Conformal $B-L$ and pseudo-Goldstone dark matter

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(Received 22 March 2023; accepted 5 May 2023; published 16 May 2023)

We show that a conformal extension of the standard model with local $B-L$ symmetry and two complex scalars breaking $B-L$ can provide a unified description of neutrino mass, origin of matter, and dark matter. There are two hierarchical $B-L$ breaking vacuum expectation value (VEV) scales in the model, the higher denoted by v_B and the lower by v_A. The higher breaking scale is dynamically implemented via the Coleman-Weinberg mechanism and plays a key role in the model since it induces electroweak symmetry breaking as well as the lower $B-L$ breaking scale. It is also responsible for neutrino masses via the seesaw mechanism and origin of matter. The imaginary part of the complex scalar with lower $B-L$ breaking VEV plays the role of a pseudo-Goldstone dark matter (DM). The DM particle is unstable with its lifetime naturally longer than 10^{28} seconds. We show that its relic density arises from the freeze-in mechanism for a wide parameter domain. Due to the pseudo-Goldstone boson nature of the DM particle, the direct detection cross section is highly suppressed. The model also predicts the dark matter to be heavier than 100 TeV and it decays to two high energy neutrinos which can be observable at the IceCube, providing a test of this model.

DOI: 10.1103/PhysRevD.107.095023

I. INTRODUCTION

Understanding the origin of the electroweak scale is a fundamental puzzle of the standard model (SM). In usual discussion of the SM, it is customary to put the scale by hand in the form of a negative mass squared for the Higgs field. A more satisfactory approach, extensively discussed in the literature (see for some examples [1–18]) is to consider conformal extensions of the SM where the mass terms vanish due to conformal symmetry and the mass scales arise dynamically via the Coleman-Weinberg radiative correction mechanism [19]. This approach when implemented in the SM leads to a very small mass for the Higgs boson and is ruled out by experiment. However, it can be implemented generally in the context of many extensions of the SM (see for instance [1–18]). It is important to explore such models and pinpoint their tests. The goal of this paper is to present the phenomenological possibilities for dark matter in one such model.

Local $B-L$ extension of the SM [20–22] has been discussed as a very highly motivated minimal scenario for neutrino masses via the seesaw mechanism [23–27] and the origin of matter. Phenomenology of these models has been also extensively discussed in the literature; see for some examples [28–37]. While most phenomenological discussion of the $B-L$ models have been done in connection with neutrino masses and collider physics, we recently pointed out another virtue [35] of these models and showed that the real part of the SM singlet Higgs field that breaks $B-L$ can be a perfectly viable candidate for decaying dark matter. This dark matter model works only for very small values of the gauge coupling and low mass of the dark scalar (with masses in the MeV to a few keV range). The dark matter relic density in this case arises via the freeze-in mechanism [38] and has interesting experimental tests. The FASER experiment [39] for example is ideally suited for testing this model. Here we present a different approach where we impose conformal invariance on this theory so that we can understand the origin of masses and explore if there is a dark matter candidate. We find that we need to extend the minimal $B-L$ model by enlarging the Higgs sector to include two complex SM singlet scalars which carry $B-L$ quantum numbers. In this case, the imaginary part of one of the two $B-L$ breaking scalars can be a viable dark matter candidate. In fact the model predicts which particle can be the dark matter. It turns out that the dark matter is a pseudo-Goldstone boson [40–49], which explains why it has escaped direct detection.

The two $B-L$ nonsinglet scalars in our model have different VEVs. The higher scale $B-L$ breaking, which we call the primary breaking scale in the theory, arises from the Coleman-Weinberg mechanism by dimensional transmutation. The neutrino masses and the electroweak symmetry breaking [9] as well as the lower scale $B-L$ breaking VEV are all induced dynamically by the high scale $B-L$ breaking VEV. We show that in a wide parameter range, the imaginary part of the second $B-L$ breaking complex scalar is constrained to have a very long life [50–52] and plays the role of dark matter. An interesting prediction of the model is that the dark matter mass is more than 100 TeV with its dominant decay mode being to two neutrinos. The model can therefore be tested by observation of TeV neutrinos by the Ice cube experiment. The baryon asymmetry of the universe in this model is generated by the leptogenesis mechanism [53].

The paper is organized as follows: after briefly introducing the model and discussing $B-L$ symmetry breaking in Sec. II. We discuss the pseudo-Goldstone dark matter (σ) in Sec. III and its lifetime in Sec. IV. We show that the lightest $B-L$ breaking scalar is naturally long lived enough for it to play the role of dark matter. In Sec. V, we show how the freeze-in mechanism determines the relic density of dark matter and the various constraints which imply on the parameter space of the model. In Sec. VI, we discuss the constraints on the parameter space of the model and in Sec. VII, we give some tests and comment on other aspects of the model, such as leptogenesis. In Sec. VIII we summarize our results and conclude.

II. BRIEF OVERVIEW OF THE MODEL

Our model is based on the local $U(1)_{B-L}$ extension of the SM with gauge quantum numbers of fermion under $U(1)_{B-L}$ defined by their baryon or lepton number. The full gauge group of the model is $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_{B-L}$, where Y is the SM hypercharge. We need to add three right-handed neutrinos (RHNs) with $B-L=-1$ to cancel the $B-L$ gauge anomaly. The RHNs being SM singlets do not contribute to SM anomalies. The electric charge formula in this case is same as in the SM.

For the Higgs sector, in addition to the SM Higgs doublet H which has zero $B-L$, we include two SM singlet Higgs fields: Φ_B with $B-L=-2$ and Φ_A with $B-L=+6$. The tree level potential is given by

$$V(H,\Phi_A,\Phi_B)=\lambda_H(H^\dagger H)^2+\lambda_A(\Phi_A^\dagger\Phi_A)^2+\lambda_B(\Phi_B^\dagger\Phi_B)^2+\lambda_{HA}(H^\dagger H)(\Phi_A^\dagger\Phi_A)+\lambda_{HB}(H^\dagger H)(\Phi_B^\dagger\Phi_B)$$
$$+\lambda_{\rm mix}(\Phi_A^\dagger\Phi_A)(\Phi_B^\dagger\Phi_B)-\lambda_{AB}\Phi_A\Phi_B^3+{\rm H.c.} \tag{1}$$

Note that there are no mass parameters in the potential neither any mass terms in the fermion sector of the Lagrangian making the theory conformal invariant.

We break the $B-L$ symmetry in stages: the first stage is by giving VEV to the $B-L=2$ field Φ_B, with $\langle\Phi_B\rangle=v_B/\sqrt{2}$; at the second stage, Φ_A acquires a lower scale VEV, $\langle\Phi_A\rangle=v_A/\sqrt{2}$. As we will see, v_A is induced by the VEV v_B. We will show that $v_B\gg v_A$ naturally. The second VEV is induced by the high scale VEV via the λ_{AB} term in the scalar potential. To induce the SM electroweak symmetry breaking, we choose the potential parameter $\lambda_{HB}<0$ and adjust $|\lambda_{HB}|v_B^2$ to be of order of the electroweak VEV squared [9]. Similarly, for the model to lead to a dark matter will imply further constraints on the parameters of the model. We discuss all these below.

III. SYMMETRY BREAKING AND PSEUDO-GOLDSTONE DARK MATTER

The first stage of the symmetry breaking is induced by the Coleman-Weinberg mechanism as follows: We write the one-loop potential involving the $B-L$ breaking fields $\Phi_{A,B}$ near the Φ_B VEV as

$$V^{1-{\rm loop}}=\frac{3g_{BL}^4}{16\pi^2}(36|\Phi_A|^2+4|\Phi_B|^2)^2\left[\ln\left(\frac{36|\Phi_A|^2+4|\Phi_B|^2}{\mu^2}\right)-\frac{5}{6}\right]. \tag{2}$$

Now if we choose $\lambda_B\sim g_{BL}^4$ and $\lambda_A\gg g_{BL}^4$, then the field Φ_B will acquire VEV ($\langle\Phi_B\rangle=v_B/\sqrt{2}$) while Φ_A will have zero VEV. We choose the value of $g_{BL}\sim 0.01$ as a typical value, which requires that $\lambda_B\sim 10^{-8}$. This leads to $m_{\varphi_B}\sim 8\times 10^{-5}v_B$. The contributions to the effective potential from other couplings such as λ_{HB}, f_N (Yukawa coupling of the right-handed neutrinos to Φ_B) and λ_{AB} are much smaller.

Note that now, if $\lambda_{HB}<0$, it will induce a negative mass term for the SM Higgs field H and will generate a VEV for H which will break the electroweak symmetry. The magnitude of this VEV is adjusted by choosing the value of λ_{HB} appropriately depending on what we choose for v_B. For example, for $v_B\sim 10^{11}$ GeV, we need $\lambda_{HB}\sim 10^{-18}$. This is a very small number but being a renormalizable coupling, we are free to adjust its value. In any case, the one loop radiative corrections to this coupling does not come from gauge loops but only the Right-handed neutrino loop and is of order $\frac{f^2h_\nu^2}{32\pi^2}$. For $f_N\sim 10^{-5}$ and $h_\nu\sim 10^{-4}$ required to get the neutrino masses of right order, we find this leading one loop correction to be $\sim 10^{-20}$, which is smaller than the required value. Thus, we see that the origin of the electroweak scale can be explained in our model though not its magnitude. At the three loop level, the leading contribution with a SM fermion loop in the middle is of order $\frac{g_Y^4 g_{BL}^4}{16(16\pi^2)^3} \sim 10^{-18}$, which is of the same order as our assumed value of λ_{HB} for $g_{BL} \simeq 0.01$.

To induce the Φ_A VEV, we minimize the effective potential below the v_B scale. We write $\Phi_A = \frac{1}{\sqrt{2}}(\varphi_A + i\chi_A)$. Assuming $\lambda_{\rm mix}$ very small and neglecting quantum corrections to λ_A, the effective potential below the mass scale v_B can be written as

$$V_{\rm eff} = -\frac{\lambda_{AB}}{2} v_B^3 \varphi_A + \frac{\lambda_A}{4}(\varphi_A^2 + \chi_A^2)^2. \quad (3)$$

Minimizing this effective potential, we find that

$$v_A \simeq \left(\frac{\lambda_{AB}}{2\lambda_A}\right)^{1/3} v_B;$$
$$m_{\varphi_A'} \simeq \sqrt{3\lambda_A} v_A; \qquad m_\sigma \simeq \sqrt{\lambda_A} v_A, \quad (4)$$

where $\varphi_A' \simeq \varphi_A + \beta\varphi_B$ and $\sigma \simeq \chi_A + \alpha\chi_B$ are the mass eigenstates with β, α being of order $\frac{v_A}{v_B} \ll 1$, obtained through the mixing with φ_B and χ_B defined by $\Phi_B = \frac{1}{\sqrt{2}}(v_B + \varphi_B + i\chi_B)$. Note that σ is a pseudo-Goldstone boson. To see the pseudo-Goldstone boson nature of σ, note that in the limit of $\lambda_{AB} = 0$, the theory has global $U(1) \times U(1)$ symmetry and there are two massless Nambu-Goldstone bosons; however once the λ_{AB} term is switched on, the symmetry reduces to only one, the $U(1)_{B-L}$ gauge symmetry and the σ field picks up mass proportional to λ_{AB}. It has the lowest mass among the $B-L$ breaking scalar fields and is a pseudo-Goldstone boson.

Now adjusting λ_{AB} we can make the v_A much lower than the primary $B-L$ breaking scale v_B. What we need for the model is that $v_A/v_B < 10^{-6}$ to get the desired lifetime for the dark matter (see later). This requires that the coupling $\lambda_{AB} \leq 10^{-18}$ for $\lambda_A \sim 1$. Again, we note that this is a very small number but being a renormalizable coupling, we are free to adjust its value. However in our model, in any case, the one loop corrections to this coupling are proportional to λ_{AB}. As a result, if the tree level λ_{AB} is small, the loop corrections are also small. There is indeed a symmetry $\Phi_A \to -\Phi_A$ with all other fields unchanged, in the model when $\lambda_{AB} = 0$. This is like a chiral symmetry in fermionic theories. We therefore feel that the small value of λ_{AB} is stable under radiative corrections and is not unnatural.

From the mass calculation, we see that σ is the lighter of the two particles φ_A and σ and can be the dark matter of the universe as we see in detail in the next section. It is a pseudo-Goldstone dark matter [40–47]. In this discussion we have neglected the mixing between the χ_A and χ_B which will occur when both v_A and v_B are nonzero. This mixing is proportional to $\alpha \sim \frac{v_A}{v_B}$, which we take into account in our discussion below. The linear combination of fields $\chi_B' \simeq \chi_B - \alpha\chi_A$ becomes the longitudinal mode of the $B-L$ gauge boson Z' and the orthogonal combination $\sigma \simeq \chi_A + \alpha\chi_B$ becomes the dark matter. This is an unstable dark matter, and we study its detailed properties in the next section.

Note that the Yukawa coupling $f_N NN\Phi_B^\dagger$ gives mass to the right-handed neutrinos of magnitude $M_N \sim f_N v_B$ implementing the seesaw mechanism for neutrinos. Unlike the model in [35], the real part of Φ_B field decays rapidly as the universe evolves. The out-of-equilibrium decay of RHNs in our model is responsible for leptogenesis.

IV. DARK MATTER LIFETIME

As noted above, the field σ can play the role of dark matter of the model, if it satisfies the lifetime constraints. There are two lifetime constraints: one from the search for cosmic ray neutrinos from decaying dark matter with IceCube [51]. This puts a lower bound on $\tau_{\rm DM} \gtrsim 10^{28}$ sec for the mass of decaying dark matter in the range of $10^4 < m_{\rm DM}[\text{GeV}] < 10^9$. A second limit comes from the Fermi-Lat search for gamma rays from dwarf spheroidal galaxies [50]. There are also limits from deep gamma ray survey from Perseus Galaxy Cluster by MAGIC collaboration [52]. As we will see below, the first one is directly applicable to our case and not the others. Those limits of $\tau_{\rm DM} \geq 10^{26}$ sec. apply to $b\bar{b}$, W^+W^-, $\tau^+\tau^-$, and $\mu^+\mu^-$ modes while we have $\tau_{\rm DM} \geq 10^{24}$ sec, for the $\gamma\gamma$ decay mode.

The dark matter interactions are given by the following:

$$\mathcal{L}_\sigma = 6g_{BL}Z'_\mu(\sigma\partial^\mu\varphi_A - \varphi_A\partial^\mu\sigma) + 18g_{BL}^2\sigma^2(Z'_\mu)^2 + 18g_{BL}^2(Z'_\mu)^2(v_A + \varphi_A)^2 + f_N\frac{v_A}{v_B}\sigma NN + \frac{\lambda_{AH}v_A}{2\sqrt{2}}\sigma hh + \text{H.c.} \quad (5)$$

Note that σ does not couple to SM fermions at the tree level. Also, the Higgs coupling of σ is given by $\lambda_{AH}v_A$ can be chosen to be very small (see below). To estimate its lifetime, we first study its decay properties. We consider the following decays which are the dominant ones for our choice of parameters.

In our model, it is natural that $M_N > m_\sigma$, since $M_N \sim f_N v_B$ and $m_\sigma \simeq \sqrt{\lambda_A} v_A$ with $v_A \ll v_B$. The possible decay modes of σ are as follows:

(i) $\sigma \to NN \to \nu\nu$ (see Fig. 1). Here, the N is a virtual state. The decay proceeds via the mixing of χ_A with χ_B proportional to $\alpha \sim \frac{v_A}{v_B}$ in amplitude. It is followed

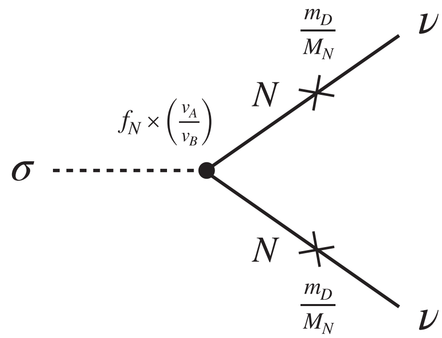

FIG. 1. Dominant decay mode of the dark matter σ in the model to two neutrinos is shown. The decay proceeds through the mixing $\sim v_A/v_B$ of the dark matter with χ_B and the mixings of right-handed neutrinos to light neutrinos in the seesaw mechanism $\sim m_D/M_N$.

by the mixing between N and ν through the seesaw mechanism $\sim m_D/M_N$, where m_D is the neutrino Dirac mass. The decay width in this case can be estimated to be

$$\Gamma_{\sigma\to\nu\nu} \simeq \frac{1}{4\pi}\left(\frac{m_\nu}{v_B}\right)^2\left(\frac{v_A}{v_B}\right)^2 m_\sigma. \qquad (6)$$

Here, we have used the seesaw formula for the light neutrino mass, $m_\nu \sim m_D^2/M_N$. For $m_\sigma \sim 10^5$ GeV, $f_N \sim 10^{-5}$, $M_N \sim 10^6$ GeV, we find the lifetime of σ dark matter to be 10^{28} sec. as required by astrophysical constraints.

(ii) A second important decay mode is $\sigma \to hh$, whose amplitude is proportional to $\lambda_{HA} v_A$. We choose the tree level coupling to be zero by setting $\lambda_{HA} = 0$ since it is a renormalizable coupling. Again, λ_{HA} can be induced at the one loop level by a combination of $\lambda_{\rm mix}\lambda_{HB}$ and is given by $\lambda_{HA}^{\rm ind} \sim \frac{\lambda_{\rm mix}\lambda_{HB}}{32\pi^2}$. For $\lambda_{HB} \sim 10^{-18}$ needed to get the weak scale right and for $\lambda_{\rm mix} \sim 10^{-8}$, we find $\lambda_{HA}^{\rm ind} \sim 10^{-28}$. The gauge loops do not contribute at the one loop level since the SM Higgs field H has zero $B-L$. This leads to $\tau_\sigma \sim 10^{28}$ sec, which is compatible with the astrophysical bounds on DM lifetime from gamma decay mode of dark matter.

(iii) Another possible mode is $\sigma \to \varphi'_A + Z' \to NNf\bar{f} \to \nu\nu f\bar{f}$, which is highly suppressed compared to $\Gamma_{\sigma\to\nu\nu}$ as is found to be

$$\Gamma^{Z'}_{\sigma\to\nu\nu f\bar{f}} \simeq \frac{1}{(4\pi)^5}\left(\frac{m_\nu}{v_B}\right)^2\left(\frac{v_A}{v_B}\right)^2\frac{m_\sigma^5}{v_B^4}. \qquad (7)$$

The bottom line of the above discussion is that $\frac{\lambda_{AB}}{\lambda_A} \sim (\frac{v_A}{v_B})^3$ must be very small for σ to be a viable dark matter. We also have assumed that λ_{AH} is very small.

(iv) Another mode which arises via φ_A mixing with φ_B and φ_B mixing with h is highly suppressed. This decay channel leads to $\sigma \to f\bar{f}f\bar{f}$ and has an amplitude given by

$$A_{\sigma\to f\bar{f}f\bar{f}} \simeq \left(\frac{v_A}{v_B}\right)\left(\frac{v_w}{m_{\varphi_B}}\right)^2\left(\frac{v_w}{v_B}\right)^2 \sim 10^{-42} \qquad (8)$$

which is very small indeed giving $\Gamma_{\sigma\to f\bar{f}f\bar{f}} \ll \Gamma_{\sigma\to\nu\nu}$.

V. DARK MATTER RELIC DENSITY

In order to discuss how relic density of DM arises in this model, we use the freeze-in mechanism [38]. The requirement for the freeze-in mechanism is that the dark matter must be out of equilibrium with the cosmic soup of the SM particles. We first require that the reheat temperature of the universe after inflation $T_R < M_{Z'}$. This is to avoid a resonance enhancement of the dark matter production by the Z' mediated process which leads to an over DM production unless the $B-L$ gauge coupling (g_{BL}) is extremely small.

For $m_\sigma^2, m_{\varphi'_A}^2 \ll s \ll M_{Z'}^2$, the DM annihilation process with φ'_A to a pair of SM fermions ($f\bar{f}$) is given by

$$\sigma_{\rm DM} \equiv \sigma_{\sigma\varphi'_A\to f\bar{f}} \simeq \frac{g_{BL}^4 s}{12\pi M_{Z'}^4} Q_{\Phi_A}^2\left(\sum_f N_f Q_f^2\right), \qquad (9)$$

where $(Q_f, N_f) = (1/3, 3)$ for a quark, $(-1, 1)$ for a charged lepton, and $(-1, 1/2)$ for an SM neutrino, and $Q_{\Phi_A} = 6$. Counting all SM fermions and RHNs for the final states $(\sum_f N_f Q_f^2 = 8)$ and using $M_{Z'} \simeq 2 g_{BL} v_B$, this reduces to

$$\sigma v_{\rm rel} \simeq \frac{3s}{\pi v_B^4}, \qquad (10)$$

where $v_{\rm rel}$ is the relative velocity of the initial particles. For a temperature of the early universe $m_\sigma^2, m_{\varphi'_A}^2 \ll T^2 \ll M_{Z'}^2$, the thermal average of the above cross section is found to be

$$\langle\sigma v_{\rm rel}\rangle \simeq \frac{36T^2}{\pi v_B^4} = \frac{36 m_\sigma^2}{\pi v_B^4} x^{-2}, \qquad (11)$$

where $x \equiv \frac{m_\sigma}{T}$.

The freeze-in build-up happens via the reaction $f\bar{f} \to \sigma\varphi'_A$ via Z' exchange and the DM yield Y obeys the Boltzmann equation:

$$\frac{dY}{dx} \simeq \frac{\langle\sigma v_{\rm rel}\rangle}{x^2}\frac{s(m_\sigma)}{H(m_\sigma)} Y_{\rm eq}^2, \qquad (12)$$

where $Y_{\rm eq} = \frac{n_{\rm eq}(T)}{s(T)}$ with the DM number density in thermal equilibrium $n_{\rm eq}(T) \simeq \frac{T^3}{\pi^2}$, and the entropy density of the universe $s(T)=\frac{2\pi^2 g_*}{45}T^3$, and $H(T)=\sqrt{\frac{\pi^2}{90}g_*}\frac{T^2}{M_P}$ is the Hubble parameter with the reduced Planck mass $M_P=2.43\times10^{18}$ GeV and the effective degrees of freedom of the thermal plasma g_* (we set $g_*=106.75$ in our analysis). We solve the Boltzmann equation from $x_R=m_\sigma/T_R\ll1$ to $x=1$ with $Y(x_R)=0$. Note that for $T<m_\sigma\sim m_{\varphi'_A}=\sqrt{3}m_\sigma$, or equivalently $x>1$, the production of DM particles is no longer effective since the averaged kinetic energy of the SM particles in the thermal plasma becomes lower than m_σ. Using $\frac{s(m_\sigma)}{H(m_\sigma)}\simeq13.7m_\sigma M_P$, $Y_{\rm eq}\simeq2.16\times10^{-3}$ and Eq. (11), we get

$$Y(x)\simeq2.45\times10^{-4}\frac{M_P m_\sigma^3}{v_B^4}\left(\frac{1}{x_R^3}-\frac{1}{x^3}\right)$$
$$\to Y(\infty)\simeq Y(x=1)\simeq2.45\times10^{-4}\frac{M_P T_R^3}{v_B^4}. \quad (13)$$

Note that the resultant $Y(\infty)$ value is determined by the reheating temperature, and this result is valid as long as $1/x_R^3=(T_R/m_\sigma)^3\gg1$. We then use the formula for the current dark matter abundance to be

$$\Omega_{\rm DM}h^2=\frac{m_\sigma s_0 Y(\infty)}{\rho_{\rm crit}/h^2}, \quad (14)$$

where $s_0\simeq2890/{\rm cm}^3$ is the current entropy density, and $\rho_{\rm crit}/h^2\simeq1.05\times10^{-5}$ GeV/cm^3. Using Eqs. (13) and (14), we find

$$T_R\simeq9.02\times10^{-9}v_B\left(\frac{v_B}{m_\sigma}\right)^{1/3} \quad (15)$$

to reproduce the observed DM relic abundance of $\Omega_{\rm DM}h^2=0.12$ [54].

Let us now consider the out-of-equilibrium condition for the dark matter σ. The observed DM relic abundance $\Omega_{\rm DM}h^2=0.12$ leads to $Y(\infty)\simeq Y(1)\simeq\frac{4.36\times10^{-10}}{m_\sigma[{\rm GeV}]}$. Considering the fact that $Y(x)$ is a monotonically increasing function for $x_R\le x\le1$ [see Eq. (13)], we conclude that $Y(x)\le Y(\infty)<Y_{eq}$ for $m_\sigma[{\rm GeV}]>2.01\times10^{-7}$. This means that as long as m_σ satisfies this lower bound, the yield $Y(x)$ starting from $Y(x_R)=0$ can never reach Y_{eq} and therefore, the dark matter σ has never been in thermal equilibrium.

VI. SUMMARY OF CONSTRAINTS ON THE MODEL AND PARAMETER SCAN

In this section, we summarize the constraints on the parameters of the model that add to the constraint in Eq. (15) from the observed DM relic density. The two main constraints that we have discussed are from the reheat temperature and the DM lifetime.

A. Constraint from the freeze-in mechanism

For the freeze-in mechanism to work, we need,

$$m_{\varphi'_A,\sigma}<T_R<M_{Z'}. \quad (16)$$

B. DM lifetime constraint

We next give the constraint from the dark matter lifetime:

$$\Gamma_{\sigma\to\nu\nu}\simeq\frac{1}{4\pi}\left(\frac{m_\nu}{v_B}\right)^2\left(\frac{v_A}{v_B}\right)^2 m_\sigma<6.58\times10^{-53}\ {\rm GeV}, \quad (17)$$

corresponding to the IceCube constraint [51] on $\tau_\sigma>10^{28}$ sec for $10^4<m_\sigma[{\rm GeV}]<10^9$. Note that this constraint implies $v_A\ll v_B$.

C. φ'_A lifetime constraint

In addition to (A) and (B), we consider the constraint coming from the lifetime of φ'_A. The main decay mode of φ'_A is $\varphi'_A\to\sigma Z'\to\sigma f\bar{f}$ via off-shell Z'. This three-body decay width is calculated to be

$$\Gamma_{\varphi'_A\to\sigma f\bar{f}}=\frac{g_{BL}^4}{24\pi^3}Q_{\Phi_A}^2\left(\sum_f N_f Q_f^2\right)\left(\frac{m_\sigma}{M_{Z'}}\right)^4 m_{\varphi'_A}C_I, \quad (18)$$

where

$$C_I=\int_1^{\frac{m_{\varphi'_A}^2+m_\sigma^2}{2m_{\varphi'_A}m_\sigma}}dz(z^2-1)^{3/2}\simeq0.0115, \quad (19)$$

for $m_{\varphi'_A}=\sqrt{3}m_\sigma$. The partial decay width of $\varphi'_A\to f\bar{f}f\bar{f}$ via virtual $Z'Z'$ is very highly suppressed compared to the above mode.

Note that $\Gamma_{\varphi'_A\to\sigma f\bar{f}}\propto(v_A/v_B)^4$ from $M'_Z\simeq2g_{BL}v_B$ and $m_\sigma\propto v_A$, and φ'_A can be long lived for $v_A\ll v_B$, which is required by the DM lifetime constraint. If φ'_A decays after the big-bang nucleosynthesis (BBN), the energetic final state SM fermions may destroy light nucleons successfully synthesized. To avoid this danger, we impose the following constraint that φ'_A decays before the BBN era, at which the age of the universe is $\tau_{\rm BBN}\simeq1$ sec.

$$\Gamma_{\varphi'_A\to\sigma f\bar{f}}\simeq4.84\times10^{-4}\left(\frac{m_\sigma}{v_B}\right)^4 m_\sigma>6.58\times10^{-25}\ {\rm GeV}, \quad (20)$$

where we have used $Q_{\Phi_A}^2=6$, $\sum_f N_f Q_f^2=8$, $M_{Z'}\simeq2g_{BL}v_B$, and $m_{\varphi'_A}=\sqrt{3}m_\sigma$ in evaluating $\Gamma_{\varphi'_A\to\sigma f\bar{f}}$.

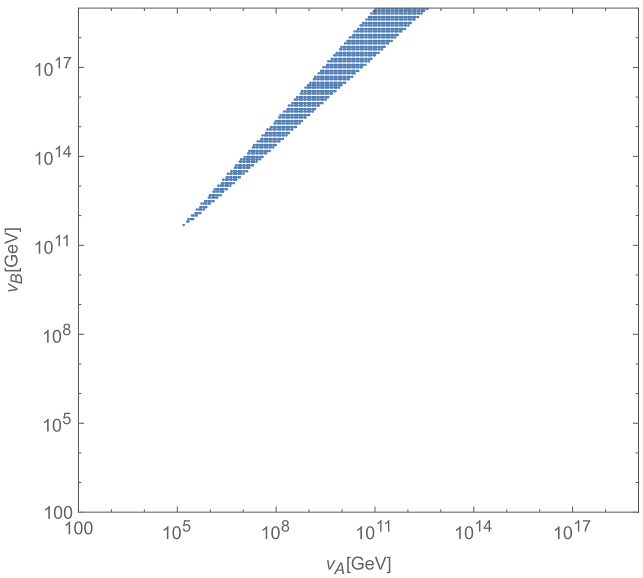

FIG. 2. Allowed values of v_A and v_B parameters in the model are shown as the shaded region.

Imposing the conditions in Eqs. (15)–(17) and (20), we perform the parameter scan for two free parameters in the range of 100 GeV $\le v_A$, $v_B \le 10^{19}$ GeV. In this analysis, we set $m_\sigma = v_A$ for simplicity, and $T_R > 3m_\sigma$ for the validity of Eq. (13). Figure 2 shows the allowed region in (v_A, v_B)-plane. Note that $m_\sigma = v_A \ll v_B$ is satisfied in the allowed region and the DM mass is constrained to be in the range of 10^5 GeV $\lesssim m_\sigma \lesssim 10^{13}$ GeV. In Fig. 3, we show the range of the DM lifetime and $v_{A,B}$. We can see that the IceCube constraint on $\tau_\sigma > 10^{28}$ sec is most severe for $m_\sigma \sim 10^5$ GeV. Figure 4 shows the allowed range of the reheating temperature and $v_{A,B}$. The condition $T_R = M_{Z'}$ sets the minimum value of gauge coupling $g_{BL}|^{\rm Min}$ for a chosen values of (v_A, v_B).

In Table I, we give a typical benchmark parameter choice for our model from the Figs. 2–4 for $v_B \sim 10^{11}$ GeV. While we have given only a single benchmark point, there is a broad range of parameters where the model works as can be seen from Figs. 2–4. For example, v_A goes from 10^5 GeV up to about 10^{11} GeV as v_B is increased and as other parameters are varied. In particular, from Eqs. (15) and (16), we conclude that g_{BL} has a minimum value of around 10^{-6} when $T_R = M_{Z'}$ (see right panel of Fig. 4) and a maximum value of 10^{-2} for the Coleman-Weinberg mechanism to work.

VII. COMMENTS AND PHENOMENOLOGY

In this section we make a few comments on the model.

(i) We first note that the model can explain the origin of matter via the usual leptogenesis mechanism. The temperature at which lepton asymmetry is generated depends on the particle spectrum of the model. For instance, note the important requirement of the model that $T_R < M_{Z'}$. So if $v_B \sim 10^9$ GeV and $g_{BL} \lesssim 0.3$ to avoid the coupling blowing up before the Planck scale, then the RHN mass must be less than T_R, which is too light to generate the observed baryon asymmetry by the usual leptogenesis. Thus, our scenario must use resonant leptogenesis mechanism [55,56].

(ii) Through the $B - L$ gauge interaction, RHNs can stay in thermal equilibrium with the SM particle plasma. As a result, the generation of lepton asymmetry is suppressed until the $B - L$ interaction is frozen. To avoid the suppression, we impose $n_{\rm eq}\langle\sigma v_{\rm rel}\rangle < H$ at $T \sim M_N$, where $n_{\rm eq} \simeq 2M_N^3/\pi^2$ is the RHN number density, and $\langle\sigma v_{\rm rel}\rangle$ is the thermal averaged cross section for the process $NN \leftrightarrow f\bar{f}$ via a virtual Z', roughly given by

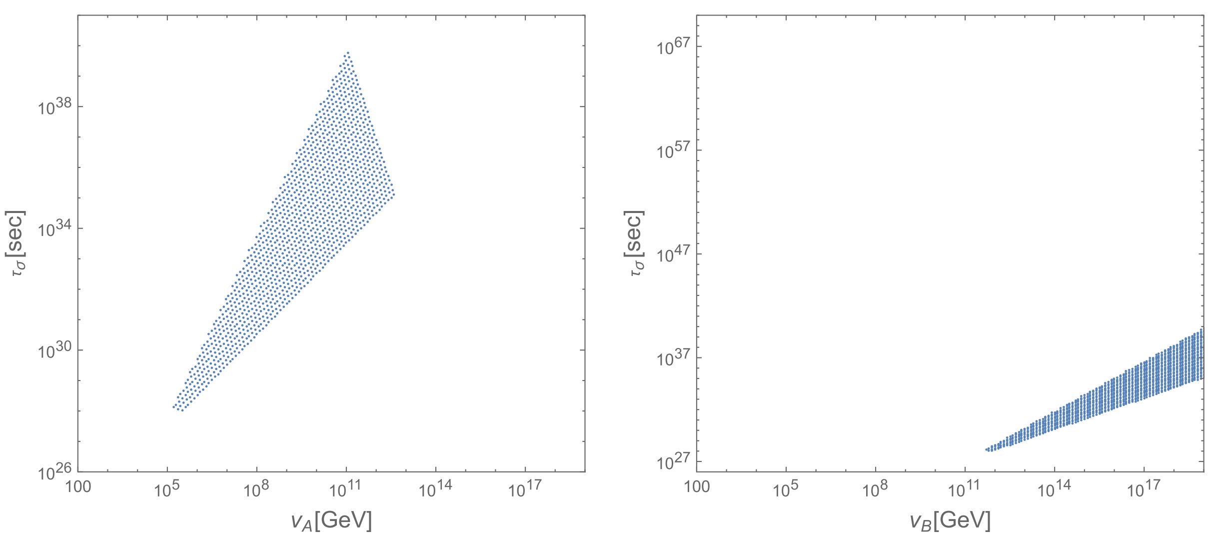

FIG. 3. Allowed range of the DM lifetime τ_σ and $v_{A,B}$.

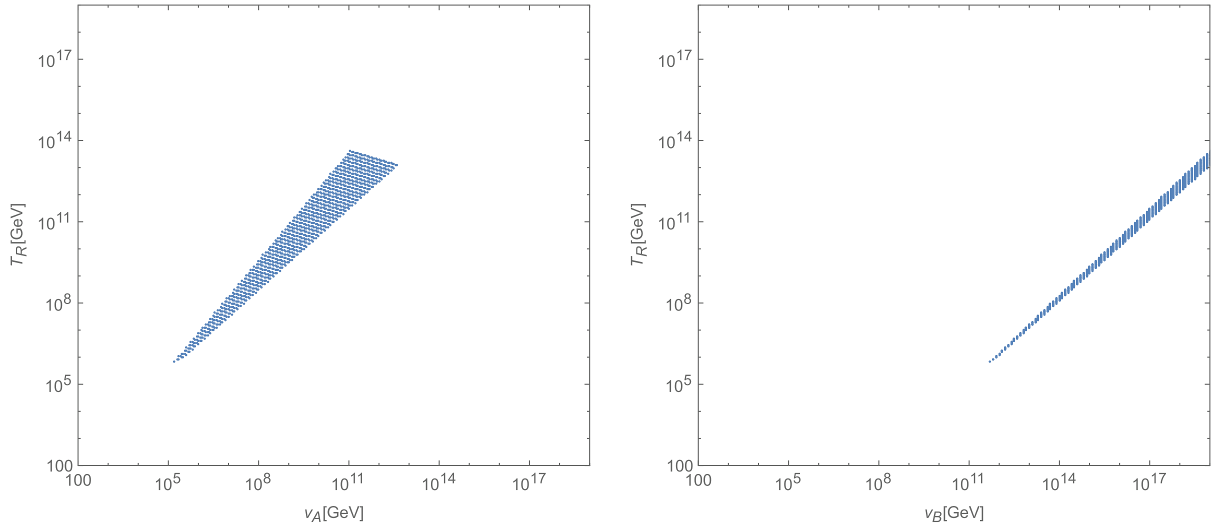

FIG. 4. Allowed range of the reheat temperature T_R and $v_{A,B}$.

$$\langle \sigma v_{\rm rel}\rangle \simeq \frac{g_{BL}^4}{4\pi}\frac{M_N^2}{M_{Z'}^4} \simeq \frac{M_N^2}{64\pi v_B^4}. \tag{21}$$

We then find the condition,

$$M_N < (32\pi^3)^{1/3}\left(\frac{\pi^2}{90}g_*\right)^{1/6} v_B \left(\frac{v_B}{M_P}\right)^{1/3} \simeq 10.3\times v_B\left(\frac{v_B}{M_P}\right)^{1/3}. \tag{22}$$

In Fig. 2, we have found the lower bound on $v_B \gtrsim 10^{11}$ GeV, for which the condition reads $M_N \lesssim 3.54\times 10^9$ GeV. For the successful (resonant) leptogenesis scenario, the lightest RHN must be in thermal equilibrium for $T_R \geq T > M_N$. Our result shown in Fig. 4 indicates that the condition of Eq. (22) is satisfied for the resultant T_R.

If $M_N > m_{\varphi_B'}$, the generation of lepton asymmetry can be suppressed by the process, $NN \leftrightarrow \varphi_B'\varphi_B'$ [57]. To avoid this suppression, we impose this process to be decoupled at $T\sim M_N$. The thermal averaged cross section for this process is given by

$$\langle \sigma v_{\rm rel}\rangle \simeq \frac{f^4}{4\pi M_N^2} \simeq \frac{M_N^2}{4\pi v_B^4}. \tag{23}$$

This formula is the same as Eq. (21) up to a factor, so that the resultant constraint is similar to Eq. (22). The similar processes, $NN\leftrightarrow \varphi_A'\varphi_A'$, $\sigma\sigma$, have no effect, since they are suppressed by the mixing $v_A/v_B \ll 1$ and ϕ_A' and σ are out-of-equilibrium.

(iii) One possible test of the model is that the dark matter decays into two neutrinos with energy $E_\nu = \frac{m_\sigma}{2}$ for each neutrinos and for dark matter masses in the multi-TeV range, there will be high energy monoenergetic neutrinos from the DM decay with a probability of τ_U/τ_σ, where $\tau_U \sim 10^{17}$ sec is the age of the universe.

(iv) We also note that due to the pseudo-Goldstone nature of the DM, direct detection cross section arises only at one loop level and is highly suppressed. At the tree level the DM behaves like an inelastic dark matter since Z' exchange by incident DM connects to a φ_A' field which is $\sqrt{3}$ times heavier. This explains why the DM has not been seen in the laboratory experiments.

(v) This model can be extended to allow the real part of the Φ_B field to play the role of inflaton while maintaining conformal invariance, as has been shown in Ref. [58–60]. In this case as well as in general, there is an upper limit on the reheat temperature coming from the power spectrum and upper limit on the tensor-to-scalar ratio $r\leq 0.036$ at 95% confidence level [61]. Typically, $T_R \gtrsim 6\times 10^{15}$ GeV is ruled out,

TABLE I. This table provides a benchmark set of parameters drawn from the allowed regions in Figs. 2–4. As explained in the text, the small numbers for the couplings λ_{HB}, λ_{AB} are radiatively stable up to two loops in the model.

λ_A	~ 1
λ_B	$\sim 10^{-8}$
λ_{HB}	10^{-18}
λ_{HA}	0
λ_{AB}	10^{-18}
g_{BL}	~ 0.01
v_B	10^{11} GeV
$v_A \sim m_\sigma$	10^5 GeV
f_N	10^{-5}
M_N	10^6 GeV

assuming the total inflaton energy is transmitted to the SM thermal plasma right after inflation.

(vi) Finally, we note that the consistency of the model requires certain couplings in the potential to be small so that the lifetime of the dark matter is long. These small couplings are however radiatively stable or guaranteed by symmetries in the theory. The model is therefore technically natural.

VIII. SUMMARY

In this brief note, we have presented a minimal conformal $B - L$ extension of the standard model with two SM singlet Higgs fields which explains the neutrino masses, origin of matter and dark matter that is produced in the early universe by the freeze-in mechanism. We have presented the allowed set of points where the model works and a table with one benchmark set of parameters. We find that the dark matter, which is a pseudo-Goldstone boson, must be heavier than 100 TeV in order to ensure that its scalar partner φ'_A must decay before the big bang nucleosynthesis. The model predicts energetic neutrinos from dark matter decay (with $E_\nu \geq 50$ TeV) which can be observed at the IceCube experiment, providing a test.

ACKNOWLEDGMENTS

The work of N. O. is supported in part by the U.S. Department of Energy Grant No. DE- SC0012447.

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