Diquarkyonic matter: Quarks, diquarks, and baryons

Aaron Park [*] and Su Houng Lee [†]
Department of Physics and Institute of Physics and Applied Physics, Yonsei University, Seoul 03722, Korea

(Received 30 December 2022; accepted 5 May 2023; published 25 May 2023)

In this work, we investigate the color-spin interaction of a quark, a diquark and a baryon with their surrounding baryons and/or quark matter. We extend our previous work by increasing the maximum number of surrounding baryons to five and additionally consider all possible diquark probes that are immersed in such surroundings. This is accomplished by classifying all possible flavor and spin states of the resulting multiquark configuration in both the flavor SU(2) and SU(3) symmetric cases. We also discuss the three-body confinement potential and show that this does not contribute to the outcome. Furthermore, we find that a quark becomes more stable than a baryon when the number of surrounding baryons is three or more. Finally, when we consider the internal color-spin factor of a probe, our results show that the effects of the color-spin interaction of a multiquark configuration is consistent with the so-called diquarkyonic configuration.

DOI: 10.1103/PhysRevD.107.094033

I. INTRODUCTION

Recent neutron star observations [1–3], which constrain the equation of state (EOS) of matter, lead us to understand the properties of dense matter in a new perspective. Gravitational wave astrophysics also has begun to provide EOS constraints measuring tidal deformability from the neutron star merger [4,5]. Additionally, recent analyses by NICER from pulsar J0740 + 6620 [6,7] estimated the radii of neutron stars as $R_{1.4} \simeq 12.45$ km, $R_{2.08} \simeq 12.35$ km, whose large mass and radius supports the stiff evolution of EOS around the core density. The quarkyonic matter configuration was introduced to explain this stiff evolution of EOS [8].

The quarkyonic matter was originally discussed in the limit of large N_C quantum chromodynamics (QCD) description of cold dense matter. The quarkyonic configuration with shell-like phase space distribution of baryon can be generated through the hard-core repulsive interaction between nucleons [9–11]. This new picture explains the stiffening of an EOS in a high density region before the transition to quark matter. The essential ingredient of this configuration is the hard-core repulsion and this can be well understood in terms of quark level interaction with Pauli blocking [12].

In order to probe possible phases in a quark model point of view, we showed that the short distance repulsion between the quark and the baryon is smaller than that between two baryons in the lowest energy channel using a constituent quark model [13]. Also, the point where the behavior occurs is consistent with the quarkyonic configuration. When the relevant scales are such that the quark mass differences can be neglected so that all quarks involved have similar distributions, the color-color interaction can be neglected. In this case, the most important factor determining the repulsive force at a short distance is the color-spin interaction. The color-spin interaction is a key factor explaining the repulsive core between nucleons [12] and also is an important ingredient in examining the bound state of exotic hadrons such as tetraquark [14].

In Ref. [13], by calculating the energy of a quark and a baryon using both the color-spin and color-color interaction, it was shown that a quarkyoniclike phase can appear when the density is high. This state differs from the existing quarkyonic configuration as we only considered the interaction energy of the d quark with the surrounding baryons leaving out the interaction of the (ud) diquark that together with the d quark comprised the initial neutron in matter. This was so because such configuration costs the least amount of excitation energy: hence called a quarkyoniclike configuration to distinguish it from the quarkyonic configuration where all interaction of all three quarks inside the neutron is important for long range excitation mode in the momentum space. As a result, if not only quarks but also diquarks can appear together, then it can be called a so-called diquarkyonic matter, a new phase that is distinct from the existing configuration. However, if the diquark can exist as an independent state, then the interaction that the diquark experiences from the surrounding baryons needs to be calculated as well.

In this paper, we extend our previous work [15] in several respects. First of all, the maximum number of surrounding

[*] aaron.park@yonsei.ac.kr
[†] suhoung@yonsei.ac.kr

baryons is increased from three to five. Second, in addition to considering the interaction of the quark and the most attractive diquark, which we will henceforth call probes, with the surrounding, we will consider all additional diquark probes to investigate which one is the most stable diquark in dense matter. Third, when we discuss color-color interactions, we add the investigation on three-body confinement potential. Fourth, a case for a flavor decuplet baryon is added. Also, we describe in more detail how we determine the allowed states for each case.

In this work, we assume the spatial part of the multiquark wave function to be totally symmetric. Also, since the color state of entire multiquark state is determined from the probe, we can determine the possible flavor and spin state to satisfy the Pauli exclusion principle.

This paper is organized as follows. In Sec. II, the formulas for the color-spin interaction factor are introduced for both flavor SU(2) and SU(3). In Sec. III, we discuss three-body confinement potentials induced by a three-gluon exchange. In Sec. IV, we construct the multibaryon states. These states are used for both correlated surrounding baryons and the multiquark states when a probe is a baryon. In Sec. V, we construct the multiquark states when a probe is a quark, a diquark, three correlated diquarks, and a baryon, respectively. In Sec. VI, we consider the case where the surrounding is a free quark gas. In Sec. VII, we analyze the results using all the states we constructed. Sec. VIII is devoted to summary and concluding remarks.

II. COLOR-SPIN INTERACTION

In this work, we will assume that all quarks occupy the same spatial configuration. This means that the spatial potential will be universal for any pair so that the relative strength is determined only by the color and/or spin factors. This factor for the color-spin interaction is defined as follows:

$$V_{CS} = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j, \\ \equiv \frac{1}{m_u^2} H_{CS}, \tag{1}$$

where λ_i^c, m_i, m_u are, respectively, the color SU(3) Gell-Mann matrices, the constituent quark mass of the ith quark, and the constituent quark mass of u, d quarks. For flavor SU(3) symmetric cases, the color-spin factor H_{CS} can be easily calculated by the following formula:

$$H_{CS} = -\sum_{i<j}^{n} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j, \\ = n(n-10) + \frac{4}{3} S(S+1) + 4C_F + 2C_C, \\ 4C_F = \frac{4}{3}(p_1^2 + p_2^2 + 3p_1 + 3p_2 + p_1 p_2), \tag{2}$$

where C_F is the first kind of the Casimir operator of the flavor SU(3) and p_i is the number of columns containing i boxes in a column in Young diagram. For the flavor SU(2) case, Eq. (2) reduces to the following formula:

$$H_{CS} = \frac{4}{3} n(n-6) + \frac{4}{3} S(S+1) + 4I(I+1) + 2C_C, \tag{3}$$

where I is the total isospin.

III. COLOR-COLOR INTERACTION

We can also consider the color-color interaction, which is responsible for the confining and Coulomb type of interactions. However, as we showed in Ref. [15], this interaction between the quarks in the probe and those in the color singlet configuration cancel out. Additionally, we can also consider color interaction induced by three-gluon exchange [16–18]. There are two types of three-body color operators in color SU(3): $d^{abc} F_i^a F_j^b F_k^c$ and $f^{abc} F_i^a F_j^b F_k^c$. We can represent these operators using the permutations of symmetric group as follows [19]:

$$d^{abc} F_i^a F_j^b F_k^c = \frac{1}{4}[(ijk) + (ikj)] + \frac{1}{9} \\ - \frac{1}{6}[(ij) + (ik) + (jk)], \tag{4}$$

$$f^{abc} F_i^a F_j^b F_k^c = \frac{i}{4}[(ijk) - (ikj)], \tag{5}$$

where (ij) and (ijk) are two-cycle and three-cycle permutations, respectively.

Let us first consider d-type three-body confinement potential. Using the fact that $\sum_{i<j<k} d^{abc} F_i^a F_j^b F_k^c$ is Casimir operator, we can calculate its eigenvalue considering only the normal Young-Yamanouchi basis and the axial distance among i, j, and k.

$$\sum_{i<j<k} d^{abc} F_i^a F_j^b F_k^c = \frac{p_1}{27} - \frac{p_1^2}{18} + \frac{p_1^3}{54} + \frac{13 p_2}{54} - \frac{5 p_1 p_2}{36} + \frac{p_1^2 p_2}{36} \\ - \frac{2 p_2^2}{9} - \frac{p_1 p_2^2}{36} - \frac{p_2^3}{54} + \frac{10 p_3}{9}, \tag{6}$$

where p_i is the number of columns containing i boxes in a column in Young diagram. As we can see, the above formula is linear in p_3, which is the number of singlet baryons in the corresponding color state. It shows that the d-type three-body confinement potentials between the singlet and the others cancel out. This holds not only for SU(3) but in general. We represent the eigenvalue of d-type three-body confinement potential for SU(4), SU(5), and SU(6) and how to calculate it in the Appendix.

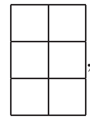
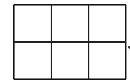
Now, let us consider the f-type three-body confinement potential. As we can see in Eq. (5), it is not difficult to check that the diagonal component of it vanishes. However, since f-type three-body confinement potential is not a Casimir operator, we need to check its off-diagonal element. By calculating it directly using the permutation matrices, we can check that all elements vanish only if the color state is $[m, m, \ldots, m]$ type for $\mathrm{SU}(N)$, where m is any integer and the number of rows are smaller or equal to N. However, in this work, since we focus on the diagonal component of interaction, we only consider the color-spin factor to study the interaction between a probe and surrounding baryons in dense matter.

IV. MULTIBARYON STATES

In this section, we first construct the multibaryon states. To calculate the interaction which a probe experiences from the surrounding baryons, we subtract the internal color-spin factor of surrounding baryons from the entire multiquark state. When there are two or more baryons, there can be multiple possible states so it is necessary to consider all possible cases.

A. Two baryons

The color state of two baryons should be color singlet. Hence, the remaining part of the wave function can be determined using the conjugate form of the color state. The color and flavor ⊗ spin states of two baryons are as follows:

Color : , Flavor ⊗ Spin : .

The flavor ⊗ spin coupling state [3,3] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

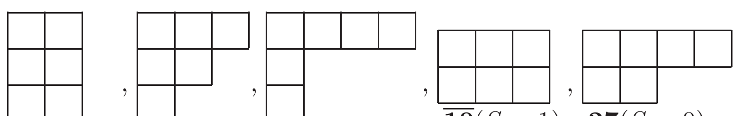
$$\begin{aligned}[3,3]_{FS} = {} & [6]_F \otimes [3,3]_S + [5,1]_F \otimes [4,2]_S + [4,2]_F \otimes [5,1]_S \\ & + [4,2]_F \otimes [3,3]_S + [3,3]_F \otimes [6]_S \\ & + [3,3]_F \otimes [4,2]_S + [3,2,1]_F \otimes [5,1]_S \\ & + [3,2,1]_F \otimes [4,2]_S + [2,2,2]_F \otimes [3,3]_S. \end{aligned} \tag{7}$$

However, all flavor and spin states in the above are not possible as two baryons states. Since we only consider the flavor octet baryon in this work, the possible flavor states should be determined using the outer product as follows:

Flavor states of two baryons:

$$\mathbf{8} \times \mathbf{8} = \mathbf{1} + \mathbf{8}_{(m=2)} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}. \tag{8}$$

Then the possible flavor and spin states of two baryons are listed as follows:

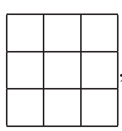
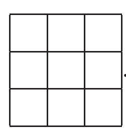
Flavor and spin : 

$\mathbf{1}(S=0)$, $\mathbf{8}(S=1)$, $\mathbf{10}(S=1)$, $\overline{\mathbf{10}}(S=1)$, $\mathbf{27}(S=0)$.

Here, the Young diagrams are the flavor states and the possible spin states are shown in the parentheses.

B. Three baryons

The color and flavor ⊗ spin states of three baryons are as follows:

Color : , Flavor ⊗ Spin : .

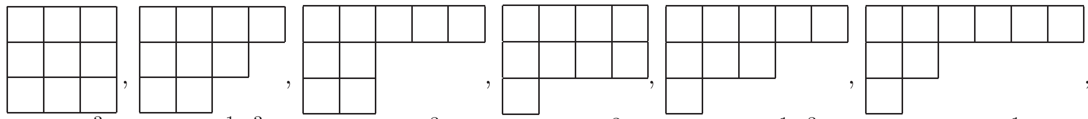
The flavor ⊗ spin coupling state [3,3,3] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$\begin{aligned}[3,3,3]_{FS} = {} & [6,3]_F \otimes [6,3]_S + [6,2,1]_F \otimes [5,4]_S + [5,4]_F \otimes [5,4]_S + [5,3,1]_F \otimes [7,2]_S + [5,3,1]_F \otimes [6,3]_S \\ & + [5,3,1]_F \otimes [5,4]_S + [5,2,2]_F \otimes [6,3]_S + [4,4,1]_F \otimes [6,3]_S + [4,3,2]_F \otimes [8,1]_S + [4,3,2]_F \otimes [7,2]_S \\ & + [4,3,2]_F \otimes [6,3]_S + [4,3,2]_F \otimes [5,4]_S + [3,3,3]_F \otimes [9]_S + [3,3,3]_F \otimes [7,2]_S + [3,3,3]_F \otimes [6,3]_S. \end{aligned} \tag{9}$$

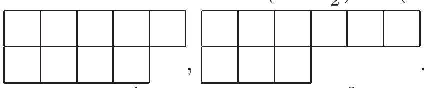
Similar to the two baryons case, the possible flavor states of three baryons are determined as follows:
Flavor states of three baryons:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}=\mathbf{1}_{(m=2)}+\mathbf{8}_{(m=8)}+\mathbf{10}_{(m=4)}+\overline{\mathbf{10}}_{(m=4)}+\mathbf{27}_{(m=6)}+\mathbf{35}_{(m=2)}+\overline{\mathbf{35}}_{(m=2)}+\mathbf{64}. \tag{10}$$

Flavor and spin :



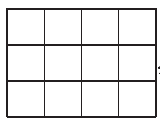
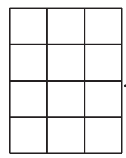
$\mathbf{1}(S=\frac{3}{2})$, $\mathbf{8}(S=\frac{1}{2},\frac{3}{2})$, $\mathbf{10}(S=\frac{3}{2})$, $\bar{\mathbf{1}}\mathbf{0}(S=\frac{3}{2})$, $\mathbf{27}(S=\frac{1}{2},\frac{3}{2})$, $\mathbf{35}(S=\frac{1}{2})$,



$\bar{\mathbf{3}}\mathbf{5}(S=\frac{1}{2})$, $\mathbf{64}(S=\frac{3}{2})$.

C. Four baryons

The color and flavor ⊗ spin states of four baryons are as follows:

Color : , Flavor ⊗ Spin : .

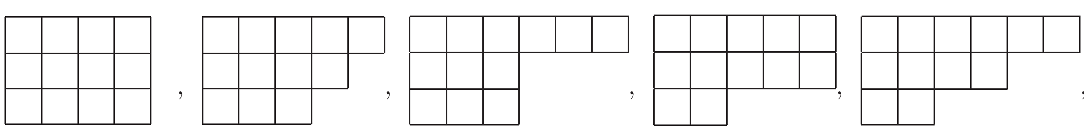
The flavor ⊗ spin coupling state [3,3,3,3] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$\begin{aligned}[3,3,3,3]_{FS}=&[6,6]_F\otimes[6,6]_S+[6,5,1]_F\otimes[7,5]_S+[6,4,2]_F\otimes[8,4]_S+[6,4,2]_F\otimes[6,6]_S+[6,3,3]_F\otimes[9,3]_S\\&+[6,3,3]_F\otimes[7,5]_S+[5,5,2]_F\otimes[7,5]_S+[5,4,3]_F\otimes[8,4]_S+[5,4,3]_F\otimes[7,5]_S\\&+[4,4,4]_F\otimes[6,6]_S.\end{aligned} \tag{11}$$

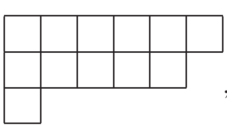
The flavor and spin states of four octet baryons are as follows:
Flavor states of four baryons:

$$\begin{aligned}\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}=&\mathbf{1}_{(m=8)}+\mathbf{8}_{(m=32)}+\mathbf{10}_{(m=20)}+\overline{\mathbf{10}}_{(m=20)}+\mathbf{27}_{(m=33)}+\mathbf{28}_{(m=2)}+\overline{\mathbf{28}}_{(m=2)}+\mathbf{35}_{(m=15)}+\overline{\mathbf{35}}_{(m=15)}\\&+\mathbf{64}_{(m=12)}+\mathbf{81}_{(m=3)}+\mathbf{125}.\end{aligned} \tag{12}$$

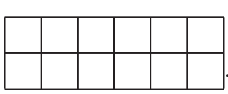
Flavor and spin :



$\mathbf{1}(S=0)$, $\mathbf{8}(S=1,2)$, $\mathbf{10}(S=1)$, $\bar{\mathbf{1}}\mathbf{0}(S=1)$, $\mathbf{27}(S=0,2)$,



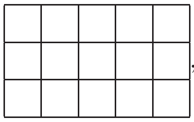
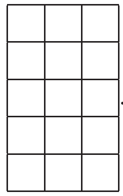
$\bar{\mathbf{3}}\mathbf{5}(S=1)$,



$\overline{\mathbf{28}}(S=0)$.

D. Five baryons

The color and flavor ⊗ spin states of five baryons are as follows:

Color : , Flavor ⊗ Spin : .

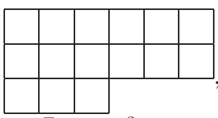
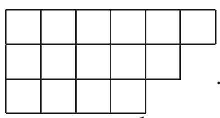
The flavor ⊗ spin coupling state [3,3,3,3,3] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$[3,3,3,3,3]_{FS} = [6,6,3]_F \otimes [9,6]_S + [6,5,4]_F \otimes [8,7]_S. \tag{13}$$

Flavor states of five baryons:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8} = \mathbf{1}_{(m=32)} + \mathbf{8}_{(m=145)} + \mathbf{10}_{(m=100)} + \overline{\mathbf{10}}_{(m=100)} + \mathbf{27}_{(m=180)} + \mathbf{28}_{(m=20)} + \overline{\mathbf{28}}_{(m=20)} + \mathbf{35}_{(m=100)} + \overline{\mathbf{35}}_{(m=100)} + \mathbf{64}_{(m=94)} + \mathbf{80}_{(m=5)} + \overline{\mathbf{80}}_{(m=5)} + \mathbf{81}_{(m=36)} + \overline{\mathbf{81}}_{(m=36)} + \mathbf{125}_{(m=20)} + \mathbf{154}_{(m=4)} + \overline{\mathbf{154}}_{(m=4)} + \mathbf{216}. \tag{14}$$

The flavor and spin states of four octet baryons are as follows:

Flavor and spin : , .

$\overline{\mathbf{10}}(S=\frac{3}{2})$ $\mathbf{8}(S=\frac{1}{2})$

Theoretically, we can construct six baryons states with totally symmetric spatial wave function. However, if we add a probe to the six baryons state, then the total state does not satisfy the Pauli principle. Therefore, in this study, we limit the number of surrounding baryons up to five.

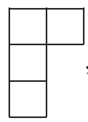
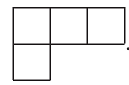
V. FLAVOR, COLOR, AND SPIN STATES OF A MULTIQUARK SYSTEM

In this section, we construct the entire multiquark states that are composed of multibaryons and a probe. As a probe, we consider a quark, a baryon, a diquark, and three correlated diquarks.

A. Quark case

1. One baryon + one quark

Let us consider the multiquark system consisting of one baryon and one quark. In this work, we assume that the spatial part of a wave function is totally symmetric and the surrounding baryons are in flavor octet states. Since a baryon is a color singlet, the color state of four quarks should be a triplet. Then, the flavor ⊗ spin coupling state of four quarks should be the conjugate of the color state to satisfy the Pauli exclusion principle.

Color : , Flavor ⊗ Spin : .

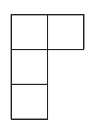
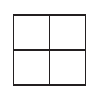
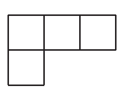
The flavor ⊗ spin coupling state [3,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$[3,1]_{FS} = [4]_F \otimes [3,1]_S + [3,1]_F \otimes [4]_S + [3,1]_F \otimes [3,1]_S + [3,1]_F \otimes [2,2]_S + [2,2]_F \otimes [3,1]_S + [2,1,1]_F \otimes [3,1]_S + [2,1,1]_F \otimes [2,2]_S. \tag{15}$$

Meanwhile, we can determine the possible flavor states of multiquark system as follows: Flavor states of four quarks:

$$\mathbf{8}\times\mathbf{3} = \mathbf{3} + \bar{\mathbf{6}} + \mathbf{15}. \tag{16}$$

Since only the above flavor states are allowed, we can classify all possible flavor and spin states of a four quarks configuration from the Eqs. (15) and (16) as follows:

Flavor and spin: , , .

$\mathbf{3}(S=0,1)$ $\bar{\mathbf{6}}(S=0,1)$ $\mathbf{15}(S=0,1)$

Now, we investigate the relative magnitude of the interaction which a quark inside the probe sees from the surrounding n baryons using the following formula:

$$\Delta H_{CS}^{n\mathrm{b}+p} = H_{CS}^{n\mathrm{b}+p} - H_{CS}^{n\mathrm{b}} - H_{CS}^{p}, \tag{17}$$

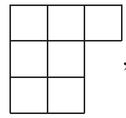
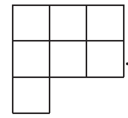
$$\Delta H_{CS}^{\mathrm{avg}} = \frac{1}{n_p n \sum_{C,F,S} d_{CFS}} \sum_{C,F,S} d_{CFS} \Delta H_{CS}^{\mathrm{nb}+p}, \quad (18)$$

$$d_{CFS} = d_C d_F d_S m_{FS}. \quad (19)$$

Here, nb and p in the superscripts represent n external baryons and the probe, respectively. The probe will be a baryon, a quark, a diquark, or three correlated diquarks. n_p is the number of quarks in the probe. We will investigate cases with $n = 1, 2, 3, 4, 5$, and also consider the case where nb is replaced by a single quark so as to study the deconfined phase. d_C, d_F, and d_S are the dimensions of the color, flavor, and spin states of $3n + n_p$ quarks, respectively, m_{FS} is the multiplicity of the flavor and spin states, and the summation is taken for all possible states. Here, we divide it by n_p to normalize the result with respect to the single quark case. We also divide by n to keep the surrounding baryon at constant density for comparison at the same density.

2. Two baryons + one quark

We now consider a quark around two baryons. The color and flavor ⊗ spin states of the seven quarks configuration are as follows:

Color : , Flavor ⊗ Spin : .

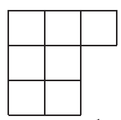
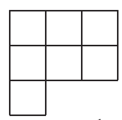
The flavor ⊗ spin coupling state [3,3,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$\begin{aligned}[3,3,1]_{FS} =\ & [6,1]_F \otimes [4,3]_S + [5,2]_F \otimes [5,2]_S + [5,2]_F \otimes [4,3]_S + [5,1,1]_F \otimes [5,2]_S + [5,1,1]_F \otimes [4,3]_S \\ & + [4,3]_F \otimes [6,1]_S + [4,3]_F \otimes [5,2]_S + [4,3]_F \otimes [4,3]_S + [4,2,1]_F \otimes [6,1]_S + [4,2,1]_F \otimes [5,2]_{S(m=2)} \\ & + [4,2,1]_F \otimes [4,3]_{S(m=2)} + [3,3,1]_F \otimes [7]_S + [3,3,1]_F \otimes [6,1]_S + [3,3,1]_F \otimes [5,2]_{S(m=2)} \\ & + [3,3,1]_F \otimes [4,3]_S + [3,2,2]_F \otimes [6,1]_S + [3,2,2]_F \otimes [5,2]_S + [3,2,2]_F \otimes [4,3]_S. \end{aligned} \quad (20)$$

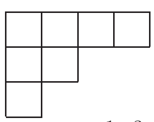
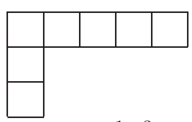
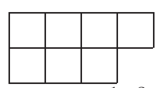
Here we consider the seven quarks system consisting of two octet baryons and one quark. Then, we can determine the possible flavor states of seven quarks as follows: Flavor states of seven quarks:

$$\mathbf{8} \times \mathbf{8} \times \mathbf{3} = \mathbf{3}_{(m=3)} + \bar{\mathbf{6}}_{(m=3)} + \mathbf{15}_{(m=4)} + \mathbf{15}' + \mathbf{24}_{(m=2)} + \mathbf{42}. \quad (21)$$

Selecting flavor states given in Eq. (21) from Eq. (20), we can represent the possible flavor and spin states as follows:

Flavor and spin : , ,

$\mathbf{3}(S = \frac{1}{2}, \frac{3}{2})$ $\bar{\mathbf{6}}(S = \frac{1}{2}, \frac{3}{2})$

, , ,

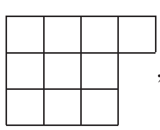
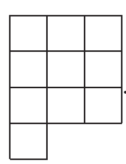
$\mathbf{15}(S = \frac{1}{2}, \frac{3}{2})$ $\mathbf{15}'(S = \frac{1}{2}, \frac{3}{2})$ $\mathbf{24}(S = \frac{1}{2}, \frac{3}{2})$

.

$\mathbf{42}(S = \frac{1}{2}, \frac{3}{2})$

3. Three baryons + one quark

The color and flavor ⊗ spin states of 10 quarks configuration are as follows:

Color : , Flavor ⊗ Spin : .

The flavor ⊗ spin coupling state [3,3,3,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

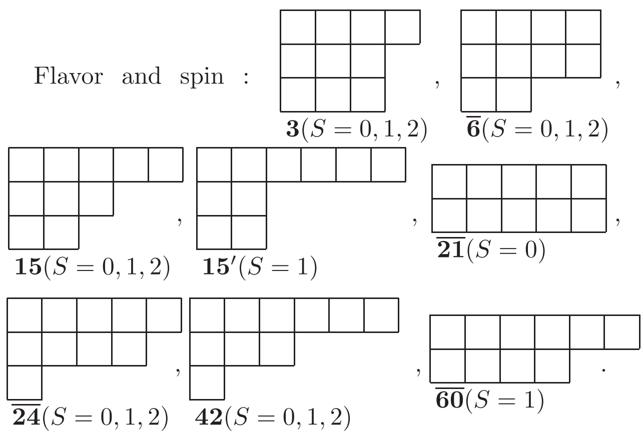
$$\begin{aligned}[3,3,3,1]_{FS} = {} & [6,4]_F \otimes [6,4]_S + [6,3,1]_F \otimes [7,3]_S + [6,3,1]_F \otimes [6,4]_S + [6,3,1]_F \otimes [5,5]_S + [6,2,2]_F \otimes [6,4]_S \\ & + [5,5]_F \otimes [5,5]_S + [5,4,1]_F \otimes [7,3]_S + [5,4,1]_F \otimes [6,4]_{S(m=2)} + [5,4,1]_F \otimes [5,5]_S \\ & + [5,3,2]_F \otimes [8,2]_S + [5,3,2]_F \otimes [7,3]_{S(m=2)} + [5,3,2]_F \otimes [6,4]_{S(m=2)} + [5,3,2]_F \otimes [5,5]_S \\ & + [4,4,2]_F \otimes [8,2]_S + [4,4,2]_F \otimes [7,3]_S + [4,4,2]_F \otimes [6,4]_{S(m=2)} + [4,3,3]_F \otimes [9,1]_S \\ & + [4,3,3]_F \otimes [8,2]_S + [4,3,3]_F \otimes [7,3]_{S(m=2)} + [4,3,3]_F \otimes [6,4]_S + [4,3,3]_F \otimes [5,5]_S. \end{aligned} \tag{22}$$

Similar to the two baryons and one quark case, we can determine the possible flavor states of 10 quarks as follows:

Flavor states of 10 quarks:

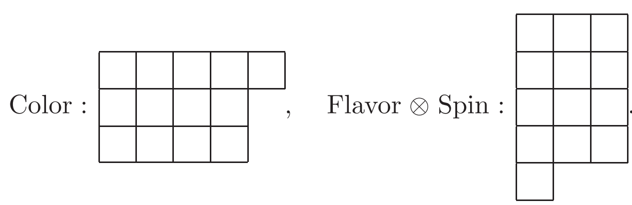
$$\begin{aligned}\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{3} = {} & \mathbf{3}_{(m=10)} + \bar{\mathbf{6}}_{(m=12)} + \mathbf{15}_{(m=18)} + \mathbf{15'}_{(m=6)} + \overline{\mathbf{21}}_{(m=2)} + \overline{\mathbf{24}}_{(m=12)} + \mathbf{42}_{(m=9)} + \mathbf{48}_{(m=2)} \\ & + \overline{\mathbf{60}}_{(m=3)} + \mathbf{90}. \end{aligned} \tag{23}$$

Combining the two equation above, we find the following possible states:



4. Four baryons + one quark

The color and flavor ⊗ spin states of 13 quarks configuration are as follows:



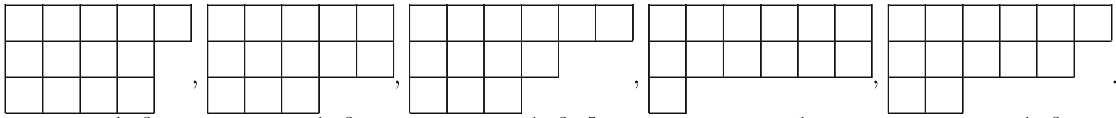
The flavor ⊗ spin coupling state [3,3,3,3,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$\begin{aligned}[3,3,3,3,1]_{FS} = {} & [6,6,1]_F \otimes [7,6]_S + [6,5,2]_F \otimes [8,5]_S + [6,5,2]_F \otimes [7,6]_S + [6,4,3]_F \otimes [9,4]_S + [6,4,3]_F \otimes [8,5]_S \\ & + [6,4,3]_F \otimes [7,6]_S + [5,5,3]_F \otimes [8,5]_S + [5,5,3]_F \otimes [7,6]_S + [5,4,4]_F \otimes [8,5]_S \\ & + [5,4,4]_F \otimes [7,6]_S. \end{aligned} \tag{24}$$

Flavor states of 13 quarks:

$$\begin{aligned}\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{3} = {} & \mathbf{3}_{(m=40)} + \bar{\mathbf{6}}_{(m=52)} + \mathbf{15}_{(m=85)} + \mathbf{15'}_{(m=35)} + \mathbf{21}_{(m=17)} + \mathbf{24}_{(m=68)} + \mathbf{36}_{(m=2)} + \mathbf{42}_{(m=60)} \\ & + \mathbf{48}_{(m=20)} + \overline{\mathbf{60}}_{(m=30)} + \mathbf{63}_{(m=5)} + \mathbf{90}_{(m=16)} + \mathbf{105}_{(m=3)} + \mathbf{120}_{(m=4)} + \mathbf{165}. \end{aligned} \tag{25}$$

Flavor and spin :

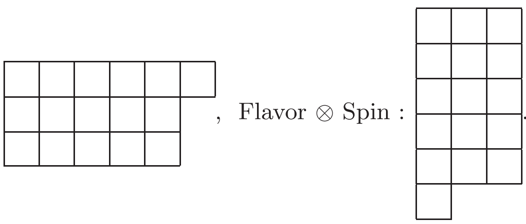


$\mathbf{3}(S=\frac{1}{2},\frac{3}{2})$, $\bar{\mathbf{6}}(S=\frac{1}{2},\frac{3}{2})$, $\mathbf{15}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$, $\mathbf{21}(S=\frac{1}{2})$, $\mathbf{24}(S=\frac{1}{2},\frac{3}{2})$.

5. Five baryons + one quark

The color and flavor ⊗ spin states of 16 quarks configuration are as follows:

Color : , Flavor ⊗ Spin : .



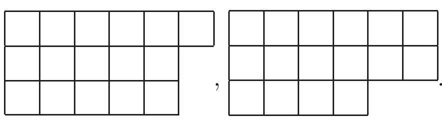
The flavor ⊗ spin coupling state [3,3,3,3,3,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$[3,3,3,3,3,1]_{FS} = [6,6,4]_F \otimes [9,7]_S + [6,5,5]_F \otimes [8,8]_S. \tag{26}$$

Flavor states of 16 quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{3} = \mathbf{3}_{(m=177)} + \bar{\mathbf{6}}_{(m=245)} + \mathbf{15}_{(m=425)} + \mathbf{15'}_{(m=200)} + \mathbf{21}_{(m=120)} + \mathbf{24}_{(m=380)} + \mathbf{36}_{(m=25)} + \mathbf{42}_{(m=374)} + \mathbf{45}_{(m=5)} + \mathbf{48}_{(m=156)} + \mathbf{60}_{(m=230)} + \mathbf{63}_{(m=61)} + \mathbf{90}_{(m=150)} + \mathbf{99}_{(m=5)} + \mathbf{105}_{(m=45)} + \mathbf{120}_{(m=60)} + \mathbf{132}_{(m=9)} + \mathbf{165}_{(m=25)} + \mathbf{192}_{(m=4)} + \mathbf{210}_{(m=5)} + \mathbf{273}. \tag{27}$$

Flavor and spin :



$\mathbf{3}(S=0)$, $\bar{\mathbf{6}}(S=1)$.

B. Diquark case ($C=\bar{\mathbf{3}}, F=\bar{\mathbf{3}}, S=0$)

From Secs. V B to V E, we construct the multiquark state containing all possible diquarks as a probe. There are four possible color-flavor-spin states of diquark satisfying the Pauli principle. Among them, in this section, we construct the multiquark state containing the most stable diquark.

1. One baryon + one diquark

Let us consider the multiquark system consisting of one baryon and one diquark. The color state of five quarks should be an antitriplet. Then, the flavor ⊗ spin coupling state of five quarks can be determined as follows:

Color : , Flavor ⊗ Spin : .

We can decompose the SU(6) flavor ⊗ spin coupling state into SU(3) flavor and SU(2) spin states as follows:

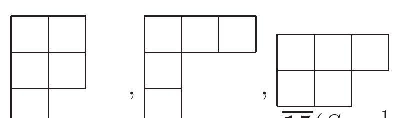
$$\begin{aligned}[3,2]_{FS} = {}& [5]_F \otimes [3,2]_S + [4,1]_F \otimes [4,1]_S \\ &+ [4,1]_F \otimes [3,2]_S + [3,2]_F \otimes [5]_S \\ &+ [3,2]_F \otimes [4,1]_S + [3,2]_F \otimes [3,2]_S \\ &+ [3,1,1]_F \otimes [4,1]_S + [3,1,1]_F \otimes [3,2]_S \\ &+ [2,2,1]_F \otimes [4,1]_S + [2,2,1]_F \otimes [3,2]_S.\end{aligned} \tag{28}$$

Now, we can classify all possible flavor and spin states of a five quarks configuration as follows:

Flavor states of five quarks:

$$\mathbf{8}\times\bar{\mathbf{3}} = \bar{\mathbf{3}} + \mathbf{6} + \overline{\mathbf{15}}. \tag{29}$$

Flavor and spin :



$\bar{\mathbf{3}}(S=\frac{1}{2})$, $\mathbf{6}(S=\frac{1}{2})$, $\overline{\mathbf{15}}(S=\frac{1}{2})$.

2. Two baryons + one diquark

The color state of eight quarks should be a antitriplet. Then, the flavor ⊗ spin coupling state of eight quarks should be its conjugate as follows:

Color : , Flavor ⊗ Spin : .

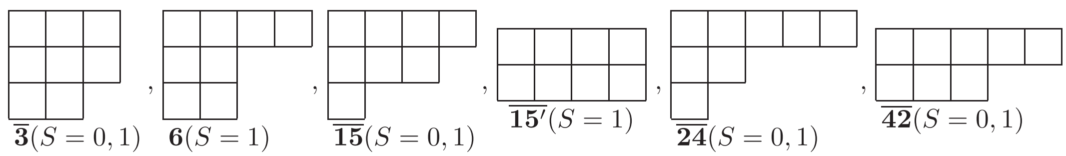
We can decompose the SU(6) flavor ⊗ spin coupling state into flavor SU(3) and spin SU(2) states as follows:

$$\begin{aligned}[3,3,2]_{FS} = {}& [6,2]_F \otimes [5,3]_S + [6,1,1]_F \otimes [4,4]_S + [5,3]_F \otimes [6,2]_S + [5,3]_F \otimes [5,3]_S + [5,3]_F \otimes [4,4]_S \\ &+ [5,2,1]_F \otimes [6,2]_S + [5,2,1]_F \otimes [5,3]_{S(m=2)} + [5,2,1]_F \otimes [4,4]_S + [4,4]_F \otimes [5,3]_S + [4,3,1]_F \otimes [7,1]_S \\ &+ [4,3,1]_F \otimes [6,2]_{S(m=2)} + [4,3,1]_F \otimes [5,3]_{S(m=2)} + [4,3,1]_F \otimes [4,4]_S + [4,2,2]_F \otimes [7,1]_S \\ &+ [4,2,2]_F \otimes [6,2]_S + [4,2,2]_F \otimes [5,3]_{S(m=2)} + [3,3,2]_F \otimes [8]_S + [3,3,2]_F \otimes [7,1]_S + [3,3,2]_F \otimes [6,2]_S \\ &+ [3,3,2]_F \otimes [5,3]_S + [3,3,2]_F \otimes [4,4]_S. \end{aligned} \tag{30}$$

Now, we can classify all possible flavor and spin states of eight quarks configuration as follows:
Flavor states of eight quarks:

$$\mathbf{8} \times \mathbf{8} \times \bar{\mathbf{3}} = \bar{\mathbf{3}}_{(m=3)} + \mathbf{6}_{(m=3)} + \overline{\mathbf{15}}_{(m=4)} + \bar{\mathbf{15}}'_{(m=2)} + \overline{\mathbf{24}}_{(m=2)} + \overline{\mathbf{42}}. \tag{31}$$

Flavor and spin :



3. Three baryons + one diquark

The color and flavor ⊗ spin states of 11 quarks configuration are as follows:

Color : , Flavor ⊗ Spin : .

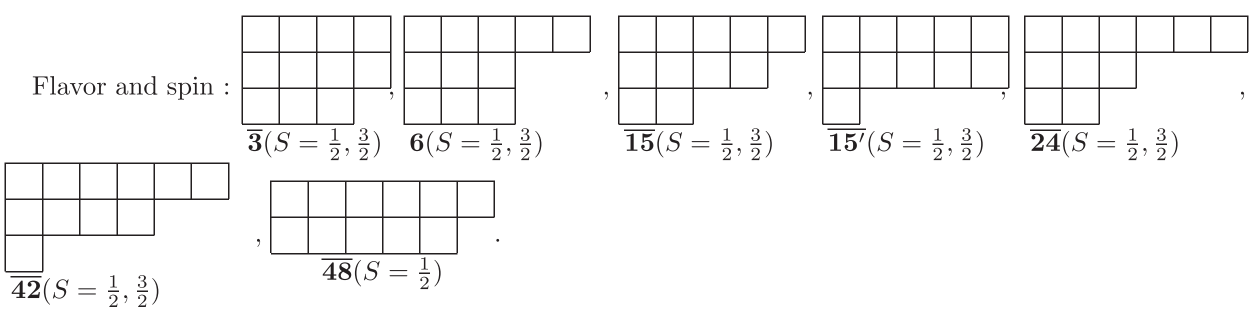
We can decompose flavor ⊗ spin coupling SU(6) state into flavor SU(3) and spin SU(2) states as follows:

$$\begin{aligned}[3,3,3,2]_{FS} = {}& [6,5]_F \otimes [6,5]_S + [6,4,1]_F \otimes [7,4]_S + [6,4,1]_F \otimes [6,5]_S + [6,3,2]_F \otimes [8,3]_S + [6,3,2]_F \otimes [7,4]_S \\ &+ [6,3,2]_F \otimes [6,5]_S + [5,5,1]_F \otimes [7,4]_S + [5,5,1]_F \otimes [6,5]_S + [5,4,2]_F \otimes [8,3]_S \\ &+ [5,4,2]_F \otimes [7,4]_{S(m=2)} + [5,4,2]_F \otimes [6,5]_{S(m=2)} + [5,3,3]_F \otimes [9,2]_S + [5,3,3]_F \otimes [8,3]_S \\ &+ [5,3,3]_F \otimes [7,4]_{S(m=2)} + [5,3,3]_F \otimes [6,5]_S + [4,4,3]_F \otimes [8,3]_S + [4,4,3]_F \otimes [7,4]_S \\ &+ [4,4,3]_F \otimes [6,5]_S. \end{aligned} \tag{32}$$

Now, we can classify all possible flavor and spin states of a 11 quarks configuration as follows: Flavor states of 11 quarks:

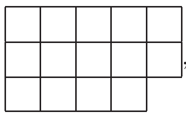
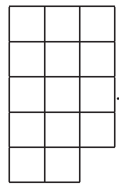
$$\mathbf{8} \times \mathbf{8} \times \mathbf{8} \times \bar{\mathbf{3}} = \bar{\mathbf{3}}_{(m=10)} + \mathbf{6}_{(m=12)} + \overline{\mathbf{15}}_{(m=18)} + \bar{\mathbf{15}}'_{(m=6)} + \overline{\mathbf{24}}_{(m=12)} + \overline{\mathbf{42}}_{(m=9)} + \overline{\mathbf{21}}_{(m=2)} + \overline{\mathbf{60}}_{(m=3)} + \overline{\mathbf{48}}_{(m=2)} + \overline{\mathbf{90}}. \tag{33}$$

Flavor and spin :

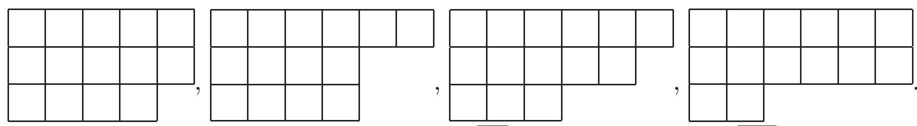


4. Four baryons + one diquark

The color and flavor ⊗ spin states of 14 quarks configuration are as follows:

Color : , Flavor ⊗ Spin : .

The flavor ⊗ spin coupling state [3,3,3,3,2] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

Flavor and spin : 

$\bar{\mathbf{3}}(S=0,1)$ $\mathbf{6}(S=1)$ $\overline{\mathbf{15}}(S=0,1,2)$ $\overline{\mathbf{15}}'(S=1)$

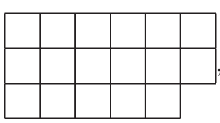
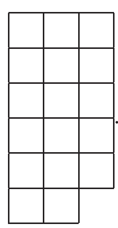
$$\begin{aligned}[3,3,3,3,2]_{FS} = {} & [6,6,2]_F \otimes [8,6]_S + [6,5,3]_F \otimes [9,5]_S \\ & + [6,5,3]_F \otimes [8,6]_S + [6,5,3]_F \otimes [7,7]_S \\ & + [6,4,4]_F \otimes [8,6]_S + [5,5,4]_F \otimes [8,6]_S \\ & + [5,5,4]_F \otimes [7,7]_S. \end{aligned} \tag{34}$$

Flavor states of 14 quarks:

$$\begin{aligned}\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\bar{\mathbf{3}} = {} & \bar{\mathbf{3}}_{(m=40)} + \mathbf{6}_{(m=52)} + \overline{\mathbf{15}}_{(m=85)} \\ & + \overline{\mathbf{15}}'_{(m=35)} + \overline{\mathbf{21}}_{(m=17)} + \overline{\mathbf{24}}_{(m=68)} \\ & + \overline{\mathbf{36}}_{(m=2)} + \overline{\mathbf{42}}_{(m=60)} + \overline{\mathbf{48}}_{(m=20)} \\ & + \overline{\mathbf{60}}_{(m=30)} + \overline{\mathbf{63}}_{(m=5)} + \overline{\mathbf{90}}_{(m=16)} \\ & + \overline{\mathbf{105}}_{(m=3)} + \overline{\mathbf{120}}_{(m=4)} + \overline{\mathbf{165}}. \end{aligned} \tag{35}$$

5. Five baryons + one diquark

The color and flavor ⊗ spin states of 17 quarks configuration are as follows:

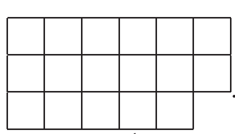
Color : , Flavor ⊗ Spin : .

The flavor ⊗ spin coupling state [3,3,3,3,3,2] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$[3,3,3,3,3,2]_{FS} = [6,6,5]_F \otimes [9,8]_S. \tag{36}$$

Flavor states of 17 quarks:

$$\begin{aligned}\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\bar{\mathbf{3}} = {} & \bar{\mathbf{3}}_{(m=177)} + \mathbf{6}_{(m=245)} + \overline{\mathbf{15}}_{(m=425)} + \overline{\mathbf{15}}'_{(m=200)} + \overline{\mathbf{21}}_{(m=120)} + \overline{\mathbf{24}}_{(m=380)} + \overline{\mathbf{36}}_{(m=25)} \\ & + \overline{\mathbf{42}}_{(m=374)} + \overline{\mathbf{45}}_{(m=5)} + \overline{\mathbf{48}}_{(m=156)} + \overline{\mathbf{60}}_{(m=230)} + \overline{\mathbf{63}}_{(m=61)} + \overline{\mathbf{90}}_{(m=150)} + \overline{\mathbf{99}}_{(m=5)} \\ & + \overline{\mathbf{105}}_{(m=45)} + \overline{\mathbf{120}}_{(m=60)} + \overline{\mathbf{132}}_{(m=9)} + \overline{\mathbf{165}}_{(m=25)} + \overline{\mathbf{192}}_{(m=4)} + \overline{\mathbf{210}}_{(m=5)} + \overline{\mathbf{273}}. \end{aligned} \tag{37}$$

Flavor and spin : .

$\bar{\mathbf{3}}(S=\frac{1}{2})$

C. Diquark case ($C=\bar{\mathbf{3}}, F=\mathbf{6}, S=1$)

As a second diquark state, we consider color triplet, flavor sextet, and spin one diquark.

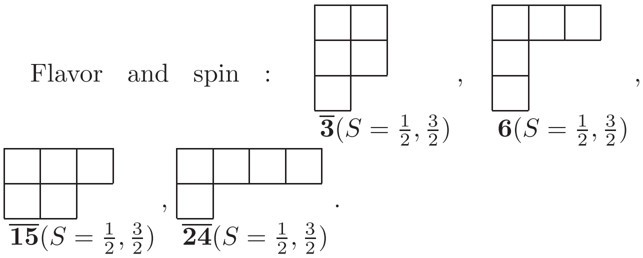
1. One baryon + one diquark

The color and flavor ⊗ spin states of five quarks configuration are same as in Sec. V B 1. However, the possible flavor and spin states of multiquark are different after the decomposition. In the case of the four possible diquarks, there are many overlapping parts in relation to decomposition or outer product, so only nonoverlapping results are listed here.

Flavor states of five quarks:

$$\mathbf{8}\times\mathbf{6} = \bar{\mathbf{3}} + \mathbf{6} + \overline{\mathbf{15}} + \overline{\mathbf{24}}. \tag{38}$$

Flavor and spin :



$\bar{\mathbf{3}}(S=\frac{1}{2},\frac{3}{2})$, $\mathbf{6}(S=\frac{1}{2},\frac{3}{2})$, $\overline{\mathbf{15}}(S=\frac{1}{2},\frac{3}{2})$, $\overline{\mathbf{24}}(S=\frac{1}{2},\frac{3}{2})$.

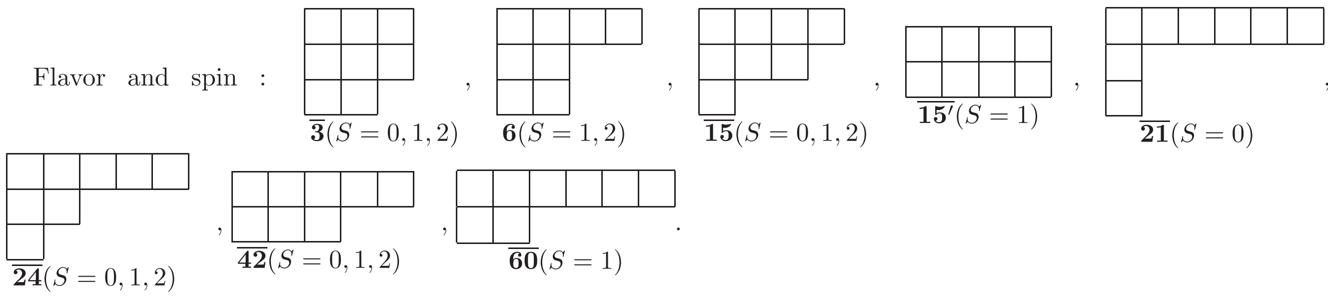
2. Two baryons + one diquark

The color and flavor ⊗ spin states of eight quarks configuration are same as in Sec. V B 2.

Flavor states of eight quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{6}=\bar{\mathbf{3}}_{(m=3)}+\mathbf{6}_{(m=4)}+\overline{\mathbf{15}}_{(m=5)}+\overline{\mathbf{15}}'+\overline{\mathbf{21}} +\overline{\mathbf{24}}_{(m=4)}+\overline{\mathbf{42}}_{(m=2)}+\overline{\mathbf{60}}. \tag{39}$$

Flavor and spin :



$\bar{\mathbf{3}}(S=0,1,2)$, $\mathbf{6}(S=1,2)$, $\overline{\mathbf{15}}(S=0,1,2)$, $\overline{\mathbf{15}}'(S=1)$, $\overline{\mathbf{21}}(S=0)$, $\overline{\mathbf{24}}(S=0,1,2)$, $\overline{\mathbf{42}}(S=0,1,2)$, $\overline{\mathbf{60}}(S=1)$.

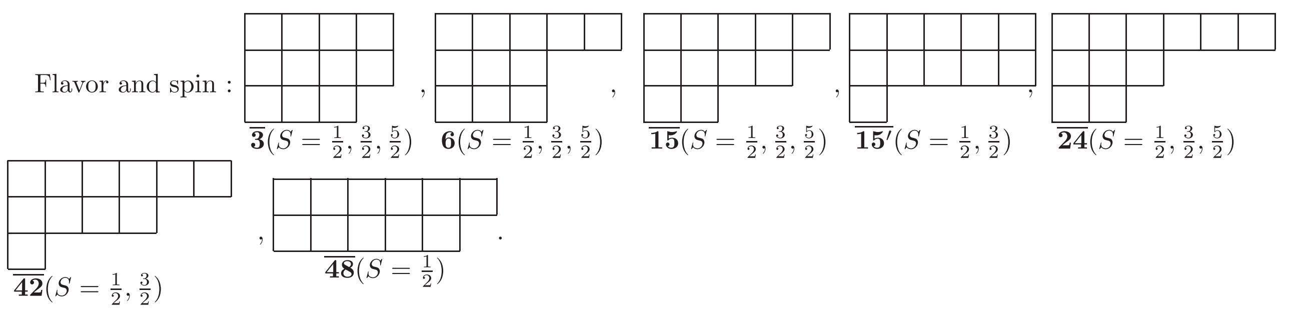
3. Three baryons + one diquark

The color and flavor ⊗ spin states of 11 quarks configuration are same as in Sec. V B 3.

Flavor states of 11 quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{6}=\bar{\mathbf{3}}_{(m=12)}+\mathbf{6}_{(m=16)}+\overline{\mathbf{15}}_{(m=24)}+\overline{\mathbf{15}}'_{(m=8)}+\overline{\mathbf{21}}_{(m=6)}+\overline{\mathbf{24}}_{(m=21)}+\overline{\mathbf{42}}_{(m=15)}+\overline{\mathbf{48}}_{(m=3)}+\overline{\mathbf{60}}_{(m=9)} +\overline{\mathbf{63}}_{(m=2)}+\overline{\mathbf{90}}_{(m=3)}+\overline{\mathbf{120}}. \tag{40}$$

Flavor and spin :



$\bar{\mathbf{3}}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$, $\mathbf{6}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$, $\overline{\mathbf{15}}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$, $\overline{\mathbf{15}}'(S=\frac{1}{2},\frac{3}{2})$, $\overline{\mathbf{24}}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$, $\overline{\mathbf{42}}(S=\frac{1}{2},\frac{3}{2})$, $\overline{\mathbf{48}}(S=\frac{1}{2})$.

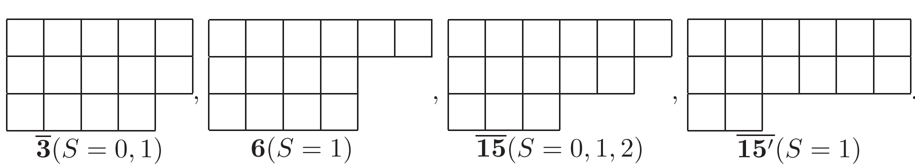
4. Four baryons + one diquark

The color and flavor ⊗ spin states of 14 quarks configuration are same as in Sec. V B 4.

Flavor states of 14 quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{6}=\bar{\mathbf{3}}_{(m=52)}+\mathbf{6}_{(m=73)}+\overline{\mathbf{15}}_{(m=120)}+\overline{\mathbf{15}}'_{(m=50)}+\overline{\mathbf{21}}_{(m=38)}+\overline{\mathbf{24}}_{(m=112)}+\overline{\mathbf{36}}_{(m=3)}+\overline{\mathbf{42}}_{(m=98)} +\overline{\mathbf{45}}_{(m=2)}+\overline{\mathbf{48}}_{(m=32)}+\overline{\mathbf{60}}_{(m=66)}+\overline{\mathbf{63}}_{(m=20)}+\overline{\mathbf{90}}_{(m=34)}+\overline{\mathbf{105}}_{(m=6)}+\overline{\mathbf{120}}_{(m=16)} +\overline{\mathbf{132}}_{(m=3)}+\overline{\mathbf{165}}_{(m=4)}+\overline{\mathbf{210}}. \tag{41}$$

Flavor and spin :

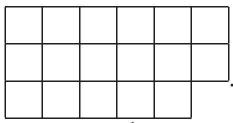


$\bar{\mathbf{3}}(S=0,1)$, $\mathbf{6}(S=1)$, $\overline{\mathbf{15}}(S=0,1,2)$, $\overline{\mathbf{15}}'(S=1)$.

5. Five baryons + one diquark

The color and flavor ⊗ spin states of 17 quarks configuration are same as in Sec. V B 5.
Flavor states of 17 quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{6}=\bar{\mathbf{3}}_{(m=245)}+\mathbf{6}_{(m=357)}+\overline{\mathbf{15}}_{(m=625)}+\bar{\mathbf{15}}'_{(m=300)}+\overline{\mathbf{21}}_{(m=236)}+\overline{\mathbf{24}}_{(m=619)}+\overline{\mathbf{36}}_{(m=41)}$$
$$+\overline{\mathbf{42}}_{(m=610)}+\overline{\mathbf{45}}_{(m=25)}+\overline{\mathbf{48}}_{(m=255)}+\overline{\mathbf{60}}_{(m=450)}+\overline{\mathbf{63}}_{(m=165)}+\overline{\mathbf{90}}_{(m=290)}+\overline{\mathbf{99}}_{(m=9)}$$
$$+\overline{\mathbf{105}}_{(m=85)}+\overline{\mathbf{120}}_{(m=160)}+\overline{\mathbf{120}'}_{(m=5)}+\overline{\mathbf{132}}_{(m=45)}+\overline{\mathbf{165}}_{(m=65)}+\overline{\mathbf{192}}_{(m=10)}+\overline{\mathbf{210}}_{(m=25)}$$
$$+\overline{\mathbf{234}}_{(m=4)}+\overline{\mathbf{273}}_{(m=5)}+\overline{\mathbf{336}}. \tag{42}$$

Flavor and spin :  .

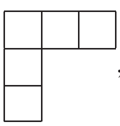
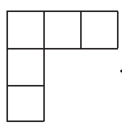
$\bar{\mathbf{3}}(S=\frac{1}{2})$

D. Diquark case ($C=6$, $F=\bar{3}$, $S=1$)

As a third diquark state, we consider the color sextet, flavor antitriplet, and spin one diquark.

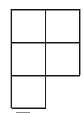
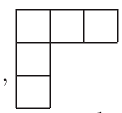
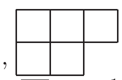
1. One baryon + one diquark

The color and flavor ⊗ spin states of five quarks configuration are as follows:

Color :  , Flavor ⊗ Spin :  .

The flavor ⊗ spin coupling state [3,1,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

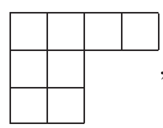
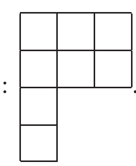
$$[3,1,1]_{FS}=[4,1]_F\otimes[4,1]_S+[4,1]_F\otimes[3,2]_S$$
$$+[3,2]_F\otimes[4,1]_S+[3,2]_F\otimes[3,2]_S$$
$$+[3,1,1]_F\otimes[5]_S+[3,1,1]_F\otimes[4,1]_S$$
$$+[3,1,1]_F\otimes[3,2]_{S(m=2)}+[2,2,1]_F\otimes[4,1]_S$$
$$+[2,2,1]_F\otimes[3,2]_S. \tag{43}$$

Flavor and spin :  ,  ,  .

$\bar{\mathbf{3}}(S=\frac{1}{2},\frac{3}{2})$ $\mathbf{6}(S=\frac{1}{2},\frac{3}{2})$ $\overline{\mathbf{15}}(S=\frac{1}{2},\frac{3}{2})$

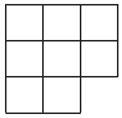
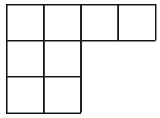
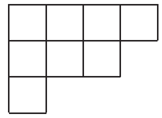
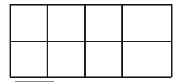
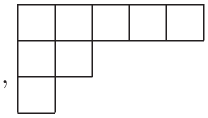
2. Two baryons + one diquark

The color and flavor ⊗ spin states of eight quarks configuration are as follows:

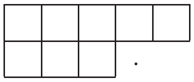
Color :  , Flavor ⊗ Spin :  .

The flavor ⊗ spin coupling state [3,3,1,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$[3,3,1,1]_{FS}=[6,2]_F\otimes[4,4]_S+[6,1,1]_F\otimes[5,3]_S+[5,3]_F\otimes[5,3]_S+[5,2,1]_F\otimes[6,2]_S+[5,2,1]_F\otimes[5,3]_{S(m=2)}$$
$$+[5,2,1]_F\otimes[4,4]_S+[4,4]_F\otimes[6,2]_S+[4,4]_F\otimes[4,4]_S+[4,3,1]_F\otimes[7,1]_S+[4,3,1]_F\otimes[6,2]_{S(m=2)}$$
$$+[4,3,1]_F\otimes[5,3]_{S(m=3)}+[4,3,1]_F\otimes[4,4]_S+[4,2,2]_F\otimes[6,2]_{S(m=2)}+[4,2,2]_F\otimes[5,3]_S$$
$$+[4,2,2]_F\otimes[4,4]_{S(m=2)}+[3,3,2]_F\otimes[7,1]_S+[3,3,2]_F\otimes[6,2]_S+[3,3,2]_F\otimes[5,3]_{S(m=2)}. \tag{44}$$

Flavor and spin :  ,  ,  ,  ,  ,

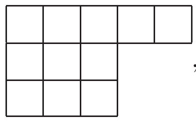
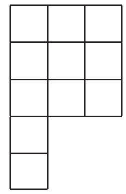
$\overline{\mathbf{3}}(S=1,2)$ $\mathbf{6}(S=0,1,2)$ $\overline{\mathbf{15}}(S=0,1,2)$ $\overline{\mathbf{15'}}(S=0,2)$ $\overline{\mathbf{24}}(S=0,1,2)$

 .

$\overline{\mathbf{42}}(S=1)$

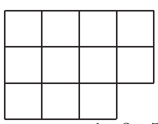
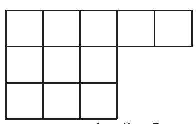
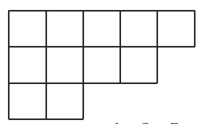
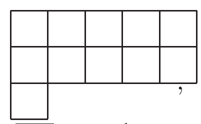
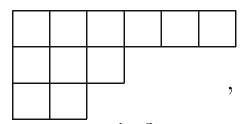
3. Three baryons + one diquark

The color and flavor ⊗ spin states of 11 quarks configuration are as follows:

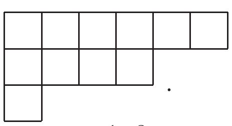
Color :  , Flavor ⊗ Spin : 

The flavor ⊗ spin coupling state [3,3,3,1,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$\begin{aligned}[3,3,3,1,1]_{FS} = {} & [6,4,1]_F \otimes [7,4]_S + [6,4,1]_F \otimes [6,5]_S + [6,3,2]_F \otimes [7,4]_S + [6,3,2]_F \otimes [6,5]_S + [5,5,1]_F \otimes [6,5]_S \\ & + [5,4,2]_F \otimes [8,3]_S + [5,4,2]_F \otimes [7,4]_{S(m=2)} + [5,4,2]_F \otimes [6,5]_{S(m=2)} + [5,3,3]_F \otimes [8,3]_S \\ & + [5,3,3]_F \otimes [7,4]_S + [5,3,3]_F \otimes [6,5]_S + [4,4,3]_F \otimes [9,2]_S + [4,4,3]_F \otimes [8,3]_S \\ & + [4,4,3]_F \otimes [7,4]_{S(m=2)} + [4,4,3]_F \otimes [6,5]_S. \end{aligned} \tag{45}$$

Flavor and spin :  ,  ,  ,  ,  ,

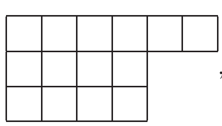
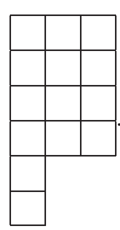
$\overline{\mathbf{3}}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$ $\mathbf{6}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$ $\overline{\mathbf{15}}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$ $\overline{\mathbf{15'}}(S=\frac{1}{2})$ $\overline{\mathbf{24}}(S=\frac{1}{2},\frac{3}{2})$

 .

$\overline{\mathbf{42}}(S=\frac{1}{2},\frac{3}{2})$

4. Four baryons + one diquark

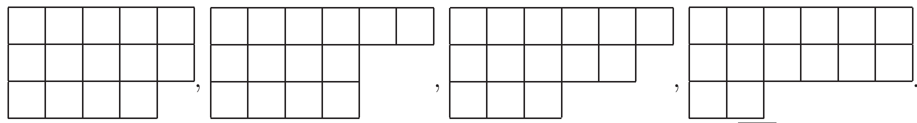
The color and flavor ⊗ spin states of 14 quarks configuration are as follows:

Color :  , Flavor ⊗ Spin :  .

The flavor ⊗ spin coupling state [3,3,3,3,1,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$[3,3,3,3,1,1]_{FS} = [6,6,2]_F \otimes [7,7]_S + [6,5,3]_F \otimes [8,6]_S + [6,4,4]_F \otimes [9,5]_S + [6,4,4]_F \otimes [7,7]_S + [5,5,4]_F \otimes [8,6]_S. \quad (46)$$

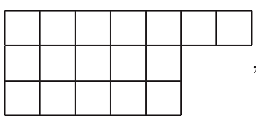
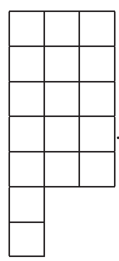
Flavor and spin :



$\overline{\mathbf{3}}(S=1)$, $\mathbf{6}(S=0,2)$, $\overline{\mathbf{15}}(S=1)$, $\overline{\mathbf{15}}'(S=0)$.

5. Five baryons + one diquark

The color and flavor ⊗ spin states of 17 quarks configuration are as follows:

Color :  , Flavor ⊗ Spin : .

There are no allowed states that satisfy the Pauli principle.

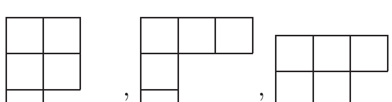
E. Diquark case ($C=6$, $F=6$, $S=0$)

As a final diquark case, we consider color sextet, flavor sextet, and spin zero diquark.

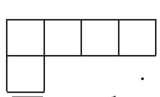
1. One baryon + one diquark

The color and flavor ⊗ spin states of five quarks configuration are same as in Sec. V D 1. In addition, flavor states of five quarks are same as in Sec. V C 1. Therefore, for the rest of the cases, we only represent the possible flavor and spin states.

Flavor and spin :



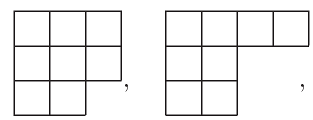
$\overline{\mathbf{3}}(S=\frac{1}{2})$, $\mathbf{6}(S=\frac{1}{2})$, $\overline{\mathbf{15}}(S=\frac{1}{2})$,



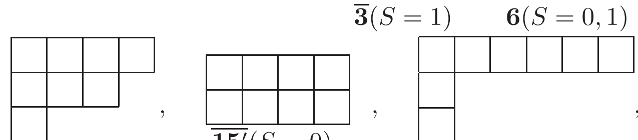
$\overline{\mathbf{24}}(S=\frac{1}{2})$.

2. Two baryons + one diquark

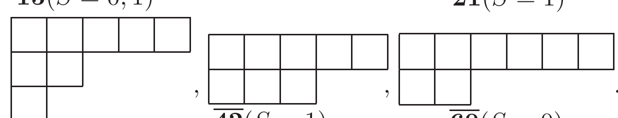
Flavor and spin :



$\overline{\mathbf{3}}(S=1)$, $\mathbf{6}(S=0,1)$,



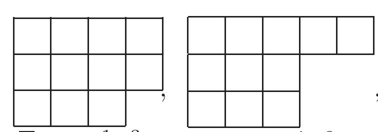
$\overline{\mathbf{15}}(S=0,1)$, $\overline{\mathbf{15}}'(S=0)$, $\overline{\mathbf{21}}(S=1)$,



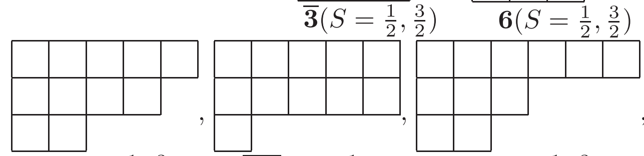
$\overline{\mathbf{24}}(S=0,1)$, $\overline{\mathbf{42}}(S=1)$, $\overline{\mathbf{60}}(S=0)$.

3. Three baryons + one diquark

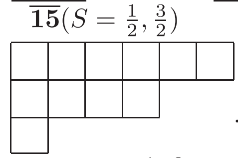
Flavor and spin :



$\overline{\mathbf{3}}(S=\frac{1}{2},\frac{3}{2})$, $\mathbf{6}(S=\frac{1}{2},\frac{3}{2})$,

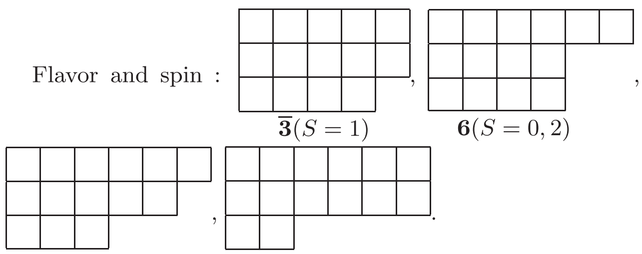


$\overline{\mathbf{15}}(S=\frac{1}{2},\frac{3}{2})$, $\overline{\mathbf{15}}'(S=\frac{1}{2})$, $\overline{\mathbf{24}}(S=\frac{1}{2},\frac{3}{2})$,



$\overline{\mathbf{42}}(S=\frac{1}{2},\frac{3}{2})$.

4. Four baryons + one diquark



$\bar{\mathbf{3}}(S=1)$, $\mathbf{6}(S=0,2)$, $\overline{\mathbf{15}}(S=1)$, $\overline{\mathbf{15}}'(S=0)$.

5. Five baryons + one diquark

There are no allowed states that satisfy the Pauli principle.

F. Three correlated diquarks

When we consider the interaction involving a free quark or a free diquark, the problem of the infinite thermal Wilson line in the confining phase arises. To avoid that problem, we consider the color singlet state of three diquarks additionally. Among several possible states, the most attractive one is the flavor singlet and spin zero state that is the same quantum number as H-dibaryon. In this study, we construct the three correlated diquarks using the most stable diquark state, which is color antitriplet, flavor antitriplet, and spin zero. Also, it should be noted that this state is not allowed in the flavor SU(2) case because it should contain two strange quarks.

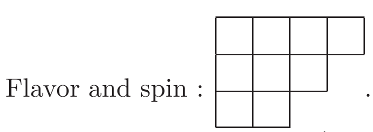
1. One baryon + three diquarks

The color and flavor ⊗ spin states of nine quarks composed of one baryon and three correlated diquarks are as follows:

Flavor states of nine quarks:

$$\mathbf{8}\times\mathbf{1}=\mathbf{8}. \tag{47}$$

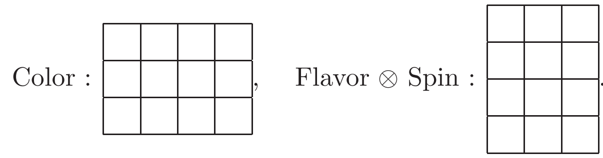
Using the decomposition in octet baryon case, we find that there is one possible flavor and spin states as follows:



$\mathbf{8}(S=\frac{1}{2})$

2. Two baryons + three diquarks

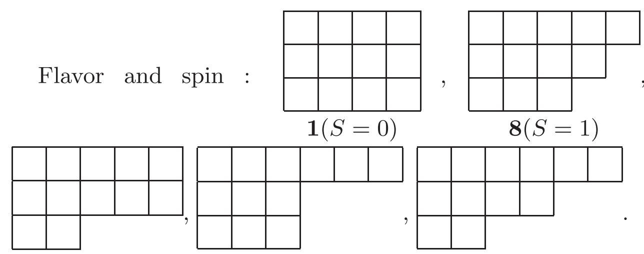
The color and flavor ⊗ spin states of 12 quarks are as follows:



Flavor states of 12 quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{1}=\mathbf{1}+\mathbf{8}_{(m=2)}+\mathbf{10}+\overline{\mathbf{10}}+\mathbf{27}. \tag{48}$$

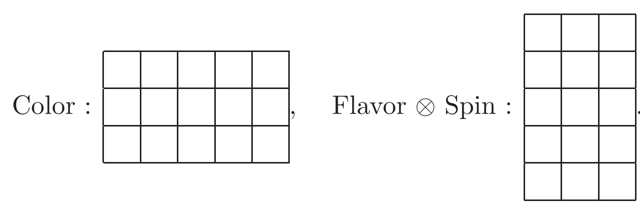
Using the decomposition in octet baryon case, we can determine the possible flavor and spin states as follows:



$\mathbf{1}(S=0)$, $\mathbf{8}(S=1)$, $\overline{\mathbf{10}}(S=1)$, $\mathbf{10}(S=1)$, $\mathbf{27}(S=0)$.

3. Three baryons + three diquarks

The color and flavor ⊗ spin states of 15 quarks are as follows:



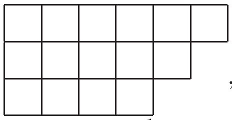
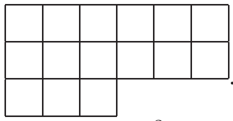
Using the decomposition in octet baryon case, we can determine the possible flavor and spin states as follows:

Flavor states of 15 quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{1}=\mathbf{1}_{(m=2)}+\mathbf{8}_{(m=8)}+\mathbf{10}_{(m=4)}+\overline{\mathbf{10}}_{(m=4)}+\mathbf{27}_{(m=6)}+\mathbf{35}_{(m=2)}+\overline{\mathbf{35}}_{(m=2)}+\mathbf{64}. \tag{49}$$

Now, we can determine the possible flavor and spin states as follows:

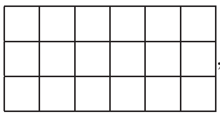
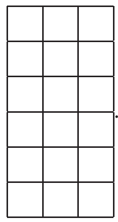
Flavor and spin :


,

.

$\mathbf{8}(S=\frac{1}{2})$ $\overline{\mathbf{10}}(S=\frac{3}{2})$

4. Four baryons + three diquarks

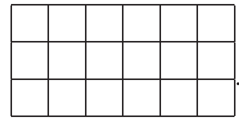
The color and flavor ⊗ spin states of 18 quarks consist of four baryons and three correlated diquarks are as follows:

Color : , Flavor ⊗ Spin : .

Flavor states of 18 quarks:

$$\begin{aligned}\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8} = {} & \mathbf{1}_{(m=8)} + \mathbf{8}_{(m=32)} + \mathbf{10}_{(m=20)} + \overline{\mathbf{10}}_{(m=20)} \\ & + \mathbf{27}_{(m=33)} + \mathbf{28}_{(m=2)} + \overline{\mathbf{28}}_{(m=2)} \\ & + \mathbf{35}_{(m=15)} + \overline{\mathbf{35}}_{(m=15)} + \mathbf{64}_{(m=12)} \\ & + \mathbf{81}_{(m=3)} + \overline{\mathbf{81}}_{(m=3)} + \mathbf{125}. \end{aligned} \tag{50}$$

Using the decomposition in octet baryon case, we can find that there is one possible flavor and spin states as follows:

Flavor and spin : .

$\mathbf{1}(S=0)$

5. Five baryons + three diquarks

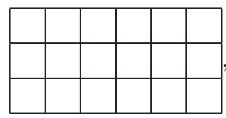
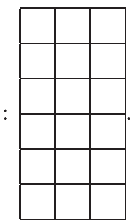
There are no allowed states that satisfy the Pauli principle.

G. Baryon (octet) case

Now, we consider the case that a probe is a flavor octet baryon. Since we can refer the multiquark states composed of baryons only in Sec. IV, here we represent the five baryons plus one baryon case only.

1. Five baryons + one baryon

The color and flavor ⊗ spin states of 18 quarks configuration are as follows:

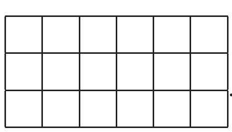
Color : , Flavor ⊗ Spin : .

The flavor ⊗ spin coupling state [3,3,3,3,3,3] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$[3,3,3,3,3,3]_{FS} = [6,6,6]_F \otimes [9,9]_S. \tag{51}$$

Flavor states of 18 quarks:

$$\begin{aligned}\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8} = {} & \mathbf{1}_{(m=145)} + \mathbf{8}_{(m=702)} + \mathbf{10}_{(m=525)} + \overline{\mathbf{10}}_{(m=525)} + \mathbf{27}_{(m=999)} + \mathbf{28}_{(m=161)} + \overline{\mathbf{28}}_{(m=161)} \\ & + \mathbf{35}_{(m=630)} + \overline{\mathbf{35}}_{(m=630)} + \mathbf{55}_{(m=5)} + \overline{\mathbf{55}}_{(m=5)} + \mathbf{64}_{(m=660)} + \mathbf{80}_{(m=70)} + \overline{\mathbf{80}}_{(m=70)} \\ & + \mathbf{81}_{(m=315)} + \overline{\mathbf{81}}_{(m=315)} + \mathbf{125}_{(m=215)} + \mathbf{154}_{(m=70)} + \overline{\mathbf{154}}_{(m=70)} + \mathbf{162}_{(m=9)} + \overline{\mathbf{162}}_{(m=9)} \\ & + \mathbf{216}_{(m=30)} + \mathbf{260}_{(m=5)} + \overline{\mathbf{260}}_{(m=5)} + \mathbf{343}. \end{aligned} \tag{52}$$

Flavor and spin : .

$\mathbf{1}(S=0)$

H. Baryon (decuplet) case

In this subsection, we additionally consider the flavor decuplet baryon case. In Ref. [20], we studied the delta isobar in normal nuclear matter using a constituent quark model. Investigating the stability of the decuplet baryon in dense matter could be interesting subject, so we include this case here.

1. One baryon + one baryon (decuplet)

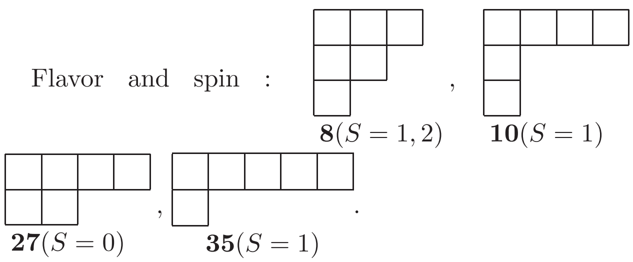
Let us consider the six quarks state, which is composed of one flavor octet baryon and one flavor decuplet baryon. The color and flavor ⊗ spin states are same as in Sec. IV A. However, the possible flavor and spin states are different after the decomposition; we need the following outer product:

Flavor states of six quarks:

$$\mathbf{8}\times\mathbf{10}=\mathbf{35}+\mathbf{27}+\mathbf{10}+\mathbf{8}. \tag{53}$$

Then, using the decomposition in the octet baryon case, we can determine the possible flavor and spin states as follows:

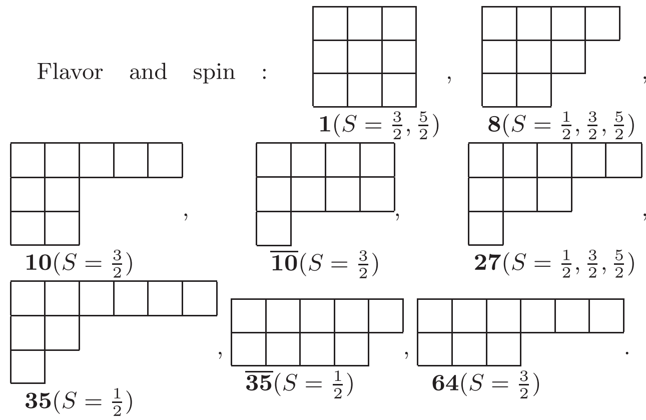
Flavor and spin :



Flavor states of nine quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{10}=\mathbf{1}+\mathbf{8}_{(m=4)}+\mathbf{10}_{(m=4)}+\overline{\mathbf{10}}_{(m=2)}+\mathbf{27}_{(m=5)}+\mathbf{28}+\mathbf{35}_{(m=4)}+\overline{\mathbf{35}}+\mathbf{64}_{(m=2)}+\mathbf{81}. \tag{54}$$

Flavor and spin :



2. Two baryons + one baryon (decuplet)

The color and flavor ⊗ spin states are same as in Sec. IV B.

3. Three baryons + one baryon (decuplet)

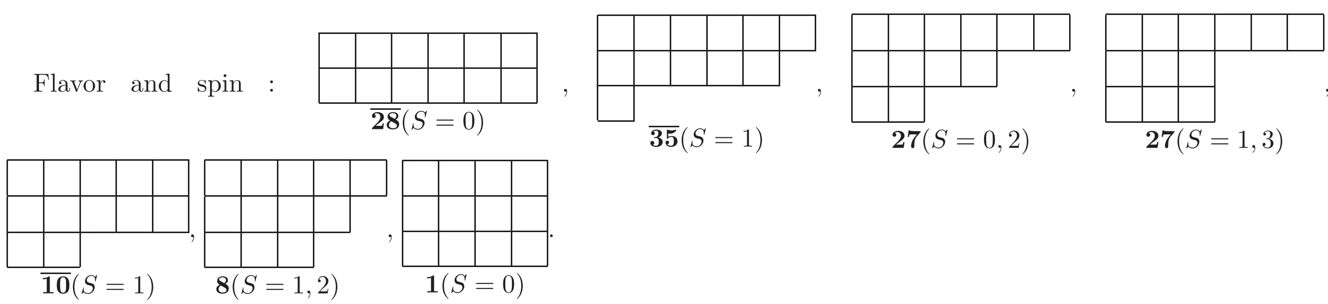
The color and flavor ⊗ spin states are same as in Sec. IV C.

Flavor states of 12 quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{10}=\mathbf{1}_{(m=4)}+\mathbf{8}_{(m=20)}+\mathbf{10}_{(m=17)}+\overline{\mathbf{10}}_{(m=12)}+\mathbf{27}_{(m=27)}+\mathbf{28}_{(m=6)}+\overline{\mathbf{28}}+\mathbf{35}_{(m=21)}+\overline{\mathbf{35}}_{(m=11)}+\mathbf{64}_{(m=15)}+\mathbf{80}_{(m=2)}+\mathbf{81}_{(m=9)}\overline{\mathbf{81}}_{(m=3)}+\mathbf{125}_{(m=3)}+\mathbf{154}. \tag{55}$$

Using the decomposition in the octet baryon case, we can determine the possible flavor and spin states as follows:

Flavor and spin :

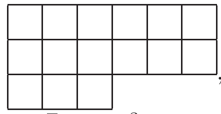
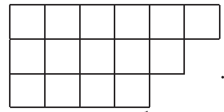


4. Four baryons + one baryon (decuplet)

The color and flavor ⊗ spin states are same as in Sec. IV D.

Flavor states of 15 quarks:

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{10}=\mathbf{1}_{(m=20)}+\mathbf{8}_{(m=100)}+\mathbf{10}_{(m=85)}+\overline{\mathbf{10}}_{(m=70)}+\mathbf{27}_{(m=150)}+\mathbf{28}_{(m=38)}+\overline{\mathbf{28}}_{(m=15)}+\mathbf{35}_{(m=116)}+\overline{\mathbf{35}}_{(m=80)}+\mathbf{55}_{(m=2)}+\mathbf{64}_{(m=104)}+\mathbf{80}_{(m=20)}+\overline{\mathbf{80}}_{(m=4)}+\mathbf{81}_{(m=66)}+\overline{\mathbf{81}}_{(m=36)}+\mathbf{125}_{(m=34)}+\mathbf{154}_{(m=16)}+\overline{\mathbf{154}}_{(m=6)}+\mathbf{162}_{(m=3)}+\mathbf{216}_{(m=4)}+\mathbf{260}. \tag{56}$$

Flavor and spin : ,  .

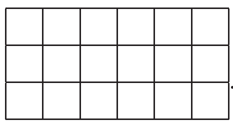
$\bar{\mathbf{10}}(S=\frac{3}{2})$ $\mathbf{8}(S=\frac{1}{2})$

5. Five baryons + one baryon (decuplet)

The color and flavor ⊗ spin states are same as in Sec. V G 1.

Flavor states of 18 quarks:

$$\begin{aligned}\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{10} = {} & \mathbf{1}_{(m=100)} + \mathbf{8}_{(m=525)} + \mathbf{10}_{(m=451)} + \overline{\mathbf{10}}_{(m=400)} + \mathbf{27}_{(m=855)} + \mathbf{28}_{(m=240)} + \overline{\mathbf{28}}_{(m=135)} \\ & + \mathbf{35}_{(m=675)} + \overline{\mathbf{35}}_{(m=535)} + \mathbf{55}_{(m=25)} + \overline{\mathbf{55}}_{(m=4)} + \mathbf{64}_{(m=690)} + \mathbf{80}_{(m=165)} + \overline{\mathbf{80}}_{(m=65)} \\ & + \mathbf{81}_{(m=460)} + \overline{\mathbf{81}}_{(m=315)} + \mathbf{125}_{(m=300)} + \mathbf{143}_{(m=5)} + \mathbf{154}_{(m=160)} + \overline{\mathbf{154}}_{(m=90)} \\ & + \mathbf{162}_{(m=45)} + \overline{\mathbf{162}}_{(m=10)} + \mathbf{216}_{(m=65)} + \mathbf{260}_{(m=25)} + \overline{\mathbf{260}}_{(m=10)} + \mathbf{280}_{(m=4)} + \mathbf{343}_{(m=5)} \\ & + \mathbf{405}. \end{aligned} \tag{57}$$

Flavor and spin : .

$\mathbf{1}(S=0)$

VI. FREE QUARK GAS

Finally, we consider the case where the surrounding is a free quark gas. In such a case, we assume that the surrounding free quarks are not correlated with each other but are correlated with the probe to satisfy the Pauli principle.

A. Quark case

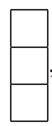
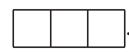
When a probe is a quark, we only need to calculate the average value of the color-spin interactions for all possible diquark configurations. There are four diquark states satisfying the Pauli principle. We represent them in the Table I. If we compare it with the results for baryons and a quark, then we should multiply it by 3 to ensure comparison at the same density.

TABLE I. Classification of two quark interaction due to the Pauli exclusion principle. We denote the antisymmetric and symmetric state as A and S, respectively. The symbols inside the parentheses represent the multiplet state.

	q_iq_j			
Flavor	A	S	A	S
Color	$A(\bar{3})$	$A(\bar{3})$	$S(6)$	$S(6)$
Spin	$A(1)$	$S(3)$	$S(3)$	$A(1)$
$-\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j$	-8	$\frac{8}{3}$	$-\frac{4}{3}$	4
$\lambda_i^c\lambda_j^c$	$-\frac{8}{3}$	$-\frac{8}{3}$	$\frac{4}{3}$	$\frac{4}{3}$

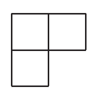
B. Diquark (C_A, F_A, S_A) case

For a diquark with color antitriplet and a free quark, since $\mathbf{3}\times\bar{\mathbf{3}} = \mathbf{1}+\mathbf{8}$, there are two possible color states of three quarks, which will come with the flavor ⊗ spin configuration as below.

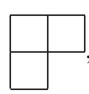
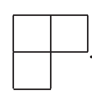
1. Color : , Flavor ⊗ Spin : .

The flavor ⊗ spin coupling state [3] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$[3]_{FS} = [3]_F \otimes [3]_S + [2,1]_F \otimes [2,1]_S. \tag{58}$$

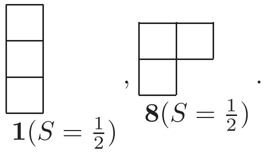
Flavor and spin :  .

$\mathbf{8}(S=\frac{1}{2})$

2. Color : , Flavor ⊗ Spin : .

The flavor ⊗ spin coupling state [2,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

$$\begin{aligned}[2,1]_{FS} = {} & [3]_F \otimes [2,1]_S + [2,1]_F \otimes [3]_S + [2,1]_F \otimes [2,1]_S \\ & + [1,1,1]_F \otimes [2,1]_S. \end{aligned} \tag{59}$$

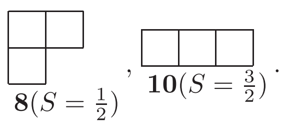
Flavor and spin :


Considering both cases, we can calculate the average value of the color-spin interaction. We multiply it by $\frac{3}{2}$ to compare this results with that for baryons and a quark.

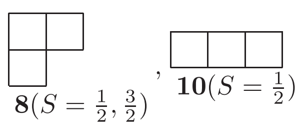
C. Diquark (C_A, F_S, S_S) case

In this case, there are two possible color states as in Sec. VI B.

1. Color : , Flavor $\otimes$ Spin :


Flavor and spin :


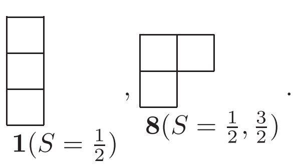
2. Color : , Flavor $\otimes$ Spin :


Flavor and spin :


D. Diquark (C_S, F_A, S_S) case

For a diquark with color sextet, since $\mathbf{6} \times \mathbf{3} = \mathbf{8} + \mathbf{10}$, there are two possible color states of three quarks, which will come with the flavor $\otimes$ spin configuration as below:

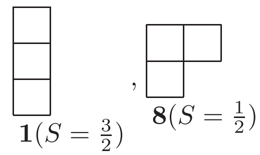
1. Color : , Flavor $\otimes$ Spin :


Flavor and spin :


2. Color : , Flavor $\otimes$ Spin :


The flavor $\otimes$ spin coupling state [1,1,1] with SU(6) can be decomposed into the states with the flavor SU(3) and the spin SU(2) as follows:

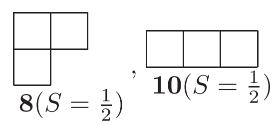
$$[1,1,1]_{FS} = [2,1]_F \otimes [2,1]_S + [1,1,1]_F \otimes [3]_S. \quad (60)$$

Flavor and spin :


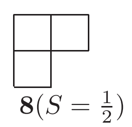
E. Diquark (C_S, F_S, S_A) case

Here as elsewhere, there are two possible color states.

1. Color : , Flavor $\otimes$ Spin :


Flavor and spin :


2. Color : , Flavor $\otimes$ Spin :


Flavor and spin :


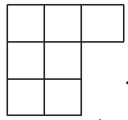
F. Three correlated diquarks case

For the color-spin interaction between three correlated diquarks and a free quark, the color and flavor $\otimes$ spin coupling state is as follows:

Color : , Flavor $\otimes$ Spin :


Flavor states of 7 quarks : $\mathbf{1} \times \mathbf{3} = \mathbf{3}$.

Flavor states of seven quarks: $\mathbf{1}\times\mathbf{3}=\mathbf{3}$.

Flavor and spin :  .

$\mathbf{3}(S=\frac{1}{2})$

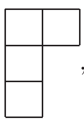
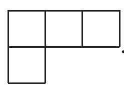
In order to compare the result with the interaction factor when one baryon looks at one quark, we need to divide by 2.

G. Baryon (octet) case

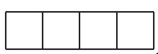
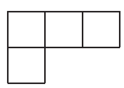
The multiquark state of this case is the same as in Sec. V A 1.

H. Baryon (decuplet) case

The color and flavor ⊗ spin states of four quarks configuration are as follows:

Color :  , Flavor ⊗ Spin :  .

Flavor states of four quarks: $\mathbf{10}\times\mathbf{3}=\mathbf{15'}+\mathbf{15}$.

Flavor and spin :  $\mathbf{15'}(S=1)$,  $\mathbf{15}(S=1,2)$.

VII. RESULTS

Table II shows the averaged color-spin interaction factors calculated using the multiquark states and Eq. (18). In the case of flavor SU(2), the result was obtained by excluding the states that must contain a strange quark from among the possible states in flavor SU(3). In the case of flavor SU(2), since there is no strange quark, the combinations that make up an antisymmetric flavor state are quite limited. Therefore, from four surrounding baryons, it is not possible to create a state that satisfies the Pauli principle regardless of the probe type.

There are a few points on these results. First of all, as we can see in the Table II, when the number of surrounding baryons is increased to two, the strength of the interaction increases, but from three it steadily decreases for most cases. Also, comparing the interaction between a quark and a octet baryon, it can be seen that the quark is more repulsive when the number of surrounding baryons is up to two, but a baryon becomes more repulsive relatively from three onwards. It shows that the interaction experienced by a quark is relatively more attractive than that of a baryon once the density increases to a point where three or more baryons are correlated.

It should be noted that, in the case of three correlated diquarks, the averaged interaction factor does not differ when the number of surrounding baryons changes. The reason is that the states of the three correlated diquarks are all singlets, so when we multiply it by the multibaryon state, the multiplet state of multibaryon remains intact.

The results when the surrounding is a free quark gas are also noteworthy. In the case of free quark gas, it can be seen

TABLE II. $\Delta H_{CS}^{\text{avg}}$ for different probes (column) in various surroundings (row). The upper and lower tables are for flavor SU(2) and SU(3), respectively. "None" represents that there is no state that satisfies the Pauli principle.

$SU(2)_F$	1b	2b	3b	4b	5b	Free quarks
Quark	8	8.533	6.133	None	None	4.364
Diquark(C_A, F_A, S_A)	8	8	8	None	None	8
Diquark(C_A, F_S, S_S)	8	8.267	4.889	None	None	2.667
Diquark(C_S, F_A, S_S)	8	8.533	None	None	None	4.471
Diquark(C_S, F_S, S_A)	8	8.4	None	None	None	2.118
Baryon(octet)	7.111	7.111	7.111	None	None	8
Baryon(decuplet)	7.111	7.111	3.556	None	None	2.872

$SU(3)_F$	1b	2b	3b	4b	5b	Free quarks
Quark	6	6.446	4.644	4.167	3.657	2.823
Diquark(C_A, F_A, S_A)	6	6.176	5.551	5.257	4.8	6.3
Diquark(C_A, F_S, S_S)	6	6.107	3.799	3.359	2.933	1.12
Diquark(C_S, F_A, S_S)	6	6.4	4.884	4.376	None	3.28
Diquark(C_S, F_S, S_A)	6	6.185	3.869	3.304	None	0.643
Three correlated diquarks	6	6	6	6	None	6
Baryon(octet)	5.714	5.78	4.944	4.667	4.267	6
Baryon(decuplet)	4.647	4.456	2.916	2.26	2.133	1.455

TABLE III. Same as Table II after the internal interactions within the probe are considered.

$SU(2)_F$	1b	2b	3b	4b	5b	Free quarks
Quark	8	8.533	6.133	None	None	4.364
Diquark(C_A, F_A, S_A)	4	4	4	None	None	4
Diquark(C_A, F_S, S_S)	9.333	9.6	6.222	None	None	4
Diquark(C_S, F_A, S_S)	7.333	7.867	None	None	None	3.804
Diquark(C_S, F_S, S_A)	10	10.4	None	None	None	4.118
Baryon(octet)	4.444	4.444	4.444	None	None	5.333
Baryon(decuplet)	9.778	9.778	6.222	None	None	5.538

$SU(3)_F$	1b	2b	3b	4b	5b	Free quarks
Quark	6	6.446	4.644	4.167	3.657	2.823
Diquark(C_A, F_A, S_A)	2	2.176	1.551	1.257	0.8	2.3
Diquark(C_A, F_S, S_S)	7.333	7.44	5.132	4.692	4.267	2.45
Diquark(C_S, F_A, S_S)	5.333	5.734	4.217	3.709	None	2.613
Diquark(C_S, F_S, S_A)	8	8.185	5.869	5.304	None	2.643
Three correlated diquarks	2	2	2	2	None	2
Baryon(octet)	3.048	3.113	2.277	2	1.6	3.333
Baryon(decuplet)	7.313	7.123	5.583	4.927	4.8	4.121

that the interaction factor of quark is much more attractive than that of baryon as we expected. Additionally, for free quark gas, the interaction factor is more attractive compared to the case where the surroundings are multibaryon states. However, there are exceptions that will be explained below when discussing the internal structure of a probe.

When we compare the octet baryon and the decuplet baryon, the interaction of the decuplet baryon seems more attractive. Even in the case of diquarks, it can be seen that the diquark with (C_A, F_S, S_S) is more attractive than the diquark with (C_A, F_A, S_A), which is considered the most attractive. However, these results are because of the internal color-spin factor of the probe. The diquark with (C_A, F_S, S_S) and the decuplet baryon have one thing in common: the flavor and spin states are totally symmetric.

As we can find out in Eq. (2), the color-spin factor is more attractive when there are more antisymmetric combinations in the flavor state because the corresponding C_F is small. The possibility that the surrounding baryons can form an antisymmetric combination with the probe increases when the flavor state of the probe is symmetric. Then the color-spin interaction between a probe and the surrounding baryons becomes attractive. However, in this case, we need to consider their internal structures because when the flavor state is symmetric then the color-spin factors are repulsive. Therefore, we show the results of considering the probe's internal color-spin factor as well as the interaction between the probe and the surrounding baryons in Table III.

Considering the internal color-spin factors of probes, it can be seen that when the surrounding is baryons, and more baryons are correlated, the most stable state is the diquark with (C_A, F_A, S_A). Therefore, if a baryon is decomposed into a quark and a diquark and the accumulated quarks further form diquarks it can be seen that this result is consistent with the so-called diquarkyonic matter configuration in which quark and diquark can coexist at high density. Even at higher density when the surrounding turns into the free quark gas, the most stable state is three correlated diquarks.

VIII. SUMMARY

In this work, we constructed the multiquark states to calculate the interaction energy of a probe. To examine the behavior of dense matter, we calculated the relative magnitude of the interaction experienced by the probe as the number of correlated surrounding baryons increased but density kept constant. As a result, we found that a quark experiences the less repulsive interaction than a baryon when the number of correlated surrounding baryons is three or more.

On the other hand, we showed that the diquark may be the most stable state in dense matter when the internal interactions of a probe are considered. These results show the possibility of a new phase called diquarkyonic matter.

There are a few additional things that need to be done in relation to this. If a diquark can exist as an independent state, then the interaction between diquarks cannot be ignored. Investigation on the interactions between diquarks can be an important factor in the study of diquark condensation as well as diquarkyonic configuration. Additionally, it is also necessary to consider the inhomogeneous matter. It can be important to look at how the behavior of an interaction changes when the spatial part of a multiquark state is not totally symmetric.

ACKNOWLEDGMENTS

This work was supported by Samsung Science and Technology Foundation under Project No. SSTF-BA1901-04. The work of A. P. was supported by the

Korea National Research Foundation under Grant No. 2021R1I1A1A01043019.

APPENDIX: THREE-BODY CONFINEMENT POTENTIAL IN SU(N)

The commutation and anticommutation relations for generators of SU(N) are as follows:

$$[T^a, T^b] = if^{abc}T^c, \tag{A1}$$

$$\{T^a, T^b\} = \frac{1}{N}\delta^{ab} + d^{abc}T^c, \tag{A2}$$

where $a, b, c = 1, 2, \ldots, N^2 - 1$. Normalization condition is as follows:

$$\mathrm{Tr}(T^aT^b) = \frac{1}{2}\delta^{ab}. \tag{A3}$$

We can also represent the color-color interaction between ith and jth quarks using the permutation

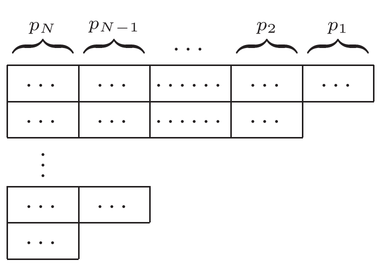
$$T_i^aT_j^a = \frac{1}{2}(ij) - \frac{1}{2N}I, \tag{A4}$$

where I is the (N^2-1) by (N^2-1) identity matrix. Then, we can represent the d-type and f-type three-body confinement forces as follows:

$$d^{abc}T_i^aT_j^bT_k^c = \frac{1}{4}[(ijk)+(ikj)] + \frac{1}{N^2}I - \frac{1}{2N}[(ij)+(ik)+(jk)], \tag{A5}$$

$$f^{abc}T_i^aT_j^bT_k^c = \frac{i}{4}[(ijk)-(ikj)]. \tag{A6}$$

Now, consider the following general color state of SU(N):



Since the d-type three-body confinement potential is Casimir operator, we can get the eigenvalue of it by calculating the diagonal component for the normal Young-Yamanouchi basis.

There are four cases: (1) i, j, k are in the same row. (2) i, j are in the same row and k is in the lower row. (3) j, k are in the same row and i is in the upper row. (4) i, j, k are in different rows. Here we show work for cases (1) and (2). For cases (3) and (4), we can calculate it in a similar way.

For case (1), since i, j, k are in the same row, any permutation between i, j, k is just the identity. And the number of this case is as follows:

$$N_1 = \binom{p_1+\cdots+p_N}{3} + \binom{p_2+\cdots+p_N}{3} + \cdots + \binom{p_N}{3}. \tag{A7}$$

For case (2), we can calculate it step by step. First, consider the case when i, j are in the first row. Then the possible number of this case is as follows:

$$N_{2,1} = \binom{p_1+\cdots+p_N}{2}\left[\binom{p_2+\cdots+p_N}{1} + \cdots + \binom{p_N}{1}\right]. \tag{A8}$$

Also, the diagonal components of each permutation are as follows:

$$(ij) = 1, \qquad (ik) = (jk) = -\frac{1}{p_1+\cdots+p_N},$$
$$(ijk) = (ij)(jk) = (jk) = (ikj). \tag{A9}$$

We can continue this calculation when i, j are in the second row, third row, $\cdots$ and finally the $(N-1)$th row. The possible number of the final cases and the diagonal components of each permutation are as follows:

$$N_{2,N-1} = \binom{p_{N-1}+p_N}{2}\binom{p_N}{1}. \tag{A10}$$

$$(ij) = 1, (ik) = (jk) = -\frac{1}{p_{N-1}+p_N},$$
$$(ijk) = (ij)(jk) = (jk) = (ikj). \tag{A11}$$

For cases (3) and (4) we can use a similar method. Collecting all the terms, we can get the eigenvalue of d-type three-body confinement force. Here, we represent it for SU(4), SU(5), and SU(6).

For SU(4),

$$\sum_{i<j<k} d^{abc}T_i^aT_j^bT_k^c = \frac{p_1}{16} - \frac{3p_1^2}{32} + \frac{p_1^3}{32} + \frac{3p_2}{8} - \frac{p_1p_2}{4} + \frac{p_1^2p_2}{16} - \frac{3p_2^2}{8} + \frac{23p_3}{16} - \frac{3p_1p_3}{16} + \frac{p_1^2p_3}{32} - \frac{p_2p_3}{2} - \frac{15p_3^2}{32} - \frac{p_1p_3^2}{32} - \frac{p_2p_3^2}{16} - \frac{p_3^3}{32} + \frac{15p_4}{4}. \tag{A12}$$

For SU(5),

$$\begin{aligned}\sum_{i<j<k} d^{abc}T_i^aT_j^bT_k^c &= \frac{2p_1}{25} - \frac{3p_1^2}{25} + \frac{p_1^3}{25} + \frac{23p_2}{50} - \frac{33p_1p_2}{100} + \frac{9p_1^2p_2}{100} - \frac{12p_2^2}{25} + \frac{3p_1p_2^2}{100} + \frac{p_2^3}{50} + \frac{41p_3}{25} - \frac{8p_1p_3}{25} + \frac{3p_1^2p_3}{50}\\ &- \frac{21p_2p_3}{25} + \frac{p_1p_2p_3}{25} + \frac{p_2^2p_3}{25} - \frac{39p_3^2}{50} - \frac{p_1p_3^2}{50} - \frac{p_2p_3^2}{25} - \frac{p_3^3}{50} + \frac{103p_4}{25} - \frac{21p_1p_4}{100} + \frac{3p_1^2p_4}{100}\\ &- \frac{13p_2p_4}{25} + \frac{p_1p_2p_4}{50} + \frac{p_2^2p_4}{50} - \frac{93p_3p_4}{100} - \frac{p_1p_3p_4}{50} - \frac{p_2p_3p_4}{25} - \frac{3p_3^2p_4}{100} - \frac{18p_4^2}{25} - \frac{3p_1p_4^2}{100} - \frac{3p_2p_4^2}{50}\\ &- \frac{9p_3p_4^2}{100} - \frac{p_4^3}{25} + \frac{42p_5}{5}.\end{aligned} \tag{A13}$$

For SU(6),

$$\begin{aligned}\sum_{i<j<k} d^{abc}T_i^aT_j^bT_k^c &= \frac{5p_1}{54} - \frac{5p_1^2}{36} + \frac{5p_1^3}{108} + \frac{14p_2}{27} - \frac{7p_1p_2}{18} + \frac{p_1^2p_2}{9} - \frac{5p_2^2}{9} + \frac{p_1p_2^2}{18} + \frac{p_2^3}{27} + \frac{16p_3}{9} - \frac{5p_1p_3}{12} + \frac{p_1^2p_3}{12}\\ &- \frac{13p_2p_3}{12} + \frac{p_1p_2p_3}{12} + \frac{p_2^2p_3}{12} - p_3^2 - \frac{118p_4}{27} - \frac{13p_1p_4}{36} + \frac{p_1^2p_4}{18} - \frac{8p_2p_4}{9} + \frac{p_1p_2p_4}{18} + \frac{p_2^2p_4}{18}\\ &- \frac{19p_3p_4}{12} - \frac{11p_4^2}{9} - \frac{p_1p_4^2}{36} - \frac{p_2p_4^2}{18} - \frac{p_3p_4^2}{12} - \frac{p_4^3}{27} + \frac{475p_5}{54} - \frac{2p_1p_5}{9} + \frac{p_1^2p_5}{36} - \frac{19p_2p_5}{36} + \frac{p_1p_2p_5}{36}\\ &+ \frac{p_2^2p_5}{36} - \frac{11p_3p_5}{12} - \frac{25p_4p_5}{18} - \frac{p_1p_4p_5}{36} - \frac{p_2p_4p_5}{18} - \frac{p_3p_4p_5}{12} - \frac{p_4^2p_5}{18} - \frac{35p_5^2}{36} - \frac{p_1p_5^2}{36} - \frac{p_2p_5^2}{18}\\ &- \frac{p_3p_5^2}{12} - \frac{p_4p_5^2}{9} - \frac{5p_5^3}{108} + \frac{140p_6}{9}.\end{aligned} \tag{A14}$$

Note that the eigenvalue of d-type three-body confinement potentials for SU(4), SU(5), and SU(6) are linear in p_4, p_5, and p_6, respectively.

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