Slowly rotating and accelerating α'-corrected black holes in four and higher dimensions

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We consider the low-energy effective action of string theory at order α', including R^2 corrections to the Einstein-Hilbert gravitational action and nontrivial dilaton coupling. By means of a convenient field redefinition, we manage to express the theory in a frame that enables us to solve its field equations analytically and perturbatively in α' for a static spherically symmetric ansatz in an arbitrary number of dimensions. The set of solutions we obtain is compatible with asymptotically flat geometries exhibiting a regular event horizon at which the dilaton is well behaved. For the four-dimensional case, we also derive the stationary black hole configuration at first order in α' and in the slowly rotating approximation. This yields string theory modifications to the Kerr geometry, including terms of the form a, a^2, α', and $a\alpha'$. In addition, we obtain the first α' correction to the C metrics, which accommodates accelerating black holes. We work in the string frame and discuss the connection to the Einstein frame, for which rotating black holes have already been obtained in the literature.

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I. INTRODUCTION

Higher-curvature corrections to Einstein's general relativity (GR) are ubiquitous in any sensible approach to quantum gravity, and they are a solid prediction of string theory [1]. Even before the formulation of the latter, effective actions containing higher-order contractions of the Riemann tensor were known to emerge in quantum field theory on curved spacetime [2] and in the semiclassical approach to quantum gravity. Such actions are the natural generalization of the Einstein-Hilbert action, thus correcting GR in the UV regime. Also, from the mathematical point of view it was understood early on that higher-curvature terms were natural in higher dimensions [3–5] and, on general grounds, it is widely accepted that any attempt to formulate a sensible UV-complete theory will involve higher-curvature corrections in one way or another. In 1976, Stelle argued that gravitational actions that include terms quadratic in the curvature tensor are renormalizable [6]. This is due to the fact that nonlinear renormalization of the graviton and the ghost fields suffices to absorb the non-gauge-invariant divergences that might arise. Stelle explained how these and other divergences may be eliminated in a way that simplifies the renormalization procedure, even when matter fields are coupled. Nevertheless, renormalizability is not the only issue: the inclusion of quadratic-curvature terms in the gravitational action typically introduces massive local degrees of freedom, apart from the massless graviton of GR [7]. These extra modes organize themselves as a massive spin-2 excitation and a massive spin-0 excitation, yielding a total of eight local degrees of freedom. The massive spin-2 part of the field has negative energy, and this is the reason why it is usually asserted that, with the exception of a few remarkable cases [4,5,8,9], augmenting the Einstein-Hilbert action with a finite set of higher-curvature terms yields ghosts when the theory is expanded about maximally symmetric vacua. In the early 1980s, the observations of Ref. [7] motivated the search for ghost-free higher-curvature theories and consistent UV completions. Since then, actions containing higher-curvature terms were considered in the context of cosmology [10], black hole physics [11], and string theory [12]. In 1985, Zwiebach studied the compatibility between the presence of curvature-squared terms and the absence of ghost modes in the low-energy limit of string theory [12]. He argued that the so-called Einstein-Gauss-Bonnet (EGB) action was a good candidate for the string effective action as it yields a ghost-free nontrivial gravitational self-interacting theory in any dimension greater than four, $d > 4$. The EGB action is made from a dimensionally extended version of the quadratic Chern-Gauss-Bonnet topological invariant which, while being dynamically trivial in $d \leq 4$, does yield a UV correction to GR in $d > 4$ with a single massless spin-2 excitation and with field equations of second order. The latter property makes the EGB theory free of Ostrogradsky instabilities. Still, in Ref. [11] Boulware and Deser showed that the EGB model proposed in Ref. [12] as a stringy action contains, in addition to flat spacetime, a second nonperturbative anti–de Sitter (AdS) vacuum which turns out to be unstable due to the presence of ghosts. This is nothing but the fact that actions that are polynomials of degree k in contractions of the Riemann tensor generically yield k different vacua, many of them being artifacts of the truncation of the effective theory. In ref. [13] the authors noticed that the inclusion of the dilaton field in the EGB effective action suffices to remove the spurious (A)dS vacuum permitted in its absence. They also showed that the spherically symmetric static solutions to the dilatonic EGB theory might have a well-defined asymptotic behavior, being nontrivial, and being compatible with the existence of a regular event horizon at which the dilaton is well behaved.[1] This was later confirmed by explicit examples, and here we also provide a concrete realization of it.

Soon after Ref. [12], in a foundational paper of string theory [24], Gross and Witten finally proved that the gravitational field equations of string theory actually contain higher-curvature corrections to GR. More precisely, they derived the modifications of the classical gravitational equations for the type II string theory by studying tree-level gravitational scattering amplitudes, and they determined the effective gravitational action up to quartic order in the curvature tensor, which corresponds to order $\mathcal{O}(\alpha^3)$ string corrections. Unlike bosonic string theory, type II superstring theory in $d = 10$ dimensions does not contain quadratic corrections to GR, and the cubic ones can be set to zero by field redefinitions, although quadratic corrections can actually appear in Calabi-Yau compactification of the quartic actions, with the moduli playing the role of the couplings, cf. Refs. [25–27]. In contrast, quadratic corrections do appear in critical bosonic and heterotic string theories. They were studied in Refs. [28,29] by Metsaev and Tseytlin, who checked the equivalence of the string equations of motion and the σ-model Weyl invariance conditions at order $\mathcal{O}(\alpha)$. They obtained the functional dependence on the dilaton, graviton, and antisymmetric tensor. To do so, they first determined the $\mathcal{O}(\alpha)$ terms in the string effective action starting from the expressions for the three- and four-point string scattering amplitudes; then, they computed the two-loop β function in the world-sheet σ model. This resulted in an effective gravity action with quadratic-curvature (R^2) corrections coupled to the other massless fields of the theory; see also the important works in Refs. [30,31], and for modern developments on α' corrections in relation to T duality and double field theory see Refs. [32–42] and references therein and thereof.

In recent years, with the advent of the AdS/CFT correspondence and its ramifications, higher-curvature terms were reconsidered in the context of holography and the interest in them was revived. Probably the best-known example of this is the discussion of the higher-curvature terms in relation to the Kovtun-Starinets-Son (KSS) viscosity bound [43,44] which showed that, for a class of conformal field theories (CFTs) with a gravity dual with the EGB action, the shear viscosity to entropy density ratio could violate the conjectured KSS lower bound. This proved that the presence of higher-curvature terms could result in qualitatively new phenomena; see also Refs. [45,46]. Microcausality violation in the CFT was also studied in the same type of scenario [43], which was rapidly interpreted as evidence supporting the idea of a universal lower bound on the shear viscosity to entropy density ratio for all consistent theories. This triggered a long series of works devoted to checking the consistency conditions of effective theories with higher-curvature modifications. For example, in Ref. [47] the authors discussed causality conditions in R^2 theories, and they studied causality violation in holographic hydrodynamics focusing on the EGB theory as a working example. In the latter theory, the value of the only R^2 coupling constant is related to the difference between the two central charges of the dual four-dimensional CFT, and the authors of Ref. [47] showed that, when such a difference is sufficiently large, causality is violated. This problem was also studied in Ref. [48], where the author discussed the relation between causality constraints in the bulk theory and the condition of energy positivity in the dual CFT. He specifically argued that special care is needed when solving the classical equations of motion in the higher-curvature gravity theory, for which the study of causality problems may be subtle. Holography in the presence of EGB gravity actions was further studied in Ref. [15] and references thereof. The authors of Ref. [15] studied the problem in an arbitrary number of dimensions d and established a holographic dictionary that relates the couplings of the gravitational theory to the universal numbers in the correlators of the stress tensor of the dual CFT, cf. Ref. [49]. This allowed the authors to examine constraints on the gravitational couplings by demanding the

[1]Higher-curvature black holes were also studied in the context of thermodynamics [14] and many other subjects, like holography [15] and the weak-gravity conjecture [16], among others. For related early works on this subject, see Refs. [17–23].

consistency of the CFT, and this yielded a much more general set of causality constraints.

Both in the context of AdS/CFT and in other scenarios, the consistency conditions for higher-curvature theories were intensively studied over the last 15 years. This line of research has continued and a much more general picture of the set of consistency conditions has been obtained. Causality, locality, stability, hyperbolicity, and other aspects were revisited. In Ref. [50], it was shown how causality constrains the sign of the stringy R^4 corrections to the Einstein-Hilbert action, giving a general restriction on candidate theories of quantum gravity. In Ref. [51], a special type of pathology that the truncated EGB theory exhibits was studied. This is a phase transition driven by nonperturbative effects that might take place in gravitational theories whenever higher-curvature corrections with no extra fields are considered. In Ref. [52], Maldacena *et al.* studied causality constraints on corrections to the graviton three-point coupling. They considered higher-curvature corrections to the graviton vertex in a weakly coupled gravity theory and derived stringent causality constraints. By considering high-energy scattering processes, they noticed a potential causality violation that might occur whenever additional Lorentz-invariant structures are included in the graviton three-point vertex. They argued that such a violation could be cured by the addition of an infinite tower of extra massive higher-spin fields such as those predicted by string theory. This problem was later reconsidered by many authors, cf. Ref. [53].

Motivated by this renewed interest in higher-curvature gravity, in the last years there have been important developments in the subject, and many new higher-curvature models were proposed and studied. The list includes the quasitopological theories [54–56], the critical gravity theories in AdS [8,9], the so-called Einsteinian cubic gravity [57,58], and their generalizations [59,60]. Black holes were recently studied in all of these setups [55,61–63], as well as in string-theory-inspired scenarios [64–66]; see also Refs. [67–69] and references therein and thereof. Here we present and study analytic, static, spherically symmetric solutions to the α'-corrected gravity action in arbitrary dimension d and including a nonvanishing dilaton coupling. We consider the graviton-dilaton sector of the low-energy effective action of string theory with R^2 terms in a specific frame that enable us to solve the problem explicitly to order $\mathcal{O}(\alpha')$ in the entire spacetime. Our solutions manifestly show that the theory is compatible with static, spherically symmetric solutions that are asymptotically flat and exhibit a regular event horizon at which the dilaton is well behaved. The paper is organized as follows. In Sec. II we present the gravity theory in a convenient frame. We briefly discuss the field redefinition ambiguity to the relevant order, and we use it to solve the adequate ansatz. The field equations are written down and solved, and the black hole solution for $d = 4$ is presented. In Sec. III we study the black hole thermodynamics. This amounts to working out the Wald entropy formula which, as is usual in this type of setup, yields corrections to the Bekenstein-Hawking area law. The mass of the solution may then be inferred from the first law of black hole mechanics. In Sec. IV we perform a consistency check of the previous formulas by explicitly computing the black hole mass by means of the Iyer-Wald method for conserved charges, which shows perfect agreement. We also show the agreement with the Euclidean action approach. In Sec. V we generalize our result by introducing angular momentum in the slowly rotating approximation. We derive a stationary metric that represents stringy modifications to the Kerr geometry. In Sec. VI we obtain the α' correction to the C metric, which accommodates accelerating black holes. While we work in the string frame, in Sec. VII we discuss the frame transformation that maps our theory to the Einstein frame, including the higher-curvature corrections. In the latter frame, rotating solutions were already studied in the literature, and we discuss the precise relation between the two frames. In Sec. VIII we generalize the static solution by presenting the explicit form of the dilatonic black hole solution in arbitrary dimension d.

II. DILATONIC BLACK HOLE

We consider the low-energy effective action of string theory including α' corrections to the graviton-dilaton sector, namely [28,29],

$$I[g_{\mu\nu},\phi] = \int_M d^d x\sqrt{-g}e^{-2\phi}[R+4(\nabla\phi)^2+\alpha R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}+\mathcal{O}(\alpha^2)], \tag{1}$$

where we denote $\alpha = \frac{1}{8}\alpha'$. We are not considering the dependence on the B field here. Performing the field redefinition $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$, $\phi \to \phi + \delta\phi$ with

$$\delta\phi = -\frac{\alpha}{2}(R + 4(2d-5)\partial_\mu\phi\partial^\mu\phi), \tag{2}$$

$$\delta g_{\mu\nu} = -4\alpha(R_{\mu\nu} - 4\partial_\mu\phi\partial_\nu\phi + 4g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi), \tag{3}$$

one obtains the action in a frame that is convenient for the computation we want to undertake, namely,

$$I[g_{\mu\nu},\phi] = \int d^d x\sqrt{-g}e^{-2\phi}[R+4(\nabla\phi)^2 + \alpha(R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2 - 16(\partial_\mu\phi\partial^\mu\phi)^2) + \mathcal{O}(\alpha^2)], \tag{4}$$

up to six-derivative operators of order $\mathcal{O}(\alpha'^2)$, cf. Ref. [70]. As the R^2 terms take the form of the $4d$ Euler characteristic, the field equations of the theory in this frame are of second order in a explicit manner. Let us first consider the case in $d = 4$. The field equations derived from Eq. (4) are given by

$$G_{\mu\nu} + 4\partial_\mu\phi\partial_\nu\phi - 2g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi + 2S_{\mu\nu} - 2g_{\mu\nu}S^\rho{}_\rho + \alpha H_{\mu\nu} = 0, \tag{5}$$

$$R + 4\partial_\rho\phi\partial^\rho\phi + 4S^\mu{}_\mu + \alpha L_{\mathrm{GB}} - 32\alpha\Big(\nabla^\mu(\partial_\rho\phi\partial^\rho\phi)\partial_\mu\phi + (\partial_\rho\phi\partial^\rho\phi)S^\mu{}_\mu + \frac{1}{2}(\partial_\rho\phi\partial^\rho\phi)^2\Big) = 0, \tag{6}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the Einstein tensor, and where

$$S_{\rho\sigma} \equiv e^{2\phi}\nabla_\rho(e^{-2\phi}\nabla_\sigma\phi),$$
$$L_{\mathrm{GB}} \equiv R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2, \tag{7}$$

and

$$H_{\mu\nu} = S_{\mu\nu}R - 4S^\sigma{}_{(\mu}R_{\nu)\sigma} + 2S^\sigma{}_\sigma R_{\mu\nu} + 2S^{\sigma\lambda}R_{\mu\sigma\lambda\nu} - 8(\partial_\rho\phi\partial^\rho\phi)\partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}(2(\partial_\rho\phi\partial^\rho\phi)^2 - S^\sigma{}_\sigma R + 2S^\sigma{}_\lambda R^\lambda{}_\sigma). \tag{8}$$

The Lagrangian L_{GB} is the integrand of the four-dimensional Chern-Gauss-Bonnet topological invariant which, in the absence of the dilaton and in $d = 4$, yields the Euler characteristic; this is the EGB quadratic gravity Lagrangian.

We are interested in solving the equations above for a static spherically symmetric spacetime with nontrivial dilaton profile. (In Sec. V, we will generalize the solution to the stationary, nonstatic case.) In order to do so, we work perturbatively at order $\mathcal{O}(\alpha)$, and propose the ansatz

$$\phi(r) = \phi_0 + \alpha\phi_1(r), \tag{9}$$

$$ds^2 = -(1+\alpha N_1(r))^2\left(1 - \frac{\mu}{r} + \alpha f_1(r)\right)dt^2 + \frac{dr^2}{1 - \frac{\mu}{r} + \alpha f_1(r)} + r^2 d\Omega^2, \tag{10}$$

where $\phi_1(r)$, $N_1(r)$, and $f_1(r)$ are functions of the radial coordinate r to be determined, μ is an arbitrary constant, and $d\Omega^2$ is the constant-curvature metric on the unit sphere. The solution that we find in this way will be valid up to order $\mathcal{O}(\alpha)$. Plugging this ansatz into the field equations and expanding up to first order in α, we obtain a remarkably simple system of equations that lead to the following general solution:

$$\phi(r) = \phi_0 + \alpha\left(A + B\log\left(\frac{r-\mu}{r}\right) - \frac{2}{\mu r} - \frac{1}{r^2} - \frac{2\mu}{3r^3}\right), \tag{11}$$

with A and B being two arbitrary constants. The former constant appears merely as a shift of ϕ_0 which does not enter in the metric, and so it can be absorbed by redefining $\bar{\phi}_0 = \phi_0 + \alpha A$, which gives the value that the dilaton takes at infinity; notice that, at infinity, Eq. (11) goes like $\phi \simeq \bar{\phi}_0 + \mathcal{O}(1/r)$. Up to $\mathcal{O}(\alpha)$ terms, for the metric we find

$$g^{rr} = 1 - \frac{\mu}{r} + \alpha\left(-\frac{\mu B}{r}\log\left(\frac{r-\mu}{r}\right) + \frac{C}{r} + \frac{2}{r^2} + \frac{\mu}{r^3} - \frac{10}{3}\frac{\mu^2}{r^4}\right),$$
$$g_{tt} = \frac{\mu}{r} - 1 - \alpha\left(\frac{B(2r-3\mu)}{r}\log\left(\frac{r-\mu}{r}\right) + D + \frac{4}{r^2} + \frac{5\mu}{3r^3} + \frac{2\mu^2}{r^4} - \frac{\mu^2 D - \mu C + 2\mu^2 B + 8}{\mu r}\right),$$

where D and C are two other integration constants. The former can be eliminated by rescaling the time coordinate as $t \to t/(1+\alpha D)$.

If we define $r_+ = \mu + \alpha\mu_1$, we can easily find the α-corrected location of the event horizon by solving for μ_1 as a function of the integration constants. This amounts to demanding $g^{rr}(r_+) = g_{tt}(r_+) = 0$, which is actually required for the horizon to be regular. Expanding up to first order in α, this yields

$$\left(B\log\left(\frac{\mu}{\alpha\mu_1}\right) + \frac{\mu_1 + C}{\mu} - \frac{1}{3\mu^2}\right)\alpha + O(\alpha^2) = 0, \tag{12}$$

$$\left(B\log\left(\frac{\mu}{\alpha\mu_1}\right) - 2B + \frac{\mu_1 + C}{\mu} - \frac{1}{3\mu^2}\right)\alpha + O(\alpha^2) = 0, \tag{13}$$

from which we consequently obtain that $B = 0$. Therefore, the α'-corrected black hole configuration reads

$$g^{rr}(r) = 1 - \frac{\mu}{r} + \alpha\left(\frac{C}{r} + \frac{2}{r^2} + \frac{\mu}{r^3} - \frac{10}{3}\frac{\mu^2}{r^4}\right) + \mathcal{O}(\alpha^2), \tag{14}$$

$$g_{tt}(r) = \frac{\mu}{r} - 1 - \alpha\left(\frac{4}{r^2} + \frac{5\mu}{3r^3} + \frac{2\mu^2}{r^4} - \frac{8 - \mu C}{\mu r}\right) + \mathcal{O}(\alpha^2), \tag{15}$$

$$\phi(r) = \bar{\phi}_0 - \alpha\left(\frac{2}{\mu r} + \frac{1}{r^2} + \frac{2\mu}{3r^3}\right) + \mathcal{O}(\alpha^2), \tag{16}$$

and, up to $\mathcal{O}(\alpha)$ corrections, the location of the horizon is

$$r_+ = \mu + \alpha\left(\frac{1}{3\mu} - C\right). \tag{17}$$

The solution we have just derived is asymptotically flat, and it exhibits a smooth event horizon at r_+, where the dilaton remains finite:

$$\phi(r_+) = \bar{\phi}_0 - \frac{11}{3}\frac{\alpha}{r_+^2} + \mathcal{O}(\alpha^2), \qquad \phi(\infty) = \bar{\phi}_0 + \mathcal{O}(\alpha^2). \tag{18}$$

In the following sections we will analyze the physical properties of this solution, compute its conserved charges, and generalize it to $d \geq 4$ dimensions.

III. THERMODYNAMICS

The thermodynamics of higher-curvature black holes has been studied for a long time [14,71,72] and in a vast number of contexts. Here we focus on the properties of the black hole solution we just presented. The Wald formula gives the entropy as a Noether charge computed at the horizon. This is given by an integral on the horizon $\mathcal{H}$,

$$S = \frac{\beta}{4}\int_{\mathcal{H}} \sqrt{-g}\epsilon_{\mu\nu\rho\sigma} q^{\mu\nu} dx^\rho \wedge dx^\sigma, \tag{19}$$

in $d = 4$ spacetime dimensions, with β being the periodicity of the Euclidean time. The Noether prepotential associated with this charge is given by

$$q^{\mu\nu} \equiv -2(E^{\mu\nu\rho\sigma}\nabla_\rho \xi_\sigma + 2\xi_\rho \nabla_\sigma E^{\mu\nu\rho\sigma}) \tag{20}$$

and

$$E^{\mu\nu}{}_{\rho\sigma} \equiv \frac{\partial \mathcal{L}}{\partial R^{\rho\sigma}{}_{\mu\nu}}. \tag{21}$$

For the action (4), the tensor (21) and the Noether prepotential take the form

$$E^{\mu\nu}{}_{\rho\sigma} = \frac{1}{2}e^{-2\phi}\left(\delta^{\mu\nu}_{\rho\sigma} + \alpha\delta^{\mu\nu\mu_3\mu_4}_{\rho\sigma\nu_3\nu_4}R^{\nu_3\nu_4}{}_{\mu_3\mu_4}\right) + \mathcal{O}(\alpha^2), \tag{22}$$

$$q^{\mu\nu} = 2e^{-2\phi}T^{\mu\nu} + \alpha e^{-2\phi}(4T^{\mu\nu}R + 16T^{\sigma[\mu}R^{\nu]}{}_\sigma + 4T^{\rho\sigma}R^{\mu\nu}{}_{\rho\sigma}) + \mathcal{O}(\alpha^2), \tag{23}$$

respectively, where we have defined $T^{\rho\sigma} \equiv 4\xi^{[\rho}\nabla^{\sigma]}\phi - \nabla^{[\rho}\xi^{\sigma]}$. Evaluating the Wald entropy (19) for our solution, we obtain

$$S = 16\pi^2 e^{-2\bar{\phi}_0}\mu^2 - 32\pi^2\alpha e^{-2\bar{\phi}_0}(C\mu - 8) + \mathcal{O}(\alpha^2). \tag{24}$$

If we naturally identify

$$e^{-2\bar{\phi}_0} = \frac{1}{16\pi G}, \tag{25}$$

with G being the $4d$ Newton constant, the leading term in Eq. (24) reproduces the Bekenstein-Hawking entropy, while the order $\mathcal{O}(\alpha)$ terms yield corrections to it. More precisely, we find

$$S = \frac{\pi\mu^2}{G} + \frac{16\pi\alpha}{G}\left(1 - \frac{1}{8}C\mu\right) + \mathcal{O}(\alpha^2). \tag{26}$$

Notice that Eq. (26) depends on both μ and C. The dependence of C can be traced back to the fact that $\alpha(8 - C\mu)/\mu$ is the $\mathcal{O}(\alpha)$ correction to the parameter in front of the Newtonian piece $\sim 1/r$ in the component g_{tt} of the metric, cf. Eq. (15). Then, using Eq. (17), the entropy can also be written as

$$S = \frac{\pi r_+^2}{G} + \frac{46\pi\alpha}{3G} + \mathcal{O}(\alpha^2). \tag{27}$$

Notice that the potential term linear in r_+ [i.e., the one that could come from the term linear in μ in Eq. (26)] has canceled out. In fact, at order $\mathcal{O}(\alpha)$, by virtue of the field equations, the computation reduces to that of the full action evaluated on the undeformed GR solution $f_1 = N_1 = \phi_1 = 0$. This means that, at that order, the only correction to the area law $S = \frac{A}{4G}$ is given by a positive constant. On the same grounds, corrections of the form $\mathcal{O}(\alpha r_+^{d-4}/G)$ are expected in higher dimensions.

Next, let us compute the Hawking temperature. We can do this by resorting to the Euclidean formalism. However, it is convenient to first simplify the expressions a bit. We can write μ as a function of r_+ by simply inverting Eq. (17), which yields

$$g_{tt}(r) = -1 + \frac{1}{r}\left(r_+ + \frac{23\alpha}{3r_+}\right) - \alpha\left(\frac{4}{r^2} + \frac{5r_+}{3r^3} + \frac{2r_+^2}{r^4}\right) + \mathcal{O}(\alpha^2), \tag{28}$$

$$g^{rr}(r) = 1 - \frac{1}{r}\left(r_+ - \frac{\alpha}{3r_+}\right) + \alpha\left(\frac{2}{r^2} + \frac{r_+}{r^3} - \frac{10r_+^2}{3r^4}\right) + \mathcal{O}(\alpha^2), \tag{29}$$

$$\phi(r) = \bar{\phi}_0 - \alpha\left(\frac{2}{r_+ r} + \frac{1}{r^2} + \frac{2r_+}{3r^3}\right) + \mathcal{O}(\alpha^2). \tag{30}$$

This gives the periodicity condition for the real section of the Euclidean geometry to be regular at $r = r_+$, namely,

$$\beta = 4\pi r_+ + \frac{44\pi\alpha}{3r_+} + \mathcal{O}(\alpha^2), \tag{31}$$

which results in the black hole temperature

$$T = \frac{1}{4\pi r_+}\left(1 - \frac{11}{3}\frac{\alpha}{r_+^2}\right) + \mathcal{O}(\alpha^2). \tag{32}$$

This corrects the Hawking formula for GR at scales $r_+ \simeq \alpha^{1/2}$. This result, together with the expression (27) for the entropy, yields the first-law-type relation

$$\delta E \equiv T\delta S = \delta\left(\frac{r_+}{2G} + \frac{11\alpha}{6Gr_+}\right) + \mathcal{O}(\alpha^2), \tag{33}$$

from which, up to subleading orders in α, we can obtain the gravitational energy

$$E - E^{(0)} = \frac{r_+}{2G}\left(1 + \frac{11}{3}\frac{\alpha}{r_+^2}\right), \tag{34}$$

with E_0 being an integration constant that corresponds to the energy of the reference background. Below, we will confirm this result by rederiving the gravitational energy using the Iyer-Wald method for computing Noether charges. It is also worth noticing that, if we insist on extrapolating the formulas above for small values of r_+, which is not well justified as higher-order terms are expected to be relevant in that regime, then the formula obtained for the specific heat changes its sign and becomes positive within the range $\frac{11}{3}\alpha < r_+^2 < 11\alpha$; the black hole temperature (32) vanishes at the lower bound $r_+^2 = \frac{11}{3}\alpha$.

IV. CONSERVED CHARGES

In order to compute the gravitational energy of the solution, we have to supplement the bulk action with the appropriate boundary terms. In the case of the higher-curvature action (4), the boundary term to be added reads

$$I_{BT} \equiv \int_{\partial M} d^3x\sqrt{-h}e^{-2\phi}\left[2K + 4\alpha\delta^{\mu_1\mu_2\mu_3}_{\nu_1\nu_2\nu_3}K^{\nu_1}_{\mu_1}\left(\frac{1}{2}\mathcal{R}^{\nu_2\nu_3}{}_{\mu_2\mu_3} - \frac{1}{3}K^{\nu_2}_{\mu_2}K^{\nu_3}_{\mu_3}\right)\right] \equiv \int_{\partial M} d^3x\sqrt{-h}\mathcal{B}, \tag{35}$$

where K is the trace of the extrinsic curvature K^{μ}_{ν}, and $\mathcal{R}^{\mu\nu}{}_{\rho\sigma}$ and $h_{\mu\nu}$ are the intrinsic curvature and induced metric on ∂M, respectively, cf. Ref. [73]. The contribution (35) renders the variational principle well posed. Then, the energy of the spacetime, which corresponds to the black hole mass, is given by the following integral on the sphere at infinity, S^2_∞:

$$M = \int_{S^2_\infty}(\boldsymbol{Q}[\boldsymbol{t}] - \boldsymbol{t}\cdot\boldsymbol{B}), \tag{36}$$

where $\boldsymbol{Q}[\boldsymbol{t}]$ is the Hodge dual of the Noether prepotential for the killing vector $\boldsymbol{t} = \partial_t$ and

$$\boldsymbol{B} = \frac{1}{3!}\mathcal{B}n^\sigma\sqrt{-g}\epsilon_{\mu\nu\rho\sigma}dx^\mu \wedge dx^\nu \wedge dx^\rho. \tag{37}$$

In flat spacetime, the trace of the extrinsic curvature is $K^{(0)} = \frac{2}{r}$ which gives a divergent piece in the action principle as the volume element contributes with r^2. To obtain a finite action principle and a finite energy definition, we have to subtract the extrinsic curvature of flat spacetime to each piece of the extrinsic curvature appearing in the formulas above. In other words, we have to define

$$\bar{K}_{\mu\nu} \equiv K_{\mu\nu} - K^{(0)}_{\mu\nu},$$

using flat space as a background reference; this corresponds to setting $E^{(0)} = 0$ for Minkowski spacetime. According to this, the energy content of the spacetime, as defined in Eq. (36), precisely gives

$$M = \frac{r_+}{2G}\left(1 + \frac{11}{3}\frac{\alpha}{r_+^2}\right), \tag{38}$$

which agrees with Eq. (34).

Another cross-check for this result can be done by means of the Euclidean action formalism. In the saddle-point approximation, the on-shell Euclidean action gives the partition function, namely,

$$\log Z \simeq I^E + I^E_{BT}, \tag{39}$$

where the superscript E stands for Euclidean. It is worth emphasizing that, at order $\mathcal{O}(\alpha)$, the computation of the Euclidean action reduces to the evaluation of the full action $I^E + I^E_{BT}$ on the undeformed GR solution. Therefore, the energy of the configuration can be simply derived from Eq. (39) by computing

$$\bar{E} = -\frac{\partial \log Z}{\partial\beta}. \tag{40}$$

The on-shell action computed with $\bar{K}_{\mu\nu}$ for the configuration (28)–(30) with the Euclidean time periodicity (31) turns out to be finite, and it reads

$$I^E + I^E_{BT} = -\frac{\pi r_+^2}{G} - \frac{10\alpha\pi}{3G}. \tag{41}$$

From this expression, we easily find

$$\bar{E} = \frac{r_+}{2G}\left(1 + \frac{11}{3}\frac{\alpha}{r_+^2}\right), \tag{42}$$

which, again, exactly reproduces Eq. (34) at the right order. This results in an $\mathcal{O}(\alpha)$, r_+-dependent correction to the GR Smarr formula, namely,

$$TS - \frac{1}{2}\bar{E} = \frac{2\alpha}{Gr_+}. \tag{43}$$

At order $\mathcal{O}(\alpha)$ this is equivalent to an additive constant in the entropy.

V. ADDING ANGULAR MOMENTUM

The black hole solution (10) can be generalized to the stationary nonstatic case, and the analytic expression in the slowly rotating approximation can also be found following a similar perturbative method as before. At first order in α and including the rotation parameter in linear and quadratic terms as well as in terms of the form $a\alpha$, the solutions reads

$$\begin{aligned} ds^2 = &-\left(1 - \frac{\mu}{r} + \frac{\mu a^2 \cos^2\theta}{r^3} + \alpha f_1(r)\right)dt^2 \\ &+ 2a\left(-\frac{\mu \sin^2\theta}{r} + \alpha h_{t\varphi}(r,\theta)\right)dt d\varphi \\ &+ \left(\frac{1}{1-\frac{\mu}{r}+\alpha g_1(r)} - \frac{((\mu - r)\cos^2\theta + 2r)a^2}{(r-\mu)^2 r}\right)dr^2 \\ &+ (r^2 + a^2\cos^2\theta)d\theta^2 \\ &+ \left((r^2 + a^2)\sin^2\theta + \frac{a^2\mu \sin^4\theta}{r}\right)d\varphi^2, \end{aligned} \tag{44}$$

with

$$f_1(r) = \frac{2\mu^2}{r^4} + \frac{5\mu}{3r^3} + \frac{4}{r^2} - \frac{8}{\mu r}, \tag{45}$$

$$g_1(r) = -\frac{40}{3r^4} + \frac{\mu}{r^3} + \frac{2}{r^2}, \tag{46}$$

$$h_{t\varphi}(r) = \sin^2\theta\left(\frac{\hat{C}}{r} + \frac{2\mu^2 + 3\mu r + 6r^2}{r^4}\right), \tag{47}$$

and with $\hat{C}$ being a new integration constant that, at this order, comes to renormalize the angular momentum; see Eq. (51) below. The scalar configuration is

$$\phi(r) = \phi_0 - \alpha\left(\frac{2\mu}{3r^3} + \frac{1}{r^2} + \frac{2}{\mu r}\right). \tag{48}$$

One can verify that, expanding in both the Gauss-Bonnet coupling α and the rotation parameter a, all of the field equations are solved at the right order, namely,

$$E_{\mu\nu} = \mathcal{O}(\alpha a^2, \alpha^2). \tag{49}$$

The angular momentum can be computed by using the Wald formalism, which yields a form

$$J = -\int_{S^2_\infty} \boldsymbol{Q}[\partial_\varphi], \tag{50}$$

with $\boldsymbol{Q}[\partial_\varphi]$ representing the Hodge dual of the Noether prepotential for the Killing vector ∂_φ. The angular momentum of the spacetime is given by

$$J = \frac{a\mu}{2G}\left(1 - \frac{\alpha\hat{C}}{\mu}\right). \tag{51}$$

The solution (44)–(47) gives a string theory modification to Kerr geometry. In particular, we see order $\mathcal{O}(a\alpha)$ modifications to the off-diagonal term in the Boyer-Lindquist coordinates. This will result in deviations from the GR prediction of the Lense-Thirring precession. It will also induce modifications to the spheroidal shape of the shadow of a rotating black hole; see Ref. [74] and references thereof.

VI. ACCELERATING BLACK HOLES

Let us consider the following ansatz for the metric and the dilaton:

$$ds^2 = \frac{\Omega(x,y)}{A^2(x+y)^2}\left(-F(y)dt^2 + \frac{dy^2}{F(y)} + \frac{dx^2}{G(x)} + G(x)d\varphi^2\right), \tag{52}$$

$$\phi = \phi(x,y). \tag{53}$$

assuming the expansion $\phi = \phi_0(x,y) + \alpha\phi_1(x,y) + \mathcal{O}(\alpha^2)$, $F = F_0(y) + \alpha F_1(y) + \mathcal{O}(\alpha^2)$, $G = G_0(x) + \alpha G_1(x) + \mathcal{O}(\alpha^2)$, and $\Omega(x,y) = 1 + \alpha\omega(x,y) + \mathcal{O}(\alpha^2)$. In GR, the ansatz (52) leads to the C metric which accommodates accelerating black holes (see Ref. [75] for a modern interpretation as well as a historical review), even in the presence of minimally coupled, self-interacting scalar fields [76]. Here A, stands for the acceleration and $\Omega(x,y) = 1$ in GR in vacuum.

To the lowest order in the string tension α, the Einstein equations lead to

$$\begin{aligned} &F(y) = F_0(y) = f_3 y^3 + f_2 y^2 + f_1 y + f_0 \quad \text{and} \\ &G(x) = G_0(x) = f_3 x^3 - f_2 x^2 + f_1 x - f_0, \end{aligned} \tag{54}$$

fulfilling $G(\xi) = -F(-\xi)$. Here, f_i with $i = \{0, \ldots, 3\}$ are integration constants. The quadratic, linear, or f_0 term in the polynomials (54) can be removed by a simultaneous, constant shift of the independent variables (x, y), maintaining the form of the metric (52). For future purposes, it is better to keep all of the f_i as nonvanishing at the moment.

The field equations of the α'-corrected theory (4) at linear order in α are solved by

$$F_1(y) = d_3y^3 + d_2y^2 + d_1y + d_0, \tag{55}$$

$$G_1(x) = f_3h_1x^3 + 3f_3h_2x^2 + \frac{(3d_3f_1 - 3f_1f_3h_1 - 6f_2f_3h_2 + 3d_1f_3 - 2d_2f_2)}{3f_3}x \tag{56}$$

$$-\frac{(-6f_0f_3h_1 - 3f_1f_3h_2 + 3d_0f_3 - d_2f_1 + 6d_3f_0)}{3f_3}, \tag{57}$$

$$\omega(x,y) = 2\phi_1(x,y) + \frac{3f_3j_1x + 3(2f_3h_1 + f_3j_1 - 2d_3)y - 6f_3h_2 - 2d_2}{3(x+y)f_3}, \tag{58}$$

leading to the following inhomogeneous partial differential equation for $\phi_1(x,y)$:

$$0 = (x+y)\left(G_0(x)\frac{\partial^2\phi_1}{\partial x^2} + F_0(y)\frac{\partial^2\phi_1}{\partial y^2}\right) + (f_3x^3 + 3f_3x^2y - 2f_2xy - f_1x + f_1y + 2f_0)\frac{\partial\phi_1}{\partial x} \tag{59}$$

$$+ (f_3y^3 + 3f_3xy^2 + 2f_2xy + f_1x - f_1y - 2f_0)\frac{\partial\phi_1}{\partial y} - 6A^2f_3^2(x+y)^5. \tag{60}$$

Here the constants (d_i, f_j, h_k, j_l) are new integration constants that emerge from the integration of the field equations at linear order in α. Even though Eq. (60) seems not to admit an analytic solution, it can be solved as a power series in the acceleration A, around $A = 0$. In order to be able to take the limit $A = 0$ in Eq. (52), it is useful to perform the change of coordinates (see Chapter 14 of Ref. [75])

$$x = -\cos\theta, \qquad y = \frac{1}{Ar}, \qquad t = A\tau, \tag{61}$$

and choosing

$$f_2 = -f_0 = 1 \quad \text{and} \quad f_3 = -f_1 = -2mA \tag{62}$$

leads to the following parametrization for the C metric in GR:

$$ds_0^2 = \frac{1}{(1 - Ar\cos\theta)^2}\left(-Q_0(r)d\tau^2 + \frac{dr^2}{Q_0(r)} + \frac{r^2d\theta^2}{P_0(\theta)} + P_0(\theta)r^2\sin^2\theta d\varphi^2\right), \tag{63}$$

with

$$Q_0(r) = \left(1 - \frac{2m}{r}\right)(1 - A^2r^2), \tag{64}$$

$$P_0(\theta) = 1 - 2mA\cos\theta. \tag{65}$$

In terms of (r, θ), and choosing the constants f' as in Eq. (62), Eq. (60) is integrated order by order in the acceleration A. For such a purpose, it is convenient to choose

$$\phi_1(r,\theta) = (1 - Ar\cos\theta)H(r,\theta), \tag{66}$$

with

$$H(r,\theta) = \sum_{i=0} H_i(r,\theta)A^i,$$

which leads to the following functions at the lowest orders:

$$H_0(r,\theta) = -\frac{4m}{3r^3} - \frac{1}{r^2} - \frac{1}{mr}, \tag{67}$$

$$H_1(r,\theta) = \left(\frac{26m}{3r^2} - \frac{1}{r}\right)\cos\theta, \tag{68}$$

$$H_2(r,\theta) = \frac{2m(2\sin^2\theta - 23)}{3r}. \tag{69}$$

Other solutions are possible, but they lead to logarithmic or divergent behavior for the dilaton as $r \to \infty$.

In order to clarify the meaning of the plethora of integration constants that remain arbitrary in the metric functions, it is useful to reconstruct the full, corrected spacetime (52) in (r, θ) coordinates. The change of coordinates (61) induces the presence of A^{-1} terms in the metric coming from the terms (55)–(58) written in terms of (r, θ), which are removed by setting $d_3 = 0$. Imposing the absence of divergences at $\theta = 0$ and $\theta = \pi$ on the metric functions suffices to fix all of the remaining integration constants except for j_1, leading to $d_2 = d_1 = d_0 = h_1 = h_2 = 0$, which in consequence leads to vanishing corrections of the functions F and G, namely,

$$F_1(r,\theta) = 0, \qquad G_1(r,\theta) = 0, \tag{70}$$

and to a conformal factor $\omega(r,\theta)$ given by

$$\omega(r,\theta) = 2\phi_1(r,\theta), \tag{71}$$

where we have also set $j_1 = 0$ since a nonvanishing value of j_1 can be absorbed into the dilaton's additive, arbitrary constant ϕ_0.

Putting all of these ingredients together leads to the corrected metric, which is given by

$$ds^2 = \frac{1+2\alpha\phi_1(r,\theta)}{(1-Ar\cos\theta)^2}\left(-Q_0(r)d\tau^2 + \frac{dr^2}{Q_0(r)} + \frac{r^2d\theta^2}{P_0(\theta)} + P_0(\theta)r^2\sin^2\theta d\varphi^2\right). \tag{72}$$

Here $\phi_1(r,\theta)$ is given by Eq. (66) and $Q_0(r)$ and $P_0(\theta)$ are given by Eqs. (64) and (65), respectively. One can check that the metric (72), with $\phi_1(r,\theta)$ in Eq. (66), solves the field equations of the theory (4), disregarding terms of the forms $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha A^3)$, i.e., when evaluated on the corrected C metric, the field equations vanish up to

$$E_{\mu\nu} = \mathcal{O}(\alpha^2) + \mathcal{O}(\alpha A^3). \tag{73}$$

It is very interesting to notice that the regularity conditions lead us to move the whole effect of the α correction to the conformal factor. The solution can be found to higher orders in the acceleration, by performing the integration of the partial differential equation (60), at the desired order in A, after moving to (τ, r, θ) coordinates via Eq. (61), in such a manner that the limit of vanishing acceleration is regular.

VII. MAPPING TO THE EINSTEIN FRAME

Recently, in Ref. [66] the authors constructed the dimensional reduction of the heterotic string on a flat torus, to dimension four, and constructed rotating solutions, perturbatively in the rotation parameter, including the first α' correction, in the Einstein frame. It is interesting to compare the setup we consider here, defined by the action (4), with that of Ref. [66], where disregarding the contribution of the $B_{\mu\nu}$ field leads to an action of the form

$$I[g'_{\mu\nu},\tilde{\phi}] = \int d^4x\sqrt{-g'}\left(R' - \frac{1}{4}\nabla_\mu\tilde{\phi}\nabla_\nu\tilde{\phi}g'^{\mu\nu} + \alpha e^{-\tilde{\phi}}(R'^{\mu\nu}{}_{\rho\sigma}R'^{\rho\sigma}{}_{\mu\nu} - 4R'^{\nu}{}_{\sigma}R'^{\sigma}{}_{\nu} + R'^2)\right). \tag{74}$$

Considering a Weyl transformation of the form

$$g_{\mu\nu} \mapsto g'_{\mu\nu} = e^{\Phi}g_{\mu\nu}, \tag{75}$$

where Φ is some scalar function on the spacetime, the transformations of the quadratic scalars constructed with the Riemann tensor are given by

$$R'^{\mu\nu}{}_{\rho\sigma}R'^{\rho\sigma}{}_{\mu\nu} = e^{-2\Phi}\Big[R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu} - 4R^{\nu}{}_{\sigma}\nabla_\nu\Phi^\sigma + 2R^{\nu}{}_{\rho}\Phi^\rho\Phi_\nu - R\Phi^\lambda\Phi_\lambda + D_2\nabla_\sigma\Phi^\nu\nabla_\nu\Phi^\sigma + (\Box\Phi)^2 - D_2\nabla_\sigma\Phi_\nu\Phi^\sigma\Phi^\nu + D_2\Box\Phi\Phi^\lambda\Phi_\lambda + \frac{1}{8}D_2D_1(\Phi^\lambda\Phi_\lambda)^2\Big],$$

$$R'^{\nu}{}_{\sigma}R'^{\sigma}{}_{\nu} = e^{-2\Phi}\Big[R^{\nu}{}_{\sigma}R^{\sigma}{}_{\nu} - D_2R^{\nu\sigma}\nabla_\nu\Phi_\sigma - R\Box\Phi + \frac{1}{2}D_2R^{\nu\sigma}\Phi_\nu\Phi_\sigma - \frac{1}{2}D_2R\Phi^\lambda\Phi_\lambda + \frac{1}{4}D_2^2\nabla_\sigma\Phi^\nu\nabla_\nu\Phi^\sigma + \frac{1}{4}(3D-4)(\Box\Phi)^2 + \frac{1}{16}D_2^2D_1(\Phi^\lambda\Phi_\lambda)^2 - \frac{1}{4}D_2^2\nabla_\sigma\Phi_\nu\Phi^\sigma\Phi^\nu + \frac{1}{4}D_2(2D-3)\Box\Phi\Phi^\lambda\Phi_\lambda\Big],$$

$$R'^2 = e^{-2\Phi}\Big[R^2 - 2D_1R\Box\Phi - \frac{1}{2}D_1D_2R(\partial\Phi)^2 + D_1^2(\Box\Phi)^2 + \frac{1}{2}D_1^2D_2\Box\Phi(\partial\Phi)^2 + \frac{1}{16}D_1^2D_2^2(\partial\Phi)^4\Big],$$

where $\Phi_\lambda = \nabla_\lambda\Phi$ and $D_p \coloneqq (D-p)$. These expressions lead to the following transformation of the Gauss-Bonnet density, which we had actually worked out in arbitrary dimension D:

$$\begin{aligned} R'^{\mu\nu}{}_{\rho\sigma}R'^{\rho\sigma}{}_{\mu\nu} - 4R'^{\nu}{}_{\sigma}R'^{\sigma}{}_{\nu} + R'^2 &= e^{-2\Phi}\Big[R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu} - 4R^{\nu}{}_{\sigma}R^{\sigma}{}_{\nu} + R^2 + 4D_3G^{\nu\sigma}\nabla_\nu\Phi_\sigma - 2D_3R^{\nu\sigma}\Phi_\nu\Phi_\sigma - \frac{1}{2}D_4D_3R(\partial\Phi)^2 - D_3D_2\nabla_\sigma\Phi^\nu\nabla_\nu\Phi^\sigma \\ &\quad + D_3D_2\nabla_\sigma\Phi_\nu\Phi^\sigma\Phi^\nu + D_3D_2(\Box\Phi)^2 + \frac{1}{2}D_3^2D_2\Box\Phi(\partial\Phi)^2 + \frac{1}{16}D_4D_3D_2D_1(\partial\Phi)^4\Big], \\ &= e^{-2\Phi}(\mathcal{G}+P), \end{aligned} \tag{76}$$

where

$$\mathcal{G} = R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu} - 4R^{\nu}{}_{\sigma}R^{\sigma}{}_{\nu} + R^2 \tag{77}$$

and $\mathcal{P}$ stands for the remaining terms. Replacing these expressions in Eq. (74), choosing the scalar Φ as

$$\Phi = -\frac{4}{D-2}\phi, \tag{78}$$

identifying the scalar field in Eq. (74) $\tilde{\phi}$ as

$$\tilde{\phi} = \frac{4}{D-2}\phi, \tag{79}$$

and setting $D = 4$ leads to

$$\begin{aligned} I[g'_{\mu\nu} = e^{-2\phi}g_{\mu\nu}, \tilde{\phi} = 2\phi] &= \int d^4x\sqrt{-g}e^{-2\phi}[R + 4(\partial\phi)^2 + \alpha(\mathcal{G} - 32(\partial\phi)^4 \\ &\quad - 16G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 24\Box\phi(\partial\phi)^2)], \end{aligned} \tag{80}$$

where we have disregarded boundary terms. Notice that this action belongs to the family of the most general α'-corrected string theory actions, which after field redefinitions lead to second-order field equations, since each of the derivative terms for the scalar sector belongs to the Horndeski family [77]. Indeed, upon comparison with Eq. (2.6) of Ref. [70] one can read from the action (80) that the coefficients (λ, μ, ν) of Ref. [70] are given by $\lambda = -32$, $\mu = -16$, and $\nu = 24$, and they indeed fulfill the consistency constraint $\lambda + 2(\mu + \nu) + 16 = 0$. The relation among the relative coefficients of the higher-derivative operators of the scalar attests about the UV finiteness of the action. In consequence, using the results of Ref. [70], one can see that the action (74) of Ref. [66] in the Einstein frame and our action (4) in the string frame are related by a field redefinition, composed with a change of frame.

Therefore, our static and rotating solutions of Secs. II and IV correspond to a change of frame of the solutions found in Refs. [19,66], respectively, composed with a field redefinition. On the other hand, the solution corresponding to accelerating black holes we presented in Sec. VI is completely new.

VIII. d-DIMENSIONAL SOLUTION

The analytic solution that we constructed in $d = 4$ can be generalized to arbitrary dimension d in a similar manner, although, as we will see, the form of the general case is a bit more involved.

Consider the ansatz

$$ds^2 = -(1 + \alpha N_1(r))^2\left(1 - \left(\frac{\mu}{r}\right)^{d-3} + \alpha f_1(r)\right)dt^2 + \frac{dr^2}{1 - (\frac{\mu}{r})^{d-3} + \alpha f_1(r)} + r^2 d\Omega^2_{d-2},$$

where now $d\Omega^2_{d-2}$ is the constant-curvature metric on the unit $(d-2)$-sphere. By plugging this ansatz into the field equations for generic d, we find the following general solution:

$$N_1(r) = -\frac{C_3}{\mu^2 f_0(r)}\left(\frac{\mu}{r}\right)^{d-3} + \frac{(d-3)(d-2)}{(d-1)\mu^2}\left(\frac{\mu}{r}\right)^{d-1}\left[F(r) + \frac{1}{f_0(r)}\left(\left(\frac{\mu}{r}\right)^{d-3}d - 1\right)\right] + \frac{C_3}{(d-3)\mu^2}\log f_0(r) + \frac{C_2}{\mu^2},$$

$$f_1(r) = -\frac{(d-3)}{(d-1)\mu^2}\left(\frac{\mu}{r}\right)^{2d-4}[(d-3)(d-2)F(r) + 2(2d-3)] + \frac{C_1}{\mu^2}\left(\frac{\mu}{r}\right)^{d-3} - \frac{C_3}{\mu^2}\left(\frac{\mu}{r}\right)^{d-3}\log f_0(r),$$

and

$$\phi_1(r) = \frac{(d-3)(d-2)^2}{2(d-1)r^2}\left(\frac{\mu}{r}\right)^{d-3}(F(r) - 1) + \frac{(d-2)C_3}{2(d-3)\mu^2}\log f_0(r) + \frac{C_4}{\mu^2},$$

where $f_0(r) = 1 - (\frac{\mu}{r})^{d-3}$ and $F(r)$ is given in terms of the hypergeometric function,

$$F(r) = {}_2F_1\left(1, \frac{d-1}{d-3}, 2\frac{d-2}{d-3}, \left(\frac{\mu}{r}\right)^{d-3}\right).$$

C_1, C_2, C_3, and C_4 are integration constants, analogous to the constants A, B, C, and D of the $d = 4$ case; for example, one can identify $C_3 = \mu^2 B + 2$, $C_4 = \mu^2\bar{\phi}_0$, and so on. Some of these constants can be fixed as in the four-dimensional case, i.e., by rescaling the time coordinate t, shifting the zero mode of ϕ, neglecting $\mathcal{O}(\alpha^2)$ remnants, imposing a globally flat asymptotic behavior as $r \to \infty$, and requiring regularity at the event horizon. One can easily check that the four-dimensional solution studied in the previous sections is recovered when $d = 4$. To see this, it is convenient to consider the relation

$${}_2F_1(1, 3, 4, z) = -\frac{3}{2z^3}[z(z+2) + 2\log(1-z)], \tag{81}$$

with $z = 1 - f_0(r)$. The presence of logarithmic terms $\log f_0(r) = \log(1-z)$ in the functions $N_1(r)$, $f_1(r)$, and

$\phi_1(r)$ is related to the fact that the third argument of the hypergeometric function, $c = 2\frac{(d-2)}{(d-3)}$, turns out to be an integer for some dimensions ($d = 4, 5$). The logarithm, which in any case tends to zero at large r, disappears if one chooses C_3 appropriately.

IX. CONCLUSIONS

In summary, the solutions we presented in this paper describe static, spherically symmetric configurations in the graviton-dilaton sector of the d-dimensional low-energy stringy effective action (4). This includes square-curvature terms and a nonvanishing dilaton coupling. We used the freedom of field redefinitions to recast the action in a form that leads to second-order field equations, while still working in the string frame. The set of solutions includes asymptotically flat black holes with regular event horizons, which behave as thermodynamic objects, just like expected. As a working example, we first focused on the four-dimensional case, which is given by Eqs. (28)–(30). We derived the corrections to the thermodynamic variables introduced by the higher-curvature effects; we computed the Bekenstein-Hawking entropy (27), the Hawking temperature (32), and the mass formula (42) including the $\mathcal{O}(\alpha)$ effects. The computation of the Noether charges was shown to be in exact agreement with the first law of black hole mechanics as derived from the Wald entropy formula; the Euclidean action formalism also reproduces these results. We also obtained the correction to the C metric, which contains accelerating black holes. We have shown that regularity conditions imply that the whole modification is contained within the conformal factor of the spacetime. We also integrated the equations of motion for the stationary, nonstatic solution in the slowly rotating approximation. This yields stringy corrections to the Kerr geometry in four dimensions. Although, in contrast to the four-dimensional case, the field equations in arbitrary dimension d are more involved, we showed that in the static case they can still be solved explicitly in terms of hypergeometric functions. Static and rotating black holes in these theories have already been considered in the literature, when the theory is expressed in a different fashion, which is possible due to the freedom of field redefinition. Considering such freedom, we found the precise relation between our setup and the frames previously considered in the literature. In particular, we mapped our theory to the Einstein frame, including the higher-curvature corrections, and we showed the equivalence of our setup and that of Ref. [66], where rotating solutions were already presented. These solutions may serve as working examples to investigate higher-curvature stringy effects in a concrete setup.

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