Manifestation of the electric dipole moment in the decays of τ leptons produced in e^+e^- annihilation

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CP-odd asymmetries in the processes $e^+e^-\to\tau^+\pi^-\nu_\tau$, $e^+e^-\to\pi^+\tau^-\bar{\nu}_\tau$, $e^+e^-\to\tau^+\rho^-\nu_\tau$, $e^+e^-\to\rho^+\tau^-\bar{\nu}_\tau$, $e^+e^-\to\tau^+e^-\nu_\tau\bar{\nu}_e$, and $e^+e^-\to\tau^-e^+\nu_e\bar{\nu}_\tau$ are investigated with account for longitudinal polarization of electron (or positron) beam. These asymmetries are a manifestation of electric dipole form factor $F_3^\tau\equiv b$ in the $\gamma\tau^+\tau^-$ vertex. It is shown that to measure Imb in the specified processes, polarization is not needed, while to measure Reb it is required. The processes $e^+e^-\to\pi^+\pi^-\nu_\tau\bar{\nu}_\tau$, $e^+e^-\to e^+e^-\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_e$, $e^+e^-\to\mu^+\mu^-\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_\mu$, $e^+e^-\to\mu^+e^-\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_e$, and $e^+e^-\to\mu^-e^+\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_\mu$ are also discussed for the case of unpolarized electron and positron beams. In the latter cases it is possible to measure Reb using the differential cross section over momenta of both registered particles.

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I. INTRODUCTION

One of the ways to search for new physics is precision measurement of the electric dipole moment d_l of a charged lepton ($l=e,\ \mu,\ \tau$). The value of d_l predicted by the Standard Model (SM) is too small for experimental measurement. Therefore, the observation of electric dipole moment or its manifestation would directly demonstrate the existence of new physics.

The manifestation of the lepton electric dipole moment can be sought in the process of $l\bar{l}$ pair production in e^+e^- annihilation. The general form of $\gamma l\bar{l}$ vertex can be represented as

$$\Gamma^\mu=-ie\left\{F_1^l(k^2)\gamma^\mu+\frac{\sigma^{\mu\nu}k_\nu}{2m_l}[iF_2^l(k^2)+F_3^l(k^2)\gamma_5]+\left(\gamma^\mu-\frac{2k^\mu m_l}{k^2}\right)\gamma_5F_4^l(k^2)\right\},\tag{1}$$

where m_l is the lepton mass, $e<0$ is the electron charge, k is the 4-momentum of a photon, $F_1^l(k^2)$ is the Dirac form factor, $F_2^l(k^2)$ is the Pauli form factor, $F_3^l(k^2)$ is the electric dipole form factor, $F_4^l(k^2)$ is the anapole form factor, and $\sigma^{\mu\nu}=i/2[\gamma^\mu,\gamma^\nu]$, $\gamma_5=-i\gamma^0\gamma^1\gamma^2\gamma^3$. In the limit $k^2\to 0$ these form factors are

$$F_1^l(0)=1,\qquad F_2^l(0)=\mu_l'\frac{2m_l}{e},$$
$$F_3^l(0)=d_l\frac{2m_l}{e},\qquad F_4^l(0)=0,\tag{2}$$

where μ_l' is the anomalous magnetic moment. It follows from (1) that a violation of P- and T parities leads to the appearance of $F_3^l(k^2)$, while $F_4^l(k^2)$ is related to violation of P parity alone. Assuming that the CPT theorem holds, violation of T parity is equivalent to violation of CP parity. Thus, d_l occurs due to CP violations.

For all charged leptons, predictions of μ_l' in SM [1–4] can be experimentally verified [5–7]. For d_l the situation is essentially different. An estimate of d_l in SM [8–11] gives $|F_3^e(0)|<|F_3^\mu(0)|<|F_3^\tau(0)|\approx 10^{-23}\ll 1$. The sensitivity of modern experiments does not allow one to measure $F_3^\tau(0)$ with an accuracy of 10^{-23}. Therefore, extracting a nonzero value of $F_3^\tau(0)$ from the experiment would be a discovery of new physics. In Refs. [7,12–21] upper limits were set to $|F_3^\tau(k^2)|$, and in Refs. [22–24] upper limits were set to Re$F_3^\tau(k^2)$ and Im$F_3^\tau(k^2)$ separately.

In our work, the processes $e^+e^-\to\tau^+\pi^-\nu_\tau$, $e^+e^-\to\pi^+\tau^-\bar{\nu}_\tau$, $e^+e^-\to\tau^+\rho^-\nu_\tau$, $e^+e^-\to\rho^+\tau^-\bar{\nu}_\tau$, $e^+e^-\to\tau^+e^-\nu_\tau\bar{\nu}_e$, and $e^+e^-\to\tau^-e^+\nu_e\bar{\nu}_\tau$ are studied for longitudinally polarized electron beam. The processes $e^+e^-\to\pi^+\pi^-\nu_\tau\bar{\nu}_\tau$, $e^+e^-\to e^+e^-\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_e$, $e^+e^-\to\mu^+\mu^-\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_\mu$, $e^+e^-\to\mu^+e^-\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_e$, and $e^+e^-\to\mu^-e^+\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_\mu$ are discussed for unpolarized electron and positron beams. The asymmetric with respect to CP transformation parts of the

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corresponding cross sections are obtained for e^+e^- invariant masses $\sqrt{s} \ll m_Z$. To derive these results, it is sufficient to consider $\gamma\tau^+\tau^-$ vertex in the form

$$\Gamma^\mu = -ie\left[\gamma^\mu + \frac{\sigma^{\mu\nu}k_\nu}{2M}F_3^\tau(k^2)\gamma_5\right], \tag{3}$$

where M is the τ lepton mass and $k^2 = s$. Measurement of CP odd parts of the cross sections can diminish the upper limits of both $\mathrm{Re}F_3^\tau(s)$ and $\mathrm{Im}F_3^\tau(s)$.

At present, there is an important question whether it is necessary to provide the longitudinal polarization of electrons at the Super-Charm-Tau factory (SCTF) [25] (see also [26]). This collider, having high luminosity $\sim 10^{35}$ cm^{-2} s^{-1} and $\sqrt{s}$ from 3 to 5–7 GeV, will become an intense source of τ leptons. It is important that we do not register τ^+ and τ^- themselves, but only the particles into which τ^+ and τ^- decay. If we measure the cross-section differential over momenta of particles in τ^- decay (or τ^+ decay), then it is impossible to measure $\mathrm{Re}F_3^\tau(s)$ without electron (positron) beam polarization. Note that $\mathrm{Im}F_3^\tau(0) = 0$ due to the CPT theorem. If one assumes that a typical size of new physics $\Lambda_{NP} \gg M$, then one should expect that $\mathrm{Im}F_3^\tau(s) \ll \mathrm{Re}F_3^\tau(s)$ at $s \gtrsim M^2$. We show that longitudinal polarization of electron beam allows one to measure $\mathrm{Re}F_3^\tau(s)$ using the cross-section differential over momenta of particles in τ^- (or τ^+) decay. Note that the cross-section differential over momenta of all final particles, except for neutrinos and antineutrinos, allows one to measure both $\mathrm{Re}F_3^\tau(s)$ and $\mathrm{Im}F_3^\tau(s)$ without electron and positron polarizations. However, such measurements are essentially more complicated with respect to number of events and accuracy than for the case of polarized beams. This is why the use of electron longitudinal polarization is very important to study the dipole moment of τ lepton. Note that in the experiment [24] polarization was absent, so we expect that polarization at SCTF will permit one to improve essentially an upper limit for $\mathrm{Re}F_3^\tau(s)$.

II. $e^+e^- \to \tau^+\tau^-$

To study the effect of polarization, let us consider a longitudinally polarized electron beam and an unpolarized positron beam. Since $\sqrt{s} \ll m_Z$, we neglect the contribution of the Z boson. Then, the cross section $d\sigma_0$ of the process $e^+e^- \to \tau^+\tau^-$ in the center-of-mass frame is

$$\begin{aligned} d\sigma_0 &= \frac{\beta\alpha^2}{4s}|\phi_\tau^\dagger H\chi_\tau|^2 d\Omega_{\boldsymbol{q}}, \\ H &= \boldsymbol{\sigma}\cdot\boldsymbol{e}_\lambda - \frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q})(\boldsymbol{e}_\lambda\cdot\boldsymbol{q})}{E(E+M)} + i\frac{b}{M}(\boldsymbol{e}_\lambda\cdot\boldsymbol{q}), \\ b &= F_3^\tau(s),\ \ s = 4E^2,\ \ \beta = q/E,\ \ \boldsymbol{e}_\lambda = \frac{1}{\sqrt{2}}(\boldsymbol{e}_x + i\lambda\boldsymbol{e}_y). \end{aligned} \tag{4}$$

Here, α is the fine-structure constant, E is the electron energy, $\boldsymbol{q}$ is the momentum of the τ^- lepton, the vector $\boldsymbol{e}_z$ is directed along the electron momentum, λ is the electron helicity, and ϕ_τ and χ_τ are two-component spinors entering, respectively, into the positive-frequency and negative-frequency solutions of the Dirac equation for a τ lepton,

$$\begin{aligned} U_{\mathbf{q}} &= \sqrt{\frac{E_q+M}{2E_q}}\begin{pmatrix} \phi_\tau \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{q}}{E_q+M}\phi_\tau \end{pmatrix}, \\ V_{-\mathbf{q}} &= \sqrt{\frac{E_q+M}{2E_q}}\begin{pmatrix} \frac{-\boldsymbol{\sigma}\cdot\boldsymbol{q}}{E_q+M}\chi_\tau \\ \chi_\tau \end{pmatrix}, \end{aligned} \tag{5}$$

where $E_q = \sqrt{\boldsymbol{q}^2 + M^2}$. Using the relation

$$e_\lambda^i e_\lambda^{*j} = \frac{1}{2}(\delta^{ij} - \Lambda^i\Lambda^j - i\epsilon^{ijk}\Lambda^k), \qquad \boldsymbol{\Lambda} = \lambda\boldsymbol{e}_z,$$

we obtain the cross section $d\sigma_0$ summed over polarizations of τ^+,

$$\begin{aligned} d\sigma_0 &= \frac{\beta\alpha^2}{4s}\left[1 - \frac{q_\perp^2}{2E^2} + \boldsymbol{\zeta}\cdot\boldsymbol{Z}\right]d\Omega_{\boldsymbol{q}}, \\ \boldsymbol{Z} &= \mathrm{Im}b\,\frac{q_\perp^2\boldsymbol{q}}{ME(E+M)} - \mathrm{Im}b\,\frac{\boldsymbol{q}_\perp}{M} - \mathrm{Re}b\,\frac{[\boldsymbol{q}_\perp\times\boldsymbol{\Lambda}]}{M} \\ &\quad + \frac{M}{E}\boldsymbol{\Lambda} + \frac{(\boldsymbol{q}\cdot\boldsymbol{\Lambda})\boldsymbol{q}}{E(E+M)}, \end{aligned} \tag{6}$$

where $\boldsymbol{q}_\perp = \boldsymbol{q} - \boldsymbol{\Lambda}(\boldsymbol{q}\cdot\boldsymbol{\Lambda})$, $\boldsymbol{\zeta}$ is the spin of τ^-, and terms quadratic in b are omitted. It is seen that the linear in b terms contribute only to the $\boldsymbol{\zeta}$-dependent part of the cross section. Besides, the term with $\mathrm{Re}b$ is proportional to λ so this contribution vanishes for unpolarized electron beam. Study of various τ decay channels is a way to measure the polarization, which in turn makes it possible to measure b. The total cross section summed over the τ^- polarization is

$$\sigma_0 = \frac{\pi\beta\alpha^2}{3E^2}\left(1 + \frac{M^2}{2E^2}\right). \tag{7}$$

As should be, here the term $\propto b$ vanishes.

III. $e^+e^- \to \tau^+\pi^-\nu_\tau$, $e^+e^- \to \tau^-\pi^+\bar{\nu}_\tau$

Consider the cross section of the process $e^+e^- \to \tau^+\tau^-$ followed by the decay $\tau^- \to \pi^-\nu_\tau$. Taking into account the smallness of the τ lepton width, $\Gamma_\tau \approx 2.27$ meV [27], we can make the substitutions

$$\begin{aligned} \frac{1}{\hat{q} - M} &\to \frac{2E_q}{\mathcal{E}^2 - E_q^2 + i\Gamma_\tau M}\sum_\mu U_{\boldsymbol{q},\mu}\bar{U}_{\boldsymbol{q},\mu}, \\ \frac{4E_q^2}{(\mathcal{E}^2 - E_q^2)^2 + \Gamma_\tau^2M^2} &\to \frac{2\pi E_q}{M\Gamma_\tau}\delta(\mathcal{E} - E_q), \end{aligned} \tag{8}$$

where $\mathcal{E} = q^0$. After that, the cross section of the process $e^+e^- \to \tau^+\pi^-\nu_\tau$ can be represented as

$$d\sigma_\pi^{(-)}(\boldsymbol{k}) = B_\pi \frac{\beta\alpha^2(E+M)d\Omega_{\boldsymbol{q}}d\boldsymbol{k}}{4\pi s M^2\omega_k} R^{(-)}\delta(E-\omega_k-|\boldsymbol{q}-\boldsymbol{k}|),$$
$$R^{(-)} = \left|\phi_\nu^+\left[1+\frac{\boldsymbol{\sigma}\cdot\boldsymbol{q}}{E+M}\right]H\chi_\tau\right|^2, \qquad (9)$$

where $B_\pi \approx 10.8\%$ [27] is the branching ratio of $\tau\to\pi\nu$ decay, $\omega_k = \sqrt{\boldsymbol{k}^2+m_\pi^2}$ and $\boldsymbol{k}$ are the pion energy and momentum, respectively, ϕ_ν is a two-component spinor in the Dirac spinor $U_{\boldsymbol{Q}}$ for neutrino, $\boldsymbol{Q}=\boldsymbol{q}-\boldsymbol{k}$, and $(\boldsymbol{\sigma}\cdot\boldsymbol{Q})\phi_\nu = -Q\phi_\nu$. We also neglected the pion mass m_π compared to M and took into account that the matrix element of the $\tau\to\pi\nu$ is proportional to $\bar{U}_{\boldsymbol{Q}}U_{\boldsymbol{q}}$ [28]. Similar, for the cross section of the process $e^+e^-\to\tau^-\pi^+\bar{\nu}_\tau$ we obtain

$$d\sigma_\pi^{(+)}(\boldsymbol{k}) = B_\pi \frac{\beta\alpha^2(E+M)d\Omega_{\boldsymbol{q}}d\boldsymbol{k}}{4\pi s M^2\omega_k} R^{(+)}\delta(E-\omega_k-|\boldsymbol{q}+\boldsymbol{k}|),$$
$$R^{(+)} = \left|\phi_\tau^+ H\left[1-\frac{\boldsymbol{\sigma}\cdot\boldsymbol{q}}{E+M}\right]\chi_\nu\right|^2. \qquad (10)$$

Then, we define the differential asymmetry dA_π as follows:

$$dA_\pi = \frac{d\sigma_\pi^{(-)}(\boldsymbol{k}) - d\sigma_\pi^{(+)}(-\boldsymbol{k})}{2\sigma_0}, \qquad (11)$$

where σ_0 is defined in (7). Integrating over the angles of vector $\boldsymbol{q}$ and taking a sum over polarizations, we get

$$dA_\pi = \frac{B_\pi \mathrm{Im}b\, d\omega_k d\Omega_{\boldsymbol{k}}}{4\pi q(1+M^2/2E^2)}\left[1-\frac{2\omega_k}{E} + (3\cos^2\theta-1)\left(1-\frac{\omega_k}{2E}-\frac{3M^2}{8E\omega_k}\right)\right]. \qquad (12)$$

Here, $\cos\theta = \boldsymbol{k}\cdot\boldsymbol{\Lambda}/k = \lambda k_z/k$; the available pion energy range is determined by the relation $|2\omega_k - E| \le q$.

The asymmetry dA_π contains only the imaginary part of b, and its measurement does not require a nonzero electron polarization.

After integration over $d\Omega_{\boldsymbol{k}}$, we obtain

$$dA_\pi = \frac{B_\pi \mathrm{Im}b\, d\omega_k}{q(1+M^2/2E^2)}\left(1-\frac{2\omega_k}{E}\right). \qquad (13)$$

As it should be, after integration over the pion energy, the asymmetry vanishes. Therefore, we define the total asymmetry A_π as dA_π (13) integrated over ω_k from $(E-q)/2$ to $E/2$ (half of the allowed energy range),

$$A_\pi = \frac{B_\pi \mathrm{Im}b}{4(1+M^2/2E^2)}\sqrt{1-\frac{M^2}{E^2}}. \qquad (14)$$

Taking in Eq. (12) the integral over ω_k in the region $(E-q)/2 \le \omega_k \le (E+q)/2$, we obtain the angular asymmetry,

$$dA_\pi = -\frac{3B_\pi \mathrm{Im}b\, d\Omega_{\boldsymbol{k}}}{16\pi(1+M^2/2E^2)}(3\cos^2\theta-1) \times\left[\frac{M^2}{2Eq}\ln\left(\frac{E+q}{E-q}\right)-1\right]. \qquad (15)$$

IV. $e^+e^-\to\tau^+\rho^-\nu_\tau$, $e^+e^-\to\tau^-\rho^+\bar{\nu}_\tau$

To measure Reb, consider the decay of one τ lepton into ρ meson with the momentum $\boldsymbol{p}$, energy $\varepsilon_p = \sqrt{\boldsymbol{p}^2+m_\rho^2}$ and 4-polarization vector $f = (f_0, \boldsymbol{f})$, where m_ρ is the mass of ρ meson. We define the differential asymmetry dA_ρ as

$$dA_\rho = \frac{d\sigma_\rho^{(-)}(\boldsymbol{p},\boldsymbol{f}) - d\sigma_\rho^{(+)}(-\boldsymbol{p},-\boldsymbol{f})}{2\sigma_0}, \qquad (16)$$

where $d\sigma_\rho^{(-)}(\boldsymbol{p},\boldsymbol{f})$ and $d\sigma_\rho^{(+)}(-\boldsymbol{p},-\boldsymbol{f})$ are the cross sections of the processes $e^+e^-\to\tau^+\rho^-\nu_\tau$ and $e^+e^-\to\tau^-\rho^+\bar{\nu}_\tau$, respectively. Using the matrix element of decay $\tau\to\rho\nu$ [28], we obtain as a result of straightforward calculations

$$dA_\rho = \frac{3B_\rho d\varepsilon_p d\Omega_{\boldsymbol{p}}[C_1\mathrm{Re}b + C_2\mathrm{Im}b]}{2\pi p M^2(1+M^2/2E^2)(2+M^2/m_\rho^2)(1-m_\rho^2/M^2)^2},$$
$$C_1 = \left[\frac{q}{p}\varepsilon_p P_2(x_0) - EP_1(x_0)\right]([\boldsymbol{\Lambda}\times\boldsymbol{f}]\cdot\boldsymbol{p})f_0,$$
$$C_2 = \frac{qp}{3}\left[\left(2+\frac{\varepsilon_p}{E}\right)f_0^2 + \left(1-\frac{\varepsilon_p}{E}\right)\boldsymbol{f}^2 - (\boldsymbol{\Lambda}\cdot\boldsymbol{f})^2\right] + P_1(x_0)\left[\frac{1}{2}[p^2-(\boldsymbol{\Lambda}\cdot\boldsymbol{p})^2](\boldsymbol{f}^2-f_0^2) + Ef_0[(\boldsymbol{\Lambda}\cdot\boldsymbol{p})(\boldsymbol{\Lambda}\cdot\boldsymbol{f}) - (\boldsymbol{p}\cdot\boldsymbol{f})]\right.$$
$$\left.+\frac{2q^2}{5E}f_0[(\boldsymbol{\Lambda}\cdot\boldsymbol{p})(\boldsymbol{\Lambda}\cdot\boldsymbol{f}) - 2(\boldsymbol{p}\cdot\boldsymbol{f})]\right] + \frac{q}{p}P_2(x_0)\left\{\left(\frac{\varepsilon_p}{2E}(\boldsymbol{f}^2-f_0^2)-f_0^2\right)\left[(\boldsymbol{\Lambda}\cdot\boldsymbol{p})^2-\frac{p^2}{3}\right] + \left[(\boldsymbol{f}\cdot\boldsymbol{p})^2 - \frac{p^2\boldsymbol{f}^2}{3}\right]\right.$$
$$\left.-(\boldsymbol{f}\cdot\boldsymbol{\Lambda})\left[(\boldsymbol{\Lambda}\cdot\boldsymbol{p})(\boldsymbol{f}\cdot\boldsymbol{p}) - \frac{p^2}{3}(\boldsymbol{f}\cdot\boldsymbol{\Lambda})\right]\right\} + P_3(x_0)\frac{q^2}{Ep^2}f_0\left\{(\boldsymbol{f}\cdot\boldsymbol{p})(\boldsymbol{\Lambda}\cdot\boldsymbol{p})^2 - \frac{p^2}{5}[(\boldsymbol{f}\cdot\boldsymbol{p}) + 2(\boldsymbol{f}\cdot\boldsymbol{\Lambda})(\boldsymbol{p}\cdot\boldsymbol{\Lambda})]\right\},$$
$$x_0 = \frac{2E\varepsilon_p - M^2 - m_\rho^2}{2qp}, \qquad (17)$$

where $B_\rho \approx 25.5\%$ [27] is the branching ratio of $\tau \to \rho\nu$ decay and $P_n(x)$ are Legendre polynomials. Thus, for a polarized electron beam and a polarized ρ meson, the contribution of Reb does not vanish. Note that

$$dA_\rho|_{\lambda=+1} - dA_\rho|_{\lambda=-1} = \frac{3B_\rho \mathrm{Re}b C_1 d\varepsilon_p d\Omega_{\boldsymbol{p}}}{\pi p M^2(1+M^2/2E^2)(2+M^2/m_\rho^2)(1-m_\rho^2/M^2)^2}. \tag{18}$$

Then we perform summation over polarizations of ρ meson, using the formula

$$\sum_{\mathrm{pol}} f^\mu f^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_\rho^2},$$

and obtain

$$\sum_{pol} C_1 = 0, \qquad \sum_{pol} C_2 = C_{21}F + C_{22},$$
$$C_{21} = -\frac{qp}{3} + P_1(x_0)\left(-\frac{3}{2}m_\rho^2 + \frac{7}{5}E\varepsilon_p - \frac{2\varepsilon_p}{5E}M^2\right) + \frac{q}{p}P_2(x_0)\left(\frac{3\varepsilon_p}{2E}m_\rho^2 - \frac{5}{3}\varepsilon_p^2 + \frac{2}{3}m_\rho^2\right) + P_3(x_0)\frac{3\varepsilon_p q^2}{5E},$$
$$C_{22} = \frac{p(M^2-2m_\rho^2)[m_\rho^2 E + M^2(E-2\varepsilon_p)]}{6m_\rho^2 Eq},$$
$$F = \frac{1}{3m_\rho^2}[3(\boldsymbol{\Lambda}\cdot\boldsymbol{p})^2 - p^2]. \tag{19}$$

In this formula, the contribution $\propto$ Reb is absent even for the case of polarized electrons. After integration over the angles of the vector $\boldsymbol{p}$, the term $\propto F$ vanishes, and the asymmetry reads

$$dA_\rho = \frac{6B_\rho d\varepsilon_p C_{22}\mathrm{Im}b}{pM^2(1+M^2/2E^2)(2+M^2/m_\rho^2)(1-m_\rho^2/M^2)^2}. \tag{20}$$

The allowed region of the energy ε_p is given by the relation

$$\left|2\varepsilon_p - \left(1+\frac{m_\rho^2}{M^2}\right)E\right| \le q\left(1-\frac{m_\rho^2}{M^2}\right).$$

After integration over ε_p in Eq. (20), the asymmetry vanishes. Therefore, we define the total asymmetry A_ρ as a result of integration of Eq. (20) over ε_p in the region

$$(1+m_\rho^2/M^2)E - (1-m_\rho^2/M^2)q < 2\varepsilon_p < (1+m_\rho^2/M^2)E.$$

One has

$$A_\rho = \frac{B_\rho \mathrm{Im}b}{2(2+M^2/E^2)}\sqrt{1-\frac{M^2}{E^2}}\left(\frac{M^2-2m_\rho^2}{M^2+2m_\rho^2}\right). \tag{21}$$

Note that

$$A_\rho = \frac{B_\rho}{B_\pi}\left(\frac{M^2-2m_\rho^2}{M^2+2m_\rho^2}\right)A_\pi. \tag{22}$$

To determine the polarization of ρ meson, it is possible to measure the main decay channel of ρ meson with momentum $\boldsymbol{p}$ into two pions with momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. The corresponding asymmetry can be obtained from Eq. (17) by the obvious substitution

$$\frac{d\boldsymbol{p}}{2\varepsilon_p(2\pi)^3}f^\mu f^\nu \to \frac{f_{\rho\pi\pi}^2 d\boldsymbol{k}_1\, d\boldsymbol{k}_2(k_1-k_2)^\mu(k_1-k_2)^\nu}{4\omega_1\omega_2(2\pi)^6[(s_1-m_\rho^2)^2+\Gamma_\rho^2 m_\rho^2]}. \tag{23}$$

Here, $s_1 = (k_1+k_2)^2$, $\Gamma_\rho = 149.1$ MeV [27] is the ρ meson width, $\omega_1 = |\boldsymbol{k}_1|$, $\omega_2 = |\boldsymbol{k}_2|$, and the constant $f_{\rho\pi\pi}^2$ is

$$f_{\rho\pi\pi}^2 = \frac{48\pi\Gamma_\rho}{m_\rho(1-4m_\pi^2/m_\rho^2)^{3/2}}.$$

V. $e^+e^- \to \tau^+e^-\nu_\tau\bar{\nu}_e$, $e^+e^- \to \tau^-e^+\nu_e\bar{\nu}_\tau$

Let us consider the cross sections $d\sigma_e^{(-)}(\boldsymbol{k})$ and $d\sigma_e^{(+)}(\boldsymbol{k})$ of the processes $e^+e^- \to \tau^+e^-\nu_\tau\bar{\nu}_e$ and $e^+e^- \to \tau^+\tau^- \to \tau^-e^+\nu_e\bar{\nu}_\tau$, respectively, where $\boldsymbol{k}$ is the electron (positron) momentum. We define the asymmetry dA_e as

$$dA_e = \frac{d\sigma_e^{(-)}(\boldsymbol{k}) - d\sigma_e^{(+)}(-\boldsymbol{k})}{2\sigma_0}. \tag{24}$$

Then we use the matrix element of decay $\tau^- \to e^-\nu_\tau\bar{\nu}_e$ [28] and perform the integration of cross sections over the neutrino and antineutrino momenta. We have

$$dA_e = \frac{6B_e d\Omega_q d\boldsymbol{k}}{(2\pi)^2M^6(1+M^2/2E^2)k}[4(kE-\boldsymbol{k}\cdot\boldsymbol{q})-M^2] \times\left[\mathrm{Re}b[\boldsymbol{k}\times\boldsymbol{q}]\cdot\boldsymbol{\Lambda} + \mathrm{Im}b\left(\boldsymbol{k}\cdot\boldsymbol{q}_\perp - \frac{k}{E}q_\perp^2\right)\right], \tag{25}$$

where $B_e \approx 18\%$ [27] is the branching ratio of $\tau^- \to e^-\nu_\tau\bar{\nu}_e$ decay. The allowed region of the parameters is given by the relation

$$2(kE-\boldsymbol{k}\cdot\boldsymbol{q}) \le M^2.$$

Integrating over the angles of vector $\boldsymbol{q}$, we find

$$dA_e = \frac{B_e \mathrm{Im} b d\boldsymbol{k}}{\pi E q M^6 (1 + M^2/2E^2)} \Big\langle 4q^3 [M^2 + 2kE(\boldsymbol{\Lambda}\cdot\boldsymbol{n_k})^2 - 6kE]\theta(k_0 - k) + \Big\{\frac{1}{2}(\boldsymbol{\Lambda}\cdot\boldsymbol{n_k})^2 \Big[k(E-q)^3(E+3q) + \frac{M^6}{2k^2}\Big(\frac{3M^2}{8k} - E\Big)\Big] - \frac{3}{2}k(E-q)^3(E+3q) + M^2(E-q)^2(E+2q) - \frac{M^8}{32k^3}\Big\}\theta(k-k_0)\Big\rangle. \tag{26}$$

Here, $\theta(x)$ is the Heaviside step function, $\boldsymbol{n_k} = \boldsymbol{k}/k$, $k_0 = (E-q)/2$, and $0 \le k \le k_{\max}$, where $k_{\max} = (E+q)/2$. Thus, the contribution of Reb vanishes. After integration over angles of vector $\boldsymbol{k}$ the asymmetry reads

$$dA_e = \frac{4B_e \mathrm{Im} b k^2 dk}{E q M^6 (1 + M^2/2E^2)} \Big\{4q^3\Big(M^2 - \frac{16}{3}kE\Big)\theta(k_0 - k) + \Big[M^2(E-q)^2(E+2q) - \frac{4}{3}k(E-q)^3(E+3q) - \frac{EM^6}{12k^2}\Big]\theta(k-k_0)\Big\}. \tag{27}$$

The dependence of asymmetry dA_e/dk on k is shown in Fig. 1 for a few values of energy E. It is seen that this dependence is very nontrivial.

The integration of dA_e/dk over k in all allowed region $0 \le k \le k_{\max}$ gives zero. Therefore, it is natural to define the total asymmetry A_e as the integral over k in the region $k_{\max}/2 \le k \le k_{\max}$. We have

$$A_e = \frac{B_e \mathrm{Im} b (E+q)}{192 E q (1 + M^2/2E^2)} \Big[(11q - 3E)\theta(E - E_0) + \frac{16q^4(E+q)^3}{M^6}\theta(E_0 - E)\Big], \tag{28}$$

where $E_0 = 3M/\sqrt{8}$. The energy dependence of A_e is shown in Fig. 3.

If one takes in Eq. (25) first the integral over k in the region $0 \le k \le M^2/[2(E - \boldsymbol{n_k}\cdot\boldsymbol{q})]$ and then over $d\Omega_{\boldsymbol{q}}$, we obtain a simple result:

$$dA_e = \frac{B_e \mathrm{Im} b d\Omega_{\boldsymbol{k}}}{16\pi(1 + M^2/2E^2)} [3(\boldsymbol{n_k}\cdot\boldsymbol{\Lambda})^2 - 1]\Big[\frac{M^2}{2Eq}\ln\Big(\frac{E+q}{E-q}\Big) - 1\Big]. \tag{29}$$

VI. $e^+e^- \to \tau^+\tau^- \to \pi^+\pi^-\nu_\tau\bar{\nu}_\tau$

If each τ lepton decays into a pion and a neutrino (antineutrino), both the imaginary and real parts of b can be measured even in the case of an unpolarized electron beam. It is this case that we consider in this section. We define the asymmetry as

$$dA_{\pi\pi} = \frac{d\sigma_{\pi\pi}(\boldsymbol{k}_1, \boldsymbol{k}_2) - d\sigma_{\pi\pi}(-\boldsymbol{k}_2, -\boldsymbol{k}_1)}{2\sigma_0}, \tag{30}$$

where $d\sigma_{\pi\pi}(\boldsymbol{k}_1, \boldsymbol{k}_2)$ is the cross section of the process $e^+e^- \to \pi^+\pi^-\nu_\tau\bar{\nu}_\tau$; $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the momenta of π^- and π^+, respectively. For $dA_{\pi\pi}$ we obtain

$$dA_{\pi\pi} = \frac{3B_\pi^2 d\boldsymbol{k}_1 d\boldsymbol{k}_2}{(2\pi)^3 M^4 (1 + M^2/2E^2)\omega_1^2\omega_2^2\sqrt{(1-x^2)q^2(q^2 - P^2)}} \times \Big\langle \mathrm{Re} b(\boldsymbol{\Lambda}\cdot[\boldsymbol{k}_1 \times \boldsymbol{k}_2])\Big[2(q^2 - P^2)(\boldsymbol{N}_2\cdot\boldsymbol{\Lambda}) + \frac{(\boldsymbol{P}\cdot\boldsymbol{\Lambda})}{E}\Big(M^2 + 2P^2 - q^2 - \frac{Ea_2}{1-x}\Big)\Big] + \mathrm{Im} b \frac{M^2}{2}\Big[\frac{(\omega_2 - \omega_1)}{E}(M^2 + (\boldsymbol{N}_3\cdot\boldsymbol{\Lambda})^2(q^2 - P^2) + (\boldsymbol{P}\cdot\boldsymbol{\Lambda})^2) + (\boldsymbol{P}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Lambda}, \boldsymbol{k}_1 + \boldsymbol{k}_2)\Big]\Big\rangle. \tag{31}$$

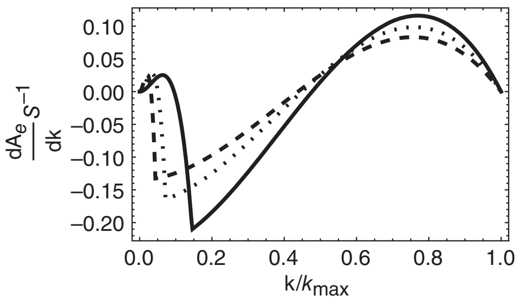

FIG. 1. Asymmetry dA_e/dk in units of $S = B_e\mathrm{Im}b/M$ as a function of $k/k_{\max}$ for a few values of E, $k_{\max} = (E+q)/2$. Solid curve: $E = 1.5M$, dotted curve: $E = 2M$, dashed curve: $E = 2.5M$.

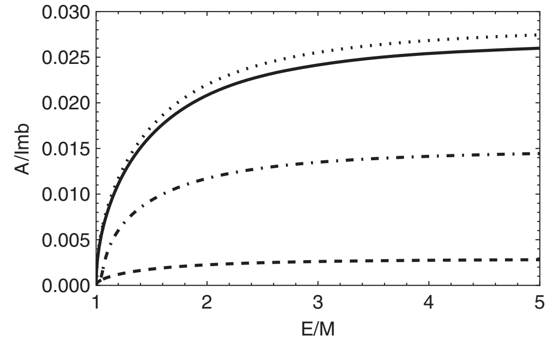

FIG. 3. Total asymmetry A in units of Imb as a function of energy E. Solid curve: A_π, dotted curve: A_ρ, dashed-dotted curve: A_e, dashed curve: $A_{\pi\pi}$.

Here, the following notation is introduced:

$$\boldsymbol{N}_1 = \frac{\boldsymbol{n}_1 + \boldsymbol{n}_2}{2(1+x)}, \qquad \boldsymbol{N}_2 = \frac{\boldsymbol{n}_1 - \boldsymbol{n}_2}{2(1-x)}, \qquad \boldsymbol{N}_3 = \frac{[\boldsymbol{n}_2 \times \boldsymbol{n}_1]}{\sqrt{1-x^2}},$$
$$\boldsymbol{P} = a_1\boldsymbol{N}_1 + a_2\boldsymbol{N}_2, \qquad a_1 = \frac{M^2(\omega_1 - \omega_2)}{2\omega_1\omega_2},$$
$$a_2 = 2E - \frac{M^2(\omega_1 + \omega_2)}{2\omega_1\omega_2},$$
$$x = (\boldsymbol{n}_1 \cdot \boldsymbol{n}_2), \qquad \boldsymbol{n}_1 = \frac{\boldsymbol{k}_1}{\omega_1}, \qquad \boldsymbol{n}_2 = \frac{\boldsymbol{k}_2}{\omega_2}. \tag{32}$$

Note that the coefficient in front of Imb changes its sign at the replacement $\boldsymbol{n}_1 \leftrightarrow \boldsymbol{n}_2$, $\omega_1 \leftrightarrow \omega_2$, while the coefficient in front of Reb does not change its sign.

It is also interesting to consider the asymmetry that remains after the integration over the angles of vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. The corresponding result reads

$$dA_{\pi\pi} = \frac{2B_\pi^2 \mathrm{Im}b(\omega_2 - \omega_1)d\omega_1 d\omega_2}{Eq^2(1 + M^2/2E^2)}. \tag{33}$$

Here, only the imaginary part of b contributes. Integrating over ω_2 in the region $|2\omega_2 - E| < q$, we obtain the result

$$dA_{\pi\pi} = \frac{B_\pi^2 \mathrm{Im}b(1 - 2\omega_1/E)d\omega_1}{q(1 + M^2/2E^2)}, \tag{34}$$

which is consistent with (13). Integrating (34) over ω_2 in the range $(E-q)/2 < \omega_2 < E/2$, we get

$$A_{\pi\pi} = \frac{B_\pi^2 \mathrm{Im}b}{4(1 + M^2/2E^2)}\sqrt{1 - \frac{M^2}{E^2}}. \tag{35}$$

Naturally, $A_{\pi\pi} = B_\pi A_\pi$.

From our point of view, the most convenient for measurement is the asymmetry integrated over the energies of emitted pions. We find for this quantity

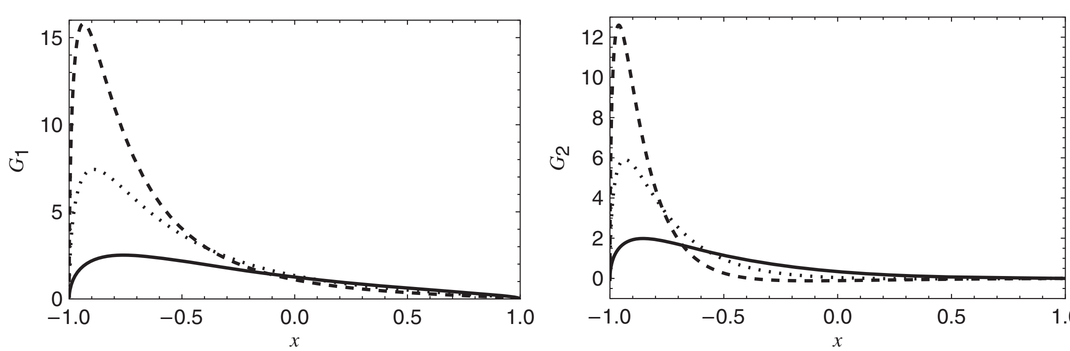

FIG. 2. Dependence of functions G_1 (left) and G_2 (right) on $x = \boldsymbol{n}_1 \cdot \boldsymbol{n}_2$; see (37) for $E = 1.5M$ (solid curve), $E = 2M$ (dotted curve), and $E = 2.5M$ (dashed curve).

$$dA_{\pi\pi} = \frac{q^2 B_\pi^2 d\Omega_1 d\Omega_2}{(32\pi)^2 M^4 (1+M^2/2E^2) a^2 (1+a)^4} \{G_{\pi\pi}^{(1)} [(\boldsymbol{\Lambda}\cdot\boldsymbol{n}_1)^2 - (\boldsymbol{\Lambda}\cdot\boldsymbol{n}_2)^2] \mathrm{Im} b + G_{\pi\pi}^{(2)} [(\boldsymbol{\Lambda}\cdot\boldsymbol{n}_1) - (\boldsymbol{\Lambda}\cdot\boldsymbol{n}_2)]([\boldsymbol{n}_1\times\boldsymbol{n}_2]\cdot\boldsymbol{\Lambda}) \mathrm{Re} b\},$$

$$G_{\pi\pi}^{(1)} = 3(1+a)\left\{[a(4a^2+16a-3)E^2 - (a+1)(4a^2+4a+3)q^2] + \frac{3\ln(\sqrt{a}+\sqrt{1+a})}{\sqrt{a(1+a)}}[a(6a+1)E^2 + (a+1)(2a+1)q^2]\right\},$$

$$G_{\pi\pi}^{(2)} = -\frac{3}{4}\left\{[a(8a^2-94a+3)E^2 + (a+1)(8a^2-10a-3)q^2] + \frac{3\ln(\sqrt{a}+\sqrt{1+a})}{\sqrt{a(1+a)}}[a(24a^2-12a-1)E^2 + (a+1)(8a^2+4a+1)q^2]\right\},$$

$$a = \frac{q^2}{2M^2}[1+(\boldsymbol{n}_1\cdot\boldsymbol{n}_2)]. \tag{36}$$

It is seen that the coefficient in front of Imb changes its sign at the replacement $\boldsymbol{n}_1 \leftrightarrow \boldsymbol{n}_2$, in contrast to the coefficient in front of Reb. This circumstance makes it easier to separate the contributions of Im b and Re b to the asymmetry. The dependence of the functions,

$$G_1 = \frac{2q^2\sqrt{1-x^2}G_{\pi\pi}^{(1)}}{64M^4(1+M^2/2E^2)a^2(1+a)^4},$$

$$G_2 = \frac{q^2\sqrt{2(1-x)(1-x^2)}G_{\pi\pi}^{(2)}}{64M^4(1+M^2/2E^2)a^2(1+a)^4}, \tag{37}$$

on $x = \boldsymbol{n}_1\cdot\boldsymbol{n}_2$ is shown in Fig. 2 for a few values of the energy E.

VII. $e^+e^- \to \tau^+\tau^- \to e^+e^-\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_e$

Similar to the asymmetry $dA_{\pi\pi}$, it is possible to measure the asymmetry dA_{ee} in the cross section $d\sigma_{ee}$ of the process $e^+e^- \to \tau^+\tau^- \to e^+e^-\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_e$,

$$dA_{ee} = \frac{d\sigma_{ee}(\boldsymbol{k}_1,\boldsymbol{k}_2) - d\sigma_{ee}(-\boldsymbol{k}_2,-\boldsymbol{k}_1)}{2\sigma_0}, \tag{38}$$

where $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the momenta of electron and positron, respectively. Again, both Reb and Imb can be extracted from this asymmetry without using of initial electron polarization. The most convenient from the experimental point of view is the asymmetry in the angular distribution. The straightforward calculations give

$$dA_{ee} = \frac{q^2 B_e^2 d\Omega_1 d\Omega_2}{(32\pi)^2 M^4 (1+M^2/2E^2) a^2 (1+a)^4} \{G_{ee}^{(1)} [(\boldsymbol{\Lambda}\cdot\boldsymbol{n}_1)^2 - (\boldsymbol{\Lambda}\cdot\boldsymbol{n}_2)^2] \mathrm{Im}\, b + G_{ee}^{(2)} [(\boldsymbol{\Lambda}\cdot\boldsymbol{n}_1) - (\boldsymbol{\Lambda}\cdot\boldsymbol{n}_2)]([\boldsymbol{n}_1\times\boldsymbol{n}_2]\cdot\boldsymbol{\Lambda}) \mathrm{Re}\, b\},$$

$$G_{ee}^{(1)} = -\frac{1}{3}G_{\pi\pi}^{(1)}, \qquad G_{ee}^{(2)} = \frac{1}{9}G_{\pi\pi}^{(2)}. \tag{39}$$

Here, $G_{\pi\pi}^{(1)}$ and $G_{\pi\pi}^{(2)}$ are given in Eq. (36), $\boldsymbol{n}_1 = \boldsymbol{k}_1/k_1$ and $\boldsymbol{n}_2 = \boldsymbol{k}_2/k_2$.

Neglecting the muon mass compared to M, we obtain the same result (39) for asymmetry in the cross section of the processes $e^+e^- \to \mu^+\mu^-\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_\mu$, $e^+e^- \to \mu^+e^-\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_e$, and $e^+e^- \to \mu^-e^+\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_\mu$.

VIII. DISCUSSION OF THE RESULTS

In our work we have obtained the asymmetries which contain both Reb and Imb. Asymmetries (11), (16), (24), (30), and (38) are the odd quantities with respect to CP transformations. Indeed, as a result of this transformation

$$d\sigma_\pi^{(-)}(\boldsymbol{k}) \to d\sigma_\pi^{(+)}(-\boldsymbol{k}), \qquad d\sigma_\pi^{(+)}(-\boldsymbol{k}) \to d\sigma_\pi^{(-)}(\boldsymbol{k}),$$
$$d\sigma_\rho^{(-)}(\boldsymbol{p},\boldsymbol{f}) \to d\sigma_\rho^{(+)}(-\boldsymbol{p},-\boldsymbol{f}), \qquad d\sigma_\rho^{(+)}(-\boldsymbol{p},-\boldsymbol{f}) \to d\sigma_\rho^{(-)}(\boldsymbol{p},\boldsymbol{f}),$$
$$d\sigma_e^{(-)}(\boldsymbol{k}) \to d\sigma_e^{(+)}(-\boldsymbol{k}), \qquad d\sigma_e^{(+)}(-\boldsymbol{k}) \to d\sigma_e^{(-)}(\boldsymbol{k}),$$
$$d\sigma_{\pi\pi}(\boldsymbol{k}_1,\boldsymbol{k}_2) \to d\sigma_{\pi\pi}(-\boldsymbol{k}_2,-\boldsymbol{k}_1), \qquad d\sigma_{\pi\pi}(-\boldsymbol{k}_2,-\boldsymbol{k}_1) \to d\sigma_{\pi\pi}(\boldsymbol{k}_1,\boldsymbol{k}_2),$$
$$d\sigma_{ee}(\boldsymbol{k}_1,\boldsymbol{k}_2) \to d\sigma_{ee}(-\boldsymbol{k}_2,-\boldsymbol{k}_1), \qquad d\sigma_{ee}(-\boldsymbol{k}_2,-\boldsymbol{k}_1) \to d\sigma_{ee}(\boldsymbol{k}_1,\boldsymbol{k}_2).$$

The term $\propto \gamma^\mu$ in (3) is CP-even while the term $\propto b$ is CP-odd. Therefore, asymmetries appear due to interference between CP-odd and CP-even terms and are linear in b. The energy dependence of total asymmetries is shown in Fig. 3. The solid curve corresponds to A_π, the dotted curve to A_ρ, the dashed-dotted curve to A_e, and the dashed curve to $A_{\pi\pi}$. The curves corresponding to A_π, A_ρ, and $A_{\pi\pi}$ have the same energy dependence and differ only in scale. Indeed, it follows from Eqs. (14), (21), and (35) that

$$A_\rho = 1.06 A_\pi, \qquad A_{\pi\pi} = 0.108 A_\pi.$$

Though the formulas for A_e and A_π are completely different, the corresponding curves have similar shapes.

IX. CONCLUSION

In conclusion, we have considered the processes $e^+e^- \to \tau^+\pi^-\nu_\tau$, $e^+e^- \to \pi^+\tau^-\bar{\nu}_\tau$, $e^+e^- \to \tau^+\rho^-\nu_\tau$, $e^+e^- \to \rho^+\tau^-\bar{\nu}_\tau$, $e^+e^- \to \tau^+e^-\nu_\tau\bar{\nu}_e$, and $e^+e^- \to \tau^-e^+\nu_e\bar{\nu}_\tau$ with longitudinally polarized electrons, as well as the processes $e^+e^- \to \pi^+\pi^-\nu_\tau\bar{\nu}_\tau$, $e^+e^- \to e^+e^-\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_e$, $e^+e^- \to \mu^+\mu^-\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_\mu$, $e^+e^- \to \mu^+e^-\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_e$, and $e^+e^- \to \mu^-e^+\nu_\tau\bar{\nu}_\tau\nu_e\bar{\nu}_\mu$ with unpolarized electrons for the invariant masses $\sqrt{s} \ll m_Z$ of the initial electron and positron. We have calculated analytically the CP-odd asymmetries $\propto \mathrm{Re}b$ and $\propto \mathrm{Im}b$. Measuring these quantities can improve the upper limits for Reb and Imb. It is shown that to measure Imb, polarization is not needed, and to measure Reb, the polarization is not necessary, but simplifies measurements.

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