# New reaction approach to reflect exotic structure of hadronic molecular state

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With the accumulation of experimental data, more and more exotic hadrons are observed. Among the interpretations of these exotic hadrons, molecular state and compact multiquark are two of the most popular pictures. However, it is still difficult to determine the structure of an exotic hadron. In this work, we propose a possible way to detect the internal structure of an exotic state. When a molecular state composed of two consistent hadrons is attacked by another particle, one of the constituents should be kicked out while another quasifree constituent stays almost unaffected. It is different from a compact multiquark which has no obvious subcluster. In this work, taking the $X(3872)$ as an example, we perform a Dalitz plot analysis of such reaction to find the effect of the different internal structures. Under the assumption of the $X(3872)$ as a molecular state or a compact tetraquark state, with the help of the effective Lagrangians, the Dalitz plot and the invariant mass spectrum are estimated with different total invariant mass of three final particles, and the effect of different binding energies is also discussed. Obvious event concentration can be observed as strips in the Dalitz plot and sharp peaks in the invariant mass spectrum for the $X(3872)$ with a small binding energy under the molecular state picture, while such concentration cannot be observed under the compact tetraquark picture. Such phenomenon can be applied to identify the internal structure of a new hadron state.

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## I. INTRODUCTION

The study of exotic hadrons is one of the most important topics in hadron physics. Theoretically, the basic theory of the strong interaction, quantum chromodynamics (QCD), allows the existence of exotic hadrons beyond the conventional picture where the hadrons are composed of three quarks or a quark-antiquark pair. Experimentally, with the development of the experimental techniques and accumulation of data, more and more exotic particles are observed but cannot be put into the frames of the conventional quark model [1–3]. If we deem these new particles as a genuine state composed of quarks, there exist two main interpretations: compact multiquark and hadronic molecular state. For most of the exotic states, both interpretations exist simultaneously in the literature [4]. It is an interesting and difficult problem to determine the real internal structure of an exotic hadron.

The molecular state is a loosely bound state of two hadrons [5]. Such idea is considerably easy understand if we take the deuteron as a molecular state composed of two hadrons, that is, a nucleon. It is also natural to expect the existence of bound states from other hadrons. The experimental observation seems to support such assumptions also. Considerable $XYZ$ particles are close to the thresholds of two hadrons [1]. The most popular interpretation about such phenomenon is that these particles are composed of the corresponding hadrons with a small binding energy as deuteron [4,5]. The hadronic molecular state is, in fact, a picture in the hadron level. Different from the molecular state, the compact multiquark is a real bound state of quarks [6]. In a compact multiquark, no obvious subcluster can be found, and its radius is usually assumed to be much smaller than a molecular state. Theoretically, the mass of a compact multiquark is irrelevant to the thresholds of hadrons. It seems to be used to judge whether an exotic hadron is a molecular state.

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However, practically, due to the uncertainty of both theory and experiment, many exotic states near thresholds can be also explained as a compact multiquark. Moreover, the multiquark is still an important picture to explain the states which are far from any threshold.

In the literature, the masses and the decay patterns are the most important ways to detect the internal structure of an exotic state. However, as said above, the accordance of the theoretical mass with the experimental mass is not enough to determine the internal structure of an exotic state because it often can be explained in both pictures. The decay pattern is most promising to reflect the quark distributions in the exotic hadrons. However, the uncertainties from both experiment and theory make it difficult to reach a determinative conclusion. Hence, it is helpful to find more ways to detect the internal structure of the exotic states.

The main difference between a molecular state and a compact multiquark is the spatial distribution of the quarks. In a molecular state, the quarks are grouped into two hadrons that have a distance of several more than 10 fm. In addition to effects on the decay pattern, such structure should be reflected when being attacked by a particle. For a molecular state, the incoming particle, which is usually about 1 fm can be easily attacked into the molecular state and even pass through it. Since the distance of two constituent hadrons is separated by long distances, the collision happens only on one constituent. When a constituent hadron is attacked, another one should be little affected. However, for a compact multiquark, which is usually about 1 fm, the results of the collision of an incoming particle is to excite the multiquark and induce its decay. Because of the compactness, the momenta of the incoming particle should be transferred to all quarks and then all final particles. Hence, the molecular state should have quite different behavior after collision.

Such difference of the behavior of collision should be a promising way to detect the internal structure of an exotic state. However, there is an obvious difficulty. We do not have enough stable exotic states to make a target or a beam to perform a collision with another particle. The direct measurement of such collision is impossible with the current or near future experimental technology. However, such collisions can happen in the production of the exotic hadrons in a nucleon-rich environment, which is the realistic scene at facilities, such as LHCb and PANDA. If we can extract the information of collision of the nucleon with produced exotic states, it is still promising to obtain enough events to study such different behaviors of the molecular state and compact multiquark.

In the current work, we try to propose a scheme to realize such idea with the well-known exotic particle $X(3872)$ as an example. Despite being studied by hundreds of experimental physicists and theorists, the structure of $X(3872)$ is not yet fully understood. The most preferred interpretation of the structure of $X(3872)$ nowadays is $c\bar{c} - D\bar{D}^*$ mixing state [7,8]. There also exist other interpretations, such as pure molecular state [9–11], compact tetraquark structure ($c\bar{c}q\bar{q}$) of this exotic state [12,13], radial excitation of the $P$-wave charmonium [14], and a vector glueball mixed with neighboring vector states of charmonium [15]. Among these interpretations, except for the nongenuine particle explanations such as triangular singularities, the $X(3872)$ is a compact quark pair or tetraquark, or a molecular state, or their mixing. In this work, we will study the behavior of the $X(3872)$ attacked by a nucleon in two pictures, the compact quark state and molecular state, to study collision of the $X(3872)$ with a proton as $p + X(3872) \to p + \bar{D}^0 + D^0$.

This article is organized as follows. We present the theoretical formalism to study the reaction of the $p + X(3872) \to p + \bar{D}^0 + D^0$ in two pictures in Sec. II. The numerical results will be given in Sec. III. Finally, the article ends with a summary in Sec. IV.

## II. COLLISION OF THE $X(3872)$ WITH A PROTON

In the current work, we will consider the process $p + X(3872) \to p + \bar{D}^0 + D^0$ to detect the internal structure of the $X(3872)$ with a nucleon. The LHCb experiment definitively established that the $X(3872)$ has $J^{PC} = 1^{++}$ [16], which means that, even if the $X(3872)$ is the $D^0\bar{D}^{*0}$ molecular state, it cannot decay into a $D$ and a $\bar{D}$ meson due to the conservation of spin parity. But collision by a proton may make this process possible. In Ref. [17], we studied the nucleon-induced fissionlike process of the $T_{cc}^+$. Under an assignment of the $T_{cc}^+$ as a molecular state, when induced by a proton, the $T_{cc}^+$ could decay into a $D$ and $D$ pair. Since the $T_{cc}^+$ and $X(3872)$ have some similar features, such as the small binding energy and narrow width, one can legitimately predict that the $X(3872)$ is possible to decay into a $D$ and $\bar{D}$ pair [18]. Furthermore, this reaction can be used to reveal underlying structures of the $X(3872)$. Here, we consider both pictures of the molecular state and compact quark state as shown in Fig. 1.

In the hypothesis of the $X(3872)$ as a loosely molecular state with wave function $(\bar{D}^{*0}D^0 - D^{*0}\bar{D}^0)/\sqrt{2}$, its radius is supposed to be about 10 fm [19] (hereafter, we use the first part of the wave function for explanations; the results for the second part can be obtained analogously). The nucleon and the constituent $\bar{D}^{*0}$ and $D^0$ mesons have a radius smaller than 1 fm. Hence, the proton should attack on one of the constituent mesons of the $X(3872)$. If we only consider the process with three final particles—proton and the $D$ and $\bar{D}$ mesons—the proton should attack on the $\bar{D}^{*0}$ meson, as shown in Fig. 1(a). After the $\bar{D}^{*0}$ meson is attacked, it is transformed to a $\bar{D}^0$ meson. Since the $X(3872)$ is a loosely bound state, it forms a quasi-two-body scattering, $p\bar{D}^{*0} \to p\bar{D}^0$. The final $\bar{D}^0$ meson should be obviously affected by the energy transferred from the incoming proton and that released from the transition of the $\bar{D}^{*0}$ to $\bar{D}^0$ meson. However, another constituent of the

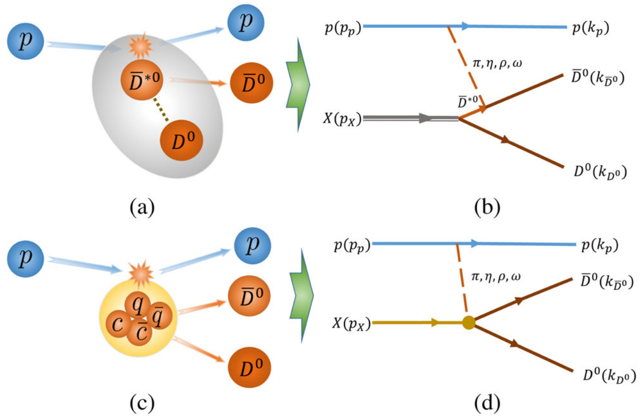


FIG. 1. The sketch map (a),(c) and Feynman diagram (b),(d) of reaction $p + X(3872) \to p + \bar{D}^0 + D^0$ with assumption of the $X(3872)$ as a loosely bound molecular state (a),(b) or a compact multiquark state (c),(d). The denotations of the momenta of particles are also given.

$X(3872)$, $D^0$ meson, should be little affected due to the weak binding. Such process can be rewritten as the Feynman diagram in Fig. 1(b).

If the $X(3872)$ is a compact quark state, the collision behavior is quite different, as shown in Fig. 1(c). The radius of the $X(3872)$ should be smaller than 1 fm for both quark-antiquark pair $[c\bar{c}]$ and tetraquark $[c\bar{c}q\bar{q}]$ explanations of the $X(3872)$ (in Fig. 1(c) and hereafter, we mainly adopt the tetraquark picture). Additionally, such state is composed of quarks binding tightly. In such picture, the $X(3872)$ is excited by the collision of the proton. The excited $X(3872)$ decays to a $\bar{D}^0$ meson and a $D^0$ meson. The energy transferred from the incoming proton will be acquired by both final mesons, which is different from the molecular state picture. The Feynman diagram can be written as Fig. 1(d).

The collision above is difficult to be performed due to lack of the $X(3872)$ target or beam. Here, we propose to consider the produced $X(3872)$ in a nucleon-rich environment. Instead of considering the initial proton and $X(3872)$, three final particles, the proton and $\bar{D}^0$ and $D^0$ mesons, can be collected with certain total invariant mass obtained as $W = \sqrt{s} = \sqrt{P^2} = \sqrt{(k_p + k_{\bar{D}^0} + k_{k^0})^2}$ with $k_{p,\bar{D}^0,D^0}$ being the momenta of final particles, which is independent of the coordinate frames. Among these events, the Dalitz plot against invariant masses $m_{pD^0} = \sqrt{(p_p + p_{D^0})^2}$ and $m_{p\bar{D}^0} = \sqrt{(p_p + p_{\bar{D}^0})^2}$ can be obtained by selecting the corresponding event. In such treatment, all observations are invariant.

The laboratory frame with the static $X(3872)$ will be adopted to perform explicit deduction and numerical calculation. In this reference frame, the cross section for the reaction $p + X(3872) \to p + \bar{D}^0 + D^0$ reads as

$$d\sigma = \frac{1}{4[(p_p \cdot p_X)^2 - m_p^2 m_X^2]^{1/2}} \frac{1}{6} \sum_{\lambda_p \lambda_X \lambda'_p} |\mathcal{M}_{\lambda_p \lambda_X, \lambda'_p}|^2 d\Phi_3, \quad (1)$$

where $p_{p,X}$ and $m_{p,X}$ are the momentum and mass of the incoming proton or the $X(3872)$. Practically, the GENEV code in FAWL is adopted to generate the event of the three-body final state by the Monte Carlo method, that is, the phase space

$$R_3 = (2\pi)^5 d\Phi_3 = \prod_i^3 \frac{d^3 k_i}{2E_i} \delta^4\left(\sum_i^n k_i - P\right),$$

where $k_i$ and $E_i$ are the momentum and energy of final particle $i$. The mechanism can be described by an amplitude $\mathcal{M}_{\lambda_p \lambda_X, \lambda'_p}$ with $\lambda$ being the helicity of the incoming proton, $X(3872)$, or final proton. It will be derived with the Feynman diagrams in Fig. 1. Here the interaction between the proton and the $X(3872)$ is described by light meson exchange. It has the same form in two pictures and can be obtained with the help of effective Lagrangians,

$$\mathcal{L}_{\mathbb{P}NN} = -\frac{g_{\mathbb{P}NN}}{\sqrt{2} m_N} \bar{N}_b \gamma_5 \gamma_\mu \partial_\mu \mathbb{P}_{ba} N_a, \quad (2)$$

$$\mathcal{L}_{\mathbb{V}NN} = -\sqrt{2} g_{\mathbb{V}NN} \bar{N}_b \left(\gamma_\mu + \frac{\kappa}{2m_N} \sigma_{\mu\nu} \partial^\nu\right) \mathbb{V}^\mu_{ba} N_a, \quad (3)$$

where $\mathbb{P}$ and $\mathbb{V}$ are two-by-two pseudoscalar and vector matrices. $N^T = (p, n)$ is a field for the nucleon. The coupling constants $g^2_{\pi NN}/(4\pi) = 13.6$, $g^2_{\rho NN}/(4\pi) = 0.84$, $g^2_{\omega NN}/(4\pi) = 20$ with $\kappa = 6.1(0)$ for the $\rho(\omega)$ meson, which are used in the Bonn nucleon-nucleon potential [20] and meson productions in nucleon-nucleon collision [21–23]. The $\eta$ exchange is neglected in the current work due to the weak coupling of $\eta$ or $\phi$ mesons to nucleons, as indicated in many previous works [20,21]. Here, a factor $f_i(q^2) = (m_i^2 - \Lambda^2)/(q^2 - \Lambda^2)$ is also introduced to the propagator of each exchanged meson with cutoff $\Lambda = 1$ GeV. The left part of the amplitudes in two pictures is different, as given below.

In the molecular state picture as shown in Fig. 1(a), the exchanged light meson interacts with the $\bar{D}^{*0}$ meson in the $X(3872)$. In terms of heavy quark limit and chiral symmetry, the corresponding Lagrangians have been constructed in the literature as [24]

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{P}} = -\frac{2g}{f_\pi} (\mathcal{P}_b \mathcal{P}^{*\dagger}_{a\lambda} + \mathcal{P}^*_{b\lambda} \mathcal{P}^\dagger_a) \partial^\lambda \mathbb{P}_{ba}, \quad (4)$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{V}} = -2\sqrt{2} \lambda g_V v^\lambda \varepsilon_{\lambda\alpha\beta\mu} (\mathcal{P}_b \mathcal{P}^{*\mu\dagger}_a + \mathcal{P}^{*\mu}_b \mathcal{P}^\dagger_a) \partial^\alpha \mathbb{V}^\beta_{ba}, \quad (5)$$

where $\mathcal{P}^{(*)T} = (D^{(*)0}, D^{(*)+})$ is the field for the $D^{(*)}$ meson. The parameters involved here were determined in the literature as $g = 0.59$, $\beta = 0.9$, $\lambda = 0.56\ \mathrm{GeV}^{-1}$, $g_V = 5.9$, and $f_\pi = 132$ MeV [24,25].

In the molecular state picture, the amplitude $\mathcal{A}_{\lambda_X,\lambda_{\bar{D}^{*0}}}$ for the split of the $X(3872) \to \bar{D}^{*0}D^0$ for the first term of the wave function is [17]

$$\frac{\mathcal{A}_{\lambda_X,\lambda_{\bar{D}^{*0}}}}{p^2 - m^2_{\bar{D}^{*0}}} \simeq -\frac{\sqrt{8m_X m_{\bar{D}^{*0}} m_{D^0}}}{m_X - m_{D^0} + m_{\bar{D}^{*0}}} \psi(\boldsymbol{k}_3)\epsilon_{\lambda_X} \cdot \epsilon^*_{\lambda_{\bar{D}^{*0}}}, \quad (6)$$

where $\lambda_X$ and $\lambda_{\bar{D}^{*0}}$ are helicities for the initial $X(3872)$ state and intermediate $\bar{D}^{*0}$ meson. The $p$ and $m_{\bar{D}^{*0}}$ are the momentum and mass of the intermediate $\bar{D}^{*0}$ meson. Here $m_{X,\bar{D}^{*0},D^0}$ is the mass of the $X(3872)$, $\bar{D}^{*0}$, and $D^0$. The $\epsilon_{\lambda_X}$ and $\epsilon_{\lambda_{\bar{D}^{*0}}}$ are the polarized vectors of the $X(3872)$ and $\bar{D}^{*0}$ meson, respectively. The wave function $\psi(\boldsymbol{k}) = \sqrt{8\pi/a}/(\boldsymbol{k}^2 + 1/a^2)$ with normalization $\int d^3k/(2\pi)^3|\psi(k)|^2 = 1$ [26]. Scattering length $a = 1/\sqrt{2\mu E_B}$ with the reduced mass $\mu = m_{\bar{D}^{*0}} m_{D^0}/(m_{\bar{D}^{*0}} + m_{D^0})$ with $E_B$ being the binding energy.

Different from the molecular state, the $X(3872)$ has no obvious subcluster in the compact quark state picture. In the molecular state picture, the large radius and distance between two constituents make the collision happen on one of the constituents. If the $X(3872)$ is a compact binding state of quarks with small radius, the proton should attack the $X(3872)$ in the whole, which will be excited by the light meson emitted by the proton and decays into two $D$ mesons. In the current work, we do not consider its explicit mechanism. However, such interaction should happen in a small space and short time, which can be taken as a four particle vertex as shown in Fig. 1(d). Such vertex can be written as effective Lagrangians,

$$\mathcal{L}_{X\mathcal{P}\mathcal{P}\mathbb{P}} = g_X X^\mu \partial_\mu \mathbb{P}\mathcal{P}\mathcal{P}, \quad (7)$$

$$\mathcal{L}_{X\mathcal{P}\mathcal{P}\mathbb{V}} = g_X \varepsilon_{\alpha\beta\zeta\eta} \partial^\alpha \mathbb{V}^\beta \partial^\zeta X^\eta \mathcal{P}\mathcal{P}. \quad (8)$$

Since no explicit mechanism is introduced, the coupling constant $g_X$ for each exchange cannot be determined, which will be discussed later.

## III. NUMERICAL RESULTS

Since the incoming momentum cannot be measured in the scene of the nucleon-rich environment, we consider the process $p + X(3872) \to p + \bar{D}^0 + D^0$ with a total invariant mass $W = \sqrt{s}$. With certain $W$, the event distribution can be obtained as the Dalitz plot and invariant mass spectrum against $m_{pD^0}$ and $m_{p\bar{D}^0}$. The different internal structures of the $X(3872)$ will be exhibited in the Dalitz plot and invariant mass spectrum. The experimental binding energy of the $X(3872)$ is every small and even above the thresholds. In the current work, we would like to take it as an example to explain how to detect the internal structure of an exotic hadron. Hence, different values of binding energy will be adopted to show the variation of the Dalitz plot and spectrum with the binding energy. The results in the molecular state picture are shown in Fig. 2, where the Dalitz plot in the $m_{pD^0}$-$m_{p\bar{D}^0}$ plane for the reaction $p + X(3872) \to p + \bar{D}^0 + D^0$ is calculated with different binding energies of the $X(3872)$ as $E_B = 0.1, 1,$ and 10 MeV. Correspondingly, the invariant mass spectrum against $m_{p\bar{D}^0}$ is also presented under each Dalitz plot. The momentum of the incoming proton $\mathrm{p}_p = |\boldsymbol{p}_p|$ will affect the event distributions and is chosen as 0.1, 1, and 3 GeV, which corresponds to total invariant mass $W = 4.814$, 5.147, and 6.341 GeV, respectively.

Because of the symmetry in the wave function, analogical distributions for $m_{p\bar{D}^0}$ and $m_{pD^0}$ can be observed.

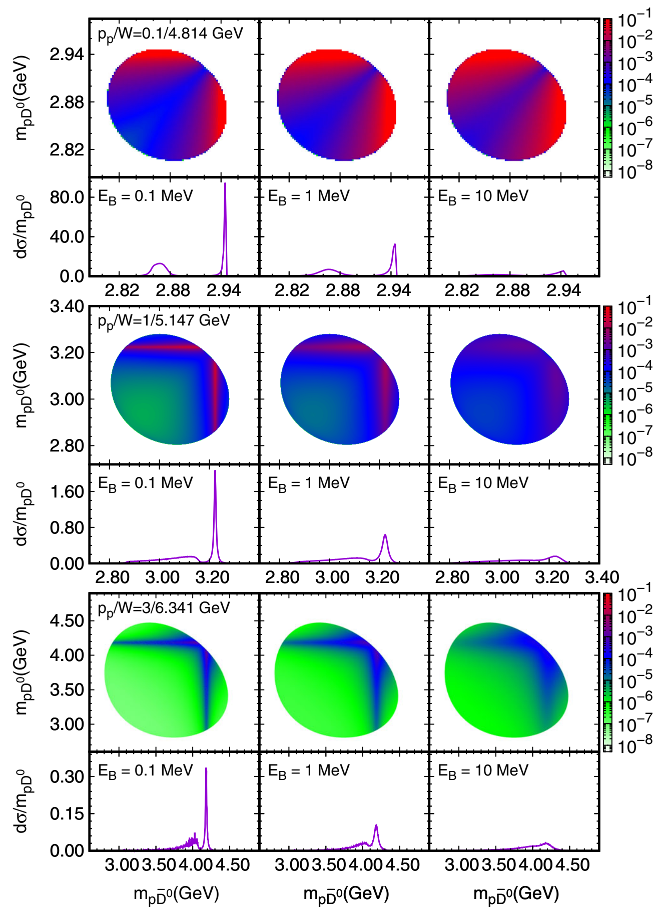


FIG. 2. Event distribution for the $p + X(3872) \to p + \bar{D}^0 + D^0$ reaction assuming the $X(3872)$ as a loosely molecular state. The momentum of the incoming proton or total invariant mass $\mathrm{p}_p/W = 0.1/4.814$, $1/5.147$, and $3/6.341$ GeV, and the binding energy $E_B = 0.1$, 1, and 10 MeV, respectively. For each example choice of $\mathrm{p}_p/W$, the figures represent the Dalitz plot $d\sigma/dm_{p\bar{D}^0}dm_{pD^0}$ in the $m_{p\bar{D}^0}$-$m_{pD^0}$ plane in a bin of $\mu b/0.002 \times 0.002$ GeV (upper) and invariant mass spectrum $d\sigma/dm_{p\bar{D}^0}$ against $m_{pD^0}$ in a bin of $\mu b/0.002$ GeV (lower). The results are obtained with $10^{11}$ simulations.

For example, the Dalitz plot in the first column of the third row in Fig. 2, with an incoming momentum or total invariant mass $\mathrm{p}_p/W = 3/6.341$ GeV, obvious and similar strips can be found at both $m_{p\bar{D}^0}$ and $m_{pD^0}$ of about 4.19 GeV in the Dalitz plot. With the elevation of the incoming proton $\mathrm{p}_p/W$, the phase space of the final states becomes larger. The area of Dalitz plots in the $m_{pD^0}$-$m_{p\bar{D}^0}$ plane for $\mathrm{p}_p/W = 0.1/4.814$ GeV is much smaller than that for 3/6.341 GeV. Hence, the results are rescaled; that is, different ranges are adopted for three momenta.

The $X(3872)$ has a very small binding energy. Here, different binding energies are adopted to discuss the effect of the binding energy on the Dalitz plot and invariant mass spectrum. The plots from the first to the third column in Fig. 2 are for binding energies 0.1, 1, and 10 MeV, respectively (the results of 100 MeV are shown in the second column of Fig. 3 and will be discussed later). An obvious concentration of events can be found for the small binding energy 0.1 MeV in the first column. For $\mathrm{p}_p/W$ of 1/4.814 and 3/6.341 GeV, sharper strips can be found at $m_{p\bar{D}^0}$ at about 3.2 and 4.2 GeV, respectively. The invariant mass spectrum against $m_{p\bar{D}^0}$ also exhibits a very sharp peak. Such concentration of events is from the mechanism of collision as shown in Fig. 1(a). The peak against invariant mass $m_{p\bar{D}^0}$ is due to the quasifree $D^0$ meson in the $X(3872)$. The energy from the incoming proton and the transition of the $\bar{D}^{*0}$ to $\bar{D}^0$ meson is mostly carried by the final proton and $\bar{D}^0$ meson. The small binding energy means a large radius, that is, the large distance and weak attraction between two constituents. With the increase of the binding energy, the attraction of the constituents becomes stronger, and the radius becomes smaller. More momentum will be transferred to the $D^0$ meson, which will be dragged by the stronger attraction of the $\bar{D}^{*0}$ meson. It is confirmed by comparing the results in the first and the third columns of Fig. 2, where, with the increase of binding energy, the strips in the Dalitz plots become vague, and the peaks in the invariant mass spectra become wider also. In addition, because the radius decreases with the increase of the binding energy, the cross section is also reduced.

The results with different incoming momenta or total invariant masses are also presented in the first to third rows in Fig. 2. Generally speaking, if the incoming proton moves fast, the quasifree $D^0$ meson will be less affected. It can be reflected in the Dalitz plots where the strips become more sharp with the increase of the momentum of the incoming proton. The invariant mass spectra have more sharp peaks (for binding energy of 0.1 GeV, the peak is more sharp because the strips are near the edge). However, sharp peaks can be found for all incoming momenta. Additionally, the cross section decreases rapidly with the increase of the incoming momentum due to shorter interaction time.

Now we turn to the compact quark state picture. The Dalitz plots and the invariant mass spectra with different

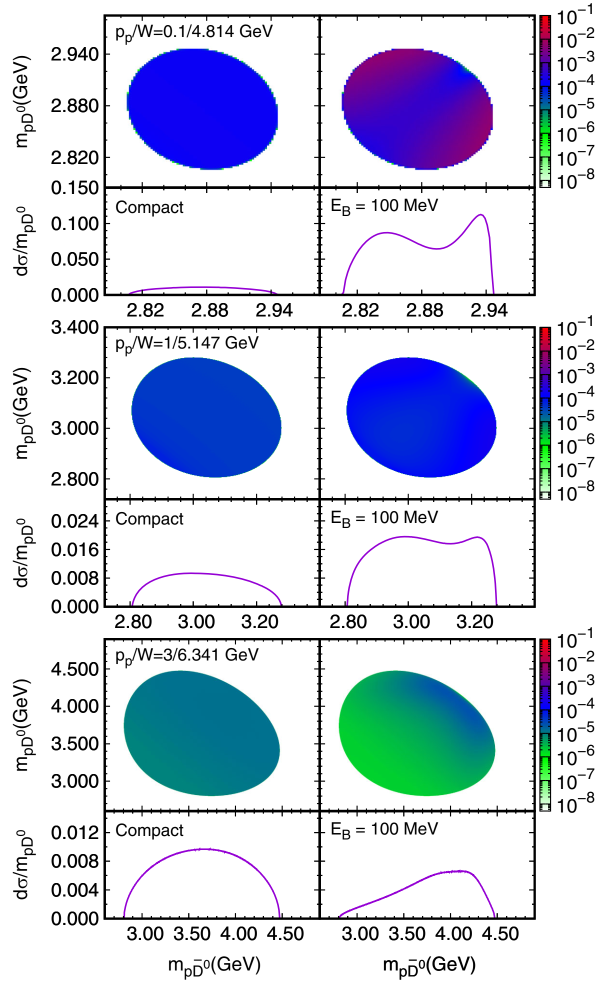


FIG. 3. Event distribution for the $p + X(3872) \to p + \bar{D}^0 + D^0$ reaction under the assumption of the $X(3872)$ as a compact quark state in the first column and under the assumption of the $X(3872)$ as a molecular state with binding energy $E_B = 100$ MeV in the second column. The $\mathrm{p}_p/W$ is chosen as 0.1/4.814, 1/5.147, and 3/6.341 GeV. For each example choice of $\mathrm{p}_p/W$, the figures represent the Dalitz plot $d\sigma/dm_{p\bar{D}^0}dm_{pD^0}$ in the $m_{p\bar{D}^0}$-$m_{pD^0}$ plane in a bin of $\mu$b/0.002 × 0.002 GeV (upper) and invariant mass spectrum $d\sigma/dm_{p\bar{D}^0}$ against $m_{pD^0}$ in a bin of $\mu$b/0.002 GeV (lower). The results are obtained with $10^{11}$ simulations.

total invariant masses are shown in the first column of Fig. 3. Because the coupling constant $g_X$ is not determined, we choose a value of 30 GeV$^{-1}$ to scale the invariant mass spectrum. The result shows that, with each momentum of incoming proton or total invariant mass, the events are almost evenly distributed in the Dalitz plot and no strip, as in the molecular state picture, can be found in the Dalitz plot. There is also no peak or structure in the invariant mass spectrum. If the binding energy of a molecular state is very large, the small radius and strong attraction between the constituents will make the behavior of such state in the reaction considered similar to a compact state.

For comparison, in the second column of Fig. 3, the Dalitz plots for the $X(3872)$ as a molecular state with a large binding energy $E_B = 100$ MeV are also presented. As we can see, these two cases give quite similar results, especially in the row with $p_p/W = 3/6.341$ GeV, which is because the hadronic molecular state tends to a compact tetraquark state with increasing binding energy.

The radius is an important metric for the internal structure of a hadronic state, which will also influence the probability of collision of the incoming proton in our work. As we know, the radius of a $D$ or $D^*$ meson is smaller than 1 fm. For a molecular state of binding energy smaller than 10 MeV, the distance between the $D$ and $\bar{D}^*$ meson in the $X(3872)$ is larger than 1 fm. In this case, the incoming proton will easily collide with the $\bar{D}^*$ meson, and a $\bar{D}$ meson is produced, such as shown in Fig. 2. If the binding energy is larger than 10 MeV, the distance between the $D$ and $\bar{D}^*$ meson in the $X(3872)$ is smaller than 1 fm, which makes the probability of collision of the incoming proton with $\bar{D}^*$ in the $X(3872)$ very small. The radius also affects the relative values of the differential cross section of different total invariant masses $W$. Comparing the results in Fig. 2 and the second column of Fig. 3, the decrease of the invariant mass spectra $d\sigma/dm_{pD^0}$ with the increase of $p_p/W$ becomes slower if the binding energy becomes larger. If the $X(3872)$ is a compact state, the total cross section even increases with the increase of the $p_p/W$, as shown in the first column of Fig. 3.

The strips and peaks are obviously from the internal structure of the exotic state. As a molecular state, the large radius and two-constituent structure makes the incoming proton only attack on one constituent and the other one remains almost unaffected. However, a compact binding state of quarks, either a quark-antiquark pair or tetraquark, the exchanged light meson should affect the state in the whole, and no obvious strip and peak can be produced. Though in the current work we do not consider the explicit mechanism of the reaction of the light meson with the compact state, the conclusion should be unaffected with different explicit models. Such conclusion can be further confirmed with a calculation of an unphysical large binding energy as 100 MeV, which corresponds to a very small radius.

Although the direct collision is unrealistic due to the lack of exotic state target or beam, we suggest observing such phenomenon in the production of an exotic state in a nucleon-rich environment, such as at LHCb and PANDA. The produced exotic state will interact with the surrounding nucleons and decay into two final particles, for example, $D^0$ and $\bar{D}^0$ mesons here, combined with a proton. With different total invariant masses, the strips in Dalitz plots and the peaks in invariant mass spectra appear in different invariant masses of two final particles. Such proposal can provide a more determinative confirmation of the molecular state structure of an exotic state and exclude the compact multiquark assignment.

## IV. SUMMARY

In the current work, we propose a new possible reaction approach to reflect an exotic structure of a hadronic molecular state. Taking the $X(3872)$ as an example, the Dalitz plot and the invariant mass spectrum are estimated for $p + X(3872) \to p + \bar{D}^0 + D^0$. Two pictures of the internal structure of the $X(3872)$, the molecular state and compact multiquark state, are considered in the calculation. In the molecular state picture, obvious strips and peaks can be observed in the Dalitz plot and invariant mass spectrum, respectively. With the increase of binding energy, the strips and peaks vanish gradually, and the Dalitz plot and invariant mass spectrum tend to these in the compact multiquark picture.

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[1] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).
[2] S. Godfrey and N. Isgur, Mesons in a relativized quark model with chromodynamics, Phys. Rev. D **32**, 189 (1985).
[3] S. Capstick and N. Isgur, Baryons in a relativized quark model with chromodynamics, Phys. Rev. D **34**, 2809 (1986).
[4] H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, The hidden-charm pentaquark and tetraquark states, Phys. Rep. **639**, 1 (2016).
[5] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao, and B. S. Zou, Hadronic molecules, Rev. Mod. Phys. **90**, 015004 (2018); **94**, 029901(E) (2022).

[6] Y. R. Liu, H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Pentaquark and tetraquark states, Prog. Part. Nucl. Phys. **107**, 237 (2019).

[7] A. M. Badalian, V. D. Orlovsky, Y. A. Simonov, and B. L. G. Bakker, The ratio of decay widths of X(3872) to $\psi'\gamma$ and $J/\psi\gamma$ as a test of the X(3872) dynamical structure, Phys. Rev. D **85**, 114002 (2012).

[8] T. H. Wang and G. L. Wang, Radiative E1 decays of X (3872), Phys. Lett. B **697**, 233 (2011).

[9] E. S. Swanson, Short range structure in the X(3872), Phys. Lett. B **588**, 189 (2004).

[10] S. K. Choi *et al.* (Belle Collaboration), Observation of a Narrow Charmoniumlike State in Exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ Decays, Phys. Rev. Lett. **91**, 262001 (2003).

[11] N. A. Tornqvist, Isospin breaking of the narrow charmonium state of Belle at 3872-MeV as a deuson, Phys. Lett. B **590**, 209 (2004).

[12] A. Esposito, L. Maiani, A. Pilloni, A. D. Polosa, and V. Riquer, From the line shape of the X(3872) to its structure, Phys. Rev. D **105**, L031503 (2022).

[13] R. Aaij *et al.* (LHCb Collaboration), Observation of Multiplicity Dependent Prompt $\chi_{c1}(3872)$ and $\psi(2S)$ Production in $pp$ Collisions, Phys. Rev. Lett. **126**, 092001 (2021).

[14] T. Barnes, S. Godfrey, and E. S. Swanson, Higher charmonia, Phys. Rev. D **72**, 054026 (2005).

[15] K. K. Seth, An alternative interpretation of X(3872), Phys. Lett. B **612**, 1 (2005).

[16] R. Aaij *et al.* (LHCb Collaboration), Determination of the $X(3872)$ Meson Quantum Numbers, Phys. Rev. Lett. **110**, 222001 (2013).

[17] J. He and X. Liu, The quasi-fission phenomenon of double charm $T_{cc}^{+}$ induced by nucleon, Eur. Phys. J. C **82**, 387 (2022).

[18] J. He, D. Y. Chen, Z. W. Liu, and X. Liu, Induced fission-like process of hadronic molecular states, Chin. Phys. Lett. **39**, 091401 (2022).

[19] Y. R. Liu, X. Liu, W. Z. Deng, and S. L. Zhu, Is $X(3872)$ really a molecular state?, Eur. Phys. J. C **56**, 63 (2008).

[20] R. Machleidt, The high precision, charge dependent Bonn nucleon-nucleon potential (CD-Bonn), Phys. Rev. C **63**, 024001 (2001).

[21] X. Cao, B. S. Zou, and H. S. Xu, Phenomenological analysis of the double pion production in nucleon-nucleon collisions up to 2.2 GeV, Phys. Rev. C **81**, 065201 (2010).

[22] K. Tsushima, A. Sibirtsev, A. W. Thomas, and G. Q. Li, Resonance model study of kaon production in baryon baryon reactions for heavy ion collisions, Phys. Rev. C **59**, 369 (1999); **61**, 029903(E) (2000).

[23] A. Engel, A. K. Dutt-Mazumder, R. Shyam, and U. Mosel, Pion production in proton proton collisions in a covariant one boson exchange model, Nucl. Phys. **A603**, 387 (1996).

[24] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phenomenology of heavy meson chiral Lagrangians, Phys. Rep. **281**, 145 (1997).

[25] R. Chen, Z. F. Sun, X. Liu, and S. L. Zhu, Strong LHCb evidence supporting the existence of the hidden-charm molecular pentaquarks, Phys. Rev. D **100**, 011502 (2019).

[26] M. B. Voloshin, Interference and binding effects in decays of possible molecular component of X(3872), Phys. Lett. B **579**, 316 (2004).