Invisible dilaton

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We analyze the dynamics of a light scalar field responsible for the μ term of the Higgs potential and coupled to matter via the Higgs-portal mechanism. We find that this dilaton model is stable under radiative corrections induced by the standard model particle masses. When the background value of the scalar field is stabilized at the minimum of the scalar potential, the scalar field fluctuations only couple quadratically to the massive fields of the standard model preventing the scalar direct decay into standard model particles. Cosmologically and prior to the electroweak symmetry breaking, the scalar field rolls down along its effective potential before eventually oscillating and settling down at the electroweak minimum. These oscillations can be at the origin of dark matter due to the initial misalignment of the scalar field compared to the electroweak minimum, and we find that, when the mass of the scalar field is less than the electron volt scale and acts as a condensate behaving like dark matter on large scales, the scalar particles cannot thermalize with the standard model thermal bath. As matter couples in a composition-dependent manner to the oscillating scalar, this could lead to a violation of the equivalence principle aboard satellites such as the MICROSCOPE experiment and the next generation of tests of the equivalence principle. Local gravitational tests are evaded thanks to the weakness of the quadratic coupling in the dark matter halo, and we find that, around other sources, these dilaton models could be subject to a screening akin to the symmetron mechanism.

I. INTRODUCTION

Dark matter (DM) is a basic constituent of our standard cosmological model. A large number of astrophysical observations constrain the amount of such a component up to the percent level. However, there is little information about its nature. Thanks to measurements of large scale structures, we know that the pressure or kinetic energy of DM is negligible. In this sense, it is said that DM is cold (CDM). However, there are observations associated with small scales that challenge this standard approach and emphasize potential issues such as the cusp-core problem [1–6].

In order to explore the properties of DM from the point of view of particle physics, it is interesting to study its relation with the electroweak sector. This sector is

associated with the mass generation of elementary particles through the Higgs mechanism. Indeed, the electroweak scale is the highest one in the Standard Model (SM) of particles and interactions, and also corresponds almost to the limit of the energy range that can be probed by present particle colliders; i.e. current particle experiments only probe energies slightly larger than the electroweak scale. In addition, this sector suffers from hierarchy problems that need to be understood better from a theoretical point of view (see for instance [7]). In part motivated by these issues, a large number of DM models expressed in terms of weakly interacting massive particles (WIMPs) have been proposed in the past decades. These candidates have the advantage of being produced in the early universe with the observed order of magnitude for the DM abundance through the so-called freeze-out mechanism. However, no experimental evidence has been found for the existence of WIMPs (see [8] for instance for a recent review).

In this work, we consider an alternative DM model directly related to the electroweak sector. In our case, DM arises from the coherent oscillations of a light scalar field responsible for the μ term of the Higgs potential. In this

sense, it has similarities with relaxion models, discussed in Refs. [9–13]. The first proposals about coherent bosonic DM date back to the late 1970s in terms of QCD-axion models [14–20]. In general, a coherent DM framework can be parametrized, from an effective-field-theory point of view, by the DM particle masses and its self-interactions [21]. The simplest case relies on a massive oscillating scalar field without self-interactions [20,22–25]. This type of generic coherent DM theories are typically indistinguishable from cold DM in relation to the formation of large-scale structures [23,26–32]. However, distinctive features at smaller scales can arise from many different causes [1–6,30,31,33–40]. Generically, coherent dark matter requires the DM particle mass to be less than the electron volt scale [41]. We will see that the electron volt scale plays a significant role in our scenario; e.g. screening of the Sun is only valid for masses larger than 10 eV.

As the new scalar field determines the Higgs potential, it is coupled to the SM via the Higgs portal. This fact determines its coupling with the ordinary matter content of the standard model. As such, this light scalar resembles a dilaton field associated with the breaking of conformal symmetry [42]. As we have commented above, our scenario has similarities with relaxion models [9–12] although there are notable differences. For instance, at the electroweak minimum of the dilaton potential, the linear coupling to matter vanishes and only the quadratic coupling remains. This structure is stable under radiative corrections and the phenomenological signatures change drastically. For instance, we find that the scalar DM cannot thermalize with the standard model bath when the DM mass is lower than the electron volt scale.

The lack of linear coupling leads to another distinctive feature of the model. Matter couples naturally with a different strength depending on its nature. Therefore, we analyze the possible observational constraints associated with violations of the equivalence principle in experiments such as MICROSCOPE and the prospects for the next generation of experiments. We find that Solar System constraints are evaded due to the weakness of the coupling. But in other environments the quadratic coupling induces a screening mechanism reducing the constraints on the parameters of the model [43–46]. In other contexts the scalar field could behave in ways reminiscent of scalarization [47]. The constraints on quadratic couplings of ultralight dark matter fields have recently been thoroughly explored in Refs. [48,49].

The paper is arranged as follows. In Sec. II, we introduce the model based on a new scalar degree of freedom. We describe the low energy action, its stability against radiative corrections, and the coupling of the scalar field to the SM. In Sec. III, we analyze the cosmological evolution supported by the dynamics of this scalar field. The main phenomenological consequences of the model are studied in Sec. IV. In particular, we discuss the violation of the equivalence

principle and the Eötvös parameter in Sec. V. Finally, we summarize the main conclusions of our work in Sec. VI.

II. SCALAR-DEPENDENT μ-TERM

A. The low energy action

We consider a simple model of electroweak symmetry breaking where one real Higgs field h gives a mass to one Dirac fermion ψ. This model is meant to reproduce in a toy-model fashion some aspects of the physics of the electroweak symmetry breaking. We will be interested in the regime where the scalar field ϕ is much lighter than the Higgs field h, i.e. $m_\phi \ll m_h$. The Lagrangian of the full theory is given by

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial h)^2 + \frac{\mu^2(\phi)}{2}h^2 - \frac{\lambda}{4}h^4 - V(\phi)$$
$$- i\bar{\psi}\slashed{\partial}\psi - \lambda_\psi h\bar{\psi}\psi. \tag{1}$$

The electroweak scale is determined by the $\mu^2(\phi)$ mass term in the broken phase which depends on the light field ϕ. As long as $\mu^2(\phi) < 0$, the Higgs field does not acquire a vacuum expectation value (VEV) and no symmetry breaking occurs. The potential $V(\phi)$ is chosen such that the scalar field induces a change from values where $\mu^2 < 0$ to $\mu^2 > 0$. In the following we will assume that as long as $\mu^2 < 0$ and large, the μ^2 function is mostly linear and the field massless. More precisely, we write the $\mu^2(\phi)$ term as

$$\mu^2(\phi) = -\Lambda_0^2 + \Lambda^2\frac{\phi}{f} + M^2\mu_I\left(\frac{\phi}{f}\right), \tag{2}$$

where μ_I is a subdominant term that we can take to vanish at the transition point $\mu_I(\Lambda_0^2/\Lambda^2) = 0$. The scales Λ_0, Λ, and M are lower than the cutoff scale of the model Λ_c. We assume that the following hierarchy is realized:

$$v \ll \Lambda_c. \tag{3}$$

This corresponds to requiring that the electroweak symmetry breaking happens at low energy compared to the cutoff scale of the theory. The scale f determines the dynamics of ϕ. This could be for instance the VEV of a $U(1)$ breaking field if ϕ were a pseudo-Goldstone mode. Notice that we assume that the correction μ_I is present in the whole range of validity of the effective field theory. In particular, it does not only appear when the electroweak symmetry takes place, and we will assume that the effective description is valid from inflation down to lower energies.

We also assume that the field ϕ couples to the inflaton in the Jordan frame through the metric

$$g_{\mu\nu}^J = A^2(\phi)g_{\mu\nu}, \tag{4}$$

where $g_{\mu\nu}$ is the Einstein frame metric. This implies that the scalar potential of the scalar is corrected and becomes [50]

$$V_{\text{eff}}(\phi) = V(\phi) - T[A(\phi) - 1], \qquad (5)$$

where we choose

$$A(\phi) = 1 + \frac{(\phi - \phi_e)^2}{2m_{\text{Pl}}^2}, \qquad (6)$$

and T is the trace of the energy momentum tensor of the inflaton. During inflation $T = -4V_{\text{inf}}$ where V_{inf} is the potential energy leading to a de Sitter phase. This coupling forces the scalar field to a value $\phi \simeq \phi_e$ at the end of inflation, which then provides an initial condition for the evolution of the field in the postinflationary universe. Other mechanisms could be invoked to regulate the early universe behavior of the field and slow it down after inflation. Here we consider this simplified description as a proxy for potentially more complex mechanisms which are beyond the scope of this paper (see for instance [41]). We will discuss the cosmological evolution of the model, including the dynamics of the scalar field during inflation, further in Sec. III.

B. The Higgs phase

After inflation, the field will evolve until a point where $\mu^2(\phi) > 0$, and the Higgs field acquires a large mass compared to that of the scalar field ϕ, as a result one can "integrate out" the Higgs field using the classical equations of motion

$$\lambda h^2(\phi) = \mu^2(\phi) - \frac{\lambda_\psi}{h(\phi)} \bar{\psi}\psi. \qquad (7)$$

This method of removing the Higgs degree of freedom is valid as the Higgs-scalar mass matrix does not have a massless eigenvalue [51]. Let us first work at the classical level by solving Eq. (7) to lowest order in a perturbative expansion in $\bar{\psi}\psi$ and obtain

$$h(\phi) = \frac{\mu(\phi)}{\sqrt{\lambda}} - \frac{\lambda_\psi}{2\mu^2(\phi)} \bar{\psi}\psi. \qquad (8)$$

At lowest order this gives the VEV of the Higgs field as

$$v = \frac{\mu(\bar{\phi})}{\sqrt{\lambda}}, \qquad (9)$$

where the dilaton field ϕ is stabilized at $\phi = \bar{\phi}$ with a mass m_ϕ. We can also obtain the effective Lagrangian for ϕ at the classical level

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{\mu^4(\phi)}{4\lambda} - i\bar{\psi}\partial\!\!\!/\psi - \lambda_\psi \frac{\mu(\phi)}{\sqrt{\lambda}} \bar{\psi}\psi. \qquad (10)$$

This Lagrangian contains the classical part of the potential for ϕ

$$V_{\text{clas}}(\phi) = V(\phi) - \frac{\mu^4(\phi)}{4\lambda}, \qquad (11)$$

which determines the dynamics of ϕ after electroweak symmetry breaking. In the following, we will see that the term in μ^4 can be neglected.

C. Radiative corrections

A potential for the dilaton field $V(\phi)$ is naturally present as it can be induced by radiative corrections of the Higgs field h. Closing the Higgs loop in the coupling provided by Eq. (1), i.e. $\mu^2(\phi)h^2/2$, gives

$$V_{\text{one loop}}(\phi) \supset \frac{\Lambda_c^2}{32\pi^2} \mu^2(\phi), \qquad (12)$$

where Λ_c is the scale at which the Higgs quadratic divergence gets cut off. In the spirit of effective field theories where allowed couplings should be present, we will assume that the potential for the scalar ϕ is corrected at the one loop level by the Higgs loop to

$$V(\phi) = V_0(\phi) - a\frac{\Lambda_c^2}{32\pi^2} \mu^2(\phi), \qquad (13)$$

where a is a dimensionless constant. We will assume that $a > 0$ in order to ensure that this contribution to the scalar potential decreases from the symmetric phase to the electroweak breaking one. We will also simplify the model by taking $V_0(\phi) = V_0$ which makes sure that the vacuum energy vanishes at the minimum of the potential.

There are also logarithmic corrections to the potential. As the mass of the Higgs field is $m_h^2 = 2\mu^2(\phi)$ when the electroweak symmetry is broken, the corrections to the scalar potential are proportional to $\mu^4(\phi)$. At the one loop level this yields for the total effective potential

$$V_{\text{eff}}(\phi) = V(\phi) - \frac{\mu^4(\phi)}{4\lambda} + \frac{\mu^4(\phi)}{16\pi^2} \ln\frac{\Lambda_c^2}{2\mu^2(\phi)}, \quad (14)$$

where Λ_c is the UV renormalization scale of the Higgs-scalar theory. This is the usual Coleman-Weinberg correction at one loop calculated in dimensional regularization [52]. The corrections due to the masses of matter particles have the same form. Indeed, the masses are all proportional to $\mu(\phi)$ and therefore lead to the same type of one loop corrections. Higher order loops should also contribute to the effective potential in μ^4 as μ is the only mass scale in the theory below the cutoff. In Eq. (14) we see that the corrections in μ^4 can be incorporated in a redefinition of the self-coupling λ which becomes $\lambda(v)$ due to the logarithmic corrections

$$\lambda \to \lambda(v) = \lambda\left(1 + \frac{\lambda}{4\pi^2}\ln\frac{\Lambda_c^2}{2\lambda v^2}\right), \qquad (15)$$

and similarly for loops coming from matter fields. Loops induced by the scalar field itself will scale in m_ϕ^4 which is assumed to be very small compared to μ^4. Hence we neglect the self-loops of ϕ in the following and work at the classical level when it comes to the scalar field ϕ. The self-coupling will always be taken to be the renormalized one $\lambda(v)$ at the electroweak scale.

In the following, we will focus on a scenario where the scalar field ϕ evolves cosmologically toward the minimum $\bar\phi$ of the scalar potential and eventually oscillates around this extremum, such that oscillations are described by $\varphi = \phi - \bar\phi$. In particular, we will focus on models where

$$V(\phi) = -g\Lambda_c^2\mu^2(\phi) + V_0, \qquad (16)$$

where $g = \frac{a}{32\pi^2} > 0$. The minimum of the scalar potential in the radiation era corresponds to

$$\partial_\phi\mu^2(\bar\phi) = 0. \qquad (17)$$

The contribution in $-\mu^4/4\lambda$ from the Higgs phenomenon does not change this result and is always negligible as $\mu \ll \Lambda_c$ close to the electroweak transition.

D. Couplings to bosons

So far we have only considered the coupling of the scalar φ to fermions via the Higgs portal. Couplings to photons and gluons are induced by triangular anomalous diagrams where massive fermions run in the loop. This affects the low energy theory of the dark matter field each time a particle of the standard model decouples [42]. This induces an effective interaction Lagrangian of the type

$$\delta L = -\frac{\alpha_F(E)e^2}{4}\frac{\varphi^2}{\Lambda_f^2}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha_G(E)g_3^2}{4}\frac{\varphi^2}{\Lambda_f^2}G_{\mu\nu}G^{\mu\nu}, \quad (18)$$

where $\alpha_{F,G}(E)$ are numerical constants depending on the charges of the decoupled fermions, i.e. fermions more massive than E, under the electromagnetic $U(1)$ symmetry and the quantum chromodynamics (QCD) $SU(3)$ group; see for instance [42,53] for an explicit discussion.[1] The coupling constant e and g_3 are the electromagnetic and QCD coupling, respectively, evaluated at the energy scale E. The sign of the interaction Lagrangian is given by the

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[1]The coefficients are given by $\alpha_F(E) = \frac{\sum_{i=1}^{N_f^>} q_i^2}{24\pi^2}$, where $N_f^>$ is the number of particles and antiparticles of charges q_i which have decoupled at the energy E. Similarly we have $\alpha_G(E) = \frac{\sum_{i=1}^{N_f^>} T(R_i)}{12\pi^2}$, where the fermions are in the representation R_i such that the Lie algebra generators are normalized by $\mathrm{Tr}(T^aT^b) = T(R_i)\delta^{ab}$.

sign of the interaction between the scalar and fermions. In the following we will be interested in the low energy effects of the coupling to photons at energies well below the electron mass. As a result, the coefficient α_F will take into account the decoupling of all the standard model particles. On the other hand, the coupling to the gluons is relevant to determining the QCD condensation scale. In this case, α_G depends on the decoupling of the heavy quarks c, b, and t. We will return to the consequences of these couplings in Sec. V C.

E. Backreaction

Using the effective Lagrangian, Eq. (10), we can see that in the Higgs phase, when standard model fermions have acquired a mass, the scalar potential is modified by the average fermion number

$$V_{\mathrm{matter}}(\phi) = V_{\mathrm{eff}}(\phi) + \frac{\lambda_\psi}{\sqrt{\lambda}}\mu(\phi)n_\psi, \qquad (19)$$

where $n_\psi = \langle\bar\psi\psi\rangle$ is the fermion number density. This is only valid when $\mu^2 > 0$, and we are in the Higgs phase. When $\mu^2 < 0$, the matter effect disappears and the potential is simply

$$V_{\mathrm{no\ matter}}(\phi) = V_{\mathrm{eff}}(\phi). \qquad (20)$$

This backreaction behavior is reminiscent of a coupling to the trace of the energy momentum tensor for scalar-tensor theories [50]. Close to the electroweak transition, the matter term is much smaller than the effective potential as $n_\psi \sim T^3$ where $T \sim v$ and $\Lambda_c \gg v$. This implies that in the vicinity of the electroweak transition, the matter corrections are negligible.

In addition to the coupling in Eq. (10), we could include higher order couplings between the Higgs field and matter of the type

$$\delta\mathcal{L} \subset -\frac{h^n}{\Lambda_h^{n-1}}\bar\psi\psi, \qquad (21)$$

where Λ_h is a cutoff scale in the Higgs sector. Typically this type of operator leads to a matter correction to the scalar potential

$$\delta V_{\mathrm{matter}} \subset \frac{v^n}{\Lambda_h^{n-1}}n_\psi, \qquad (22)$$

after the electroweak transition. This is always a negligible contribution to the potential at the electroweak scale for $T \sim v$ as $v \ll \Lambda_h \lesssim \Lambda_c$.

In the following, we will consider models where the field ϕ can escape the vicinity of the minimum at $\bar\phi$ if the field ϕ reaches values such as $\mu^2 \simeq -M^2$ where M is a large scale taken to be smaller than the cutoff scale Λ_c. Now the matter

backreaction would stop the field from jumping over the barrier associated with the potential $V(\phi) = -g\Lambda_c^2\mu^2$ if the μ term were prevented from reaching a large value of order M. We assume that the scale M is the natural scale $M \gg v$ of $|\mu(\phi)|$ far away in field space from the minimum of the potential where $\mu(\bar{\phi}) \ll M$. In this case, this stopping mechanism would be reminiscent of the Damour-Nordtvelt effect [43,44] for the models considered here, whereby the electroweak scale should be driven close to the zero of μ cosmologically in an attractor fashion. The higher order terms in Eq. (21) provide a stopping correction to the scalar potential of the form

$$\delta V(\phi) \simeq \frac{\mu^n(\phi)}{\Lambda_h^{n-1}} v^3, \qquad (23)$$

where $h(\phi) = \mu(\phi)/\sqrt{\lambda}$ and $n_\psi \simeq v^3$. This term dominates compared to $V(\phi)$ when $\mu \sim M$ provided

$$\left(\frac{M}{\Lambda_h}\right)^{n-2} \gtrsim \frac{\Lambda_c^2 \Lambda_h}{v^3}. \qquad (24)$$

This can be realized if $M \gg \Lambda_h$ where higher and higher corrections to the Higgs portal would backreact strongly on the dynamics of the dilaton. Of course this is beyond the realm of the effective field theory setup we have adopted here as the full nonperturbative series has to be known. The only conclusion we can draw is that a full analysis of the Higgs sector and its coupling to matter is necessary to probe the large $\mu^2 > 0$ region of the theory. In particular, it is quite likely that after the electroweak phase transition the effects of such matter couplings could be efficient enough to stop the dilaton and guarantee that the field simply oscillates around the minimum where $\mu \sim v$ close to $\mu = 0$. A full discussion of this issue goes beyond the present paper, and we refer the reader to Ref. [41]. In addition to the higher order corrections to the Higgs coupling, thermal effects must be taken into account, as it has been discussed in Ref. [45]. As the coupling of the scalar field is proportional to the trace of the energy-momentum tensor, it is commonly assumed to vanish during the radiation dominated epoch in the early universe. However, finite radiative corrections to the coupling impact generically on the evolution of the field, modifying the allowed region of its parameter space, changing the abundance of different cosmological relics and producing early phases of contracting evolution (in the Jordan frame) [45]. In the following, we will analyze the dynamics of the theory by neglecting these thermal effects, which will be taken into account in future works.

F. The coupling to matter

The light scalar field ϕ couples to matter via the Higgs portal. In the Higgs phase where matter fields acquire a mass, one can expand the light scalar field $\phi = \bar{\phi} + \varphi$ to obtain the resulting interaction between φ and matter

$$\mathcal{L}_{\text{int}} = -\frac{\beta}{m_{\text{Pl}}} m_\psi \varphi \bar{\psi}\psi - \frac{m_\psi}{2\Lambda_f^2} \varphi^2 \bar{\psi}\psi, \qquad (25)$$

where we have identified $m_\psi = \lambda_\psi v$. The scalar has a Yukawa type coupling

$$\beta \equiv m_{\text{Pl}} \frac{\partial_\phi \mu(\phi)|_{\bar{\phi}}}{\sqrt{\lambda} v} = 0, \qquad (26)$$

where the Higgs vev v was defined in Eq. (9) and a quadratic coupling

$$\frac{1}{2\Lambda_f^2} = \frac{\partial_\phi^2 \mu}{2\mu} = -\frac{m_\phi^2}{4g\mu^2\Lambda_c^2}, \qquad (27)$$

which is composition independent at the fundamental fermion level; i.e. it depends not on the fermion species ψ. On the other hand, the coupling to nucleons will depend on the species, and this could induce violations of the equivalence principle. We will investigate this possibility below.

As a result the light scalar field φ whose mass is given by

$$m_\phi^2 = -\frac{\mu^2 \partial_\phi^2 \mu^2}{2\lambda} + V''|_{\bar{\phi}} \qquad (28)$$

is stable quantum mechanically; i.e. there is no decay into two fermions at tree level in vacuum. The scalar mass is dominated by

$$m_\phi^2 \simeq -g\Lambda_c^2 \partial_\phi^2 \mu^2 = -2g\Lambda_c^2\mu\partial_\phi^2\mu, \qquad (29)$$

where the last equality is valid at the minimum where $\partial_\phi\mu = 0$. We will see below how this mass scale can be much smaller than m_h.

In the following we will be interested in φ as a candidate for dark matter where φ is light, i.e. $m_\phi \lesssim 1$ eV [41]. In principle the coupling Eq. (25) could induce the thermal equilibrium between ϕ and the standard model fields, eventually leading to the freezing out of the ϕ abundance which could then be adjusted to match the present amount of dark matter in the universe. But as we will see in Sec. III C, it is not possible for the scalar field in our model to be in thermal equilibrium with the thermal bath. Another possibility, as m_ϕ is small, could be that the scalar ϕ decoupled when relativistic. The remaining abundance of hot dark matter behaves as $\Omega_\phi h^2 \simeq 10^{-3}(\frac{100}{g_\star})(\frac{m_\phi}{1 \text{ eV}})$, where $g_\star$ is the number of relativistic species at decoupling. For very light scalars, the abundance of hot dark matter becomes negligible.

So the scalar field in our model can only play the role of dark matter in a nonthermal fashion and decouples from quarks and leptons at the electroweak symmetry breaking. In this case, decoupling happens before the scalar acquires a mass and therefore causes no issue with the abundance of dark matter, whether hot or cold. In the following we will consider the natural situation where the scalar field rolls down along its potential from small values before oscillating around its minimum. This mechanism is similar to the misalignment mechanisms used for axion [18,20,54–56] or scalar dark matter models such as fuzzy dark matter [25]. The abundance of dark matter in these cases is related to the amplitude of the oscillations around the minimum.

G. An explicit dilaton model

As we expect fluctuations of the scalar around the minimum of its potential to play the role of dark matter, we now return to finding the minimum of the scalar potential and imposing that electroweak symmetry breaking takes place at the scale v. For this, let us notice that in the radiation dominated era the minimum of the effective potential, Eq. (16) with μ defined in Eq. (2), is such that

$$\left.\frac{d\mu_I(y)}{dy}\right|_{\bar{y}} = -\frac{2\Lambda^2}{\pi M^2}, \tag{30}$$

in terms of the rescaled scalar field $y = \pi\phi/(2f)$ and its VEV $\bar{y}$. The electroweak scale is determined by imposing that at the minimum we have $\mu^2(\bar{\phi}) = \lambda v^2$ which implies that

$$\mu_I(\bar{y}) - \bar{y}\left.\frac{d\mu_I}{dy}\right|_{\bar{y}} = \frac{\Lambda_0^2}{M^2} + \lambda\frac{v^2}{M^2}. \tag{31}$$

As a typical example we choose a correction to the μ^2 term of the axion type where

$$\mu_I(y) = \cos y, \tag{32}$$

where we have assumed that $\Lambda_0 = \Lambda$. The axionic contribution vanishes for $y = \pi/2$ so that the transition from $\mu^2 < 0$ to $\mu^2 > 0$ takes place where both the linear part of μ^2 and μ_I vanish. Explicitly we have

$$\mu^2(\phi) = \Lambda^2\left(\frac{\phi}{f} - 1\right) + M^2\cos\left(\frac{\pi}{2}\frac{\phi}{f}\right). \tag{33}$$

We find that Eq. (30) implies that

$$\sin\bar{y} = \frac{2}{\pi}\frac{\Lambda^2}{M^2}. \tag{34}$$

The tuning of the electroweak symmetry breaking, Eq. (34), can be satisfied provided

$$\cos\bar{y} + \bar{y}\sin\bar{y} = \lambda\frac{v^2}{M^2} + \frac{\Lambda_0^2}{M^2}. \tag{35}$$

This can easily be analyzed by expanding the μ^2 term around $y = \pi/2$ using $\bar{y} = \pi/2(1 + \delta)$. This gives

$$\mu^2(\phi) = \left(\Lambda^2 - \frac{\pi}{2}M^2\right)\delta + \frac{M^2}{6}\left(\frac{\pi}{2}\delta\right)^3. \tag{36}$$

The minimum of the potential is then given by

$$\delta^2 = 2\left(\frac{2}{\pi}\right)^3\left(\frac{\pi}{2} - \frac{\Lambda^2}{M^2}\right). \tag{37}$$

Imposing that $\mu^2(\bar{\phi}) = \lambda v^2$ implies that

$$\delta = \frac{3}{2}\frac{\lambda v^2}{\Lambda^2 - \frac{\pi}{2}M^2}. \tag{38}$$

These two conditions, Eqs. (37) and (38), are compatible provided the two scales Λ and M are related by

$$\frac{\Lambda^2}{M^2} = \frac{\pi}{2}\left[1 - \left(\frac{3\lambda v^2}{2\sqrt{2}M^2}\right)^{2/3}\right]. \tag{39}$$

This may appear as a tuning of the potential although we have seen that radiative corrections preserve the shape of μ^2 and therefore the ratio between Λ and M.

Defining

$$\epsilon = \left(\frac{3\lambda v^2}{2\sqrt{2}M^2}\right)^{2/3}, \tag{40}$$

we find that the value of δ at the minimum of the potential is

$$\frac{\pi}{2}\delta_{\min} = -\sqrt{2}\left(\frac{3\lambda v^2}{2\sqrt{2}M^2}\right)^{1/3} = -\sqrt{2\epsilon}. \tag{41}$$

Notice that this extremum is a maximum of μ^2. As a result we get a minimum for the scalar potential and a mass for the scalar field

$$m_\phi^2 = -g\left(\frac{\pi}{2}\right)^3\delta_{\min}\frac{M^2\Lambda_c^2}{f^2}. \tag{42}$$

This mass is reduced compared to the naive expectation of the mass that one might obtain from considering Eq. (33) and assuming $\cos(\pi\phi/2f) \sim 1$, which would give

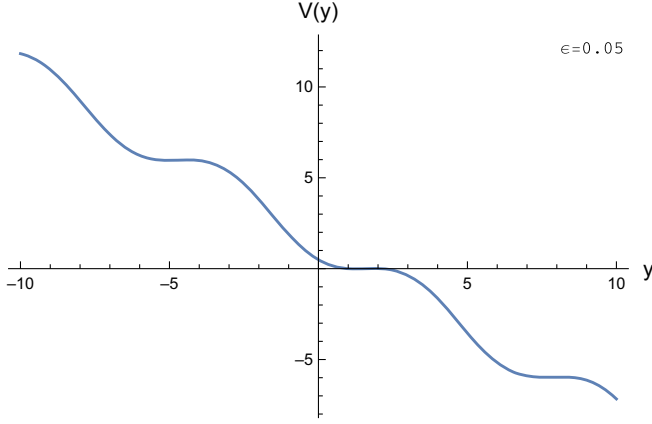$$m_0^2 = g\frac{\pi^2}{4}\frac{M^2\Lambda_c^2}{f^2}. \tag{43}$$

FIG. 1. The normalized potential $V(y)$ where $y = \frac{\pi\phi}{2f}$ for $\epsilon = 0.05$. The oscillations of the dark matter field take place on the flat part of the potential close to the first minimum on the positive real axis. The field is first stabilized during inflation and then released in the postinflationary era when the Hubble rate drops below the mass of the scalar field. The oscillatory behavior is guaranteed as long as the potential is not too flat. The mass on the steep part of the potential is typically $1/\sqrt{\epsilon}$ larger than close to the minimum.

In fact the mass is reduced by a factor of $\delta_{\min}$ [9], i.e.

$$m_\phi^2 = -\frac{\pi}{2}\delta_{\min}m_0^2 = \sqrt{2\epsilon}m_0^2. \tag{44}$$

The mass m_0 is suppressed compared to the cutoff scale by a factor of f. Moreover the mass of the scalar field m_ϕ in the shallow part of the potential close to the minimum is reduced by a factor of $\epsilon^{1/4}$ which is also small (see Fig. 1). This small mass will eventually be identified with the mass of the scalar dark matter in the Universe. We are also interested in the location of the field where the potential vanishes. The potential vanishes where $\mu^2 = 0$ corresponding to $\delta = 0$ and also for

$$\frac{\pi}{2}\delta_0 = -\sqrt{6}\left(\frac{3\lambda v^2}{2\sqrt{2}M^2}\right)^{1/3} = -\sqrt{6\epsilon}. \tag{45}$$

In the interval between δ_0 and the origin, the mass of the field is of order of m_ϕ. In this region, the potential is very flat as can be seen in Fig. 1.

III. COSMOLOGICAL EVOLUTION

A. Inflation

During inflation, the electroweak symmetry is preserved and $\langle h \rangle = 0$. The dynamics of ϕ are determined by

$$V_{\text{eff}}(\phi) = -g\Lambda_c^2\mu^2(\phi) + 6H_{\text{inf}}^2(\phi - \phi_e)^2 + V_0, \tag{46}$$

where the form of the coupling to the inflaton Hubble rate, H_{inf}, is assumed to be given in Eq. (6). The large quadratic
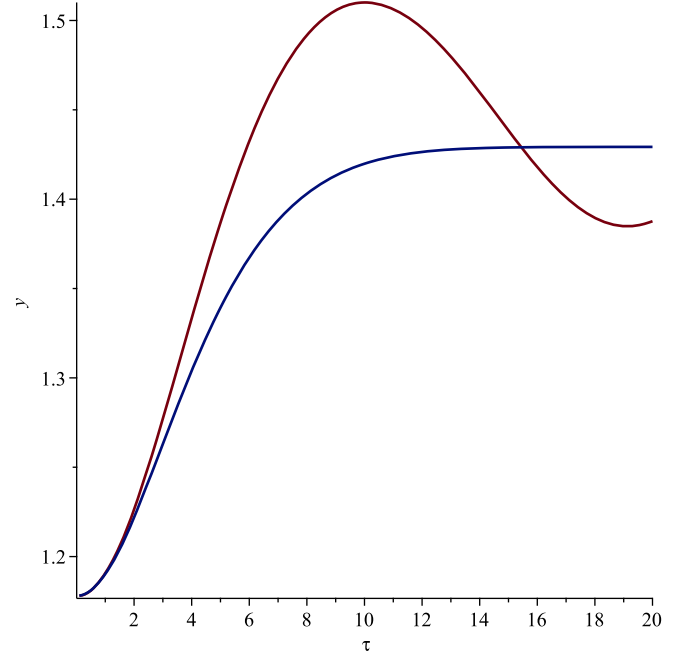

FIG. 2. The normalized $y = \pi\phi/2f$ field as a function of τ for $\epsilon = 0.01$. The blue curve is the numerical solution. The red one is the approximate solution. One can see that the slow roll approximation is valid for a few Hubble times.

term due to the coupling to inflation forces ϕ is close to ϕ_e and stabilized. The minimum of the effective potential is obtained for

$$\phi_{\text{inf}} = \phi_e + \frac{g\Lambda_c^2}{12H_{\text{inf}}^2}\partial_\phi\mu^2|_{\phi=\phi_{\text{inf}}}. \tag{47}$$

This is of order

$$\frac{\phi_{\text{inf}} - \phi_e}{f} \propto \frac{g\Lambda_c^2 M^2}{f^2 H_{\text{inf}}^2} \sim \frac{m_0^2}{H_{\text{inf}}^2} \ll 1, \tag{48}$$

which is very small as long as we assume that $m_0 \ll H_{\text{inf}}$, and in addition we will assume that $H_{\text{inf}} \ll \Lambda_c$. In particular, this implies that the field does not move at the end of inflation until the Hubble rate goes down to $H \simeq m_0$ in the postinflationary era, an expectation that is confirmed by our numerical solutions (see Fig. 2). This is independent of the choice of ϕ_e. Notice that during inflation the mass of the scalar field is $m_{\text{inf}}^2 = 12H_{\text{inf}}^2$ implying that the field is heavy and no isocurvature fluctuations are generated.

B. Postinflation evolution

The equation of motion of the scalar field is the Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \tag{49}$$
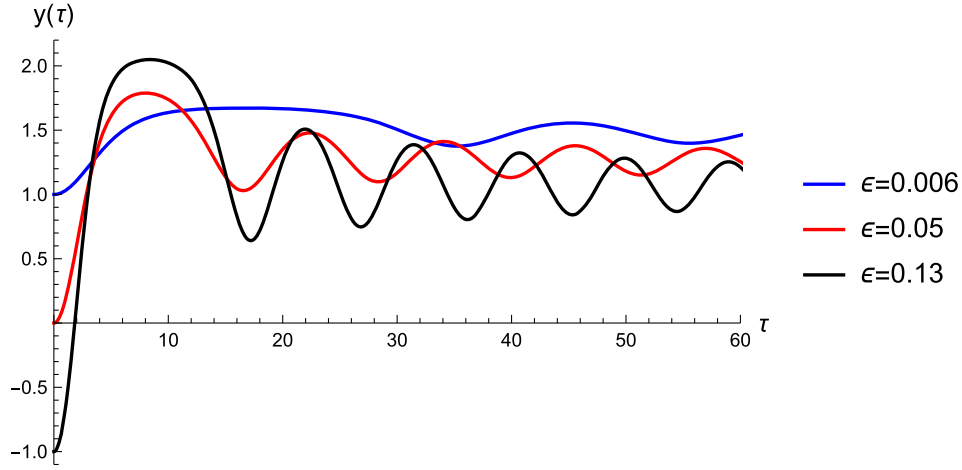
FIG. 3. Scalar field trajectories $y(\tau) = \frac{\pi\phi}{2f}$ as a function of τ for the initial conditions $y_0 = 1, 0, -1$, and the smallest associated values of ϵ that ensure the field remains trapped in the first local minimum.

with the specific potential

$$V(\phi) = -g\Lambda_c^2\mu^2 + V_0$$
$$= -g\Lambda_c^2\left[\Lambda^2\left(\frac{\phi}{f} - 1\right) + M^2\cos\left(\frac{\pi\,\phi}{2\,f}\right)\right] + V_0. \quad (50)$$

It is useful to change the time variable to $\tau = m_0 t$, to obtain the reduced equation in the radiation era

$$y'' + \frac{3}{2\tau}y' + \sin(y) = 1 - \epsilon, \quad (51)$$

where, as before, $y = \pi\phi/(2f)$, ϵ was defined in Eq. (40), and the primes denote derivatives with respect to the rescaled time τ. This corresponds to the motion of the particle $y(\tau)$ in the potential $V(y) = -(1-\epsilon)y - \cos(y)$ shown in Fig. 1.

The field starts rolling at the end of inflation, at time $\tau_i \ll 1$, with zero velocity. In the limit $\tau_i \to 0$, there is a regular solution with a Taylor expansion of the form

$$y(\tau) = y_0 + y_2\tau^2 + y_4\tau^4 + y_6\tau^6 + \cdots. \quad (52)$$

Substituting into the equation of motion gives the coefficients

$$y_2 = \frac{1 - \epsilon - \sin(y_0)}{5},$$
$$y_4 = \frac{\cos(y_0)[-1 + \epsilon + \sin(y_0)]}{90}, \ldots, \quad (53)$$

and we can see that the field only moves significantly after a time $\tau \gtrsim 1$, that is, after $H \lesssim m_0$.[2]

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[2]A more detailed study of the slow roll evolution of the field after the end of inflation can be found in Appendix B.

In this paper, we assume that the field starts near the first positive minimum, located at $y \simeq \pi/2$, that is, $-1 \lesssim y_0 \lesssim \pi/2$. Then, for $\epsilon > 0$ not too small, because of the Hubble friction, the field will remain trapped inside this first shallow potential well and oscillate at late times around the equilibrium value

$$\bar{y} = \arcsin(1-\epsilon) = \frac{\pi}{2} - \sqrt{2\epsilon} + \cdots,$$
$$V''(\bar{y}) = \sqrt{2\epsilon} + \cdots. \quad (54)$$

We show in Fig. 3 the trajectories obtained for different values of the initial condition y_0 with, in each case, the smallest value of ϵ that keeps the field trapped in the first local minimum. We see that as y_0 is decreased from $\pi/2$, the parameter ϵ must increase to enhance the barrier at the right of the local minimum. However, thanks to the Hubble friction, which slows down the rolling down the potential, small values of $\epsilon \lesssim 0.1$ are sufficient to keep the field in the local potential well for a reasonably large range of initial conditions, $-1 \lesssim y_0 \lesssim 1$. From the definition of ϵ in Eq. (40) we see that this is guaranteed as long as $M \gtrsim 1$ TeV. For larger values of M, ϵ would be smaller, and as a result the basin of attraction of the minimum would shrink.

Writing $y = \bar{y} + \frac{\pi}{2}\delta$, as before, we obtain for the late-time small oscillations the equation of motion

$$\delta'' + \frac{3}{2\tau}\delta' + \sqrt{2\epsilon}\delta = 0. \quad (55)$$

This gives decaying oscillations of the form $\delta = \tau^{-1/4}J_{\pm 1/4}((2\epsilon)^{1/4}\tau)$. As $\tau \gg 1$ this corresponds to harmonic oscillations with an amplitude that slowly decays as $\tau^{-3/4} \propto a^{-3/2}$, where $a(\tau)$ is the cosmological scale factor. This gives an energy density ρ_ϕ that decays as a^{-3},

as for dark matter. The potential $V(\phi)$ of Eq. (50) reads $V(\bar{\phi}) + g\Lambda_c^2 M^2 \sqrt{\epsilon/2}(y - \bar{y})^2 + \cdots$. The first local maximum for $y > \bar{y}$ is for $y_{\max} = \pi/2 + \sqrt{2\epsilon} + \cdots$. If the scalar field first turns around at a value smaller but of the order of $y_{\max}$, we obtain that at $\tau \sim 1/2$ the dark matter density is

$$H \sim m_0: \; \rho \sim g\Lambda_c^2 M^2 \epsilon^{3/2} \sim g\lambda \Lambda_c^2 v^2. \qquad (56)$$

At later times this energy density evolves like cold dark matter and decays as a^{-3}. The initial dark matter density is then

$$\rho_{\rm in} = \frac{1}{2} m_\phi^2 \phi_0^2, \qquad (57)$$

which is then redshifted up to now to become the dark matter density in the present Universe ρ_0.

C. Thermalization

We can now come back to the thermalization of the scalar particles and impose that the scalar is never in equilibrium with the particles of the Standard Model. This requires that the decoupling temperature $T_{\rm dec} \gtrsim v$. Let us examine this scenario.

The coupling between the scalar and matter is quadratic in the scalar with a coupling constant $1/\Lambda_f^2$ of order $m_\phi^2/v^2 \Lambda_c^2$. In the relativistic regime, the square of the scattering amplitude $\phi + \phi \rightarrow \psi + \psi$ behaves like[3]

$$|\mathcal{M}|^2 \simeq \frac{m_\psi^2}{4\Lambda_f^4}(u\bar{v})^2, \qquad (58)$$

where in the relativistic regime the external spinors are such that $u\bar{v} \simeq T$. The cross section is of order

$$\sigma \simeq \frac{|\mathcal{M}|^2}{s}, \qquad (59)$$

where the Mandelstam variable $s \simeq T^2$, which yields

$$\sigma \simeq \frac{m_\psi^2}{128\pi^2 \Lambda_f^4}, \qquad (60)$$

which is constant.[4] In this regime the number of relativistic species is given by $\tilde{g}_\star$ and the number density of massive

particles reads $n \simeq \tilde{g}_\star \zeta(3) T^3/\pi^2$ where $\tilde{g}_\star = \sum_{\rm bosons} g_i^B + \sum_{\rm fermions} \frac{3}{4} g_i^F$ and $g_i^{B,F}$ are the degeneracy factors for bosons and fermions. The reaction rate is therefore

$$\Gamma = \frac{\tilde{g}_\star \zeta(3)}{\pi^2} \sigma T^3 \simeq \frac{m_\psi^2}{4\Lambda_f^4} T^3, \qquad (61)$$

and the Hubble rate is given by

$$H \simeq \sqrt{\frac{g_\star \pi^2}{30}} \frac{T^2}{m_{\rm Pl}}, \qquad (62)$$

where $g_\star = \sum_{\rm bosons} g_i^B + \sum_{\rm fermions} \frac{7}{8} g_i^F$. The sum is taken over all the particles of the Standard Model. Equilibrium is maintained when $\Gamma \gtrsim H$ which implies

$$T \geq T_{\rm dec} \simeq \frac{4\Lambda_f^4}{m_\psi^2 m_{\rm Pl}}, \qquad (63)$$

where we have taken $\tilde{g}_\star \sim g_\star \sim 100$.

The scalar is never in thermal equilibrium when $T_{\rm dec} \gtrsim v$. We use this criterion as at higher temperatures the electroweak transition has not occurred and the dilaton field does not behave like dark matter. This corresponds to

$$\frac{m_\psi}{2\Lambda_f^2} \lesssim (m_{\rm Pl} v)^{-1/2}, \qquad (64)$$

or more appropriately

$$\frac{m_\phi}{v} \lesssim \frac{\Lambda_c}{m_{\rm Pl}} \left(\frac{m_{\rm Pl}^3}{m_\psi^2 v}\right)^{1/4} \simeq 10^{13} \frac{\Lambda_c}{m_{\rm Pl}}, \qquad (65)$$

where the final approximate equality arises when $m_\psi \sim$ GeV corresponding to the b quarks. Indeed, the most severe constraint comes from the heaviest quark with a mass m_ψ of a few GeV. As the cutoff scale, Λ_c, must be larger than 1 TeV, the bound in Eq. (65) is always satisfied as soon as $m_\phi \lesssim 1$ GeV. This is always satisfied, as the range of masses for which the occupation number of the oscillating scalar is large enough to describe dark matter is $m_\phi \lesssim 1$ eV [57]. As a result we conclude that the oscillating dilaton scalar never thermalizes with the standard model and can describe dark matter via the misalignment mechanism.

IV. VIOLATION OF THE EQUIVALENCE PRINCIPLE

A. The coupling to matter

The oscillating scalar field around the minimum of the φ scalar potential induces a quadratic coupling to fermions of the type

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[3]Note that in this expression v is a spinor, whereas elsewhere in this article v refers to the vacuum expectation value of the Higgs field.

[4]The cross section is given by $\sigma = \frac{m_\psi^2}{128\pi^2 \Lambda_f^4} \frac{s - 4m_\psi^2}{s} \sqrt{\frac{s - 4m_\psi^2}{s - 4m_\phi^2}}$. Taking the limit $s \sim T \gtrsim m_\psi, m_\phi$, we find $\sigma = \frac{m_\psi^2}{128\pi^2 \Lambda_f^4}$.

$$\mathcal{L} \supset \frac{m_\psi}{2\Lambda_f^2} \varphi^2 \bar{\psi}\psi, \tag{66}$$

which implies a universal dependence of the fermion masses,

$$m_\psi(\varphi) = m_\psi \left(1 - \frac{\varphi^2}{2\Lambda_f^2}\right). \tag{67}$$

Notice that the coupling to matter tends to destabilize the scalar field as in scalarization models [47]. As such the weak equivalence principle is respected at the level of elementary particles as each fermion couples universally to the dilaton with a Jordan frame metric

$$g_{\mu\nu}^\psi = \left(1 - \frac{\varphi^2}{2\Lambda_f^2}\right)^2 g_{\mu\nu}, \tag{68}$$

where $g_{\mu\nu}$ is the Einstein frame metric. On the other hand, as macroscopic matter is composed of atoms themselves comprising a nucleus and electrons, the coupling to a particular species depends on the number of nucleons A and the number of electrons Z of the material. This is important as tests of the equivalence principle are carried out with two different bodies with different numbers of electrons and nucleons. The Klein-Gordon equation for the scalar φ in the presence of matter species A of density ρ_A now reads

$$\Box\varphi = m_\phi^2\varphi + \frac{\alpha_A(\varphi)}{m_{\text{Pl}}}\rho_A. \tag{69}$$

The species-dependent coupling function is defined as

$$\alpha_A(\varphi) = m_{\text{Pl}}Q_A \frac{\partial_\varphi m_\psi(\varphi)}{m_\psi} = -Q_A \frac{m_{\text{Pl}}}{\Lambda_f^2}\varphi, \tag{70}$$

where Q_A is a dimensionless phenomenological coefficient. Concentrating on the contributions from the fermion masses, Q_A is given by

$$Q_A = Q + [Q_A']_{\hat{m}} + [Q_A']_{m_e} + [Q_A']_{\delta m}, \tag{71}$$

where the couplings to the average mass of the u and d quarks is $[Q_A']_{\hat{m}}$, the coupling to the electron mass is $[Q_A']_{m_e}$, and the coupling to the u-d mass difference is $[Q_A']_{\delta m}$. They are tabulated for different metals [48]. The universal coupling $Q \simeq 0.093$ is also phenomenological. Depending on the sign of Q_A, the model will behave like a symmetron ($Q_A < 0$) [46] or scalarization [47] ($Q_A > 0$). The effects of the coupling to the gluons will be analyzed below.

The field at infinity oscillates as

$$\varphi_\infty(t) = \varphi_0 \cos(m_\phi t), \tag{72}$$

and must be regular at the center of the ball of uniform density ρ_A, mass M_A, and radius R_A. These boundary conditions determine the field profile, which takes the form [48]

$$\varphi = \varphi_0 \cos(m_\phi t) f_\pm(r/R_A). \tag{73}$$

The sign of the charge Q_A selects either the function f_+ or f_-, with

$$
\begin{aligned}
Q_A < 0 : x < 1, \quad & f_+(x) = \frac{1}{\cosh u}\frac{\sinh ux}{ux} \text{ and for } x > 1, \quad f_+(x) = 1 - \frac{1}{x}\left(1 - \frac{\tanh u}{u}\right), \\
Q_A > 0 : x < 1, \quad & f_-(x) = \frac{1}{\cos u}\frac{\sin ux}{ux} \text{ and for } x > 1, \quad f_-(x) = 1 - \frac{1}{x}\left(1 - \frac{\tan u}{u}\right),
\end{aligned}
\tag{74}
$$

with

$$u = \sqrt{|Q_A|\frac{\rho_A R_A^2}{\Lambda_f^2}} = \sqrt{|Q_A|\Phi_A \frac{6m_{\text{Pl}}^2}{\Lambda_f^2}}, \tag{75}$$

where we introduced the gravitational potential at the surface of the object, $\Phi_A = G_N M_A/R_A$. In both cases, outside the object we have

$$r > R_A : \varphi = \varphi_0\left(1 - s_A \frac{G_N M_A}{r}\right)\cos(m_\phi t) = \varphi_\infty(t) - \frac{\beta_A(t)}{4\pi m_{\text{Pl}}}\frac{M_A}{r}, \tag{76}$$

where we introduced the couplings s_A and $\beta_A(t)$, which are related by

$$\beta_A(t) = \frac{s_A \varphi_\infty(t)}{2m_{\text{Pl}}}. \tag{77}$$

In the unscreened regime, the couplings that give the amplitude of the fifth force outside of the objects are

$$\text{unscreened: } |u| \ll 1, \qquad s_A = \frac{2Q_A m_{\text{Pl}}^2}{\Lambda_f^2}, \qquad \beta_A(t) = \frac{Q_A m_{\text{Pl}} \varphi_\infty(t)}{\Lambda_f^2} = \alpha_A(\varphi(t)), \qquad (78)$$

and the field is almost constant inside the object. This coupling is the same as the one of a point particle; i.e. no effect results from the finite size of the object.

On the other hand, in the screened regime associated with large Newtonian potential Φ_A, the couplings are

$$\text{screened: } |u| \gg 1, \qquad s_A = \frac{1}{\Phi_A}, \qquad \beta_A(t) = \frac{\varphi_\infty(t)}{2m_{\text{Pl}} \Phi_A}, \qquad (79)$$

and the field shows a steep decay or fast oscillations inside the object. Notice that objects with large Newtonian potentials tend to be screened, a result reminiscent of screening mechanisms in modified gravity [58]. We will comment on this analogy below.

The coupling of the scalar to matter differs in the screened and unscreened cases. It is only species dependent in the unscreened regime, through the charge Q_A. On the other hand, it is object dependent in the screened regime through the dependence on the Newtonian potential Φ_A. Objects with various Newton potentials have differing trajectories depending on the couplings $\beta_A \propto 1/\Phi_A$.

B. Screened modified gravity

The screening regime and the resulting screening mechanism for the dark scalar is similar to the symmetron screening mechanism, where the coupling to matter vanishes in regions of high density. Here and contrary to the symmetron case, the solution outside the object is the time-varying φ_∞. The screening criterion $|u| \gg 1$ is analogous to the one for all nonderivative screening mechanisms [58], i.e.

$$|\beta_A^{\text{screened}}| \le |\beta_A^{\text{unscreened}}|: \quad \frac{|\varphi_\infty|}{2m_{\text{Pl}} \Phi_A} \le \frac{m_{\text{Pl}} |Q_A| |\varphi_\infty|}{\Lambda_f^2}. \qquad (80)$$

This corresponds to requiring that the effective coupling β_A is less than the coupling α_A that an unscreened object such as a point particle would experience. For static objects, the scalar force is proportional to the gradient of the scalar field. Focusing on solar system tests where the test objects such as the Cassini satellite have very small Newtonian potentials and behave like point particles in the scalar dark matter background, screening may occur if the Earth or the Sun themselves are screened. Thus, if the Earth is screened, the field gradient is suppressed and no deviation from general relativity will take place in the vicinity of the Earth. The Cassini bound [59] implies that

$$\langle \beta_\oplus \alpha_S \rangle \le 2 \times 10^{-5}, \qquad (81)$$

where $\oplus$ denotes quantities evaluated for the Earth, S for the satellite, $Q_\oplus \sim 0.1$ for a model of the Earth made of silicon and iron, and $Q_S \sim 0.1$ for a metallic satellite. We have taken the average over the rapid oscillations of the scalar field. This bound depends on the dark matter density locally $\rho_0 = m_\phi^2 \varphi_0^2 / 2$, which is of the order of 10^6 times the cosmological matter density. This becomes the numerical constraint

$$\frac{Q_S \rho_0}{m_\phi^2 \Phi_\oplus \Lambda_f^2} \le 2 \times 10^{-5}. \qquad (82)$$

Using Eq. (75), we see that the Earth is screened provided

$$\frac{\Lambda_f}{m_{\text{Pl}}} \ll \sqrt{2Q_\oplus \Phi_\oplus}, \qquad (83)$$

where $\Phi_\oplus \simeq 10^{-9}$. For the typical H/He composition of the Sun we have $|Q_\odot| \simeq 0.15$ and $\Phi_\odot \sim 10^{-6}$. We see that the Sun is automatically screened if the Earth is screened.

C. The Eötvös parameter

The contribution to the acceleration of an unscreened body A in the field φ due to the environment is given by

$$\vec{a}_A^\varphi = -\frac{\beta_A(\varphi)}{m_{\text{Pl}}} (\vec{\nabla} \varphi + \vec{v}_A \dot{\varphi}), \qquad (84)$$

where $\vec{v}_A$ is the nonrelativistic velocity of body A. Here we have

$$\frac{\beta_A(\varphi)}{m_{\text{Pl}}} = \frac{Q_A \varphi}{\Lambda_f^2}. \qquad (85)$$

The acceleration of the test body A due to the scalar field φ generated by a distant massive body C is then given by

$$\vec{a}_A^\varphi = Q_A \frac{\varphi_0^2}{\Lambda_f^2} \cos^2(m_\phi t) \left(1 - s_C \frac{G_N M_C}{r} \right)$$
$$\times \left[s_C \frac{G_N M_C}{r^3} \vec{r} - m_\phi \vec{v}_A \left(1 - s_C \frac{G_N M_C}{r} \right) \tan(m_\phi t) \right]. \qquad (86)$$

This is only the acceleration due to the scalar to which the gravitational acceleration should be added.

Let us now consider two test bodies A and B at the same location r from a distant object C, e.g. the two cylinders of the MICROSCOPE experiment aboard a satellite at 710 km from the Earth in a nearly circular orbit and falling in the terrestrial gravitational field [60]. Then the difference between their accelerations toward the third object C is given by

$$\vec{a}_A^\varphi - \vec{a}_B^\varphi = (Q_A - Q_B)\frac{\varphi_0^2}{\Lambda_f^2}\cos^2(m_\phi t)\left(1 - s_C\frac{G_N M_C}{r}\right)$$
$$\times \left[s_C\frac{G_N M_C}{r^3}\vec{r} - m_\phi \vec{v}\left(1 - s_C\frac{G_N M_C}{r}\right)\tan(m_\phi t)\right],$$

(87)

where we have taken that the centers of mass of the two objects coincide. We see that the factor $(1 - s_c G_N M_C/r)$ modulates the Newtonian acceleration and vanishes when $r = R_C$, the radius of object C, if object C is screened. Hence the violations of the equivalence principle are maximized for satellite experiments and minimized on the Earth, which must be screened for usual modified-gravity scenarios to pass terrestrial tests of gravity. There is an extra modulation when the objects move around C with a common velocity $\vec{v}$.

We define the Eötvös parameter

$$\eta_{AB} = 2\left|\frac{\vec{a}_A - \vec{a}_B}{\vec{a}_A + \vec{a}_B}\right| \simeq \left|\frac{\vec{a}_A^\varphi - \vec{a}_B^\varphi}{\vec{a}^N}\right|,$$

(88)

where $\vec{a}^N = -G_N M_C \vec{r}/r^3$ is the common Newtonian acceleration of the two bodies. For quasicircular orbits and on average this gives

$$\eta_{AB} = \frac{\varphi_0^2}{2\Lambda_f^2}|Q_A - Q_B|s_C\left(1 - s_C\frac{G_N M_C}{r}\right),$$

(89)

where r is the radius of the orbit. If the Earth is screened, then for experiments such as MICROSCOPE we have $s_C = s_\oplus \simeq 1/\Phi_\oplus \simeq 10^9$. This implies that

$$\eta_{AB} = \frac{\varphi_0^2}{\Lambda_f^2}\frac{|Q_A - Q_B|}{\Phi_\oplus}\left(1 - \frac{R_\oplus}{r}\right).$$

(90)

Moreover in this experiment two cylinders of platinum and titanium alloys were used with $Q_{Ti} - Q \sim -10^{-2}$ and $Q_{Pt} - Q \sim -7.5 \times 10^{-3}$ and $|Q_{Ti} - Q_{Pt}| \sim 2.9 \times 10^{-3}$. As a result we get a bound from $|\eta_{Pt-Ti}| \leq 5 \times 10^{-15}$ [61] on the amplitude

$$\frac{\varphi_0}{\Lambda_f} \leq 10^{-10}.$$

(91)

This can be used to put bounds on the parameters of the model as we can write

$$\frac{\sqrt{\rho_0}}{m_\phi \Lambda_f} \leq 10^{-10},$$

(92)

which relates the cutoff of the theory, the mass of the dark matter field, and the local density of dark matter. Fixing the local dark matter density also implies

$$\sqrt{m_\phi \Lambda_f} > 10^{-5} \text{ GeV},$$

(93)

which correlates the suppression scale of the quadratic coupling to matter and the scalar mass.

D. Constraints on the dilaton model parameters

1. Unscreened case

For scalar masses below 1 eV, it is straightforward to check that the Earth and the Sun are both unscreened implying that the scalar field behaves like a nearly massless (when the range is large enough) field in the solar system with an effective coupling to matter

$$\beta_A = \frac{Q_A m_{Pl}\varphi_0}{\Lambda_f^2}.$$

(94)

Using $\varphi_0 = \sqrt{2\rho_0}/m_\phi$, the effective coupling is of order

$$\beta_A \simeq \frac{Q_A}{g\lambda}\frac{m_\phi m_{Pl}\sqrt{\rho_0}}{8\sqrt{2}v^2\Lambda_c^2} \sim 10^{-24}\frac{1}{g\lambda}\frac{Q_A}{0.1}\frac{m_\phi}{1 \text{ eV}}\left(\frac{\Lambda_c}{1 \text{ TeV}}\right)^{-2}.$$

(95)

This small value of β_A guarantees that the MICROSCOPE results are hardly affected by the scalar field.

Moreover, solar system tests are evaded when the coupling is smaller than the Cassini bound of order 10^{-5} for β_A^2. This is easily satisfied due to the very small value of the dark matter density and the smallness of the scalar mass. Therefore, even when the scalar is very light and the screening mechanism does not operate, the fact that the coupling to matter is proportional to φ_0 which is very small implies that the scalar is effectively decoupled from matter. The coupling becomes large enough to lead to possibly detectable effects only if locally the density of dark matter were to increase, for example due to the presence of a dark matter clump, significantly above the dark matter halo density [62]. The investigation of this possibility is left for future work.

2. Screened case

To see where in the dilaton parameter space screening might be relevant for terrestrial and satellite experiments, we start from the quadratic coupling

$$\frac{1}{\Lambda_f^2} = -\frac{\partial_\phi^2 \mu^2}{2\mu^2},$$

(96)

where we recall that the linear coupling vanishes as $\partial_\phi \mu = 0$ at the minimum of the potential. Explicitly we obtain that

$$\frac{1}{\Lambda_f} = \frac{\pi^{3/2}}{4} \frac{M}{\sqrt{\lambda} f v} \sqrt{-\delta_{\min}}, \tag{97}$$

or in terms of the scalar mass

$$\frac{1}{\Lambda_f} = \frac{1}{4\sqrt{g\lambda}} \frac{m_\phi}{v\Lambda_c}. \tag{98}$$

We get the bound from Eq. (93),

$$\sqrt{g\lambda}\Lambda_c > 10^{-13} \text{ GeV}. \tag{99}$$

This is always easily satisfied for theories with a cutoff scale larger than the electroweak scale.

As a result, the MICROSCOPE bound can always be respected due to the screening of the Earth if

$$m_\phi > 1.9 \times 10^8 \frac{\sqrt{g\lambda}\Lambda_c}{m_{\text{Pl}}} \text{ GeV}. \tag{100}$$

The lowest admissible cutoff scale from the particle physics point of view is $\Lambda_c \gtrsim 10$ TeV which implies that

$$m_\phi > 1 \text{ keV}. \tag{101}$$

If the Earth were unscreened, the Sun could still be screened itself. This happens typically when

$$m_\phi > 10 \text{ eV}, \tag{102}$$

as the Newtonian potential of the Sun is 10^3 stronger than the Earth's (but with a scalar charge that is 5 times larger). Screening of the Sun would guarantee that the Cassini experiment was insensitive to scalar interactions. In conclusion, we find that screening could only occur for large scalar masses. Unfortunately, this is not allowed for scalar fields generating dark matter via the misalignment mechanism. If the scalar dark matter represented only a fraction of the dark matter density, the constraint on its mass would be relaxed and therefore one could envisage that the Earth could be screened. In all cases, the dilaton studied here turns out to be invisible gravitationally. We now turn to other potential probes of the models under investigation.

V. FURTHER PHENOMENOLOGY

A. Atomic clocks

Oscillating dark matter fields coupled to matter could lead to changes in atomic structure [48,63,64]. In particular, tiny oscillations in the electron to proton mass ratio could be detected using atomic clocks. The variation of the atomic frequencies for various atomic transitions are sensitive to the coupling of the scalar field to particles such as the electrons and could probe very small couplings for very light scalars of masses less than 10^{-18} eV. In our model and in the background of the local dark matter density, the coupling of the scalar to matter particles is universal with a value[5]

$$\beta = \frac{m_{\text{Pl}}\varphi_0}{\Lambda_f^2}, \tag{103}$$

of order $\beta \simeq 10^{-24}(\frac{m_\phi}{1 \text{ eV}})$ which is very much lower than the expected sensitivity of atomic clock experiments [48].

B. Large scale structure

The coupling of dark matter to baryons in cosmology could lead to an increase in the rate of growth for baryonic structures. Indeed and as long as the mass of the dark matter scalar is small enough, structures characterized by their wave number k would be affected as long as $k/a \gtrsim m_\phi$, where a is the scale factor of the Universe normalized to unity now. Gravity would be enhanced corresponding to a rescaling of Newton's constant by a factor $(1 + 2\beta^2(\rho))$ where the coupling is $\beta(\rho) = a^{-3/2}\beta$. As this coupling is valid from the electroweak scale time characterized by a redshift z_{EW}, where $a^{-1} = 1 + z$, $z_{\text{EW}} \simeq v/\Lambda_{\text{DE}} \simeq 10^{14}$, we find that the smallness of the coupling (103) cannot be compensated by the large redshift-dependent factor. This implies that no effects on the growth of structures is expected.

C. Consequences of the coupling to bosons

The interactions between the light scalar and photons and gluons in Eq. (18) induce a dependence of the electromagnetic and the QCD couplings on the scalar field as

$$\frac{1}{e^2(\varphi)} = \frac{1}{e^2} + \alpha_F e^2 \frac{\varphi^2}{\Lambda_f^2},$$

$$\frac{1}{g_3^2(\varphi)} = \frac{1}{g_3^2} + \alpha_G g_3^2 \frac{\varphi^2}{\Lambda_f^2}. \tag{104}$$

In the dark matter background this leads to linear couplings to the scalar field proportional to β. As a result both the fine structure constant and the QCD condensation scale, defined as the point where the QCD gauge coupling becomes large, become dependent on the scalar field. This leads to contributions to the masses of the nucleons coming from the electromagnetic and gluonic energies [66,67]. It turns

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[5]The coupling to fermions in [63] d_i where i labels the different fermions is universal in our model and equal to $\sqrt{2}\beta$. The same factor of $\sqrt{2}$ relates also the coupling to the QCD condensate and photons in the parametrization of [48,65] and the couplings $(\beta_{\text{QCD}}, \beta_\gamma)$; see below.

out that the gluonic contribution dominates as the masses of nucleons are mostly due to the gluon condensate, and we now concentrate on this effect.

QCD condensation takes place below the c, b, t quark masses in terms of energy scale at a value around 250 MeV. As a result, we only take into account the quadratic coupling of the scalar to the gluons when the heavy quarks have been integrated out. Writing the renormalization group equation for the QCD coupling between the charm decoupling scale m_c and a lower scale E we have

$$\frac{4\pi}{g_3^2(E)} = \frac{4\pi}{g_3^2(\varphi)} - \frac{b_3}{2\pi} \ln\left(\frac{m_c}{E}\right). \tag{105}$$

The coefficient $b_3 > 0$ is the QCD beta function coefficient due to the gluons and the u, d, s quarks. The QCD scale is such that $g_3(\Lambda_{\text{QCD}})$ diverges leading to

$$\Lambda_{\text{QCD}}(\varphi) = m_c e^{-8\pi^2/b_3 g_3^2(\varphi)} \simeq \Lambda_{\text{QCD}}\left(1 - \frac{8\pi^2 \alpha_G g_3^2}{b_3} \frac{\varphi^2}{\Lambda_f^2}\right), \tag{106}$$

where in the last term g_3 is the QCD coupling at the energy scale m_c. The quadratic dependence of the QCD scale on φ^2 can be tested using atomic clocks [48] and places the constraint that for scalar masses $m_\phi \lesssim 10^{-18}$ eV one must require that $\Lambda_f \gtrsim m_{\text{Pl}}$. This is easily achieved as $\Lambda_f \simeq \frac{v\Lambda_c}{m_\phi}$ where the scalar mass is lower than 10^{-18} eV and the cutoff scale is above the 10 TeV range.

In the dark matter background, the dependence on the scalar variation $\delta\phi$ of Λ_{QCD} compared to the dark matter background φ_0 can be parametrized as

$$\Lambda_{\text{QCD}}(\delta\varphi) = \Lambda_{\text{QCD}}\left(1 + \beta_{\text{QCD}} \frac{\delta\varphi}{m_{\text{Pl}}}\right), \tag{107}$$

where

$$\beta_{\text{QCD}} = -\frac{16\pi^2 \alpha_G g_3^2}{b_3} \beta. \tag{108}$$

As this coupling is also very small and proportional to β, the conclusion that the scalar field hardly couples to matter and is therefore invisible in gravitational experiments remains.[6]

Finally the dark matter scalar can also decay to photons where the coupling is induced at one loop. This can only happen when the scalar field gets a VEV, which can occur

in the dark matter halo and also in the early universe. The decay rate to photons due to fermion loops is given by

$$\Gamma_{\phi\to\gamma\gamma} \simeq \beta_\gamma^2 \frac{m_\phi^3}{m_{\text{Pl}}^2}, \tag{109}$$

where $\beta_\gamma = \alpha_F e^2 \frac{m_{\text{Pl}}\varphi_0}{2\Lambda_f^2}$. This is of order

$$\Gamma_{\phi\to\gamma\gamma} \simeq e^4 \frac{m_\phi \rho_0}{\Lambda_f^4} \sim e^4 \frac{m_\phi^5 \rho_0}{v^4 \Lambda_c^4}. \tag{110}$$

This should be much smaller than $H_0 \sim \frac{\sqrt{\rho_0}}{m_{\text{Pl}}}$, leading to a very weak bound on

$$m_\phi \lesssim \left(\frac{v^4 \Lambda_c^4}{e^4 m_{\text{Pl}} \rho_0^{1/2}}\right)^{1/5}, \tag{111}$$

which is always satisfied easily for small $m_\phi \lesssim 1$ eV. Even in the early universe where the constraint on a very slow decay rate is obtained by substituting $\rho_0 \to \rho$, we find that the decay is essentially nonexistent. This confirms that dark matter for these models is stable.

VI. CONCLUSION

If we do not take into account the massive sector of neutrinos, the SM of particles and interactions only has one term with an explicit dimensional parameter. It is the μ term, which determines the vacuum expectation value of the Higgs field at low energies. In this work, we have studied the phenomenology associated with a dynamical μ term related to a new scalar degree of freedom. This field is coupled through the energy-momentum tensor of the matter content and can be identified with a dilaton associated with the conformal symmetry breaking of the theory in the matter sector only. We have discussed this framework by assuming a Higgs singlet and modeling the matter content with a unique fermion field. Interestingly, when the scalar sector of the model is stabilized at its fundamental state, the linear coupling of the dilaton to matter disappears. This fact provides a very distinctive phenomenology for this new scalar degree of freedom, whose main coupling is quadratic. Another feature that we have explicitly discussed in this work is the stability against radiative corrections of this model.

After analyzing the main theoretical characteristics of this dilaton model, we have studied its cosmological evolution. The cosmological evolution leads the dilaton to lie close to the minimum of its potential, and the corresponding oscillations are described by an harmonic approximation. In this limit, the energy-momentum tensor of the dilaton behaves as DM if its value can be averaged over many oscillations. In fact, this is what happens, and we have found explicitly that this dilaton can be a candidate

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[6]The coupling α_A in the unscreened case is modified and is not given by (70) anymore [48]. It becomes $\alpha_A = Q_A \beta$ where $Q_A = -\frac{16\pi^2 \alpha_G g_3^2}{b_3} + (0.093 + [Q'_A]_{\hat{m}} + [Q'_A]_{m_e} + [Q'_A]_{\delta m})(1 + \frac{16\pi^2 \alpha_G g_3^2}{b_3})$.

for DM. As we have commented, the dilaton is coupled quadratically to matter, so it is stable. In addition, the strength of its coupling is typically suppressed, which means that it does not thermalize for a broad range of the parameter space of the model.

Finally, we have studied the phenomenological signatures of the model. The quadratic coupling effectively couples matter to the dilaton in a composition-dependent way. We have explored signals related to experiments measuring violations of the equivalence principle aboard satellites such as the MICROSCOPE experiment and the future generation of tests related to this signature. We have found that the quadratic coupling provides naturally a type of screening mechanism similar to those studied for string motivated frameworks or symmetron models [43–46]. For masses below the electron volt, the Sun and other planetary objects are not screened. Nevertheless, post-Newtonian parameter tests are easily fulfilled due to the weakness of the strength of the effective linear coupling, even in the Solar System. This makes the dilaton invisible. Visibility would be granted if the dark matter density were locally much larger such as in a dark matter clump. Such clumps could result from the balance between the quantum pressure and the attraction due to the negative quartic interaction of the dilaton close to the electroweak minimum. The study of this possibility is left for future work.

APPENDIX A: A POSSIBLE MODEL

We consider a model with a massive field X of large mass m coupled to the Higgs field according to the Lagrangian

$$\mathcal{L} = -g'(mX - uH^2)^2 - \lambda' X \bar{\psi}_H \psi_H, \qquad (A1)$$

where the fermions ψ_H are charged under a gauge group which condenses in a similar way to QCD. At energies $E \ll m$, we can integrate out the field X according to

$$X = \frac{uH^2}{m}, \qquad (A2)$$

implying the Higgs field couples to the gauged fermions as

$$\mathcal{L} = -\lambda' u \frac{H^2}{m} \bar{\psi}_H \psi_H. \qquad (A3)$$

Assuming that the fermions condense according to

$$\langle \bar{\psi}_H \psi_H \rangle = \Lambda_H^3 e^{i\frac{\pi}{2}\frac{\phi}{f}}, \qquad (A4)$$

at a scale $\Lambda_H \ll m$, we have at low energy a potential term

$$\mathcal{L} = -2\lambda' u \frac{\Lambda_H^3}{m} H^2 \cos \frac{\pi}{2} \frac{\phi}{f}, \qquad (A5)$$

of the type used in the main text. We will assume that this transition happens before the end of inflation so that the cos term is realized as soon as the field evolves at the end of inflation.

APPENDIX B: SLOW ROLLING IN THE RADIATIVE ERA

Initially, when the field is released at the end of inflation, the field starts moving slowly. Let us look for a simplified solution of the Klein-Gordon equation in this regime. We assume that in this slow roll regime we have

$$H\dot{\phi} = \alpha V', \qquad (B1)$$

where α is nearly constant. This is what would happen in the slow roll regime during inflation although here α will not be equal to $-1/3$. Using this ansatz and identifying $m^2 = V''$ we get

$$\frac{\ddot{\phi}}{H\dot{\phi}} = \alpha \frac{m^2}{H^2} - \frac{\dot{H}}{H^2}, \qquad (B2)$$

and $\dot{H} = -\frac{3(1+\omega)}{2} H^2$ where the equation of state in the radiation era is $\omega = 1/3$. Now using the Klein-Gordon equation $\ddot{\phi} + 3H\dot{\phi} + V' = 0$ we obtain

$$\frac{m^2}{H^2}\alpha^2 + \frac{9 + 3\omega}{2}\alpha + 1 = 0. \qquad \text{(B3)}$$

This implies that

$$\alpha = \frac{H^2}{m^2}\left(-\frac{9 + 3\omega}{4} + \frac{1}{2}\sqrt{\left(\frac{9 + 3\omega}{2}\right)^2 - 4\frac{m^2}{H^2}}\right). \qquad \text{(B4)}$$

Obviously, this is only valid when m/H is small enough and varies very slowly. In the slow roll regime as well as having initially $m/H_{\text{inf}} \ll 1$ implies that

$$\alpha \simeq -\frac{2}{9 + 3\omega} = -\frac{1}{5}. \qquad \text{(B5)}$$

When the equation of state is close to -1 during inflation, we retrieve that slow roll is realized with $\alpha = -1/3$. During radiation domination the slow roll evolution is governed by

$$\frac{dx}{d\tau} = -\frac{4\alpha}{\pi}\tau\left(1 - \epsilon - \sin\frac{\pi}{2}x\right), \qquad \text{(B6)}$$

where we have denoted $\tau = m_0 t = \frac{m_0}{2H}$ and the potential reads

$$V(\phi) = \left(\frac{2}{\pi}\right)^2 m_0^2 f^2\left(\frac{\pi}{2}(1 - \epsilon)(1 - x) - \cos\frac{\pi}{2}x\right). \qquad \text{(B7)}$$

This differential equation (B6) fails when $\tau = \mathcal{O}(1)$.

The solution to (B6) with the initial condition $x(\tau = \tau_i) \simeq x_e$ where the slow roll regime starts is given by

$$x(\tau) = \frac{4}{\pi}\arctan\left(\frac{1 - \sqrt{2\epsilon - \epsilon^2}X}{1 - \epsilon}\right), \qquad \text{(B8)}$$

where

$$X = \frac{a + b}{1 - ab}, \qquad \text{(B9)}$$

and

$$a = \tanh\left(\frac{\alpha}{2}\sqrt{2\epsilon - \epsilon^2}(\tau^2 - \tau_i^2)\right),$$
$$b = \frac{1 - (1 - \epsilon)\tan\frac{\pi x_e}{4}}{\sqrt{2\epsilon - \epsilon^2}}. \qquad \text{(B10)}$$

The field starts moving when $\tau \sim 0.5$ corresponds to $H \simeq m_0$.

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