Higgs sector phenomenology in the 3-3-1 model with an axionlike particle

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The scalar sector of the 3-3-1 model with an axionlike particle is studied in detail. In the model under consideration, there are two kinds of scalar fields: the bilepton scalars carrying lepton number two and the ordinary ones without lepton number. We show that there is no mixing among these two kinds of scalar fields. We analyze in detail the *CP*-odd scalar sector of the model to find the physical fields of the axionlike particle and a pseudoscalar with mass in the range 100 GeV to 1 TeV. The results are different from others which have been published before. The *CP*-even scalar sector of the model is analyzed as well. The results of our analysis of the scalar sector allow us to accommodate scalar masses in the 100 GeV–1 TeV region. Furthermore we analyze the implications of the model in several flavor changing neutral decays of the top quark as well as in rare top quark decays. Besides that, the leptonic decays of the SM like Higgs boson as well as the meson oscillations are also analyzed. Our numerical analysis show that the model under consideration is consistent with the experimental constraints imposed by these processes.

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I. INTRODUCTION

Nowadays, it is well known that the standard model (SM) has to be extended. Among the extended models of the SM, the versions based on the $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ gauge group (called 3-3-1 models in short) [1–10] are of interest with the following intriguing features such as the explanation on the number of fermion generations, the electric charge quantization [11,12], source of *CP* violation [13,14] as well as the automatic fulfillment [15] of the Peccei-Quinn symmetry [16,17]. The Peccei-Quinn symmetry for the economical 3-3-1 model [18–23] are discussed in Refs. [24,25]. The models contain self-interacting dark matter [26,27].

The models are classified by a parameter β appearing in the electric charge operator

$$Q = T_3 + \beta T_8 + X, \tag{1}$$

where T_3 and T_8 are $SU(3)_L$ generators, X is the $U(1)_X$ charge. The 3-3-1 model with arbitrary beta is presented in Ref. [28] (see also [29]). There are two main versions of

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the 3-3-1 models. The first one is the minimal model with $\beta = \pm\sqrt{3}$ which requires three $SU(3)_L$ scalar triplets and one $SU(3)_L$ scalar sextet [1–3]. Moreover, this version has a Landau pole around 5 TeV leading to a loss of perturbativity around that scale. There exist efforts to solve this puzzle [30]. In the recent work [31], the Landau pole, in the minimal version by addition of octet leptoquarks, can be around 100 TeV. The second one is the model with $\beta = \pm\frac{1}{\sqrt{3}}$ which just requires three $SU(3)_L$ scalar triplets to provide masses for all fermions and bosons [5–9]. This kind of model is more attractive due to its simpler scalar content and its lack of Landau pole at the TeV scale.

About two decades ago, the axion have been introduced in the 3-3-1 models [32–34]. The new nice property of the 3-3-1 model is found in a recent paper [35], where the cosmological inflation, axionlike particle (ALP) and see-saw mechanism are simultaneously addressed with a minimal scalar content. However, the above-mentioned paper contains some mistakes and does not address phenomenological aspects related with flavor changing neutral process such as the $t \to hu$, $t \to hc$, $t \to u\gamma$, and $t \to c\gamma$ decays as well as the $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$, and $B_s^0 - \bar{B}_s^0$ meson oscillations whose explanations, analysis, and discussions are the purpose of this work.

II. BRIEF REVIEW OF THE MODEL

A. Particle content and discrete symmetries

To provide masses for fermions and to account for the existence of the ALP, the scalar sector of the model requires three $SU(3)_L$ scalar triplets η, ρ, χ as well as an electrically neutral $SU(3)_L$ scalar singlet ϕ. The scalar content of the model with their corresponding $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ assignments are given by:

$$\begin{aligned} \chi^T &= (\chi_1^0, \chi_2^-, \chi_3^0) \sim \left(1, 3, -\frac{1}{3}\right), \\ \eta^T &= (\eta_1^0, \eta_2^-, \eta_3^0) \sim \left(1, 3, -\frac{1}{3}\right), \\ \rho^T &= (\rho_1^+, \rho_2^0, \rho_3^+) \sim \left(1, 3, \frac{2}{3}\right), \\ \phi &\sim (1, 1, 0). \end{aligned} \tag{2}$$

To provide masses for the fermions and gauge bosons, the above scalar fields have vacuum expectation values (VEVs) as follows

$$\begin{aligned} \langle\chi\rangle &= \frac{1}{\sqrt{2}}(0, 0, v_\chi)^T, \qquad \langle\eta\rangle = \frac{1}{\sqrt{2}}(v_\eta, 0, 0)^T \\ \langle\rho\rangle &= \frac{1}{\sqrt{2}}(0, v_\rho, 0)^T, \qquad \langle\phi\rangle = \frac{1}{\sqrt{2}} v_\phi. \end{aligned} \tag{3}$$

where the VEV v_χ triggers the spontaneous breaking of the $SU(3)_L \times U(1)_X$ gauge symmetry down to the SM electroweak gauge group. The remaining $SU(3)_L$ scalar triplets η and ρ break the SM electroweak gauge group.

On the other hand, the fermion spectrum of the model and their $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ assignments are:

$$\begin{aligned} &\psi_{aL} = (\nu_a, e_a, (\nu_R^c)^a)_L^T \sim (1, 3, -1/3), \quad e_{aR} \sim (1, 1, -1), \\ &N_{aR} \sim (1, 1, 0), \quad Q_{3L} = (u_3, d_3, U)_L^T \sim (3, 3, 1/3), \\ &Q_{nL} = (d_n, -u_n, D_n)_L^T \sim (3, 3^*, 0), \\ &u_{aR}, U_R \sim (3, 1, 2/3), \quad d_{aR}, D_{nR} \sim (3, 1, -1/3), \end{aligned} \tag{4}$$

where $n = 1$, 2 and $a = \{n, 3\}$ are family indices. The U and D are exotic quarks with ordinary electric charges, whereas N_{aR} are right-handed Majorana neutrinos.

The typical trouble of the 3-3-1 model with $\beta = \pm\frac{1}{\sqrt{3}}$ is that there are two triplets η and χ with identical quantum numbers by $\mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ gauge group leading to the term $\mu_{\eta\chi}^2 \eta^\dagger \chi$, which complicates the structure of the square scalar mass matrices, thus making the analysis of the scalar sector very tedious. To avoid this kind of terms, one imposes the Z_2 discrete symmetry under which the $SU(3)_L$ scalar triplets η and χ have opposite numbers, as done in Ref. [33]. To provide Dirac and Majorana mass terms for ν_L and N_R we have the above described particle content, shown in Table I. The particle assignments under the $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{U}(1)_X \times Z_{11} \times Z_2$ group are summarized in Table I. Here we have used a notation $\omega_k \equiv e^{i2\pi\frac{k}{11}}, k = 0, \pm 1 \cdots \pm 5$.

TABLE I. $SU(3)_C \times SU(3)_L \times U(1)_X \times Z_{11} \times Z_2$ charge assignments of the particle content of the model. Here $a = 1, 2, 3$ and $\alpha = 1, 2$.

	Q_{nL}	Q_{3L}	u_{aR}	d_{aR}	U_{3R}	D_{nR}	ψ_{aL}	e_{aR}	N_{aR}	η	χ	ρ	ϕ
$SU(3)_C$	**3**	**3**	**3**	**3**	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$SU(3)_L$	$\bar{\mathbf{3}}$	**3**	**1**	**1**	**1**	**1**	**3**	**1**	**1**	**3**	**3**	**3**	**1**
$U(1)_X$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	0
Z_{11}	ω_4^{-1}	ω_0	ω_5	ω_2	ω_3	ω_4	ω_1	ω_3	ω_5^{-1}	ω_5^{-1}	ω_3^{-1}	ω_2^{-1}	ω_1^{-1}
Z_2	1	1	-1	-1	1	1	1	-1	-1	-1	1	-1	1

From Table I, one recognizes that under Z_2 symmetry, the following fields are *odd*

$$(\eta, \rho, u_R, d_{nR}, e_{nR}, N_R) \to -(\eta, \rho, u_R, d_{nR}, e_{nR}, N_R). \quad (5)$$

B. Yukawa couplings

With the above specified particle content, the following Yukawa interactions invariant under the $SU(3)_C \times SU(3)_L \times U(1)_X \times Z_{11} \times Z_2$ symmetry, arise [35]:

$$\begin{aligned} -\mathcal{L}^Y = {} & y_1 \bar{Q}_{3L} U_R \chi + \sum_{n,m=1}^{2} (y_2)_{n,m} \bar{Q}_{nL} D_{mR} \chi^* \\ & + \sum_{a=1}^{3} (y_3)_{3a} \bar{Q}_{3L} u_{aR} \eta + \sum_{n=1}^{2}\sum_{a=1}^{3} (y_4)_{na} \bar{Q}_{nL} d_{aR} \eta^* \\ & + \sum_{a=1}^{3} (y_5)_{3a} \bar{Q}_{3L} d_{aR} \rho + \sum_{n=1}^{2}\sum_{a=1}^{3} (y_6)_{na} \bar{Q}_{nL} u_{aR} \rho^* \\ & + \sum_{a=1}^{3}\sum_{b=1}^{3} g_{ab} \bar{\Psi}_{aL} e_{bR} \rho + \sum_{a=1}^{3}\sum_{b=1}^{3} (y_\nu^D)_{ab} \bar{\Psi}_{aL} \eta N_{bR} \\ & + \sum_{a=1}^{3}\sum_{b=1}^{3} (y_N)_{ab} \phi \bar{N}_{aR}^C N_{bR} + \text{H.c.} \end{aligned} \quad (6)$$

Let us note that the above given Yukawa interactions in (6) are invariant only under the Z_2 assignment given above. It is emphasized that the transformation under the Z_2 in this paper is different from than the one given in Ref. [35] where χ is odd.

The exotic quarks get masses from v_χ, top quark get mass from v_η, charged leptons get masses from v_ρ, while new Majorana neutrino N_R gets mass through v_ϕ. The Dirac neutrino mass term arises from v_η, while the Majorana mass term arises from v_ϕ [see last two terms in (6)]. From the last two terms of Eq. (6), it follows that the tiny masses for the light active neutrinos are generated from a type I seesaw mechanism mediated by right handed Majorana neutrinos, thus implying that the resulting light active neutrino mass matrix has the form:

$$M_\nu = M_\nu^D M_N^{-1} (M_\nu^D)^T, \quad M_\nu^D = y_\nu^D \frac{v_\eta}{\sqrt{2}}, \quad M_N = \sqrt{2} y_N v_\phi. \quad (7)$$

C. Gauge bosons

First of all, the model has nine electroweak gauge bosons arising from the $SU(3)_L \times U(1)_X$ symmetry. Their interactions with the $SU(3)_L$ scalar triplets are included in the following kinetic terms:

$$\mathcal{L}_{Higgs} = \sum_{H=\chi,\eta,\rho,\phi} (D^\mu H)^\dagger D_\mu H, \quad (8)$$

where the covariant derivative is given by

$$D_\mu \equiv \partial_\mu - igT^a W_\mu^a - ig_X X T^9 X_\mu, \quad (9)$$

where $T^9 = 1/\sqrt{6} I_{3\times3}$ being $I_{3\times3}$ the 3×3 identity matrix and g, g_X are gauge couplings of the two groups $SU(3)_L$ and $U(1)_X$, respectively. Secondly, the matrix $W^a T^a$, where $T^a = \lambda_a/2$ corresponds to a triplet representation, is written as follows:

$$W_\mu^a T^a = \frac{1}{2} \begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8 & \sqrt{2} W_\mu^+ & \sqrt{2} Y_\mu^+ \\ \sqrt{2} W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8 & \sqrt{2} X_\mu^0 \\ \sqrt{2} Y_\mu^- & \sqrt{2} X_\mu^{0*} & -\frac{2}{\sqrt{3}} W_\mu^8 \end{pmatrix}, \quad (10)$$

in which we have defined the mass eigenstates of the charged gauge bosons as

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2), \qquad Y_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^4 \mp i W_\mu^5), \\ X_\mu^0 &= \frac{1}{\sqrt{2}} (W_\mu^6 - i W_\mu^7), \qquad X_\mu^{0*} = \frac{1}{\sqrt{2}} (W_\mu^6 + i W_\mu^7). \end{aligned} \quad (11)$$

After spontaneous symmetry breaking, the mass spectrum of the gauge bosons arise from the following terms:

$$\mathcal{L}_{\text{mass}} = \sum_{H=\chi,\eta,\rho} (D^\mu \langle H \rangle)^\dagger (D_\mu \langle H \rangle). \quad (12)$$

The charged and *bilepton* gauge bosons get masses given by:

$$\begin{aligned} m_W^2 &= \frac{g^2}{4} (v_\eta^2 + v_\rho^2), \qquad m_{X^0}^2 = \frac{g^2}{4} (v_\chi^2 + v_\eta^2), \\ m_Y^2 &= \frac{g^2}{4} (v_\chi^2 + v_\rho^2). \end{aligned} \quad (13)$$

W is identical to that of the standard model, while (X, Y) form a new, heavy gauge vector doublet with a mass splitting [36]

$$|m_Y^2 - m_{X^0}^2| < m_W^2.$$

From (13), it follows

$$v_\eta^2 + v_\rho^2 = v_{ew}^2 = 246^2 \text{ GeV}^2. \quad (14)$$

Finally, there is a mixing among the W_3, W_8, B components. In the basis of these elements, the mass matrix is given by

$$M^2_{\text{neural}} = \frac{g^2}{4}\begin{pmatrix} v_\eta^2 + v_\rho^2 & \frac{v_\eta^2 - v_\rho^2}{\sqrt{3}} & -\frac{2t}{3\sqrt{6}}(v_\eta^2 + 2v_\rho^2) \\ \frac{v_\eta^2 - v_\rho^2}{\sqrt{3}} & \frac{1}{3}(4v_\chi^2 + v_\eta^2 + v_\rho^2) & \frac{\sqrt{2}t}{9}(2v_\chi^2 - v_\eta^2 + 2v_\rho^2) \\ -\frac{2t}{3\sqrt{6}}(v_\eta^2 + 2v_\rho^2) & \frac{\sqrt{2}t}{9}(2v_\chi^2 - v_\eta^2 + 2v_\rho^2) & \frac{2t^2}{27}(v_\chi^2 + v_\eta^2 + 4v_\rho^2) \end{pmatrix}, \tag{15}$$

where

$$t = \frac{3\sqrt{2}s_W}{\sqrt{3 - 4s_W^2}}. \tag{16}$$

Diagonalization proceeds through two steps, in the first step the 3×3 matrix reduces to one block diagonalized which yields a 2×2 matrix in the bottom. The eigenstates are now rewritten as follows

$$A_\mu = s_W W_{3\mu} + c_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1 - \frac{t_W^2}{3}}B_\mu\right),$$
$$Z_\mu = c_W W_{3\mu} - s_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1 - \frac{t_W^2}{3}}B_\mu\right),$$
$$Z'_\mu = \sqrt{1 - \frac{t_W^2}{3}}W_{8\mu} + \frac{t_W}{\sqrt{3}}B_\mu. \tag{17}$$

From the analysis of the gauge sector, we found one massless gauge boson, which corresponds to the photon A. Furthermore, besides the bilepton gauge bosons, the neutral gauge boson spectrum contains two massive neutral gauge bosons Z and Z'. The elements of the neutral squared gauge boson mass matrix in the (Z, Z') basis is given by

$$m_Z^2 = \frac{g^2}{4c_W^2}(v_\rho^2 + v_\eta^2), \tag{18}$$

$$m_{ZZ'}^2 = \frac{g^2[(t_W^2 - 1)v_\rho^2 + (t_W^2 + 1)v_\eta^2]}{4\sqrt{3}c_W\sqrt{1 - \frac{1}{3}t_W^2}}, \tag{19}$$

$$m_{Z'}^2 = \frac{g^2[4v_\chi^2 + (t_W^2 - 1)^2 v_\rho^2 + (t_W^2 + 1)^2 v_\eta^2]}{4(3 - t_W^2)}. \tag{20}$$

Finally, this matrix is diagonalized by the following field transformations

$$Z^1_\mu = c_{\theta_Z}\mathcal{Z}_\mu - s_{\theta_Z}\mathcal{Z}'_\mu,$$
$$Z^2_\mu = s_{\theta_Z}\mathcal{Z}_\mu + c_{\theta_Z}\mathcal{Z}'_\mu \tag{21}$$

where [37]

$$t_{2\theta_Z} = \frac{s_{\theta_Z}}{c_{\theta_Z}} = \frac{2m_{ZZ'}^2}{m_{Z'}^2 - m_Z^2} \simeq \frac{\sqrt{(3 - t_W^2)}[(t_W^2 - 1)v_\rho^2 + (t_W^2 + 1)v_\eta^2]}{2v_\chi^2 c_W}, \tag{22}$$

$$m_{Z_1}^2 = \frac{1}{2}\left[m_Z^2 + m_{Z'}^2 - \sqrt{(m_Z^2 - m_{Z'}^2)^2 + 4m_{ZZ'}^4}\right] \simeq m_Z^2 - \frac{m_{ZZ'}^4}{m_{Z'}^2} \simeq \frac{g^2}{4c_W^2}\left\{v_\rho^2 + v_\eta^2 - \frac{[(t_W^2 - 1)v_\rho^2 + (t_W^2 + 1)v_\eta^2]^2}{4v_\chi^2}\right\} \approx \frac{m_W^2}{c_W^2},$$
$$m_{Z_2}^2 = \frac{1}{2}\left[m_Z^2 + m_{Z'}^2 + \sqrt{(m_Z^2 - m_{Z'}^2)^2 + 4m_{ZZ'}^4}\right] \simeq m_{Z'}^2 \approx \frac{g^2 c_W^2}{(3 - 4s_W^2)}v_\chi^2. \tag{23}$$

Note that exotic quarks U and D_α as well as gauge bosons $X^0, Y^\pm$ carry lepton number two [38–40]. The gauge boson couplings of this model are the same in Refs. [41,42]. Due to quark family discrimination, there are flavor changing neutral currents mediated by Z' at the tree level [43–46].

III. HIGGS POTENTIAL

The model scalar potential has the form:

$$V = \mu_\phi^2\phi^*\phi + \mu_\chi^2\chi^\dagger\chi + \mu_\rho^2\rho^\dagger\rho + \mu_\eta^2\eta^\dagger\eta + \lambda_1(\chi^\dagger\chi)^2 + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\rho^\dagger\rho)^2 + \lambda_4(\chi^\dagger\chi)(\eta^\dagger\eta) + \lambda_5(\chi^\dagger\chi)(\rho^\dagger\rho)$$
$$+ \lambda_6(\eta^\dagger\eta)(\rho^\dagger\rho) + \lambda_7(\chi^\dagger\eta)(\eta^\dagger\chi) + \lambda_8(\chi^\dagger\rho)(\rho^\dagger\chi) + \lambda_9(\eta^\dagger\rho)(\rho^\dagger\eta) + \lambda_{10}(\phi^*\phi)^2 + \lambda_{11}(\phi^*\phi)(\chi^\dagger\chi)$$
$$+ \lambda_{12}(\phi^*\phi)(\rho^\dagger\rho) + \lambda_{13}(\phi^*\phi)(\eta^\dagger\eta) + (\lambda_\phi\epsilon^{ijk}\eta_i\rho_j\chi_k\phi + \text{H.c.}) \tag{24}$$

The VEV v_ϕ is responsible for the PQ symmetry breaking resulting in the existence of invisible ALP due to very high scale around 10^{10}–10^{11} GeV. Then $SU(3)_L \times U(1)_X$ breaks to the SM group by v_χ and two others v_ρ, v_η are needed for the usual $U(1)_Q$ symmetry. Hence $v_\phi \gg v_\chi \gg v_\rho, v_\eta$. The constraint conditions of such scalar potential were analyzed in Ref. [33]. From (24), it is reasonable to assume: $\lambda_2 \approx \lambda_3$, $\lambda_4 \approx \lambda_5$, $\lambda_7 \approx \lambda_8$,

$\lambda_{12} \approx \lambda_{13}$. According Ref. [47], $v_\chi \geq 10357$ GeV for $M_{Z'} \geq 4.1$ TeV.

Let us expand these scalar fields around their VEVs.

$$\rho_2^0 = \frac{1}{\sqrt{2}}(v_\rho + R_\rho + iI_\rho), \quad \eta_1^0 = \frac{1}{\sqrt{2}}(v_\eta + R_\eta^1 + iI_\eta^1),$$
$$\chi_3^0 = \frac{1}{\sqrt{2}}(v_\chi + R_\chi^3 + iI_\chi^3), \quad \phi = \frac{1}{\sqrt{2}}(v_\phi + R_\phi + iI_\phi). \tag{25}$$

Substitution of (25) into (24) leads to the following constraints at the tree level as follows

$$\mu_\rho^2 + \lambda_3 v_\rho^2 + \frac{\lambda_5}{2}v_\chi^2 + \frac{\lambda_6}{2}v_\eta^2 + \frac{\lambda_{12}}{2}v_\phi^2 + \frac{A}{2v_\rho^2} = 0,$$
$$\mu_\eta^2 + \lambda_2 v_\eta^2 + \frac{\lambda_4}{2}v_\chi^2 + \frac{\lambda_6}{2}v_\rho^2 + \frac{\lambda_{13}}{2}v_\phi^2 + \frac{A}{2v_\eta^2} = 0,$$
$$\mu_\chi^2 + \lambda_1 v_\chi^2 + \frac{\lambda_4}{2}v_\eta^2 + \frac{\lambda_5}{2}v_\rho^2 + \frac{\lambda_{11}}{2}v_\phi^2 + \frac{A}{2v_\chi^2} = 0,$$
$$\mu_\phi^2 + \lambda_{10} v_\phi^2 + \frac{\lambda_{11}}{2}v_\chi^2 + \frac{\lambda_{12}}{2}v_\rho^2 + \frac{\lambda_{13}}{2}v_\eta^2 + \frac{A}{2v_\phi^2} = 0, \tag{26}$$

where $A \equiv \lambda_\phi v_\phi v_\chi v_\eta v_\rho$.

A. Charged scalar sector

There are four charged scalar fields: $\eta_2^-, \rho_1^-, \rho_3^-$, and χ_2^-.

(i) In the basis (η_2^-, ρ_1^-), the corresponding squared mass matrix is given by:

$$M_c = \begin{pmatrix} \frac{\lambda_9 v_\rho^2}{2} - \frac{A}{2v_\eta^2} & \frac{\lambda_9 v_\rho v_\eta}{2} - \frac{A}{2v_\rho v_\eta} \\ \frac{\lambda_9 v_\rho v_\eta}{2} - \frac{A}{2v_\rho v_\eta} & \frac{\lambda_9 v_\eta^2}{2} - \frac{A}{2v_\rho^2} \end{pmatrix}$$
$$= -\frac{(A - \lambda_9 v_\rho^2 v_\eta^2)}{2}\begin{pmatrix} \frac{1}{v_\eta^2} & \frac{1}{v_\eta v_\rho} \\ \frac{1}{v_\eta v_\rho} & \frac{1}{v_\rho^2} \end{pmatrix}. \tag{27}$$

From this matrix, we get the massless $G_1^\pm$ states and two massive ones, i.e., $H_1^\pm$ with mass equal to

$$m_{H_1^\pm}^2 = -\frac{(A - \lambda_9 v_\rho^2 v_\eta^2)}{2} \cdot \frac{(v_\rho^2 + v_\eta^2)}{v_\rho^2 v_\eta^2} \tag{28}$$

Let us note that the $G_1^\pm$ massless charged scalar fields correspond to the SM charged Goldstone bosons associated with the longitudinal components of the $W^\pm$ gauge bosons.

The physical fields are given by

$$\begin{pmatrix} G_1^\pm \\ H_1^\pm \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \rho_1^\pm \\ \eta^\pm \end{pmatrix}, \tag{29}$$

where

$$\tan\alpha = \frac{v_\eta}{v_\rho}. \tag{30}$$

From (28) it follows

$$\lambda_9 > \lambda_\phi \frac{v_\phi v_\chi}{v_\rho v_\eta} = \frac{A}{v_\rho^2 v_\eta^2} = \frac{A}{(v_{ew}^2 - v_\eta^2)v_\eta^2}. \tag{31}$$

From (31), one gets the condition for the perturbative coupling as follows

$$\frac{|A|}{(v_{ew}^2 - v_\eta^2)v_\eta^2} < 1. \tag{32}$$

Then, the constraint for the important coupling λ_ϕ is given by

$$|A| < (v_{ew}^2 - v_\eta^2)v_\eta^2 \Rightarrow |\lambda_\phi| < \frac{(v_{ew}^2 - v_\eta^2)\tan\alpha}{v_\phi v_\chi}. \tag{33}$$

For simplicity, let us assume $v_\eta = v_\rho = v_{ew}/\sqrt{2} \simeq 174$ GeV, $v_\phi = 10^{10}$ GeV, and $v_\chi = 10^5$ GeV, then $|\lambda_\phi| < 10^{-11}$. It is interesting to note that such tiny couplings (Yukawa couplings responsible for proton instability) arise also in the supersymmetric 3-3-1 model [48].

(ii) For the charged scalars, in the basis (χ_2^-, ρ_3^-), the corresponding squared scalar mass matrix has the form:

$$M_{c2} = \begin{pmatrix} \frac{\lambda_8 v_\rho^2}{2} - \frac{A}{2v_\chi^2} & \frac{\lambda_8 v_\rho v_\chi}{2} - \frac{A}{2v_\rho v_\chi} \\ \frac{\lambda_8 v_\rho v_\chi}{2} - \frac{A}{2v_\rho v_\chi} & \frac{\lambda_8 v_\chi^2}{2} - \frac{A}{2v_\rho^2} \end{pmatrix}$$
$$= -\frac{(A - \lambda_8 v_\rho^2 v_\chi^2)}{2}\begin{pmatrix} \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} \\ \frac{1}{v_\chi v_\rho} & \frac{1}{v_\rho^2} \end{pmatrix}. \tag{34}$$

This matrix has the massless scalar states $G_2^\pm$ and the massive one $H_2^\pm$ with mass equal to

$$m_{H_2^\pm}^2 = -\frac{(A - \lambda_8 v_\rho^2 v_\chi^2)}{2} \cdot \frac{(v_\rho^2 + v_\chi^2)}{v_\rho^2 v_\chi^2} \tag{35}$$

The physical fields are given as

$$\begin{pmatrix} G_2^\pm \\ H_2^\pm \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix}\begin{pmatrix} \chi_2^\pm \\ \rho_3^\pm \end{pmatrix}, \tag{36}$$

where

$$\tan\theta_1 = \frac{v_\rho}{v_\chi}. \tag{37}$$

It is worth mentioning that the bilepton massless $G_2^{\pm}$ correspond to the Goldstone boson associated with the longitudinal component of the $Y^{\pm}$ bilepton gauge boson.

From (35) it follows

$$\lambda_8 > \lambda_\phi \frac{v_\phi v_\eta}{v_\chi v_\rho}. \tag{38}$$

B. Complex neutral scalar sector

There are two neutral scalars: one χ_1^0 with mass

$$m^2_{\chi_1^0} = (\lambda_7 v_\eta^2 v_\chi^2 - A)\frac{(v_\eta^2 + v_\chi^2)}{v_\eta^2 v_\chi^2}. \tag{39}$$

and one massless η_3^0 which is identified with Goldstone boson eaten by massive X^0. Hence

$$\eta_3^0 \equiv G_{X^0}. \tag{40}$$

From (39), it follows

$$\lambda_7 v_\eta^2 v_\chi^2 > A. \tag{41}$$

It is to be noted that in the framework of 3-3-1 model with right-handed neutrinos, χ_1^0 is bilepton scalar which can play a role of DM [49].

C. *CP*-odd scalar sector

There are four *CP*-odd scalars with VEVs: $(I_\phi, I_\chi^3, I_\rho^2, I_\eta^1)$. In the following we describe the corrections to Ref. [35].

(1) The squared mass matrix for the electrically neutral *CP* odd scalars in the basis $(I_\phi, I_\chi^3, I_\rho, I_\eta^1)$ has the form:

$$M^2_{\rm odd} = -\frac{A}{2}\begin{pmatrix} \frac{1}{v_\phi^2} & \frac{1}{v_\phi v_\chi} & \frac{1}{v_\phi v_\rho} & \frac{1}{v_\phi v_\eta} \\ & \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} & \frac{1}{v_\chi v_\eta} \\ & & \frac{1}{v_\rho^2} & \frac{1}{v_\eta v_\rho} \\ & & & \frac{1}{v_\eta^2} \end{pmatrix}. \tag{42}$$

As seen from Eq. (42), there are nontrivial mixings among the *CP* odd scalars $(I_\phi, I_\chi^3, I_\rho, I_\eta^1)$ in the interaction basis. Note that an element at the first row and third columns in (42) have to be $\frac{1}{v_\rho v_\phi}$, instead of $\frac{1}{v_\rho v_\eta}$ reported in Eq. (16) of Ref. [35].

(2) The *CP* odd squared mass matrix $M^2_{\rm odd}$ in (42) can be exactly diagonalized by the Euler diagonalization method. The *CP* odd scalar fields in the physical and interaction basis are related through the following transformation:

$$\begin{pmatrix} a \\ G_{Z'} \\ G_Z \\ A_5 \end{pmatrix} = \begin{pmatrix} \cos\theta_\phi & -\sin\theta_3\sin\theta_\phi & -\sin\alpha\cos\theta_3\sin\theta_\phi & -\cos\alpha\cos\theta_3\sin\theta_\phi \\ 0 & \cos\theta_3 & -\sin\alpha\sin\theta_3 & -\cos\alpha\sin\theta_3 \\ 0 & 0 & \cos\alpha & -\sin\alpha \\ \sin\theta_\phi & \sin\theta_3\cos\theta_\phi & \sin\alpha\cos\theta_3\cos\theta_\phi & \cos\alpha\cos\theta_3\cos\theta_\phi \end{pmatrix}\begin{pmatrix} I_\phi \\ I_\chi^3 \\ I_\rho \\ I_\eta^1 \end{pmatrix}, \tag{43}$$

where the mixing angles in the *CP* odd scalar sector take the forms:

$$\tan\alpha = \frac{v_\eta}{v_\rho}, \qquad \tan\theta_3 = \frac{v_\eta}{v_\chi\sqrt{1+\frac{v_\eta^2}{v_\rho^2}}} \approx \frac{v_\eta}{v_\chi},$$
$$\tan\theta_\phi = \frac{v_\chi}{v_\phi\sqrt{1+v_\chi^2(\frac{1}{v_\rho^2}+\frac{1}{v_\eta^2})}} \approx \frac{v_\chi}{v_\phi}. \tag{44}$$

Note that the matrix in (42) depends on four VEVs namely, v_ρ, v_η, v_χ and v_ϕ. The derived mixing matrix in (43) has three angles $\alpha, \theta_3, \theta_\phi$ given in (44) and one parameter is $(\frac{1}{v_\phi^2}+\frac{1}{v_\chi^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\eta^2})$ which is entered to the expression of A_5 mass in (46). It is worth mentioning that the rotation matrix that diagonalizes the *CP* odd squared mass matrix has three mixing angles instead of four because of the VEV hierarchy $v_\rho, v_\eta \ll v_\chi \ll v_\phi$.

It is worth mentioning that our result is completely different from the ones given in Ref. [33], where the mixing matrix is not unitary.

Here the ALP is massless and is given by the following combination of four *CP* odd neutral scalar fields I_ϕ, I_χ^3, I_ρ, and I_η^1:

$$a = I_\phi\cos\theta_\phi - I_\chi^3\sin\theta_\phi\sin\theta_3 - I_\rho\cos\theta_3\sin\alpha\sin\theta_\phi - I_\eta^1\cos\alpha\cos\theta_3\sin\theta_\phi, \tag{45}$$

which cannot be the same expression for an a given in Refs. [33,35].[1] It is worth mentioning that due to $v_\chi \ll v_\phi$, it follows that $\tan\theta_\phi \to 0$ as well as $\sin\theta_\phi$ then $\cos\theta_\phi \simeq 1$. This leads to $a \simeq I_\phi$.

[1]It is possible to get the mixing matrix in which ALP contains only two components as in Refs. [33,35], but in this case both Goldstone bosons G_Z and $G_{Z'}$ contain a component along I_ϕ.

Furthermore, the mass of new massive field CP odd scalar field A_5 is given by

$$m^2_{A_5} = -\frac{A}{2}\left(\frac{1}{v^2_\phi}+\frac{1}{v^2_\chi}+\frac{1}{v^2_\rho}+\frac{1}{v^2_\eta}\right) \approx -\frac{1}{2}\lambda_\phi v_\phi v_\chi(\tan\alpha+\cot\alpha) = -\frac{\lambda_\phi v_\phi v_\chi}{\sin 2\alpha}. \quad (46)$$

From (46), we can see that the value of λ_ϕ should be negative. It is emphasized that the squared mass matrix in Eq. (42) as well as mass of the A_5 are only available due to the last term in (24) which just appears because of specific discrete symmetry in this paper (for discussion on this, the reader is referred to Ref. [33]).

Summary: in the CP-odd sector we have 6 fields: two Goldstone bosons for Z and Z', one axion like particle a, one massless field G_1 being eaten by one component of the massive X^0 and one massive pseudoscalar A_5.

D. CP-even scalar sector

As same as the CP-odd scalar sector, there are four fields in the CP-even scalar sector with VEVs: ($R_\phi, R^3_\chi, R^2_\rho$ and R^1_η).

In basis $(R^1_\eta, R_\rho, R^3_\chi, R_\phi)$, the squared mass matrix of CP-even has form as below:

$$M^2_R = 2\begin{pmatrix} \lambda_2 v^2_\eta - \frac{A}{4v^2_\eta} & \frac{1}{2}\left(\lambda_6 v_\eta v_\rho + \frac{\lambda_\phi v_\chi v_\phi}{2}\right) & \frac{1}{2}\left(\lambda_4 v_\eta v_\chi + \frac{\lambda_\phi v_\rho v_\phi}{2}\right) & \frac{1}{2}\left(\lambda_{13} v_\eta v_\phi + \frac{\lambda_\phi v_\rho v_\chi}{2}\right) \\ \frac{1}{2}\left(\lambda_6 v_\eta v_\rho + \frac{\lambda_\phi v_\chi v_\phi}{2}\right) & \lambda_3 v^2_\rho - \frac{A}{4v^2_\rho} & \frac{1}{2}\left(\frac{\lambda_\phi v_\eta v_\phi}{2} + \lambda_5 v_\rho v_\chi\right) & \frac{1}{2}\left(\frac{\lambda\phi v_\eta v_\chi}{2} + \lambda_{12} v_\rho v_\phi\right) \\ \frac{1}{2}\left(\lambda_4 v_\eta v_\chi + \frac{\lambda_\phi v_\rho v_\phi}{2}\right) & \frac{1}{2}\left(\frac{\lambda_\phi v_\eta v_\phi}{2} + \lambda_5 v_\rho v_\chi\right) & \lambda_1 v^2_\chi - \frac{A}{4v^2_\chi} & \frac{1}{2}\left(\frac{\lambda_\phi v_\eta v_\rho}{2} + \lambda_{11} v_\chi v_\phi\right) \\ \frac{1}{2}\left(\lambda_{13} v_\eta v_\phi + \frac{\lambda_\phi v_\rho v_\chi}{2}\right) & \frac{1}{2}\left(\frac{\lambda_\phi v_\eta v_\chi}{2} + \lambda_{12} v_\rho v_\phi\right) & \frac{1}{2}\left(\frac{\lambda_\phi v_\eta v_\rho}{2} + \lambda_{11} v_\chi v_\phi\right) & \lambda_{10} v^2_\phi - \frac{A}{4v^2_\phi} \end{pmatrix}. \quad (47)$$

Comparing with a similar matrix in Ref. [35], we see that the first three elements in the fourth column of CP even mass matrix in Ref. [33] have the *extra* terms: $\frac{\lambda_{11} v_\phi v_{\chi'}}{2}, \frac{\lambda_{13} v_\phi v_\eta}{2}$ and $\frac{\lambda_{12} v_\phi v_\rho}{2}$, respectively. To recognize the existence of these terms, let us write them explicitly

$$\lambda_{11}(\phi^\dagger\phi)(\chi^\dagger\chi) \supset v_\phi v_{\chi'} R_\phi R_{\chi'},$$
$$\lambda_{12}(\phi^\dagger\phi)(\rho^\dagger\rho) \supset v_\phi v_\rho R_\phi R_\rho,$$
$$\lambda_{13}(\phi^\dagger\phi)(\eta^\dagger\eta) \supset v_\phi v_\eta R_\phi R_\eta.$$

The matrix which is used to diagonalize M^2_R is

$$U_R = \begin{pmatrix} -\cos\alpha_2 & -\sin\alpha_2\cos\alpha_3 & -\sin\alpha_2\sin\alpha_3\cos\alpha_\phi & \sin\alpha_2\sin\alpha_3\sin\alpha_\phi \\ \sin\alpha_2 & -\cos\alpha_2\cos\alpha_3 & -\cos\alpha_2\sin\alpha_3\cos\alpha_\phi & \cos\alpha_2\sin\alpha_3\sin\alpha_\phi \\ 0 & \sin\alpha_3 & -\cos\alpha_3\cos\alpha_\phi & \cos\alpha_3\sin\alpha_\phi \\ 0 & 0 & \sin\alpha_\phi & \cos\alpha_\phi \end{pmatrix} \quad (48)$$

in which, the mixing angles in the CP even scalar sector are defined as below:

$$\tan 2\alpha_2 = \frac{4\cos\alpha_3 v_\eta v_\rho (A + \lambda_6 v^2_\eta v^2_\rho)}{A\cos^2\alpha_3 v^2_\eta - A v^2_\rho + 4v^2_\eta v^2_\rho(\lambda_2 v^2_\eta - \lambda_3\cos^2\alpha_3 v^2_\rho)} \quad (49)$$

$$\tan 2\alpha_3 = \frac{4v_\chi(A + 2\lambda_5 v^2_\rho v^2_\chi)}{\cos\alpha\phi(A - 4\lambda_1 v^4_\chi)^2}, \quad (50)$$

$$\tan 2\alpha_\phi = \frac{\lambda_{11} v_\chi}{\lambda_{10} v_\phi}. \quad (51)$$

Changing the signs of h, h_5, and H_χ, the physical fields are given by:

$$\begin{pmatrix} h_5 \\ h \\ H_\chi \\ \Phi \end{pmatrix} = \begin{pmatrix} \cos\alpha_2 & \sin\alpha_2\cos\alpha_3 & \sin\alpha_2\sin\alpha_3\cos\alpha_\phi & -\sin\alpha_2\sin\alpha_3\sin\alpha_\phi \\ -\sin\alpha_2 & \cos\alpha_2\cos\alpha_3 & \cos\alpha_2\sin\alpha_3\cos\alpha_\phi & -\cos\alpha_2\sin\alpha_3\sin\alpha_\phi \\ 0 & -\sin\alpha_3 & \cos\alpha_3\cos\alpha_\phi & -\cos\alpha_3\sin\alpha_\phi \\ 0 & 0 & \sin\alpha_\phi & \cos\alpha_\phi \end{pmatrix}\begin{pmatrix} R^1_\eta \\ R_\rho \\ R^3_\chi \\ R_\phi \end{pmatrix}. \quad (52)$$

In the limit $v_\phi \gg v_\chi \gg v_\rho, v_\eta$ it follows

$$h_5 \approx R_\eta^1 \cos\alpha_2 + R_\rho \sin\alpha_2, \tag{53}$$

$$h \approx -R_\eta^1 \sin\alpha_2 + R_\rho \cos\alpha_2, \tag{54}$$

$$H_\chi \approx R_\chi^3 \cos\alpha_\phi, \tag{55}$$

$$\Phi \approx R_\phi \cos\alpha_\phi, \tag{56}$$

and their respective masses are shown in Appendix B.

Note that comparing to the 4×4 matrix of CP-odd sector containing only four parameters with three massless solution, the matrix in (47) having 10 parameters is not exactly diagonalized. To solve this problem we have used the Hartree-Fock method where some conditions such as $v_\phi \gg v_\chi \gg v_\rho, v_\eta$, $\lambda_\phi \ll 1$ and $\sin\alpha_3 \approx 0$. As a consequence of the aforementioned VEV hierarchy, the derived matrix contains three angles α_2, α_3 and α_ϕ and three parameters associated with masses of new fields Φ, H_χ and h_5.

In the limit $v_\phi \gg v_\chi \gg v_\rho \gg v_\eta$, one has

$$\chi \simeq \begin{pmatrix} \chi_1^0 \\ G_{Y^-} \\ \frac{1}{\sqrt{2}}(v_\chi + H_\chi + iG_{Z'}) \end{pmatrix}, \quad \eta \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(u + h_5 + iA_5) \\ H_1^- \\ G_{X^0} \end{pmatrix},$$

$$\rho \simeq \begin{pmatrix} G_{W^+} \\ \frac{1}{\sqrt{2}}(v + h + iG_Z) \\ H_2^+ \end{pmatrix}, \quad \phi \simeq \frac{1}{\sqrt{2}}(v_\phi + \Phi + ia). \tag{57}$$

In the CP-even scalar sector, there are six fields. One massless field is part of G_{X^0}, another massive in TeV scale is associated to χ_1^0. One heavy field with mass in the range of 10^{11} GeV and associated with singlet ϕ is identified to inflaton Φ. One SM-like Higgs boson h with mass $\sim$125 GeV. Two remain fields include one heavy with mass at TeV scale (H_χ) and another with mass at EW scale (h_5).

Combination of Table I and (57) leads to some interesting consequences

(1) SM-like Higgs boson h has Yukawa couplings with only SM fermions.
(2) ALP a can have Yukawa couplings with only exotic quarks.
(3) The pseudoscalar A_5 and H_χ can have Yukawa couplings with not only exotic quarks but also SM quarks and leptons.

IV. NUMERICAL ANALYSIS OF THE SCALAR SECTOR

To find particle content in CP-even sector namely the SM-like Higgs boson, and another one close to it H_5 is the aim in this section.

(1) In order to successfully reproduce the W gauge boson mass, the VEVs of the $SU(3)_L$ scalar triplets η and ρ should obey the following constraint:

$$v_\eta = \sqrt{v^2 - v_\rho^2}. \tag{58}$$

where $v = 246$ GeV is the electroweak symmetry breaking scale.

(2) Charged sector
 (a) From Eq. (28), it follows that positive squared scalar masses are obtained provided that the following relation is fulfilled:

$$\lambda_9 v_\rho^2 v_\eta^2 > A \tag{59}$$

 (b) From Eq. (35), it follows that

$$\lambda_8 v_\rho^2 v_\chi^2 > A \tag{60}$$

(3) CP-odd sector
 (i) From Eq. (39) it follows that the requirement of obtaining positive squared mass for the massive complex scalar φ^0 implies:

$$\lambda_7 v_\eta^2 v_\chi^2 > A. \tag{61}$$

 (ii) From Eq. (46), it follows

$$m_{A_5}^2 = -\frac{A}{2}\left(\frac{1}{v_\phi^2} + \frac{1}{v_\chi^2} + \frac{1}{v_\rho^2} + \frac{1}{v_\eta^2}\right) \simeq -\frac{\lambda_\phi v_\phi v_\chi}{\sin 2\alpha}. \tag{62}$$

If $v_\eta = v_\rho$ in EW scale, then we may have $(m_{A_5}^2)_{\min} = -\lambda_\phi v_\phi v_\chi$, which implies $\lambda_\phi < 0$. From Eq. (62), we get $\lambda_\phi = -\frac{m_{A_5}^2 \sin 2\alpha}{v_\phi v_\chi}$. With $m_{A_5} \sim 10^3$ GeV, $v_\phi \sim 10^{10}$ GeV, and $v_\chi = 10^5$ GeV, then we get $|\lambda_\phi| < 10^{-9}$. Moreover, from the condition for λ_9 and assuming $v_\eta = v_\rho \simeq 174$ GeV, $v_\phi = 10^{10}$ GeV, and $v_\chi = 10^5$ GeV, then we get $|\lambda_\phi| < 10^{-10}$. The tiny value of the quartic scalar coupling λ_ϕ can be qualitatively understood from the requirement of having a physical pseudoscalar A_5 with a mass at the TeV or subTeV scale. It is worth mentioning that the Z_{11} symmetry is spontaneously broken at a very large scale $\sim 10^{10}$ GeV by the VEV of the singlet scalar field ϕ, which also generates the mass for the physical pseudoscalar A_5. Another more formal way to justify the smallness of λ_ϕ is by considering an accidental Peccei-Quinn symmetry $U(1)_{PQ}$ under which ϕ has charge equal to -2, whereas the right handed Majorana neutrinos, the $SU(3)_L$ leptonic triplets and the right handed leptons will have charges equal to 1. Under that assignment the quartic scalar interaction involving λ_ϕ will be forbidden at tree level, however the mass of the pseudoscalar A_5 can be radiatively generated from a box diagram involving the one loop level exchange of the neutral components of the $SU(3)_L$ scalar triplets as well as the exchange of the scalar singlet ϕ. That loop suppression together with the large mass scale of the CP even component of ϕ can be interpreted as dynamical sources for the tiny values of the λ_ϕ coupling. Besides that, it is worth mentioning that low energy effective theory below the scale of breaking of the $SU(3)_L \times U(1)_X$ gauge symmetry corresponds to a two Higgs doublet model, where the consistency with allowed experimental ranges for the oblique T, S and U parameters, requires that the masses of the non SM scalars should not differ significantly [50]. In view of the above, it is required that the pseudoscalar A_5 should acquire a mass at the subTeV or TeV scale, not far from the masses of the physical scalar states arising from the η and ρ scalar triplets.

(4) CP-even sector

(i) Mass of inflaton

$$m_\Phi = \sqrt{2\lambda_{10}}v_\phi \approx 10^{11}\ \text{GeV} \Rightarrow \lambda_{10} \approx 1 \quad \text{if } v_\phi \approx 10^{10}\ \text{GeV}. \tag{63}$$

(ii) Mass of heavy scalar: The Eq. (B20) yields

$$m^2_{H_\chi} \approx 2\lambda_1 v_\chi^2 + \frac{\lambda_5^2}{2\lambda_1} v_\rho^2. \tag{64}$$

(iii) Two light scalars: From the Eq. (B29) and use the approximation $\lambda_2 \simeq \lambda_3 \simeq \lambda_6$ we have:

$$m^2_{h,h5} \approx \lambda_3 v^2 + \frac{m^2_{A_5}}{2} \pm \sqrt{m^4_{A_5} + \lambda_3^2(v^4 - 3v_\eta^2 v_\rho^2) - \frac{\lambda_3 m^2_{A_5}(v^4 - 2v_\eta^2 v_\rho^2)}{v^2}}. \tag{65}$$

In case $v_\eta = v_\rho = \frac{v}{\sqrt{2}}$, the model predicts

$$m^2_{h,h5} \simeq \lambda_3 v^2 + \frac{m^2_{A_5}}{2} \pm \frac{\lambda_3 v^2 - m^2_{A_5}}{2}. \tag{66}$$

Then we have:

$$m_h^2 \simeq \frac{3}{2}\lambda_3 v^2, \tag{67}$$

$$m^2_{h_5} \simeq \frac{\lambda_3 v^2}{2} + m^2_{A_5} \tag{68}$$

One scalar is the SM like Higgs boson h with mass of 125 GeV. One another scalar is a new one h_5 with mass takes the values of 150 GeV [51–57] or 96 GeV [58–61], respectively. The mass value of h_5 depends on some parameters such as $\lambda_2, \lambda_3, \lambda_\phi$ and the VEVs of the scalar fields in this model. From (67) and (68), we have the correlation between A_5, h, and h_5 as below:

$$|m^2_{h_5} - m^2_{A_5}| = \mathcal{O}(m_h^2). \tag{69}$$

From Eq. (69), it follows that in the case $v_\eta = v_\rho$, the splitting by masses of h_5 and A_5 is about few hundreds GeV.

V. YUKAWA COUPLINGS AND TOP QUARK FCNC DECAYS

In the quark sector, there are two parts: exotic quarks without mass mixing and ordinary quarks with mass mixing. Because of having no mass mixing, the mass eigenstates of exotic quarks are their original states. Then, we just consider on the mass mixing of ordinary quarks. The mass matrices of ordinary quarks are

$$M_u = \begin{pmatrix} (y_6)_{11}\frac{v_\rho}{v_\eta} & (y_6)_{12}\frac{v_\rho}{v_\eta} & (y_6)_{13}\frac{v_\rho}{v_\eta} \\ (y_6)_{21}\frac{v_\rho}{v_\eta} & (y_6)_{22}\frac{v_\rho}{v_\eta} & (y_6)_{23}\frac{v_\rho}{v_\eta} \\ (y_3)_{31} & (y_3)_{32} & (y_3)_{33} \end{pmatrix} \frac{v_\eta}{\sqrt{2}} = V_{uL}\tilde{M}_u V^\dagger_{uR}, \tag{70}$$

with

$$\tilde{M}_u = \text{diag}(m_u, m_c, m_t) \tag{71}$$

and

$$M_d = \begin{pmatrix} (y_4)_{11}\frac{v_\eta}{v_\rho} & (y_4)_{12}\frac{v_\eta}{v_\rho} & (y_4)_{13}\frac{v_\eta}{v_\rho} \\ (y_4)_{21}\frac{v_\eta}{v_\rho} & (y_4)_{22}\frac{v_\eta}{v_\rho} & (y_4)_{23}\frac{v_\eta}{v_\rho} \\ (y_5)_{31} & (y_5)_{32} & (y_5)_{33} \end{pmatrix} \frac{v_\rho}{\sqrt{2}} = V_{dL}\tilde{M}_d V_{dR}^\dagger, \quad (72)$$

with

$$\tilde{M}_d = \mathrm{diag}(m_d, m_s, m_b). \quad (73)$$

$$K = V_{uL}^\dagger V_{dL}. \quad (74)$$

In these matrices above, all Yukawa couplings of the form $(y_i)_{ab}$ a, $b = 1, 2, 3$; $i = 3, 4, 5, 6$ are real and positive. With $\alpha = 1, 2$ and $a = \alpha, 3$, these couplings can be defined by the following equations:

$$(y_6)_{na} = \frac{\sqrt{2}}{v_\rho}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{na},$$
$$(y_3)_{3a} = \frac{\sqrt{2}}{v_\eta}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{3a} \quad (75)$$

$$(y_4)_{na} = \frac{\sqrt{2}}{v_\eta}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{na},$$
$$(y_5)_{3a} = \frac{\sqrt{2}}{v_\rho}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{3a}. \quad (76)$$

From (70) and (72), the diagonalized mass matrix of ordinary quarks are defined as below:

$$\tilde{M}_{u,d} = (V_L^{(u,d)})^\dagger M_{u,d} V_R^{(u,d)}. \quad (77)$$

In general, we get:

$$\begin{aligned} \tilde{M}_f &= (M_f)_{\mathrm{diag}} = V_{fL}^\dagger M_f V_{fR}, \\ f_{(L,R)} &= V_{f(L,R)}\tilde{f}_{(L,R)}, \\ \bar{f}_{aL}(M_f)_{ab}f_{bR} &= \bar{\tilde{f}}_{kL}(V_{fL}^\dagger)_{ka}(M_f)_{ab}(V_{fR})_{bl}\tilde{f}_{lR} \\ &= \bar{\tilde{f}}_{kL}(V_{fL}^\dagger M_f V_{fR})_{kl}\tilde{f}_{lR} = \bar{\tilde{f}}_{kL}(\tilde{M}_f)_{kl}\tilde{f}_{lR} \\ &= m_{f_k}\bar{\tilde{f}}_{kL}\tilde{f}_{kR}, \quad k = 1, 2, 3. \end{aligned} \quad (78)$$

Here, $\tilde{f}_{k(L,R)}$ and $f_{k(L,R)}$ ($k = 1, 2, 3$) are the SM fermionic fields in the mass and interaction bases, respectively. Hence, the SM up and down type quark Yukawa interactions are given by:

$$\begin{aligned} -\mathcal{L}_Y^{(u)} &= \sum_{n=1}^{2}\sum_{a=1}^{3}(y_6)_{na}\bar{u}_{nL}\frac{v_\rho + R_\rho - iI_\rho}{\sqrt{2}}u_{aR} + \sum_{a=1}^{3}(y_3)_{3a}\bar{u}_{3L}\frac{v_\eta + R_\eta^1 + iI_\eta^1}{\sqrt{2}}u_{bR} + h.c \\ &= \sum_{n=1}^{2}\sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{c=1}^{3}(y_6)_{na}\bar{\tilde{u}}_{cL}((V_L^{(u)})^\dagger)_{cn}\frac{v_\rho + R_\rho - iI_\rho}{\sqrt{2}}(V_R^{(u)})_{ab}\tilde{u}_{bR} \\ &\quad + \sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{c=1}^{3}(y_3)_{3a}\bar{\tilde{u}}_{cL}((V_L^{(u)})^\dagger)_{c3}\frac{v_\eta + R_\eta^1 + iI_\eta^1}{\sqrt{2}}(V_R^{(u)})_{ab}\tilde{u}_{bR} + h.c \\ &= \sum_{n=1}^{2}\sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{c=1}^{3}\frac{\sqrt{2}}{v_\rho}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{na}\bar{\tilde{u}}_{cL}((V_L^{(u)})^\dagger)_{cn}\frac{v_\rho + R_\rho - iI_\rho}{\sqrt{2}}(V_R^{(u)})_{ab}\tilde{u}_{bR} \\ &\quad + \sum_{a=1}^{3}\frac{\sqrt{2}}{v_\eta}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{3a}\bar{\tilde{u}}_{cL}((V_L^{(u)})^\dagger)_{c3}\frac{v_\eta + R_\eta^1 + iI_\eta^1}{\sqrt{2}}(V_R^{(u)})_{ab}\tilde{u}_{bR} + \mathrm{H.c.} \end{aligned} \quad (79)$$

$$\begin{aligned} -\mathcal{L}_Y^{(d)} &= \sum_{n=1}^{2}\sum_{a=1}^{3}(y_4)_{na}\bar{d}_{nL}\frac{v_\eta + R_\eta^1 - iI_\eta^1}{\sqrt{2}}d_{aR} + \sum_{a=1}^{3}(y_5)_{3a}\bar{d}_{3L}\frac{v_\rho + R_\rho + iI_\rho}{\sqrt{2}}d_{bR} + h.c \\ &= \sum_{n=1}^{2}\sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{c=1}^{3}(y_4)_{na}\bar{\tilde{d}}_{cL}((V_L^{(d)})^\dagger)_{cn}\frac{v_\eta + R_\eta^1 - iI_\eta^1}{\sqrt{2}}(V_R^{(d)})_{ab}\tilde{d}_{bR} \\ &\quad + \sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{c=1}^{3}(y_5)_{3a}\bar{\tilde{d}}_{cL}((V_L^{(d)})^\dagger)_{c3}\frac{v_\rho + R_\rho + iI_\rho}{\sqrt{2}}(V_R^{(d)})_{ab}\tilde{d}_{bR} + h.c \\ &= \sum_{n=1}^{2}\sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{c=1}^{3}\frac{\sqrt{2}}{v_\eta}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{na}\bar{\tilde{d}}_{cL}((V_L^{(d)})^\dagger)_{cn}\frac{v_\eta + R_\eta^1 - iI_\eta^1}{\sqrt{2}}(V_R^{(d)})_{ab}\tilde{d}_{bR} \\ &\quad + \sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{c=1}^{3}\frac{\sqrt{2}}{v_\rho}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{3a}\bar{\tilde{d}}_{cL}((V_L^{(d)})^\dagger)_{c3}\frac{v_\rho + R_\rho + iI_\rho}{\sqrt{2}}(V_R^{(d)})_{ab}\tilde{d}_{bR} + \mathrm{H.c.} \end{aligned} \quad (80)$$

Replacing Eqs. (43) and (52) in (79) and (80), we found that the Yukawa couplings of h, h_5, and A_5 with up and down -type SM quarks are given by:

$$(\Gamma_u^h)_{ij} = \frac{\cos\alpha_2}{v_\rho}\sum_{n=1}^{2}\sum_{a=1}^{3}((V_L^{(u)})^\dagger)_{in}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{na}(V_R^{(u)})_{aj} - \frac{\sin\alpha_2}{v_\eta}\sum_{a=1}^{3}((V_L^{(u)})^\dagger)_{i3}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{3a}(V_R^{(u)})_{aj} \tag{81}$$

$$(\Gamma_u^{h_5})_{ij} = \frac{\sin\alpha_2}{v_\rho}\sum_{n=1}^{2}\sum_{a=1}^{3}((V_L^{(u)})^\dagger)_{in}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{na}(V_R^{(u)})_{aj} + \frac{\cos\alpha_2}{v_\eta}\sum_{a=1}^{3}((V_L^{(u)})^\dagger)_{i3}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{3a}(V_R^{(u)})_{aj} \tag{82}$$

$$(\Gamma_u^{A_5})_{ij} = -i\frac{\sin\alpha}{v_\rho}\sum_{n=1}^{2}\sum_{a=1}^{3}((V_L^{(u)})^\dagger)_{in}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{na}(V_R^{(u)})_{aj} + i\frac{\cos\alpha}{v_\eta}\sum_{a=1}^{3}((V_L^{(u)})^\dagger)_{i3}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{3a}(V_R^{(u)})_{aj} \tag{83}$$

$$(\Gamma_d^h)_{ij} = -\frac{\sin\alpha_2}{v_\eta}\sum_{n=1}^{2}\sum_{a=1}^{3}((V_L^{(d)})^\dagger)_{in}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{na}(V_R^{(d)})_{aj} + \frac{\cos\alpha_2}{v_\rho}\sum_{a=1}^{3}((V_L^{(d)})^\dagger)_{i3}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{3a}(V_R^{(d)})_{aj} \tag{84}$$

$$(\Gamma_d^{h_5})_{ij} = \frac{\cos\alpha_2}{v_\eta}\sum_{n=1}^{2}\sum_{a=1}^{3}((V_L^{(d)})^\dagger)_{in}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{na}(V_R^{(d)})_{aj} + \frac{\sin\alpha_2}{v_\rho}\sum_{a=1}^{3}((V_L^{(d)})^\dagger)_{i3}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{3a}(V_R^{(d)})_{aj} \tag{85}$$

$$(\Gamma_d^{A_5})_{ij} = -i\frac{\cos\alpha}{v_\eta}\sum_{n=1}^{2}\sum_{a=1}^{3}((V_L^{(d)})^\dagger)_{in}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{na}(V_R^{(d)})_{aj} + i\frac{\sin\alpha}{v_\rho}\sum_{a=1}^{3}((V_L^{(d)})^\dagger)_{i3}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{3a}(V_R^{(d)})_{aj} \tag{86}$$

Rewriting the couplings (81) and (84) in another form, one gets:

$$(\Gamma_{u,d}^h)_{ij} = \frac{\cos\alpha_2}{v_\rho}(\tilde{M}_{u,d})_{ij} - \frac{\cos\alpha_2}{v_\eta}(\tan\alpha + \tan\alpha_2)(\Gamma_h'^{(u,d)})_{ij}. \tag{87}$$

The first term in (87) is a flavor conserving. The second term in (87) is a flavor changing. In order to have flavor conservation for SM-Higgs interactions, the second term should be vanished. Then, one gets the condition below:

$$\tan\alpha = -\tan\alpha_2 \tag{88}$$

The Eq. (88) gives the condition among v_ρ and $v_\phi, v_\chi, \lambda_\phi, \lambda_2, \lambda_3, \lambda_6$ which guarantees the flavor conservation of SM-Higgs at tree level. In the SM, the resulting top quark FCNCs are strongly suppressed. But in this model, the FCNCs of top quark appear and can be used to look for new physics. The Yukawa couplings of up-type quarks $\Gamma_{ut,ct}^{h,h_5}$ allow some decays at tree-level such as: $t \to hu$ or $t \to hc$. These processes get the branching ratios limited by ATLAS [62]: at 95% C.L. upper limits on the $\mathrm{Br}(t \to hc) = 1.1 \times 10^{-3}(8.3 \times 10^{-4})$ and $\mathrm{Br}(t \to hu) = 1.2 \times 10^{-3}(8.3 \times 10^{-4})$, respectively. The corresponding combined observed (expected) upper limits on the couplings $|\Gamma_{tc}^h| = 0.064(0.055)$ and $|\Gamma_{tu}^h| = 0.066(0.055)$, respectively.

Considering the process $t \to hc$, its branching ratio is given by:

$$\mathrm{Br}(t \to hc) = \frac{\frac{g_{thc}^2}{4\pi}\frac{(m_t^2 - m_h^2)^2}{2m_t m_h}}{\Gamma_t}, \tag{89}$$

with $\Gamma_t = 1.32$ GeV is the decay width for top quark ($m_t = 172.5$ GeV) predicted by SM. And g_{thc} is the coupling defined by [63]:

$$g_{thc}^2 = \left(\frac{\cos\alpha_2}{v_\rho}((V_L^{(u)})^\dagger)_{23}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{32}(V_R^{(u)})_{23} - \frac{\sin\alpha_2}{v_\eta}((V_L^{(u)})^\dagger)_{23}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{32}(V_R^{(u)})_{23}\right)^2 + \left(\frac{\cos\alpha_2}{v_\rho}((V_L^{(u)})^\dagger)_{32}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{23}(V_R^{(u)})_{32} - \frac{\sin\alpha_2}{v_\eta}((V_L^{(u)})^\dagger)_{32}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{23}(V_R^{(u)})_{32}\right)^2 \tag{90}$$

We can also get the branching ratio for the process $t \to hu$ as follows:

$$\mathrm{Br}(t \to hu) = \frac{\frac{g_{thu}^2}{4\pi}\frac{(m_t^2 - m_h^2)^2}{2m_t m_h}}{\Gamma_t}, \tag{91}$$

with g_{thu} is the coupling that is similarly defined by:

$$g^2_{thu} = \left(\frac{\cos\alpha_2}{v_\rho}((V_L^{(u)})^\dagger)_{13}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{31}(V_R^{(u)})_{13} - \frac{\sin\alpha_2}{v_\eta}((V_L^{(u)})^\dagger)_{13}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{31}(V_R^{(u)})_{13}\right)^2 + \left(\frac{\cos\alpha_2}{v_\rho}((V_L^{(u)})^\dagger)_{31}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{13}(V_R^{(u)})_{31} - \frac{\sin\alpha_2}{v_\eta}((V_L^{(u)})^\dagger)_{31}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{13}(V_R^{(u)})_{31}\right)^2 \quad (92)$$

With $m_h = 125$ GeV, $m_t = 172.9$ GeV and the branching ratios limited by ATLAS that we mentioned above, we plot the correlation between the mixing angle in range $(-\frac{\pi}{2}, -\frac{\pi}{4})$ and the branching ratios of $t \to hq$ decay with $q = u$, c. Moreover, in this model, we have a light non SM CP even scalar field such as h_5 then the decays $t \to qh_5$ $(q = c, u)$ can be under the consideration as well as the decays $t \to hq$. The couplings of the decays $t \to qh_5$ $(q = c, u)$ are defined by:

$$g^2_{th_5q_i} = \left(\frac{-\sin\alpha_2}{v_\rho}((V_L^{(u)})^\dagger)_{i3}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{3i}(V_R^{(u)})_{i3} - \frac{\cos\alpha_2}{v_\eta}((V_L^{(u)})^\dagger)_{i3}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{3i}(V_R^{(u)})_{i3}\right)^2 + \left(\frac{-\sin\alpha_2}{v_\rho}((V_L^{(u)})^\dagger)_{3i}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{i3}(V_R^{(u)})_{3i} - \frac{\cos\alpha_2}{v_\eta}((V_L^{(u)})^\dagger)_{3i}(V_{uL}\tilde{M}_u V_{uR}^\dagger)_{i3}(V_R^{(u)})_{3i}\right)^2, \quad (93)$$

with $q_1 = u, q_2 = c, i = 1, 2$. Hence, the branching ratios of $t \to qh_5$ $(q = c, u)$ are

$$\mathrm{Br}(t \to h_5 u) = \frac{\frac{g^2_{th_5u}}{4\pi}\frac{(m_t^2 - m_{h_5}^2)^2}{2m_t m_{h_5}}}{\Gamma_t}, \quad \mathrm{Br}(t \to h_5 c) = \frac{\frac{g^2_{th_5c}}{4\pi}\frac{(m_t^2 - m_{h_5}^2)^2}{2m_t m_{h_5}}}{\Gamma_t}. \quad (94)$$

Considering a benchmark scenario where the h_5 non-SM scalar has a mass around 150 GeV, we have numerically checked that the branching ratios for the $t \to h_5 q$ decays (with $q = u$, c) can acquire values of the order of 10^{-3}, which are within of the future experimental sensitivities.

In this section, we discuss the implications of the model in meson oscillations, in the $h \to \bar{b}b$, $h \to \bar{l}l$ decays as well as in the rare top decays $t \to c\gamma$ and $t \to u\gamma$. Furthermore, we also determine the couplings of the ALP a and pseudoscalar A_5 and we provide the corresponding discussion.

A. SM-like Higgs decays

1. SM-like Higgs decays into two down-type quarks $h \to \bar{b}b$

Use (C10), the decay rate of the process $h \to \bar{b}b$ is

$$\Gamma(h \to \bar{b}b) = \int d\Gamma = \frac{g^2_{hbb}}{8\pi} m_h \left(1 - \frac{4m_b^2}{m_h^2}\right)^{\frac{3}{2}} \quad (95)$$

with

$$\begin{aligned} g_{hb\bar{b}} &= \frac{\cos\alpha_2}{v_\rho}((V_L^{(d)})^\dagger)_{33}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{33}(V_R^{(d)})_{33} - \frac{\sin\alpha_2}{v_\eta}((V_L^{(d)})^\dagger)_{33}(V_{dL}\tilde{M}_d V_{dR}^\dagger)_{33}(V_R^{(d)})_{33} \\ &= \left(\frac{\cos\alpha_2}{v_\rho} - \frac{\sin\alpha_2}{v_\eta}\right) m_b = \left(\frac{\cos\alpha_2}{\cos\alpha} - \frac{\sin\alpha_2}{\sin\alpha}\right)\frac{m_b}{v} \\ &= \frac{\cos\alpha_2}{\sin\alpha}(\tan\alpha - \tan\alpha_2)\frac{m_b}{v} = \frac{2\cos\alpha_2}{\cos\alpha}\frac{m_b}{v} \\ &= a_{h\bar{b}b} g^{\mathrm{SM}}_{hb\bar{b}}. \end{aligned} \quad (96)$$

where $a_{hb\bar{b}}$ is the deviation factor from the SM Higgs bottom quark coupling (in the SM this factor is unity). The experimental data constraint on the $a_{h\bar{b}b}$ parameter is given by:

$$a^{\exp}_{hb\bar{b}} = 0.91^{+0.17}_{-0.16}, \quad (97)$$

2. SM-like Higgs decays into two charged leptons $h \to \bar{l}l$

Concerning the lepton sector, the Yukawa interaction for charged leptons are given by:

$$-\mathcal{L}_Y^{(l)} = \sum_{a=1}^{3}\sum_{b=1}^{3} g_{ab}\bar{l}_{aL}\frac{v_\rho + R_\rho + iI_\rho}{\sqrt{2}} l_{bR} + \mathrm{H.c.} \quad (98)$$

Replacing Eqs. (43) and (52) in Eq. (98), we get the Yukawa couplings of h with leptons as below:

$$\begin{aligned} -\mathcal{L}_Y^{(l)} &\supset \sum_{a=1}^{3}\sum_{b=1}^{3}\frac{g_{ab}\cos\alpha_2}{\sqrt{2}}\bar{l}_{aL} h l_{bR} \\ &\supset \sum_{a=1}^{3}\frac{(M_l)_{aa}\cos\alpha_2}{v_\rho}\bar{l}_{aL} h l_{aR} \\ &\supset \sum_{a=1}^{3}\frac{v\cos\alpha_2}{v_\rho}\frac{(M_l)_{aa}}{v}\bar{l}_{aL} h l_{aR}. \end{aligned} \quad (99)$$

$$g_{h\bar{l}l} = \sum_{a=1}^{3}\frac{v\cos\alpha_2}{v_\rho}\frac{(M_l)_{aa}}{v} = a_{h\bar{l}l} g^{\mathrm{SM}}_{h\bar{l}l}. \quad (100)$$

where $a_{h\bar{l}l}$ is the deviation of the $h\bar{l}l$ coupling with respect to the SM prediction (in the SM this factor is unity).

Using (C10), the decay rate of the process $h \to \mu\mu$ and $h \to \tau\tau$ are

$$\Gamma(h \to \mu\mu) = \int d\Gamma = \frac{g^2_{(h,\mu,\mu)}}{8\pi} m_h \left(1 - \frac{4m_\mu^2}{m_h^2}\right)^{\frac{3}{2}} = \left(\frac{v \cos\alpha_2}{v_\rho}\right)^2 \frac{m_\mu^2}{v^2}\frac{m_h}{8\pi}\left(1 - \frac{4m_\mu^2}{m_h^2}\right)^{\frac{3}{2}} = \left(\frac{\cos\alpha_2}{\cos\alpha}\right)^2 \frac{m_\mu^2}{v^2}\frac{m_h}{8\pi}\left(1 - \frac{4m_\mu^2}{m_h^2}\right)^{\frac{3}{2}} \quad (101)$$

$$\Gamma(h \to \tau\tau) = \int d\Gamma = \frac{g^2_{(h,\tau,\tau)}}{8\pi} m_h \left(1 - \frac{4m_\tau^2}{m_h^2}\right)^{\frac{3}{2}} = \left(\frac{v \cos\alpha_2}{v_\rho}\right)^2 \frac{m_\tau^2}{v^2}\frac{m_h}{8\pi}\left(1 - \frac{4m_\tau^2}{m_h^2}\right)^{\frac{3}{2}} = \left(\frac{\cos\alpha_2}{\cos\alpha}\right)^2 \frac{m_\tau^2}{v^2}\frac{m_h}{8\pi}\left(1 - \frac{4m_\tau^2}{m_h^2}\right)^{\frac{3}{2}} \quad (102)$$

From (101) and (102), one can get the constraints of the mixing angle α_2 in this model. Using the following experimental allowed values of the parameters [64]:

$$a_{h\mu\mu}^{\rm exp} = 0.72^{+0.50}_{-0.72}, \qquad a_{h\tau\tau}^{\rm exp} = 0.93^{+0.13}_{-0.13}, \quad (103)$$

we can obtain plots where the allowed range of the mixing angle in the CP even scalar sector is shown. Furthermore, we have found the our obtained values for the $a_{h\mu\mu,\tau\tau}$ parameters range from about 0.6 up to about 1.2, which is consistent with their current experimental bounds. This is shown in Fig. 1, which displays a linear correlation between the $a_{h\tau\tau}$ and $a_{h\mu\mu}$ parameters.

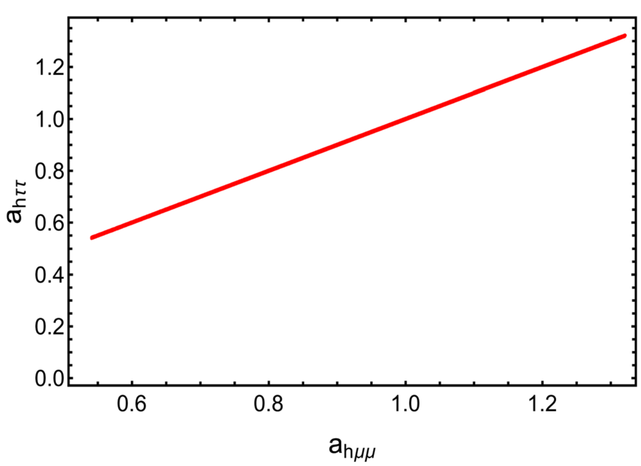

FIG. 1. Correlation between the $a_{h\tau\tau}$ and $a_{h\mu\mu}$ parameters.

Requiring the consistency of the rates for the $h \to \bar{\mu}\mu$, $h \to \bar{\tau}\tau$ and $h \to \bar{b}b$ decays with their corresponding experimentally allowed ranges, we display in Fig. 2 the correlation between the mixing angles α and α_2.

From the Fig. 2, with α in range $38^o \le \alpha \le 70^o$, we get the following constraints for the mixing angle α_2:

$$0^o \le \alpha_2 \le 75^o, \quad \text{or} \quad 280^o \le \alpha_2 \le 360^o. \quad (104)$$

We will use this constraint to analyze the meson oscillations of this model in the subsection below.

B. Meson oscillations

In this section, we analyze the consequences of the model under consideration in the $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ meson oscillations. These meson oscillations are caused by flavor violating scalar and Z' interactions in the down type quark sector. The $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ meson mixings are described by the following effective Hamiltonians:

$$\mathcal{H}_{\rm eff}^{(K^0-\bar{K}^0)} = \frac{G_F^2 m_W^2}{16\pi^2}\sum_{i=1}^{3} C_i^{(K^0-\bar{K}^0)}(\mu) O_i^{(K^0-\bar{K}^0)}(\mu) + \frac{4\sqrt{2}G_F c_W^4 m_Z^2}{(3-4s_W^2)m_{Z'}^2}|(V_{DL}^*)_{32}(V_{DL})_{31}|^2 O_4^{(K^0-\bar{K}^0)}, \quad (105)$$

$$\mathcal{H}_{\rm eff}^{(B_d^0-\bar{B}_d^0)} = \frac{G_F^2 m_W^2}{16\pi^2}\sum_{i=1}^{3} C_i^{(B_d^0-\bar{B}_d^0)}(\mu) O_i^{(B_d^0-\bar{B}_d^0)}(\mu) + \frac{4\sqrt{2}G_F c_W^4 m_Z^2}{(3-4s_W^2)m_{Z'}^2}|(V_{DL}^*)_{31}(V_{DL})_{33}|^2 O_4^{(B_d^0-\bar{B}_d^0)}, \quad (106)$$

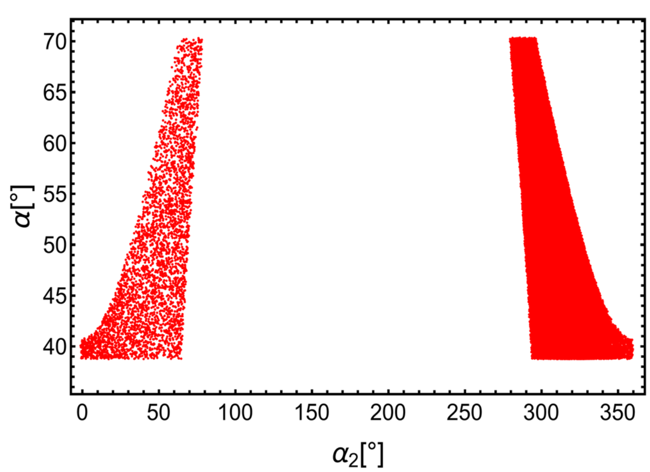

FIG. 2. Correlation between the mixing angles α and α_2 consistent with the experimental values of the $h \to \bar{\mu}\mu$, $h \to \bar{\tau}\tau$ and $h \to \bar{b}b$ decay rates.

$$\mathcal{H}_{\rm eff}^{(B_s^0-\bar{B}_s^0)}=\frac{G_F^2 m_W^2}{16\pi^2}\sum_{i=1}^{3} C_i^{(B_s^0-\bar{B}_s^0)}(\mu)O_i^{(B_s^0-\bar{B}_s^0)}(\mu) + \frac{4\sqrt{2}G_F c_W^4 m_Z^2}{(3-4s_W^2)m_{Z'}^2}|(V_{DL}^*)_{32}(V_{DL})_{33}|^2 O_4^{(B_s^0-\bar{B}_s^0)}, \tag{107}$$

where V_{DL} is the rotation matrix that diagonalizes $M_D M_D^\dagger$ according to $V_{DL}^\dagger M_D M_D^\dagger V_{DL} = \mathrm{diag}(m_d^2, m_s^2, m_b^2)$ being M_D the SM down type quark mass matrix. Furthermore, the operators appearing in Eqs. (105), (106), and (107) are given by:

$$O_1^{(K^0-\bar{K}^0)} = (\bar{s}P_L d)(\bar{s}P_L d),\quad O_2^{(K^0-\bar{K}^0)} = (\bar{s}P_R d)(\bar{s}P_R d), \tag{108}$$

$$O_3^{(K^0-\bar{K}^0)} = (\bar{s}P_L d)(\bar{s}P_R d),\quad O_4^{(K^0-\bar{K}^0)} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d), \tag{109}$$

$$O_1^{(B_d^0-\bar{B}_d^0)} = (\bar{d}P_L b)(\bar{d}P_L b),\quad O_2^{(B_d^0-\bar{B}_d^0)} = (\bar{d}P_R b)(\bar{d}P_R b), \tag{110}$$

$$O_3^{(B_d^0-\bar{B}_d^0)} = (\bar{d}P_L b)(\bar{d}P_R b),\quad O_4^{(B_d^0-\bar{B}_d^0)} = (\bar{d}\gamma_\mu P_L b)(\bar{d}\gamma^\mu P_L b), \tag{111}$$

$$O_1^{(B_s^0-\bar{B}_s^0)} = (\bar{s}P_L b)(\bar{s}P_L b),\quad O_2^{(B_s^0-\bar{B}_s^0)} = (\bar{s}P_R b)(\bar{s}P_R b), \tag{112}$$

$$O_3^{(B_s^0-\bar{B}_s^0)} = (\bar{s}P_L b)(\bar{s}P_L b),\quad O_4^{(B_s^0-\bar{B}_s^0)} = (\bar{s}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L b), \tag{113}$$

and the Wilson coefficients read:

$$C_1^{(K^0-\bar{K}^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g^2_{h\bar{s}_R d_L}}{m_h^2} + \frac{g^2_{h_5\bar{s}_R d_L}}{m_{h_5}^2} - \frac{g^2_{A_5\bar{s}_R d_L}}{m_{A_5}^2}\right), \tag{114}$$

$$C_2^{(K^0-\bar{K}^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g^2_{h\bar{s}_L d_R}}{m_h^2} + \frac{g^2_{h_5\bar{s}_L d_R}}{m_{h_5}^2} - \frac{g^2_{A_5\bar{s}_L d_R}}{m_{A_5}^2}\right), \tag{115}$$

$$C_3^{(K^0-\bar{K}^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g_{h\bar{s}_R d_L}g_{h\bar{s}_L d_R}}{m_h^2} + \frac{g_{h_5\bar{s}_R d_L}g_{h_5\bar{s}_L d_R}}{m_{h_5}^2} - \frac{g_{A_5\bar{s}_R d_L}g_{A_5\bar{s}_L d_R}}{m_{A_5}^2}\right), \tag{116}$$

$$C_1^{(B_d^0-\bar{B}_d^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g^2_{h\bar{d}_R b_L}}{m_h^2} + \frac{g^2_{h_5\bar{d}_R b_L}}{m_{h_5}^2} - \frac{g^2_{A_5\bar{d}_R b_L}}{m_{A_5}^2}\right), \tag{117}$$

$$C_2^{(B_d^0-\bar{B}_d^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g^2_{h\bar{d}_L b_R}}{m_h^2} + \frac{g^2_{h_5\bar{d}_L b_R}}{m_{h_5}^2} - \frac{g^2_{A_5\bar{d}_L b_R}}{m_{A_5}^2}\right), \tag{118}$$

$$C_3^{(B_d^0-\bar{B}_d^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g_{h\bar{d}_R b_L}g_{h\bar{d}_L b_R}}{m_h^2} + \frac{g_{h_5\bar{d}_R b_L}g_{h_5\bar{d}_L b_R}}{m_{h_5}^2} - \frac{g_{A_5\bar{d}_R b_L}g_{A_5\bar{d}_L b_R}}{m_{A_5}^2}\right), \tag{119}$$

$$C_1^{(B_s^0-\bar{B}_s^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g^2_{h\bar{s}_R b_L}}{m_h^2} + \frac{g^2_{h_5\bar{s}_R b_L}}{m_{h_5}^2} - \frac{g^2_{A_5\bar{s}_R b_L}}{m_{A_5}^2}\right), \tag{120}$$

$$C_2^{(B_s^0-\bar{B}_s^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g^2_{h\bar{s}_L b_R}}{m_h^2} + \frac{g^2_{h_5\bar{s}_L b_R}}{m_{h_5}^2} - \frac{g^2_{A_5\bar{s}_L b_R}}{m_{A_5}^2}\right), \tag{121}$$

$$C_3^{(B_s^0-\bar{B}_s^0)} = \frac{16\pi^2}{G_F^2 m_W^2}\left(\frac{g_{h\bar{s}_R b_L}g_{h\bar{s}_L b_R}}{m_h^2} + \frac{g_{h_5\bar{s}_R b_L}g_{h_5\bar{s}_L b_R}}{m_{h_5}^2} - \frac{g_{A_5\bar{s}_R b_L}g_{A_5\bar{s}_L b_R}}{m_{A_5}^2}\right), \tag{122}$$

with g_{abc} are the couplings between the scalar $a = h, h_5, A_5$ and down-type quarks $b = \bar{d}^i_{L,R}$, $c = d^j_{L,R}$, $i, j = 1, 2, 3$, $i \neq j$. On the other hand, the $K - \bar{K}$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mass splittings are given by:

$$\Delta m_K = (\Delta m_K)_{\rm SM} + \Delta m_K^{(NP)},\quad \Delta m_{B_d} = (\Delta m_{B_d})_{\rm SM} + \Delta m_{B_d}^{(NP)},\quad \Delta m_{B_s} = (\Delta m_{B_s})_{\rm SM} + \Delta m_{B_s}^{(NP)}, \tag{123}$$

where $(\Delta m_K)_{\rm SM}$, $(\Delta m_{B_d})_{\rm SM}$ and $(\Delta m_{B_s})_{\rm SM}$ are the SM contributions, whereas $\Delta m_K^{(NP)}$, $\Delta m_{B_d}^{(NP)}$ and $(\Delta m_{B_s})_{\rm SM}$ are new physics contributions.

In the model under consideration, the new physics contributions to the meson differences are given by:

$$\Delta m_K^{(NP)} = \frac{4\sqrt{2}G_F c_W^4 m_Z^2}{(3-4s_W^2)m_{Z'}^2}|(V_{DL}^*)_{32}(V_{DL})_{31}|^2 f_K^2 B_K \eta_K m_K + \frac{G_F^2 m_W^2}{6\pi^2} m_K f_K^2 \eta_K B_K [P_2^{(K^0-\bar{K}^0)} C_3^{(K^0-\bar{K}^0)} + P_1^{(K^0-\bar{K}^0)}(C_1^{(K^0-\bar{K}^0)} + C_2^{(K^0-\bar{K}^0)})],$$

$$\Delta m_{B_d}^{(NP)} = \frac{4\sqrt{2}G_F c_W^4 m_Z^2}{(3-4s_W^2)m_{Z'}^2}|(V_{DL}^*)_{31}(V_{DL})_{33}|^2 f_{B_d}^2 B_{B_d} \eta_{B_d} m_{B_d} + \frac{G_F^2 m_W^2}{6\pi^2} m_{B_d} f_{B_d}^2 \eta_{B_d} B_{B_d} [P_2^{(B_d^0-\bar{B}_d^0)} C_3^{(B_d^0-\bar{B}_d^0)} + P_1^{(B_d^0-\bar{B}_d^0)}(C_1^{(B_d^0-\bar{B}_d^0)} + C_2^{(B_d^0-\bar{B}_d^0)})],$$

$$\Delta m_{B_s}^{(NP)} = \frac{4\sqrt{2}G_F c_W^4 m_Z^2}{(3-4s_W^2)m_{Z'}^2}|(V_{DL}^*)_{32}(V_{DL})_{33}|^2 f_{B_s}^2 B_{B_s}\eta_{B_s} m_{B_s} + \frac{G_F^2 m_W^2}{6\pi^2} m_{B_s} f_{B_s}^2 \eta_{B_s} B_{B_s}[P_2^{(B_s^0-\bar{B}_s^0)} C_3^{(B_s^0-\bar{B}_s^0)} + P_1^{(B_s^0-\bar{B}_s^0)}(C_1^{(B_s^0-\bar{B}_s^0)} + C_2^{(B_s^0-\bar{B}_s^0)})]$$

Using the following parameters [65]:

$$(\Delta m_K)_{\rm exp} = (3.484 \pm 0.006) \times 10^{-12}\ \text{MeV},$$
$$(\Delta m_K)_{\rm SM} = 3.483 \times 10^{-12}\ \text{MeV}$$
$$f_K = 155.7\ \text{MeV}, \qquad B_K = 0.85, \qquad \eta_K = 0.57,$$
$$P_1^{(K^0-\bar{K}^0)} = -9.3, \qquad P_2^{(K^0-\bar{K}^0)} = 30.6,$$
$$m_K = (497.611 \pm 0.013)\ \text{MeV}, \tag{124}$$

$$(\Delta m_{B_d})_{\rm exp} = (3.334 \pm 0.013) \times 10^{-10}\ \text{MeV},$$
$$(\Delta m_{B_d})_{\rm SM} = (3.653 \pm 0.037 \pm 0.019) \times 10^{-10}\ \text{MeV},$$
$$f_{B_d} = 188\ \text{MeV}, \qquad B_{B_d} = 1.26, \qquad \eta_{B_d} = 0.55,$$
$$P_1^{(B_d^0-\bar{B}_d^0)} = -0.52, \qquad P_2^{(B_d^0-\bar{B}_d^0)} = 0.88,$$
$$m_{B_d} = (5279.65 \pm 0.12)\ \text{MeV}, \tag{125}$$

$$(\Delta m_{B_s})_{\rm exp} = (1.1683 \pm 0.0013) \times 10^{-8}\ \text{MeV},$$
$$(\Delta m_{B_s})_{\rm SM} = (1.1577 \pm 0.022 \pm 0.051) \times 10^{-8}\ \text{MeV},$$
$$f_{B_s} = 225\ \text{MeV}, \qquad B_{B_s} = 1.33, \qquad \eta_{B_s} = 0.55,$$
$$P_1^{(B_s^0-\bar{B}_s^0)} = -0.52, \qquad P_2^{(B_s^0-\bar{B}_s^0)} = 0.88,$$
$$m_{B_s} = (5366.9 \pm 0.12)\ \text{MeV}, \tag{126}$$

We plot in Fig. 3 the correlation between of the Δm_K meson mass splitting with the non-SM CP even scalar mass m_{h_5}, whereas in Fig. 5 we display the allowed region in the $m_{A_5} - m_{h_5}$ plane consistent with the constraints on Δm_K, Δm_{B_d} and Δm_{B_s} meson mass splittings, whose obtained values are within the experimentally allowed range. As seen from Figs. 3 and 5, if one keeps the other parameters fixed, an increase of the non-SM CP even scalar mass m_{h_5} yields a decrease of the Δm_K meson mass difference. Besides that, Fig. 3 indicates that the number of solutions consistent with the meson oscillation constraints is increased when the mass m_{h_5} of the non SM CP even scalar h_5 acquires larger values close to the TeV scale. This is due to the fact that the scalar contributions to the meson mass splittings are inversely proportional to the square of the scalar and pseudoscalar masses m_{h_5} and m_{A_5}, then making easier to find more solutions consistent with the meson oscillation constraints in the large mass region than in the low mass region of the non SM scalars. Here the CP even and CP odd scalar masses have been varied in the ranges $200\ \text{GeV} \leq m_{h_5} \leq 1\ \text{TeV}$ and $100\ \text{GeV} \leq m_{A_5} \leq 1\ \text{TeV}$, respectively. In our numerical analysis we have varied the mixing angles α, α_2 in a range of values consistent with the experimental constraints of the $h\tau\bar{\tau}$, $h\mu\bar{\mu}$, and $hb\bar{b}$ couplings (being h is the 126 SM like Higgs boson) as well as with the meson oscillation constraints. Besides that, the VEV v_η of the neutral component of the $SU(3)_L$ scalar triplet have been varied in window around 200 GeV, which is consistent with the experimental constraints on meson mass splittings. Moreover, we have considered a simplified benchmark scenario of real down type quark sector parameters so that the CP violation in the quark sector entirely arises from the up type quark sector. Furthermore, we have set the Z' mass to be equal to 6 TeV, which is

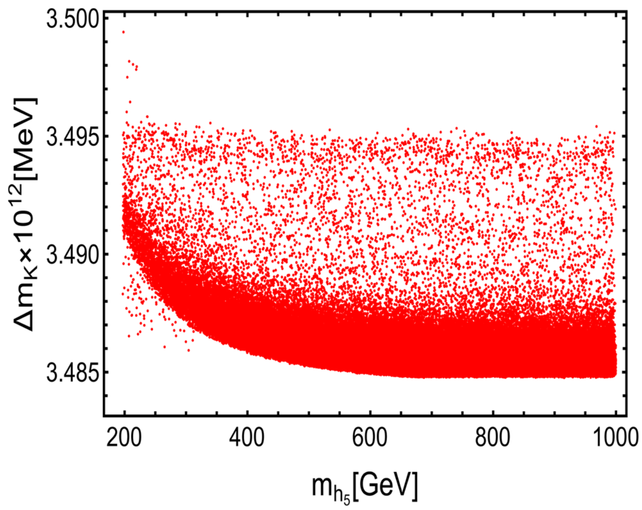

FIG. 3. Correlation of the Δm_K meson mass splitting with the heavy CP even scalar mass m_{h^5}.

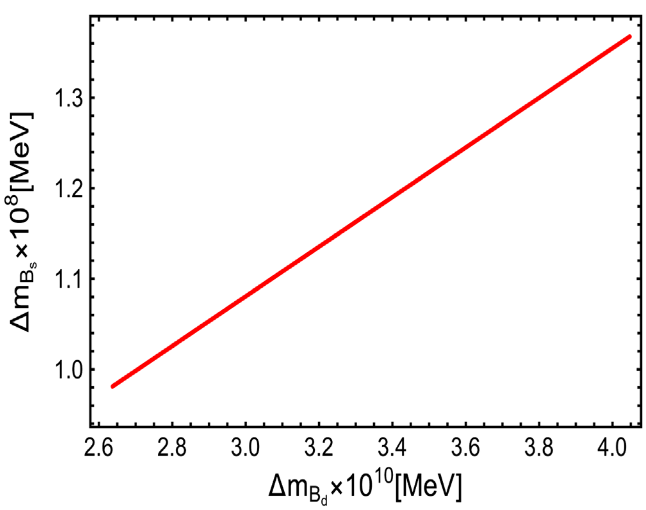

FIG. 4. Correlation between Δm_{B_d} and Δm_{B_s} meson mass splittings.

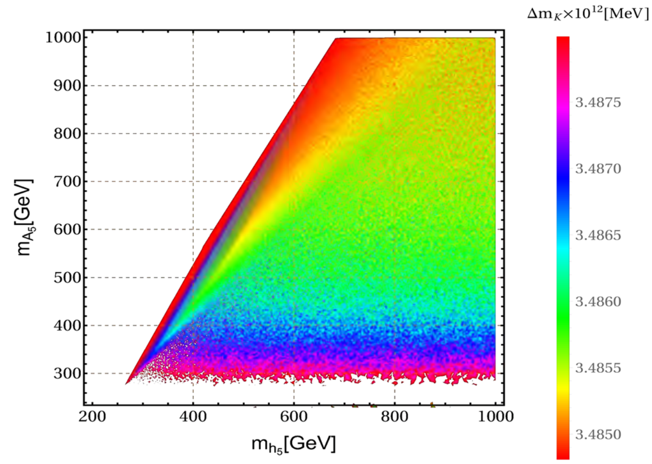

FIG. 5. Allowed region in the $m_{A_5} - m_{h_5}$ plane consistent with the constraints on Δm_K, Δm_{B_d}, and Δm_{B_s} meson mass splittings.

consistent with the constraints arising from collider searches [66,67]. Moreover, a linear correlation between Δm_{B_d} and Δm_{B_s} meson mass splittings is displayed in Fig. 4. As seen from Figs. 3 and 5, the model under consideration successfully fulfills the constraints arising from the meson oscillation experimental data and the obtained meson mass differences $K - \bar{K}$, $B_d^0 - \bar{B}_d^0$, and $B_s^0 - \bar{B}_s^0$ reach values within the reach of experimental sensitivity. Given that we are considering the case of real down type quark sector parameters, the constraints that are usually imposed on any possible new contributions to the $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$, and $B_s^0 - \bar{B}_s^0$ meson oscillations arising from CP-violating processes are not relevant for our case.

C. Rare top decays $t \to c\gamma$ and $t \to u\gamma$ with flavor changing neutral scalar interactions

In this section we discuss about the implications of the model under consideration in the rare top quark decays $t \to u\gamma$ and $t \to c\gamma$. In the SM these decays have very tiny branching ratios, however in extensions of the SM, like the 331 model considered in this paper, the branching ratios of these decays can be significantly enhanced with respect to the SM prediction. This is due to the flavor changing neutral scalar interactions in the quark sector, which provide the dominant contributions to the are top quark decays $t \to u\gamma$ and $t \to c\gamma$.

The one loop Feynman diagram is with a neutral Higgs boson in the internal line. This diagram shows the flavor changing neutral scalar contribution [68]. The rare top quark decays $t \to u\gamma$ and $t \to c\gamma$ also receive contributions from electrically charged scalars and down type quarks, however those contributions are subleading. Thus, the decay rate for the $t \to c\gamma$ and $t \to u\gamma$ processes have the form [68]:

$$\Gamma(t \to c\gamma) = \frac{\alpha G_F m_t^3 |y_{hct}|^2}{192\pi^4} \left| \left(f_1\left(\frac{m_h}{m_t}\right) + f_2\left(\frac{m_h}{m_t}\right) \right) A_h B_h + \left(f_1\left(\frac{m_{h_5}}{m_t}\right) + f_2\left(\frac{m_{h_5}}{m_t}\right) \right) A_{h_5} B_{h_5} \right|^2$$
$$\Gamma(t \to u\gamma) = \frac{\alpha G_F m_t^3 |y_{hut}|^2}{192\pi^4} \left| \left(f_1\left(\frac{m_h}{m_t}\right) + f_2\left(\frac{m_h}{m_t}\right) \right) A_h B_h + \left(f_1\left(\frac{m_{h_5}}{m_t}\right) + f_2\left(\frac{m_{h_5}}{m_t}\right) \right) A_{h_5} B_{h_5} \right|^2 \quad (127)$$

where:

$$A_h = -\frac{\sin\alpha_2}{\sin\beta}, \qquad A_{h_5} = \frac{\cos\alpha_2}{\sin\beta},$$
$$B_h = \frac{\sin\alpha_2}{\sin\beta} + \frac{\cos\alpha_2}{\cos\beta}, \qquad B_{h_5} = -\frac{\cos\alpha_2}{\sin\beta} + \frac{\sin\alpha_2}{\cos\beta}. \quad (128)$$

and the loop integrals are given by:

$$f_1(z) = \int_0^1 dx \int_0^{1-x} dy \frac{x(x+y-1)}{x^2 + xy - (2 - z^2) + 1},$$
$$f_2(z) = \int_0^1 dx \int_0^{1-x} dy \frac{x-1}{x^2 + xy - (2 - z^2) + 1} \quad (129)$$

It is worth mentioning that, in order to simplify our analysis, we have considered a simplified benchmark scenario where the neutral CP odd scalar A_5 has a mass close to the TeV scale, whereas the CP even neutral scalar h_5 has a mass in the range 100 GeV $\leq m_{h_5} \leq$ 200 GeV. Then, in this scenario, the leading contributions to the $t \to u\gamma$ and $t \to c\gamma$ decays will arise from the virtual exchange of the top quark and neutral CP even scalars h and h_5, being h the 126 GeV SM like Higgs boson. Furthermore, we have varied the flavor changing top quark Yukawa couplings y_{hct} and y_{hut} in the range 10^{-2} GeV $\leq y_{hct}, y_{hut} \leq 1.2 \times 10^{-2}$. The branching ratio for the rare top quark decays $t \to c\gamma$ and $t \to u\gamma$ are given by:

$$\mathrm{Br}(t \to c\gamma) = \frac{\Gamma(t \to c\gamma)}{\Gamma_{\mathrm{top}}}, \quad Br(t \to u\gamma) = \frac{\Gamma(t \to u\gamma)}{\Gamma_{\mathrm{top}}}, \quad (130)$$

where $\Gamma_{\mathrm{top}} = 1.42^{+0.19}_{-0.15}$ GeV is the total top quark decay width. We have numerically checked that the branching ratios for the $t \to c\gamma$ and $t \to u\gamma$ decays acquire values of the order of 10^{-10}, several orders of magnitude lower than their corresponding experimental upper bounds of 2.2×10^{-4} and 6.1×10^{-5}, respectively. On the other hand, our obtained values for the $t \to c\gamma$ and $t \to u\gamma$ decay branching ratios are 4 and 6 orders of magnitude larger than their corresponding SM values of 4.6×10^{-14} and 3.7×10^{-16}, respectively.

D. Couplings of ALP a and pseudoscalar A_5

1. Coupings with exotic quarks

Due to Z_2 symmetry, all terms containing the Yukawa interactions of ordinary quarks with ALP a are forbidden. The ALP a just interact with exotic quarks. Hence, one has

$$\mathcal{L}_a^Y = \sqrt{2} i a \sin\theta_\phi \sin\theta_3 \left(\frac{m_U}{v_\chi} \bar{U}\gamma_5 U - \sum_{\alpha=1}^{2} \frac{m_{D_\alpha}}{v_\chi} \bar{D}_\alpha \gamma_5 D_\alpha \right). \tag{131}$$

About the interactions between the pseudoscalar A_5 with quarks in the model, this A_5 interacts with not only exotic quarks but also ordinary quarks. The Yukawa interaction between A_5 with exotic quarks can be defined by the equation below:

$$\mathcal{L}_{A_5}^Y \approx \sqrt{2} i A_5 \cos\theta_\phi \sin\theta_3 \left(-\frac{m_U}{v_\chi} \bar{U}\gamma_5 U + \sum_{\alpha=1}^{2} \frac{m_{D_\alpha}}{v_\chi} \bar{D}_\alpha \gamma_5 D_\alpha \right). \tag{132}$$

So, the ALP interacts only with exotic quarks with tiny strength ($\propto \sin\theta_\phi \sin\theta_3$). This property is suitable with one of properties of dark matter. This is the reason why ALP a can be regarded as a candidate of dark matter. Remember that $\sin\theta_3$ is also very small, so the strength of interactions between the pseudoscalar A_5 and exotic quarks are also tiny($\propto \sin\theta_3$). From Eqs. (131) and (132), one gets the couplings of ALP a and pseudoscalar A_5 with exotic quarks as below:

$$g_a^{Q_i} = i\gamma_5 \sqrt{2} \sin\theta_\phi \sin\theta_3 \frac{m_{q_i}}{v_\chi}, \tag{133}$$

$$g_{A_5}^{Q_i} = i\gamma_5 \sqrt{2} \cos\theta_\phi \sin\theta_3 \frac{m_{q_i}}{v_\chi}, \tag{134}$$

with $i = \alpha, 3, \quad \alpha = 1, \quad 2, \quad Q_\alpha = D_\alpha, \quad Q_3 = U$. From Eqs. (133) and (134), we have:

$$g_a^{Q_i} \ll g_{A_5}^{Q_i}. \tag{135}$$

2. Coupings with SM-like Higgs h and new light Higgs h_5

The coupling of h and two ALP a is defined from (C4) as below:

$$g_{haa} \approx \frac{v_\rho v_\eta}{2\sqrt{2}} \left(\frac{\lambda_6 \lambda_{12}}{\sqrt{V_{236}^2 + (\lambda_3 v_\rho^2 - \lambda_2 v_\eta^2) V_{236}}} - \lambda_{13} \sqrt{V_{236} + \lambda_3 v_\rho^2 - \lambda_2 v_\eta^2} \right), \tag{136}$$

with $V_{236} = \sqrt{(\lambda_2 v_\eta^2 - \lambda_3 v_\rho^2)^2 + \lambda_6^2 v_\eta^2 v_\rho^2}$.

We also get the coupling of h and two pseudoscalar A_5 from (C5):

$$g_{hA_5A_5} \approx \frac{1}{2\sqrt{2}} \left(v_\rho (2\lambda_3 v_\eta^2 + \lambda_6 v_\rho^2) \sqrt{\frac{V_{236} - \lambda_3 v_\rho^2 + \lambda_2 v_\eta^2}{V_{236}}} - v_\eta (2\lambda_2 v_\rho^2 + \lambda_6 v_\eta^2) \sqrt{\frac{V_{236} + \lambda_3 v_\rho^2 - \lambda_2 v_\eta^2}{V_{236}}} \right). \tag{137}$$

Similarly with the new light Higgs h_5, use (C6) and (C7) one gets:

$$g_{h_5aa} \approx \frac{1}{2\sqrt{2}} v_\rho \left(\lambda_{12} \sqrt{V_{236} + \lambda_3^2 v_\rho^2 - \lambda_2 v_\eta^2} + \frac{\lambda_6 \lambda_{13} v_\eta^2}{\sqrt{V_{236}^2 + V_{236}(\lambda_3^2 v_\rho^2 - \lambda_2 v_\eta^2)}} \right) \tag{138}$$

$$g_{h_5A_5A_5} \approx \frac{v_\eta^4}{2\sqrt{2}(v_\eta^2 + v_\rho^2)(v_\eta^2 + 2v_\rho^2)^2} \times \left(v_\eta (2v_\rho^2 + \lambda_6 v_\eta^2) \sqrt{\frac{V_{236} + \lambda_2 v_\eta^2 - \lambda_3 v_\rho^2}{V_{236}}} + v_\rho (2\lambda_3 v_\eta^2 + \lambda_6 v_\rho^2) \sqrt{\frac{V_{236} + \lambda_3 v_\rho^2 - \lambda_2 v_\eta^2}{V_{236}}} \right). \tag{139}$$

From Eq. (136) to Eq. (139), we can see that couplings $g_{haa}, g_{hA_5A_5}, g_{h_5aa}, g_{h_5A_5A_5}$ depend on v_ρ, v_η in EW scale.

VI. CONCLUSIONS

We have analyzed in detail the scalar sector of the 3-3-1 model with ALP. In the model under consideration, there are two kinds of scalar fields: the bilepton scalars carrying lepton number two and ordinary ones without lepton number. We show that there is no mixing among these two kinds of scalar fields. Moreover, relations among VEVs are related to the self-interactions of scalar fields. The physical fields of ALP a and pseudoscalar A_5 are defined exactly to help us show that they just interact with exotic quarks in this model with very tiny strength. As a result, ALP is regarded as a candidate of dark matter. Our numerical analysis of the scalar sector allows to successfully accommodate a pseudoscalar A_5 with a mass ranging from 100 GeV to 1 TeV. The results are different from the others which have been published before. The CP-even scalar sector of the model was analyzed as well. Its results allow the existence of a non SM scalar boson with mass in a similar range as the pseudoscalar field A_5. Numerical analysis has shown the constraints on the couplings $\lambda_2, \lambda_3, \lambda_\phi$ with $\tan\alpha = \frac{v_\eta}{v_\rho}$ and VEVs of scalar fields ϕ, χ, η, ρ to raise the new CP even scalar h_5 and CP odd scalar A_5 with masses in the TeV or subTeV scale.

Furthermore, we analyzed the consequences of the model in several flavor changing top quark decays, in rare top quark decays, in the leptonic decays of the SM like Higgs boson as well as in the $K^0-\bar{K}^0$, $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ meson oscillations. We have found that the model under consideration is consistent with the experimental constraints arising from these processes.

ACKNOWLEDGMENTS

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APPENDIX A: DIAGONALIZATION OF *CP*-ODD MASS MIXING MATRIX IN BASIS $(I_\phi, I_\chi^3, I_\eta^1, I_\rho)$

Step by step, the matrix M^2_{odd} in (42) can be exactly diagonalized by the Euler method.

(1) In the basis (I_η^1, I_ρ), the squared mass matrix has form:

$$M^2_{I_{\eta\rho}} = \begin{pmatrix} -\frac{A}{4v_\eta^2} & -\frac{A}{4v_\eta v_\rho} \\ -\frac{A}{4v_\eta v_\rho} & -\frac{A}{4v_\rho^2} \end{pmatrix} \tag{A1}$$

The matrix in (A1) has 2 eigenvalues which are 0 and $\frac{-A(v_\eta^2+v_\rho^2)}{4v_\eta^2 v_\rho^2}$. This matrix is diagonalized by the matrix below:

$$U_{I_{\eta\rho}} = \begin{pmatrix} -\frac{v_\rho}{v_\eta\sqrt{\frac{v_\rho^2}{v_\eta^2}+1}} & \frac{1}{\sqrt{\frac{v_\rho^2}{v_\eta^2}+1}} \\ \frac{v_\eta}{v_\rho\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}} & \frac{1}{\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}} \end{pmatrix} \tag{A2}$$

Then we receive the 4×4 matrix which is used to diagonalize the matrix M^2_{odd} as following:

$$U_I^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{v_\rho}{v_\eta\sqrt{\frac{v_\rho^2}{v_\eta^2}+1}} & \frac{1}{\sqrt{\frac{v_\rho^2}{v_\eta^2}+1}} \\ 0 & 0 & \frac{v_\eta}{v_\rho\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}} & \frac{1}{\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}} \end{pmatrix} \tag{A3}$$

where the mixing angle α is defined by:

$$\tan\alpha = \frac{v_\eta}{v_\rho}. \tag{A4}$$

Under the effect of the matrix U_1 in (A3), the matrix M^2_{odd} becomes:

$$M^2_{I_\rho^{\text{diag}}} = U_I^1.M^2_{\text{odd}}.(U_I^1)^T$$
$$= \begin{pmatrix} -\frac{A}{4v_\phi^2} & -\frac{A}{4v_\chi v_\phi} & 0 & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\phi} \\ -\frac{A}{4v_\chi v_\phi} & -\frac{A}{4v_\chi^2} & 0 & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\chi} \\ 0 & 0 & 0 & 0 \\ -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\phi} & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\chi} & 0 & -\frac{A(v_\eta^2+v_\rho^2)}{4v_\eta^2 v_\rho^2} \end{pmatrix} \tag{A5}$$

(2) Continuously, we consider the 3×3 mixing matrix in (A5):

$$M^2_{I_{33}} = \begin{pmatrix} -\frac{A}{4v_\phi^2} & -\frac{A}{4v_\chi v_\phi} & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\phi} \\ -\frac{A}{4v_\chi v_\phi} & -\frac{A}{4v_\chi^2} & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\chi} \\ -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\phi} & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\chi} & -\frac{A(v_\eta^2+v_\rho^2)}{4v_\eta^2 v_\rho^2} \end{pmatrix} \tag{A6}$$

The matrix $M^2_{I_{33}}$ in (A6) has got 3 eigenvalues: $0, 0, \frac{-A}{4}\left(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\chi^2}+\frac{1}{v_\phi^2}\right)$. We use the second eigenstate that corresponds to the basis A_3, I_η^1.

In the basis A_3, I_χ^1, the squared mass matrix has the form:

$$M^2_{I_{A_3\chi}} = \begin{pmatrix} -\frac{A}{4v_\chi^2} & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\chi} \\ -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{4v_\eta v_\chi} & -\frac{A(v_\eta^2+v_\rho^2)}{4v_\eta^2 v_\rho^2} \end{pmatrix} \tag{A7}$$

The matrix $M^2_{I_{A_3\chi}}$ in (A7) has 2 eigenvalues: 0 and $\frac{1}{4}A(-\frac{1}{v_\eta^2}-\frac{1}{v_\rho^2}-\frac{1}{v_\chi^2})$. This matrix is diagonalized by the matrix below:

$$U_{A_3\chi} = \begin{pmatrix} -\frac{v_\chi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{v_\eta\sqrt{v_\chi^2(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2})+1}} & \frac{1}{\sqrt{v_\chi^2(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2})+1}} \\ \frac{v_\eta}{v_\chi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}} & \frac{1}{\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}} \end{pmatrix} \tag{A8}$$

As a result, we receive the 4×4 matrix which is used to diagonalize $M^2_{I_\phi^{\text{diag}}}$ as follows:

$$U_I^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{v_\chi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}{v_\eta\sqrt{v_\chi^2(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2})+1}} & 0 & \frac{1}{\sqrt{v_\chi^2(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2})+1}} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{v_\eta}{v_\chi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}} & 0 & \frac{1}{\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}} \end{pmatrix}. \tag{A9}$$

The mixing angle θ_3 is defined by:

$$\tan\theta_3 = \frac{v_\eta}{v_\chi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}}. \tag{A10}$$

Under the effect of the matrix U_2 in (A9), the matrix $M^2_{I_\eta^{\text{diag}}}$ changes into:

$$M^2_{I_{\eta\rho}^{\text{diag}}} = U_I^2.M^2_{I_\rho^{\text{diag}}}(U_I^2)^T = \begin{pmatrix} -\frac{A}{4v_\phi^2} & 0 & 0 & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}}{4v_\eta v_\phi} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}}{4v_\eta v_\phi} & 0 & 0 & \frac{1}{4}A(-\frac{1}{v_\eta^2}-\frac{1}{v_\rho^2}-\frac{1}{v_\chi^2}) \end{pmatrix} \tag{A11}$$

Next, we consider the matrix 2×2 in (A11) corresponding to the basis (A_4, I_ϕ):

$$M_{I_{22}} = \begin{pmatrix} -\frac{A}{4v_\phi^2} & -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}}{4v_\eta v_\phi} \\ -\frac{A\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}}{4v_\eta v_\phi} & \frac{-A}{4}\left(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\chi^2}\right) \end{pmatrix} \tag{A12}$$

The matrix in (A12) is a squared mass matrix in basis (A_4, I_ϕ) and has got 2 eigenvalues which are 0 and $\frac{-A}{4}(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\chi^2}+\frac{1}{v_\phi^2})$. The matrix $M_{I_{22}}$ is diagonalized by the matrix below:

$$U_{A_4\phi} = \begin{pmatrix} -\frac{v_\phi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}}{v_\eta\sqrt{v_\phi^2(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\chi^2})+1}} & \frac{1}{\sqrt{v_\phi^2(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\chi^2})+1}} \\ \frac{v_\eta}{v_\phi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}\sqrt{\frac{v_\eta^2 v_\rho^2 v_\chi^2}{v_\phi^2(v_\eta^2(v_\rho^2+v_\chi^2)+v_\rho^2 v_\chi^2)}+1}} & \frac{1}{\sqrt{\frac{v_\eta^2 v_\rho^2 v_\chi^2}{v_\phi^2(v_\eta^2(v_\rho^2+v_\chi^2)+v_\rho^2 v_\chi^2)}+1}} \end{pmatrix} \tag{A13}$$

Hence, we receive the 4×4 matrix which is used to diagonalized $M^2_{I^{\rm diag}_{\eta\rho}}$ in the following form:

$$U_I^3 = \begin{pmatrix} -\frac{v_\phi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}}{v_\eta\sqrt{v_\phi^2(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\chi^2})+1}} & 0 & 0 & \frac{1}{\sqrt{v_\phi^2(\frac{1}{v_\eta^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\chi^2})+1}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{v_\eta}{v_\phi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}\sqrt{\frac{v_\eta^2 v_\rho^2 v_\chi^2}{v_\phi^2(v_\eta^2(v_\rho^2+v_\chi^2)+v_\rho^2 v_\chi^2)}+1}} & 0 & 0 & \frac{1}{\sqrt{\frac{v_\eta^2 v_\rho^2 v_\chi^2}{v_\phi^2(v_\eta^2(v_\rho^2+v_\chi^2)+v_\rho^2 v_\chi^2)}+1}} \end{pmatrix}. \tag{A14}$$

As the mixing angle θ_ϕ is defined as below:

$$\tan\theta_\phi = \frac{v_\eta}{v_\phi\sqrt{\frac{v_\eta^2}{v_\rho^2}+1}\sqrt{\frac{v_\eta^2 v_\rho^2}{v_\chi^2(v_\eta^2+v_\rho^2)}+1}} = \frac{v_\chi}{v_\phi\sqrt{1+v_\chi^2(\frac{1}{v_\rho^2}+\frac{1}{v_\eta^2})}}. \tag{A15}$$

Under the effect of the matrix U_3 in (A14), the matrix $M^2_{I^{\rm diag}_{\eta\rho}}$ becomes:

$$M^2_{I^{\rm diag}} = U_I^3.M^2_{I^{\rm diag}_{\eta\rho}}.(U_I^3)^T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{A}{4}(-\frac{1}{v_\eta^2}-\frac{1}{v_\rho^2}-\frac{1}{v_\chi^2}-\frac{1}{v_\phi^2}) \end{pmatrix} \tag{A16}$$

(3) Finally, the matrix which is used to diagonalize the matrix $M^2_{\rm odd}$ is

$$U_I = U_I^3.U_I^2.U_I^1, \tag{A17}$$

and gets the trigonometric form as below:

$$U_{Is} = \begin{pmatrix} \cos\theta_\phi & -\sin\theta_3\sin\theta_\phi & \sin\alpha(-\cos\theta_3)\sin\theta_\phi & -\cos\alpha\cos\theta_3\sin\theta_\phi \\ 0 & \cos\theta_3 & -\sin\alpha\sin\theta_3 & -\cos\alpha\sin\theta_3 \\ 0 & 0 & \cos\alpha & -\sin\alpha \\ -\sin\theta_\phi & \sin\theta_3(-\cos\theta_\phi) & \sin\alpha(-\cos\theta_3)\cos\theta_\phi & -\cos\alpha\cos\theta_3\cos\theta_\phi \end{pmatrix}. \tag{A18}$$

The CP odd squared mass matrix M^2_{odd} in (42) can be exactly diagonalized by the Euler diagonalization method. Then the physical CP odd scalar fields are related with the CP odd scalars in the interaction basis via the following transformation:

$$\begin{pmatrix} a \\ G_{Z'} \\ A_5 \\ G_Z \end{pmatrix} = \begin{pmatrix} \cos\theta_\phi & -\sin\theta_\phi & 0 & 0 \\ \cos\theta_3\sin\theta_\phi & \cos\theta_3\cos\theta_\phi & -\sin\theta_3 & 0 \\ \sin\theta_3\cos\theta_4\sin\theta_\phi & \sin\theta_3\cos\theta_4\cos\theta_\phi & \cos\theta_3\cos\theta_4 & \sin\theta_4 \\ -\sin\theta_3\sin\theta_4\sin\theta_\phi & -\sin\theta_3\sin\theta_4\cos\theta_\phi & -\cos\theta_3\sin\theta_4 & +\cos\theta_4 \end{pmatrix} \begin{pmatrix} I_\phi \\ I_{\chi'} \\ I_\eta \\ I_\rho \end{pmatrix}, \tag{A19}$$

Note that the mixing matrix has three angles and one parameter which is entered in expression of the pseudoscalar A_5 mass given in (46).

APPENDIX B: DIAGONALIZATION OF *CP*-EVEN MASS MIXING MATRIX IN BASIS $(R^1_\eta, R_\rho, R^3_\chi, R_\phi)$

The matrix M^2_R in (47) is diagonalized by the Hartree-Fock method. It is split into two matrices: M^2_{R0}—the main contribution and M^2_{Rp}—the perturbation. Those are satisfied the below equation:

$$M^2_R = M^2_{R0} + M^2_{Rp}, \tag{B1}$$

with

$$M^2_{R0} = 2\begin{pmatrix} 0 & 0 & 0 & \frac{A}{4v_\eta v_\phi} + \frac{1}{2}\lambda_{13}v_\eta v_\phi \\ 0 & 0 & 0 & \frac{A}{4v_\rho v_\phi} + \frac{1}{2}\lambda_{12}v_\rho v_\phi \\ 0 & 0 & 0 & \frac{A}{4v_\chi v_\phi} + \frac{1}{2}\lambda_{11}v_\chi v_\phi \\ \frac{A}{4v_\eta v_\phi} + \frac{1}{2}\lambda_{13}v_\eta v_\phi & \frac{A}{4v_\rho v_\phi} + \frac{1}{2}\lambda_{12}v_\rho v_\phi & \frac{A}{4v_\chi v_\phi} + \frac{1}{2}\lambda_{11}v_\chi v_\phi & \lambda_{10}v_\phi^2 - \frac{A}{4v_\phi^2} \end{pmatrix}, \tag{B2}$$

and

$$M^2_{Rp} = 2\begin{pmatrix} \lambda_2 v_\eta^2 - \frac{A}{4v_\eta^2} & \frac{A}{4v_\eta v_\rho} + \frac{1}{2}\lambda_6 v_\eta v_\rho & \frac{A}{4v_\eta v_\chi} + \frac{1}{2}\lambda_4 v_\eta v_\chi & 0 \\ \frac{A}{4v_\eta v_\rho} + \frac{1}{2}\lambda_6 v_\eta v_\rho & \lambda_3 v_\rho^2 - \frac{A}{4v_\rho^2} & \frac{A}{4v_\rho v_\chi} + \frac{1}{2}\lambda_5 v_\rho v_\chi & 0 \\ \frac{A}{4v_\eta v_\chi} + \frac{1}{2}\lambda_4 v_\eta v_\chi & \frac{A}{4v_\rho v_\chi} + \frac{1}{2}\lambda_5 v_\rho v_\chi & \lambda_1 v_\chi^2 - \frac{A}{4v_\chi^2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{B3}$$

In the limits $v_\rho, v_\eta \ll v_\chi \ll v_\phi$, both of v_ρ and v_η can be considered approximately as zero. This makes the main contribution (B2) change into the below matrix:

$$M^2_{R_{00}} \approx \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{11}v_\chi v_\phi \\ 0 & 0 & \lambda_{11}v_\chi v_\phi & 2\lambda_{10}v_\phi^2 \end{pmatrix} \tag{B4}$$

The matrix (B4) is diagonalized by the matrix:

$$U_{44} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2} + \lambda_{10}v_\phi}{\lambda_{11}v_\chi\sqrt{\frac{\left(\sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2} + \lambda_{10}v_\phi\right)^2}{\lambda_{11}^2 v_\chi^2} + 1}} & \frac{1}{\sqrt{\frac{\left(\sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2} + \lambda_{10}v_\phi\right)^2}{\lambda_{11}^2 v_\chi^2} + 1}} \\ 0 & 0 & -\frac{\lambda_{10}v_\phi - \sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2}}{\lambda_{11}v_\chi\sqrt{\frac{\left(\sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2} - \lambda_{10}v_\phi\right)^2}{\lambda_{11}^2 v_\chi^2} + 1}} & \frac{1}{\sqrt{\frac{\left(\sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2} - \lambda_{10}v_\phi\right)^2}{\lambda_{11}^2 v_\chi^2} + 1}} \end{pmatrix}, \tag{B5}$$

and the diagonalized matrix of main contribution has form as below:

$$M^2_{R_{00}} = U_{44}.M^2_{R0}.U^T_{44} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & v_\phi\left(\lambda_{10}v_\phi - \sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2}\right) & 0 \\ 0 & 0 & 0 & v_\phi\left(\sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2} + \lambda_{10}v_\phi\right) \end{pmatrix}. \tag{B6}$$

From (B6), the squared mass of inflaton is defined by:

$$m^2_\phi = v_\phi\left(\sqrt{\lambda_{11}^2 v_\chi^2 + \lambda_{10}^2 v_\phi^2} + \lambda_{10}v_\phi\right) \approx 2\lambda_{10}v_\phi^2. \tag{B7}$$

On the other hand, the matrix U_{44} in (B5) can be presented by another form such as:

$$U^1_R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\cos\alpha_\phi & \sin\alpha_\phi \\ 0 & 0 & \sin\alpha_\phi & \cos\alpha_\phi \end{pmatrix}, \tag{B8}$$

with

$$\tan 2\alpha_\phi = \frac{\lambda_{11}v_\chi}{\lambda_{10}v_\phi}. \tag{B9}$$

The perturbation M^2_{Rp} is effected by the diagonal matrix U^1_R in (B8) so that it has form:

$$M^2_{Rp_{44}} = \begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & \frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho & -\cos\alpha_\phi\left(\frac{A}{2v_\eta v_\chi} + \lambda_4 v_\eta v_\chi\right) & \sin\alpha_\phi\left(\frac{A}{2v_\eta v_\chi} + \lambda_4 v_\eta v_\chi\right) \\ \frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho & 2\lambda_3 v_\rho^2 - \frac{A}{2v_\rho^2} & -\cos\alpha_\phi\left(\frac{A}{2v_\rho v_\chi} + \lambda_5 v_\rho v_\chi\right) & \sin\alpha_\phi\left(\frac{A}{2v_\rho v_\chi} + \lambda_5 v_\rho v_\chi\right) \\ -\cos\alpha_\phi\left(\frac{A}{2v_\eta v_\chi} + \lambda_4 v_\eta v_\chi\right) & -\cos\alpha_\phi\left(\frac{A}{2v_\rho v_\chi} + \lambda_5 v_\rho v_\chi\right) & -\frac{\cos^2\alpha_\phi(A-4\lambda_1 v_\chi^4)}{2v_\chi^2} & \frac{\sin\alpha_\phi\cos(\alpha\phi)(A-4\lambda_1 v_\chi^4)}{2v_\chi^2} \\ \sin\alpha_\phi\left(\frac{A}{2v_\eta v_\chi} + \lambda_4 v_\eta v_\chi\right) & \sin\alpha_\phi\left(\frac{A}{2v_\rho v_\chi} + \lambda_5 v_\rho v_\chi\right) & \frac{\sin\alpha_\phi\cos\alpha_\phi(A-4\lambda_1 v_\chi^4)}{2v_\chi^2} & -\frac{\sin^2(\alpha\phi)(A-4\lambda_1 v_\chi^4)}{2v_\chi^2} \end{pmatrix} \tag{B10}$$

Because α_ϕ is defined by (B9), then $\sin\alpha_\phi \to 0$ when $v_\chi \ll v_\phi$ and $\lambda_{10} > 0$. This helps the matrix $M^2_{Rp_{44}}$ reduce an order and can be rewritten like the form after:

$$M^2_{Rp_{44}} = \begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & \frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho & -\cos\alpha_\phi\left(\frac{A}{2v_\eta v_\chi} + \lambda_4 v_\eta v_\chi\right) & 0 \\ \frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho & 2\lambda_3 v_\rho^2 - \frac{A}{2v_\rho^2} & -\cos\alpha_\phi\left(\frac{A}{2v_\rho v_\chi} + \lambda_5 v_\rho v_\chi\right) & 0 \\ -\cos\alpha_\phi\left(\frac{A}{2v_\eta v_\chi} + \lambda_4 v_\eta v_\chi\right) & -\cos\alpha_\phi\left(\frac{A}{2v_\rho v_\chi} + \lambda_5 v_\rho v_\chi\right) & -\frac{\cos^2\alpha_\phi(A-4\lambda_1 v_\chi^4)}{2v_\chi^2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{B11}$$

From (B11), one gets a 3×3 matrix below:

$$M^2_{Rp_{33}} = M^2_{Rp^0_{33}} + M^2_{Rp^p_{33}}, \tag{B12}$$

with

$$M^2_{Rp^0_{33}} = \begin{pmatrix} 0 & 0 & -\cos\alpha_\phi\left(\frac{A}{2v_\eta v_\chi}+\lambda_4 v_\eta v_\chi\right) \\ 0 & 0 & -\cos\alpha_\phi\left(\frac{A}{2v_\rho v_\chi}+\lambda_5 v_\rho v_\chi\right) \\ -\cos\alpha_\phi\left(\frac{A}{2v_\eta v_\chi}+\lambda_4 v_\eta v_\chi\right) & -\cos\alpha_\phi\left(\frac{A}{2v_\rho v_\chi}+\lambda_5 v_\rho v_\chi\right) & -\frac{\cos^2\alpha_\phi(A-4\lambda_1 v_\chi^4)}{2v_\chi^2} \end{pmatrix} \tag{B13}$$

is considered as the main contribution and

$$M^2_{Rp^p_{33}} = \begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & \frac{A}{2v_\eta v_\rho}+\lambda_6 v_\eta v_\rho & 0 \\ \frac{A}{2v_\eta v_\rho}+\lambda_6 v_\eta v_\rho & 2\lambda_3 v_\rho^2 - \frac{A}{2v_\rho^2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{B14}$$

is a perturbation of $M^2_{Rp_{33}}$.

Consider the main contribution $M^2_{Rp^0_{33}}$ in the limit $v_\chi \gg v_\rho, v_\eta$, we get $-\frac{\cos\alpha_\phi(A+\lambda_4 v_\eta^2 v_\chi^2)}{2v_\eta v_\chi} \to 0$ then $M^2_{Rp^0_{33}}$ approximately has form:

$$M^2_{Rp^{00}_{33}} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\cos\alpha_\phi\left(\frac{A}{2v_\rho v_\chi}+\lambda_5 v_\rho v_\chi\right) \\ 0 & -\cos\alpha_\phi\left(\frac{A}{2v_\rho v_\chi}+\lambda_5 v_\rho v_\chi\right) & -\frac{\cos^2\alpha_\phi(A-4\lambda_1 v_\chi^4)}{2v_\chi^2} \end{pmatrix}. \tag{B15}$$

The matrix $M^2_{Rp^{00}_{33}}$ in (B15) is diagonalized by the following matrix:

$$U_{33} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\cos\alpha_3 & \sin\alpha_3 \\ 0 & \sin\alpha_3 & \cos\alpha_3 \end{pmatrix}, \tag{B16}$$

in which α_3 is defined by:

$$\tan 2\alpha_3 = \frac{4v_\chi(A+2\lambda_5 v_\rho^2 v_\chi^2)}{\cos\alpha\phi(A-4\lambda_1 v_\chi^4)^2}. \tag{B17}$$

After being diagonalized, $M^2_{Rp^{00}_{33}}$ has the form:

$$M^2_{Rp^{\mathrm{diag}}_{33}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m^2_{H_{\chi_1}} & 0 \\ 0 & 0 & m^2_{H_{\chi_2}} \end{pmatrix}, \tag{B18}$$

with

$$m^2_{H_{\chi_{1,2}}} = \frac{-\cos\alpha_\phi}{4v_\rho v_\chi^2}\left(Av_\rho\cos\alpha_\phi - 4\lambda_1 v_\rho v_\chi^4\cos\alpha_\phi \pm \sqrt{4v_\chi^2(A+2\lambda_5 v_\rho^2 v_\chi^2)^2 + (Av_\rho\cos\alpha_\phi - 4\lambda_1 v_\rho v_\chi^4\cos\alpha_\phi)^2}\right). \tag{B19}$$

Because of the condition $m^2_{H_\chi} > 0$, $v_\phi \gg v_\chi$ and λ_ϕ is very tiny then one gets:

$$m^2_{H_\chi} = \left(\lambda_1 v_\chi^2 + v_\chi\sqrt{\lambda_5^2 v_\rho^2 + \lambda_1^2 v_\chi^2}\right) \approx 2\lambda_1 v_\chi^2 + \frac{\lambda_5^2}{2\lambda_1}v_\rho^2. \tag{B20}$$

With U_{33}, we get the 4×4 diagonal matrix below:

$$U_R^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\cos\alpha_3 & \sin\alpha_3 & 0 \\ 0 & \sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{B21}$$

Under the effect of U_{33}, the perturbation $M^2_{Rp^p_{33}}$ changes into the following form:

$$M^2_{Rp^{p'}_{33}} = \begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & -\cos\alpha_3\left(\frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho\right) & \sin\alpha_3\left(\frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho\right) \\ -\cos\alpha_3\left(\frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho\right) & -\frac{\cos^2\alpha_3(A-4\lambda_3 v_\rho^4)}{2v_\rho^2} & \frac{\sin\alpha_3\cos\alpha_3(A-4\lambda_3 v_\rho^4)}{2v_\rho^2} \\ \sin\alpha_3\left(\frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho\right) & \frac{\sin\alpha_3\cos\alpha_3(A-4\lambda_3 v_\rho^4)}{2v_\rho^2} & -\frac{\sin^2\alpha_3(A-4\lambda_3 v_\rho^4)}{2v_\rho^2} \end{pmatrix}. \tag{B22}$$

With the limit $v_\chi \gg v_\rho, v_\eta$, we get $\sin\alpha_3 \to 0$. So that $M^2_{Rp^{p'}_{33}}$ approximately has form:

$$M^2_{Rp^{p'}_{33}0} = \begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & -\cos\alpha_3\left(\frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho\right) & 0 \\ -\cos\alpha_3\left(\frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho\right) & -\frac{\cos^2\alpha_3(A-4\lambda_3 v_\rho^4)}{2v_\rho^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{B23}$$

From (B23), one get the 2×2 matrix below:

$$M^2_{Rp_{22}} = \begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & -\cos\alpha_3\left(\frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho\right) \\ -\cos\alpha_3\left(\frac{A}{2v_\eta v_\rho} + \lambda_6 v_\eta v_\rho\right) & -\frac{\cos^2\alpha_3(A-4\lambda_3 v_\rho^4)}{2v_\rho^2} \end{pmatrix}. \tag{B24}$$

Assuming that $\cos\alpha_3 \approx 1$, the matrix $M^2_{Rp_{22}}$ in (B24) can be diagonalized by the 2×2 matrix below:

$$U_{22} = \begin{pmatrix} -\cos\alpha_2 & \sin\alpha_2 \\ \sin\alpha_2 & \cos\alpha_2 \end{pmatrix}, \tag{B25}$$

in which we have

$$\tan 2\alpha_2 = \frac{4\cos\alpha_3 v_\eta v_\rho (A + \lambda_6 v_\eta^2 v_\rho^2)}{A\cos^2\alpha_3 v_\eta^2 - A v_\rho^2 + 4v_\eta^2 v_\rho^2(\lambda_2 v_\eta^2 - \lambda_3\cos^2\alpha_3 v_\rho^2)}. \tag{B26}$$

After being diagonalized, the matrix $M^2_{Rp_{22}}$ has form:

$$M^2_{Rp^{\mathrm{diag}}_{22}} = \begin{pmatrix} m^2_{h_5} & 0 \\ 0 & m_h^2 \end{pmatrix}, \tag{B27}$$

with

$$m^2_{h,h_5} = \lambda_2 v_\eta^2 - \frac{A}{4v_\eta^2} - \frac{\cos^2\alpha_3(A-4\lambda_3 v_\rho^4)}{4v_\rho^2} \pm \sqrt{\lambda_6\cos^2\alpha_3(A+\lambda_6 v_\eta^2 v_\rho^2) + \lambda_2 v_\eta^4(A-4\lambda_3 v_\rho^4) + \lambda_3 A v_\rho^4 + \frac{(A(v_\eta^2\cos 2\alpha_3 + v_\eta^2 + 2v_\rho^2) - 8\lambda_3 v_\eta^2 v_\rho^4\cos^2\alpha_3 - 8\lambda_2 v_\eta^4 v_\rho^2)^2}{64 v_\eta^4 v_\rho^4}} \tag{B28}$$

With the approximations $\cos\alpha_3 \approx 1$, one gets:

$$m^2_{h,h_5} = \lambda_2 v_\eta^2 + \lambda_3 v_\rho^2 - \frac{A v^2}{4 v_\eta^2 v_\rho^2} \pm \frac{1}{4 v_\eta v_\rho}\sqrt{16 v_\eta^2 v_\rho^2((\lambda_2 v_\eta^2 - \lambda_3 v_\rho^2)^2 + \lambda_6^2 v_\eta^2 v_\rho^2) + \lambda\phi^2 v_\chi^2 v_\phi^2 (v_\eta^2 + v_\rho^2)^2 + 8\lambda\phi v_\eta v_\rho v_\chi v_\phi (\lambda_2 v_\eta^4 - v_\eta^2 v_\rho^2(\lambda_2 + \lambda_3 - 2\lambda_6) + \lambda_3 v_\rho^4)} \tag{B29}$$

With U_{22}, we get the 4×4 matrix below:

$$U_R^3 = \begin{pmatrix} -\cos\alpha_2 & \sin\alpha_2 & 0 & 0 \\ \sin\alpha_2 & \cos\alpha_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{B30}$$

Finally, the matrix which is used to diagonalize M_R^2 is

$$U_R = U_R^3.U_R^2.U_R^1 = \begin{pmatrix} -\cos\alpha_2 & -\sin\alpha_2\cos\alpha_3 & -\sin\alpha_2\sin\alpha_3\cos\alpha_\phi & \sin\alpha_2\sin\alpha_3\sin\alpha_\phi \\ \sin\alpha_2 & -\cos\alpha_2\cos\alpha_3 & -\cos\alpha_2\sin\alpha_3\cos\alpha_\phi & \cos\alpha_2\sin\alpha_3\sin\alpha_\phi \\ 0 & \sin\alpha_3 & -\cos\alpha_3\cos\alpha_\phi & \cos\alpha_3\sin\alpha_\phi \\ 0 & 0 & \sin\alpha_\phi & \cos\alpha_\phi \end{pmatrix}. \tag{B31}$$

Note that comparing to the 4×4 matrix of CP-odd sector containing only four parameters with three massless solutions, the matrix in (47) having 10 parameters are not exactly diagonalized. To solve this problem we have used the Hartree-Fock method where some conditions such as $v_\phi \gg v_\chi \gg v_\rho, v_\eta$, $\lambda_\phi \ll 1$ and $\sin\alpha_3 \approx 0$. As a consequence, derived matrix contains three angles α_2, α_3 and α_ϕ and three parameters associated with masses of new fields Φ, H_χ, and h_5.

APPENDIX C: DECAY RATE OF THE SM-LIKE HIGGS BOSON INTO A PAIR OF FERMIONS

1. SM-like Higgs couplings

We focus on the coupling of SM-like boson h with two ALP a which is a part of V in (24):

$$V \supset \mathcal{V}(h,a,a), \tag{C1}$$

where

$$\begin{aligned} \frac{2\mathcal{V}(h,a,a)}{haa} = & -\frac{2\lambda_2 v_\eta}{\cos^2 2\alpha}\cos^2\alpha\sin\alpha_2\cos^2\theta_3\sin^2\theta_\phi - \frac{\lambda_6 v_\eta}{\cos^2 2\alpha}\sin^2\alpha\sin\alpha_2\cos^2\theta_3\sin^2\theta_\phi - \lambda_4 v_\eta\sin\alpha_2\sin^2\theta_3\sin^2\theta_\phi \\ & - \lambda_{13}(v_\eta\sin\alpha_2\cos^2\theta_\phi + v_\phi\cos^2\alpha\sec^2 2\alpha\cos\alpha_2\sin\alpha_3\sin\alpha_\phi\cos^2\theta_3\sin^2\theta_\phi) \\ & + \frac{2\lambda_3 v_\rho}{\cos^2 2\alpha}\sin^2\alpha\cos\alpha_2\cos\alpha_3\cos^2\theta_3\sin^2\theta_\phi + \frac{\lambda_6 v_\rho}{\cos^2 2\alpha}\cos^2\alpha\cos\alpha_2\cos\alpha_3\cos^2\theta_3\sin^2\theta_\phi \\ & + \lambda_5 v_\rho\cos\alpha_2\cos\alpha_3\sin^2\theta_3\sin^2\theta_\phi + \lambda_{12} v_\rho\cos\alpha_2\cos\alpha_3\cos^2\theta_\phi \\ & + \frac{\lambda_4 v_\chi}{\cos^2 2\alpha}\cos^2\alpha\cos\alpha_2\sin\alpha_3\cos\alpha_\phi\cos^2\theta_3\sin^2\theta_\phi + \frac{\lambda_5 v_\chi}{\cos^2 2\alpha}\sin^2\alpha\cos\alpha_2\sin\alpha_3\cos\alpha_\phi\cos^2\theta_3\sin^2\theta_\phi \\ & + 2\lambda_1 v_\chi\cos\alpha_2\sin\alpha_3\cos\alpha_\phi\sin^2\theta_3\sin^2(\theta\phi) + \lambda_{11} v_\chi\cos\alpha_2\sin\alpha_3\cos\alpha_\phi\cos^2\theta_\phi \\ & - \frac{\lambda_{12} v_\phi}{\cos^2 2\alpha}\sin^2\alpha\cos\alpha_2\sin\alpha_3\sin\alpha_\phi\cos^2\theta_3\sin^2\theta_\phi - \lambda_{11} v_\phi\cos\alpha_2\sin\alpha_3\sin\alpha_\phi\sin^2\theta_3\sin^2\theta_\phi \\ & - 2\lambda_{10} v_\phi\cos\alpha_2\sin\alpha_3\sin\alpha_\phi\cos^2\theta_\phi. \end{aligned} \tag{C2}$$

In the limits $v_\phi \gg v_\chi \gg v_\rho, v_\eta$ and $\lambda_\phi \approx 0$, the mixing angles in (B17), (B26) approximately get:

$$\tan\alpha_3 \approx \frac{\lambda_5 v_\rho}{\cos\alpha_\phi v_\chi}, \qquad \tan 2\alpha_2 \approx \frac{\lambda_6 \cos\alpha_3 v_\eta v_\rho}{\lambda_2 v_\eta^2 - \lambda_3 \cos^2\alpha_3 v_\rho^2}. \tag{C3}$$

Since $\sin\theta_\phi \approx 0$ and $\sin\alpha_3 \approx 0$, hence we neglect the terms associated with them. Then

$$\begin{aligned}\mathcal{V}(h,a,a) &\approx \frac{haa}{2}\cos^2\theta_\phi(\lambda_{12} v_\rho \cos\alpha_2 \cos\alpha_3 - \lambda_{13} v_\eta \sin\alpha_2)\\ &\approx \frac{haa}{2\sqrt{2}} v_\rho v_\eta \left(\frac{\lambda_6\lambda_{12}}{\sqrt{V_{236}^2 + (\lambda_3 v_\rho^2 - \lambda_2 v_\eta^2)V_{236}}} - \lambda_{13}\sqrt{V_{236} + \lambda_3 v_\rho^2 - \lambda_2 v_\eta^2}\right),\end{aligned} \tag{C4}$$

in which, $V_{236} = \sqrt{(\lambda_2 v_\eta^2 - \lambda_3 v_\rho^2)^2 + \lambda_6^2 v_\eta^2 v_\rho^2}$.

Similarly about the coupling of SM-like boson h with two pseudscalar A_5, with the limits $v_\phi \gg v_\chi \gg v_\rho, v_\eta$ and $\lambda_\phi \approx 0$, one has:

$$\begin{aligned}\mathcal{V}(h,A_5,A_5) &\approx \frac{hA_5A_5}{2}\cos^2\theta_\phi\left[\frac{-2\lambda_2 v_\eta}{\cos^2 2\alpha}\cos^2\alpha\cos^2\theta_3 \sin\alpha_2 - \lambda_4 v_\eta \sin\alpha_2 \sin^2\theta_3\right.\\ &\quad\left. + v_\rho \cos\alpha_2\cos\alpha_3\left(\frac{2\lambda_3}{\cos^2 2\alpha}\cos^2\theta_3\sin^2\alpha + \lambda_5\sin^2\theta_3\right) + \frac{\lambda_6\cos^2\theta_3}{\cos^2 2\alpha}(v_\rho\cos^2\alpha\cos\alpha_2\cos\alpha_3 - v_\eta\sin^2\alpha\sin\alpha_2)\right]\\ &\approx \frac{hA_5A_5}{2\sqrt{2}}\left(v_\rho(2\lambda_3 v_\eta^2 + \lambda_6 v_\rho^2)\sqrt{\frac{V_{236} - \lambda_3 v_\rho^2 + \lambda_2 v_\eta^2}{V_{236}}} - v_\eta(2\lambda_2 v_\rho^2 + \lambda_6 v_\eta^2)\sqrt{\frac{V_{236} + \lambda_3 v_\rho^2 - \lambda_2 v_\eta^2}{V_{236}}}\right).\end{aligned} \tag{C5}$$

The new light boson h_5 also has couplings with ALP a and pseudoscalar A_5. The potential of (h_5, a, a) coupling is

$$\begin{aligned}\mathcal{V}(h_5,a,a) &\approx \frac{h_5aa}{2}\cos^2\theta_\phi(\lambda_{12} v_\rho \cos\alpha_3 \sin\alpha_2 + \lambda_{13} v_\eta \cos\alpha_2)\\ &\approx \frac{h_5aa}{2\sqrt{2}} v_\rho\left(\lambda_{12}\sqrt{V_{236} + \lambda_3^2 v_\rho^2 - \lambda_2 v_\eta^2} + \frac{\lambda_6\lambda_{13} v_\eta^2}{\sqrt{V_{236}^2 + V_{236}(\lambda_3^2 v_\rho^2 - \lambda_2 v_\eta^2)}}\right).\end{aligned} \tag{C6}$$

The coupling (h_5, A_5, A_5) is given by:

$$\begin{aligned}\mathcal{V}(h_5,A_5,A_5) &\approx \frac{h_5A_5A_5}{2}\cos^2\theta_\phi\left[\frac{2\lambda_2 v_\eta}{\cos^2 2\alpha}\cos^2\alpha\cos\alpha_2\cos^2\theta_3 + \frac{\lambda_6 v_\eta}{\cos^2 2\alpha}\sin^2\alpha\cos\alpha_2\cos^2\theta_3 + \lambda_4 v_\eta\cos\alpha_2\sin^2\theta_3\right.\\ &\quad\left. + \frac{2\lambda_3 v_\rho}{\cos^2 2\alpha}\sin^2\alpha\sin\alpha_2\cos\alpha_3\cos^2\theta_3 + \frac{\lambda_6 v_\rho}{\cos^2 2\alpha}\cos^2\alpha\sin\alpha_2\cos\alpha_3\cos^2\theta_3 + \lambda_5 v_\rho\sin\alpha_2\cos\alpha_3\sin^2\theta_3\right]\\ &\approx \frac{h_5A_5A_5}{2\sqrt{2}}\frac{v_\eta^4}{(v_\eta^2 + v_\rho^2)(v_\eta^2 + 2v_\rho^2)^2}\left(v_\eta(2v_\rho^2 + \lambda_6 v_\eta^2)\sqrt{\frac{V_{236} + \lambda_2 v_\eta^2 - \lambda_3 v_\rho^2}{V_{236}}}\right.\\ &\quad\left. + v_\rho(2\lambda_3 v_\eta^2 + \lambda_6 v_\rho^2)\sqrt{\frac{V_{236} + \lambda_3 v_\rho^2 - \lambda_2 v_\eta^2}{V_{236}}}\right)\end{aligned} \tag{C7}$$

2. SM-like boson h decays to two fermions

Let us consider the decay:

$$h(\vec{p}) \to f(\vec{k}_1) + \tilde{f}(\vec{k}_2), \qquad f = u, d, c, s, \tau, \mu, e. \tag{C8}$$

Amplitude of the above process is given by

$$M_{fi}(h \to f\bar{f}) = g_{(h,f,f)} \bar{u}(\vec{k}_1, s_1) v(\vec{k}_2, s_2). \tag{C9}$$

Then, the decay rate of $h \to \bar{f}f$ process is

$$\Gamma(h \to \bar{f}f) = \int d\Gamma = \frac{g^2_{(h,f,f)}}{8\pi} m_h \left(1 - \frac{4m_f^2}{m_h^2}\right)^{\frac{3}{2}}. \tag{C10}$$

Hence

$$\Gamma(h \to \bar{e}e) = \frac{\cos^2\alpha_2 \cos^2\alpha_3 \frac{m_e^2}{v_\rho^2}}{8\pi} m_h \left(1 - \frac{4m_e^2}{m_h^2}\right)^{\frac{3}{2}} \tag{C11}$$

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