Hidden physics in the decays of pions and other mesons

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It has been commonly assumed that pseudoscalar contributions to the leptonic decay of charged mesons, like pions and kaons, is strongly constrained due to the helicity suppression present in the ratio $R_{l/l'} = \Gamma(P \to l\nu[\gamma])/\Gamma(P \to l'\nu[\gamma])$, where P are the charged pseudoscalar meson and $l, l' = e, \mu, \tau$. Here we show that if the effective couplings are proportional to the corresponding charged lepton masses (and also the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix), the constraints from $R_{l/l'}$ are entirely avoided, and a rather new large allowed region is permitted in the parameter space. In the case of the electron, we found a nontrivial region in the range $10^{-4} \lesssim (G^\eta/G_F) \lesssim 10^{-3}$, where G^η is the effective pseudoscalar coupling associated with a novel charged scalar field, η, and G_F is the Fermi constant. Furthermore, we show that this dependence of the pseudoscalar couplings on the charged lepton masses can naturally be associated with a critical class beyond the standard model physics, namely models without (leptonic) flavor-changing neutral currents in the scalar sector. The most known examples are the models that satisfy the so-called Glashow-Weinberg-Paschos theorem. Finally, we also point out that, in those cases, the decay rate is degenerated with the Standard Model prediction, possibly hiding the new physics effects in those decays.

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I. INTRODUCTION

In most possible new physics beyond the Standard Model (BSM), even when considering their minimal versions, a more complex scalar sector is encountered than the simple neutral Higgs present in the particle spectrum of the Standard Model (SM) [1]. Even in the context of the $SU_L(2) \times U_Y(1)$ gauge symmetry, nothing limits the number of scalar fields. However, at least a single doublet is necessary for the usual spontaneous symmetry-breaking pattern. Thus, one cannot rule out the possibility that extra scalars, heavier than the observed Higgs (or lighter, but with sufficiently weak couplings), exist. Moreover, many mechanisms to generate neutrino masses require additional scalars [2–4]. Nevertheless, in principle, such particles can solve some anomalies in high-energy experiments, like those in B-meson decays [5,6] or the muon anomalous magnetic moment [7,8], for example.

If those extra scalars exist, they may modify several well-known processes, such as leptonic pion decay. Such a process has an astonishing agreement between the experimental results and the SM theoretical calculations, often used as a hallmark of the weak interactions' V-A structure. Moreover, its helicity suppression explains the dominant decay in muons (99.99%), not electrons. Therefore, strong constraints on new physics (especially from pseudoscalar interactions) are possible in this decay [5,9–39].

The main goal of this work is to stress that this parameter space region in the leptonic decays with pseudoscalar interactions could be hidden in well-motivated scenarios. Theoretically, in models with a Glashow-Weinberg-Paschos (GWP) mechanism implemented and phenomenological using the helicity-suppressed ratio as observable, we automatically cancel new physics effects, rendering these well-known tests for charged scalar new physics ineffective. Therefore, we choose to use a fundamental Lagrangian, in order to analyze this kind of decays under the model building approach.

II. CHARGED MESON DECAY

Consider the leptonic decay of charged mesons, $P^+ \to \ell^+\nu[\gamma]$ (henceforth denoted by $P_{\ell 2}$), in the presence of a

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novel pseudoscalar interaction among the SM fermions. The low-energy effective Lagrangian, in this case, is given by

$$-\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ij} \Big\{ \bar{u}_i \gamma_\mu (1-\gamma_5) d_j \cdot \overline{\ell}_l \gamma^\mu (1-\gamma_5) \nu_l + \frac{\mathcal{G}^\eta_{ij,ll'}}{G_F} (\bar{u}_i \gamma_5 d_j) \cdot \overline{\ell}_l (1-\gamma_5) \nu_{l'} \Big\} + \text{H.c.}, \tag{1}$$

where G_F is the tree-level Fermi constant, V_{ij} is an element of the Cabbibo-Kobayashi-Maskawa (CKM) matrix [40,41], and $\mathcal{G}^\eta$ is the effective coupling matrix of the new four-fermion interaction in the neutrino interaction basis. Notice that we also factored out a CKM matrix element from the new physics term. Each meson P fixes the corresponding quark indices i, j, while the lepton indices assume the values $l, l' = e, \mu, \tau$. The SM effective contribution, mediated by a W-boson exchange, corresponds to the first term between curly brackets in the above equation. Unless explicitly stated otherwise, all repeated indices are summed over throughout this paper.

As is well known, the left-handed neutrino fields $\nu_{lL} = \frac{1}{2}(1-\gamma_5)\nu_l$ are actually a linear combination of the mass eigenstates ν_{kL},

$$\nu_{lL} = U_{lk} \nu_{kL}, \tag{2}$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [42,43]. Therefore, in the neutrino mass basis, the effective matrix coupling for the new interaction is given by

$$G^\eta_{ij,lk} \equiv \mathcal{G}^\eta_{ij,ll'} U_{l'k}. \tag{3}$$

The most simple realization of the new effective operator in Eq. (1), and the one we will be interested in, is through the Yukawa interaction of the SM fields with a new charged scalar field η

$$-\mathcal{L} = \bar{u}_i (c^s_{ij} + c^p_{ij} \gamma_5) d_j \eta^+ + X_{lk} \overline{\ell}_{lR} \nu_{kL} \eta^- + \text{H.c.}, \tag{4}$$

where c^s and c^p are, respectively, the scalar and pseudoscalar Yukawa couplings in the quark sector and X is the matrix of Yukawa couplings in the lepton sector. Matching the interactions in Eqs. (1) and (4), and using the relation in Eq. (3), we have that

$$\frac{G^\eta_{ij,lk}}{\sqrt{2}} = \frac{c^p_{ij}}{V_{ij}} \frac{X_{lk}}{m^2_\eta}, \tag{5}$$

with m_η being the mass of the new scalar field.

To calculate the amplitude for $P_{\ell 2}$, the following matrix elements are needed (all other matrix elements are null for pseudoscalar mesons)

$$\langle 0|\bar{u}_i \gamma^\mu \gamma_5 d_j |P^+(k)\rangle = i k^\mu f_P, \qquad \langle 0|\bar{u}_i \gamma_5 d_j |P^+(k)\rangle = i\tilde{f}_P, \tag{6}$$

where k^μ is the meson linear momentum, f_P is the corresponding meson decay constant, and $\tilde{f}_P$ and f_P are related by the identity [44]

$$\frac{\tilde{f}_P}{f_P} = \frac{m^2_P}{m_{u_i} + m_{d_j}} \equiv B_P, \tag{7}$$

with m_P being the charged meson mass, and m_{u_i} and m_{d_j} the bare masses of its constituent quarks. Notice that, due to the quark masses, the B_P factor depends on both the renormalization scale and scheme, that is, $B_P = B_P(\mu)$ [13,16,45–50]. Our analysis were entirely made using $\mu = 2$ GeV in the $\overline{\rm MS}$ scheme, and we only consider the pseudoscalar contribution. The only exception being the analysis of the B-meson, where we first calculated using $\mu = m_b$, and then we go to $\mu = 2$ GeV, using the renormalization group equations. For the running of the Wilson coefficients, we follow Ref. [51]. Using their notation, the relevant coefficients are ε_S, ε_P, and ε_T, which corresponds to effective couplings of scalar $[(\bar{u}d)(\overline{\ell}P_L\nu)]$, pseudoscalar $[(\bar{u}\gamma^5 d)(\overline{\ell}P_L\nu)]$, and tensor $[(\bar{u}\sigma^{\alpha\beta}P_L d)(\overline{\ell}\sigma_{\alpha\beta}P_L\nu)]$ interactions, respectively (notice that their ε_P is equivalent to G^η in our notation). In particular, we need the running of a_P from $\mu = 2$ GeV to $\mu = m_b$. From Ref. [51], we have that $\varepsilon_P(\mu = 2\text{ GeV}) \approx 1.178\varepsilon_P(\mu = m_b)$, which directly translates to our case as $a_P(\mu = 2\text{ GeV}) \approx 1.178 a_P(\mu = m_b)$. Another effect we might expect by the running of renormalization group equation is the generation of new Wilson coefficients in a different scale. In this case, we have from Ref. [51] that $\varepsilon_S(\mu = 2\text{ GeV})/\varepsilon_P(\mu = m_b) \approx 10^{-8}$ and $\varepsilon_T(\mu = 2\text{ GeV})/\varepsilon_P(\mu = m_b) \approx 10^{-5}$, hence the scalar and tensor contributions are heavily suppressed with respect to the pseudoscalar contribution.

With the above assumptions, the total decay rate for $P_{\ell 2}$, in the meson rest frame, is given by

$$\Gamma_l = \Gamma^{\rm SM}_l \times (1 + \Delta_l), \tag{8}$$

where

$$\Gamma^{\rm SM}_l = r_l \frac{G^2_F}{8\pi m^3_P} f^2_P V_{ij} m^2_l (m^2_P - m^2_l)^2 \tag{9}$$

corresponds to the usual Standard Model rate, including radiative corrections r_l for soft photons [52,53],

$$\Delta_l = \frac{B_P}{m_l} \sum_{k=1}^{3} \left[\left(\frac{|G^\eta_{lk}|}{G_F} \right)^2 \frac{B_P}{m_l} - 2\text{Re}\left(U_{lk} \frac{(G^\eta)^*_{lk}}{G_F} \right) \right], \quad (l = e, \mu, \tau) \tag{10}$$

quantifies the presence of new physics beyond the SM, and m_l is the mass of the final state-charged lepton. For simplicity, we omitted the quark indices in G^η, since they are fixed for each meson. Terms proportional to the neutrino masses were neglected, and for each charged lepton state, we summed over all the active neutrino mass eigenstates ($k = 1, 2, 3$). In Eq. (10), the first term inside square brackets comes purely from the pseudoscalar interaction, and the latter corresponds to the interference between the SM contribution and the new interaction.

Experimental results require $|\Delta_l| \ll 1$. For example, for pion decay, we must have $|\Delta_l| \lesssim 10^{-3}$, using the central value of the experimental result and the current experimental uncertainties. Therefore, an agreement of the experimental data and the SM prediction can be possible in BSM scenarios if we set $\Delta_l \equiv 0$, rendering the new physics contributions hidden for these observable. For example, most analyses present in the literature, as in [9,10,16], assume that, for each meson P, the effective couplings G^η_{lk} has a similar size for all charged leptons and neutrino states. Therefore, due to the enhancement factor B_P/m_l present in Eq. (10), the most sensible channel for new physics is the decay with electrons in the final state. Here, in this paper, we make two different assumptions, namely (i) that the effective coupling G^η_{lk} depends directly on the charged lepton masses, and (ii) that the neutrino flavor is conserved. That is, *we assume* that, in the neutrino mass basis,

$$G^\eta_{lk} = a_P m_l U_{lk}, \quad (\text{no sum in } l) \tag{11}$$

where a_P depends only on the decaying charged meson and the scalar field η that mediates the interaction. Although we, again, omitted the quark indices in the above expression, a subscript P was added to a_P to remind the reader that this quantity can be different to each meson. Using Eq. (3), the corresponding expression in the neutrino flavor basis is given by

$$\mathcal{G}^\eta_{ll'} = a_P m_l \delta_{ll'}, \quad (\text{no sum in } l) \tag{12}$$

where $\delta_{ll'}$ is the usual Kronecker delta.

Using Eq. (11), and the unitarity of the PMNS matrix, the new physics contribution given in Eq. (10) becomes

$$\Delta_l = B_P\left[\left(\frac{|a_P|}{G_F}\right)^2 B_P - 2\mathrm{Re}\left(\frac{a_P^*}{G_F}\right)\right] \equiv \Delta, \tag{13}$$

Furthermore, all the charged leptons are equal, although it can differ for each meson.

In the literature, in order to avoid uncertainties coming from f_P, the ratio

$$R_{l/l'} = \frac{\Gamma(P \to l\nu[\gamma])}{\Gamma(P \to l'\nu[\gamma])} = R^{\mathrm{SM}}_{l/l'}\left(\frac{1+\Delta_l}{1+\Delta_{l'}}\right), \quad (l, l' = e, \mu, \tau), \tag{14}$$

is commonly used to constrain new physics [9,10,16]. However, as we just saw, for our solution in Eq. (11), Δ is independent of the final lepton states. Therefore, all new contributions for these ratios are automatically canceled, that is

$$R_{l/l'} = R^{\mathrm{SM}}_{l/l'}, \tag{15}$$

irrespective of the value of a_P. With this, the usual helicity suppression in meson decays within the SM is recovered, and the strong constraints usually assumed to come from these observables will not apply. Although the fact that, with an *Ansatz* as in Eq. (11), the ratios coincide with its SM value had already been mentioned in the literature cited above, no statistical analysis of the allowed parameter space using the individual rates has ever been performed, to our knowledge, being also a novelty of this work.

Besides automatic canceling new contributions for the ratio $R_{l/l'}$, the effective coupling of Eq. (11) can be chosen such that the effects on the individual rates Γ_l also vanish, rendering the new physics contribution on the leptonic decays of pseudoscalar mesons completely hidden. Then, taking $\Delta = 0$, we find that a_P must satisfy

$$|a_P|^2 - \frac{2G_F}{B_P}\mathrm{Re}(a_P) = 0. \tag{16}$$

A trivial solution for Eq. (16) would be $a_P = 0$, and perturbations around this solution correspond to a weakly coupled scalar, either because the associated Yukawa couplings in Eq. (5) are small or because the scalar has a huge mass. However, other nontrivial solutions are possible and will be discussed in the following subsections. However, other nontrivial solutions are possible and will be discussed in the following subsections. Moreover, as we will show in Sec. III, such a nontrivial solution can naturally arise in models where flavor-changing neutral currents (FCNC) are absent in the scalar sector at tree level. In such models, constraints coming from charged meson decays can be less stringent than one would first assume. Nevertheless, before we enter such a discussion, let us analyze the allowed parameter space for a nontrivial solution of Eq. (11).

A. Real solutions

If we assume that a_P is a (nonzero) real parameter, Eq. (16) simplifies to

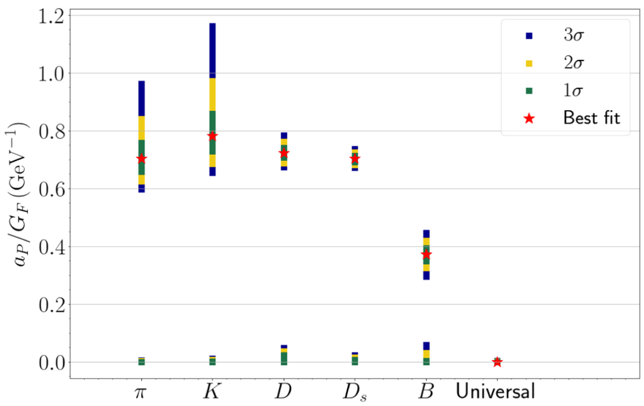

FIG. 1. Allowed region for new physics for each pseudoscalar meson π, K, D, D_s, and B, up to 3σ, assuming a real parameter a_P in Eq. (11). We also show the analysis fitting the experimental data (as detailed in the Appendix) for all five mesons considering a universal parameter on the quark sector. The allowed region, in this case, is hidden behind the best-fit marker.

$$\frac{a_P}{G_F} = \frac{2}{B_P}. \tag{17}$$

Figure 1 shows the allowed region for a_P constrained by the experimental values of the decay rates for each meson $P = \pi, K, D, D_s$, and B. As can be seen, two distinct regions arise. The first one, with $a_P \approx 0$, is compatible with a full dominant SM process, as discussed above. The second region, with a_P/G_F values of the order of GeV^{-1}, corresponds to the nontrivial solution given in Eq. (17), and will be called, henceforth, the *nontrivial region*. A nontrivial region not only appears as a possible solution to all the mesons considered but is slightly preferred by our statistical analysis, as indicated by the best points.

As seen from Fig. 1, the nontrivial regions occur within the same range for all the mesons except for the B meson. Since the quark condensates are approximately equal for the lightest quarks, the factor B_P, defined in Eq. (7), turns out to be almost identical for the lightest mesons. The deviations in B_P are larger for mesons containing heavier quarks like the B-meson [20]. This feature explains why the nontrivial region for the B-meson is considerably lower than the other mesons. In the last column of Fig. 1, we present a global fit for the leptonic decays of the five considered mesons considering a single universal parameter independent of the meson type, that is,

$$a_\pi = a_K = a_D = a_{D_s} = a_B. \tag{18}$$

No nontrivial universal solution was found in this last analysis considering all five mesons, and only solutions around the SM value appear. However, if we consider an "universal" solution for π and K, that is, $a_\pi = a_K$, the ratio $\pi_{\mu 2}/K_{\mu 2}$, used in some nonstandard searches analyses in the literature to reduce lattice uncertainties [54,55], cancels the new physics contribution for any value of $a_P = a_\pi = a_K$, similarly from what happens in the case of $R_{l/l'}$. Therefore, we see that it can avoid strong constraints from this observable in this case. It is also possible to note from Fig. 1 that the parameter space for pion and kaon overlap at 1σ even at the nontrivial region, so an "universal" solution of this kind is not excluded by these decays alone.

As a numerical example, for the particular case of pion decay, and using the best-fit point present in the nontrivial region in Fig. 1, we have that the effective coupling G^η_{lk} is given by

$$\frac{G^\eta_{lk}}{G_F} \approx \begin{pmatrix} 3\times 10^{-4} & 2\times 10^{-4} & 5\times 10^{-5} \\ -3\times 10^{-2} & 4\times 10^{-2} & 5\times 10^{-2} \\ 4\times 10^{-1} & 8\times 10^{-1} & 8\times 10^{-1} \end{pmatrix} \tag{19}$$

in the neutrino mass basis, or

$$\frac{\mathcal{G}^\eta_{ll'}}{G_F} \approx \begin{pmatrix} 4\times 10^{-4} & 0 & 0 \\ 0 & 7\times 10^{-2} & 0 \\ 0 & 0 & 1.2 \end{pmatrix} \tag{20}$$

in the respective neutrino flavor basis. We point out that, in the neutrino mass basis, for any fixed line, each column element differ from the others due to the presence of the PMNS matrix in our assumption given in Eq. (11), which includes the negative signs appearing in Eq. (19). Finally, in both neutrino bases, the hierarchy between every two lines directly results from the charged lepton masses dependence.

B. Complex solutions

In general, the a_P parameter will be complex. In this case, Eq. (16) can be written as

$$\left(\frac{a^r_P}{G_F} - \frac{1}{B_P}\right)^2 + \left(\frac{a^i_P}{G_F}\right)^2 = \left(\frac{1}{B_P}\right)^2, \tag{21}$$

where $a^r_P = \mathrm{Re}(a_P)$ and $a^i_P = \mathrm{Im}(a_P)$ are the real and imaginary parts of a_P, respectively. Equation (21) describes a circle of radius B_P^{-1}, centered in $(B_P^{-1}, 0)$ in the $a^r_P/G_F \times a^i_P/G_F$ plane. From Eq. (21), it is easy to see that, for $a^i_P = 0$, we can recover the real nontrivial solution of Eq. (17), while for $a^r_P = 0$ the only possible solution is $a^i_P = 0$.

Figure 2 shows the allowed region for a complex a_P for the case π_{l2}. As can be seen, apart from the uncertainties, a connected circular region is encountered. Now, solutions compatible with a full-dominant SM contribution correspond to small perturbations around the point (0,0) in the $a^r_P/G_F \times a^i_P/G_F$ plane, while the rest of the circle corresponds to a nontrivial solution of Eq. (21).

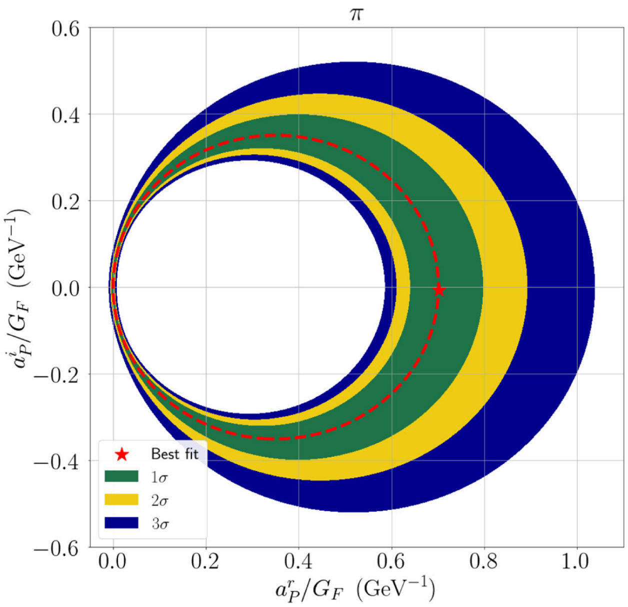

FIG. 2. Allowed region for the meson π, up to 3σ, assuming a complex parameter a_P. The red dashed curve corresponds to the fine-tuned (exact) solution of Eq. (21). However, as can be seen from the above figure, a larger allowed region appears beyond this fine-tuned solution.

III. MODELS WITH FLAVOR CONSERVING NEUTRAL CURRENTS

In the previous section, we propose and study a nontrivial solution for the new physics, given in Eq. (11), that is fully compatible with the experimental data, that is, with $\Delta_l = 0$. In this section, we want to show that, within reasonable assumptions, the structure proposed in Eq. (11) can naturally arise in a broad class of models. Namely, those assumptions are: (i) the charged scalar field η^+, introduced in Eq. (4), can be associated with a neutral scalar field, and (ii) the corresponding scalar neutral interactions with the leptons both conserves flavor and generates the charged lepton masses.

To illustrate our arguments, we restrict ourselves to a model with the same fermion content as the SM but with N-Higgs doublets in the scalar sector. Generalizations to more complex models satisfying the two hypotheses above can easily be made. In this model, the most general Yukawa interactions are given by

$$-\mathcal{L}_{\text{Yuk}} = \sum_{n=1}^{N}\{\lambda_{ab}^{\ell,n}(\bar{L}_a\phi_n)\ell'_{bR} + \lambda_{ab}^{u,n}(\bar{Q}_a\tilde{\phi}_n)u'_{bR} + \lambda_{ab}^{d,n}(\bar{Q}_a\phi_n d'_{bR})\} + \text{H.c.}, \tag{22}$$

where $L_a = (\nu'_{aL}, \ell'_{aL})^T$ and $Q_a = (u'_{aL}, d'_{aL})^T$ are the usual fermion doublets; the right-handed fields are all singlets; $\phi_n = (\varphi_n^+, \varphi_n^0)^T$ are the N scalar doublets, and $\tilde{\phi}_n = i\sigma_2\phi_n^*$. Finally, the indices a, $b = 1, 2, 3$ label the different fermion generations, and all fields corresponds to gauge symmetry states.

From Eq. (22) above, we see that when the neutral component of each scalar doublet develop a nonzero vacuum expectation value (VEV), $\langle\phi_n^0\rangle = v_n/\sqrt{2}$, the mass matrices for fermions of each charge sector are

$$M^f = \sum_{n=1}^{N}\frac{v_n}{\sqrt{2}}\lambda^{f,n}, \quad (f = \ell, u, d). \tag{23}$$

In general, the mass matrices given above are nondiagonal. However, they can be diagonalized by biunitary transformations such as

$$\hat{M}^f = (V_L^f)^\dagger M^f V_R^f, \quad (f = \ell, u, d), \tag{24}$$

where $\hat{M}^f$ is a diagonal matrix. For example, $\hat{M}^\ell = \text{diag}(m_e, m_\mu, m_\tau)$, and so forth.

Defining the fermion mass states[1] as $f_{L,R} = (V_{L,R})^\dagger f'_{L,R}$ $(f = \ell, u, d)$, the Yukawa interactions in Eq. (22) can be written in the fermion mass basis as (omitting the generation indices of the fermions)

$$\begin{aligned}-\mathcal{L}_{\text{Yuk}} = \sum_{n=1}^{N}\{&(\bar{\ell}_L Y^{\ell,n}\ell_R)\varphi_n^0 + (\bar{u}_L Y^{u,n}u_R)\varphi_n^{0*} \\ &+ (\bar{d}_L Y^{d,n}d_R)\varphi_n^0 + (\bar{\nu}_L U^\dagger Y^{\ell,n}\ell_R)\varphi_n^+ \\ &- (\bar{d}_L V^\dagger Y^{u,n}u_R)\varphi_n^- + (\bar{u}_L V Y^{d,n}d_R)\varphi_n^+\} + \text{H.c.},\end{aligned} \tag{25}$$

where $U \equiv (V_L^\ell)^\dagger V_L^\nu$ and $V \equiv (V_L^u)^\dagger V_L^d$ are, respectively, the PMNS and CKM mixing matrices, and

$$Y^{f,n} = (V_L^f)^\dagger\lambda^{f,n}V_R^f, \tag{26}$$

At this point, since the coupling matrices $Y^{f,n}$ are not diagonal, we see from Eq. (25) that flavor-changing neutral currents (FCNC) may occur in the scalar sector. This happened because the unitary transformations in Eq. (24) do not diagonalize individually the coupling matrices $\lambda^{f,n}$, they only diagonalize the combination given in Eq. (23).

Experimental results indicates that FCNC interactions must be heavily suppressed [56,57]. As shown by Glashow and Weinberg [58] and, independently, by Paschos [59], a natural way to avoid these FCNC interactions in the scalar sector, at tree-level, is to assume that only one Higgs multiplet couples to each charged sector, due to an appropriately chosen discrete or continuous symmetry. This result is part of what is known in the literature as the Glashow-Weinberg-Paschos (GWP) theorem.

[1]Although the N-Higgs model without RH neutrinos cannot generate neutrino masses, we also define $\nu_L = (V_L^\nu)^\dagger\nu'_L$.

A. Applying the GWP theorem to the lepton sector

First, we will explore the consequences of applying the GWP theorem to the lepton sector. Let ϕ_ℓ be the only doublet that couples to the charged lepton singlets, ℓ_R (notice that ϕ_ℓ corresponds to one, and only one, of the ϕ_n doublets). Then, as a result of the theorem, the only nonzero Yukawa coupling in the lepton sector is

$$Y^\ell = \frac{\sqrt{2}}{v_\ell}\hat{M}^\ell, \tag{27}$$

where v_ℓ is the VEV associated with ϕ_ℓ.

Since Y^ℓ is now diagonal, no scalar FCNC appears at tree-level in the lepton sector. Equation (27) also hold true in other situations where the GWP theorem does not apply, but FCNC are still suppressed. For example, in the so-called aligned models [19,31,60–65], where the Yukawa couplings are proportional one to another, or the Branco-Grimus-Lavoura (BGL) model [66], where the Yukawa entries are dependent only on CKM matrix elements and on the lepton masses [24,30,61,66].

Finally, we can write the charged scalar symmetry states φ_n^+ as a linear combination of the physical fields

$$\varphi_n^+ = \sum_{m=1}^{N-1} O_{nm}\eta_m^+ + O_{n,N-1}G^+, \tag{28}$$

where G^+ is the (massless) would-be Goldstone mode associated with the W^+-boson, and η_m^+ are $N-1$ massive scalar fields. In general, O is a $N\times N$ unitary matrix (or orthogonal if the scalar potential conserves CP).

Therefore, applying the GWP theorem to the lepton sector only, the Yukawa interactions with the novel physical fields η_m^+ are given by

$$-\mathcal{L}_{\rm Yuk}^{\eta^+} = \sum_{m=1}^{N-1}\left\{\frac{\sqrt{2}}{v_\ell}O_{\ell m}\bar{\nu}_L(U^\dagger\hat{M}^\ell)\ell_R + \bar{u}\left[V\sum_{n=1}^{N}(O_{nm}Y^{d,n}P_R - O_{nm}^*(Y^{u,n})^\dagger P_L)\right]d\right\}\eta_m^+ + \text{H.c.}, \tag{29}$$

where $P_{L,R} = \frac{1}{2}(1\mp\gamma^5)$ are the usual chiral projectors.

Comparing Eq. (29) with Eqs. (4) and (5), we find that

$$\frac{G^\eta_{ij,lk}}{\sqrt{2}} = \sum_{m=1}^{N-1}\frac{1}{m^2_{\eta_m^+}}\left[\sum_{n=1}^{N}(O_{nm}Y^{d,n}_{ij} + O^*_{nm}(Y^{u,n}_{ji})^*)\right]O_{\ell m}\left(\frac{m_l}{v_\ell}\right)U_{lk}\quad (\text{no sum in } l), \tag{30}$$

where $m_{\eta_m^+}$ are the scalar masses, and we remember the reader that only the pseudoscalar Yukawa coupling in the quark sector contribute to the meson decay $P_{2\ell}$.

For simplicity, if we assume that one of the scalar fields (which we will denote by η^+) have a dominant contribution, we got

$$\frac{G^\eta_{ij,lk}}{\sqrt{2}} = \frac{1}{m_\eta^2}c^p_{ij}\left(\frac{m_l}{v_\ell}\right)U_{lk},\quad (\text{no sum in } l) \tag{31}$$

with

$$c^p_{ij} = \sum_{n=1}^{N}(O_{n\eta}Y^{d,n}_{ij} + O^*_{n\eta}(Y^{u,n}_{ji})^*)O_{\ell\eta}. \tag{32}$$

In this way, we have shown that in the broad class of models that satisfy the GWP theorem (and hence avoid scalar FCNC in the lepton sector) the nontrivial solution given in Eq. (11), with

$$a_P = \frac{c^p}{m_\eta^2 v_\ell}, \tag{33}$$

can naturally arise.

B. Applying the GWP theorem to the quark sector

Although, as shown above, the main structure in Eq. (11) arises just by applying the GWP theorem to the lepton sector, even stronger bounds can be placed in FCNC from the quarks. For this reason, extending the above analysis to the quark sector is sensible.

Consider again the simple N-Higgs doublet model of the previous section. Assuming that the subset $\{\phi_\ell, \phi_u, \phi_d\}$ are the only doublets that give mass to the corresponding charge sector, we have that the only nonzero Yukawa couplings in Eq. (25) are

$$Y^f = \frac{\sqrt{2}}{v_f}\hat{M}^f,\quad (f = \ell, u, d), \tag{34}$$

where Y^f and v_f are the Yukawa coupling matrix and VEV associated with ϕ_f, respectively.

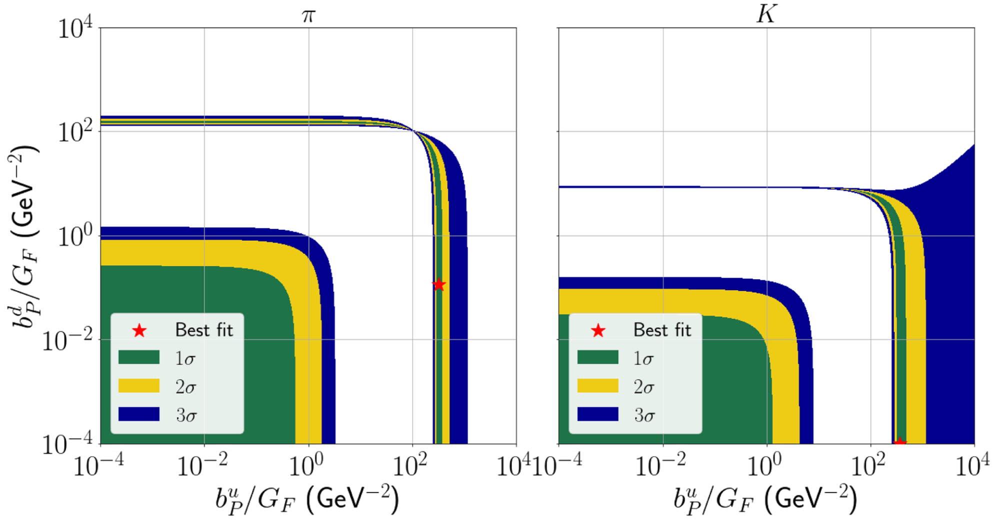

FIG. 3. Allowed region using pion (kaon) data left (right) considering the GWP mechanism in both the quark and the lepton sectors for the effective couplings b_P^u/G_F and b_P^d/G_F.

Hence, the Lagrangian in Eq. (29) becomes

$$-\mathcal{L}_{\rm Yuk}^{\eta^+} = \sum_{m=1}^{N-1} \frac{\sqrt{2}}{v} \{\bar{\nu}_L (U^\dagger \xi_m^\ell \hat{M}^\ell) \ell_R + \bar{u} V (\xi_m^d \hat{M}^d P_R - \xi_m^u \hat{M}^u P_L) d\} \eta_m^+ + \text{H.c.}, \quad (35)$$

where $v^2 = \sum_{n=1}^N v_n^2$, and

$$\xi_m^f \equiv \frac{v}{v_f} O_{fm}, \quad (f = \ell, u, d). \quad (36)$$

We immediately find that

$$\frac{G^\eta}{\sqrt{2}} = \sum_{m=1}^{N-1} \frac{1}{v^2 m_{\eta_m^+}^2} \left(\xi_m^d \hat{M}^d + \xi_m^u \hat{M}^u\right) \left(\xi_m^\ell \hat{M}^\ell U\right). \quad (37)$$

Or, if we assume that one of the scalar (denoted by η^+) dominates the new physics contribution, we have

$$\frac{G^\eta}{\sqrt{2}} = \frac{1}{v^2 m_\eta^2} \left(\xi^d \hat{M}^d + \xi^u \hat{M}^u\right) \left(\xi^\ell \hat{M}^\ell U\right). \quad (38)$$

Note that the effective coupling constant defined in Eq. (38) does not include the CKM matrix, as its elements were factored out on the effective Lagrangian in Eq. (1).

Comparing Eq. (38) with Eq. (11), we find that, for models where the theorem is valid in both the quark and lepton sectors,

$$a_P = b_P^u m_{u_i} + b_P^d m_{d_j}, \quad (39)$$

where u_i and d_j are the quarks present in the P meson, and we have defined

$$b_P^u = \frac{\xi^u \xi^\ell}{m_\eta^2 v^2}, \qquad b_P^d = \frac{\xi^d \xi^\ell}{m_\eta^2 v^2}, \quad (40)$$

for later convenience. Notice that the dimension of $b_P^{u,d}$ is different from that of a_P, due to the factorization of the quark masses.

For a_P real, we already saw that an exact nontrivial solution occurs for $a_P/G_F = 2/B_P$ [cf. Eq. (17)]. Now, using Eqs. (7) and (39), we can rewrite this solution as

$$\frac{1}{G_F}(b_P^u m_{u_i} + b_P^d m_{d_j}) = \frac{2}{m_P^2}(m_{u_i} + m_{d_j}). \quad (41)$$

Moreover, we see that a similar structure in the quark masses appears in both sides of Eq. (41).

The allowed parameter space for this case is shown in Fig. 3, for the most precise measurements of $\pi_{\ell 2}$ and $K_{\ell 2}$. Figure 4 shows the same analysis for the heavier mesons D, D_s, and B. Now, three different situations can occur (i) both $b_P^u \approx b_P^d \approx 0$, (ii) only one of the parameters $b_P^{u,d}$ are nonzero, and (iii) both parameters are nonzero. The first situation corresponds just to an SM-compatible solution, where the new physics is small compared to the SM contribution. In the second case, we are in a similar situation as that given in Fig. 1, and the nontrivial solution is given in Eq. (17) with the replacement $a_P \to b_P^u m_{u_i}$, $b_P^d m_{d_j}$. Comparing the allowed regions for pions and kaons in Fig. 3, we see that, for $b_P^u \neq 0$ and $b_P^d \to 0$, both corresponds to a similar range, and we have that $b_\pi^u = b_K^u$ is a possible solution; while for the opposite case, $b_P^u \to 0$ and $b_P^d \neq 0$, the asymptotic value for b_P^d is very different, and no solution of the form $b_\pi^d = b_K^d$ can occur. This happens due to the quark content of those mesons, while they share the same up-type quark content, the down-type

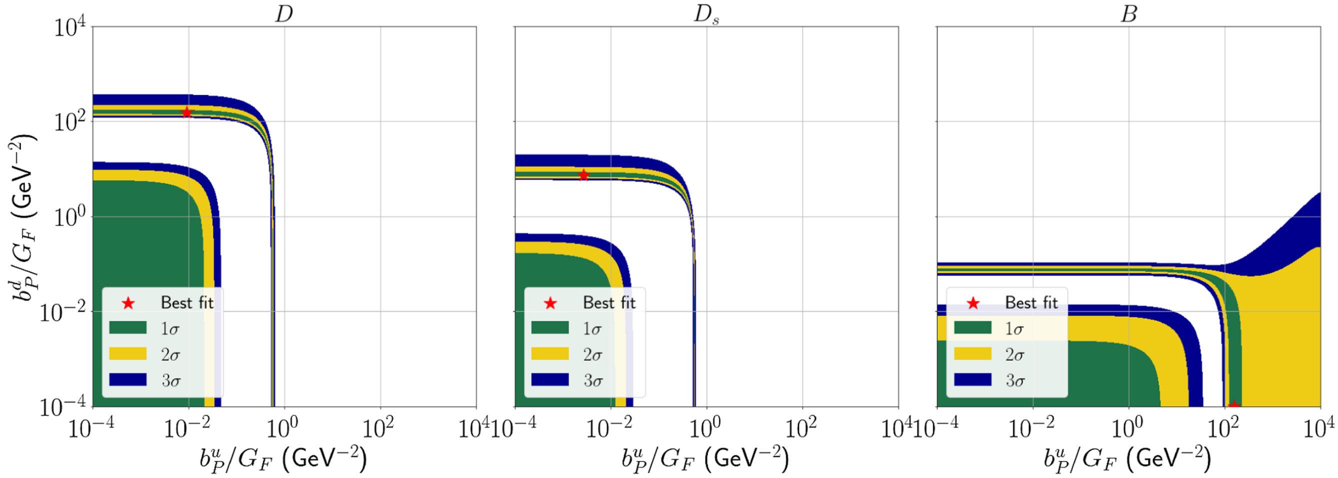

FIG. 4. The same as Fig. 3 for the heavy mesons D (left), D_s (center) and B (right).

quark is different, with a down quark for the pions and a strange quark in the kaons. Finally, for case (iii) above, we have the full solution given in Eq. (41), and it differ for each meson P.

Again, no universal solution in the quark sector is possible for the parameters b_P^u and b_P^d in the nontrivial region. However, in this nontrivial region, as it is possible to see from Eq. (39) and the almost constant factor B_P for all mesons, couplings related to the same quark asymptotic to the same values, e.g., π and K coupling b_P^u, which is related to the up-quark go to the same value when $b_P^d \to 0$. This result suggests that nonuniversality is preferred for quarks in this region, but couplings related to the same generation of quarks are consistent among different mesons. Remembering that B_P is not so close to the value of the ratio for the other mesons, we can also understand why b_P^u asymptotic to a slightly different value for B than for π and K. We also point out that the parameters a_P and $b_P^{u,d}$ have different dimensions since the quark masses have been factored out in Eq. (39).

IV. CONSTRAINTS ON THE PSEUDOSCALAR MASS

This section uses experimental and theoretical constraints to restrict the charged Higgs mass in this new region. For the lower limit, we use current searches for charged scalars on collider experiments, specifically from the large electron-positron (LEP) collider experiment [52]. We use the perturbative limits on the effective Lagrangian couplings for the upper limit on the Higgs mass.

Assuming that the quark couplings in Eq. (33), c^P, respects the perturbative limits, that is $c^P < \sqrt{4\pi}$, we have that

$$\frac{a^P}{G_F} \equiv \frac{c^P}{m_\eta^2 v_\ell G_F} < \frac{\sqrt{4\pi}}{m_\eta^2 v_\ell G_F}. \tag{42}$$

Hence, we must have

$$m_\eta < \sqrt{\frac{\sqrt{4\pi}}{(a^P/G_F) v_\ell G_F}} \tag{43}$$

Now, for example, if we use the best-fit point for $\pi_{\ell 2}$ in Fig. 1, $a^P/G_F = 0.7\ \text{GeV}^{-1}$, we have that

$$m_\eta < \left(658.33\sqrt{\left(\frac{1\ \text{GeV}}{v_\ell}\right)}\right)\ \text{GeV} \tag{44}$$

If we take $v_\ell = v_{\text{SM}}$, we have that

$$m_\eta < 42\ \text{GeV}, \tag{45}$$

and this case is already excluded by the LEP low limit of 80 GeV.

But, if we use $v_\ell = 2$ GeV, corresponding to the perturbative limit necessary for the tau mass, we have that

$$m_\eta < 466\ \text{GeV}. \tag{46}$$

Therefore, we found that the scalar mass should be in the range

$$80(181)\ \text{GeV} < m_\eta < 466\ \text{GeV}, \tag{47}$$

where the lower bound comes from charged scalar searches at LEP as reported on PDG [52].

V. CONCLUSIONS

In this work, we studied the leptonic decays of charged pseudoscalar mesons, $P_{\ell 2}$, which is mediated by a novel scalar field. Although such decays have already been widely analyzed in the literature, we have proposed a new nontrivial solution, given in Eq. (11), that depends only on one free parameter, a_P, for each meson. We have shown that such a solution can altogether avoid the constraints coming from the ratio $R_{l/l'} = \Gamma(P \to l\nu[\gamma])/\Gamma(P \to l'\nu[\gamma])$ and still allows a relatively large region in parameter space when the individual decay rates are used in the statistical analysis. As can be seen in Figs. 1 and 2, a larger allowed region appears beyond the fine-tuned nontrivial solution. Taking π_{e2} as an example, we see that our proposed solution permits an effective coupling for the new contribution in the range of $10^{-4} \lesssim (G^\eta/G_F) \lesssim 10^{-3}$, as can be seen in Fig. 1. On the other hand, we also have shown that no universal solution for all the pseudoscalar mesons, that is, $a_\pi = a_K = a_D = a_{D_s} = a_B$, could be found.

Moreover, we have shown that such a nontrivial solution can naturally emerge in models where flavor-changing neutral currents in the scalar sector are avoided in interactions with leptons. The most notorious examples are the models that satisfy the so-called Glashow-Weinberg-Paschos theorem. However, our results remain valid in other models where FCNC are avoided, such as, for example, the aligned models or the Branco-Grimus-Lavoura model. We then study the case where the FCNC in the scalar sector is avoided in interactions with quarks and leptons. Finally, we have also estimated the mass of the new scalar to be in the range $80(181)$ GeV $< m_\eta <$ 466 GeV.

To conclude, the nontrivial solution may also impact other relevant physical processes, for example, beta decay, the semileptonic meson decays, and tau and muon decays. Although such processes are out of the scope of this current work, to consolidate this solution as a candidate for new physics or to exclude it definitively, their analysis will be done in the future.

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APPENDIX: STATISTICAL ANALYSIS

As mentioned, the ratio $R_{l/l'}$ given in Eq. (14) is unsuitable for statistical analysis for the particular structure of Eq. (11), as it will always cancel out the new physics terms. With that in mind, we use the individual decay rates for each decay channel to fit the experimental data to our calculated rates.

The statistical analyses were performed using the following definition of the χ^2 function,

$$\chi^2(x) = \sum_l \frac{(\Gamma_l(x) - \Gamma_l^{\rm exp})^2}{(\sigma_{\Gamma_l}^{\rm SM})^2 + (\sigma_{\Gamma_l}^{\rm exp})^2 + (\sigma_{\Gamma_l}^{\eta}(x))^2}, \quad \text{(A1)}$$

where $\sigma_{\Gamma_l}^{\rm SM}$ is the uncertainty in SM theoretical calculations for the leptonic meson rate given in Eq. (8), $\sigma_{\Gamma_l}^{\eta}$ is the propagated uncertainty in the new physics terms due the charged scalar, and $\sigma_{\Gamma_l}^{\rm exp}$ is the experimental uncertainty of decay rate, $\Gamma_l^{\rm exp}$ [52]. Finally, we have $x = a_P/G_F$ for the analysis present in Fig. 1, $x = a_P^r/G_F$, a_P^i/G_F for Fig. 2 and $x = b_P^d/G_F$, b_P^u/G_F for Figs. 3 and 4. We use the confidence regions for one free parameter for the real case and two free parameters for the complex and the quark sector GWP structure.

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