

# Phenomenology of a gauge boson triplet with hypercharge one

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A vector boson  $W_1^\mu$  with the quantum numbers  $(\mathbf{3}, 1)$  under  $SU(2)_L \times U(1)_Y$  could in principle couple with the Higgs field via the renormalizable term  $W_1^{\mu*} H D_\mu H$ . This interaction is known to affect the  $T$  parameter and, in so doing, it could potentially explain the recent CDF measurement of the  $W$ -boson mass. As it is often the case with vectors, building a viable model with a  $W_1$  gauge boson is nontrivial. In this work I will describe two variations of a minimal setup containing this field; they are based on an extended  $SO(5) \times SU(2) \times U(1)$  electroweak group. I will nevertheless show that interactions such as  $W_1^{\mu*} H \partial_\mu H$  are not generated in a Yang-Mills theory. A coupling between  $W_1$ ,  $H$  and another Higgs doublet  $H'$  is possible though. Finally, I will provide an explicit recipe for the construction of viable models with gauge bosons in arbitrary representations of the Standard Model group which may couple to a Standard Model fermion and an exotic one.

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## I. INTRODUCTION

There is abundant speculation on what may lie beyond the Standard Model (SM). Out of many approaches, there have been attempts to systematically parametrize the effect of new particles without reference to specific models. For example, one may look for those fields which can interact with Standard Model particles—such as a pair of fermions—in a renormalizable way [1,2]. At low energies, they give rise to four-fermion interactions much like those in the Fermi theory, which led to the discovery of weak interactions.

A field which has received fairly little attention is  $W_1$ , with the  $SU(2)_L \times U(1)_Y$  quantum numbers  $(\mathbf{3}, 1)$  and transforming as a 4-vector under the Lorentz group. It can couple only to the Higgs doublet, doing so via the interaction [1–3]

$$\frac{\kappa}{2} W_1^{\mu,a*} H^T (i\sigma_2 \sigma_a) D_\mu H. \quad (1)$$

Once integrated out, the fermionphobic  $W_1^\mu$  gives rise to the effective dimension six interaction

$$-\frac{|\kappa|^2}{4m_{W_1}^2} (\mathcal{O}_H^1 + \mathcal{O}_H^3) \quad (2)$$

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with  $\mathcal{O}_H^1 \equiv [(D^\mu H)^\dagger (D_\mu H)](H^\dagger H)$  and  $\mathcal{O}_H^3 \equiv |H^\dagger (D^\mu H)|^2$ . In turn, this last operator contributes to the  $\hat{T} = \alpha T$  parameter [4–6] as follows:

$$\hat{T} \equiv \frac{\Pi_{W^3 W^3}(0) - \Pi_{W^+ W^-}(0)}{m_W^2} = \frac{|\kappa|^2 v^2}{4 m_{W_1}^2};$$

$$v^2 \equiv \langle H^\dagger H \rangle \approx (174 \text{ GeV})^2. \quad (3)$$

Recently, the CDF collaboration published a surprising large value of the  $W$  mass [7],

$$m_W = 80433.5 \pm 9.4 \text{ MeV}, \quad (4)$$

which seems to be well fitted by  $\hat{T} \approx (8.8 \pm 1.4) \times 10^{-4}$  [8] (see also [9]). This would correspond to a value of  $m_{W_1}/|\kappa| \approx 2.9 \text{ TeV}$  [3], which is not ruled out by LHC searches. The authors of [10] find that  $W_1$  is one of the single-field extensions of the Standard Model which best fits the CDF data, adding nonetheless that this field is not commonly found in unified gauge theories. Notice that unlike a  $Z'$  or a  $W'$ , a single  $W_1$  cannot be produced in a proton-proton collision, neither through vector-boson fusion nor through the Drell-Yan process. For this reason, searches for new vector fields by ATLAS and CMS [11–14]—which in some cases exclude masses up to 5 TeV—do not apply to  $W_1$ . On the other hand, mass limits from pair production searches reach at most 1 to 2 TeV (see for instance [15,16]) for colored fields. For uncolored ones, the LHC reach is even lower, as can be seen from the limits on the scalar analog of  $W_1$ , which is present in seesaw type-II mechanism for neutrino masses [17,18].

It is far from clear that this new determination of the  $W$  mass will resist the test of time, and in fact the CDF measurement is in tension with other direct and indirect determinations of  $m_W$  [10,19,20]. Nonetheless—independently of the validity of this result—it is worth considering in detail the phenomenology associated to a  $W_1$  vector, which will inevitably depend on the ultraviolet origin of the field. To my knowledge, there have been no previous attempts of incorporating this vector in a complete model. With that in mind, in this work I will present a minimal setup where  $W_1$  is a gauge boson associated to an extended electroweak group, which is spontaneously broken at the TeV scale or above. Another possibility, not considered here, is that  $W_1$  is a composite field rather than a fundamental one.

It turns out that the trilinear interaction between a gauge field and two scalars,  $\phi$  and  $\phi'$ , must be anti-symmetric under the exchange of the scalars. This means that for  $\phi = \phi' = H$ , which is an  $SU(2)$  doublet, there can be no coupling to the triplet  $W_1$ . I will discuss this point in Sec. II, after which I will detail a realistic model for  $W_1$  based on the gauge group  $SO(5) \times SU(2) \times U(1)$  (Sec. III). In it the new vector does not couple to matter; however, one can modify the setup so that it does (Sec. IV).

Looking beyond  $W_1$ , it is not straightforward to build Yang-Mills theories with gauge bosons transforming according to random representations of the Standard Model group. In many cases, there might be no model in the literature containing such vector fields. To improve on this situation, in Sec. V I describe an explicit recipe for building models with gauge bosons assigned to arbitrary representations of the Standard Model group. Section VI summarizes the main conclusions in this work.

## II. $W_1^{\mu*} H D_\mu H$ AND SIMILAR COUPLINGS IN A YANG-MILLS THEORY

In order for a gauge boson to potentially couple to the combination  $H D_\mu H$ , rather than to  $H^* D_\mu H$ , it must be that the Higgs is a linear combination of at least two fields—let us call them  $H_u = (2, 1/2)$  and  $H_d = (2, -1/2)$ —which are part of the same irreducible representation  $\Omega$  of the gauge group. Then, from the kinetic term of this last field, one might hope to get the sought after interaction:

$$(D^\mu \Omega)^\dagger (D_\mu \Omega) \propto \dots + (W_1^{\mu*} H_d^* D_\mu H_u + \text{H.c.}) \\ \propto \dots + (W_1^{\mu*} H D_\mu H + \text{H.c.}) \quad (5)$$

I have used here the proportionality sign to avoid distractions with the prefactors of each expression; likewise I also did not track carefully how the  $SU(2)$  indices are contracted. However, by tracking attentively relative signs, the reader will see that while one can indeed achieve a  $W_1^{\mu*} H_d^* D_\mu H_u$  coupling which goes on to contribute to the interaction  $W_1^{\mu*} H D_\mu H$ , we must also take into account a term

$W_1^{\mu*} H_u D_\mu H_d^*$  with the opposite effect. All in all, this means that the prefactor of the Higgs-Higgs- $W_1$  coupling is null.

Perhaps—one might think—this cancellation is specific to the minimalist scalar setup described above. That is not true: I will argue in the following that in a Yang-Mills theory an interaction  $\mathcal{A}^\mu \phi \phi'$  between a gauge boson  $\mathcal{A}^\mu$  and two scalars  $\phi$  and  $\phi'$ , through a derivative, must be antisymmetric under an exchange of these two scalars. Since in our particular example the  $SU(2)$  quantum numbers force the two  $H$ 's to be contracted symmetrically with the triplet  $W_1$ , it must be that the coefficient of the interaction is zero, regardless of the details of the model.

To see this, we may start by decomposing all scalars in a model in real components, and collecting them in a column vector  $\Phi (= \Phi^*)$ . Any gauge transformation can be represented through a matrix  $\exp(i\varepsilon_a T_a)$  which must be both real and unitary, hence

$$T_a = T_a^\dagger = -T_a^* \quad (6)$$

These anti-symmetric  $T_a$  generators regulate the interaction<sup>1</sup>

$$\Phi^T (ig T_a \mathcal{A}_a^\mu) (D_\mu \Phi) \quad (7)$$

obtained from the kinetic term  $(D^\mu \Phi)^T (D_\mu \Phi)/2$ , so in any other basis (such as the electroweak one, or perhaps the mass basis), with  $\Phi = U \Phi'$ , the all-important matrices  $ig U^T T_a U$  remains antisymmetric. Therefore, for any pair of irreducible representations  $\phi$  and  $\phi'$  of the gauge group, we extract from the off-diagonal part of  $ig U^T T_a U$  a term

$$\mathcal{A}_a^\mu [\phi^T X_a (D_\mu \phi') - \phi'^T X_a (D_\mu \phi)] \quad (8)$$

for some real matrices  $X_a$ . In the particular case when  $\phi = \phi'$  we may write the interaction as

$$\mathcal{A}_a^\mu [\phi^T X_a (D_\mu \phi)] \quad \text{with} \quad X_a = -X_a^T; \quad (9)$$

the  $X_a$  are nothing but diagonal blocks of the bigger  $ig U^T T_a U$  matrices mentioned above. The last expression shows explicitly that the gauge indices of the two  $\phi$ 's must contract antisymmetrically: a coupling  $B_1^{\mu*} H D_\mu H$  with the field  $B_1 = (1, 1)$  is fine, given that two doublets contract anti-symmetrically to form a singlet; by an analogous argument, a coupling  $W_1^{\mu*} H D_\mu H$  is not.

I have followed the usual method of building gauge boson interactions, expressing them in terms of the  $T_a$  matrices. This in turn led us to antisymmetric  $X_a$ 's. But, to be clear, one can check in a more direct way that any symmetric component of the  $X_a$ 's would break gauge invariance. Indeed the relation

<sup>1</sup>There is more than one coupling constant  $g$  if the gauge group is semisimple. Nonetheless, such complication is of no consequence to the present discussion.

$$\mathcal{A}_a^\mu \phi^T (X_a + X_a^T) (\partial_\mu \phi) = \partial_\mu (\mathcal{A}_a^\mu \phi^T X_a \phi) - (\partial_\mu \mathcal{A}_a^\mu) \phi^T X_a \phi, \quad (10)$$

shows that—up to a total derivative—the leftmost expression is the same as  $-(\partial_\mu \mathcal{A}_a^\mu) \phi^T X_a \phi$ , which is not invariant under a general local gauge transformation. A hypothetical symmetric component of the  $X_a$  generated through loops must therefore be finite and gauge dependent.

As a complement to the above arguments, let us consider the spinor-helicity formalism, where it is straightforward to compute the scattering amplitude for a spin 1 field (particle #1) and two scalars (particles #2 and #3) when all three are massless. Poincaré invariance and unitarity are the only extra assumptions. In the widely used bracket notation, depending on the helicity of the spin 1 particle, the amplitude is proportional to either

$$\frac{\langle 12 \rangle \langle 31 \rangle}{\langle 23 \rangle} \quad \text{or} \quad \frac{[12][31]}{[23]}. \quad (11)$$

In both cases, permuting the two scalars ( $2 \leftrightarrow 3$ ) yields a minus sign so the spinor-helicity formalism corroborates the antisymmetry of a  $\mathcal{A}^\mu \phi \phi'$  interaction. To reach this result, all fields we assumed to be massless. However, this assumption is unnecessary: the spinor-helicity formalism can be extended to massive particles [21], and doing so one reaches the same conclusion as in the massless case (see for example [21–23]).

In conclusion, a fundamental field  $W_1$  which gets a mass through the Higgs mechanism cannot have a  $W_1^{\mu*} H D_\mu H$  coupling; such an interaction is absent if  $W_1$  is a gauge boson.

With this advance warning, I will proceed to describe the basic features of a minimal extension of the Standard Model where this field appears.

### III. A MODEL FOR $W_1^\mu$

Since  $W_1$  is charged under both  $SU(2)_L$  and  $U(1)_Y$ , this field as well as the Standard Model  $W$  and  $B$  must be gauge bosons associated to some group which includes the electroweak one,  $SU(2)_L \times U(1)_Y$ . The adjoint representation of such a group must be of size at least  $6 + 3 + 1 = 10$ , considering that this is the total number of real field components in  $W_1$ ,  $W$  and  $B$ . It turns out that the adjoint representation of the group  $SO(5)$  [whose algebra is isomorphic to  $Sp(4)$ ] is precisely 10 dimensional. Furthermore,  $SU(2) \times U(1)$  is a subgroup of  $SO(5)$ , and under it the spinor representation branches as follows<sup>2</sup>:

<sup>2</sup>In fact,  $SO(5)$  has two inequivalent  $SU(2) \times U(1)$  subgroups. The other embedding is associated to the branching rule  $\mathbf{4} \rightarrow (\mathbf{1}, 1/2) + (\mathbf{1}, -1/2) + (\mathbf{2}, 0)$ , which is not relevant for the present work.

$$\mathbf{4} \rightarrow (\mathbf{2}, 1/2) + (\mathbf{2}, -1/2). \quad (12)$$

The adjoint decomposes in the manner alluded above, namely

$$\mathbf{10} \rightarrow (\mathbf{1}, 0) + (\mathbf{3}, 0) + \underbrace{(\mathbf{3}, 1) + (\mathbf{3}, -1)}_{W_1}. \quad (13)$$

Note that the decomposition of the spinor representation implies that a scalar field transforming as a  $\mathbf{4}$  contains both an  $H_u$ - and an  $H_d$ -like field, which is precisely what one needs to generate a  $H - H' - W_1^\mu$  coupling, as discussed in the previous section.

So far everything looks promising. Nevertheless, when it comes to fermions, it is challenging to charge them non-trivially under  $SO(5)$ . For example, the left-handed leptons  $L = (\mathbf{2}, -1/2)$  necessarily interact via the  $W_1$  gauge bosons with fermions whose charges are  $(\mathbf{2}, -3/2)$ ,  $(\mathbf{2}, 1/2)$ ,  $(\mathbf{4}, -3/2)$ , or  $(\mathbf{4}, 1/2)$ , none of which are part of the Standard Model. The same thing happens with quarks, and therefore one would need to find vectorlike partners for these new fields in order to give them masses above those of  $\langle H \rangle \approx 174$  GeV. With the help of an extra  $U(1)$  which would provide more flexibility in forming the SM hypercharge group, one can certainly find  $SO(5) \times U(1)$  representations with the sought-after fermions, however these tend to propagate the problem by introducing further chiral states.

We are therefore guided to the possibility that no chiral fermion is charged under  $SO(5)$ , and instead the full electroweak group is  $SO(5) \times SU(2) \times U(1)$ , which contains  $SU(2)' \times U(1)' \times SU(2) \times U(1)$ ; in turn, its diagonal subgroup is  $SU(2)_L \times U(1)_Y$ :

$$SO(5) \times SU(2) \times U(1) \rightarrow \underbrace{SU(2)' \times SU(2)}_{\supset SU(2)_L} \times \underbrace{U(1)' \times U(1)}_{\supset U(1)_Y}. \quad (14)$$

The adjoint representation of the extended electroweak group includes the  $SU(2)_L \times U(1)_Y$  representations  $B$ ,  $Z' = (\mathbf{1}, 0)$ ,  $W$ ,  $W' = (\mathbf{3}, 0)$  and  $W_1 = (\mathbf{3}, 1)$ . These gauge bosons acquire a mass proportional to the  $SO(5)$ -symmetry breaking scale, except for the Standard Model  $B$  and  $W$ , which are less massive. The latter fields are a mixture of the  $SO(5)$  gauge bosons with those of  $SU(2) \times U(1)$ .

Note also that a scalar  $\Omega$  transforming as a spinor under  $SO(5)$  would couple to  $W_1$ , but not to fermions since they are uncharged under this group. Consequently, in order to have Yukawa interactions one also needs an  $SO(5)$ -singlet scalar  $\hat{H}$ . Finally, the extended electroweak group can be broken to  $SU(2)_L \times U(1)_Y$  with a nonzero vacuum expectation value (VEV) of some field  $\chi$ .

With the above general considerations, we are in a position to flesh out a model. The requirements discussed

TABLE I. The quantum numbers of the three scalars in the model, under the extended electroweak group  $SO(5) \times SU(2) \times U(1)$ . A nonzero vacuum expectation value of  $\chi$  can break this symmetry down to  $SU(2)_L \times U(1)_Y$ . The transformation properties of the scalars under this latter group are shown in the last column. All fermions transform trivially under  $SO(5)$ .

Scalar	$SO(5) \times SU(2) \times U(1)$	$SU(2)_L \times U(1)_Y$ decomposition
$\Omega$	$(\mathbf{4}, \mathbf{1}, 0)$	$(\mathbf{2}, -\frac{1}{2}) + (\mathbf{2}, \frac{1}{2})$
$\hat{H}$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{2}, \frac{1}{2})$
$\chi$	$(\mathbf{4}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, 0) + (\mathbf{1}, 1) + (\mathbf{3}, 0) + (\mathbf{3}, 1)$

earlier on the scalar sector are fully met by the fields in Table I. In terms of Standard Model  $SU(2)_L \times U(1)_Y$  representations,  $\chi$  contains a component with quantum numbers  $(\mathbf{1}, 0)$ ; as we shall see, its VEV preserves only the subgroup  $SU(2)_L \times U(1)_Y$  of  $SO(5) \times SU(2) \times U(1)$ . Note also that there is a total of 3 Higgs doublets in  $\Omega$  and  $\hat{H}$ , which can mix to produce the Standard Model field  $H$ .

Let us now establish a set of generators of the 4-dimensional representation of  $SO(5)$ . Notice again that this group is isomorphic to  $Sp(4)$ , which can be defined via

$$\varepsilon^a T^a \equiv \frac{1}{2} \begin{pmatrix} \varepsilon_3 + \varepsilon_{10} & \varepsilon_1 - i\varepsilon_2 & \varepsilon_8 - i\varepsilon_5 & \sqrt{2}(\varepsilon_6 - i\varepsilon_9) \\ \varepsilon_1 + i\varepsilon_2 & -\varepsilon_3 + \varepsilon_{10} & \sqrt{2}(\varepsilon_4 - i\varepsilon_7) & -\varepsilon_8 + i\varepsilon_5 \\ \varepsilon_8 + i\varepsilon_5 & \sqrt{2}(\varepsilon_4 + i\varepsilon_7) & \varepsilon_3 - \varepsilon_{10} & \varepsilon_1 - i\varepsilon_2 \\ \sqrt{2}(\varepsilon_6 + i\varepsilon_9) & -\varepsilon_8 - i\varepsilon_5 & \varepsilon_1 + i\varepsilon_2 & -\varepsilon_3 - \varepsilon_{10} \end{pmatrix}. \quad (18)$$

If we throw away  $\varepsilon_{4,\dots,9}$ , this becomes a block-diagonal matrix and in fact  $T^{1,2,3}$  are generators of an important  $SU(2)'$  subgroup [see expression (14)], with Pauli matrices on the diagonal blocks;  $T^{10}$  generates a  $U(1)'$  which commutes with this  $SU(2)'$ .

The scalar  $\chi = (\mathbf{4}, \mathbf{2}, \frac{1}{2})$  can be seen as a two index field  $\chi_{jk}$  with  $SO(5)$  acting on  $j$  and  $SU(2)$  on  $k$ . With this understanding,

$$D_\mu \chi_{jk} = \partial_\mu \chi_{jk} + i g_A A_\mu^{a,SO(5)} T_{jj'}^a \chi_{j'k} + \frac{i}{2} g_B A_\mu^{b,SU(2)} \sigma_{kk'}^b \chi_{jk'} + \frac{1}{2} g_C A_\mu^{U(1)} \chi_{jk}. \quad (19)$$

The vacuum expectation value (VEV)

the set of 4-dimensional matrices  $G$  satisfying the relation  $G^T J G = J$ ;  $J$  is a nonsingular antisymmetric matrix which is often taken to have the block form

$$J = \begin{pmatrix} \mathbf{0} & \mathbf{1}_2 \\ -\mathbf{1}_2 & \mathbf{0} \end{pmatrix}. \quad (15)$$

Rewriting  $G$  as  $\exp(i\varepsilon^a T^a)$  with real  $\varepsilon^a$  parameters—and requiring also that  $G^\dagger G = \mathbf{1}_4$ —leads to infinitesimal generators of the form

$$\varepsilon_a T^a = \begin{pmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^* & -\mathbf{B}^* \end{pmatrix} \quad (16)$$

where  $\mathbf{B}$  and  $\mathbf{C}$  are arbitrary 2-dimensional hermitian and symmetric matrices, respectively. However, it is more convenient to work on a basis where the last two entries of the 4-dimensional space are rotated with the  $\varepsilon = i\sigma_2$  matrix, such that

$$J = \begin{pmatrix} \mathbf{0} & \varepsilon \\ \varepsilon & \mathbf{0} \end{pmatrix}. \quad (17)$$

The 10 generators can then be picked to have the following form<sup>3</sup>:

$$\langle \chi \rangle \propto \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (20)$$

breaks all but 4 linear combinations of the original 14 generators of the group  $SO(5) \times SU(2) \times U(1)$ , namely

$$\begin{aligned} & \left( T_{jj'}^{1,2,3} \delta_{kk'} + \frac{1}{2} \delta_{jj'} \sigma_{kk'}^{1,2,3} \right) \langle \chi \rangle_{j'k'} \\ & = \left( T_{jj'}^{10} \delta_{kk'} + \frac{1}{2} \delta_{jj'} \delta_{kk'} \right) \langle \chi \rangle_{j'k'} = 0. \end{aligned} \quad (21)$$

They generate the  $SU(2)_L$  diagonal subgroup of  $SU(2) \times SU(2)'$ , together with  $U(1)_Y$  which is the diagonal subgroup of  $U(1) \times U(1)'$ . It follows directly from these

<sup>3</sup>This is  $\sqrt{2}$  times the matrices given by RepMatrices [SO5, 4] in GroupMath [24], after a reordering.



relations that the Standard Model gauge couplings  $g$  and  $g'$  are given by the expressions

$$g^{-2} = g_A^{-2} + g_B^{-2}, \quad (22)$$

$$(g')^{-2} = g_A^{-2} + g_C^{-2}. \quad (23)$$

We can also extract from  $(D_\mu \langle \chi \rangle_{jk})^* D_\mu \langle \chi \rangle_{jk}$  the leading contribution to the various gauge boson masses (the Higgs doublet VEVs produce corrections):

$$m_{W_1}^2 = g_A^2 \langle \chi \rangle^2, \quad (24)$$

$$m_{W'}^2 = (g_A^2 + g_B^2) \langle \chi \rangle^2, \quad (25)$$

$$m_{Z'}^2 = (g_A^2 + g_C^2) \langle \chi \rangle^2, \quad (26)$$

where  $\langle \chi \rangle^2 \equiv \langle \chi \rangle_{jk}^* \langle \chi \rangle_{jk}$ . From Eqs. (22) and (23) plus the known values of couplings at the electroweak scale ( $g \approx 0.65$  and  $g' \approx 0.36$ ) it must be that  $g_C$  is smaller than  $g_B$ , so

$$m_{W_1} < m_{Z'} < m_{W'}. \quad (27)$$

Furthermore, the two independent ratios which can be computed out of the three masses are related by the expression

$$\frac{m_{W_1}^2}{m_{Z'}^2} \left( \frac{m_{W'}^2}{m_{Z'}^2} - \tan^2 \theta_w \right) = (1 - \tan^2 \theta_w) \frac{m_{W'}^2}{m_{Z'}^2} \quad (28)$$

where  $\tan^2 \theta_w \equiv g'^2/g^2 \approx 0.30$ . Notice that while  $m_{W'}/m_{W_1}$  can be arbitrarily large,  $m_{Z'}/m_{W_1}$  is bounded between 1 and  $\sqrt{1/(1 - \tan^2 \theta_w)} \approx 1.19$ . Since  $W_1$ —the lightest new particle—is hard to produce in a collider, the most stringent constraint on this model comes from LHC searches for a  $Z'$  boson with the same couplings to fermions as the SM  $Z$  (that is what happens in the present model). Current data excludes such a particle up to a mass of 5 TeV [25,26].

Besides interacting with other gauge bosons,  $W_1$  couples to scalars. In particular  $\Omega$  contains an up- plus a down-type Higgs doublet,  $H_u = (\mathbf{2}, 1/2)$  and  $H_d = (\mathbf{2}, -1/2)$ , so—as foreseen in Sec. II—from the covariant derivative of  $\Omega$  we get the interactions

$$\frac{g_A}{\sqrt{2}} W_1^{\mu, a*} [H_d^\dagger \sigma_a (D_\mu H_u) - (D_\mu H_d)^\dagger \sigma_a H_u] + \text{H.c.} \quad (29)$$

The fields  $H_u$ ,  $H_d^*$  and  $\hat{H}$  mix, generating the 125 GeV scalar  $H$  of the Standard Model, as well as two heavier doublets:  $H'$  and  $H''$ . No matter what is the form of this

mixing, there will be no  $HH$ ,  $H'H'$  or  $H''H''$  interaction with  $W_1$ . However, we do get the term

$$\kappa_{HH'} W_1^{\mu, a*} [H^T (i\sigma_2 \sigma_a) D_\mu H' - H'^T (i\sigma_2 \sigma_a) D_\mu H] \quad (30)$$

plus similar ones for other combinations of the three scalar doublets.

In this model, fermions do not couple to  $W_1$ . They also do not couple to the  $\chi$  scalar, which is significant because this fields contains an  $(\mathbf{3}, 1)$  representation. These are the quantum numbers of the mediator in the type-II seesaw mechanism, capable of generating neutrino masses. But for that to happen,  $\chi$  would need to interact with leptons, which is not the case. Note also that the model conserves lepton number, hence neutrinos are massless. This changes, for example, with the introduction of an extra scalar with the quantum numbers  $(\mathbf{1}, \mathbf{3}, 1)$  under the extended electro-weak group.

#### IV. AN ALTERNATIVE: CHARGING FERMIONS UNDER $SO(5)$

No fermion is charged under  $SO(5)$  in the model described above. However, on top of the chiral ones, we may introduce vectorlike fermions which do couple to the gauge bosons of this group, without producing dangerous light fields. One possibility is this: for every chiral fermion  $F = Q, u^c, d^c, L, e^c$  with the quantum numbers  $(\mathbf{C}, \mathbf{1}, \mathbf{L}, y)$  under  $SU(3)_C \times SO(5) \times SU(2) \times U(1)$ , we introduce the vectorlike pair of left-handed Weyl spinors  $(\mathbb{F}, \bar{\mathbb{F}})$  with  $\mathbb{F} = (\mathbf{C}, \mathbf{4}, \mathbf{L}, y)$  and its conjugate representation  $\bar{\mathbb{F}} = (\bar{\mathbf{C}}, \mathbf{4}, \mathbf{L}, -y)$ .<sup>4</sup> In other words, apart from transforming as a spinor of  $SO(5)$ , all other quantum numbers of  $\mathbb{F}$  are the same as those of  $F$ . The scalar  $\chi$ , as before, is needed for symmetry breaking; it also participates in Yukawa interactions with the new fermions. Between the two remaining scalars ( $\hat{H}$  and  $\Omega$ ) there is need for just one: in the following I will keep  $\Omega$  (see Table II) since it induces a coupling of  $W_1$  to two scalar doublets. However, since this is not a phenomenological requirement,  $\chi + \hat{H}$  would also lead to a valid scalar sector.

With these charge assignments, we may have the following masses and interactions:

$$\begin{aligned} \sum_F (y_F^\Omega F \bar{\mathbb{F}} \Omega + m_F \mathbb{F} \bar{\mathbb{F}}) + y_{QU}^\chi Q u^c \chi + y_{QU}'^\chi Q u^c \chi \\ + y_{QD}^\chi Q d^c \chi^* + y_{QD}'^\chi Q d^c \chi^* + y_{LE}^\chi L e^c \chi^* \\ + y_{LE}'^\chi L e^c \chi^* + \text{H.c.} \end{aligned} \quad (31)$$

Note that  $\Omega$  needs to be a complex field, so terms of the form  $F \bar{\mathbb{F}} \Omega^*$  are also allowed. However, for simplicity, I will

<sup>4</sup>The spinor representation of  $SO(5)$  is pseudoreal, therefore it is isomorphic to its conjugate. The same is true for any representations of  $SU(2)$ .

TABLE II. Field content of the second model, containing vectorlike fermions ( $\mathbb{F}, \bar{\mathbb{F}}$ ) unlike the first setup. Furthermore, adequate Yukawa interactions can be achieved without the scalar  $\hat{H}$  (compare with Table I).

Field	Spin	$SU(3)_C \times SO(5) \times SU(2) \times U(1)$
$F = Q, u^c, d^c, L, e^c$	1/2	As in the SM; <b>1</b> under $SO(5)$
$\mathbb{F} = \mathbb{Q}, \mathbb{u}^c, \mathbb{d}^c, \mathbb{L}, \mathbb{e}^c$	1/2	As in the SM; <b>4</b> under $SO(5)$
$\bar{\mathbb{F}} = \bar{\mathbb{Q}}, \bar{\mathbb{u}}^c, \bar{\mathbb{d}}^c, \bar{\mathbb{L}}, \bar{\mathbb{e}}^c$	1/2	Complex conjugate of $\mathbb{F}$
$\Omega$	0	<b>(4, 1, 0)</b>
$\chi$	0	<b>(4, 2, <math>\frac{1}{2}</math>)</b>

consider that they has been removed (with a  $Z_4$  symmetry, for example).<sup>5</sup>

Let us now consider what happens under the Standard Model subgroup. The  $F$ 's do not transform under  $SO(5)$ , so for convenience one may use the same name,  $F$ , to designate their quantum numbers under the reduced  $SU(2) \times U(1)$  group. With this understanding, a quick way of grasping all the fermion subrepresentation in  $\mathbb{F}/\bar{\mathbb{F}}$  is to note that  $\Omega = H_u + H_d$ , so  $\bar{\mathbb{F}}$  contains all the states which couple to the product  $FH_u$  as well as all those which couple to  $FH_d$ . This is an unusual two-Higgs doublet model where every Standard Model fermion  $F$  couples to both  $H_u$  and  $H_d$  (not their conjugates), which implies that in some cases the remaining fermion in the interaction must be exotic. This is not a problem since the extra fields can be made heavy via the  $m_F$  mass term in Eq. (31).

Let us consider in the following the lepton sector only. The decomposition of the various representations is as follows:

$$L \rightarrow \underbrace{(2, -1/2)}_{\ell_D}, \quad (32)$$

$$\mathbb{L} \rightarrow (3, 0) + (3, -1) + (1, 0) + \underbrace{(1, -1)}_{\ell_S}, \quad (33)$$

$$\bar{\mathbb{L}} \rightarrow (3, 0) + (3, 1) + (1, 0) + \underbrace{(1, 1)}_{\ell_S^c}, \quad (34)$$

$$e^c \rightarrow \underbrace{(1, 1)}_{\ell_S^c}, \quad (35)$$

$$e^c \rightarrow (2, 3/2) + \underbrace{(2, 1/2)}_{\ell_D^c}, \quad (36)$$

$$\bar{e}^c \rightarrow (2, -3/2) + \underbrace{(2, -1/2)}_{\ell_D^c}. \quad (37)$$

<sup>5</sup>The charges  $i, 1, -i, 1, 1, 1, i, -i, 1, i, -i, -i, 1, 1$  for the fields  $\Omega, \chi, Q, \mathbb{Q}, \bar{\mathbb{Q}}, u^c, \mathbb{u}^c, \bar{\mathbb{u}}^c, d^c, \mathbb{d}^c, \bar{\mathbb{d}}^c, L, \mathbb{L}, \bar{\mathbb{L}}, e^c, \mathbb{e}^c, \bar{\mathbb{e}}^c$  successfully achieve the goal.

The Standard Model charged leptons are a mixture of the charged components of the  $SU(2) \times U(1)$  representations labeled above. The nomenclature keeps track of lepton number, which is conserved (a superscript  $c$  denotes an antilepton), and the subscripts indicate whether a field is part of a  $SU(2)$  singlet ( $S$ ) or a doublet ( $D$ ). Inserting the vacuum expectation values of the scalars, and collecting the fermions in the vectors  $\Psi = (\ell_D, \ell_D', \ell_S)^T$  and  $\Psi^c = (\ell_S^c, \ell_S^c, \ell_D^c)^T$  we get the mass term

$$\Psi^T \begin{pmatrix} 0 & y_L^\Omega \langle H_d \rangle & y_{LE}^\chi \langle \chi \rangle \\ y_E^\Omega \langle H_d \rangle & 0 & m_E \\ y_{LE}^{\chi'} \langle \chi \rangle & m_L & 0 \end{pmatrix} \Psi^c. \quad (38)$$

The VEV of the  $H_u$  doublet contained in  $\Omega$  does not appear in this expression. Presuming that  $m_E$  and  $m_L$  are substantially larger than the scalar vacuum expectation values, the last expression implies that there are two heavy Dirac fermions, with masses  $\approx m_E$  and  $\approx m_L$ , and a light one with a mass

$$m_{\text{light}} \approx \left( \frac{y_E^\Omega y_{LE}^\chi}{m_L} + \frac{y_L^\Omega y_{LE}^{\chi'}}{m_E} \right) \langle H_d \rangle \langle \chi \rangle. \quad (39)$$

The left-handed part of this field is mostly composed of  $\ell_D$ , with a small admixture of  $\ell_D'$  and  $\ell_S$ ; the right-handed part is mostly formed from  $\ell_S^c$ , but also from  $\ell_S^c$  and  $\ell_D^c$  (to a lesser extent):

$$\ell \approx \ell_D - \frac{y_{LE}^\chi \langle \chi \rangle}{m_L} \ell_D' - \frac{y_L^\Omega \langle H_d \rangle}{m_E} \ell_S, \quad (40)$$

$$\ell^c \approx \ell_S^c - \frac{y_{LE}^{\chi'} \langle \chi \rangle}{m_E} \ell_S^c - \frac{y_E^\Omega \langle H_d \rangle}{m_L} \ell_D^c. \quad (41)$$

Note that the terms proportional to  $\langle H_d \rangle / m_{E,L}$  change the lepton couplings to the  $W$  and  $Z$  bosons, with respect to the Standard Model prediction. Since the branching fractions of  $Z \rightarrow \ell^- \ell^+$  and  $W \rightarrow \ell^+ \nu$  have been measured with a precision of 1 part in  $10^3$  [27], a rough estimate would be that the  $m_{E,L}$  masses should be at least  $\sim \sqrt{10^3}$  times larger than  $\langle H_d \rangle$ .

The situation is analogous for quarks: due to small admixtures with the fields in the spinor representations of  $SO(5)$ , the Standard Model fermions can interact through  $W_1^\mu$  with heavy new fermions, the latter having exotic quantum numbers.

Note that lepton number is conserved again, and there is a total of 5 neutrinos and 4 anti-neutrinos (per generation), so we conclude without further calculations that one neutrino is massless. This comment applies to each generation of fermions so, in contradiction with oscillation data, there is a total of three massless neutrinos. Nevertheless, as in the previous model, an extra  $(1, 1, 3, 1)$  scalar solves the problem.

## V. GAUGE BOSONS WITH ARBITRARY QUANTUM NUMBERS

Most research on extensions of the Standard Model group  $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$  is based on just a handful of groups, and there is little freedom (if any) to change the way fermions transform under them. Fermion masses are the main culprit: by having these particles transform under some arbitrary representation of the gauge symmetry, it is likely that fermions with exotic quantum numbers and which are chiral under  $G_{\text{SM}}$  will also be part of the model (see for instance [28]). This is a concern because such fields can have at most an electroweak scale mass.

On the other hand, the simple exercise of picking pairs of right- and left-handed Standard Model fermions yields a list of quantum numbers for vectors fields which would have interesting phenomenology consequences [1,29,30]. However, just a few of them have been incorporated into fully fledged models, mostly because of the above difficulty. For example, while looking at vectors fields that can mediate the neutrinoless beta decay of a proton, the paper [30] argued that ultraviolet-complete models might be possible only for a few of them.

That assessment was too pessimistic. In the following, I will argue that one can build viable models containing gauge bosons in arbitrary representations of the Standard Model symmetry group. I will proceed in two steps:

- (1) It is possible to show that for any representation  $X$  of the Standard Model group  $G_{\text{SM}}$ , there is always a group  $G$  containing  $G_{\text{SM}}$  in such a way that its adjoint representation includes  $X$ .
- (2) Fermions can be assigned to representations of  $G$  such that only the Standard Model ones remain massless right before electroweak symmetry is broken. This guarantees that exotic new fermions can be made heavy. Furthermore, no gauge anomalies are generated. The vector boson mentioned above can couple to Standard Model fermions.

The first step involves group theory only. For the sake of argument, consider the following scenario which works for any  $X$ ,<sup>6</sup> even though it might not yield the smallest group  $G$ . Take  $S$  to be the trivial representation  $(\mathbf{1}, \mathbf{1}, 0)$  of  $G_{\text{SM}}$ , and  $S' = (\mathbf{1}, \mathbf{1}, y)$  with  $y$  equal to minus the total hypercharge of all components of  $X$ . In other words,  $U(1)_Y$  acts on the reducible representation  $X \oplus S'$  via a traceless matrix. If  $n$  is the dimension of  $X$ , then one can embed  $G_{\text{SM}}$  in  $SU(n+2)$  in such a way that the adjoint representation of the latter contains  $X$ . To see it, note that there is an embedding under which the fundamental representation  $F$  of  $SU(n+2)$  decompose as

$$F \rightarrow X \oplus S \oplus S'. \quad (42)$$

After all,  $X \oplus S \oplus S'$  is represented by a set of twelve  $(n+2)$ -dimensional matrices which are traceless and Hermitian, hence they form a subalgebra of  $SU(n+2)$ . Moving on to the adjoint representation, it transforms in the same way as  $F \times F^*$  with a singlet subtracted (informally, we may express this as  $\mathbf{Ad} \sim F \times F^* - \mathbf{1}$ ), so it follows directly from the previous branching rule that the adjoint representation  $\mathbf{Ad}$  of  $SU(n+2)$  decomposes as

$$\mathbf{Ad} \rightarrow X \oplus X^* \oplus \text{'more'}, \quad (43)$$

with 'more' =  $(X \times X^*) \oplus S' \oplus S'^* \oplus (X \times S'^*) \oplus (X^* \times S') \oplus S$ . As an example, we can infer immediately that a gauge boson with the unusual quantum numbers  $X = (\mathbf{1}, \mathbf{5}, 8) \equiv \mathbf{5}_8$  can be obtained from  $SU(7)$  through the embedding defined by the branching rules

$$F \equiv 7 \rightarrow \underbrace{\mathbf{5}_8}_X \oplus \underbrace{\mathbf{1}_0}_S \oplus \underbrace{\mathbf{1}_{-40}}_{S'}, \quad (44)$$

$$\begin{aligned} \mathbf{Ad} \equiv 48 &\rightarrow \underbrace{\mathbf{5}_8 \oplus \mathbf{5}_{-8}}_{X+X^*} \oplus \underbrace{\mathbf{1}_{-40} \oplus \mathbf{1}_{40}}_{S'+S'^*} \\ &\oplus \underbrace{\mathbf{1}_0 \oplus \mathbf{3}_0 \oplus \mathbf{5}_0 \oplus \mathbf{7}_0 \oplus \mathbf{9}_0}_{X \times X^*} \oplus \underbrace{\mathbf{5}_{48} \oplus \mathbf{5}_{-48}}_{X \times S'^* + X^* \times S'} \\ &\oplus \underbrace{\mathbf{1}_0}_S. \end{aligned} \quad (45)$$

The above reasoning works for any  $X$ , but it is unlikely to involve the smallest possible group. For several quantum numbers of the vector field, the reader can see in Table 1 of [30] what are the minimal groups. To illustrate the point,  $X = (\mathbf{8}, \mathbf{3}, 0)$  can be obtained from  $SU(26)$  by the above argument, however, it can also be extracted from the much smaller  $SU(6) \times U(1)$  group.<sup>7</sup>

Having settled this mathematical part of the problem, it remains to be seen whether or not one can build realistic models based on the above group embeddings. In principle, the solution adopted in this work for the  $W_1$  vector field—and which has also been used in models for the B-anomalies [31–34]—can be adapted to gauge bosons with other representations. We may start by extending  $G$  to  $G \times SU(3) \times SU(2) \times U(1)$  (some of these factors, such as  $SU(3)$  on the earlier models for  $W_1$ , might be unnecessary). The Standard Model symmetry group  $G_{\text{SM}}$  is obtained from the diagonal subgroup of an  $SU(3)' \times SU(2)' \times U(1)'$  contained in  $G$ , and  $G_{321} \equiv SU(3) \times SU(2) \times U(1)$  outside it. The model will contain fermions  $F = Q, u^c, d^c, L, e^c$  which transform as usual under  $G_{321}$  and have a trivial  $G$  charge.

<sup>6</sup>The argument actually fails when  $X$  is inert under the full  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . However, it is well known that the trivial representation  $X = (\mathbf{1}, \mathbf{1}, 0)$  is obtainable from extra  $U(1)$  factors, for example.

<sup>7</sup>It corresponds to the first branching rule (out of three) given by the command `DecomposeRep[{SU6,U1}, Adjoint[{SU6,U1}], {SU3,SU2,U1}]` in `GroupMath` [24].

One must also add some scalars to correctly break the extended gauge group and to couple to fermions via Yukawa interactions.

This is one possibility. There are no gauge anomalies and, with an appropriate scalar sector, it should be feasible to obtain the Standard Model as a low energy effective theory. Importantly, fermions will not couple directly to the gauge bosons of  $G$ .

The situation changes if we introduce for each (or at least some)  $F$  a corresponding pair of vectorlike fermions ( $\mathbb{F}, \bar{\mathbb{F}}$ ) transforming nontrivially under the group  $G$ , and with the same  $G_{321}$  quantum numbers as  $F$ . (In the case of  $F = Q$  it might be convenient to add two pairs of vectorlike fermions, as explained shortly.) The recipe can be as follows for a gauge boson in some representation  $X$  of  $G_{\text{SM}}$ :

- (1) Find a group  $G$  whose adjoint representation contains  $X$ . This requirement can always be fulfilled. The full symmetry of the model shall be given by the  $G \times G_{321}$  group.
- (2) Pick a nontrivial representation  $\mathbf{R}$  of  $G$  such that  $(\mathbf{R}, \mathbf{1}, \mathbf{1}, 0)$  of  $G \times G_{321}$  contains the Standard Model subrepresentation  $(\mathbf{1}, \mathbf{2}, -1/2)$ .
- (3) Introduce two scalars  $\Omega = (\mathbf{R}, \mathbf{1}, \mathbf{1}, 0)$  and  $\chi = (\bar{\mathbf{R}}, \mathbf{1}, \mathbf{2}, -1/2)$ ; they include at least one Higgs doublet  $(\mathbf{1}, \mathbf{2}, -1/2)$  and a singlet  $(\mathbf{1}, \mathbf{1}, 0)$ .<sup>8</sup>
- (4) Introduce Weyl fermions  $F = Q, u^c, d^c, L, e^c$  transforming as  $(\mathbf{1}, \mathbf{3}, \mathbf{2}, 1/6)$ ,  $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, -2/3)$ ,  $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, 1/3)$ ,  $(\mathbf{1}, \mathbf{1}, \mathbf{2}, -1/2)$  and  $(\mathbf{1}, \mathbf{1}, \mathbf{1}, 1)$ . For each  $F$  we need a vectorlike fermion pair  $(\mathbb{F}, \bar{\mathbb{F}})$  such that  $\mathbb{F}$  transforms as  $F$  under  $G_{321}$  and as a  $\mathbf{R}$  under  $G$ . However, since the  $\mathbf{R}$  representation contains only a downlike Higgs doublet,  $(\mathbf{R}, \mathbf{1}, \mathbf{1}, 0) \rightarrow (\mathbf{1}, \mathbf{2}, -1/2) + \dots$ ,  $(\mathbb{Q}, \bar{\mathbb{Q}})$  will contain only downlike quarks  $-(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$  and  $(\mathbf{3}, \mathbf{1}, 1/3)$ . In order to treat all quarks equally, we may want to introduce two vectorlike fermions in association to  $F = Q$ :

$$\mathbb{Q}_u \equiv (\bar{\mathbf{R}}, \mathbf{3}, \mathbf{2}, 1/6) \quad \text{and} \quad \mathbb{Q}_d \equiv (\mathbf{R}, \mathbf{3}, \mathbf{2}, 1/6). \quad (46)$$

This is not needed if  $\mathbf{R}$  contains both  $(\mathbf{1}, \mathbf{2}, -1/2)$  and  $(\mathbf{1}, \mathbf{2}, 1/2)$  (as in the  $SO(5)$  models of Secs. III and IV).

- (5) If  $\mathbf{R}$  is complex, the list of fermion masses and Yukawa terms is the following:

$$\begin{aligned} \text{masses: } & \mathbb{Q}_u \bar{\mathbb{Q}}_u, \quad \mathbb{Q}_d \bar{\mathbb{Q}}_d, \quad u^c \bar{u}^c, \quad d^c \bar{d}^c, \\ & \mathbb{L} \bar{\mathbb{L}}, \quad e^c \bar{e}^c, \end{aligned} \quad (47)$$

$$\begin{aligned} \text{singlet interactions: } & \mathbb{Q} u^c \chi^*, \quad \mathbb{Q}_u u^c \chi^*, \quad \mathbb{Q} d^c \chi, \\ & \mathbb{Q}_d d^c \chi, \quad L e^c \chi, \quad \mathbb{L} e^c \chi, \end{aligned} \quad (48)$$

<sup>8</sup>If the VEV of this last field is insufficient to correctly break  $G \times G_{321}$  down to the Standard Model group, one must add more scalars.

$$\begin{aligned} \text{doublet interactions: } & \mathbb{Q} \bar{\mathbb{Q}}_u \Omega^*, \quad u^c \bar{u}^c \Omega^*, \quad \mathbb{Q} \bar{\mathbb{Q}}_d \Omega, \\ & d^c \bar{d}^c \Omega, \quad L \bar{\mathbb{L}} \Omega, \quad e^c \bar{e}^c \Omega. \end{aligned} \quad (49)$$

By design all but the Standard Model fermions can be made heavy without any tuning. Assuming that the scalar VEVs are smaller than the vectorlike masses  $m_F \mathbb{F} \bar{\mathbb{F}}$ , the light fermion mass eigenstates are composed mostly of the  $F$ 's (that is  $Q, u^c, d^c, L$ , and  $e^c$ ). Mostly, but not entirely: a particularly important consequence is that through mixing the Standard Model fermions will couple to the gauge bosons of  $G$ . Furthermore, note that the VEV of  $\chi$  does not break  $G_{\text{SM}}$ , thus it can be comparable (or even greater than) the masses  $m_F$ ; as a consequence, fermion mixing might be large.<sup>9</sup>

An analogous list of interactions can be compiled when  $\mathbf{R}$  is (pseudo)real. In that case there is no need for both  $\mathbb{Q}_u$  and  $\mathbb{Q}_d$  (a single vectorlike  $\mathbb{Q}$  is sufficient).

Lastly, it is interesting that baryon and lepton numbers are preserved in this construction. That was also the case in the  $SO(5)$  models for  $W_1$ , where neutrinos are Dirac particles even though there are right-handed neutrinos and scalars which are capable of inducing Majorana masses.

As a further example, consider a gauge boson  $X_\mu$  with the quantum numbers  $\mathbf{X} = (\mathbf{3}, \mathbf{2}, 5/6)$ . The argument reported earlier points to the  $SU(8)$  group, but the field can also be obtained in the widely studied  $SU(5)$  model of grand unification [35].  $X_\mu$  induces proton decay via its simultaneous coupling to the Standard Model bilinears  $\mathbb{Q} \bar{u}^c$  and  $L \bar{e}^c$ , hence it must be an extremely heavy field. An alternative is to forbid one of its two problematic couplings with a symmetry that, for example, enforces baryon-number conservation. Given the stringent limits on the proton's lifetime [36], a  $X_\mu$  field at the TeV scale is problematic even if it induces nucleon decay through loops only.

Now consider an  $SU(5) \times SU(3) \times SU(2) \times U(1)$  model, with  $\mathbf{R} = \mathbf{5}$ . Despite the complexity of the list of fermion

<sup>9</sup>There is a caveat. For every  $\mathbf{X}$ , it is always possible to pick a  $G$  and an  $\mathbf{R}$  fulfilling steps 1 and 2. However, by itself this does not ensure that the gauge bosons of  $G$  transforming as  $\mathbf{X}$  will couple to the important sub-representations in the  $\mathbb{F}/\bar{\mathbb{F}}$  fields (i.e., those which can mix with the  $F$ 's). This should not be a concern as long as  $\mathbf{X}$ ,  $G$  and  $\mathbf{R}$  are not too exotic. However, in general, one has an extra requirement which—it turns out again—can always be met. I argued that an  $n$ -dimensional  $\mathbf{X}$  is contained in the adjoint representation of  $SU(n+2)$ , as shown by the branching rules (42) and (43). Those decompositions do not ensure, as we would like, that the down Higgs doublet  $H_d$  in  $\Omega$  couples to anything else through  $\mathbf{X}$ . We can fix that in  $SU(n+3)$  with  $\mathbf{R} =$  fundamental rep  $\rightarrow H_d \oplus \mathbf{X}' \oplus \mathbf{S}'$  where  $\mathbf{X}'$  is a representation in the product  $H_d \times \mathbf{X}^*$  and, as before,  $\mathbf{S}'$  makes  $H_d \oplus \mathbf{X}' \oplus \mathbf{S}'$  traceless under  $U(1)_Y$ . With this choice of group embedding, the  $H_d$  scalar in  $\Omega$  couples to something else ( $\mathbf{X}'$ ) via the gauge bosons transforming as  $\mathbf{X}$ , and so do the other important components in  $\chi$  and the  $\mathbb{F}/\bar{\mathbb{F}}$  fermions.



masses and interactions (see above), one can assign an unbroken baryon number  $B$  to all fields:  $B(Q, \mathbb{Q}_{u/d}, \overline{u^c}, \overline{d^c}) = -B(\overline{\mathbb{Q}_{u/d}}, u^c, \overline{u^c}, d^c, \overline{d^c}) = 1/3$ . The same holds for lepton number. A model constructed along these lines should therefore predict a stable proton, even if the  $SU(5) \times SU(3) \times SU(2) \times U(1)$  symmetry breaking scale is as low as a few TeV. This is true also for other groups and other  $X$ 's: more fields are needed in order to break baryon and/or lepton number.

## VI. SUMMARY

A vector field  $W_1$  with the quantum numbers  $(3,1)$  under  $SU(2)_L \times U(1)_Y$  cannot couple to pairs of Standard Model fermions, yet in principle it could interact with two Higgs doublets. While this is true, I have argued in this paper that such coupling will not be generated in a Yang-Mills theory, if  $W_1$  is a gauge boson. As a consequence, the suggestion in [3,10] that such a field could explain the recent CDF measurement of the  $W$ -boson mass becomes less appealing.

Notwithstanding the lack of the above interaction, in this paper I considered a minimal model containing  $W_1$  as a gauge boson, potentially with a TeV scale mass. In fact, I considered two closely related models: one where this field does not interact with fermions at all, and another where it does. In the latter, due to its quantum numbers, the  $W_1$  coupling to Standard Model fermions involves necessarily exotic ones as well. The two models are based on an  $SO(5) \times SU(2) \times U(1)$  extended electroweak group, and therefore they predict the existence of a  $W'$  and a  $Z'$ , with a

mass hierarchy  $m_{W_1} < m_{Z'} < m_{W'}$ . Incidentally, these two vector fields are known to affect the  $W$  mass (see Refs. [3,8,10,37–40]):  $W'$  pulls it down and  $Z'$  has the opposite effect. Since the  $Z'$  is lighter, it could in principle explain the CDF data. The two models also contain new scalars (shown in Tables I and II).

Finally, in the last part of this work I have argued that it is possible to extend the argument used here for  $W_1$ —and elsewhere for the  $U_\mu$  leptoquark [31–34]—to gauge bosons with arbitrary quantum numbers. Some phenomenological limitations do apply: for example, models with a colorless and fractionally charged field contain necessarily a stable electrically charged particle, which is a problem in astrophysics and cosmology. However, even with this kind of consideration, many viable possibilities remain, and therefore the existence of TeV-scale gauge bosons with a wide variety of quantum numbers cannot be ruled out.

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