Enhanced power of gravitational waves and rapid coalescence of black hole binaries through *k*-essence dark energy accretion

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We consider the accretion of dark energy by constituent black holes in binary formations during the present epoch of the Universe. In the context of an observationally consistent dark energy model, we evaluate the growth of black holes' masses due to accretion. However, the accretion rate may turn out to be too high if this model is naively applied to supermassive black holes. We show that the accretion leads to faster circularization of the binary orbits. We compute the average power of the gravitational waves emitted from binaries, which exhibits a considerable enhancement due to the effect of growth of masses as a result of accretion. This in turn, leads to a significant reduction of the coalescence time of the binaries. We present examples pertaining to various choices of the initial masses of the black holes in the stellar mass range and above, in order to clearly establish a possible observational signature of dark energy in the emerging era of gravitational wave astronomy.

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I. INTRODUCTION

After the first direct detection of gravitational waves from a merging binary of black holes by aLIGO [1], and subsequent series of detections from similar sources [2–4], a new era in observational astronomy has begun. Besides binaries of compact objects in bounded orbits, there are various other mechanisms of production of gravitational waves from a wide varieties of sources, such as nearby fly-pass of two compact objects in unbounded orbits [5], gravitational collapse of sufficiently massive stars [6], cosmological phase transitions [7,8], breaking of cosmic strings [9,10], inflation and pre-heating [11,12], dark sector interactions [13], etc. However, to date the observations by the aLIGO and VIRGO detectors have been carried out from mainly one type of sources, which are the binaries of compact objects, *viz.*, black holes and neutron stars.

Gravitational wave observations have been used to estimate and constrain various astrophysical and cosmological parameters associated with the generation and propagation of these gravitational waves. Among these, some important ones worth mentioning are: (i) estimating the Hubble parameter [14,15], (ii) constraining a large class of cosmological scalar-tensor theories [16,17], (iii) constraining the mass of gravitons for bimetric-gravity theories [18], (iv) investigating the state of matter inside a neutron star [19], (v) constraining higher-dimensional theories [20], and there are several others. Attempts to constrain dark energy, responsible for the accelerated expansion of the late Universe [21], have been made indirectly using gravitational wave observations, either through the estimation of the Hubble parameter, or through constraining cosmological scalar-tensor theories.

Late time acceleration of expansion of the Universe is one of the most intriguing discoveries of recent times, which was directly confirmed from supernovae Ia observations in 1998 [22,23] and was also supported by various indirect probes. Many theoretical approaches have been employed to explain the current cosmic acceleration. The component of the Universe providing the required negative pressure for driving this accelerated expansion is generically called 'dark energy' [24]. As normal matter (radiation, baryonic matter or cold dark matter) is gravitationally attractive, the standard lore is to assume the presence of a relativistic fluid which is repulsive in nature, as the dark energy candidate. The simplest candidate of dark energy is the cosmological constant Λ, which is mostly consistent with cosmological observations. However, it is plagued with conceptual problems, for example, fine-tuning and coincidence problems [25], which are theoretical in nature. With a hope to address

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these problems, cosmologists have proposed mechanisms where the role of dark energy is played by a completely different component of the Universe, which may have a variable equation-of-state parameter.

Many varieties of dark energy models have been proposed, theoretically studied and observationally constrained till now. There exist a wide class of scalar field models coupled to gravity. Among these, minimally coupled ones, called quintessence, in which cosmic acceleration is driven by the potential energy [26,27], are known to alleviate some of the problems of the cosmological constant. Scalar fields, in which the cosmic acceleration is driven by the kinetic energy, called "k-essence" [28–30], have also been studied, motivated from unification and quantum gravity scenarios. Such models may further yield a consistent picture of the complete evolution,[1] starting from the early era inflation, the subsequent dark matter domination, and finally the late time acceleration [37,38]. Other alternatives include random barotropic fluids with pre-determined forms of the equation-of-state parameter, such as the Chaplygin gas models [39], string theory motivated models [40–42] and braneworld models [43,44]. There also exist approaches without requiring additional fields [45–48].

A major difference between the scalar field and other fluid models of dark energy with the ΛCDM model (and other approaches not requiring additional fields) is that the former type of dark energy is subjected to accretion by the black holes present in the Universe. In fact, those black holes with surroundings containing insufficiently available other forms of matter-energy for accretion, would still accrete the scalar field dark energy, which is uniformly distributed throughout the Universe. Accretion of various types of dark energy by black holes has been a subject of theoretical interest for a considerable time [49–53]. On the basis of various works done till date, it is widely accepted that the mass of a black hole would increase due to steady spherical accretion if the equation-of-state parameter of the dark energy $w_{\rm DE}$ is > -1. On the other hand, accretion would result in mass loss of a black hole, for phantom type dark energy with $w_{\rm DE} < -1$ [50] in the case of fixed background. However, it has been pointed out [51] that if one takes into account the backreaction which becomes large when the dark energy background becomes large (for the case of phantom dark energy this will eventually happen), the black hole mass can increase when accreting the dark energy with $w_{\rm DE} < -1$.[2]

If dark energy exists in the Universe in a form which can be accreted by black holes, the result would not be limited to just the change of masses of the black holes. It is expected that other phenomena associated with the black holes would also be influenced. The evolution of binaries formed with the black holes, the gravitational wave emitted from those binaries and coalescence of those binaries are some of the physical processes which get directly affected if the masses of the concerned black holes are changing continuously instead of being constant. Several works have been done exploring the effect of accretion of gas on compact binaries emitting gravitational waves and detectability of the imprints of this effect [54–58]. In the context of spherically symmetric accretion of dark energy [49], the efficacy of the above effects, in particular, whether the modified variation of gravitational energy of the binary system could be detectable via the rate of change of the orbital radius, has been a subject of debate [59,60].

In the present work our motivation is to explore the problem of associated modification of black hole binary parameters due to accretion in the context of a popular k-essence model of dark energy. Specifically, we consider a string theory [61] inspired low energy effective action framework containing a dilaton scalar field [62]. The resultant k-essence dark energy scenario [63] is compatible with cosmological observations [64]. Here we study spherical accretion of the k-essence dilatonic ghost condensate dark energy by black holes. This falls within a class of models known as 'ghost condensates' [65]. Considering binary formations of black holes in the early inspiral stage, we study main aspects of evolution of the orbits, due to continuous change of masses of the component black holes of such binaries, resulting from spherical accretion of the chosen model of k-essence dark energy. More specifically, we study the pace of shrinking of such an orbit and the average power of the gravitational wave emitted from the orbit in the course of its evolution, and perform quantitative comparisons of the differences with the case when the masses of component black holes are constant. We further investigate the modification in the time required to reach the coalescence stage of such a binary, in comparison with the constant mass case.

[1]Recently, there exists an interesting debate on the impact of the Hubble tension [31,32] on the viability of dark energy models. It is claimed that a large value of local determination of the Hubble parameter may severely constrain dark energy models [33,34], though schemes for resolution of the Hubble tension have also been proposed using early dark energy (EDE) models [35]. Interestingly, the k-essence framework is also conducive to the EDE picture [36].

[2]If the background space-time is evolving, which is the case for the FLRW-Universe, then it has been shown that according to the notions of quasilocal mass and the generalized Misner-Sharp mass, the black hole mass increases due to accretion of phantom dark energy with $w_{\rm DE} < -1$ [51]. It has been argued that the discrepancy of increasing quasilocal mass and apparent horizon of a black hole, due to accretion of a phantom fluid, with the decrease in mass of a black hole in fixed background space-time, is due to the reason that the quasilocal mass does not receive contributions from the negative pressure. In the present work, we do not need to consider quasilocal mass of the black holes, hence it is not required to consider the above mentioned effect.

The paper is organized as follows. In Sec. II, we study the growth of black hole mass due to accretion of the chosen model of k-essence dark energy in the late Universe. In Sec. III, we investigate the effect of growing masses of black holes on the evolution of binaries. We compute the rate of decrease of orbital radius after circularization of orbits, and the average power of the emitted gravitational waves. We compare these results with the case of binaries with black holes of constant masses without accretion. In Sec. IV, we estimate the reduction in the time required for reaching coalescence-stage by such binaries. We present our concluding remarks in Sec. V.

II. DARK ENERGY ACCRETION BY BLACK HOLES IN A k-ESSENCE MODEL

k-essence scalar fields are the dynamical dark energy models where the acceleration is driven by kinetic term in the scalar field Lagrangian. Among many k-essence models, we choose a particular string-inspired ghost condensate model, called "k-essence dilatonic ghost condensate," which can successfully describe the cosmological evolution, while simultaneously satisfies the necessary conditions of quantum stability and sound speed [63,66]. This model has also found to be observationally consistent [64].

The condition on sound speed for any scalar field dark energy model is simply that the sound speed cannot exceed the speed of light in vacuum (c) i.e. it cannot have superluminal speed. In this regard, it is worthwhile to mention that the sound speed makes an important difference between quintessence and k-essence models. While for standard quintessence models, with canonical scalar fields, the sound speed is always equal to the speed of light; for the k-essence models it is not so. This fact of having varying sound speeds through the cosmic evolution gives various ways of distinguishing different k-essence models from one-another and from the quintessence models [67]. In fact this difference of sound speed of k-essence models with the quintessence models is one of the main reasons, for which we have chosen a k-essence model for this work.

The action of k-essence scalar field φ, along with nonrelativistic matter and radiation, can be generally written as [28]:

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R + \mathcal{L}(\varphi, X)\right\} + S_m, \tag{1}$$

where $\kappa = (8\pi G/3)^{1/2}$, R is the Ricci-scalar and $\mathcal{L}$ is a function of the k-essence scalar field φ and its kinetic energy $X = -(1/2)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$. S_m is the action contributed from the nonrelativistic matter and radiation. In case of the specific model considered here, the Lagrangian density is given by [63,66]:

$$\mathcal{L} = -X + e^{\kappa\lambda\varphi}\frac{X^2}{M^4}, \tag{2}$$

where M is a constant having the dimension of mass and λ is a constant dimensionless parameter, which is set according to stability conditions.

The set of equations governing the cosmological dynamics of this k-essence model can be conveniently written in terms of three dimensionless parameters: [63,66]:

$$x_1 = \frac{\kappa\dot{\varphi}}{\sqrt{6}H}, \qquad x_2 = \frac{\varphi^2 e^{\kappa\lambda\varphi}}{2M^4}, \qquad x_3 = \frac{\kappa\sqrt{\rho_r}}{\sqrt{3}H}, \tag{3}$$

where H is the Hubble-parameter and ρ_r is the density of radiation in the Universe. With these dimentionless parameters x_1, x_2 and x_3, the evolution equations can be cast in the following autonomous form:

$$\frac{dx_1}{dN} = -\frac{x_1}{2}\frac{6(2x_2-1)+3\sqrt{6}\lambda x_1 x_2}{(6x_2-1)} + \frac{x_1}{2}(3-3x_1^2+3x_1^2x_2+x_3), \tag{4}$$

$$\frac{dx_2}{dN} = x_2\frac{3x_2(4-\sqrt{6}\lambda x_1)-\sqrt{6}(\sqrt{6}-\lambda x_1)}{1-6x_2}, \tag{5}$$

$$\frac{dx_3}{dN} = -\frac{3}{2}\left(-x_1^2+x_1^2x_2+\frac{x_3^2}{3}+1\right). \tag{6}$$

where $N = \ln(\boldsymbol{a}) = -\ln(1+z)$; while $\boldsymbol{a}$ and z are respectively the scale-factor and the redshift. N is generally called *e-foldings*. The advantage of these autonomous equations and the dimensionless parameters is that, these are easier to solve numerically, and various important cosmological quantities can be given in terms of these dimensionless parameters x_1, x_2 and x_3 *viz.* [63,66],

$$w_{\rm eff} = -1 - \frac{2\dot{H}}{3H^2} = -x_1^2 + x_1^2x_2 + \frac{1}{3}x_3^2, \tag{7}$$

$$w_\varphi = \frac{1-x_2}{1-3x_2}, \tag{8}$$

$$c_s^2 = \frac{2x_2-1}{6x_2-1}, \tag{9}$$

$$\Omega_\varphi = -x_1^2 + 3x_1^2x_2, \tag{10}$$

$$\Omega_R = x_3^2, \tag{11}$$

$$\Omega_M = 1 + x_1^2 - 3x_1^2x_2 - x_3^2, \tag{12}$$

where $w_{\rm eff}$ and w_φ are respectively the effective equation-of-state parameter and the equation-of-state parameter of the k-essence model. c_s is the sound speed of the k-essence model. Ω_φ, Ω_M and Ω_R are respectively the fractional densities of the dark energy, nonrelativistic matter and radiation in the Universe. In the Fig. 1, the evolutions of

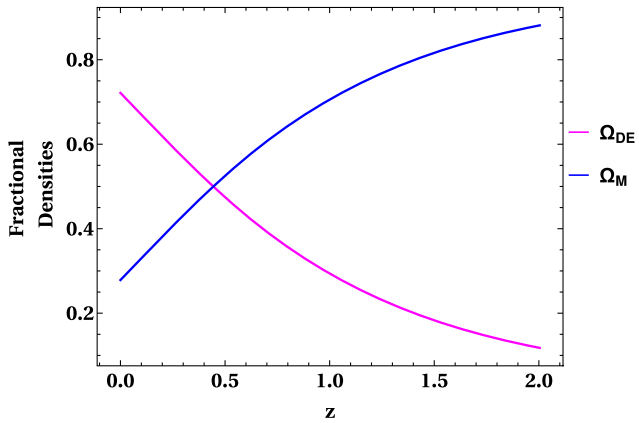

FIG. 1. Evolution of the fractional densities of k-essence dark energy denoted by $\Omega_{\rm DE} \equiv \Omega_\varphi$ and nonrelativistic matter denoted by Ω_M, with respect to redshift z. The fractional density of radiation Ω_R is negligible in this era of the Universe.

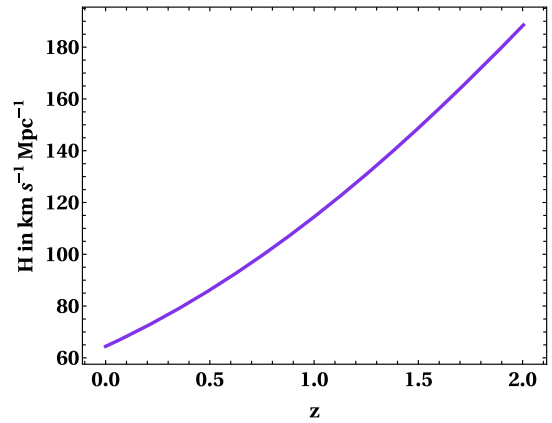

FIG. 2. Evolution of the Hubble parameter H of the Universe, for the chosen k-essence dark energy model, with respect to redshift z.

the fractional densities Ω_φ and Ω_M have been depicted with respect to redshift z, for a certain period in the late Universe, with the initial conditions taken as $x_1 = 6.0 \times 10^{-11}$, $x_2 = 0.5 + (1.0 \times 10^{-9})$, and $x_3 = 0.999$ at redshift $z \approx 10^{6.218}$ and the value of $\lambda = 0.2$ [66].

One can obtain simple equations for the Hubble-parameter H and the time t in terms of the e-foldings N, using Eq. (7):

$$\frac{1}{H^2}\frac{dH}{dt} = -\frac{3}{2}\left(-x_1^2 + x_1^2 x_2 + \frac{1}{3}x_3^2 + 1\right). \tag{13}$$

As $\frac{dN}{dt} = H$, the left-hand side (lhs) of the Eq. (13) can be expressed as

$$\frac{1}{H^2}\frac{dH}{dt} = \frac{1}{H^2}\frac{dH}{dN}\frac{dN}{dt} = \frac{1}{H}\frac{dH}{dN}.$$

Denoting $h = \ln H$, we get:

$$\frac{dh}{dN} = \frac{d}{dN}(\ln H) = \frac{1}{H}\frac{dH}{dN}. \tag{14}$$

Using the Eq. (14), we can write the equation (13) as:

$$\frac{dh}{dN} = -\frac{3}{2}\left(-x_1^2 + x_1^2 x_2 + \frac{1}{3}x_3^2 + 1\right), \tag{15}$$

by solving which we can get h and consequently $H = e^h$. After solving Eq. (15) for the Hubble-parameter, we can simply solve the equation:

$$\frac{dt}{dN} = \frac{1}{H}, \tag{16}$$

to get the time t. Also, for the scale factor $\boldsymbol{a}$, we have the equation:

$$\frac{d\boldsymbol{a}}{dN} = \boldsymbol{a}. \tag{17}$$

Solving the above Eq. (17) one can obtain the scale factor $\boldsymbol{a}$ and corresponding redshift from the relation $1 + z = 1/\boldsymbol{a}$. We numerically solve the set of Eqs. (4), (5), (6) along with the Eqs. (15), (16), (17), with appropriate initial conditions as specified here. We obtain the present values of the key cosmological parameters as: Hubble parameter $H_0 \approx 64.43$ km s^{-1} Mpc^{-1}, fractional density of the k-essence dark energy $\Omega_{\varphi,0} \approx 0.721$, fractional density of nonrelativistic matter $\Omega_{M,0} \approx 0.278$, equation-of-state parameter of the k-essence dark energy $w_{\varphi,0} \approx -0.847$, effective equation-of-state parameter $w_{\rm eff,0} \approx -0.611$. The evolutions of the Hubble parameter H with respect to redshift and equation-of-state parameters w_φ, $w_{\rm eff}$ with respect to redshift have been plotted in the Fig. 2 and Fig. 4 respectively.

We now consider accretion of dark energy by black holes in the context of the above dark energy model. It may be noted here that the rate of accretion is affected by the sound speed of the ambient fluid. The surface of accretion is defined by the black hole horizon if there is no critical point outside the horizon [49]. The fluid being accreted by a black hole has the critical point, if its speed increases from subsonic to transonic values. From the historical development of spherical accretion by black holes starting from the pioneering work by Bondi [68], it is evident that if a black hole moves through the ambient medium with a speed much lesser than speed of light in vacuum (c), and the medium, considered as a perfect fluid, has a sound speed less than c, then the accretion radius would be larger than the black hole horizon i.e. the Schwarzschild radius [69].

In the above scenario where the sound speed of the k-essence model lies in the range $0 < c_s/c < 1$, the time

rate of change of mass of a black hole spherically accreting the k-essence dark energy is obtained by using the accretion radius $r_a = Gm/(v_{\rm rel}^2 + c_s^2)$, which defines the relevant surface of accretion [69]. Hence, the rate of accretion is given by

$$\frac{dm}{dt} = 4\pi\mathcal{A}\frac{G^2m^2}{(v_{\rm rel}^2 + c_s^2)^{3/2}}(1 + w_\varphi)\rho_\varphi, \tag{18}$$

where $c_s = (\frac{dP}{d\rho})^{1/2}$ and $v_{\rm rel}$ is the relative speed of the black hole with respect to the ambient cosmic fluid being accreted. ρ_φ is the background density of the k-essence dark energy, w_φ is the equation-of-state parameter of the k-essence dark-energy. $\mathcal{A}$ is a proportionality-factor that can be taken to be of the order of ~ 1 [69]. Note that for $v_{\rm rel} \ll c_s$ and $c_s \sim c$, the equation (18) leads to the rate of change of mass derived in Ref. [49].

For the present analysis we consider that the relative speed of the moving black hole with respect to the ambient cosmic fluid, *viz.*, the k-essence dark energy, is negligible in comparison to the sound speed of the k-essence model, i.e., $v_{\rm rel} \ll c_s$. This is valid for most of the black holes in late Universe, as it can be seen that in the dark energy dominated Universe, the sound speed of the chosen k-essence model is of the order of $\sim 0.1c$ (see Fig. 3). So, in the denominator on the right-hand side (rhs) of Eq. (18), $v_{\rm rel}^2$ can be neglected in comparison to c_s^2. Thereby, the time rate of change of mass of a black hole due to spherical accretion of the k-essence dark energy is given by

$$\frac{dm}{dt} = 4\pi\mathcal{A}\frac{G^2m^2}{c_s^3}(1 + w_\varphi)\rho_\varphi. \tag{19}$$

It may be noted from the evolution of the w_φ (Fig. 4) that it decreases for higher redshift. Hence at higher redshift the w_φ makes the rate of growth of mass of a black hole lower.

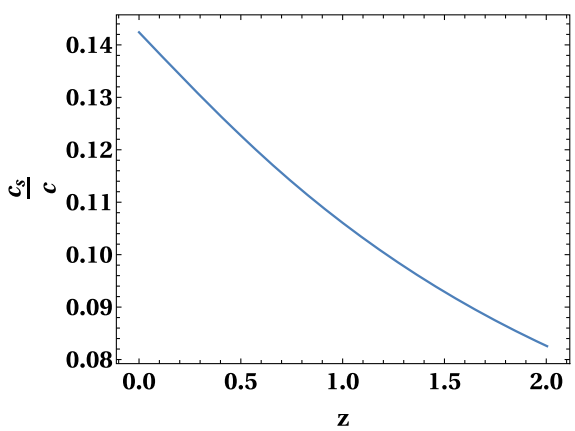

FIG. 3. The variation of the sound speed c_s of the k-essence model with respect to redshift z.

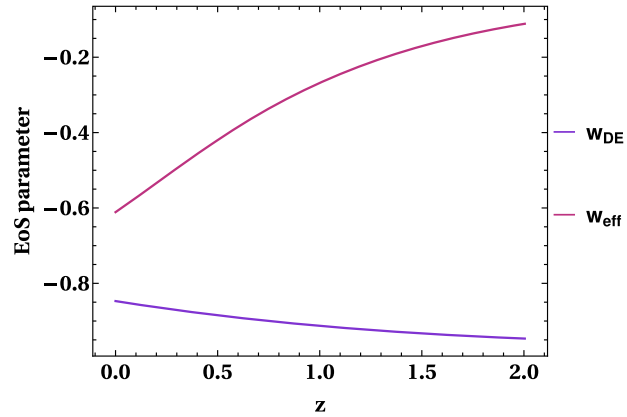

FIG. 4. The variation of the equation-of-state (EoS) parameters $w_{\rm DE} \equiv w_\varphi$ and $w_{\rm eff}$ with respect to redshift z.

We can determine the time rate of change of mass of a black hole due to spherical accretion of the chosen k-essence dark energy using equation (19), where $\rho_\varphi = \rho_T\Omega_\varphi$, and $\rho_T = (3/8\pi G)H^2$ is obtained by solving the equation (15) for getting the Hubble-parameter H. We depict the evolution of masses of black holes, due to accretion of the chosen k-essence dark energy, with four different initial masses taken as 10, 30, 50 and 70 times of the Solar-mass ($M_\odot$) respectively, with respect to the redshift z in Fig. 5. It can be seen from the Fig. 5 that the amount of growth in masses of the black holes due to the dark energy accretion increases with the increase in their initial masses. It may be noted that ordinary stellar-mass black holes, which are usually observed by electromagnetic signals emitted by various type of astrophysical mechanisms, generally have masses in the range 5–20$M_\odot$. However, aLIGO and VIGO have detected gravitational waves from mergers of binaries with component black holes having masses from 30$M_\odot$ to as large

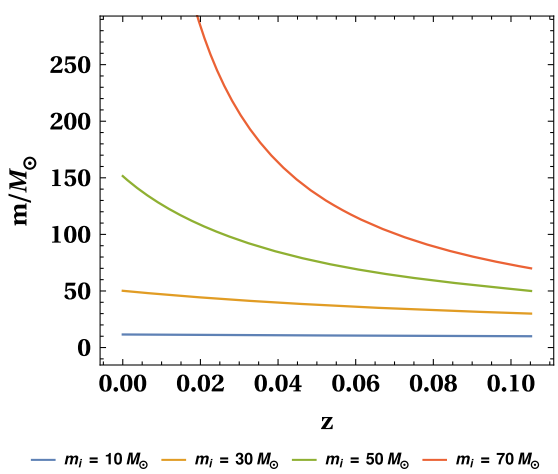

FIG. 5. Growth of mass of black holes with various initial masses due to accretion of the k-essence dark energy with respect to redshift z.

as $80M_\odot$ [70,71]. It is quite evident from the Fig. 5 that stellar-mass black holes, having mass in the range 5–20$M_\odot$, can grow to heavier ones by means of continuous spherical accretion of similar type of dark energy.

III. POWER OF GRAVITATIONAL WAVES EMITTED FROM BINARIES

The instantaneous power of gravitational radiation due to the orbital motion of two black holes of masses m_1 and m_2 in the quadrupole-approximation is given by [72,73]:

$$\mathcal{P}(t) = \frac{8}{15}\frac{G^4}{c^5}\frac{M(m_1m_2)^2}{(r_{\min}(1+e))^5}(1+e\mathrm{Cos}\phi)^4 \times \{e^2\mathrm{Sin}^2\phi + 12(1+e\mathrm{Cos}\phi)^2\}, \tag{20}$$

where $M = m_1 + m_2$, 'e' is the eccentricity of the orbit, ϕ is the angular position of the reduced mass μ on the plane of the orbit in a polar-coordinate system (r, ϕ) with origin at the center-of-mass, and $r_{\min}$ is the radial distance of closest approach. In the present case, the masses are continuously changing due to accretion of dark energy. Due to this continuous time variation of the masses of the black holes, two extra terms (having single and double time derivatives of the masses) arise along with the main term in the amplitude of gravitational radiation [74]. However, these terms are negligible in comparison to the main term in this case. Hence, the Eq. (20) needs to be considered here with time-dependent masses.

When the orbit is bounded, the total energy carried away by the gravitational radiation due to the relative motion of the system of two black holes within one complete cycle or time period, is given by

$$\Delta\mathcal{E} = \int_0^T \mathcal{P}(t)dt = \int_0^{2\pi} \mathcal{P}(t)\frac{dt}{d\phi}d\phi. \tag{21}$$

It is known that energy of gravitational waves is well-defined when the average of the energy over several time periods of the wave is taken. Also, a compact object in a Keplarian elliptical orbit emits gravitational waves with angular-frequencies, which are integral multiples of the angular-frequency $\omega_0 = (GM/a^3)^{1/2}$, where a is the semi-major axis of the elliptical orbit. Hence, the period of the gravitational waves emitted due to this orbital-motion, is a fraction of the orbital-period. Therefore, a well-defined version of the power of the emitted gravitational waves is the average of the power taken over one period of the orbit. The average of the power P_{avg} over one period of the orbit can be written as [72]

$$P_{\mathrm{avg}}(t) = \frac{1}{T}\int_0^T \mathcal{P}(t)dt = \frac{32G^4(m_1m_2)^2M}{5c^5a^5}f(e), \tag{22}$$

where the function $f(e)$ of eccentricity e is given by:

$$f(e) = \frac{1}{(1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right). \tag{23}$$

For a circular orbit $e = 0$, thereby $f(e)$ becomes 1 and a becomes the radius of the circular orbit. Note that in case of constant masses the eccentricity of the orbit changes only due to the emission of gravitational waves. However, in the present case since the masses of the black holes are continuously changing through accretion of dark energy, the change of eccentricity would be due to two different effects: (i) growth of the masses via accretion, and (ii) loss of energy and angular momentum carried away by gravitational waves. The angular momentum of the system of two black holes is not affected due to spherical accretion of the dark energy because the scalar-field dark energy model considered here does not contain angular momentum, and hence, cannot impart any angular momentum to the system.

Using the rate of change of energy and angular momentum of a binary of black holes in bounded orbit, the rate of change of the semi-major axis a and eccentricity e of the orbit can be obtained as [75],

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3(m_1m_2)M}{c^5a^3}\frac{1}{(1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right), \tag{24}$$

and

$$\frac{de}{dt} = -\frac{304}{15}\frac{G^3(m_1m_2)M}{c^5a^4}\frac{e}{(1-e^2)^{5/2}}\left(1 + \frac{121}{304}e^2\right). \tag{25}$$

It may be noted that the semimajor axis a and eccentricity e, governed by the above Eqs. (24) and (25), are averages of these quantities over one period of the orbit, not their instantaneous values, as the corresponding equations of energy and angular momentum of the system, from which these are derived, govern their averages over one period. This is quite evident from the fact that these Eqs. (24) and (25) do not contain the phase-angle ϕ. From the Eq. (25) it follows that, if the eccentricity e becomes zero(0), then $\frac{de}{dt} = 0$ implying $e =$ constant, i.e., e remains zero. Thereby, once the orbit becomes circular, it remains circular. We solve these Eqs. (24) and (25) numerically for different initial masses of the black holes forming binaries and orbiting in elliptical orbits with initial eccentricity $e_i = 0.9$. We choose the combinations of initial masses of the black holes forming the binaries to be $50, 60M_\odot$; $10, 60M_\odot$ and $10, 20M_\odot$, respectively. The initial semimajor axis a_i of the elliptical orbit has been taken as 10^6 times of the sum of their initial Schwarzschild-radii, i.e., $a_i = 10^6(2GM_i/c^2)$, (M_i being the initial total-mass of the black holes) so that the Keplarian-approximation holds

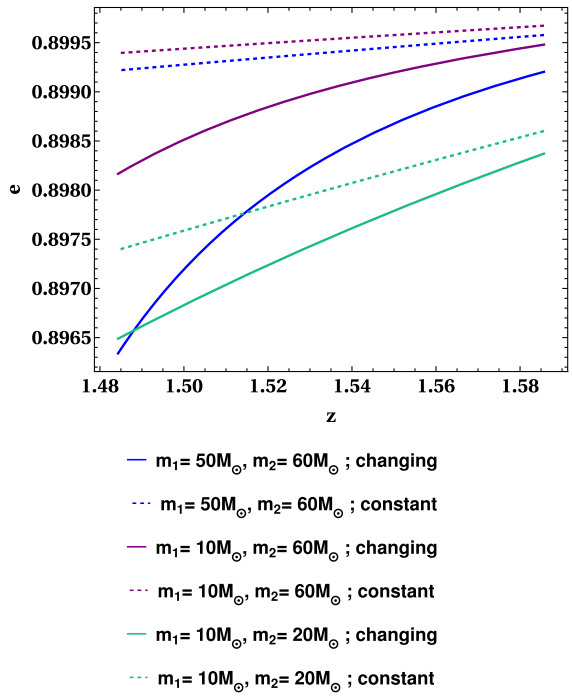

FIG. 6. Evolution of eccentricities of elliptical orbits, from the initial value 0.9, with respect to redshift z, for three different combinations of the initial masses of black holes, for two different cases, (i) growing masses and (ii) constant masses.

well. The time period of the orbit is given by: $T = 2\pi/\omega_0$, where the angular-frequency ω_0 is given by: $\omega_0 = (\frac{GM}{a^3})^{1/2}$.

The fall of the eccentricities of the elliptical orbits of the binaries, for three different combinations of initial masses of the constituent black holes, is depicted in the Fig. 6 (Only certain portions of the full evolution-profiles have been shown here so that the differences can be visualized clearly). It can be seen from Fig. 6 that the eccentricities for the orbits of binaries, where the masses of the black holes are growing due to accretion of the chosen model of k-essence dark energy, drop faster than those where the masses are constant. Moreover, the eccentricities of binaries with larger mass black holes drop faster. After the eccentricity vanishes, i.e., circularization of the orbit is achieved, the rate of change of radius r of the circular orbit is given by

$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^3(m_1 m_2)M}{c^5 r^3}. \tag{26}$$

Correspondingly, the average power of the emitted gravitational wave for the circular orbit becomes,

$$P_{\rm avg}(t) = \frac{32 G^4 (m_1 m_2)^2 M}{5 c^5 r^5}. \tag{27}$$

We first determine the patterns of shrinking of radius r, by solving the Eq. (26), for the circular orbits in which two black holes of masses m_1 and m_2 are in binary formations, for two different cases *viz.*, (i) when the masses are changing due to spherical accretion of dark energy described by our chosen model and (ii) when the masses are constant, for three specified combinations of initial values of m_1 and m_2 for each of the cases. For this, we fix the initial radius for each of the cases to be 10^5 times of the sum of the initial Schwarzschild radii of the black holes, i.e., $r_i = 10^5(2GM_i/c^2)$ (where M_i stands for the initial total mass of the black holes). This choice for the initial radii of the circular orbits for each case is considered to study the comparative evolution with similar initial conditions. The radii of the circular orbits for three different combinations of initial values of masses m_1 and m_2, and for two different cases, as mentioned above, are plotted with respect to redshift z in Fig. 7. It can be seen from the Fig. 7 that, with the increasing difference in the masses and increasing total masses of the component black holes of

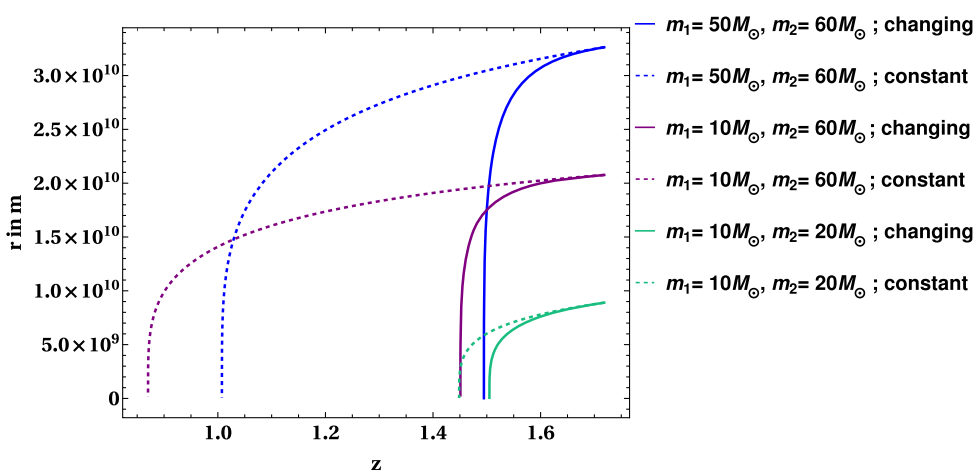

FIG. 7. Variation of the radius r of circular orbit of two black holes in binary formation with respect to redshift z, in three different combinations of initial masses and two cases *viz.*, (i) growing masses and (ii) constant masses.

the binaries, the difference in rate of shrinking of the radii of the circular orbits increases.

We next study the variation of the average power $P_{\rm avg}(t)$ with the evolution of the circular orbits for each of the cases. We depict the variation of the average power with respect to the redshift for each of the cases in Fig. 8. It can be seen from the plots in Fig. 8 that within the same interval, the average power of the emitted gravitational wave achieve significantly higher values for the binaries of black holes with growing masses, in comparison with the case when the masses of the black holes are constant. The average power of the emitted gravitational wave in case of evolving masses of black holes in the binaries, grows faster in comparison to the case of constant masses of the black holes. A certain amount of increase of the masses of the black holes of binaries results in more amplification of the average power, because of the fact that the average power of the emitted gravitational waves is proportional to the quantity $\mu^2 M^3$ (μ being the reduced-mass of the black holes forming the binary). So, a small increment in the masses of the black holes results in a comparatively greater increase in the average power of the emitted gravitational waves. Moreover, the faster shrinking of the radius of circular orbit for increasing masses of the black holes, in comparison to the case of constant masses, also contributes to the faster growth of the average power $P_{\rm avg}(t)$ in the former case, as it is proportional to r^{-5}.

IV. REDUCED COALESCENCE TIME

From the previous analysis we have seen that as the masses of the black holes forming the binary increases due to accretion of dark energy, the average power of the emitted gravitational waves becomes significantly higher with the evolution of the orbit, in comparison to the case of constancy of masses when there is no accretion. Since the power of the emitted gravitational wave increases with time, the binary loses energy faster and shrinks more rapidly. As a result, the time taken by a binary to coalesce is shorter when the black holes' masses are growing, than for the case of constant masses of the black holes. Let us now estimate the decrease in coalescence time interval of a binary, due to increasing masses of the component black holes that are spherically accreting the chosen model of k-essence dark energy.

For a binary constituted with black holes of constant masses, the rate of loss of energy by the binary is equal to the power of the emitted gravitational waves, i.e., $P_{\rm avg} = -dE_{\rm avg}/dt$, where $E_{\rm avg}$ is the average energy of the binary. Using the expression of average power from Eq. (27), the time evolution of the frequency of gravitational waves ($f_{\rm gw}$) emitted from the binary is given by [75],

$$\frac{df_{\rm gw}}{dt} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f_{\rm gw}^{11/3}, \tag{28}$$

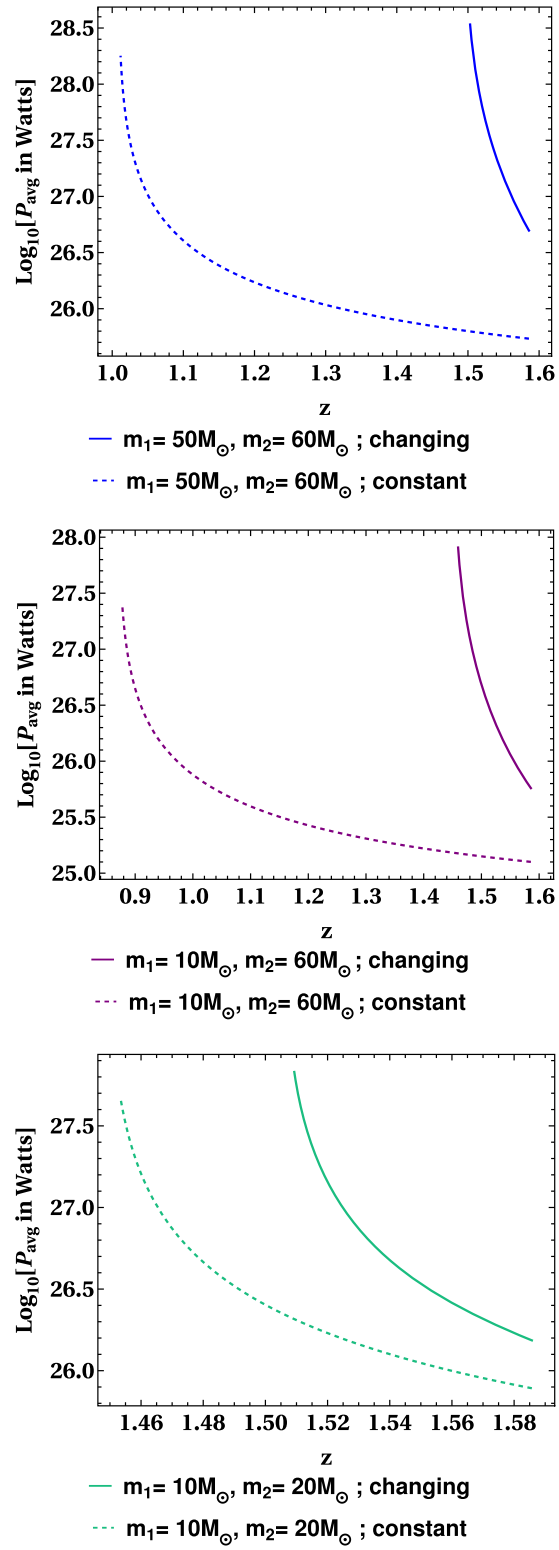

FIG. 8. Evolution of average power $P_{\rm avg}$ of the gravitational wave emitted due to the orbital-motion of two black holes in binary formation, with respect to redshift z, for three different combinations of initial masses and for two different (constant and changing mass) cases. A range of the full evolution profiles have been shown for visual clarity.

where $\mathcal{M} = (m_1 m_2)^{3/5}/M^{1/5}$ is the chirp-mass of the binary. For the case of constant masses of the component black holes of the binary, the solution of the above Eq. (28) can be written as [75]:

$$f_{\rm gw} = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{\tau}\right)^{3/8}\left(\frac{G\mathcal{M}}{c^3}\right)^{-5/8}, \tag{29}$$

where t_c is the time of coalescence of the binary and $\tau = t_c - t$ is the time interval required by the binary to reach coalescence, from any stage of its evolution at an arbitrary time t. Using the Eq. (29) evaluated at an initial time t_i and the relation $\omega_{si}^2 = (GM/r_i^3)$, where ω_{si} is the initial angular-frequency of the source, it can be shown that the time interval $\tau_i = t_c - t_i$ required by the binary to reach the coalescence stage from initial instant, is related to the initial radius of the circular-orbit r_i, as [75]

$$\tau_i = \frac{5}{256}\frac{c^5 r_i^4}{G^3 M m_1 m_2}. \tag{30}$$

Now, for the case when the masses of the black holes are changing, the counterpart of the Eq. (29) valid in the present case is given by

$$f_{\rm gw}^{-8/3} = \frac{256}{5}\pi^{8/3}\left(\frac{G}{c^3}\right)^{5/3}\int_t^{t_c}\mathcal{M}^{5/3}dt. \tag{31}$$

Making a change of variable from t to τ in the integral $\int_t^{t_c}\mathcal{M}^{5/3}dt$, we can write:

$$\int_t^{t_c}\mathcal{M}^{5/3}dt = \int_0^{\tau}\mathcal{M}^{5/3}d\tau. \tag{32}$$

In following the suffix i denotes the corresponding initial value of the quantity at initial time t_i'. Next, as was done for the case of constant masses, here also we evaluate the Eq. (31) at initial time t_i' and use the relation $\omega_{si}^2 = (GM_i/r_i^3)$. It follows that t_i' or alternatively τ_i' satisfies the equation:

$$\frac{\int_0^{\tau_i'}\mathcal{M}^{5/3}d\tau}{\mathcal{M}_i^{5/3}} = \frac{5}{256}\frac{c^5 r_i^4}{G^3 M_i m_{1i} m_{2i}}, \tag{33}$$

as the counterpart of the Eq. (30) for the case of changing masses of black holes in binaries, due to dark energy accretion. The Eqs. (30) and (33) provide the values of the coalescence time intervals for the two different cases, *viz.*, constant mass and varying mass of the black holes, respectively. Therefore, when the initial masses of the component black holes and initial radii of orbits are same for both the cases, the difference of the coalescence time intervals corresponding to the two cases is given by,

$$\Delta\tau_i = \tau_i - \tau_i'. \tag{34}$$

In order to perform a comparative estimate of the reduction in coalescence time intervals due to dark energy accretion for the examples of binaries studied in the present work, we fix the initial time to be same for both the cases (pertaining to constant and changing masses), and evaluate the corresponding times of coalescence. We obtain the times of coalescence for binaries of the three different combinations of initial black hole masses considered by us in the previous section, for studying the evolution of eccentricities of the elliptical orbits, shrinking of radii of the circular orbits and the average power of emitted gravitational waves.

We choose the initial time at the e-folding value $N = -1$, or the corresponding redshift $z \approx 1.72$. For each of the cases, we set the initial radii r_i of the circular orbits to be 10^5 times of the sum of the initial Schwarzschild radii of the black holes. We display the decrease in coalescence time intervals for these three different examples in the Table I.

From the above analysis it is evident that for binaries consisting of black holes, having masses in the stellar-mass range and few-times greater than the stellar-mass black holes, specifically those from which many merging events have been detected by the aLIGO and VIRGO detectors, the time required for coalescence gets significantly reduced due to the increase in masses of the black holes caused by accretion of the chosen model of dark energy. The magnitude of reduction in the coalescence time interval is $\sim 10^8$ years, when both the component black holes are stellar-mass black holes (note the column for the combination of 10 and $20M_\odot$ in Table I). However, for larger mass black holes (having masses few times larger than stellar-mass ones) the effect of accretion of the dark energy

TABLE I. Reduction in coalescence time intervals due to accretion of the chosen model of dark energy.

Initial masses	10 and $20M_\odot$	10 and $60M_\odot$	50 and $60M_\odot$
Initial radius of orbit r_i	$8.899 \times 10^9\ m$	$20.764 \times 10^9\ m$	$32.628 \times 10^9\ m$
t_c for constant masses	$4.817\ Gy$	$6.956\ Gy$	$6.329\ Gy$
t_c' for varying masses	$4.665\ Gy$	$4.81\ Gy$	$4.693\ Gy$
τ_i for constant masses	$66.135 \times 10^7\ y$	$2.8\ Gy$	$2.173\ Gy$
τ_i' for varying masses	$50.99 \times 10^7\ y$	$0.655\ Gy$	$0.537\ Gy$
Decrease in coalescence time interval $\Delta\tau_i = \tau_i - \tau_i'$	$15.14 \times 10^7\ y$	$2.146\ Gy$	$1.637\ Gy$

is greater. For example, the coalescence time interval gets reduced by $\sim 10^9$ years (see the column for 50 and $60M_\odot$ in Table I), and even more if there is a significant difference in the initial masses of the constituent black holes (see the column for 10 and $60M_\odot$ in Table I). Note though, that the magnitude of decrease in coalescence time interval also depends on the initial radius of the circular orbit.

It may be pertinent to mention that the black holes which are produced from stellar collapse may reside in regions of galaxies where they may also accrete other stellar matter, interstellar gas, dust etc.. During the accretion of such matter, characteristic electromagnetic radiation would be emitted. In the present era of "*multimessenger astronomy*," it should be possible to distinguish the effects of purely dark energy accretion on the gravitational wave spectrum, from the effects of stellar matter accretion, through these 'electromagnetic counterparts' of gravitational wave signals. Dark energy and dark matters, while getting accreted by black holes, should not emit any electromagnetic radiation. Besides, the existence of accretion disks around black holes accreting stellar matter leads to certain characteristic phenomena, which would be absent in case of spherical accretion of dark energy or dark matter.

In the context of our present work, it is more significant to note the timescale of the evolution of the binary from its formation to the merger. For black holes in stellar mass and few times higher than stellar mass ranges, and for sufficiently large initial separation, the coalescence time interval is $\sim 10^8$ to 10^9 years. Continuous supply of stellar matters in the same order of density for accretion by the black holes seems far less likely throughout this timescale. This is more obvious if the black holes are situated in the outer part of the galactic halo, which usually has very less density of stellar mass.

V. CONCLUSIONS

A variety of cosmological observations have revealed that the present Universe is undergoing a phase of accelerated expansion, and such observations lend support to dynamical dark energy models responsible for the present acceleration. The string theory inspired dilatonic scalar field model, chosen in this work, in which acceleration is driven by the scalar field kinetic energy [63], seems to be observationally consistent [64]. If dark energy exists in an accretable form, it is inevitable that the black holes existing in the present Universe would evolve by accreting it. This in turn, would have a natural imprint on the evolution of binaries constituted by the black holes, as we have shown in this work.

Specifically, we have studied the effect of growth of masses of black holes due to the spherical accretion of the chosen model of k-essence dark energy on several important parameters of binaries constituted by those black holes. We have investigated the effect of changing masses of the black holes on the evolution of the binaries and the average power of the emitted gravitational waves. We have found that accretion of the chosen model of k-essence dark energy leads to rapid circularization of binary orbits in comparison to the case of constancy of masses, i.e., without accretion. Further, in comparison with the constant-mass case, the average power of gravitational waves increases significantly faster due to the increase in masses of the black holes. Since the average power grows as $P_{\rm avg} \propto \mu^2 M^3$, a comparatively small amount of increase in masses due to accretion leads to a much larger increment of the average power of emitted gravitational waves within the concerned timescale. Finally, we have analyzed how the effect of the increase in masses of the black holes leads to the reduction in the coalescence time intervals of black hole binaries in the stellar mass range and above.

Our work establishes the fact that if dark energy is similar to scalar field models like the k-essence model considered here, then it would result in reduced coalescence time intervals of the binaries of black holes present in the current era of the Universe. The reduction in coalescence time intervals means increased rate of coalescences. However, the scenario of accretion of dark energy by supermassive black holes may be quite complicated. On the one hand, accretion of dark energy may itself be responsible for formation of supermassive black holes from primordial seeds leading to the present large mass contained in supermassive black holes [76]. On the other hand, continued accretion of dark energy at a very high rate during the present epoch could result in further rapid rise in the mass of supermassive black holes leading to other astrophysical and cosmological issues such as depletion in the local density of dark energy surrounding such black holes. This would consequently indeed inhibit further accretion at some stage. Moreover, the supermassive black holes, usually residing at the centers of the galaxies, have various stellar matters surrounding it in its vicinity and hence those get accreted by it. Various astrophysical phenomena take place around the supermassive black holes due to accretion of these ambient stellar matters e.g. formation of accretion disks, relativistic jets, photon spheres etc.. These astrophysical processes around a supermassive black hole may prohibit scalar field dark energy to get spherically accreted by the supermassive black hole. So, the topic of dark energy accretion by supermassive black holes is rather complex, and calls for further in-depth investigations.

A possible upshot of the effect of accretion of dark energy by black holes in binary formations is that if this effect is observationally detectable in the new era of gravitational wave astronomy, it can lead to independent constraints on the equation-of-state parameter of the dark energy model. Such observations on local candidates are associated with much less noise compared to certain other dark energy observations involving all-sky surveys such as cosmic microwave background and baryon acoustic oscillations. Observations with aLIGO and VIRGO detectors should be useful in this regard, as we have demonstrated the significance of the effect for the binaries of black

holes within mass-ranges, from which a considerable number of merging events have been detected by these detectors. Moreover, upcoming observations using the planned futuristic detector LISA, may also be able to investigate the imprint of dark energy accretion on coalescence time intervals for binaries of supermassive black holes formed during galaxy mergers or even extreme mass-ratio inspirals (EMRIs) also.

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