$X(6900)$ peak could be a molecular state

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Analyses of the LHCb data on $X(6900)$ found in di-J/ψ and $J/\psi\psi(3686)$ systems are performed using a momentum-dependent Flatté-like parametrization. The use of the pole-counting rule and spectral density function sum rule give consistent conclusions that $X(6900)$ may not be a molecule of $J/\psi\psi(3686)$. Nevertheless, it is still possible that $X(6900)$ is a molecule of higher states, such as $J/\psi\psi(3770)$, $\chi_{c0}\chi_{c2}$, etc.

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I. INTRODUCTION

The LHCb Collaboration has observed a structure named $X(6900)$ in the di-J/ψ invariant mass spectrum [1], with the signal statistical significance above 5σ. It is probably composed of four (anti)charm quarks ($c\bar{c}c\bar{c}$), and its widths [1] are determined to be $80 \pm 19(\text{stat}) \pm 33(\text{sys})$ and $168 \pm 33(\text{stat}) \pm 69(\text{sys})$ MeV in two fitting scenarios of Breit-Wigner parametrizations with constant widths. Additionally, a broad bump and a narrow bump exist in the low and high sides of the di-J/ψ mass [1], respectively, where the former might be a result of a lower broad resonant state (or several lower states) or interference effect, and the latter is found to be a hint of a state located at ~7200 MeV, named $X(7200)$. The $X(6900)$ peak in the di-J/ψ channels was also found by the CMS Collaboration [2]. More recently, the $X(6900)$ peak was reported to be observed in the $J/\psi\psi(3686)$ invariant mass spectrum [3].

The experimental observation has triggered tremendous studies; see, for example, Refs. [4–30].[1] Generally speaking, a molecular state may locate near the threshold of two color singlet hadrons, like a deuteron $Z_b(10610)$ [31], $Z_c(3900)$ [32–34], $P_c(4450)$ [35,36], etc. The $X(6900)$ state is close to the threshold of $J/\psi\psi(3770)$, $J/\psi\psi_2(3823)$, $J/\psi\psi_3(3842)$, and $\chi_{c0}\chi_{c1}$, and the $X(7200)$ is close to the threshold of $J/\psi\psi(4160)$ and $\chi_{c0}\chi_{c1}(3872)$. Inspired by this, it is studied in this paper, as an extension to the work of Ref. [4], the properties of $X(6900)$ and $X(7200)$, by assuming, for example, the $X(6900)$ coupling to the $J/\psi J/\psi$, $J/\psi\psi(2s)$, $J/\psi\psi(3770)$, $J/\psi\psi_2(3823)$, $J/\psi\psi_3(3842)$, and $\chi_{c0}\chi_{c2}$ channels, etc., and $X(7200)$ to the $J/\psi J/\psi$, $J/\psi\psi(2s)$, and $J/\psi\psi(4160)$ channels, etc. Notice that in this paper, we limit ourselves to discussing only the S-wave ($l = 0$) couplings, since only in this situation are we able to distinguish a molecular state from an elementary state (confined by color force). Such an assignment corresponds to $J^P = 0^{++}$, 2^{++} spin quantum numbers of $X(6900)$. For the S-wave $J/\psi J/\psi$ coupling, the pole-counting rule (PCR) [37], which has been applied to the studies of "XYZ" physics in Refs. [38–41], and the spectral density function sum rule [41–45] are employed to analyze the nature of the two structures in Ref. [4]. It is found that the di-J/ψ data alone are not enough to judge the intrinsic properties of the two states. It is also pointed out that $X(6900)$ is unlikely a molecule of $J/\psi\psi(3686)$ [4]—a conclusion drawn before the new data from Ref. [3].

In this paper, we conduct an upgraded analysis of $X(6900)$ by using the new $J/\psi\psi(3686)$ data. What we conclude from this reanalysis is that, even though $X(6900)$ is very unlikely a molecule of $J/\psi\psi(3686)$, it is not necessarily an elementary state, i.e., a compact $\bar{c}\,\bar{c}\,cc$ tetraquark state. Considering that the tetraquark idea is rather attractive in the literature (see, for example, Refs. [11,12,18,19,30]), it is important to make a more careful reanalysis of the nature of $X(6900)$. The present analysis points out that there still exists the possibility that the $X(6900)$ state is a molecular state composed of particles whose thresholds are closer to $X(6900)$ compared with $J/\psi\psi(3686)$, such as $J/\psi\psi(3770)$, $J/\psi\psi_2(3823)$, $J/\psi\psi_3(3842)$, $\chi_{c0}\chi_{c2}$, etc.

[1]For an incomplete list of earlier studies, see, for example, the references listed in Ref. [4].

We start with a two-channel reanalysis [i.e., di-J/ψ, $J/\psi\psi(3686)$] using the neural network program which has been recently developed in the study of XYZ states in Ref. [25]. Then we make further numerical three-channel studies by including another nearby threshold and point out that a strong coupling to the third channel is not excluded by the current data, and hence, $X(6900)$ may still be of molecule nature.

II. EXAMINATION OF THE COUPLED CHANNEL SITUATION BY USING A NEURAL NETWORK

For this study, a supervised learning scheme is adopted as in Ref. [25]. This means that we need some "molecule-type" and "elementary-type" samples beforehand. These samples are then taken into the machine learning program. The samples named "molecule" are labeled "0," and the samples named "elementary" are labeled "1." The purpose is to let the program determine the different characteristics of these input line shapes between 0 and 1. We use PCR [37] as our criteria. That is, if there exists one pole near one threshold in the s plane, the S-wave resonance can be interpreted as the molecule in that channel. On the other hand, if there exists one more nearby pole (on different sheets), then the S-wave resonance is labeled elementary. This method has been used in many works to study the nature of exotic hadron states; see, for example, Refs. [16,38,40,41]. Before proceeding, we give a brief introduction to PCR [37].

In the S-wave low energy scattering, the effective range expansion reads

$$p\cot\delta(p) = \frac{-1}{a} + \frac{r_e}{2}p^2 + \mathcal{O}(p^4), \tag{1}$$

where a is the scattering length and r_e effective range parameter. For the typical potential scattering, $r_e \simeq 1$ fm. The partial wave S matrix (for the one-channel case) is

$$S(p) = \frac{\cot\delta(p) + i}{\cot\delta(p) - i}. \tag{2}$$

Hence, the two roots of the quadratic polynomial obey the relation

$$p_1 + p_2 = \frac{2i}{r_e} \Rightarrow |p_1| + |p_2| \geq \frac{2}{|r_e|}, \tag{3}$$

which means that there only exists one pole near the corresponding threshold in a molecule picture. For the other case, if the intermediate particle is elementary, there exists one CDD (Castillejo-Dalitz-Dyson) pole [46]. At this time, the partial wave T matrix forms as

$$T \sim \frac{1}{\frac{p^2-p_0^2}{g^2} - ip}, \tag{4}$$

where p_0 is the CDD pole location, and g represents the coupling strength between the intermediate state and two scattering hadrons. For the elementary case, this g is a small number in the weak coupling situation, and the two roots of the quadratic polynomial obey the relation

$$|p_1| + |p_2| \geq g^2, \tag{5}$$

which means that for the elementary case, there will exist two poles near the threshold. It corresponds to the case that the intermediate state couples little to the two scattering hardons.

To simulate a coupled channel resonance amplitude, we chose the Flatté-like parametrization to generate the training data. For a two-channel parametrization of invariant mass spectrum, it is written as

$$\mathcal{R}_i(s) = \rho_i(s)\left|\frac{e^{i\phi_i}}{s - M^2 + iM(g_1\rho_1(s) + g_2\rho_2(s))} + bg_{1i}\right|^2 + \rho_i(s)bg_{2i}, \tag{6}$$

where ρ_i $(i = 1, 2)$ represents the two-body phase space factor for final state 1 (FS1) and final state 2 (FS2), respectively. Parameter M is the bare mass of the resonance. Coherent and incoherent background contributions are also considered as noise, which should be weak in order not to influence the judgement on the line shapes. In the region we are interested in, the background terms bg_{1i} and bg_{2i} are all first order polynomials of $\sqrt{s}$. All the parameters are tuned for generating a resonance near the FS2 threshold, and there is one or two poles near FS2 representing the 0 and 1 samples, respectively. In practice, the coupling constants g_1, g_2 are important to line shapes as well as the pole positions in the complex s plane [25].

Considering that the experimental data are affected by the energy resolution, to match the real signal better, we add a Gaussian convolution to the model. According to the experiment by LHCb [1], we fix $\Delta = 5$ MeV and

$$\frac{d\sigma_i}{d\sqrt{s}} \sim \frac{1}{\sqrt{2\pi}\Delta}\int_{\sqrt{s}-3\Delta}^{\sqrt{s}+3\Delta}\mathcal{R}_i(s')e^{-\frac{(\sqrt{s}-\sqrt{s'})^2}{2\Delta^2}}d\sqrt{s'}. \tag{7}$$

According to the above equation, when the parameter space is determined, all the training data can be generated. Some pretreatment is made to fit our machine learning program. The first is to set the window size of the signal. For the FS1 di-J/ψ signal, we set the window size from $E_{\text{th2}} - 100$ MeV to $E_{\text{th2}} + 300$ MeV to cover the relevant data and keep away from the noise from the peak $X(6200)$. For the $J/\psi\psi(2S)$ FS2 signal, we set the window size from E_{th2} to $E_{\text{th2}} + 200$ MeV. See Figs. 1 and 2 for two typical examples of the generated training data. The energy interval is fixed at 1 MeV uniformly, so it can meet the demand that all the inputs sent to a special neural network

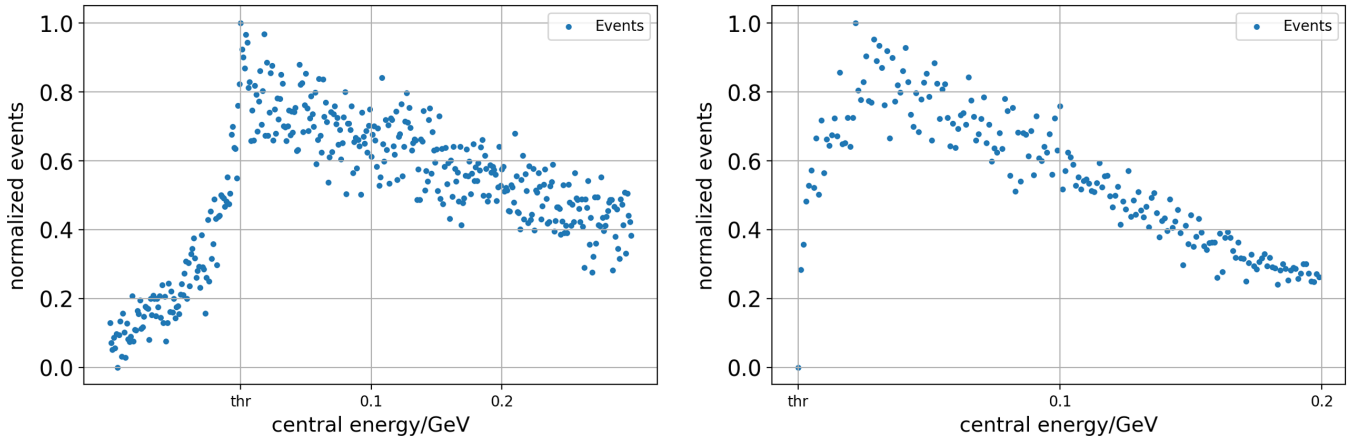


FIG. 1. An example of the 0 signal from the training data for the two-channel case.

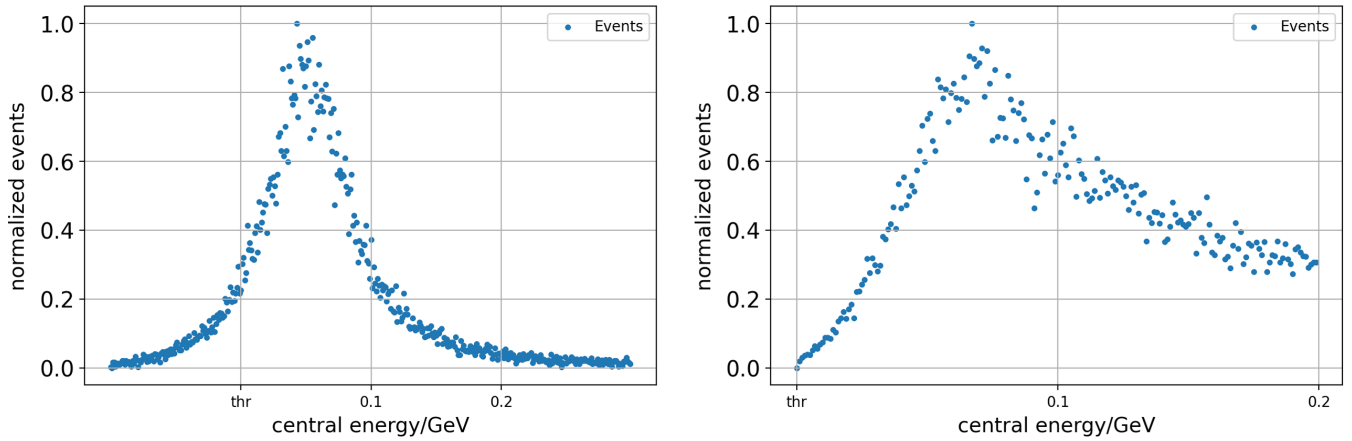


FIG. 2. An example of the 1 signal from the training data for the two-channel case.

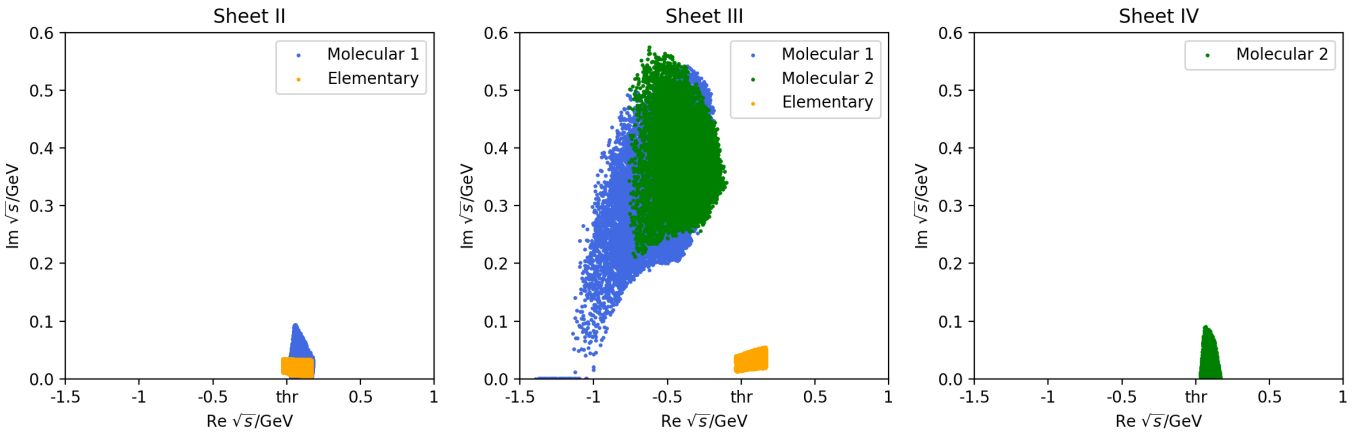
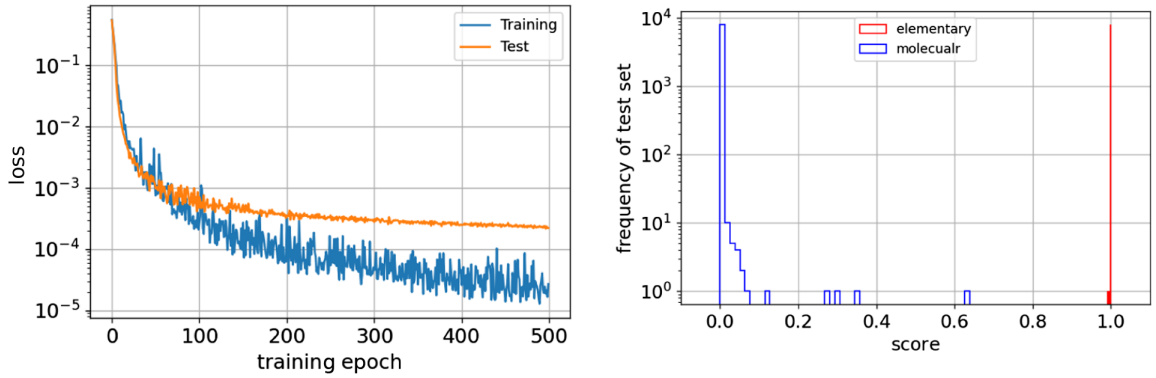


FIG. 3. The pole locations of molecule 1 (blue dots), molecule 2 (green dots), and elementary state (yellow dots) of the training data. Here, molecule 1 corresponds to a bound state of channel 2, whereas molecule 2 corresponds to a virtual pole of channel 2.

should have the same size. The values at every energy point are calculated using a linear extrapolation of the experimental data, and their errors are taken randomly as 5%, 10%, and 15% of their values. Finally, the normalization is made before sending them as learning input. In Fig. 3, examples of the pole locations of two molecules and an elementary state are drawn and labeled appropriately using PCR.



(a) (b)

FIG. 4. (a) Loss function of the training set (blue), and test set (orange). (b) The output result of the test set.

Before sending the experimental data into the neural network, it is noted that the energy interval is wider than 1 MeV. We have to manage these data to make the input the same size as the neural network. To be specific, if the energy interval of the experimental data is larger than 1 MeV, then the method of linear interpolation will be employed to supply extra points.

From Fig. 4, we find the loss function of our two-channel machine is below 10^{-4} for both training data and test data, so the data are trained well and can be used to judge whether a peak stands for a molecule of a given threshold. As shown in Fig. 5, the 100 data points from the experimental data are all judged around 1 by the trained machine. It tells us that $X(6900)$ is not a molecule composed of $J/\psi\psi(3686)$.

A short remark on neural network analysis is that it does not provide new insights into physics, but we hope our study may be helpful in making a quick judgement on the determination of molecular/elementary pictures. For example, one may conclude that if the resonance peak is left-right asymmetric, then it is likely to be a molecule as shown by Fig. 1(a), otherwise it is likely to be an elementary state, as shown by Fig. 2(a).

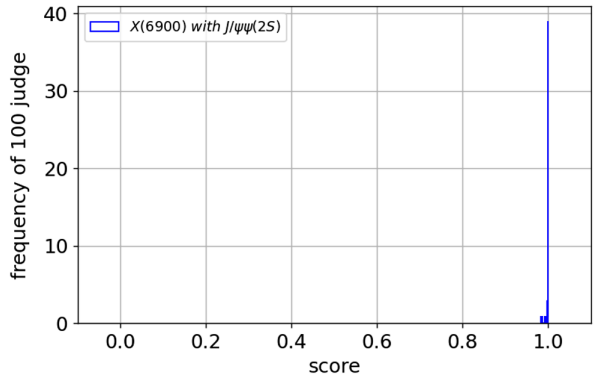


FIG. 5. Test result of the true data indicating that $X(6900)$ is not a molecule of $J/\psi\psi(2S)$.

III. A TRIPLE-CHANNEL STUDY

From the above discussion, it is concluded that $X(6900)$ is not a molecule of $J/\psi\psi(2S)$, but it does not necessarily mean that $X(6900)$ is elementary since it may still be a molecule in some other higher channels, e.g., $J/\psi\psi(3770)$, $\chi_{c0}\chi_{c2}$, etc. To examine this possibility, we extend the Flatté-like parametrization to three channels:

$$\begin{aligned}&\mathrm{Event}_i(s)\\&= \mathrm{C}_i\rho_i(s)\left|\frac{e^{i\phi_i}}{s-M^2+iM(g_1\rho_1(s)+g_2\rho_2(s)+g_3\rho_3(s))}\right.\\&\left.+(a+b(\sqrt{s}-E_{\mathrm{th}i}))\right|^2. \qquad (8)\end{aligned}$$

In this part, we try to fit the $J/\psi J/\psi$ and $J/\psi\psi(2S)$ signal directly by using above Flatté-like parametrization equation. $E_{\mathrm{th}i}$, $i=1,2$ are the threshold energies of FS1 and FS2. All the parameters g_1, g_2, g_3, ϕ_i, a, b, C_i are considered as fit parameters where C_i is a normalization constant. Here the "third channel" is taken as $\chi_{c0}\chi_{c2}$ for example [corresponding to the 2^{++} assignment of $X(6900)$].[2] As expected, the current data leave room for totally different scenarios. See Table I; apparently there exist multisolutions. One is an elementary solution, which is similar to the coupled channel solution discussed

[2]The $J/\psi\psi(3770)$ or $\chi_{c1}\chi_{c1}$ molecule (0^{++} or 2^{++}) is also possible, though slightly less preferred. In the former situation, $X(6900)$ is a virtual state composed of $J/\psi\psi(3770)$.

TABLE I. Fit parameters for two different solutions.

Parameter	g_1	g_2	g_3	M	a	b	ϕ_1	ϕ_2	C_1	C_2	χ^2/d.o.f.
Elementary	0.64	0.07	0.21	6.93	0.88	−0.39	1.49	3.82	276	275	0.69
Molecule	0.69	0.48	1	6.91	0.82	−0.46	1.31	3.78	477	263	0.73

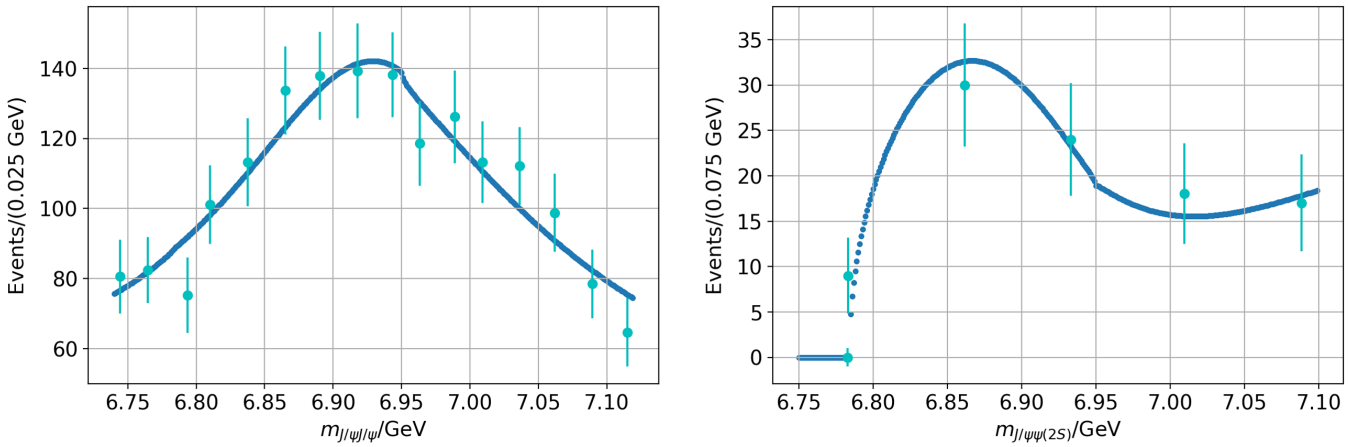


FIG. 6. Fit to the invariant mass spectrum for the elementary solution in Table I. The data are from Ref. [2].

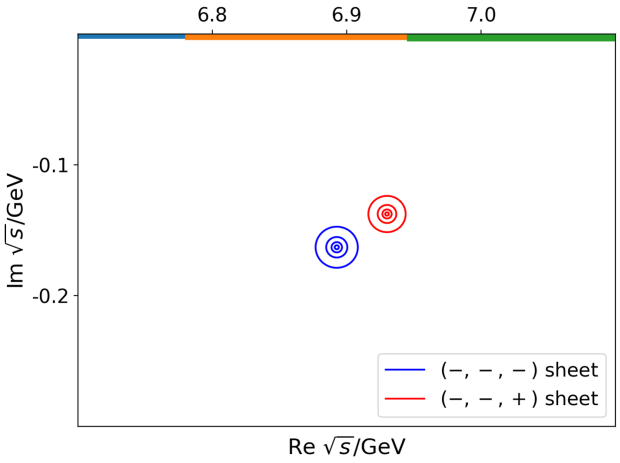


FIG. 7. Pole positions of the elementary solution in Table I.

previously. However, when the coupling to the third channel g_3 tuned large, a molecule (of the third channel) solution appears. See Figs. 6–9 for illustrations. For example, in Fig. 7, the two poles are very close to each other. Hence, according to PCR it is an elementary solution, whereas Fig. 9 reveals the molecule picture.

Based on the standard PCR analysis, we suggest that it is possible that the $X(6900)$ peak may be a molecule composed of, for example, $\chi_{c0}\chi_{c_2}$. In principle, one may also make use of a neural network to test the three-channel situation. However, in the absence of data of the third channel, it is very difficult to conduct a reliable test. Finally, we would like to comment on the work of Ref. [47], in which the authors also adopted a triple-channel study [di-J/ψ, $J/\psi\psi(2S)$, $J/\psi\psi(3770)$]. They conclude from their fit results [without including the new $J/\psi\psi(2S)$ data of

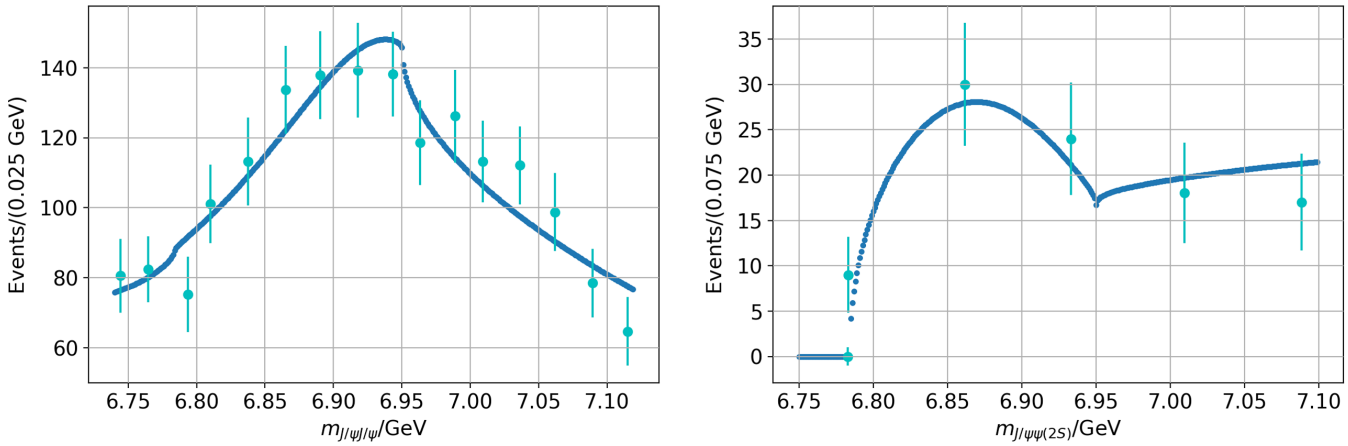


FIG. 8. Fit to the invariant mass spectrum for the molecule solution in Table I. The data are from Ref. [2].

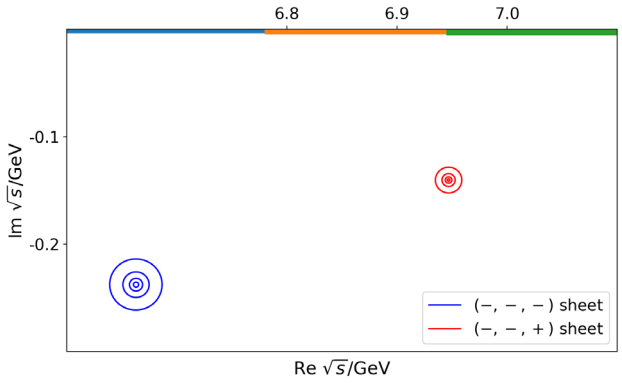


FIG. 9. Pole positions of the molecule solution in Table I.

Ref. [3]] that $X(6900)$ is likely a true tetraquark state. However, it is clearly indicated from our analysis that the fit program contains a redundancy of fit parameters, and it certainly leaves room for a molecule solution. So, additional experimental information in the $\chi_{c0}\chi_{c2}$, $J/\psi\psi(3770)$ channels, etc., is needed to further clarify the issue on whether $X(6900)$ is a molecular state or a compact tetraquark state.

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