Toward a UV model of kinetic mixing and portal matter. III. Relating portal matter and right-handed neutrino masses

Thomas G. Rizzo*

SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA

(Received 14 March 2023; accepted 24 April 2023; published 9 May 2023)

The kinetic mixing (KM) of a dark photon (DP) with the familiar one of the Standard Model (SM) requires the existence of a new set of fields, called portal matter (PM), which carry both SM and dark sector quantum numbers, some of whose masses may lie at the TeV scale. In the vanilla KM model, the dark gauge group is just the simple $G_{\rm Dark} = U(1)_D$ needed to describe the DP while the SM gauge interactions are described by the usual $G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$. However, we need to go beyond this simple model to gain a better understanding of the interplay between $G_{\rm SM}$ and $G_{\rm Dark}$ and, in particular, determine how they both might fit into a more unified construction. Following our previous analyses, this generally requires $G_{\rm Dark}$ to be extended to a non-Abelian group, e.g., $SU(2)_I \times U(1)_{Y_I}$, under which both the PM and SM fields may transform nontrivially. In this paper, also inspired by our earlier work on top-down models, we consider extending the SM gauge group to that of the left-right symmetric model (LRM) and, in doing so, through common vacuum expectation values, link the mass scales associated with the breaking of $G_{\rm Dark} \to U(1)_D$ and the PM fields to that of the RH-neutrino as well as the heavy gauge bosons of the LRM. This leads to an interesting interplay between the now coupled phenomenologies of both visible and dark sectors at least some of which may be probed at, e.g., the LHC and/or at the future FCC-hh.

DOI: 10.1103/PhysRevD.107.095014

I. INTRODUCTION AND BACKGROUND

Although strong evidence for dark matter (DM) is known to exist over many length scales, its fundamental nature remains a great mystery. In particular, the answer to the question as to just how or if DM may interact with the fields of the Standard Model (SM), apart from via the obvious gravitational interactions, is most pivotal in our attempt to understand how the DM may have achieved the relic density determined by Planck [1]. More than likely, some new non-SM force(s) must exist to help achieve this result and one might ask how such new forces and the familiar ones of the SM may be related (if at all) and if some unified interaction framework might be contemplated.

Of course these are not new questions and the searches for the "traditional" DM candidates, such as the QCD axion [2–4] and weakly interacting massive particles, i.e., WIMPS [5,6], continue to push deeper and wider into parameter space with ever greater sensitivities. So far, however, these searches by direct or indirect detection experiments, as well as those at the LHC [7–11], have produced negative results, thus excluding an ever growing region of the corresponding allowed model space. Over the last few years, the long wait for convincing axion and/or WIMP signatures has led to an ever expanding set of new ideas for the nature of DM and its interactions with the SM. In particular, it is now clear that both DM masses and coupling strengths to (at least some of) the fields of the SM can both span extremely large ranges [12–17] thus requiring a wide variety of very broad and very deep searches. In addition, the types of interactions that are possible between the SM and DM fields have also been found to be quite numerous and a very useful classification tool to describe these potential structures is via renormalizable (i.e., dimension ≤ 4) and nonrenormalizable (i.e., dimension >4) "portals." This approach posits the existence of a new set of mediator fields which link the SM to the DM and also possibly to an enlarged, potentially complex, dark sector of which the DM itself is its lightest, stable member due to the existence of some new at least approximately conserved quantum number.

Of the various portals, one that has gotten significant attention in the literature due to its parameter flexibility is the renormalizable kinetic mixing (KM)/vector portal [18–20] scenario which is based upon the existence of a new dark gauge interaction. One finds that in such a setup, even in its simplest manifestation and for a suitable range of parameters, that this scenario allows DM to reach its measured abundance via the usual WIMP-like thermal

*rizzo@slac.stanford.edu

freeze-out mechanism [21,22]. However, this now occurs for sub-GeV DM masses by employing this new non-SM dark gauge interaction that so far have evaded detection. This simplest and most familiar of these manifestation assumes only the existence of a new $U(1)_D$ gauge group, with a gauge coupling g_D, under which the SM fields are neutral, thus carrying no dark charges, i.e., having $Q_D = 0$, and where the new $U(1)_D$ gauge boson is referred to as the "dark photon" (DP) [23,24], which we will henceforth denote by V or A_I depending on context. As noted, to obtain the observed relic density by thermal means, this new $U(1)_D$ is usually assumed to be spontaneously broken at or below the ∼few GeV scale so that both the DM and DP will have comparable masses. This symmetry breaking is usually accomplished via the (sub-)GeV scale vev(s) of at least one new scalar, the dark Higgs, in complete analogy with the symmetry breaking occurring in the SM. Within such a framework, the interaction between the SM and the dark sector is generated via renormalizable kinetic mixing (KM) at the 1-loop level between the $U(1)_D$ and the SM $U(1)_Y$ gauge fields. The strength of this KM interaction is then described by a small, dimensionless parameter, ϵ. For this mixing to occur, these loops must arise from a set of new matter fields, usually being vectorlike fermions (and/or complex scalars), here called portal matter (PM) [25–37], that, unlike the SM fields, carry *both* SM hypercharge (plus other model-dependent SM quantum numbers) as well as a $U(1)_D$ dark charge. Subsequent to field redefinitions that bring us back to canonically normalized fields, after both the SM and $U(1)_D$ gauge symmetries are broken, and further noting the large ratio of the resulting Z to V masses, this KM leads to a coupling of the DP to SM fields of the form $\simeq e\epsilon Q_{\rm em}$, up to correction terms or order $m_V^2/m_Z^2 \ll 1$. The size of the parameter ϵ is constrained by phenomenology to very roughly lie in the $\epsilon \sim 10^{-(3\text{–}4)}$ range given the DM/DP lies within the sub-GeV mass region that leads to the thermal DM freeze-out mechanism of interest to us here. One also finds that in such a setup, for p–wave annihilating DM or for pseudo-Dirac DM with a sufficient mass splitting, the rather tight constraints arising from the CMB can also be rather easily avoided [1,38–40] for a similar range of parameters.

In the conventionally chosen normalization [18,19], with $c_w = \cos\theta_w$, ϵ can be expressed in terms of the properties of the PM fields themselves appearing in these vacuum polarization graphs and is given by the sum

$$\epsilon = c_w \frac{g_D g_Y}{24\pi^2} \sum_i \eta_i \frac{Y_i}{2} N_{c_i} Q_{D_i} \ln \frac{m_i^2}{\mu^2}, \qquad (1)$$

where $g_{Y,D}$ are the $U(1)_{Y,D}$ gauge couplings and $m_i(Y_i, Q_{D_i}, N_{c_i})$ are the mass (hypercharge, dark charge, number of colors) of the ith PM field. Here, we note that $\eta_i = 1(1/2)$ if the PM particle is a chiral fermion (complex scalar) and the SM hypercharge is here normalized so that the electric charge is given as $Q_{\rm em} = T_{3L} + Y/2$. It is important to note that in a somewhat more complex scenario where this effective theory is embedded into a broader UV-complete setup, such as we will describe below, this same group theory requires that the sum (for fermions and scalars separately)

$$\sum_i \eta_i \frac{Y_i}{2} N_{c_i} Q_{D_i} = 0, \qquad (2)$$

so that one finds that ϵ is both finite and, if the PM masses and couplings were also known, completely determined within the more fundamental underlying model.

Clearly it is advantageous to go beyond this rudimentary effective theory to further our understanding of how this (apparently) simple KM mechanism fits together in a single picture with the SM, something that we have begun to examine in pathfinder mode employing various bottom-up and top-down approaches in a recent series of papers [25,26,28–35]. Two specific features of our general framework are the extension of the $U(1)_D$ dark Abelian symmetry to, e.g., the non-Abelian, SM-like $G_{\rm Dark} = SU(2)_I \times U(1)_{Y_I}$ [26] gauge symmetry [41] and the appearance of at least some of the SM fields in common $SU(2)_I$ representations with the PM fields. In such setups, the PM masses are themselves generally the result of the $G_{\rm Dark} \to U(1)_D$ symmetry breaking and so, with $O(1)$ Yukawa couplings, will share a similar overall scale with the new, heavy, $Q_{\rm em} = 0$, gauge bosons, denoted by $W_I^{(\dagger)}$, Z_I, associated with the broken group generators. This was seen quite explicitly in Ref. [26] whose PM content and the $G_{\rm Dark} = SU(2)_I \times U(1)_{Y_I}$ gauge group were both inspired by E_6 [42,43]. One can also consider classes of models wherein the SM gauge group, $G_{\rm SM}$, is itself extended as was suggested by the top-down analysis in Ref. [34]. In any such setup where the SM lepton doublets and PM fields are found to lie in common representations of that group, it is easily imagined that there may exist a possible relationship between the seesaw mass scale that is responsible for generating small Majorana neutrino masses and that associated with the breaking of $G_{\rm Dark}$ down to $U(1)_D$ and producing the PM masses. This is perhaps most easily realized in scenarios loosely described by the product of gauge groups $G = G_{\rm SM} \times G_{\rm Dark}$ under which the DM is a $G_{\rm SM}$ singlet and $G_{\rm SM}$ already naturally gives rise to a seesaw mass structure. The most simple, obvious and familiar example of such a possibility is to consider identifying the augmented $G_{\rm SM}$ with the Left-Right Symmetric Model (LRM) [44–48] wherein the usual SM is extended to $G = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which also has other advantages, e.g., in making it easier to satisfy anomaly constraints and in obtaining a finite and calculable value of ϵ as described above. This is the scenario that we will consider below. As in earlier work, we will employ $G_{\rm Dark} = SU(2)_I \times U(1)_{Y_I}$ [26] as the simplest non-Abelian example which can contain an unbroken $U(1)_D$ and which allows for $Q_D = 0$ SM fields to lie in common representations with PM fields which must have $Q_D \neq 0$. Of course, this choice is hardly unique but its simplicity allows us to more clearly see the relationship between the right-handed neutrino/seesaw mass scale and that of PM, where $G_{\rm Dark}$ also breaks.

In order to satisfy our various model building requirements, it is far simpler (and more easily UV-completed) to base our model on a simplified, single-generation version of the one appearing in Ref. [29] (since here we will not be addressing any flavor issues) in the following discussion. Specifically, we'll be considering an extended version of the Pati-Salam(PS)-left-right model (LRM) [44] augmented by a non-Abelian dark sector gauge group as discussed in Ref. [29], i.e., $G = SU(4)_c \times SU(2)_L \times SU(2)_R \times SU(2)_I \times U(1)_{Y_I}$, which we will denote for brevity as $4_c 2_L 2_R 2_I 1_{Y_I}$.[1] Here it will be assumed that the breaking of $SU(4)_c \to SU(3)_c \times U(1)_{B-L}$ occurs at a very large mass scale, $M_c \gtrsim 10^6$ TeV or even at the unification scale, so that although it determines the initial representation structure of the fermion sector necessary for anomaly cancellation, etc., it will not have any phenomenological impact on the discussion that follows below [49]. This setup then implies that at accessible energy scales below ∼10's of TeV, the effective gauge group for our discussion is actually just $G_{\rm eff} = 3_c 1_{B-L} 2_L 2_R 2_I 1_{Y_I}$. In such a framework, it is now the two $U(1)$'s, $1_{B-L} 1_{Y_I}$, which will undergo KM. While this KM will still manifest itself as a DP which (weakly) couples like the SM photon at the $\lesssim 1$ GeV mass scale, additional coupling terms to SM fields of various kinds will also be present due to, e.g., mass mixing from the several steps of symmetry breaking which are necessary before the $\lesssim 1$ GeV scale is reached and due to the fact that some of the Higgs fields with vacuum expectation values (vevs) will carry both SM and nonzero values of Q_D. As we will see below, to maintain anomaly freedom in such a setup, *every* SM field is now accompanied by two sets of PM fields with similar SM electroweak quantum numbers, which will together form a complete SM/LRM vectorlike family, but whose members will transform differently under $G_{\rm Dark}$. As we will see in the analysis that follows, the masses of the PM fields, the mass of the right-handed neutrino and the breaking scale for all of the new heavy gauge bosons will all become correlated, intertwining the physics of the LRM and dark sector gauge groups and leading to a complex phenomenological structure some of whose implications we will begin to study below. For example, we will find that both of the heavy neutral Dirac PM fields in the model will be split into pairs of pseudo-Dirac states via Majorana mass terms arising from some of the same Higgs fields that are responsible for LRM-like neutrino mass generation.

[1] As we will see below, we will on some occasions refer to the product $2_L 2_R$ as simply $2_1 2_2$ whenever we need to avoid confusion with respect to the fermion assignments.

The outline of this paper is as follows: Following the present Introduction and Background discussion, in Sec. II a broad outline of the model framework will be presented to set the overall stage for the analysis that follows. Section III will then individually examine the various sectors of this setup, i.e., the generation of the Dirac and Majorana fermion masses together with the corresponding mixings between the PM and SM/LRM fermion fields. The KM and gauge symmetry breaking which takes place in several distinct steps at a hierarchy of mass scales and resulting gauge boson masses and mixings that will be important at the electroweak scale and below will then be discussed. An examination of a sample of some of the phenomenological implications and tests of this scenario will also be presented along the way throughout this section as part of the model development, although much of this model still remains to be explored in future work. A summary, a discussion of our results, possible future avenues of exploration and our subsequent conclusions can then be found in Sec. IV.

II. MODEL SETUP AND FRAMEWORK

For our study below, the specific model building requirements will be taken to be as follows: (i) Due to the dual quark-lepton and left-right symmetries of the Pati-Salam setup, all of the SM fermions will have the need of PM partners. (ii) The PM fermions, though vectorlike with respect to the SM/LRM gauge groups, should obtain their masses at the $G_{\rm Dark}$ and/or the $G_{\rm LRM}$ breaking scale. The combination of (i) and (ii), in fact, implies that there are now *two* distinct PM chiral partners, together forming a single vectorlike fermion, for each SM field as we will see below. (iii) With an eye toward a possible unification in an even larger gauge structure, this setup must be automatically anomaly-free and yield a finite and calculable value for ϵ as described above. These conditions follow automatically from the discussion in Ref. [29] when the additional family symmetry group is suppressed as will be the case below. Some additional constraints associated with the symmetry breaking hierarchy will be subsequently encountered as we move forward with our discussion.

In terms of the $4_c 2_L 2_R 2_I 1_{Y_I}$ gauge groups discussed above and denoting the quantum numbers of the fields by $(4_c 2_L 2_R 2_I)_{Y_I/2}$, a single fermion generation, here denoted by $\mathcal{F}$, will consist of the following set of fields [29]:

$$\mathcal{F} = A(4,2,1,2)_{-1/2} + B(4,1,2,2)_{-1/2} + C(4,2,1,1)_{-1} + D(4,1,2,1)_{-1}, \quad (3)$$

and, recalling that under the breaking $SU(4)_c \to SU(3)_c \times U(1)_{B-L}$ at the very large mass scale, M_c, assumed here, one has $4 \to 3_{1/3} + 1_{-1}$. Thus we see that while the familiar SM fermions and the RH-neutrino, $f_{L,R}$, which form the usual LRM doublets under $SU(2)_{L(R)}$, lie in the representations A, B, additional vectorlike (with respect to the SM/LRM) fermions, here denoted as $F_{L,R}$, $F'_{L,R}$, are also present. While $(f,F)^T_{L,R}$ combine to form the 2_I doublets, A, B, respectively, F'_R and F'_L, are both 2_I singlets that form the corresponding representations C and D. It is important to note that F'_R is a $2_1(=2_L)$ doublet and a $2_2(=2_R)$ singlet while the reverse is true for F'_L. It is this subtle and somewhat unusual fermion assignment which prevents us from completely casually referring to $2_1 2_2$ as $2_L 2_R$ in the usual manner, though we will with some caution mostly employ this later notation to make contact with the traditional LRM setup as long as the careful reader is always mindful of the subtleties involved. Within this framework, we see explicitly that the SM fermion fields will all have $Q_D = 0$ while the PM fermions will all have $Q_D = -1$. In such a setup with, e.g., $SU(2)_I$ acting vertically and $SU(2)_L$ acting horizontally, the LH SM fermions will appear as bidoublets with their unprimed PM partners, e.g.,

$$\begin{pmatrix} \nu_L & e_L \\ N_L & E_L \end{pmatrix}, \qquad \begin{pmatrix} u_L & d_L \\ U_L & D_L \end{pmatrix}, \tag{4}$$

and similarly for the RH states, while the primed PM states will be appear purely "horizontal" as they are $SU(2)_I$ singlets but $SU(2)_L$ or $SU(2)_R$ doublets, e.g., $(N',E')_{R,L}$, respectively (note the flipped helicities), and $(U',D')_{R,L}$. It is important to note that while transforming quite differently under $G_{\rm Dark}$, with the caveats mentioned above, the (chiral) fields f, F and F' will all have somewhat similar transformation properties under the $3_c 2_L 2_R 1_{B-L}$ combination of gauge groups since F, F' together will form a SM vectorlike copy of the chiral fermion, f.

As seen above, at the level of $G_{\rm eff} = 3_c 1_{B-L} 2_L 2_R 2_I 1_{Y_I}$, it will be the two $U(1)_{B-L}$ and $U(1)_{Y_I}$ Abelian gauge fields which will undergo KM due to the now familiar PM loops. Denoting the $B-L$ and Y_I kinetically mixed gauge field strengths as $\tilde{B}_{\mu\nu}$, $\tilde{D}_{\mu\nu}$, respectively, the KM piece of the Lagrangian for the two $U(1)$'s above the $2_R 2_I$ breaking scales, $M_{R,I}$, can be written as

$$\mathcal{L}_{\rm KM} = -\frac{1}{4}\tilde{B}^2_{\mu\nu} - \frac{1}{4}\tilde{D}^2_{\mu\nu} + \frac{\sigma}{2}\tilde{B}_{\mu\nu}\tilde{D}^{\mu\nu}, \tag{5}$$

where the dimensionless parameter, $\sigma \sim 10^{-(3-4)}$, describes the strength of the KM. Below, we will connect this parameter with the familiar ϵ one of similar magnitude which describes the KM of the DP with the SM photon at low energy scales. Employing similar notation to the above, one finds that σ is given by

$$\sigma = \frac{g_{B-L} g_{Y_I}}{24\pi^2} \sum_i \eta_i \frac{Y_{I_i}}{2} \frac{(B-L)_i}{2} N_{c_i} \ln \frac{m_i^2}{\mu^2}, \tag{6}$$

with g_{B-L,Y_I} being the $U(1)_{B-L,Y_I}$ gauge couplings with their associated quantum numbers and that, correspondingly, the requirement

$$\sum_i \eta_i \frac{Y_{I_i}}{2} \frac{(B-L)_i}{2} N_{c_i} = 0, \tag{7}$$

so that the requirement that σ is finite and calculable is indeed found to be satisfied for the fermion content of the setup above. We will later see that this remains true once the scalar degrees of freedom are included below as these are just "products" of the above fermion representations and so will just have the quantum numbers which are either sums and differences of those of the $A-D$ fermion fields which themselves lead to a finite σ. As is usual, since the KM parameter (in this case σ) is expected to be so small, we can safely work to linear order in this parameter *most* of the time and so we observe that the KM above is removed by the familiar field redefinitions $\tilde{B}_{\mu\nu} \to B_{\mu\nu} + \sigma D_{\mu\nu}$, $\tilde{D}_{\mu\nu} \to D_{\mu\nu}$ and leads to the following interaction structure (in obvious notation)

$$g_{B-L}\frac{B-L}{2}B_\mu + \left(g_{Y_I}\frac{Y_I}{2} + \sigma g_{B-L}\frac{B-L}{2}\right)D_\mu, \tag{8}$$

where B, D here are simply the associated canonically normalized gauge fields, and which then will appear as one of the pieces of the covariant derivative.

For the neutral, Hermitian fields (apart from QCD which remains exactly as in the SM), the part of the covariant derivative describing interactions can be suggestively written in the familiar $G_{\rm SM/LRM} \times G_{\rm Dark}$ (from Ref. [26]), but not-quite mass eigenstate, basis as (suppressing Lorentz indices)

$$\begin{aligned}\mathcal{L}^h_{\rm int} = {} & eQA + \frac{g_L}{c_w}(T_{3L} - xQ)Z + \frac{g_L}{c_w}[\kappa^2 - (1+\kappa^2)x]^{-1/2} \\ & \times (xT_{3L} + \kappa^2(1-x)T_{3R} - xQ)Z_R \\ & + \frac{g_I}{c_I}(T_{3I} - x_I Q_D)Z_I + g_D Q_D A_I \\ & + \sigma\lambda g_L \frac{B-L}{2}(c_I A_I - s_I Z_I),\end{aligned} \tag{9}$$

with $Q = Q_{\rm em}$ and, more suggestively, with the replacement $V \to A_I$ to further heighten the analogy to the SM. Here, we have introduced the usual SM relationship $e = g_L s_w$ with $s_w(c_w) = \sin\theta_w(\cos\theta_w)$, etc., as well as the abbreviations $x = x_w = s_w^2$, $\kappa = g_R/g_L$ and also

$$\lambda^2 = \frac{\kappa^2 x}{\kappa^2 - (1+\kappa^2)x} = \kappa^2 x \Omega^{-2}, \tag{10}$$

so that $g_{B-L} = g_L\lambda$; note the constraint arising from the requirement of real couplings in that κ is bounded from below, i.e., $\kappa^2 > x/(1-x) = t_w^2$ [50] so that $\kappa \gtrsim 0.55$.

Similarly, in close analogy to the SM and as we have employed in earlier work [26], we have also defined $g_D = g_I s_I = e_I$, with s_I being the analog of s_w, etc., to be the $U(1)_D$ gauge coupling of the light DP, together with $x_I = s_I^2$ and $Q_D = T_{3I} + Y_I/2$ as the usual dark charge to which the DP couples, again simply completely paralleling the SM case. We note that Z_I in this setup couples universally, i.e., independent of flavor or generation, for all choices of f, F and F', before the effects of fermion mixing are included as we will discuss below. The last term in the above expression is the one arising from the KM $\sigma g_{B-L} \frac{B-L}{2} D$ coupling term in Eq. (8) above but is now written here more suggestively in terms of the Z_I, A_I fields. Similarly, the interactions of the *non-Hermitian* gauge bosons, $W_{L,R,I}$, are controlled by the gauge group coupling structure which in this same approximate mass eigenstate basis is given by

$$\mathcal{L}_{\rm int}^{nh} = \frac{g_L}{\sqrt{2}}(T_L^+ W + {\rm H.c.}) + (L \to R) + \frac{g_I}{\sqrt{2}}(T_I^+ W_I + {\rm H.c.}), \tag{11}$$

where $T_{L,R,I}^{+(-)}$ are the corresponding isospin raising(lowering) operators for the $SU(2)_{L,R,I}$ gauge groups, respectively.

III. ANALYSIS AND PHENOMENOLOGY

To go further, we must address how the various symmetries are broken and how the corresponding gauge bosons and fermions obtain their different masses. In this construction, ignoring M_c, there are (at least) 3 distinct, widely separated mass scales. At the highest mass scales, $\gtrsim 10$'s of TeV, the LRM must break down to the SM (at M_R) and also $G_{\rm Dark} \to U(1)_D$ (at M_I), both of which may be related as we will discuss below. At the ~ 100 GeV scale, the SM undergoes the familiar electroweak symmetry breaking while $U(1)_D$ itself breaks at low energies, i.e., ~ 1 GeV or below. Note the hierarchy of roughly a factor of $\sim 10^2$ between these three scales so that it is reasonable to consider them somewhat sequentially and we note that Q_D will be a conserved quantity until quite low scales are reached. Thus the vevs of the Higgs fields which are *mainly* responsible for the first two symmetry breaking steps can only arise from neutral scalar multiplet members also having $Q_D = 0$. This will be important to remember in the following discussion.

A. Dirac fermion masses

The quantum numbers of the active set of Higgs fields, H_{1-4}, all assumed to be color singlets and having $B - L = 0$, that are needed to generate the various Dirac fermion masses are easily obtained by taking appropriate products of the fermion representations $A - D$ above and can be expressed, i.e., via the Yukawa couplings which generalizes the usual LRM structure as:

$$\mathcal{L}_{\rm Dirac} = \bar{A}_L B_R (y_1 H_1 + \tilde{y}_1 \tilde{H}_1) + y_2 \bar{A}_L C_R H_2 + y_3 \bar{D}_L B_R H_3 + \bar{D}_L C_R (y_4 H_4 + \tilde{y}_4 \tilde{H}_4) + {\rm H.c.} \tag{12}$$

where as usual $\tilde{H}_i = i\sigma_2 H_i^* \sigma_2$, with σ_2 being the Pauli matrix, and where the y_i, $\tilde{y}_i$ are Yukawa couplings, so that the H_i's $(2_L, 2_R, 2_I)_{Y_I/2}$ quantum numbers can be easily chosen to be

$$H_1(2,2,1)_0, \quad H_2(1,1,2)_{1/2}, \quad H_3(1,1,2)_{-1/2}, \quad H_4(2,2,1)_0. \tag{13}$$

Note that we will not necessarily impose any P, C or CP symmetries as might be the case in the usual LRM on these Yukawa couplings in the discussion below but for simplicity alone we will assume that all of the couplings and vevs are real. For the immediate discussion, we will focus ourselves only on the Higgs vevs which do *not* break $U(1)_D$ and so all correspond to $Q_D = 0$ components of the H_i. The effects of any small additional terms due to possible $Q_D \neq 0$ vevs can be added later on as a perturbation upon those which we will now discuss as these are relatively quite highly suppressed by factors of (at least) 10^2. Note that we will treat H_1 and H_4, which are typical LRM bidoublets, as distinct fields and we will not take H_2 and H_3 to be the complex conjugates of each other so that they too are also unrelated fields. Note further that the two vevs contained in each of $H_{1,4}$ are of the electroweak scale while the single vev in each of $H_{2,3}$ will be at the ~ 10 TeV scale or so and these will lead to the breaking of $SU(2)_I \times U(1)_{Y_I} \to U(1)_D$ as will be discussed later below.

Denoting the generic set of weak eigenstate fermion fields as $\mathcal{F}_{L,R}^0 = (f, F, F')_{L,R}^0$ in the notation employed above, the vevs within the H_i will then generate a 3×3 mass matrix of the form

$$\bar{\mathcal{F}}_L^0 \mathcal{M} \mathcal{F}_R^0, \tag{14}$$

whose entries will depend upon the location of the $Q_D = 0$ elements within the various Higgs representations and which can be diagonalized as is usual by a biunitary transformation

$$\mathcal{M}_D = U_L \mathcal{M} U_R^\dagger. \tag{15}$$

As noted, ignoring the possibility of CP-violation, etc., we can for simplicity take the elements of $\mathcal{M}$ to be real so that this 3×3 matrix can be *symbolically* (as the $2_L 2_R$ subspace itself does not appear here) written, after absorbing the various Yukawa couplings into the vevs for brevity, as

$$\mathcal{M} \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 & 0 \\ 0 & v & \Lambda \\ 0 & \Lambda' & v' \end{pmatrix}, \tag{16}$$

where v, v' represent generic weak scale vevs $\sim$100 GeV, arising from H_1 and H_4, respectively, and Λ, Λ' represent vevs at the $\sim$10 TeV scale, arising from H_2 and H_3, respectively. To clarify, it should be recalled that since both $H_{1,4}$ are standard bidoublets in the $2_L 2_R$ subspace, v and v' here are both just symbolic 'projections' of the familiar electroweak scale vevs, $k_{1,2}$ and $k'_{1,2}$, that we would usually encounter in the ordinary LRM, where we would instead write the vevs of $H_{1,4}$, now in the $2_L 2_R$-subspace language as[2]

$$\langle H_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}, \quad \langle H_4\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} k'_1 & 0 \\ 0 & k'_2 \end{pmatrix}, \tag{17}$$

which, given the coupling structure above, would allow, e.g., different masses for up- and down-type quarks. Thus v, v' can just be thought of as symbolic appropriate linear combinations of the k_i and k'_i, respectively, depending upon the $2_L 2_R$ transformation properties of the relevant fermion field. Next, we can easily determine the biunitary transformations needed to diagonalize this matrix, which here are just (almost) essentially rotations, $U_{L,R}$, via the standard relation

$$\mathcal{M}_D^2 = U_L^\dagger \mathcal{M}\mathcal{M}^\dagger U_L = U_R^\dagger \mathcal{M}^\dagger \mathcal{M} U_R, \tag{18}$$

where $\mathcal{M}_D$ is the resulting diagonal mass matrix. From the form of $\mathcal{M}$, and the corresponding products with its Hermitian conjugate, it can be seen that the generated mixing at this level of symmetry breaking lies totally within the $F - F'$ sector in a 2×2 sub-matrix and that the mass of the unmixed SM field, f, is just $\sim y_1(y'_1)v/\sqrt{2}$, as is similar to the LRM/SM.[3] With this and the assumptions we made above we can then replace U_L by a simple 2×2 rotation matrix, O_L, and U_R by a simple 2×2 rotation together with a discrete transformation, i.e., $U_R = O_R P$ where P is just

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{19}$$

and $O_{L,R}$ are each described by a single mixing angle, $\theta_{L,R}$, which are given in terms of ratios of the vevs v, v', Λ, Λ' by

$$\tan 2\theta_L = \frac{2(\Lambda v' + \Lambda' v)}{\Lambda^2 + v^2 - \Lambda'^2 - v'^2},$$
$$\tan 2\theta_R = \frac{2(\Lambda v + \Lambda' v')}{\Lambda^2 + v'^2 - \Lambda'^2 - v^2}, \tag{20}$$

both of which are of similar magnitude, $O(10^{-2})$, given the anticipated mass scales of the various vevs. These mixing angles can then be used to describe how the resulting mass eigenstate fermion fields, here termed f, $F_{1,2}$ at this stage of symmetry breaking, will interact with the many gauge bosons in the current setup, in particular, the heavy gauge fields associated with the broken $G_{\rm Dark} = SU(2)_I \times U(1)_{Y_I}$ generators. For completeness, we note that to leading order in the squared vev ratios $(v^2, v'^2)/(\Lambda^2, \Lambda'^2)$, the mass squared eigenvalues for $F_{1,2}$ are given by the expressions

$$m_{1,2}^2 \simeq \frac{1}{2}(\Lambda^2 + v^2 + \Lambda'^2 + v'^2) \pm \frac{1}{2}|(\Lambda^2 + v^2 - \Lambda'^2 - v'^2)| \pm \frac{(\Lambda v' + \Lambda' v)^2}{|\Lambda^2 - \Lambda'^2|}, \tag{21}$$

along the lines that we might have expected, i.e., that essentially $m_1 \simeq \Lambda$ while $m_2 \simeq \Lambda'$ up to few percent corrections. Note that before explicitly evaluating these expressions for the mixing angles and masses, however, we must appropriately restore all the suppressed Yukawa couplings, e.g., $v \to y_1(y'_1)v$, $\Lambda \to y_2\Lambda$, $\Lambda' \to y_3\Lambda'$, and $v' \to y_4(y'_4)v'$.

One very simple but important application of this mixing analysis is to, e.g., identify the PM mass eigenstates sharing the $SU(2)_I$ left- and right-handed doublets together with the SM fields, $f_{L,R}$, as these allow the PM states to decay via, e.g., $F_{1,2} \to fW_I$. This is easily done and one finds, defining $c(s)_{L,R} = \cos(\sin)\theta_{L,R}$, that

$$F_L = F_{1L}c_L - F_{2L}s_L,$$
$$F'_L = F_{2L}c_L + F_{1L}s_L, +(\mathrm{L} \to \mathrm{R}, F \Leftrightarrow F'). \tag{22}$$

Since the resulting $fF_{1,2}W_I$ couplings are nonchiral, one possible implication of this is that one-loop graphs can produce a significant effective dipole moment type interaction of the SM fermions with A_I at 1-loop that can have important implications for DM searches and associated phenomenology as was discussed via a toy example in Ref. [30] but here can realized in a more realistic fashion. Specifically, comparing with this earlier work, one finds the scale associated with these dipole couplings to be given by

$$\frac{1}{\Lambda_f} = \frac{\alpha_D}{32\pi s_I^2}\Sigma_i \frac{G(y_i)}{m_i}(v_i^2 - a_i^2), \tag{23}$$

where we have defined the mass squared ratio $y_i = m_i^2/m_{W_I}^2 \sim \mathrm{O}(1)$, the loop function $G(y)$ [which numerically is generally also O(1)] is given by

$$G(y) = 3y^2\left[\frac{-2(y-1) + (y+1)\ln(y)}{(y-1)^3}\right], \tag{24}$$

and the $v_i^2 - a_i^2$ factors can be directly obtained from the equations above, i.e., $v_1^2 - a_1^2 = 4c_L s_R$ and $v_2^2 - a_2^2 = -4c_R s_L$, respectively, so that the $F_{1,2}$ loop contributions

[2]Note that the overall factor of $1/\sqrt{2}$ here is associated with each of the vevs appearing in this mass matrix.

[3]Of course in actuality this is really just a weighted sum of the k_i or k'_i vevs.

relatively destructively interfere. For typical choices of the TeV scale PM and W_I masses and associated model parameters, one might then expect to obtain values for $\Lambda_f \sim 100$'s of TeV in the present setup, which is a phenomenologically interesting range.[4]

Also, it is interesting to note that since the 2 sets of fermions with $Q_D = -1$, $T_{3I} = -1/2, 0$ now mix, the $F_{1,2}$ states will also have off-diagonal, "flavor changing neutral current" -like couplings to the neutral Z_I gauge boson which will then propagate to the new gauge boson mass eigenstates that we will describe in more detail below.

Finally, as noted previously, it is important to recall that when/if at least some of the $Q_D \neq 0$ vevs that are possible in the H_i are turned on at much smaller scales below $\sim$1 GeV, the mixing as discussed above will be slightly perturbed. These new mass terms will be of order $\lesssim 10^{-2}(v, v')$ so will not alter the results obtained above very significantly in a numerical fashion *except* that they will generate $f - F_{1,2}$ mixing, which *is* phenomenologically important. In particular, we see that both $H_{2,3}$ can have such small, $\lesssim$1 GeV, vevs, $(\lambda, \lambda')/\sqrt{2}$, respectively, in obvious notation, that will directly couple the LH- and RH-handed components of f and F'. In the $f - F_1 - F_2$ basis obtained above, the resulting perturbed, now almost diagonal mass matrix will then appear as

$$\mathcal{M}_{D_P} \simeq \begin{pmatrix} m_f & a & b \\ a' & m_1 & 0 \\ b' & 0 & m_2 \end{pmatrix}, \tag{25}$$

where $(a, b) = y_2\lambda(c_R, -s_R)/\sqrt{2}$ and $(a', b') = y_3\lambda'(s_L, c_L)/\sqrt{2}$. Diagonalization of this matrix produces these $f - F_{1,2}$ mixings and also slight shifts (to lowest order in the small parameters) the fields such as $f_L \to f_L + aF_{1L}/m_1 + bF_{2L}/m_2$, $F_{1L} \to F_{1L} - af/m_1$, etc., so that A_I can now couple off-diagonally to the f and $F_{1,2}$ mass eigenstates in a generally parity violating, yet nonchiral manner, i.e.,

$$-\frac{g_D}{m_1}\bar{F}_1\gamma_\mu(aP_L + a'P_R)fA_I^\mu + (1 \to 2, a \to b) + \text{H.c.}, \tag{26}$$

which then allows for the dominant decay paths $F_{1,2} \to fA_I$. Recall that this decay mode is always found to be the most important one for the PM fields in comparison to other decay paths generated by such mixings for more conventional vectorlike fermions such as $F \to fZ, fH$ or $F \to f'W$. Although *all* of these decay modes are apparently suppressed by rather small mixing angles and/or mass ratios, the amplitude for the decay into the DP is also *enhanced* by large factors of $m_{1,2}/M_{A_I} \gg 1$ through the longitudinal couplings of the DP. Numerically, this enhancement can compensate rather completely for the presence of the small mixing angles. In particular, this is quantitatively similar to the results found in Refs. [25,26] in slightly different contexts and leads to rapid PM decays generated by via the application of the Goldstone boson equivalence theorem [51] applied in the scalar sector and/or the dominance of the longitudinal modes of the A_I (i.e., the equivalent of the Goldstone boson) since the PM fermion masses $m_{1,2}$ are so much larger than that of A_I itself. In the likely event that the DP appears in collider detectors as MET, the signatures for pair production of these PM states will then be observable pairs of SM states, e^+e^-, $\mu^+\mu^-$, $\bar{b}b$, $\bar{t}t$, etc, accompanied by this MET in a manner qualitatively similar to those employed in SUSY searches.[5]

It should be noted that W_I, $W_I^\dagger$ will also pick up a *diagonal* coupling to $\bar{f}f$ via this same tiny mixing, $\sim(a, a')/m_1 \sim 10^{-4}$, but which in this case is *not* offset by a large longitudinal enhancement in any decay process and so is not likely to be of much phenomenological relevance in, e.g., the single resonant production of W_I, $W_I^\dagger$ gauge bosons at colliders.

B. Neutral fermion Majorana masses

When we consider the $Q_{\text{em}} = 0$ leptonic components of $\mathcal{F}$, i.e., ν, N, N' in the weak basis, they can be 'self-coupled' in a manner such that these neutral, neutrinolike fields may all obtain Majorana masses from the vevs of suitably chosen Higgs scalars which will carry $|Q_D| = 0$, 1 or 2, e.g., via the Yukawa structure

$$\begin{aligned}\mathcal{L}_{\text{Majorana}} = {} & z_1^L\bar{A}_L^c i\sigma_2 A_L\Delta_L + z_2^L\bar{D}_L^c i\sigma_2 D_L\tilde{\Delta}_L \\ & + z_3^L\bar{A}_L^c i\sigma_2 D_L X_L + (A \to B, D \to C, L \to R) \\ & + \text{H.c.}\end{aligned} \tag{27}$$

where the $z_i^{L,R}$ are new Yukawa couplings, σ_2 is the Pauli matrix as above and $\Delta_{L,R}$, $\tilde{\Delta}_{L,R}$ and $X_{L,R}$ are the appropriate Higgs fields, whose active, color-singlet components that will concern us here will now all carry $|L| = 2$.[6] The quantum numbers of these Higgs representations in terms of $(2_L, 2_R, 2_I)_{Y_I/2}$ are easily seen to be just given by

[4]Note that in addition to the W_I contribution to Λ_f discussed here, there are also potential contributions arising from both CP-even and CP-odd Higgs scalar exchanges which can also yield results of a similar magnitude.

[5]For an overview of the current LHC PM search limits and future prospects, see Ref. [31]; current limits range from 0.9 to 1.5 TeV depending upon the PM flavor.

[6]Recall that $F'_{L,R}$ is a doublet under $SU(2)_{R,L}$ and a singlet under $SU(2)_{L,R}$.

$$\Delta_L(3,1,3)_1, \quad \tilde{\Delta}_L(3,1,1)_2, \quad X_L(2,2,2)_{3/2}, +(L \to R), \tag{28}$$

so that while $\Delta_{L(R)}$ and $\tilde{\Delta}_{L(R)}$ are $SU(2)_{L(R)}$ isotriplets, $X_{L,R}$ are bidoublets of $SU(2)_{L,R}$.[7] Further, $\Delta_{L,R}$ [$\tilde{\Delta}_{L,R}$] are $SU(2)_I$ triplets [singlets] while $X_{L,R}$ are $SU(2)_I$ doublets. Given these quantum numbers we see that the vevs of all of the neutral component fields contained in any of the $\tilde{\Delta}_{L,R}$ and $X_{L,R}$ will be associated with a *nonzero* value of Q_D thus will necessarily lead to a breaking of $U(1)_D$. Thus these vevs must be quite small, i.e., certainly $\lesssim 1$ GeV or so and will be ignored at the present stage of the discussion but will be returned to below. Meanwhile, only one component in each of $\Delta_{L,R}$ has both $Q_{\rm em} = Q_D = 0$ and so can obtain a vev without breaking $U(1)_D$ and these can be identified with the familiar triplet vevs, $v_{L,R}/\sqrt{2}$ ignoring potential phases, commonly appearing in the LRM. As is usual in that framework and for all the familiar reasons, e.g., ρ or oblique T parameter constraints [44–48], we will assume here that $v_L \ll v_R$ with v_R setting the breaking scale for the LRM, M_R. In fact, given such constraints, one might imagine that if v_L is nonzero, its maximum value cannot be too dissimilar from the various possible $Q_D \neq 0$ vevs we will later consider below. As above, we will not explicitly impose any P, C or CP symmetries on these vevs or Yukawa couplings but we will for simplicity of our discussion assume that all of them are real.

One difference between the current setup and the classic LRM scenario, however, is that both of these $Q_D = 0$ allowed vevs, $v_{L,R}$, arise from fields which are seen to also be $SU(2)_I$ triplets and as such these vevs will *also* lead to the breaking of $SU(2)_I \times U(1)_{Y_I} \to U(1)_D$. Comparing this to the discussion in the last subsection, we now observe that there are two limiting possibilities obtained by comparison of these multi-TeV scale vevs: if $v_R \gg \Lambda, \Lambda'$ then the $SU(2)_I \times U(1)_{Y_I} \to U(1)_D$ breaking scale is also set by v_R and so $M_I \simeq M_R$. However, if $v_R \ll \Lambda, \Lambda'$, then we instead find that $M_R \ll M_I$ and thus it must be so that $M_R \leq M_I$ is always satisfied in the present model setup. Generically, without tuning we might expect to end up in the middle of these two extremes so that all of these vevs are semiquantitatively comparable and we will treat them in all generality as such in the analysis that follows in the next subsection when we discuss the nature of the various gauge bosons masses, etc., in the current setup.

Given this discussion it is clear that only one large Majorana fermion mass term can be generated at this $Q_D = 0$ level and this is due to v_R since all of the other potentially contributing vevs are constrained to be very small, at the GeV level or below. This implies that at or above the mass scale of SM electroweak symmetry breaking, $\sim$100 GeV, the neutral fields N, N' will mix as described above to form the two Dirac mass eigenstates, $N_{1,2}$, while $\nu_L - \nu_R$ will form a Majorana mass matrix as in the LRM with the conventional seesaw mechanism being active via the hierarchal $v_L \ll v$, $v' \ll v_R$ vevs. This decoupled picture will, however, be slightly perturbed once the set of additional, small lepton-number violating, $Q_D \neq 0$ vevs get turned on. In particular, those associated with the $T_{3L(R)} = 1$ members of $\tilde{\Delta}_{L(R)}$, i.e., $\tilde{\Delta}^0_{L(R)}/\sqrt{2}$, and those of the corresponding $T_{3L(R)} = 1$, $T_{3I} = -1$ members of $\Delta_{L(R)}$, i.e., $v''_{L(R)}/\sqrt{2}$, will turn out to play the dominant roles since these are both $|Q_D| = 2$ fields which are obtaining vevs and, as we will see, produce mass terms that also lie along the diagonal of the Majorana mass matrix.

Within this setup, it is important to emphasize that while $W_{L,R}$ will couple linear combinations of the N_i with their isodoublet PM partners, $E_{L,R}$, and, correspondingly W_I will couple them to $\nu_{L,R}$, there is no direct $O(1)$ tree-level gauge coupling of the N_i to the usual SM leptons, e.g., $e_{L,R}$. This renders the study of the nature and properties of these interesting neutral PM states at colliders somewhat problematic since conventionally we need light charged leptons as decay products and/or coproduced states to probe the e.g., Dirac vs Majorana nature of any new heavy neutral lepton. Heavier gauge bosons that would play this potentially important role would need to live within a larger gauge group, G', within which $SU(2)_I$ and $SU(2)_{L/R}$ were unified with the possibilities of such a group to be discussed elsewhere.

Returning now to the Majorana masses themselves, in the original weak eigenstate basis, i.e., ν_L, N_L, $N'_L +$ (L $\to$ R), the full 6×6 Majorana mass matrix for the neutral fermions can be symbolically written as

$$M_{\rm Maj} = \begin{pmatrix} M_L & \mathcal{M} \\ \mathcal{M}^\dagger & M_R \end{pmatrix}, \tag{29}$$

where $\mathcal{M}$ is the 3×3 Dirac fermion mass matrix given above in the previous subsection and

$$M_L = \frac{1}{\sqrt{2}} \begin{pmatrix} z_1^L v_L & z_1^L v'_L & z_3^L x_L \\ z_1^L v'_L & z_1^L v''_L & z_3^L x'_L \\ z_3^L x_L & z_3^L x'_L & z_2^L \tilde{\Delta}^0_L \end{pmatrix}, \tag{30}$$

with the elements of M_R given by the same expressions but with L $\to$ R. The not previously mentioned remaining $Q_D \neq 0$ vevs that appear here correspond to the $T_{3L(R)} = 1$, $T_{3I} = \pm 1/2$ vevs, $(x_{L(R)}, x'_{L(R)})/\sqrt{2}$, of $X_{L(R)}$ and the $T_{3L(R)} = 1$, $T_{3I} = 0$ vevs, $\sim v'_{L(R)}/\sqrt{2}$, of $\Delta_{L(R)}$. Note that in the absence of any of the small $|Q_D| = 1$ vevs, the SM/LRM fields, $\nu_{L,R}$, and the neutral, N, N', PM sector fields

[7] Note that, in all generality, we allow X_L and X_R to be different scalar fields for this discussion but this need not be the case.

will become completely decoupled. After diagonalization, the most important effect of the previously noted $|Q_D| = 2$ vevs that live along the diagonal of $M_{\rm Maj}$, to leading order in the small vev ratios, is to split both of the heavy $N_{1,2}$ Dirac states into pairs of quasi-Dirac/pseudo-Dirac ones [52], i.e., $N_i \to N_i^{\pm}$. The corresponding masses, $m_i \to m_i \pm \delta m_i/2$, where the δm_i are just linear combinations of these four vevs along the diagonal with their associated Yukawa couplings. Note that for N_i masses in the expected range of roughly $m_i \sim 1$–10 TeV, the corresponding *fractional* mass splittings will then be expected to be only of order $\simeq O(10^{-(3–4)})$ making these splittings quite difficult to discern experimentally since the N_i are not directly connected to the charged SM fermions via a direct gauge interaction. By this we mean, as was noted above, that while the $W_{L,R}$ gauge boson will connect the N_i to the charged PM fields, $E_{L,R}$, and the W_I gauge bosons will connect them to the $\nu_{L,R}$ fields, there are no gauge bosons in this setup that will connect the N_i directly to, e.g., $e^{\pm}$ making the Majorana nature of the N_i quite difficult to ascertain. It is important to remember that to leading order in the small mixing angles, the N_i will dominantly decay as $N_1 \to \nu_L A_I$ and similarly, $N_2 \to \nu_R A_I$, if it is kinematically allowed, which are induced by both fermion and, as we will see below, gauge boson mixing so that the conventional vectorlike leptonic decay modes into, e.g., eW, are usually quite suppressed. However, if the decay through ν_R is *not* kinematically allowed then the $\nu_L A_I$ mode still remains open but this is suppressed by a factor of $\simeq s_{L,R}^2 \sim 10^{-4}$ in rate. Instead, the 3-body decay through a virtual ν_R and/or E may become important and this also leads to a visible final state, i.e., not just missing energy from SM neutrinos and DPs.

As was discussed in Ref. [52], it was noted that the ratio $\rho = (\delta m_i/\Gamma_i)^2$, where Γ_i is the N_i total decay width, is, in principle, an excellent probe of the Majorana/Dirac nature of such new neutral heavy leptonic states. From the arguments above, we already expect that $\delta m_i/m_i \lesssim 10^{-(3–4)}$ whereas the ratio Γ_i/m_i arising from the dominant decay into the $\nu_L A_I$ final state is roughly $\lesssim 10^{-(2–3)}$ implying that ρ may be relatively small for much of the model parameter space. A more detailed analysis of this possibility, however, lies beyond the scope of the current discussion.

We leave a further study of these effects to future work.

C. Gauge boson masses and mixings

The couplings of the many gauge bosons to the various fermions introduced above depend not only upon the mixings between these fermions states as already discussed but also on the KM and mass mixings among the gauge fields themselves which we will now consider.

The gauge bosons masses and mixings are rather complex in this scenario due to the presence of both KM as well as mass mixing at multiple Higgs vev-induced breaking scales. However, since these scales are widely separated by roughly 2 orders of magnitude, we can treat them in stages one at a time in a perturbative manner. It is natural that we will begin this analysis by working in the convenient and suggestive basis described by $\mathcal{L}_{\rm int}^h$ and $\mathcal{L}_{\rm int}^{nh}$ above and first consider the effects of the largest vevs, v_R, Λ and Λ', neglecting the effects of KM, upon the real, Hermitian gauge fields. At this level, only the Z_R and Z_I fields can obtain masses so that in this 2×2 subspace one obtains a mass squared matrix of the form

$$M_{RI}^2 = (g_L c_w z v_R)^2 \begin{pmatrix} 1 & \gamma \\ \gamma & \gamma^2(1+R) \end{pmatrix}, \qquad (31)$$

where

$$z = \frac{\kappa^2}{\sqrt{\kappa^2 - (1+\kappa^2)x}} = \kappa^2 \Omega^{-1}, \qquad \gamma = \frac{g_I/c_I}{g_L c_w z}, \\ R = \frac{\Lambda^2 + \Lambda'^2}{4v_R^2}, \qquad (32)$$

and which can be diagonalized by a rotation to the $Z_{1,2}$ mass eigenstate basis described by an angle

$$\tan 2\theta_\gamma = \frac{2\gamma}{1-(1+R)\gamma^2}, \qquad (33)$$

so that $Z_R = c_\gamma Z_1 - s_\gamma Z_2$, etc., in obvious notation. We note that for $R, \gamma \simeq 1$ and $O(1)$ Yukawa couplings, the PM fields, ν_R and $Z_{1,2}$ all will have quite comparable masses in the $\sim$ several TeV range.

Note that in terms of, perhaps, the more fundamental quantity, $r = (g_I/c_I)/(g_L/c_w)$, that we employed in our earlier work [26], and which describes the overall coupling strength relative to that of the SM Z, the ratio γ/r is found to be purely a function of κ and is always $\lesssim 1.04$ as is shown in the top panel of Fig. 1.

Without any prior input we expect that the mixing angle θ_γ to be $O(1)$ so that both $Z_{1,2}$ can now have substantial couplings to the dark sector fields carrying $Q_D \neq 0$ which may lead to important phenomenological implications. The resulting mass-squared eigenvalues (always with $M_{Z_1} \geq M_{Z_2}$) are now given by

$$\frac{2M_{Z_{1,2}}^2}{(g_L c_w z v_R)^2} = 2\lambda_{1,2}^2 = 1 + \gamma^2(1+R) \pm [(1+\gamma^2)^2 - 2\gamma^2(1-\gamma^2)R + R^2\gamma^4]^{1/2}. \qquad (34)$$

Here, the λ_i (with $\lambda_1 \geq \lambda_2$) can be thought of as the masses of these new heavy neutral gauge bosons scaled in comparison to that of the conventional LRM expectation "reference" value for M_{Z_R}, i.e., $M_{Z_R}^2 = (g_L c_w z v_R)^2 = M_0^2$.

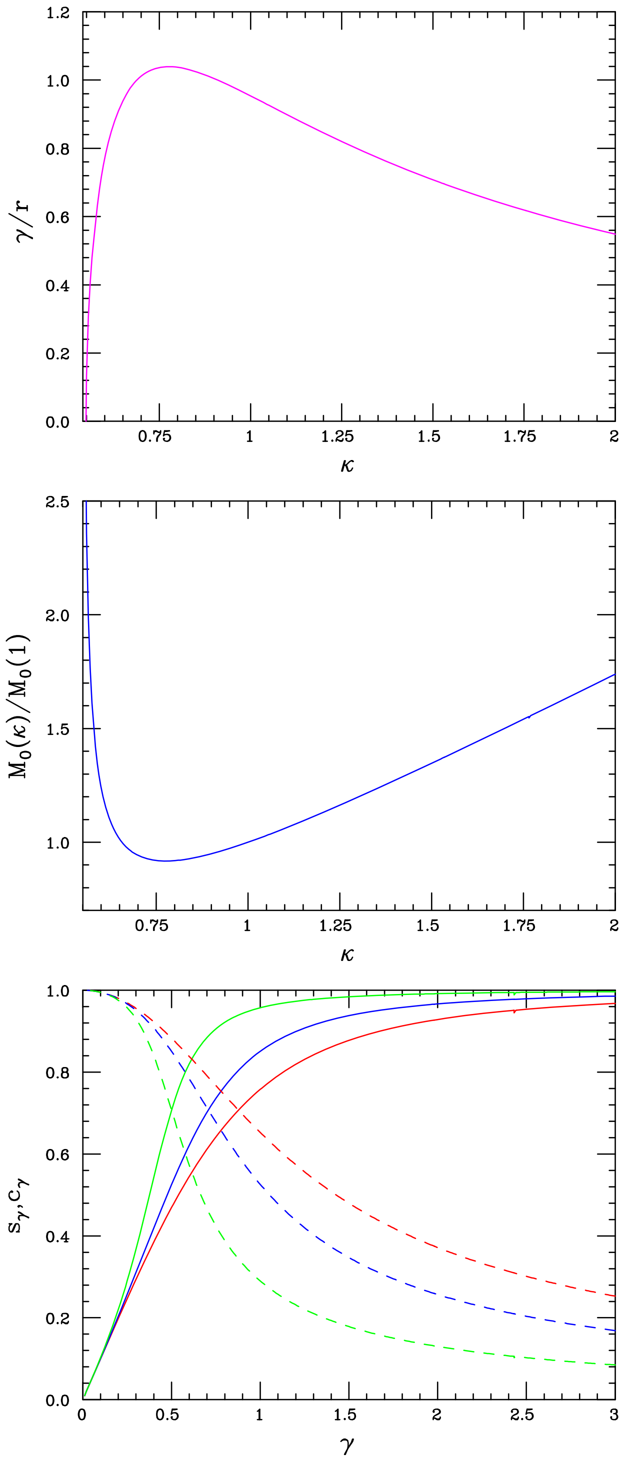

FIG. 1. The ratios γ/r (Top) and $M_0(\kappa)/M_0(1)$ (Middle) as functions of κ as described in the text. (Bottom) s_γ (solid) and c_γ (dashed) as functions of γ for values of the vev ratio $R = 0.3$ (red), 1 (blue) and 3 (green), respectively.

It is to be noted that M_0 is itself a function of the parameter κ and can vary significantly as the value of κ changes; this dependence can be seen in the middle panel of Fig. 1. Here we observe that M_0 diverges as κ approaches its minimum value, $\simeq 0.55$, and that it grows linearly with κ for larger values $\gtrsim 1$. The lower panel of this same figure displays the R, γ dependence of both s_γ and c_γ; note that very substantial mixing occurs even for values of γ below unity. While s_γ grows linearly with γ for small values, it rapidly asymptotes to unity; on the other hand, c_γ falls like $1/\gamma$ for large values.

Figure 2 shows how these scaled Z_i masses, the λ_i, vary as functions of the coupling ratio γ for fixed values of the Higgs vev ratio, R. For large values of γ, λ_1 is found to grow asymptotically as $\sqrt{(1+R)}\gamma$. Note that λ_2 vanishes when $\gamma = 0$, since the relevant gauge coupling then vanishes, and it then asymptotes at large values of γ to $\sqrt{R/(1+R)}$. Here we see, e.g., that the Z_2 is always significantly lighter than Z_1 so it is likely to be much more kinematically accessible to collider searches (for fixed M_0) although both fields generally have qualitatively distinct couplings over much of the parameter space as we will find below.

If $\kappa \simeq R \simeq 1$ and γ is relatively small so that we are not too far from the LRM limit and only decays to the SM fermion final states are kinematically allowed, then, employing the results from the 13 TeV, 139 fb^{-1} search by ATLAS [53], we find that, e.g., the Z_1 is constrained from searches from the combined $e^+e^- + \mu^+\mu^-$ dilepton channel to lie above roughly $\simeq$4.9–5.1 TeV. This constraint may increase by $\simeq$10–15% or so as the LHC integrated luminosity is increased to 3 ab^{-1} [54] if no signal is found. Of course, if these various assumptions are significantly relaxed, the present search reach will extend over a significantly larger range of masses. For the Z_1, other regions of the parameter space can generally lead to stronger constraints than the one obtained in the LRM (always under the assumption that only decays to SM final states are kinematically allowed) as both the couplings to the SM quarks as well as the Z_1 leptonic branching fraction all increase with corresponding increases in values of γ. This result for the present Z_1 search limit, assuming the validity of the narrow width approximation (NWA), is demonstrated in the upper panel of Fig. 3 where the choice $\kappa = 1$ is maintained but both R, γ are allowed to vary. If additional decay modes are present, clearly the Z_1's branching fraction to SM leptons will diminish by, certainly at least, $O(1)$ factors which will degrade the search reach in this channel somewhat but this may be partially compensated for by the additional alternate search channels that now become available. Similarly, to the Z_1 example, the mass of the Z_2 is also constrained; however, in that case, as $\gamma \to 0$, the couplings of the Z_2 to SM states all vanish so that the bound then disappears. Of course, for larger values of γ, a respectable bound is obtained as the relevant couplings (initially) grow rapidly and this is shown in the lower panel of Fig. 3 under the same assumptions as were previously made for the Z_1. Since the Z_2 couplings saturate as γ gets large, with $s_\gamma \to 1$ and γc_γ scaling approximately as $\sim(1+R)^{-1}$ independently of γ; here we see that the resulting bound flattens out in this parameter

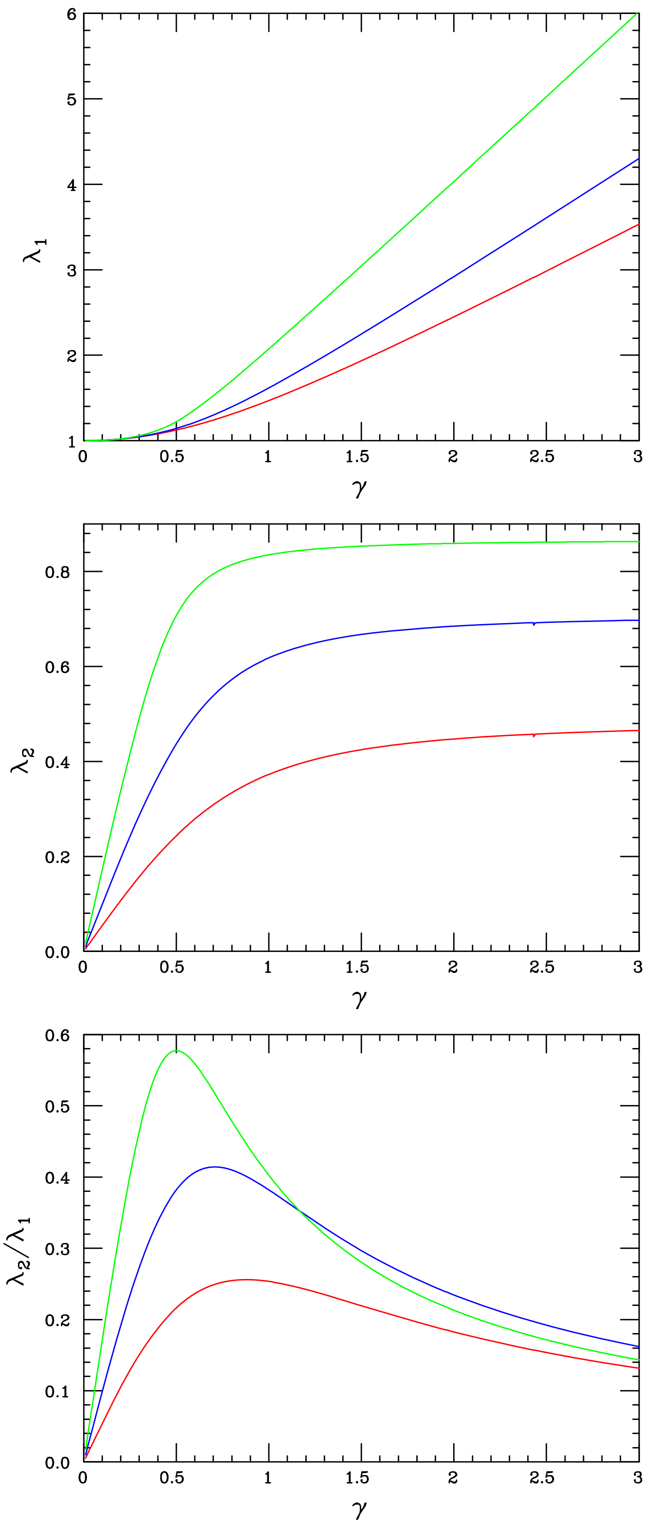

FIG. 2. The scaled masses of the $Z_{1,2}$ gauge bosons, $M_{Z_i} = M_0\lambda_i$, as described in the text, as functions of γ for values of the vev ratio $R = 0.3$ (bottom red), 1 (middle blue), and 3 (top green), respectively. The top (middle) panel shows the result for $Z_{1(2)}$ while the lower panel show the corresponding gauge boson mass ratio. Note that all κ-dependence of these masses lies in the M_0 prefactor.

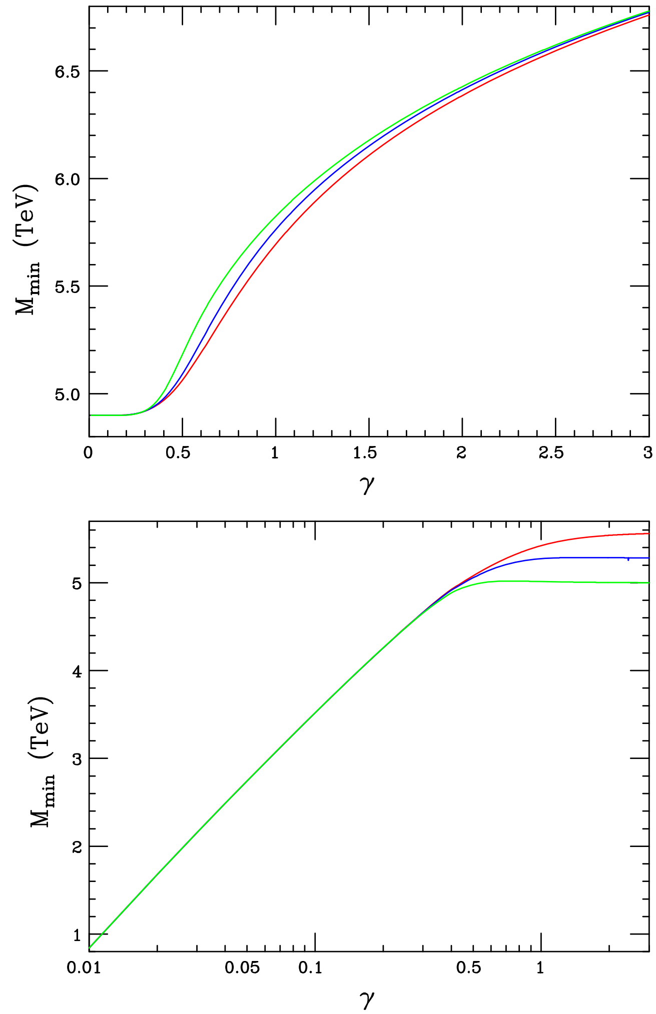

FIG. 3. Approximate present lower bound on the (top) Z_1 and (bottom) Z_2 masses assuming only decays to SM fermion final states are kinematically accessible, as discussed in the text, as a function of the parameter γ obtained by employing the dilepton search results from ATLAS [53]. The red (blue, green) curves correspond to $R = 0.3(1, 3)$, respectively, and, for demonstration purposes, all curves assume that $\kappa = 1$ as well as the applicability of the narrow width approximation.

space region. Of course, once γ becomes too large, depending upon the values of the other parameters, our reach estimate based on the NWA will fail as the Z_i's will become too wide and thus the signal to background ratio under the resonance will drop significantly so that the limit obtained here will clearly overestimate the true bound by a potentially significant factor and a far more detailed analysis will then be required.

It is also possible to extrapolate these results for the $Z_{1,2}$ mass reaches to the case of the 100 TeV FCC-hh by following the dilepton analysis as presented in Ref. [54], here assuming an integrated luminosity of 30 ab^{-1}. The results of this analysis, with the same assumptions as in the

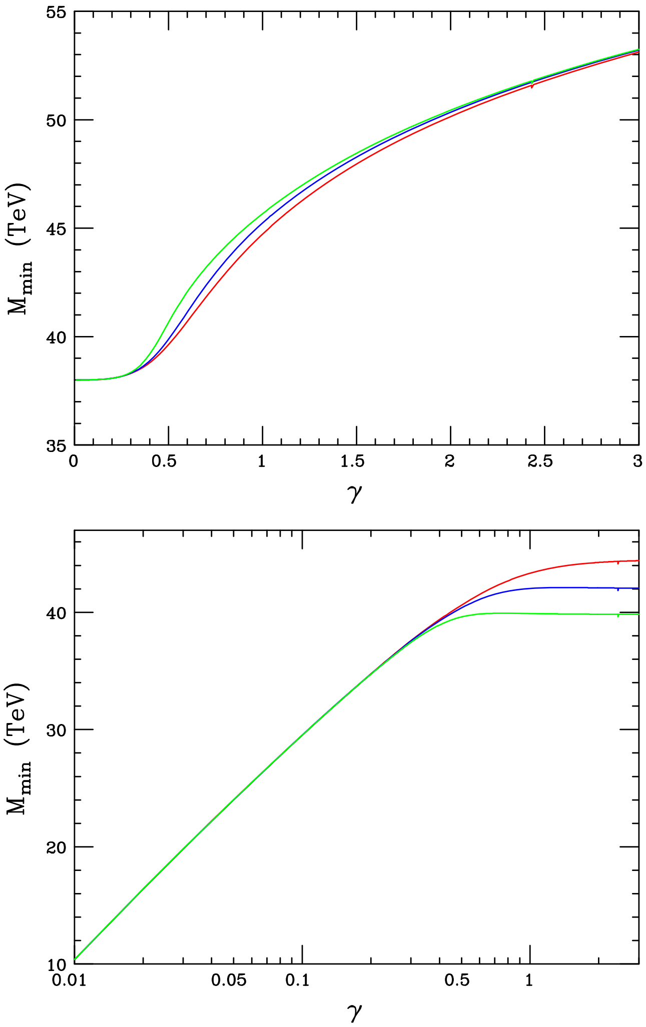

FIG. 4. Same as the previous figure, but now for the 100 TeV FCC-hh assuming an integrated luminosity of 30 ab^{-1} following Ref. [54].

case of the LHC are displayed in Fig. 4, and unsurprisingly show the same overall qualitative behavior as was seen above although at a significant higher mass scale. The same words of caution with respect to the applicability of the NWA will apply in this case as above.

Correspondingly, at this same mass scale of symmetry breaking, in the complex, non-Hermitian sector, we find the relatively simple results for the gauge boson mass eigenvalues to be

$$M_{W_R}^2 = \frac{1}{2}\kappa^2 g_L^2 v_R^2, \qquad M_{W_I}^2 = \frac{1}{2} g_I^2 v_R^2 (1+2R), \qquad (35)$$

which, of course, cannot mix together as $Q(W^\pm, W_I^{(\dagger)}) = \pm 1, 0$ while W_I also carries $|Q_D| = 1$. The W_R appearing here is just the usual one present in the LRM, except now for possible decay modes into heavy PM states and that its mass is no longer directly correlated in a simple manner with that of either of $Z_{1,2}$, for which many searches exist in multiple final states under various assumptions.[8] We note, however, that more indirectly, since all of the gauge boson masses are essentially determined by the values of $\kappa, \gamma, c_I/c_w$ and R, some correlations will exist especially in certain limits, e.g., at large values of γ when Z_1 is mostly Z_I, M_{W_I} is linearly proportional to M_{Z_1}. Roughly speaking, these lower bounds on the W_R mass for $\kappa = 1$ from these various LHC searches hover in the 4.0–5.7 TeV range depending upon the search channel and will likely be improved upon somewhat by HL-LHC. Similarly, the mass of the W_I is no longer directly correlated in a simple way with that of either $Z_{1,2}$, e.g., $M_{W_I} = c_I M_{Z_I}$, yet still must decay, if kinematically allowed, into a, $fF_{1,2}$, i.e., a SM + PM final state or, if this kinematically forbidden, into the $\bar{f}f + A_I$ final state as discussed in Ref. [31]. At the LHC, the W_I can be made in pairs via $q\bar{q}$ annihilation via s-channel $Z_{1,2}$ exchange (plus t-channel $Q_{1,2}$ PM exchange), in association with A_I (also via $t-$, u-channel Q_i exchange), or in association with a $Q_{1,2}$ PM field in gq fusion as was discussed in some detail in Ref. [26]. The situation here is slightly different, however, in that the $q = u$ channel for associated production is now also open. Since W_I decay (to a very good approximation) necessarily involves PM fields, the search reaches for these states are much more model-dependent than are those for the other gauge bosons that we have so far discussed. Clearly, since W_I production itself generally involves other heavy states at some level, the W_I search reaches are clearly suppressed in comparison to those for the more well-studied W_R.

To get an idea where these two non-Hermitian gauge boson masses may lie relative to the those of the Z_i discussed above, Fig. 5 shows both M_{W_R}/M_0 as a function of κ (as it is independent of both γ and R) and M_{W_I}/M_0 as a function of γ (since it is independent of κ) for different values of R *assuming* that an additional overall scaling factor of $c_I/c_w \lesssim 1.14$ appearing in this mass ratio has been set to unity. Here we see that while both W_R and W_I generally lie somewhat close to the $Z_{1,2}$ in mass, it is difficult to make too many universal statements that might be useful, e.g., for resonance searches and/or model testing purposes at the LHC. One obvious condition we observe is that the W_R is *always* lighter than Z_1, being somewhat closer to the Z_2 in overall mass range. Indeed, for a respectable fraction of this parameter space the decay $Z_1 \to W_R^+ W_R^-$ is kinematically allowed. On the other hand, for much of the parameter space examined here, the W_I is close to, but is always below, the Z_1 in mass even when the ratio c_I/c_w takes on its maximum allowed value of $\simeq 1.14$.

We next turn to the symmetry breaking which occurs at the electroweak scale, this time first examining the

[8]For a selection of such searches, see Refs. [55,56].

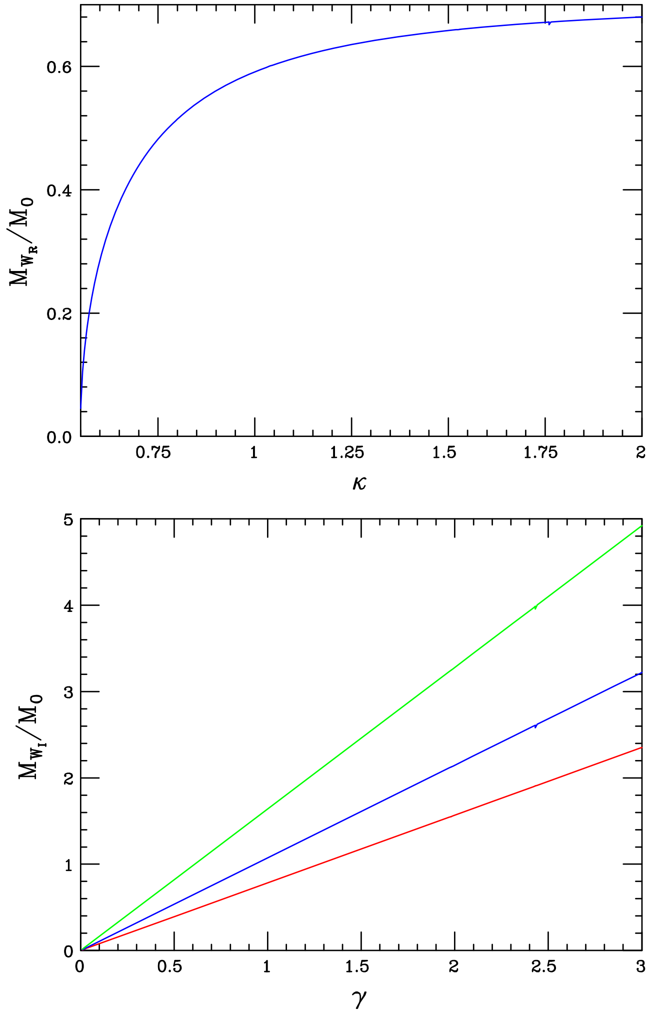

FIG. 5. Top: the W_R mass, which is independent of the values of γ, R, in units of M_0 as a function of κ. Bottom: the W_I mass, which is κ-independent, in units if M_0 as a function of γ for values of the vev ratio $R = 0.3$ (bottom red), 1 (middle blue) and 3 (top green), respectively. Note that this value must be further rescaled by the ratio $c_I/c_w \leq 1.14$ which we have taken to be unity here for demonstration purposes.

non-Hermitian sector which is somewhat simpler as W_I is unaffected by the relevant electroweak scale vevs of the $H_{1,4}$. The situation in the charged $W - W_R$ sector is essentially the same as in the LRM so we can be quite brief. Recalling the bidoublet discussion above when these are projected back into the $2_L 2_R$ subspace (i.e., the usual LRM subspace), one now generates a mass for the SM W, a shift in the W_R mass, as well as a mixing between these two states as usual as can be seen from the 2×2 mass matrix:

$$M^2_{WW_R} = \begin{pmatrix} M_W^2 & \beta_W M_W^2 \\ \beta_W M_W^2 & M^2_{W_R} + \kappa^2 M_W^2 \end{pmatrix}, \quad (36)$$

where $M^2_{W_R}$ is as given above and

$$M_W^2 = \frac{1}{4} g_L^2 (k_1^2 + k_2^2 + k_1'^2 + k_2'^2),$$
$$\beta_W = \kappa \frac{2(k_1 k_2 + k_1' k_2')}{k_1^2 + k_2^2 + k_1'^2 + k_2'^2}. \quad (37)$$

This mass squared matrix can be diagonalized via a mixing angle (which we might expect to be $\sim 10^{-(3-4)}$) given in the notation above by

$$\tan 2\phi_W = \frac{-2\beta_W M_W^2}{M^2_{W_R} + (\kappa^2 - 1) M_W^2}, \quad (38)$$

to form the mass eigenstates $W_{1,2}$ given by $W = c_{\phi_W} W_1 - s_{\phi_W} W_2$, etc., and whose corresponding mass-squared eigenvalues are given by

$$2M^2_{W_{1,2}} = M^2_{W_R} + (\kappa^2 + 1) M_W^2$$
$$\pm (M^4_{W_R} + 2M_W^2 M^2_{W_R}(\kappa^2 - 1)$$
$$+ M_W^4 [4\beta_W^2 + (\kappa^2 - 1)^2])^{1/2}. \quad (39)$$

The resulting leading order fractional *downward* shift in the SM W mass due to this mixing is then found to be very roughly of the same magnitude as the mixing angle, ϕ_W, i.e.,

$$\frac{\delta M_W^2}{M_W^2} \simeq -\beta_W^2 \frac{M_W^2}{M^2_{W_R}}. \quad (40)$$

We next examine the corresponding symmetry breaking in the Hermitian gauge boson sector at the electroweak scale where the situation is a bit more complex. Employing the SM relation $M_Z^2 = M_W^2 / c_w^2$ and recalling the $O(1)$ parameter combination employed above, $\Omega^2 = \kappa^2 - (1 + \kappa^2) x$, in the now $Z - Z_1 - Z_2$ basis the relevant 3×3 mass squared matrix now becomes

$$M^2_{Z,Z_1,Z_2} = \begin{pmatrix} M_Z^2 & -M_Z^2 \Omega c_\gamma & M_Z^2 \Omega s_\gamma \\ -M_Z^2 \Omega c_\gamma & M_1^2 + (M_Z \Omega c_\gamma)^2 & -M_Z^2 \Omega^2 s_\gamma c_\gamma \\ M_Z^2 \Omega s_\gamma & -M_Z^2 \Omega^2 s_\gamma c_\gamma & M_2^2 + (M_Z \Omega s_\gamma)^2 \end{pmatrix}, \quad (41)$$

where $M^2_{1,2}$, s_γ, etc., are all as defined above. To leading order in the small ratios $M_Z^2 / M^2_{1,2}$, the most important effects that result from the $Z - Z_{1,2}$ mixings via the angles,

$$\theta_{ZZ_{1,2}} \simeq \Omega M_Z^2 \left(\frac{-c_\gamma}{M_1^2}, \frac{s_\gamma}{M_2^2} \right), \quad (42)$$

respectively, are to slightly reduce the SM Z mass (but only by fractional factors $\sim 10^{-(3-4)}$ which are also the expected sizes of these mixing angles), i.e.,

$$\frac{\delta M_Z^2}{M_Z^2} \simeq -\Omega^2 M_Z^2 \left(\frac{c_\gamma^2}{M_1^2} + \frac{s_\gamma^2}{M_2^2}\right) = -\Omega^2 \frac{M_Z^2}{M_0^2} \frac{1+R}{R}, \tag{43}$$

and to allow this (almost) SM Z state to now pick up, at this mixing suppressed level, some of the couplings associated with both $Z_{R,I}$, e.g., a coupling to RH-neutrinos as well as to the set of dark sector fields which have $Q_D \neq 0$. It should be noted that over almost all of the model parameter space, one finds that $|\delta M_Z^2| > |\delta M_W^2|$, employing the result obtained above and this may be of some interest given the recent W boson mass measurement by CDF II [57] due to the relative displacement of the two mass eigenstates induced via this mixing.

The final stage of symmetry breaking at or above the electroweak scale arises from the effects of both the KM and the rather large set of all the possible $Q_D \neq 0$ vevs that may be nonzero in the various Higgs scalar representations we have introduced above; we will deal with the KM effects first. In the Hermitian sector, following the notation above and now accounting for the effects of the ∼10 TeV scale mass mixing discussed previously, the KM-induced interaction term in Eq. (9) above now appears in terms of the approximate mass eigenstates as

$$\mathcal{L}^h_{\rm int}({\rm KM}) = \sigma \frac{g_L s_w \kappa}{\Omega} (Q_{\rm em} - T_{3L} - T_{3R}) \times [c_I A_I - s_I (c_\gamma Z_2 + s_\gamma Z_1)], \tag{44}$$

with the further effects of the $Z - Z_{1,2}$ mass mixing at the electroweak scale being additionally suppressed by factors of order $\theta_{ZZ_{1,2}}$ that can be safely numerically neglected. Recalling the dimensionless parameters $\lambda_{1,2}$ from above, we see that largest mass mixing term generated by this interaction arises unsurprisingly from the $T_{3I} = T_{3R} = 1$, $T_{3L} = 0$ vev, v_R, resulting in the induced mass mixing of A_I with the $Z_{1,2}$ via both new diagonal and off-diagonal terms given by

$$\theta_{A_I Z_{1,2}} \simeq -\frac{\sigma t_w c_I}{\kappa} \left(\frac{c_\gamma + \gamma s_\gamma}{\lambda_1^2}, \frac{\gamma c_\gamma - s_\gamma}{\lambda_2^2}\right), \tag{45}$$

which are expected to be roughly $\sim 10^{-4}$ or so, such that to leading order in the small parameters, one essentially finds the corresponding shifts in the fields

$$A_I \to A_I + \sum_i \theta_{A_I Z_i} Z_i, \qquad Z_i \to Z_i - \theta_{A_I Z_i} A_I, \tag{46}$$

result in the diagonalization of the perturbed mass squared matrix. This implies that both $Z_{1,2}$ pick up some KM-suppressed interactions to $T_{3I} = 0$, $Q_D \neq 0$ states that they might otherwise not have coupled to, while the A_I correspondingly picks up KM-suppressed couplings to the SM fields with nonzero values of $T_{3(L,R,I)}$ and/or $Q_{\rm em}$ which all have $Q_D = 0$. Combining the $\theta_{A_I Z_i}$-induced couplings here with those in Eq. (40) (and recalling that the A_I direct coupling to Q_D is already present at leading order), after some algebra we now find that the *total* KM-induced coupling for A_I at this stage of symmetry breaking to SM/LRM states is explicitly given by (and recalling from above that $\kappa^2 > t_w^2$)

$$\sigma g_Y c_I \left(1 - \frac{t_w^2}{\kappa^2}\right)^{-1/2} \left[(\alpha - 1) T_{3R} + \left(1 - \frac{\alpha t_w^2}{\kappa^2}\right) \frac{Y}{2} + \beta\gamma T_{3I}\right], \tag{47}$$

where γ is given above, g_Y is the usual SM hypercharge coupling and, in terms of the previously defined parameters, one finds that the coefficients α, β are given by

$$\alpha = \frac{c_\gamma^2 + \gamma s_\gamma c_\gamma}{\lambda_1^2} - \frac{\gamma s_\gamma c_\gamma - s_\gamma^2}{\lambda_2^2}, \qquad \beta = \frac{\gamma s_\gamma^2 + s_\gamma c_\gamma}{\lambda_1^2} + \frac{\gamma c_\gamma^2 - s_\gamma c_\gamma}{\lambda_2^2}. \tag{48}$$

Note that in the pure LRM limit, i.e., γ, s_γ, $\beta \to 0$ so that also c_γ, $\alpha \to 1$, the DP coupling is easily seen to be only to the SM hypercharge at this stage of symmetry breaking as it would be in the familiar $U(1)_D$ DP model. In this same limit we would then easily identify the usual ϵ parameter of the $U(1)_D$ model to be given by

$$\epsilon = \sigma c_w c_I (1 - t_w^2/\kappa^2)^{1/2}. \tag{49}$$

Interestingly, using the definitions above, after some lengthy algebra one finds that the relations $\alpha = 1$, $\beta = 0$ are *always* satisfied so that the A_I in this setup indeed only has KM-induced couplings to the $Q_D = 0$ sector via the SM hypercharge as in the usual $U(1)_D$ model.

The mass of A_I, i.e., M_{A_I}, which we have not yet discussed in any detail as, before any potential mixing effects, it arises solely from the $Q_D \neq 0$ vevs, is found not to be shifted to leading order in the small parameters by KM but it is possible that the quadratic terms of order $\sigma^2 M_0^2$ can potentially be present and could be numerically significant in some regions of the parameter space as we expect $\sigma \sim \epsilon \sim 10^{-(3–4)}$ and M_0 is relatively quite large, at least several TeV. In order to address this potential problem, we must return to the analysis above and re-examine the full 3×3, $Z_R - Z_I - A_I$ mass-squared matrix including these new terms that are now generated by KM:

$$M_0^2 \begin{pmatrix} 1 & \gamma & -q \\ \gamma & \gamma^2(1+R) & -\gamma q \\ -q & -\gamma q & \tilde{M}_{A_I}^2 + q^2 \end{pmatrix}, \qquad q = \frac{\sigma t_w c_I}{\kappa}, \qquad \tilde{M}_{A_I} = M_{A_I}/M_0. \tag{50}$$

Combining all of the contributions to the squared A_I mass, one finds that, fortunately, the terms which are quadratic in σ completely cancel so that the DP mass still only arises from the vevs of the $Q_D \neq 0$ scalars that we will discuss later below. This is a generalization of the well-known result that occurs in the simple $U(1)_D$ scenario.

Next, we consider whether or not A_I and Z will correspondingly mix in the familiar manner via the electroweak scale, $B-L=0$, bidoublet vevs from $H_{1,4}$ that were discussed previously. We recall that the initial KM-induced A_I coupling to the SM fields in Eq. (40) is proportional to $B-L=Q-T_{3L}-T_{3R}$ so this coupling would vanish completely for these representations. However, the A_I-Z_i mass mixing above was seen to alter this situation as A_I now in general couples instead only to $Y/2$ implying a nonvanishing contribution from the bidoublets which appears in the conventional manner. The relevant $Z-A_I$, 2×2 part of the gauge boson mass squared matrix can be written at this level of approximation, now employing the conventional ϵ notation, as

$$\begin{pmatrix} M_Z^2 & -\epsilon t_w M_Z^2 \\ -\epsilon t_w M_Z^2 & M_{A_I}^2 + \epsilon^2 t_w^2 M_Z^2 \end{pmatrix}, \tag{51}$$

which can be diagonalized as usual by $Z \to Z + \epsilon t_w A_I$, etc. After diagonalization, the mass of A_I is unaltered but it now couples as $\simeq \epsilon e Q$ in the limit when $M_{A_I}^2/M_Z^2 \ll 1$, appearing as the conventional DP as far as the KM-suppressed couplings are concerned.

By way of contrast, the non-TeV but now *electroweak* scale vev KM-induced $A_I - Z_i$ mixing is also found to be nonzero but it is significantly smaller than that obtained in the discussion above for KM-induced mixing with the Z by factors of order $\sim M_Z^2/M_{1,2}^2 < 10^{-(3-4)}$ and so can be safely neglected in what follows.

Lastly, we must turn our attention to the set of the many possible $Q_D \neq 0$ vevs, here denoted collectively as w_i, that can occur at scales $\lesssim 1$ GeV and generally also have other additional quantum numbers, e.g., $T_{3(L,R,I)}$, associated with them depending upon which Higgs scalar representation of the many encountered above that they may come from. These will not only generate a mass (before any possible mass or kinetic mixing effects might be included) for A_I, i.e.,

$$M_{A_I}^2 = g_D^2 \sum_i Q_{D_i}^2 w_i^2, \tag{52}$$

but will also induce a small gauge boson mass mixing with the other neutral states (including now with the combination $W_I + W_I^\dagger$) as well as generating (relatively) tiny Majorana mass terms for some subset of the neutral fermions as was discussed above. The largest of the resulting Hermitian gauge boson mixings will be induced between the Z and A_I as the corresponding mixing with $Z_{1,2}$ will be further suppressed by factors of order $\sim M_Z^2/M_{1,2}^2$, and so vevs with both $T_{3L}, Q_D \neq 0$ will be the most relevant. This precludes the H_i as well as Δ_R, $\tilde{\Delta}_R$ and X_R from playing any important role in generating these mixings.[9] Thus the $Q_D \neq 0$ vevs of the three remaining Higgs scalars, Δ_L, $\tilde{\Delta}_L$ and X_L, will be the main subject of our attention and the resulting $Z-A_I$ induced mixing angle can generically be written as

$$\phi_{ZA_I} \simeq \frac{g_L}{g_D c_w}\left[\frac{\sum_i Q_{D_i} T_{3L_i} w_i^2}{\sum_i Q_{D_i}^2 w_i^2}\right]\frac{M_{A_I}^2}{M_Z^2}, \tag{53}$$

where we expect the prefactor in front of the mass squared ratio to be roughly $\lesssim O(1)$. This implies a not too uncommon additional coupling of the A_I to SM fields in a Z-like manner, i.e., proportional to $-\phi_{ZA_I}\frac{g_L}{c_w}(T_{3L}-xQ)$. Here, in principle, the A_I mass also experiences a tiny fractional shift, $\delta M_{A_I}^2/M_{A_I}^2$, due to this mixing as well from the KM-induced couplings to the $Z_{\mathrm{SM},1,2}$ associated currents so that *all* of the $Q_D \neq 0$ vevs may now contribute; however, these terms are all found to be suppressed by appropriate factors of order $M_{A_I}^2/M_{Z_{L,1,2}}^2$ and so can be safely ignored.

As has been noted several times, the A_I is also found to have a somewhat unusual induced mixing with the $Q_D \neq 0$ Hermitian combination $W_I + W_I^\dagger$ of states arising from the Higgs representations which are $SU(2)_I$ nonsinglets. In particular, the largest contributions to this mixing will arise, due to the action of the raising and lowering operators, from the product of a $Q_D = 0$ and a $Q_D \neq 0$ vev from representations wherein the largest $Q_D = 0$ vevs reside, i.e., the $SU(2)_I$ doublets $H_{2,3}$ (with vevs Λ, Λ', respectively) and the $SU(2)_I$ triplet Δ_R (with vev v_R). Let us denote the corresponding small, $Q_D = -1$, $\lesssim 1$ GeV vevs in these representations by λ, λ' and v_R', respectively, as above. Then the $(W_I + W_I^\dagger) - A_I$ mixing angle is found to given by

$$\phi_{W_I A_I} \simeq -s_I \frac{2 v_R v_R' + (\Lambda\lambda + \Lambda'\lambda')}{2 v_R^2 (1+2R)}, \tag{54}$$

which is again found to be roughly of order $\sim 10^{-4}$. This implies that A_I picks up a new, Q_D-changing coupling to the $SU(2)_I$ isospin raising and lowering operators of the form

$$g_I \phi_{W_I A_I} (T_I^+ + T_I^-) A_I. \tag{55}$$

When acting on the $f - F_1 - F_2$ fermions, this structure produces the effective interaction

$$g_I \phi_{W_I A_I} [\bar{f}\gamma_\mu P_L (F_1 c_L - F_2 s_L) + \bar{f}\gamma_\mu P_R (F_2 c_R + F_1 s_R)] A_I^\mu + \mathrm{H.c.}, \tag{56}$$

[9]This will of course *not* be the case for the A_I mass itself.

which augments those couplings of a similar Q_D-violating nature already appearing above in Eq. (26) due to SM-PM fermion mixing effects. By adding these two results, we see that to leading order in the small vev ratios, the sum of both the contributions to the effective $f - F_1 - A_I$ coupling can now be written as

$$-g_D \bar{f} \gamma_\mu P_L F_1 \left[\frac{\lambda}{\Lambda} + \frac{2v_R v'_R + (\Lambda\lambda + \Lambda'\lambda')}{2v_R^2 + \Lambda^2 + \Lambda'^2}\right] A_I^\mu + \text{H.c.}, \quad (57)$$

while that for F_2 is given in the same approximation by the same expression with the replacements $P_L \to P_R$ and $(\Lambda, \lambda) \to (\Lambda', \lambda')$. Couplings to the opposite helicities are found to be suppressed for both F_i states by factors of order $s_{L,R} \sim (v, v')/(\Lambda, \Lambda') \sim 10^{-2}$. As noted previously, this result is qualitatively similar to that found in Ref. [26] in a somewhat different (and simpler) context. Like in that case, here we also see that the contribution to the $F_{1,2} \to f A_I$ decay amplitude due to the longitudinal component of the A_I polarization is enhanced by a factor of $m_{1,2}/M_{A_I} \gg 1$ which offsets the suppression due to the small overall mixing angle factor, $\phi_{W_I A_I}$ or, more explicitly, by the set of small vev ratios appearing in the expression above.

A direct application of this analysis arising from the $(W_I + W_I^\dagger) A_I$ mixing-induced coupling is the process $Z_i \to W_I^{(\dagger)} A_I$ which occurs via the $Z_I W_I W_I^\dagger$ non-Abelian trilinear interaction at order g_I. This is an s-channel, resonance-enhanced version of a previously examined process [26] which in that case instead occurred via $t-$ (or $u-$)channel F-exchanges so that in the present case the W_I (and its decay products) would appear more centrally in the detector. As discussed above, the large mass ratio M_{Z_i}/M_{A_I} appearing in the amplitude due to the dominance of the A_I longitudinal polarization offsets the small value of $\phi_{W_I A_I}$. Note that given the scaled Z_i and W_I masses shown in the figures above, for most of the parameter space only the resonant decay with the Z_1 initial state will be kinematically allowed on-shell. To estimate the cross section for this process, we need several distinct pieces of information, e.g., the fraction of the A_I mass resulting from the vev v'_R, i.e., $f = (g_D v'_R)^2/M_{A_I}^2 < 1$. Then, we need to account for the various multiple vev ratios that enter into the $\phi_{W_I A_I}$ mixing angle expression as well as the gauge boson masses themselves; to this end we define the $O(1)$ ratio

$$\mathcal{R} = (1 + 2R)^{-1} \left[1 + \frac{\Lambda\lambda + \Lambda'\lambda'}{2v_R v'_R}\right], \quad (58)$$

which we see equals unity when $\lambda/\Lambda = \lambda'/\Lambda' = v'_R/v_R$, but can be either greater or less than one. Next, to obtain the NWA estimate for the desired cross section, we need to determine the ratio of the Z_1 partial widths for the $W_I^{(\dagger)} A_I$

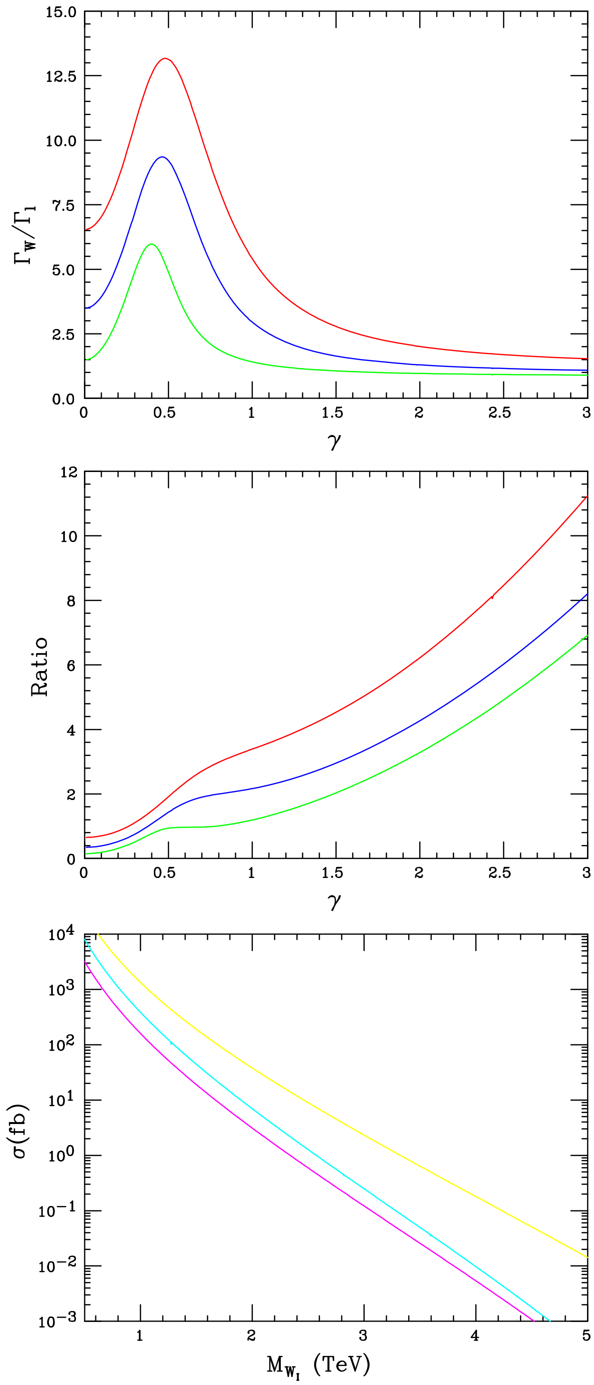

FIG. 6. Top: the ratio of the $W_I^{(\dagger)} A_I$ to dilepton partial widths of the Z_1 as a function of γ for values of the vev ratio $R = 0.3$ (top red), 1 (middle blue), and 3 (bottom green), respectively, assuming $f\mathcal{R}^2 = 1$. Middle: the ratio of the resonant $Z_1 \to W_I^{(\dagger)} A_I$ production cross section at the 13 TeV LHC to that for dileptons in the LRM with the three curves labeled as in the panel above and here assuming that $f\mathcal{R}^2 = 0.1$. Bottom: resonant $Z_1 \to W_I^{(\dagger)} A_I$ induced production cross section as a function of M_{W_I} assuming that $R = \gamma = 1$ (bottom magenta), $R = 0.3$, $\gamma = 2$ (middle cyan), and $R = 0.3$, $\gamma = 3$ (top yellow) as an extreme example, together with $f\mathcal{R}^2 = 0.1$. Note that the values $\kappa = c_I/c_w = 1$ have been assumed in all of the panels above.

final state to that for dileptons, Γ_W/Γ_ℓ, which we can write as

$$\frac{\Gamma_W}{\Gamma_\ell} = f\mathcal{R}^2 \frac{s_\gamma^2 \lambda_1^2}{R_\ell^2 + L_\ell^2} P(M_{W_I}^2/M_{Z_1}^2), \tag{59}$$

where, s_γ, λ_1 are defined above, R_ℓ, L_ℓ are the leptonic chiral couplings of the Z_1 which depend upon the parameters R, γ and κ, and $P(y)$ is a kinematic function arising from the product of the squared matrix element with the relevant phase space:

$$P(y) = (1-y)^3 \left[2 + \frac{(1+y)^2}{4y}\right]. \tag{60}$$

The top panel of Fig. 6 shows this partial width ratio as a function of γ for specific values of R and we see that it is generally $O(1)$ or larger. In the middle panel, we take the ratio of the resonant $Z_1 \to W_I^{(\dagger)} A_I$ production cross section at the 13 TeV LHC to that for dileptons as given in the LRM for reference here assuming that no other additional new Z_1 decay modes exist for simplicity (and thus avoiding a reduced branching fraction) as well as $\kappa = c_I/c_w = 1$ and, to be a bit conservative, we will also take $f\mathcal{R}^2 = 0.1$ for purposes of demonstration. Finally, combining these results, we can obtain the corresponding resonant $(W_I + W_I^\dagger)A_I$ production cross section for specific values of (γ, R) as a function of the mass of the W_I as is shown for three sample values of these pairs of parameters in the lower panel of Fig. 6, again with the assumption that $f\mathcal{R}^2 = 0.1$. Note, in particular, the very large result obtained in the case of $R = \gamma = 3$; this is due not only to the enhanced Z_1 couplings one finds for these parameter choices, but also the fact that, for a fixed value of M_{Z_1}, a larger value of M_{W_I} is obtained and here we are displaying the cross section as a function of this variable, *not* M_{Z_1}. These results compare quite favorably with those shown in the top panel of Fig. (13) in Ref. [26] for the nonresonant process mediated by quarklike PM (to which this new resonant contribution would be added) which was obtained under somewhat different assumptions.

Clearly, with such a rather complex setup with multiple moving parts in the gauge, fermion, and scalar sectors, many more interesting processes which can be probed at colliders will arise; we plan to consider these and other potential signatures in our later work.

IV. DISCUSSION AND CONCLUSIONS

The kinetic mixing portal model allows for the possibility of thermal dark matter in the sub-GeV mass range owing to the existence of a similarly light gauge boson mediator which has naturally suppressed couplings to the fields of the Standard Model. The success of this KM scenario for generating the interactions of SM fields with DM rests upon the existence of a new set of portal matter fields, at least some of which may naturally lie at the ~TeV scale [31], which carry both SM and dark charges thus allowing for the generation of the link between the ordinary and dark gauge fields at the 1-loop level via vacuum polarizationlike graphs. In the simplest Abelian realization of this possibility, the dark gauge group, $G_{\rm Dark}$, is just the $U(1)_D$ associated with the dark photon; SM fields are all neutral under $U(1)_D$, i.e., they have $Q_D = 0$ and so do not couple directly to the DP except via KM. However, the existence of PM indicates that a larger gauge structure of some kind for $G_{\rm Dark}$ is likely present and one may then ask how $G_{\rm SM}$ and this enlarged $G_{\rm Dark}$ might fit together into a more unified framework. Clearly, at least a partial answer to this question can be found through an understanding of the detailed nature and possible properties of the various scalar and/or fermionic PM fields themselves and an examination of other impacts that their existence might have beyond their essential role in the generation of KM.

In past work, we have begun an examination of the interplay of PM and a simple non-Abelian version of $G_{\rm Dark}$ together with the SM following both complementary bottom-up and top-down approaches in an effort to gain insight into these and related issues given a minimal set of model building requirements (which included a finite and calculable value for the usual KM parameter, ϵ, [25,26,28–34]. Among the findings from this set of analyses are that (i) It is likely for at least some of the SM and PM fields lie in common representations of $G_{\rm Dark}$, the simplest example of which, consistent with our constraints and the one employed here, being the SM-like $SU(2)_I \times U(1)_{Y_I}$ setup which breaks down to $U(1)_D$ at a mass scale essentially the same as that at which the PM fields acquire their masses. Again, paralleling the SM, this is achieved by having the PM masses generated by Yukawa couplings to dark Higgs fields whose vevs are also responsible for the breaking of $G_{\rm Dark}$. It was also found that (ii) It is possible to relate phenomenological issues in both the visible and dark sectors, e.g., the magnitude of a possible upward shift in the mass of W relative to SM expectations, as measured by CDF II [57], can be related to the mass of the DP while also satisfying the other model constraints. (iii) In a top-down study based upon the assumption of a unification of $G_{\rm SM} \times G_{\rm Dark}$ in a single, though hardly unique, $SU(N)$ gauge group [34], it was found that all of our model building constraints could not be satisfied when both $G_{\rm SM}$ and $G_{\rm Dark}$ take their "minimal" forms. (iv) It was shown that there are some possible model building gains to be made when addressing various experimental puzzles by also extending $G_{\rm SM}$ beyond the usual $3_c 2_L 1_Y$ while also simultaneously considering a non-Abelian $G_{\rm Dark}$ as was done earlier [29] to relate the dark sector and the KM mechanism with the flavor/mixing problem. By employing a simpler, single generation version of this same model, in this paper we have begun to examine the possible relationship between the masses of the portal matter fields and the masses of the right-handed neutrino as well as the new spin-1 fields associated with both its visible and dark extended gauge sectors when the symmetries of the SM are replaced by those of the Pati-Salam/left-right symmetric model, i.e., $G = 4_c 2_L 2_R 2_I 1_{Y_I}$ or, more simply below the color breaking scale of $M_c \gtrsim 10^6$ TeV which concerns us here, just $G_{\rm eff} = 3_c 2_L 2_R 2_I 1_{Y_I} 1_{B-L}$.

Among the many immediate implications of and results obtained from this setup that we have examined above are that (a) It is $1_{Y_I} 1_{B-L}$ that undergo Abelian kinetic mixing at the ∼few TeV scale. (b) Left-right symmetry plus anomaly cancellation requires the set of fermionic PM fields to transform as a complete vectorlike family under both the SM/LRM as well as the $U(1)_D$ symmetries and this also leads to a finite and calculable value for KM strength parameter ϵ of the desired magnitude, $\sim 10^{-(3-4)}$. (c) All of the usual chiral SM/LRM fermion fields (which still carry $Q_D = 0$) lie in doublets of $SU(2)_I$ together with a corresponding PM field (which has $Q_D = -1$) with which they share their QCD and electroweak quantum numbers. At the TeV scale these two sets of fields are connected via the exchange of the neutral, non-Hermitian gauge bosons of $SU(2)_I$, $W_I^{(\dagger)}$; however, the DP also couples these two sets of fields at the sub-GeV scale but in a suppressed manner yielding the dominant PM decay path. (d) As usual, at low energies the DP couples diagonally to the SM via KM as $\simeq e\epsilon Q_{\rm em}$ and, as occurs frequently in many setups, also proportional to the SM Z couplings via mass mixing through a small mixing angle of the same order as ϵ. (e) If the standard RH-triplet Higgs fields are employed to break the LR symmetry and generate a heavy Majorana seesaw mass for the RH-neutrino via a $|B - L| = 2$ vev, since these RH-triplet Higgs are *additionally* required to be $SU(2)_I$ triplets, they will necessarily also lead to the breaking of $2_I 1_{Y_I} \to 1_D$ at the same mass scale. The extra Higgs scalars generating the Dirac masses of the charged PM fermions will also contribute to this same symmetry breaking. (f) The same bitriplet Higgs representations also contain vevs carrying both $|Q_D| = 1, 2$, the later of which contributes to a tiny splitting in the masses of each of the two heavy neutral Dirac PM states forming pairs of pseudo-Dirac fields. (g) Loops of PM and W_I gauge bosons can realize potentially important dark dipole momentlike couplings of the SM fermions to the DP, making possibly substantial alterations in the associated phenomenology, as suggested in previous work. (h) The non-Hermitian, W_R and W_I gauge bosons have properties which are semi-quantitatively not too dissimilar from those encountered in the usual LRM and in the simpler scenario explored in Ref. [26] where the important mixing of the DP with the Hermitian combination $W_I + W_I^\dagger$ was previously noted. However, due to the mixing of the SM and PM fields at the $\sim 10^{-(3-4)}$ level some novel and yet to be explored new effects are possible. (i) The two new heavy neutral gauge bosons present in this setup, $Z_{R,I}$, generally undergo substantial mixing into the $Z_{1,2}$ mass eigenstates, one of which is always heavier (lighter) than the corresponding pure LRM Z_R "reference" state with generally stronger (weaker) couplings given the same input parameter values. Making some reasonable model assumptions for purposes of demonstration, estimates were obtained for the lower bounds on the masses of both of these states from existing ATLAS dilepton resonance search data and then these reaches were extrapolated to obtain the corresponding mass reaches for the 100 TeV FCC-hh under an identical set of assumptions.

The extension of the SM gauge group to the LRM in addition to the existence of a non-Abelian symmetry for the dark sector provides a phenomenologically rich and interesting direction to explore in our search for a more UV-complete model of the gauge interactions of the visible and dark sectors. Further steps in this direction will be taken in future work.

ACKNOWLEDGMENTS

The author would like to particularly thank J. L. Hewett and G. Wojcik for valuable discussions during the early aspects of this work. This work was supported by the Department of Energy, Contract DE-AC02-76SF00515.

[1] N. Aghanim *et al.* (Planck Collaboration), Astron. Astrophys. **641**, A6 (2020); **652**, C4(E) (2021).
[2] M. Kawasaki and K. Nakayama, Annu. Rev. Nucl. Part. Sci. **63**, 69 (2013).
[3] P. W. Graham, I. G. Irastorza, S. K. Lamoreaux, A. Lindner, and K. A. van Bibber, Annu. Rev. Nucl. Part. Sci. **65**, 485 (2015).
[4] I. G. Irastorza and J. Redondo, Prog. Part. Nucl. Phys. **102**, 89 (2018).
[5] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo, and F. S. Queiroz, Eur. Phys. J. C **78**, 203 (2018).
[6] L. Roszkowski, E. M. Sessolo, and S. Trojanowski, Rep. Prog. Phys. **81**, 066201 (2018).
[7] K. Pachal, Dark matter searches at ATLAS and CMS, in *Proceedings of the 8th Edition of the Large Hadron Collider Physics Conference* (2020).

[8] E. Aprile *et al.* (XENON Collaboration), Phys. Rev. Lett. **121**, 111302 (2018).
[9] A. Albert *et al.* (Fermi-LAT and DES Collaborations), Astrophys. J. **834**, 110 (2017).
[10] C. Amole *et al.* (PICO Collaboration), Phys. Rev. D **100**, 022001 (2019).
[11] J. Aalbers *et al.* (LZ Collaboration), arXiv:2207.03764.
[12] J. Alexander *et al.*, arXiv:1608.08632.
[13] M. Battaglieri *et al.*, arXiv:1707.04591.
[14] G. Bertone and Tim M. P. Tait, Nature (London) **562**, 51 (2018).
[15] J. Cooley, T. Lin, W. H. Lippincott, T. R. Slatyer, T. T. Yu, D. S. Akerib, T. Aramaki, D. Baxter, T. Bringmann, R. Bunker *et al.*, arXiv:2209.07426.
[16] A. Boveia, T. Y. Chen, C. Doglioni, A. Drlica-Wagner, S. Gori, W. H. Lippincott, M. E. Monzani, C. Prescod-Weinstein, B. Shakya, T. R. Slatyer *et al.*, arXiv:2210.01770.
[17] P. Schuster, N. Toro, and K. Zhou, Phys. Rev. D **105**, 035036 (2022).
[18] B. Holdom, Phys. Lett. **166B**, 196 (1986); Phys. Lett. B **178**, 65 (1986); K. R. Dienes, C. F. Kolda, and J. March-Russell, Nucl. Phys. **B492**, 104 (1997); F. Del Aguila, Acta Phys. Pol. B **25**, 1317 (1994); K. S. Babu, C. F. Kolda, and J. March-Russell, Phys. Rev. D **54**, 4635 (1996); T. G. Rizzo, Phys. Rev. D **59**, 015020 (1998).
[19] There has been a huge amount of work on this subject; see, for example, D. Feldman, B. Kors, and P. Nath, Phys. Rev. D **75**, 023503 (2007); D. Feldman, Z. Liu, and P. Nath, Phys. Rev. D **75**, 115001 (2007); M. Pospelov, A. Ritz, and M. B. Voloshin, Phys. Lett. B **662**, 53 (2008); M. Pospelov, Phys. Rev. D **80**, 095002 (2009); H. Davoudiasl, H. S. Lee, and W. J. Marciano, Phys. Rev. Lett. **109**, 031802 (2012); Phys. Rev. D **85**, 115019 (2012); R. Essig *et al.*, arXiv:1311.0029; E. Izaguirre, G. Krnjaic, P. Schuster, and N. Toro, Phys. Rev. Lett. **115**, 251301 (2015); M. Khlopov, Int. J. Mod. Phys. A **28**, 1330042 (2013); For a general overview and introduction to this framework, see D. Curtin, R. Essig, S. Gori, and J. Shelton, J. High Energy Phys. 02 (2015) 157.
[20] T. Gherghetta, J. Kersten, K. Olive, and M. Pospelov, Phys. Rev. D **100**, 095001 (2019).
[21] G. Steigman, Phys. Rev. D **91**, 083538 (2015).
[22] K. Saikawa and S. Shirai, J. Cosmol. Astropart. Phys. 08 (2020) 011.
[23] M. Fabbrichesi, E. Gabrielli, and G. Lanfranchi, *The Physics of the Dark Photon* (Springer, Cham, 2021).
[24] M. Graham, C. Hearty, and M. Williams, Annu. Rev. Nucl. Part. Sci. **71**, 37 (2021).
[25] T. G. Rizzo, Phys. Rev. D **99**, 115024 (2019).
[26] T. D. Rueter and T. G. Rizzo, Phys. Rev. D **101**, 015014 (2020).
[27] J. H. Kim, S. D. Lane, H. S. Lee, I. M. Lewis, and M. Sullivan, Phys. Rev. D **101**, 035041 (2020).
[28] T. D. Rueter and T. G. Rizzo, arXiv:2011.03529.
[29] G. N. Wojcik and T. G. Rizzo, Phys. Rev. D **105**, 015032 (2022).
[30] T. G. Rizzo, J. High Energy Phys. 11 (2021) 035.
[31] T. G. Rizzo, arXiv:2202.02222.
[32] G. N. Wojcik, arXiv:2205.11545.
[33] T. G. Rizzo, Phys. Rev. D **106**, 035024 (2022).
[34] T. G. Rizzo, Phys. Rev. D **106**, 095024 (2022).
[35] G. N. Wojcik, L. L. Everett, S. T. Eu, and R. Ximenes, arXiv:2211.09918.
[36] A. Carvunis, N. McGinnis, and D. E. Morrissey, J. High Energy Phys. 01 (2023) 014.
[37] S. Verma, S. Biswas, A. Chatterjee, and J. Ganguly, arXiv:2209.13888.
[38] T. R. Slatyer, Phys. Rev. D **93**, 023527 (2016).
[39] H. Liu, T. R. Slatyer, and J. Zavala, Phys. Rev. D **94**, 063507 (2016).
[40] R. K. Leane, T. R. Slatyer, J. F. Beacom, and K. C. Ng, Phys. Rev. D **98**, 023016 (2018).
[41] For related work on the possibilities of KM and DM physics employing this same gauge group, see M. Bauer and P. Foldenauer, Phys. Rev. Lett. **129**, 171801 (2022).
[42] See, for example, F. Gursey, P. Ramond, and P. Sikivie, Phys. Lett. **60B**, 177 (1976); Y. Achiman and B. Stech, Phys. Lett. **77B**, 389 (1978); Q. Shafi, Phys. Lett. **79B**, 301 (1978).
[43] J. L. Hewett and T. G. Rizzo, Phys. Rep. **183**, 193 (1989).
[44] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); **11**, 703(E) (1975).
[45] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975).
[46] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 2558 (1975).
[47] G. Senjanovic and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975).
[48] R. N. Mohapatra, *Unification and Supersymmetry* (Springer, New York, NY, 1986).
[49] For a recent analysis of this mass scale, see for example, T. P. Dutka and J. Gargalionis, Phys. Rev. D **107**, 035019 (2023).
[50] See, for example, T. G. Rizzo, arXiv:hep-ph/0610104.
[51] M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985); B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D **16**, 1519 (1977); J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974); **11**, 972(E) (1975); G. J. Gounaris, R. Kogerler, and H. Neufeld, Phys. Rev. D **34**, 3257 (1986).
[52] Such heavy neutral lepton states have been discussed in a number of different contexts; see for example, A. Das, P. S. Bhupal Dev, and N. Okada, Phys. Lett. B **735**, 364 (2014); A. de Gouvea, W. C. Huang, and J. Jenkins, Phys. Rev. D **80**, 073007 (2009); G. Anamiati, M. Hirsch, and E. Nardi, J. High Energy Phys. 10 (2016) 010; P. Hernández, J. Jones-Pérez, and O. Suarez-Navarro, Eur. Phys. J. C **79**, 220 (2019); D. Chang and O. C. W. Kong, Phys. Lett. B **477**, 416 (2000); S. Bahrami, M. Frank, D. K. Ghosh, N. Ghosh, and I. Saha, Phys. Rev. D **95**, 095024 (2017).
[53] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **796**, 68 (2019).
[54] C. Helsens, D. Jamin, M. L. Mangano, T. G. Rizzo, and M. Selvaggi, Eur. Phys. J. C **79**, 569 (2019).
[55] See, for example, A. M. Sirunyan *et al.* (CMS Collaboration), Phys. Lett. B **820**, 136535 (2021); G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 03 (2020) 145; A. Tumasyan *et al.* (CMS Collaboration), J. High Energy Phys. 04 (2022) 047; G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. D **100**, 052013 (2019); A. Tumasyan *et al.* (CMS Collaboration), J. High Energy

Phys. 07 (2022) 067; A. M. Sirunyan *et al.* (CMS Collaboration), J. High Energy Phys. 05 (2020) 033; ATLAS Collaboration, Search for vector boson resonances decaying to a top quark and a bottom quark in the hadronic final state using pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, Report No. ATLAS-CONF-2021-043.

[56] ATLAS Collaboration, Combination of searches for heavy resonances using 139 fb^{-1} of proton–proton collision data at $\sqrt{s} = 13$ TeV with the ATLAS detector, Report No. ATLAS-CONF-2022-028.

[57] T. Aaltonen *et al.* (CDF Collaboration), Science **376**, 170 (2022).