + \psi(1 - i\frac{M}{H})] \right] \right\}.\tag{13}$$

This quantum contribution in the field equation can be regarded as a kind of quantum-induced *k*-essence. Suppose that there exists a function of the kinetic term in the action $\int d^D x \Delta K(-\frac{1}{2} \partial_\rho \varphi \partial_\sigma \varphi g^{\rho\sigma}) \sqrt{-g}$. Then, the contribution to the effective field equation gives

$$\partial_\mu \left[\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \times \Delta K' \left(-\frac{1}{2} \partial_\rho \varphi \partial_\sigma \varphi g^{\rho\sigma} \right) \right].\tag{14}$$

We can immediately identify Eq. (13) as $\Delta K'(-\frac{1}{2} \partial_\rho \varphi \partial_\sigma \varphi g^{\rho\sigma})$. By integrating Eq. (13) back with the argument, the quantum-induced kinetic term is

$$\begin{aligned}\Delta K_f(z) &= \frac{H^4}{8\pi^2} \left\{ 2\gamma z^2 + [\gamma - \zeta(3)]z^4 + 2 \int_0^z dx [x + x^2] [\psi(1 + ix) + \psi(1 - ix)] \right\}, \\ \text{where } z \text{ is defined as } z &\equiv -\frac{\partial_\rho \varphi \partial_\sigma \varphi g^{\rho\sigma}}{2m_c^3 H} = \frac{M}{H}.\end{aligned}\tag{15}$$

To get the small-field expansion, we substitute

$$|z| \ll 1 \Rightarrow \psi(1 + z) = -\gamma - \sum_{k=1}^{\infty} \zeta(k+1) (-z)^k.\tag{16}$$

The resulting expansion is

$$\Delta K_f(z) = \frac{H^4}{4\pi^2} \sum_{n=2}^{\infty} \frac{(-1)^n}{n+1} [\zeta(2n-1) - \zeta(2n+1)] z^{2n+2}\tag{17}$$

$$= \frac{H^4}{8\pi^2} \left\{ \frac{2}{3} [\zeta(3) - \zeta(5)] z^6 - \frac{1}{2} [\zeta(5) - \zeta(7)] z^8 + \mathcal{O}(z^{10}) \right\}.\tag{18}$$

By substituting the large-argument expansion for the digamma function,

$$|z| \gg 1 \Rightarrow \psi(z) = \ln(z) - \frac{1}{2z} - \frac{1}{12z^2} + \frac{1}{120z^4} - \frac{1}{256z^6} + \mathcal{O}\left(\frac{1}{z^8}\right),\tag{19}$$

the large-field expansion can be obtained:

$$\Delta K_f(z) = \frac{H^4}{8\pi^2} \left\{ \frac{1}{2} z^4 \ln(z^2 + 1) - \left(\zeta(3) + \frac{1}{4} - \gamma \right) z^4 + z^2 \ln(z^2 + 1) - \left(\frac{4}{3} - 2\gamma \right) z^2 + \frac{11}{60} \ln(z^2 + 1) + \mathcal{O}(z^0) \right\}.\tag{20}$$

III. THE MODEL WITH DERIVATIVE COUPLING OF THE INFLATON TO SCALARS

In this section, we begin with a derivation of the quantum-induced k -essence model due to scalars with arbitrary nonminimal coupling. We then present two special cases. One is a conformal coupling, and the other is the minimally coupled one.

A. General derivation with nonminimal coupling

Derivatives of the inflaton $\varphi(x)$ might couple to another scalar $\Phi(x)$, which need not be minimally coupled to gravity,²

$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi g^{\mu\nu}\sqrt{-g} - \frac{1}{12}(1 + \Delta\xi)R\Phi^2\sqrt{-g} + \frac{1}{2m_c^2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\Phi^2\sqrt{-g}, \quad (21)$$

where $\frac{1}{m_c^2}$ is a dimensionful coupling strength. Note that $\Delta\xi = 0$ corresponds to a conformally coupled scalar. The mass term of the scalar $\Phi(x)$ can be identified from Eq. (21):

$$M_\Phi^2 = -\frac{1}{m_c^2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}. \quad (22)$$

Taking the functional derivative of the sum of Eqs. (1) and (21), and then replacing Φ^2 with the coincidence limit of the Φ propagator gives the effective field equation,

$$\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi] - V'(\varphi)\sqrt{-g} - \frac{1}{m_c^2}\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi \times i\Delta[\xi, M_\Phi^2](x; x)] = 0. \quad (23)$$

On de Sitter background ($\epsilon = 0$), the coincidence limit of the scalar propagator is [5,13]

$$i\Delta[\xi, M_\Phi^2](x; x) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Gamma\left(1 - \frac{D}{2}\right)\frac{\Gamma(\frac{D-1}{2} + \nu)\Gamma(\frac{D-1}{2} - \nu)}{\Gamma(\frac{1}{2} + \nu)\Gamma(\frac{1}{2} - \nu)}, \quad (24)$$

where ν^2 is

$$\nu^2 = \left(\frac{D-1}{2}\right)^2 - D(D-1)\xi - \frac{M_\Phi^2}{H^2}, \quad \xi \equiv \frac{1}{6}(1 + \Delta\xi). \quad (25)$$

After substituting Eq. (24) into Eq. (23), expanding around $D = 4$, and segregating finite parts from divergences, we see that two counterterms are needed to renormalize the primitive contribution:

$$\begin{aligned} \mathcal{L}_{\text{ct}} = & -\frac{1}{2}\delta Z\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - \frac{1}{4}\delta Z_1(\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu})^2\sqrt{-g}, \\ \frac{\delta\mathcal{S}_{\text{ct}}}{\delta\varphi} \Rightarrow & \delta Z\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi] \\ & + \delta Z_1\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi(\partial_\rho\varphi\partial_\sigma\varphi g^{\rho\sigma})]. \end{aligned} \quad (26)$$

Here, the finite parts of δZ and δZ_1 are chosen to cancel the $\partial_\rho\varphi\partial_\sigma\varphi g^{\rho\sigma}$ and $(\partial_\rho\varphi\partial_\sigma\varphi g^{\rho\sigma})^2$ terms in the small-field expansion,³

$$\begin{aligned} \delta Z = & \frac{1}{m_c^2}\left\{\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Gamma\left(1 - \frac{D}{2}\right)(2\Delta\xi) + \frac{H^2}{16\pi^2}\left(\frac{1}{3} + \frac{7}{3}\Delta\xi + 2\Delta\xi[\psi(\nu_+) + \psi(\nu_-)]\right)\right\}, \\ \delta Z_1 = & \frac{-1}{m_c^2}\left\{\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(1 - \frac{D}{2})}{m_c^2 H^2} + \frac{H^2}{16\pi^2}\left(\frac{[\psi(\nu_+) + \psi(\nu_-)]}{m_c^2 H^2} - \frac{2\Delta\xi[\psi'(\nu_+) - \psi'(\nu_-)]}{m_c^2 H^2\sqrt{1 - 8\Delta\xi}}\right)\right\}. \end{aligned} \quad (27)$$

Note that $\psi(z) \equiv \frac{d}{dz}\ln[\Gamma(z)]$ is the digamma function and that we define $\nu_\pm \equiv \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 8\Delta\xi}$. The renormalized result can be expressed in terms of the dimensionless quantity $y \equiv \frac{-\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}}{2m_c^2 H^2}$:

$$\begin{aligned} \Delta K(y) = & \frac{H^4}{16\pi^2}\left\{[2\Delta\xi y + y^2][\psi(\nu_+) + \psi(\nu_-)] - \frac{2\Delta\xi y^2[\psi'(\nu_+) - \psi'(\nu_-)]}{\sqrt{1 - 8\Delta\xi}}\right. \\ & \left. - 2\int_0^y dx(\Delta\xi + x)\left[\psi\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - 8(\Delta\xi + x)}\right) + \psi\left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 8(\Delta\xi + x)}\right)\right]\right\}. \end{aligned} \quad (28)$$

²The + sign is chosen for stability, because $M^2 \equiv -\frac{1}{m_c^2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}$ is positive for a time-dependent inflaton.

³The expression of the finite parts works for $\Delta\xi \geq 0$.

We Taylor-expand the digamma function in Eq. (28) to get the small-field expansion,

$$\begin{aligned} \Delta K(y) = & \frac{H^4}{16\pi^2} \left\{ \left[\frac{(1-6\Delta\xi)[\psi'(\nu_+) - \psi'(\nu_-)]}{(1-8\Delta\xi)^{\frac{3}{2}}} - \frac{\Delta\xi[\psi''(\nu_+) + \psi''(\nu_-)]}{(1-8\Delta\xi)} \right] \left(\frac{4y^3}{3} \right) \right. \\ & + \left[\frac{2(1-4\Delta\xi)[\psi'(\nu_+) - \psi'(\nu_-)]}{(1-8\Delta\xi)^{\frac{5}{2}}} - \frac{(1-4\Delta\xi)[\psi''(\nu_+) + \psi''(\nu_-)]}{2(1-8\Delta\xi)^2} \right. \\ & \left. \left. + \frac{\frac{2}{3}\Delta\xi[\psi'''(\nu_+) - \psi'''(\nu_-)]}{(1-8\Delta\xi)^{\frac{3}{2}}} \right] y^4 + \mathcal{O}(y^5) \right\}. \end{aligned} \quad (29)$$

The large-field expansion comes from using the asymptotic expansion [Eq. (19)]:

$$\begin{aligned} \Delta K(y) = & \frac{H^4}{16\pi^2} \left\{ -y^2 \ln(2y + 2\Delta\xi) + \left[\frac{1}{2} + \psi(\nu_+) + \psi(\nu_-) - \frac{2\Delta\xi[\psi'(\nu_+) - \psi'(\nu_-)]}{\sqrt{1-8\Delta\xi}} \right] y^2 \right. \\ & - 2\Delta\xi y \ln(2y + 2\Delta\xi) + \left[\frac{1}{3} + \Delta\xi + 2\Delta\xi[\psi(\nu_+) + \psi(\nu_-)] \right] y \\ & \left. + \left(\frac{1}{30} - \Delta\xi^2 \right) \ln(2y + 2\Delta\xi) + \mathcal{O}(y^0) \right\}. \end{aligned} \quad (30)$$

B. Conformal coupling

In this subsection, we specialize to the case in which derivatives of the inflaton are coupled to a massless, conformally coupled scalar. By expanding Eq. (28) for small $\Delta\xi$, and carefully dealing with singular contributions from the digamma functions and its derivatives, such as

$$\psi(\nu_-) \simeq \frac{-1}{2\Delta\xi} + 1 - \gamma + \dots, \quad (31)$$

$$\psi'(\nu_-) \simeq \frac{1}{4\Delta\xi^2} (1 - 4\Delta\xi - 4\Delta\xi^2) + \frac{\pi^2}{6} + \dots, \quad (32)$$

the quantum-induced term is seen to be

$$\Delta K_{\text{con}}(y) = \frac{H^4}{16\pi^2} \left\{ -y + (1-2\gamma)y^2 - 2 \int_0^y dx x \left[\psi \left(\frac{1}{2} + \frac{1}{2}\sqrt{1-8x} \right) + \psi \left(\frac{1}{2} - \frac{1}{2}\sqrt{1-8x} \right) \right] \right\}. \quad (33)$$

Employing the same techniques as the preceding section gives the small-field and large-field expansions:

$$\Delta K_{\text{conS}}(y) = \frac{H^4}{16\pi^2} \left\{ -\frac{4}{3}y^3 - [4 + 2\psi''(1)]y^4 + \mathcal{O}(y^5) \right\}, \quad (34)$$

$$\Delta K_{\text{conL}}(y) = \frac{H^4}{16\pi^2} \left\{ -y^2 \ln(2y) + \left(\frac{3}{2} - 2\gamma \right) y^2 - \frac{2}{3}y + \frac{1}{30} \ln(2y) + \mathcal{O}(y^0) \right\}. \quad (35)$$

C. Minimal coupling

The other special case we consider is minimal coupling, for which $\Delta\xi$ is taken to -1 in Eq. (21). We assume that M_ϕ is small and positive because the massless limit of a massive, minimally coupled propagator is not smooth [14–16]. Furthermore, in order to eliminate the two leading contributions in the small-field expansion, the finite parts of Eq. (27) need to be adjusted as follows:

$$\begin{aligned} \delta\mathcal{Z} = & \frac{1}{m_c^2} \left\{ \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left[-2\Gamma \left(1 - \frac{D}{2} \right) \right] + \frac{H^2}{16\pi^2} \left(4\gamma - \frac{29}{3} \right) \right\}, \\ \delta\mathcal{Z}_1 = & \frac{-1}{m_c^2} \left\{ \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(1 - \frac{D}{2})}{m_c^2 H^2} + \frac{H^2}{16\pi^2} \left(\frac{-2\gamma + \frac{25}{27}}{m_c^2 H^2} \right) \right\}. \end{aligned} \quad (36)$$

A straightforward but lengthy computation gives the exact result,

$$\begin{aligned} \Delta K_{\min}(y) = & \frac{H^4}{16\pi^2} \left\{ -2\gamma + \frac{182}{27} + \left(4\gamma - \frac{23}{3}\right)y + \left(-2\gamma + \frac{25}{27}\right)y^2 \right. \\ & \left. + 2 \int_1^y dx(1-x) \left[\psi\left(\frac{1}{2} + \frac{3}{2}\sqrt{1-\frac{8}{9}x}\right) + \psi\left(\frac{1}{2} - \frac{3}{2}\sqrt{1-\frac{8}{9}x}\right) \right] \right\}. \end{aligned} \quad (37)$$

The small- and large-field expansions are

$$\Delta K_{\min S}(y) = \frac{H^4}{16\pi^2} \left\{ -3 \ln(y) + C + \left[\frac{-92}{729} + \frac{8}{27} \psi''(1) \right] y^3 - \frac{2}{81} \left[\frac{44}{27} + 5\psi''(1) \right] y^4 + \dots \right\}, \quad (38)$$

$$\Delta K_{\min L}(y) = \frac{H^4}{16\pi^2} \left\{ -y^2 \ln(2y) + \left(\frac{77}{54} - 2\gamma \right) y^2 + 2y \ln(2y) + \left[4\gamma - \frac{25}{3} \right] y - \frac{29}{30} \ln(2y) + \mathcal{O}(y^0) \right\}. \quad (39)$$

Here, $C = \frac{14}{81} \left[\frac{26}{27} - \psi''(1) \right]$ is an integration constant.

IV. THE MODIFIED FRIEDMANN EQUATIONS

Our results in Eqs. (15), (28), (33), and (37) were all derived on de Sitter background because that is the only case for which the necessary propagators are known. However, for realistic inflation, the first slow-roll parameter is nonzero, and the Hubble parameter changes with time. Numerical studies of cosmological Coleman-Weinberg potentials have shown that it is reasonable to simply replace the constant de Sitter H_{dS} with the evolving $H(t)$, and ignore $\epsilon(t)$ [17–19]. That is what we shall do for quantum-induced k -essence models,

$$\mathcal{L} = \frac{R\sqrt{-g}}{16\pi G} + K(\mathcal{k}, H)\sqrt{-g} - V(\varphi)\sqrt{-g},$$

$$K(\mathcal{k}, H) \equiv \mathcal{k} + \Delta K(\mathcal{k}, H), \quad \mathcal{k} = -\frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu}. \quad (40)$$

The purpose of this section is to work out how the two Friedmann equations and the scalar evolution equation change. The section closes by converting these equations into a dimensionless form conducive to numerical work.

If the effective action is known for a general metric $g_{\mu\nu}(x)$, the modified Friedmann equations can be obtained by taking the functional derivative with respect to it and then specializing to the cosmological background [Eq. (2)]. However, what we have is (an approximation for) the effective action already specialized to Eq. (2). Because this depends only upon the single gravitational dynamical variable $a(t)$, varying can give at most one of the two Friedmann equations. The theorem of Palais [20,21] guarantees us that the single equation so obtained is at least correct. We first show that this is the second Friedmann equation, and then we use conservation to reconstruct the first Friedmann equation.

In our homogeneous, isotropic, and spatially flat geometry [Eq. (2)], the Lagrangian [Eq. (40)] becomes

$$L = -\frac{6a^3 H^2}{16\pi G} + a^3 K(\mathcal{k}, H) - a^3 V(\varphi). \quad (41)$$

The gravitational dynamical variable is $a(t)$, and the associated Euler-Lagrange equation is

$$\begin{aligned} -2\dot{H} - 3H^2 = & 8\pi G \left[K(\mathcal{k}, H) - V(\varphi) \right. \\ & \left. - H \frac{\partial K(\mathcal{k}, H)}{\partial H} - \frac{1}{3} \frac{d}{dt} \frac{\partial K(\mathcal{k}, H)}{\partial H} \right]. \end{aligned} \quad (42)$$

This is the ij Einstein equation, sometimes known as the second Friedmann equation. Varying Eq. (40) with respect to $\varphi(t)$ gives the scalar evolution equation,

$$\ddot{\varphi} \left(2\mathcal{k} \frac{\partial^2 K}{\partial \mathcal{k}^2} + \frac{\partial K}{\partial \mathcal{k}} \right) + \dot{\varphi} \left(\dot{H} \frac{\partial^2 K}{\partial \mathcal{k} \partial H} + 3H \frac{\partial K}{\partial \mathcal{k}} \right) + \frac{\partial V}{\partial \varphi} = 0. \quad (43)$$

Missing is the first Friedmann equation, which is the 00 Einstein equation. We can recover it by noting that the two Friedmann equations are related to the scalar evolution equation through conservation:

$$\begin{aligned} \frac{d}{dt} [\text{00 equation}] + 3H[(\text{00} + ij) \text{ equations}] \\ = 8\pi G \dot{\varphi} [\text{scalar evolution equation}]. \end{aligned} \quad (44)$$

From relation (44), we infer the first Friedmann equation,

$$3H^2 = 8\pi G \left[2\ell \frac{\partial K}{\partial \ell} - K(\ell, H) + H \frac{\partial K}{\partial H} + V(\varphi) \right]. \quad (45)$$

Because the scale of temporal variation changes dramatically over the course of inflation, and because the dependent variables $\varphi(t)$ and $H(t)$ are dimensionful, it is convenient to convert to dimensionless variables. We first change the comoving time t to the number of inflationary e -foldings since the beginning of inflation,

$$n \equiv \ln \left(\frac{a(t)}{a(t_i)} \right) \Rightarrow \frac{d}{dt} = H \frac{d}{dn},$$

$$\frac{d^2}{dt^2} = H^2 \left[\frac{d^2}{dn^2} - \epsilon \frac{d}{dn} \right]. \quad (46)$$

It is also useful to make the various other quantities dimensionless:

$$\begin{aligned} \phi(n) &\equiv \sqrt{8\pi G} \varphi(t), & \chi(n) &\equiv \sqrt{8\pi G} H(t), \\ \kappa(n) &\equiv (8\pi G)^2 \ell(t) = \frac{1}{2} \chi^2 \phi'^2, \\ k_c &\equiv \sqrt{8\pi G} m_c, & \mathcal{K}(\kappa, \chi) &\equiv (8\pi G)^2 K(\ell, H), \\ U(\phi) &\equiv (8\pi G)^2 V(\varphi). \end{aligned} \quad (47)$$

With these changes, the two modified Friedman equations (42) and (45) take the forms

$$3\chi^2 = 2\kappa \frac{\partial \mathcal{K}}{\partial \kappa} - \mathcal{K}(\kappa, \chi) + \chi \frac{\partial \mathcal{K}}{\partial \chi} + U(\phi), \quad (48)$$

$$\begin{aligned} -2\chi\chi' - 3\chi^2 &= \mathcal{K}(\kappa, \chi) - U(\phi) - \chi \frac{\partial \mathcal{K}}{\partial \chi} \\ &\quad - \frac{1}{3} \chi \frac{d}{dn} \frac{\partial \mathcal{K}(\kappa, \chi)}{\partial \chi}. \end{aligned} \quad (49)$$

And the scalar evolution equation becomes

$$\begin{aligned} \phi'' \left(2\kappa \frac{\partial^2 \mathcal{K}}{\partial \kappa^2} + \frac{\partial \mathcal{K}}{\partial \kappa} \right) + \phi' \left[-2\kappa\epsilon \frac{\partial^2 \mathcal{K}}{\partial \kappa^2} - \epsilon\chi \frac{\partial^2 \mathcal{K}}{\partial \kappa \partial \chi} + (3-\epsilon) \frac{\partial \mathcal{K}}{\partial \kappa} \right] \\ + \frac{1}{\chi^2} \frac{\partial U}{\partial \phi} = 0, \end{aligned} \quad (50)$$

where the first slow-roll parameter is expressed as

$$\begin{aligned} \epsilon(n) &\equiv -\frac{\chi'}{\chi} = \frac{\frac{1}{2} \phi'^2 \frac{\partial \mathcal{K}}{\partial \kappa} - \frac{1}{6} \chi \phi'' \phi' \frac{\partial^2 \mathcal{K}}{\partial \kappa \partial \chi}}{1 - \frac{1}{6} \left[\frac{\partial^2 \mathcal{K}}{\partial \chi^2} + \chi \phi'^2 \frac{\partial^2 \mathcal{K}}{\partial \kappa \partial \chi} \right]} \\ &= \frac{\frac{\phi'^2}{2} \frac{\partial \mathcal{K}}{\partial \kappa} \left[\frac{\partial \mathcal{K}}{\partial \kappa} + 2\kappa \frac{\partial^2 \mathcal{K}}{\partial \kappa^2} + \chi \frac{\partial^2 \mathcal{K}}{\partial \chi \partial \kappa} \right] + \frac{1}{6} \frac{\phi'}{\chi} \frac{\partial^2 \mathcal{K}}{\partial \chi \partial \kappa} \frac{\partial U}{\partial \phi}}{\left[\frac{\partial \mathcal{K}}{\partial \kappa} + 2\kappa \frac{\partial^2 \mathcal{K}}{\partial \kappa^2} \right] \left[1 - \frac{1}{6} \frac{\partial^2 \mathcal{K}}{\partial \chi^2} \right] + \frac{\kappa}{3} \left(\frac{\partial^2 \mathcal{K}}{\partial \chi \partial \kappa} \right)^2}. \end{aligned} \quad (51)$$

Here we have used Eqs. (48) and (49) in the first equality and eliminated ϕ'' to obtain the final expression using the scalar evolution equation (50).

V. THE FATE OF THE $m^2\varphi^2$ MODEL

Because the Hubble parameter $H(t)$ is not a local functional of the metric, we cannot completely subtract the quantum-induced k -essence $\Delta K(\ell(t), H(t))$ in expression (40) using permissible counterterms. What we could do instead is to subtract $\Delta K(\ell(t), H_i)$, which would restore the classical model at the initial time, but it would lead to quantum corrections as evolution carries $H(t)$ away from $H(t_i)$. This procedure is known as *Hubble subtraction*. The purpose of this section is to investigate the effects of quantum-induced k -essence models with Hubble subtraction in the context of the classical $m^2\varphi^2$ model. This model has the virtue of simplicity, even though it is not consistent with the current upper bound [Eq. (6)] on the tensor-to-scalar ratio r [22,23]. We begin by describing classical evolution of the $m^2\varphi^2$ model, then consider the effects of fermionic-induced k -essence [Eq. (15)], and finally close by discussing the corrections due to conformal scalars [Eq. (33)] and minimally coupled scalars [Eq. (37)].

The dimensionless expressions of the two classical Friedman equations and the scalar evolution equation are

$$\begin{aligned} 3\chi^2 &= \frac{1}{2} \chi^2 \phi'^2 + U(\phi), \\ (2\epsilon - 3)\chi^2 &= \frac{1}{2} \chi^2 \phi'^2 - U(\phi), \end{aligned} \quad (52)$$

$$\phi'' + (3 - \epsilon)\phi' + \frac{1}{\chi^2} \frac{\partial U(\phi)}{\partial \phi} = 0, \quad (53)$$

where χ, ϕ are defined in Eq. (47), and $U(\phi)$ can be expressed as

$$U(\phi) = \frac{1}{2} k^2 \phi^2, \quad k^2 \equiv 8\pi G m^2. \quad (54)$$

Under the slow-roll approximation, the time evolutions of several useful quantities can be found:

$$\begin{aligned} \phi(n) &\simeq \sqrt{\phi_0^2 - 4n}, & \phi'(n) &\simeq \frac{-2}{\phi(n)}, \\ \epsilon(n) &\simeq \frac{2}{\phi(n)^2}, & \epsilon'(n) &\simeq 2\epsilon^2, \end{aligned} \quad (55)$$

$$\Delta_{\mathcal{R}}^2 \simeq \frac{1}{8\pi^2} \frac{\chi^2}{\epsilon} \longrightarrow \frac{1}{8\pi^2} \frac{k^2}{3\epsilon^2}, \quad 1 - n_s \simeq 2\epsilon + \frac{\epsilon'}{\epsilon} \longrightarrow 4\epsilon. \quad (56)$$

The justification for the magnitude of the dimensionless mass k is to reproduce the scalar power spectrum amplitude A_s and spectral index n_s [22]:

$$k \simeq \pi(1 - n_s) \sqrt{\frac{3}{2} A_s} \simeq 6.13 \times 10^{-6}. \quad (57)$$

To make inflation last about 100 e -foldings, the slow-roll approximation suggests the initial conditions

$$\phi_0 = 20, \quad \phi'_0 = \frac{-1}{10}, \quad \chi_0 \simeq \frac{1}{\sqrt{6}} k \phi_0 \simeq 5 \times 10^{-5}. \quad (58)$$

These should continue to apply in the quantum-corrected model as long as the classical kinetic term $K_C \equiv \kappa$ dominates over the quantum correction $K_Q \equiv \Delta\mathcal{K}$. In principle, two initial conditions are enough to numerically simulate the system, even with the quantum correction. However, because the first Friedmann equation [Eq. (48)] is highly nonlinear in χ , it is simpler to evolve the quantum-corrected system using Eqs. (50) and (51). That is why we need an initial condition for χ_0 .

For the regime where quantum contributions K_Q become comparable or larger than the classical result K_C , one should fix *geometric* initial conditions, making χ_0 and ϵ_0 equal their classical values. From the first modified Friedmann equation (48), the potential still dominates the right-hand side below some coupling strength,⁴ so it leaves χ_0 unchanged. Using the second expression of Eq. (51), one can find ϵ_0 as a function of ϕ'_0 at the specific coupling strength with fixed values of ϕ_0 and χ_0 , and then solve for ϕ'_0 numerically by demanding $\epsilon_0(\phi'_0) = [\epsilon_0]_{\text{classical}} \sim 5 \times 10^{-3}$. One should use this more appropriate ϕ'_0 in the quantum-dominated regime.

A. Induced k -essence model due to fermions

If fermions quantum-correct the kinetic term and Hubble subtraction is employed, its dimensionless form is

$$\mathcal{K}(\kappa, \chi) = \kappa + \frac{\chi^4}{8\pi^2} f\left(\frac{\kappa}{k_c^3 \chi}\right) - \frac{\chi^4(0)}{8\pi^2} f\left(\frac{\kappa}{k_c^3 \chi(0)}\right), \quad (59)$$

where $f(z)$ can be identified from Eq. (15):

$$f(z) = 2\gamma z^2 + [\gamma - \zeta(3)]z^4 + 2 \int_0^z dx (x + x^3) [\psi(1 + ix) + \psi(1 - ix)]. \quad (60)$$

At this point, we digress to discuss the magnitude of the dimensionless coupling strength, $k_c \equiv \sqrt{8\pi G} m_c$. Our goal was to make the derivative coupling roughly comparable to the nonderivative coupling during reheating:

$$\frac{1}{2m_c^3} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} \bar{\Psi} \Psi \sqrt{-g} \Leftrightarrow -\lambda \phi \bar{\Psi} \Psi \sqrt{-g}. \quad (61)$$

Because the inflaton depends only on time, we infer

⁴For fermionic k -essence, the kinetic contributions become comparable with the potential contribution around $1/k_c^3 \sim 1.564 \times 10^{10}$.

$$\lambda \phi \sim \frac{\dot{\phi}^2}{2m_c^3}. \quad (62)$$

The scale of $\frac{1}{2}\dot{\phi}^2$ can be approximated as $\frac{1}{2}\omega^2\phi^2$, and it is roughly equal to $\frac{1}{2}m^2\phi^2$. Therefore, the dimensionless parameter k_c relates to the dimensionless Yukawa coupling as

$$\frac{1}{k_c^3} \sim \lambda \left(\frac{2}{k^2\phi}\right) \sim (5.32 \times 10^{12})\lambda. \quad (63)$$

Here we have used the relation in Eq. (57) and $\phi \sim 10^{-2}$, which is roughly valid during reheating. Finally, note that the cosmological Coleman-Weinberg potential due to the fermion corrections has a similar expression to Eq. (59) [5,6,11],

$$U(\phi, \chi) = \frac{1}{2}k^2\phi^2 - \frac{\chi^4}{8\pi^2} f\left(\frac{\lambda\phi}{\chi}\right) + \frac{\chi^4(0)}{8\pi^2} f\left(\frac{\lambda\phi}{\chi(0)}\right), \quad (64)$$

where the form of the function f is exactly the same as Eq. (60).

There is an interesting distinction between cosmological Coleman-Weinberg potentials and quantum-induced k -essence. Because the inflaton rolls down its potential, cosmological Coleman-Weinberg potentials become smaller as inflation progresses. However, the inflaton's kinetic energy does not show a similar decline, meaning that quantum-induced k -essence terms do not typically fall off. Obviously, increasing $1/k_c^3$ makes the quantum contribution larger. Figure 1 shows that the quantum correction begins dominating the initial kinetic energy around $1/k_c^3 \sim 3.26 \times 10^8$, which corresponds to a small Yukawa coupling $\lambda \sim 6.13 \times 10^{-5}$.

In spite of the quantum correction dominating the classical kinetic energy, it is still possible to stay close

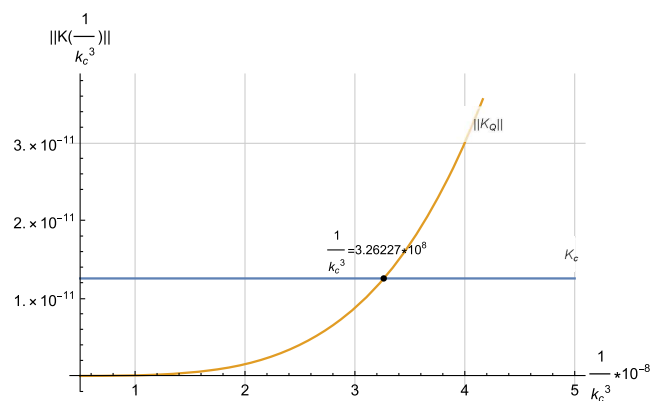


FIG. 1. Comparison of the initial ($n = 0$) magnitudes of the classical kinetic energy (in blue) with the quantum correction (in yellow) for different values of the dimensionless coupling $1/k_c^3$, assuming slow-roll initial conditions [Eq. (58)].

to the classical evolution, provided the slow-roll initial conditions [Eq. (58)] are abandoned for geometric conditions, as described at the end of Sec. V. As long as the potential term dominates over the rest of the kinetic contributions in Eq. (48), choosing $\chi_0 = 5 \times 10^{-5}$ is still a reasonable approximation, whereas ϕ'_0 needs to be solved numerically by demanding that the initial first slow-roll parameter equal its classical value. Without Hubble subtraction (the unsubtracted model), only one solution is found to give $\epsilon(\phi'_0) = 5 \times 10^{-3}$. With Hubble subtraction (the subtracted model), the nonlinearity of the function $\epsilon(\phi'_0)$ results in two solutions at each specific coupling strength. One should choose the branch which is connected to the classical value $\phi'_0 = -0.1$ for small $1/k_c^3$. This is shown by the blue dots in Fig. 2.

We chose several coupling strengths within a range such that $\chi_0 \sim 5 \times 10^{-5}$ is still valid. We present these evolutions both without and with Hubble subtraction in Fig. 3. The general trend is that reducing the coupling strength decreases the duration of inflation.

The general trends in either case can be understood in terms of a simple picture of the slow-roll approximation. Applying the slow-roll conditions to the effective field equation (13), one obtains the speed of the inflaton:

$$|\phi'(n)| \simeq \left| \frac{-U'(\phi)}{3\chi^2[1 + g(\chi, z)]} \right|,$$

$$g(\chi, z) = h(\chi, z) + \frac{1}{3} \frac{d}{dn} h(\chi, z), \quad z = \frac{\kappa}{k_c^3 \chi},$$

$$h(\chi, z) = \frac{\chi^4}{4\pi^2 k_c^3} \{2\gamma z + 2[\gamma - \zeta(3)]z^3 + (z + z^3)[\psi(1 + iz) + \psi(1 - iz)]\}. \quad (65)$$

In addition to $-U'(\phi)/3\chi^2$ from a classical evolution, an extra factor $1 + g(\chi, z)$ occurs in the denominator from quantum corrections. Because $g(\chi, z)$ is positive and $1 + g(\chi, z)$ is monotonically increasing, the quantum-corrected inflaton rolls down its potential with a smaller speed than its classical cousin, which lengthens inflation. The difference between the subtracted model

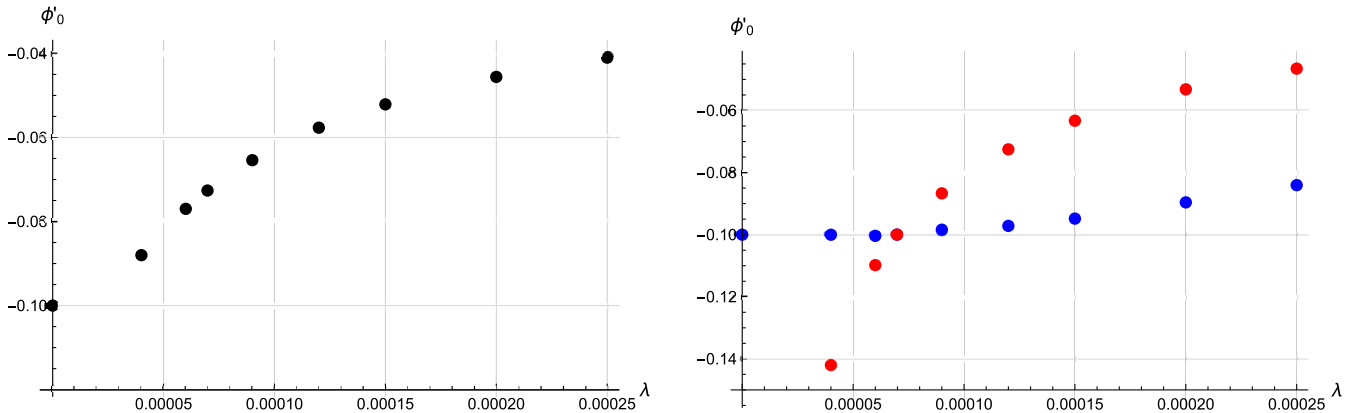


FIG. 2. The initial condition ϕ'_0 in the quantum-dominated regime versus the equivalent coupling constant λ [Eq. (63)]. The left-hand graph depicts a series of solutions for the unsubtracted model. The two branches of the subtracted model are shown in the right-hand plot.

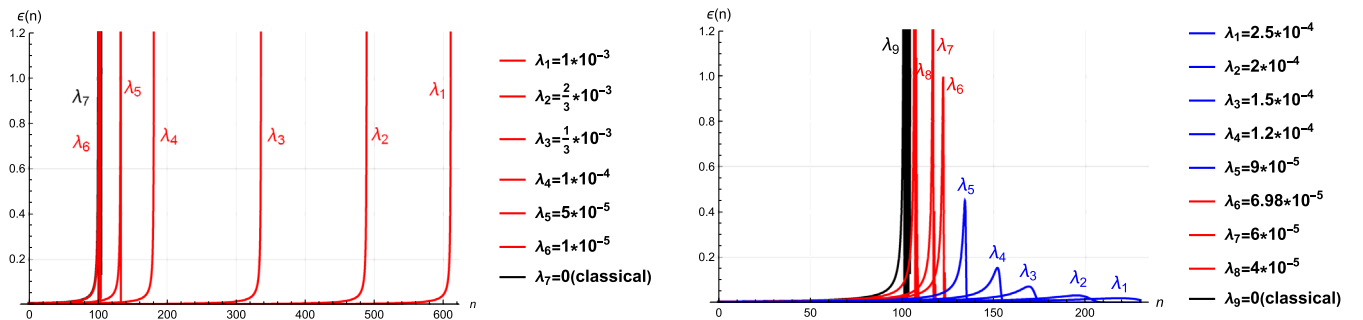


FIG. 3. The $\epsilon(n)$ versus n for various λ 's. The unsubtracted model is on the left-hand side, while the subtracted one is plotted on the right-hand side.

(the right-hand plot of Fig. 3) and the unsubtracted one (the left-hand plot of Fig. 3) is that the former tends to shorten inflation, but with a lower peak value for ϵ . The value of ϵ falls back to zero after inflation as well. Also note that the maximum of ϵ never reaches 1 until $\lambda \leq 6.98 \times 10^{-5} (1/k_c^3 \leq 3.71 \times 10^8)$.

Finally, we compare evolution in the k -essence model (derivative couplings) with evolution in the analogous cosmological Coleman-Weinberg potential (nonderivative model). These comparisons are presented in Figs. 4 and 5. One can see that the derivative model deviates from a classical evolution more than the nonderivative one at the same coupling strength and also has a bigger dimensionless Hubble parameter after inflation. For evolution with $\lambda = 6.98 \times 10^{-5}$, the dimensionless Hubble parameter in the derivative model is 1.41×10^{-6} , while it is 1.26×10^{-7} in the nonderivative model. At a coupling strength of $\lambda = 1 \times 10^{-4}$, the dimensionless Hubble parameters are $\chi_{\text{derivative}} = 1.21 \times 10^{-5}$, $\chi_{\text{non-derivative}} = 2.59 \times 10^{-7}$. According to a previous study [6], the system with a moderate Yukawa coupling constant λ tends to inflate forever. It is not clear whether we should extrapolate our current conclusion from Fig. 3 and draw a similar conclusion for the k -essence model of the fermionic coupling. The obstruction to this conclusion is that, at coupling

strength $\lambda \sim 0.1 \Rightarrow 1/k_c^3 \sim 5.32 \times 10^{11}$, the approximation $\chi_0 \sim 5 \times 10^{-5}$ is not valid, and the complexities of highly nonlinear dependence of ϵ and χ on ϕ and ϕ' become nontrivial. No analytic solution exists, and we must employ some kind of nonlinear search routine such as a Monte Carlo Markov chain method to solve for χ_0 and ϕ'_0 from Eqs. (48) and (51).

B. Induced k -essence model due to scalars

The generic expression of the k -essence from Eqs. (33) and (37) takes the form

$$\mathcal{K}(\kappa, \chi) = \kappa - \frac{\chi^4}{16\pi^2} f\left(\frac{\kappa}{k_c^2 \chi^2}\right) + \frac{\chi^4(0)}{16\pi^2} f\left(\frac{\kappa}{k_c^2 \chi^2(0)}\right). \quad (66)$$

The two explicit cases we consider are

$$f_{\text{con}}(y) = y + (2\gamma - 1)y^2 + 2 \int_0^y dx x \left[\psi\left(\frac{1}{2} + \frac{1}{2}\sqrt{1-8x}\right) + \psi\left(\frac{1}{2} - \frac{1}{2}\sqrt{1-8x}\right) \right], \quad (67)$$

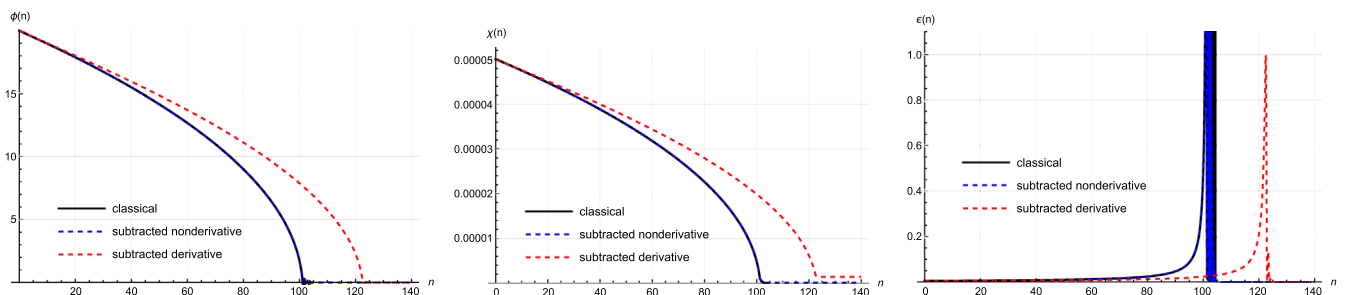


FIG. 4. Plots of the dimensionless scalar $\phi(n)$ (left), the dimensionless Hubble parameter $\chi(n)$ (middle), and the first slow-roll parameter $\epsilon(n)$ (right) for a coupling constant $\lambda = 6.98 \times 10^{-5}$ ($1/k_c^3 = 3.71 \times 10^8$) with the initial conditions $\chi_0 = 5 \times 10^{-5}$ and $\phi'_0 = -9.99737 \times 10^{-2}$.

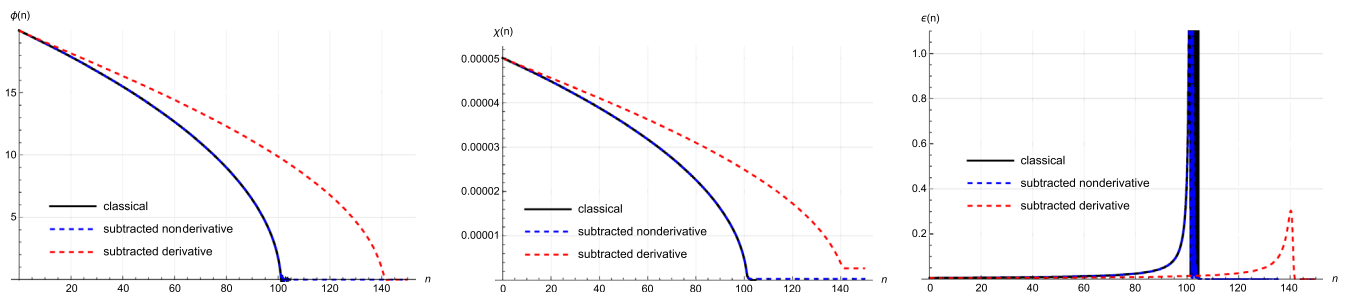


FIG. 5. Plots of the dimensionless scalar $\phi(n)$ (left), the dimensionless Hubble parameter $\chi(n)$ (middle), and the first slow-roll parameter $\epsilon(n)$ (right) for a coupling constant $\lambda = 1 \times 10^{-4}$ ($1/k_c^3 = 5.32 \times 10^8$) with the initial conditions $\chi_0 = 5 \times 10^{-5}$ and $\phi'_0 = -9.81626 \times 10^{-2}$.

$$f_{\min}(y) = 2\gamma - \frac{182}{27} + \left(-4\gamma + \frac{23}{3}\right)y + \left(2\gamma - \frac{25}{27}\right)y^2 - 2 \int_1^y dx(1-x) \left[\psi\left(\frac{1}{2} + \frac{3}{2}\sqrt{1 - \frac{8}{9}x}\right) + \psi\left(\frac{1}{2} - \frac{3}{2}\sqrt{1 - \frac{8}{9}x}\right) \right], \quad (68)$$

where $f_{\text{con}}(y)$ stands for the contribution from the conformal coupling, and the minimal coupling's contribution is denoted by $f_{\min}(y)$. Similarly, by making an analogy with the nonderivative coupling term which generates a cosmological Coleman-Weinberg potential

$$-\frac{1}{4}h^2\varphi^2\Phi^2\sqrt{-g} \Leftrightarrow \frac{1}{2m_c^2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\Phi^2\sqrt{-g}, \quad (69)$$

the dimensionless couplings h and k_c can be related:

$$\frac{1}{k_c^2} \sim \frac{h^2}{2k^2} \sim (1.33 \times 10^{10})h^2. \quad (70)$$

Also, note that the scalar corrections to Coleman-Weinberg potential have the same form as Eq. (66) [5], with the replacements of κ by $\frac{1}{2}k^2\phi^2$ and $\frac{\kappa}{k_c^2}$ by $\frac{1}{4}h^2\phi^2$:

$$U(\phi, \chi) = \frac{1}{2}k^2\phi^2 + \frac{\chi^4}{16\pi^2}f\left(\frac{h^2\phi^2}{4\chi^2}\right) - \frac{\chi^4(0)}{16\pi^2}f\left(\frac{h^2\phi^2}{4\chi^2(0)}\right). \quad (71)$$

Here, the function f is the same as either Eq. (67) or Eq. (68) for conformal scalars and minimally coupled scalars, respectively.

We first determine when evolution enters the quantum-dominated domain for conformal scalars and minimally coupled scalars. Figure 6 shows that this happens around $1/k_c^2 \gtrsim 10^6$ ($h^2 \gtrsim 10^{-5}$) in both cases. Because K_Q in the scalar k -essence model is negative, we must not go beyond the classical-dominated regime in order to avoid a disastrous kinetic instability. When K_C dominates over K_Q , it is reliable to use the slow-roll initial conditions [Eq. (58)]. Within the safe regime, we find that quantum effects due to each sort of scalar tend to shorten inflation. The result agrees with what one expects from the effective field equation using the slow-roll approximation:

$$\mu(\chi, z)\phi'(n) \sim \frac{-U'(\phi)}{3\chi^2}, \quad \mu(\chi, z) \equiv 1 + \alpha(\chi, z), \quad z = \frac{\kappa}{k_c^2\chi^2}. \quad (72)$$

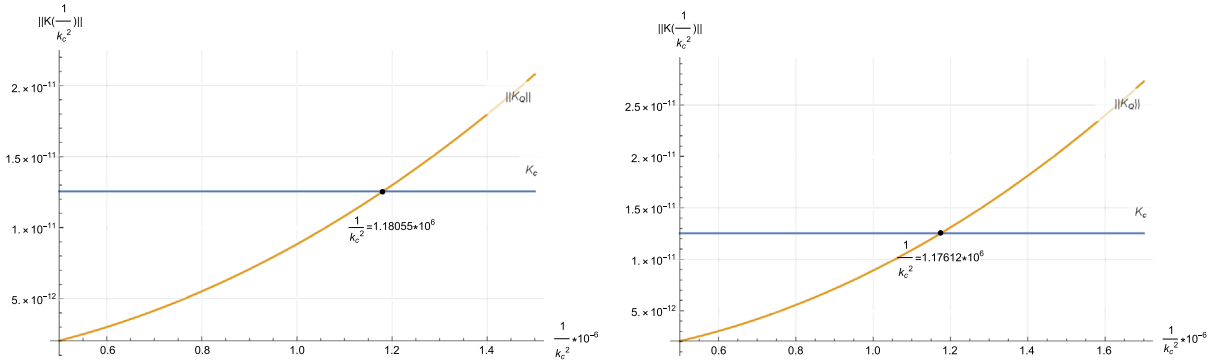


FIG. 6. Plots of K_Q and K_C versus $1/k_c^2$ at $n = 0$ with slow-roll initial conditions [Eq. (58)]. The left-hand graph is for conformally coupled scalars, while the right-hand plot is for minimally coupled scalars.

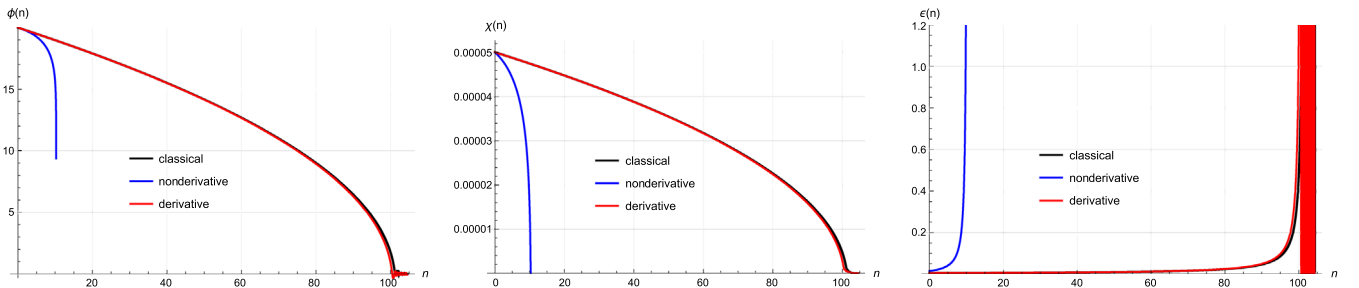


FIG. 7. Evolution for conformally coupled scalars, comparing the classical result (in black) with nonderivative coupling (in blue) and derivative coupling (in red). Plots show the dimensionless scalar $\phi(n)$ (left), the dimensionless Hubble parameter $\chi(n)$ (middle), and the first slow-roll parameter $\epsilon(n)$ (right) for an effective coupling constant of $h^2 = 1.6 \times 10^{-5}$, corresponding to $1/k_c^2 = 2.218 \times 10^5$, with slow-roll initial conditions [Eq. (58)].

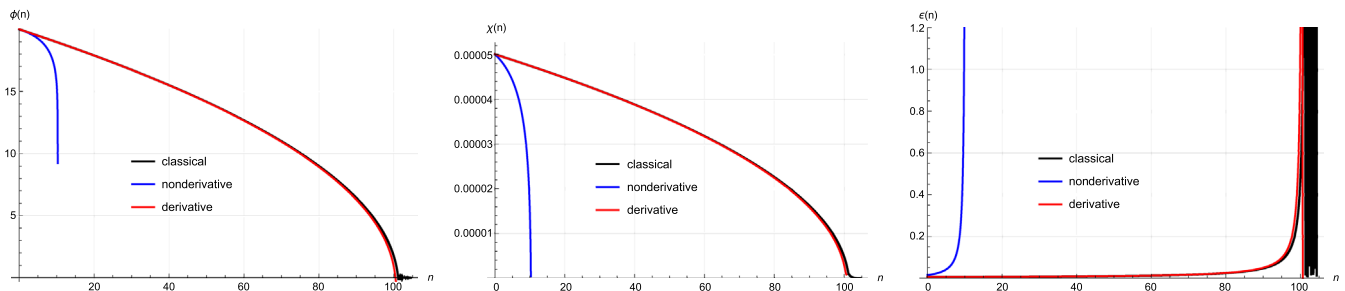


FIG. 8. Evolution for minimally coupled scalars, comparing the classical model (in black) with nonderivative couplings (in blue) and derivative coupling (in red). Plots show the dimensionless scalar $\phi(n)$ (left), the dimensionless Hubble parameter $\chi(n)$ (middle), and the first slow-roll parameter $\epsilon(n)$ (right) for an effective coupling constant of $h^2 = 1.6 \times 10^{-5}$, corresponding to $1/k_c^2 = 2.218 \times 10^5$, with slow-roll initial conditions [Eq. (58)].

Because quantum contributions are negative, they make the inertia $\mu(\chi, x)$ smaller than 1 and hence cause the inflaton to roll down its potential more rapidly than that in the classical model. Figure 7 compares the k -essence model (derivative), with the model of nonderivative couplings (nonderivative) and the classical result for an inflaton coupled to conformal scalars whose coupling strength is $h^2 = 1.6 \times 10^{-5}$ ($1/k_c^2 = 2.218 \times 10^5$). Figure 8 presents a similar comparison, at the same coupling constant, for an inflaton coupled to minimally coupled scalars. There is little distinction between conformal and minimal coupling. At this coupling strength, inflation ends at about ten e -foldings for the nonderivative model, whereas the derivative model nicely traces the classical evolution.

VI. CONCLUSIONS

Quantum-induced k -essence models, or cosmological Coleman-Weinberg potentials, are the price scalar-driven inflation pays for efficient reheating. Although coupling the differentiated inflaton to matter alters the kinetic energy instead of the potential, quantum-induced k -essence and cosmological Coleman-Weinberg potentials are the same functions of different arguments. On de Sitter background, this goes like H^4 multiplying a complicated function of the dimensionless parameters, $\mathcal{L}/m_c^3 H$ for fermions, and $\mathcal{L}/m_c^2 H^2$ for scalars. Recent work [17–19] strongly supports the idea that these de Sitter results remain approximately valid when the constant Hubble parameter of de Sitter is replaced by the evolving $H(t)$ of realistic inflation. A significant difference between quantum-induced k -essence models and cosmological Coleman-Weinberg potentials is that the dimensionless parameters of the former *increase* as time progresses, so that the system remains in the large-field domain over the course of inflation. As a result, changes due to the quantum correction are enormously large.

Because $H(t)$ is not a local functional of the metric for a general geometry, neither quantum-induced k -essence nor cosmological Coleman-Weinberg potentials can be completely eliminated using local counterterms [5]. The

technique of *Hubble subtraction* consists of subtracting the local counterterm which results from setting $H(t)$ to its initial value. One can compare the dimensionful coupling constants of quantum-induced k -essence with the dimensionless coupling constants of cosmological Coleman-Weinberg potentials by requiring the two couplings to have the same strength during reheating. This paper is devoted to comparing quantum-induced k -essence with cosmological Coleman-Weinberg potentials, with and without Hubble subtraction, regarding the competing requirements of facilitating efficient reheating without disturbing the observational constraints [Eqs. (3)–(6)] on primordial inflation.

Fermionic k -essence (derivative) and fermionic Coleman-Weinberg potentials (nonderivative) both tend to lengthen inflation, albeit for different reasons. One can understand this by looking at the scalar evolution equation simplified by the slow-roll approximation:

$$\phi'(n) \approx \frac{-1 [U'_{\text{cl}}(\phi) + \Delta U'(\phi)]}{3\chi^2 (1 + g(\chi, z))}, \quad z \equiv \frac{\kappa}{k_c^3 \chi} \left(\frac{\mathcal{L}}{m_c^3 H} \right). \quad (73)$$

Nonderivative couplings induce $\Delta U'(\phi) < 0$ and $g(\chi, z) = 0$, so they decrease $|\phi'(n)|$ (and hence increase the duration of inflation) by decreasing the force term. In contrast, derivative couplings induce $\Delta U'(\phi) = 0$ with $g(\chi, z) > 0$, so $|\phi'(n)|$ is also decreased, but now by increasing the inflaton's inertia $\mu(\chi, z) = 1 + g(\chi, z)$. However, fermionic k -essence enters the quantum-dominated regime even at a quite small effective coupling $\lambda \sim 6.3 \times 10^{-5}$, for which the cosmological Coleman-Weinberg potential would be small compared to the classical potential. At the same coupling strength, the quantum effect in the derivative model is generically stronger than it is for the nonderivative one. This is why Figs. 4 and 5 show longer durations for inflation, and larger dimensionless Hubble parameters after the end of inflation, for fermionic k -essence models than for the analogous cosmological Coleman-Weinberg potentials. For nonderivative couplings, inflation does not even end ($\epsilon = 1$)

unless the coupling is less than about $\lambda \sim 1.15 \times 10^{-4}$ [6], whereas this occurs for derivative couplings at an even smaller equivalent value of $\lambda \sim 6.98 \times 10^{-5}$. Neither fermionic model (derivative or nonderivative) begins to have a reasonable evolution until the coupling is made very small, which could endanger reheating.

For scalars, both derivative and nonderivative couplings induce models which are within the classical domain at the small effective coupling constant of $h^2 = 1.6 \times 10^{-5}$. Scalar-induced k -essence suffers from a disastrous kinetic instability [that is, $g(\chi, z) < -1$ in Eq. (73) with $z = \mathcal{L}/m_c^2 H^2$] if it leaves the classical-dominated regime. From Figs. 7 and 8, one can see that both derivative and nonderivative models tend to shorten inflation, with an especially rapid termination of inflation for the nonderivative model. One can understand this from the analog of Eq. (73) with $z = \mathcal{L}/m_c^2 H^2$. Nonderivative couplings induce $\Delta U > 0$ with $g = 0$, so they increase $|\phi'(n)|$ by strengthening the force. Derivative couplings induce $\Delta U = 0$ with $g < 0$, which increases $|\phi'(n)|$ by reducing the inertia. We found no significant differences between minimally coupled and conformally coupled scalars, presumably because the (derivative or nonderivative) coupling is larger than the conformal coupling. Generally speaking, coupling the differentiated inflaton to scalars seems to do a better job than the nonderivative coupling. However, neither coupling provides a very satisfactory resolution of the tension between facilitating efficient reheating and preserving the observational constraints [Eqs. (3)–(6)].

Because dimensionful coupling constants are introduced to quantum-induced k -essence, we would like to digress in order to comment on the validity of these models as low-energy effective field theories. Even though our couplings [Eqs. (7) and (21)] are not renormalizable, they induce no higher-derivative counterterms [Eqs. (11) and (26)] as long as the inflaton kinetic factor [Eq. (22)] is considered to be constant. Without inflaton loops, the only way one gets higher loop corrections is from the interaction with gravity. Hence, the two-loop contribution would take the form of $G(\frac{\partial\varphi\cdot\partial\varphi}{m_c^3})^6$, and its effect is suppressed by G . As a result, we recognize the cutoff of this effective field theory as the Planck mass. Inflation is assumed to occur well below the Planck mass. Note that the value of the quantum gravitational loop counting parameter GH^2 (obtained from the upper bound of the tensor-to-scalar ratio and the scalar power spectrum [4]) is about 10^{-11} . So, the work considered here should be safely within the realm of validity of low-energy effective field theory.

There is so far no consensus about the magnitude of the reheate temperature. The common belief is that it could be as low as 10^2 GeV or as high as 10^{19} GeV. If one extrapolates the initial form of the potential to the end of inflation, one finds a large reheate temperature. This is done by employing observational data on primordial perturbations to estimate

the number of e -foldings since the end of inflation, and then comparing that with a thermal estimate of the same quantity. Using WMAP data, Martin and Ringeval derived a bound of $T_R > 10^4$ GeV [24], and more recent data raise this bound. Please see the Appendix for a detailed explanation. Accommodating a lower reheat temperature requires dramatic changes in the shape of the potential after the emission of currently observable perturbations.

Leaving aside the issue of what geometric and thermal considerations imply for the value of the reheate temperature, we can estimate T_R from the dynamics of the coupling. For the fermionic k -essence model considered in this paper, if one imagines that the interaction is a sort of effective Yukawa coupling, the reheate temperature can be estimated when the Hubble parameter falls below the decay rate $\Gamma = \frac{\lambda^2 m}{8\pi}$ (where m is the mass of the inflaton) [6,25,26]:

$$T_R \simeq \frac{1}{5} \left(\frac{\Gamma^2}{G} \right)^{\frac{1}{4}} \simeq \lambda \times 10^{15} \text{ GeV}. \quad (74)$$

As we have seen, the largest value of λ which is consistent with viable inflation is $\lambda \sim 10^{-5}$. This corresponds to a reheate temperature of $T_R \sim 10^{10}$ GeV.

An alternative approach would be to make no subtractions and attempt instead to cancel the positive kinetic corrections induced by fermions with the negative ones induced by scalars. Of course, coupling the inflaton to more fields aids reheating, so there are no worries on that score. By adjusting the effective coupling constants λ and h^2 's, one could manage to get the leading terms of the large-field expansions of Eqs. (20), (35), and (39) to cancel. The question then becomes how the lower-order terms affect the observational constraints in Eqs. (3)–(6).

In addition to studies on axionic couplings like $\varphi e^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\sigma}$ by Adshead and collaborators [27], another alternative is to investigate derivative couplings to vector bosons, analogous to the undifferentiated couplings already studied for a charged inflaton [5–7,19,28],

$$\mathcal{L} = \frac{R\sqrt{-g}}{16\pi G} - \frac{1}{4} F_{\rho\sigma} F_{\mu\nu} g^{\rho\mu} g^{\sigma\nu} \sqrt{-g} - (\partial_\mu - iqA_\mu)\varphi^*(\partial_\nu + iqA_\nu)\varphi g^{\mu\nu} \sqrt{-g}. \quad (75)$$

One might explore some exotic derivative coupling of uncharged inflatons such as

$$-\frac{1}{4m_c^4} \partial_\alpha \varphi \partial_\beta \varphi g^{\alpha\beta} \times F_{\rho\sigma} F_{\mu\nu} g^{\rho\mu} g^{\sigma\nu} \sqrt{-g}. \quad (76)$$

We might consider its undifferentiated cousin to be

$$-\frac{1}{4m_c^2} \varphi^2 F_{\rho\sigma} F_{\mu\nu} g^{\rho\mu} g^{\sigma\nu} \sqrt{-g}. \quad (77)$$

To carry out studies with such derivative couplings, one first needs to figure out the coincidence limit of the field strength propagator on de Sitter background,

$$\langle F_{\rho\sigma}(x)F_{\mu\nu}(x)g^{\rho\mu}g^{\sigma\nu} \rangle. \quad (78)$$

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APPENDIX: CONNECTING DATA TO THE REHEAT TEMPERATURE T_R

In this appendix, we begin by presenting two approaches to estimate the number of e -foldings from the end of inflation to the current time. Comparison of these results implies that a large T_R is favored. To facilitate the discussion, we define $N \equiv \ln\left[\frac{a(t)}{a_i}\right]$ as the number of e -foldings since the start of inflation to comoving time t . If we follow the usual practice that the current scale factor is one ($a_0 = 1$), then the number of e -foldings between any event and now is

$$N_0 - N = \ln\left[\frac{1}{a(t)}\right]. \quad (A1)$$

We begin by using the geometry of inflation to estimate the number of e -foldings from the end of inflation to now [29]. Primordial perturbations which are today observed with a comoving wave number k experienced their first horizon crossing at a time t_k during inflation with $k \equiv H(t_k)a(t_k)$. The number of e -foldings from then to now is

$$N_0 - N_k = \ln\left[\frac{H(t_k)}{k}\right] \simeq \frac{1}{2} \ln\left[\Delta_{\mathcal{R}}^2(k) \frac{\pi\epsilon(t_k)}{Gk^2}\right], \quad (A2)$$

where the final step follows from the approximate form of the scalar power spectrum, $\Delta_{\mathcal{R}}^2(k) \simeq \frac{GH^2(t_k)}{\pi\epsilon(t_k)}$. Now, we substitute the simple power law formula used to represent the observational data in terms of a scalar amplitude $A_s \simeq 2 \times 10^{-9}$ and spectral index $n_s \simeq 1 - 0.035$ around a pivot wave number of $k_0 = 0.05 \text{ Mpc}^{-1}$:

$$\begin{aligned} \Delta_{\mathcal{R}}^2(k) &\simeq A_s \left(\frac{k}{k_0}\right)^{n_s-1} \Rightarrow N_0 - N_{k_0} = \frac{1}{2} \ln\left[\frac{A_s \pi r}{16Gk_0^2}\right] \\ &\simeq 122.33 + \frac{1}{2} \ln\left[\frac{r}{0.036}\right]. \end{aligned} \quad (A3)$$

The final logarithm would vanish if the tensor-to-scalar ratio were resolved at its current upper limit; otherwise, it would be negative. We stress that Eq. (A3) derives from geometry and observation; it cannot be evaded (within the

context of single-scalar inflation), no matter what assumptions are made about the inflationary potential or the mechanism of reheating. It is rather our assumptions about these two things which must accommodate Eq. (A3).

Consider first the duration of inflation after the pivot wave number experienced its first horizon crossing. We will see that increasing this interval increases the reheat temperature. Trivial calculus allows us to express this as an integral over the first slow-roll parameter,

$$N_e - N_{k_0} = \int_{\epsilon(t_{k_0})}^1 \frac{d\epsilon}{\epsilon'}. \quad (A4)$$

Primes on ϵ denotes derivatives with respect to the number of e -foldings. At this point, the inflationary potential becomes relevant. When these estimates were first made in 2010 [29], the quadratic potential was employed to conclude $\epsilon' = 2\epsilon^2$, and the resulting integral was evaluated to $2/(1 - n_s) - \frac{1}{2} \simeq 56.64$. Since then, the increasingly tight upper bounds on r (and hence on $\epsilon = r/16$) have favored very flat potentials, which give an even larger result. To understand this, consider the slow-roll relation for the spectral index under the assumption of very small ϵ with n_s fixed (which is supported by the absence of evidence for running of the spectral index [4]):

$$1 - n_s = 2\epsilon + \frac{\epsilon'}{\epsilon} \Rightarrow \epsilon' = (1 - n_s)\epsilon - 2\epsilon^2 \simeq (1 - n_s)\epsilon. \quad (A5)$$

Substituting this approximation into the integral (A4) gives

$$\begin{aligned} N_e - N_{k_0} &\simeq \frac{1}{1 - n_s} \int_{\epsilon(t_{k_0})}^1 \frac{d\epsilon}{\epsilon} = \frac{\ln\left(\frac{16}{r}\right)}{1 - n_s} \\ &\simeq 28.57 \left(6.10 - \ln\left[\frac{r}{0.036}\right]\right). \end{aligned} \quad (A6)$$

Of course, expression (A6) is larger than Eq. (A3), which is impossible, but it indicates the general trend towards large reheat temperatures. To facilitate the discussion, we made a numerical computation using the Einstein frame version of Starobinsky's model [30], which gives $N_e - N_{k_0} \simeq 51.32$. Because this number is smaller than either the result for quadratical potential or our general estimate in Eq. (A6), we can regard it as a sort of lower bound. Combining this figure with Eq. (A3) gives

$$\begin{aligned} \Delta N &\equiv N_0 - N_e = \frac{1}{2} \ln\left[\frac{A_s \pi r}{16Gk_0^2}\right] - 51.32 \\ &\simeq 71.01 + \frac{1}{2} \ln\left[\frac{r}{0.036}\right]. \end{aligned} \quad (A7)$$

Note that inserting the much lower value of r predicted for the Starobinsky model would decrease ΔN , thereby increasing the reheat temperature.

We can use thermodynamics to estimate the number of *e*-foldings from the end of inflation to now, $\Delta N \equiv N_0 - N_e$. The estimate is based on considering three periods [29]:

- (1) The interval from the end of inflation to the end of reheating.
- (2) The interval from the end of reheating to recombination.
- (3) The interval from recombination to now.

The (approximately matter-dominated) energy density at the end of inflation ρ_e derives from the kinetic energy of the inflaton. We assume that this kinetic energy is approximately constant throughout inflation, which implies that it can be related to conditions at the time the pivot wave number experiences first horizon crossing:

$$\rho_e = \frac{1}{2} \dot{\phi}^2 = \frac{\epsilon H^2}{8\pi G} \simeq \frac{r^2 A_s}{2^{11} G^2}. \quad (\text{A8})$$

This energy density redshifts like nonrelativistic matter until reheating converts it into the energy density of g_* relativistic species,

$$\rho_R = \rho_e \left(\frac{a_e}{a_R} \right)^3 = \frac{g_* \pi^2 T_R^4}{30}. \quad (\text{A9})$$

Hence, the number of *e*-foldings from the end of inflation to the end of reheating is

$$N_R - N_e = \frac{1}{3} \ln \left[\frac{15 r^2 A_s}{2^{10} \pi^2 g_* G^2 T_R^4} \right]. \quad (\text{A10})$$

Entropy is conserved during the second period, which means

$$\frac{a_{\text{rec}}}{a_R} = \left(\frac{g_*}{2} \right)^{\frac{1}{3}} \times \frac{T_R}{T_{\text{rec}}} \Rightarrow N_{\text{rec}} - N_R = \frac{1}{3} \ln \left[\frac{g_* T_R^3}{2 T_{\text{rec}}^3} \right]. \quad (\text{A11})$$

Finally, from recombination to now, we have

$$\frac{a_0}{a_{\text{rec}}} = \frac{T_{\text{rec}}}{T_0} \Rightarrow N_0 - N_{\text{rec}} = \frac{1}{3} \ln \left[\frac{T_{\text{rec}}^3}{T_0^3} \right]. \quad (\text{A12})$$

Adding the three intervals (A10), (A11), and (A12) gives

$$\begin{aligned} \Delta N \equiv N_0 - N_e &= \frac{1}{3} \ln \left[\frac{15 r^2 A_s}{2^{11} \pi^2 G^2 T_R T_0^3} \right] \\ &\simeq 62.65 + \frac{2}{3} \ln \left[\frac{r}{0.036} \right] - \frac{1}{6} \ln(G T_R^2). \end{aligned} \quad (\text{A13})$$

Equating Eqs. (A7) and (A13), and assuming that the tensor power spectrum is resolved at the current upper limit, we find the reheat temperature to be $T_R \sim 10^8$ GeV. One can avoid this conclusion by fine-tuning the model to reduce $N_e - N_{k_0}$, but the preference for a large reheat temperature is clear.

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