Theories of gravity with nonminimal matter-curvature coupling and the de Sitter swampland conjectures

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We discuss, in the context of alternative theories of gravity with nonminimal coupling between matter and curvature, if inflationary solutions driven by a single scalar field can be reconciled with the swampland conjectures about the emergence of de Sitter solutions in string theory. We find that the slow-roll conditions are incompatible with the swampland conjectures for a fairly generic inflationary solution in such alternative theories of gravity.

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I. INTRODUCTION

Swampland conjectures have been proposed in order to distinguish consistent-looking low-energy effective field theories that do not admit a suitable ultraviolet completion in string theory—and, therefore, are said to be in the swampland—from those that lie in the string theory landscape. This is particularly relevant as it is notoriously difficult to obtain inflation from the fundamental fields that naturally arise in string theory.

This difficulty is somewhat surprising as in $N = 1$ supergravity—which, under certain conditions, can be thought to be a low-energy limit of string theory—inflation can be rather easily setup (see, for instance, Ref. [1]). In fact, alternative routes to obtain inflation in string theory have been discussed, but they tend to be more involved (see, for instance, Ref. [2]). It is relevant to point out that some phenomenologically viable string models, the ones with an intermediate grand unified theory energy scale, ask for a period of inflation for its full implementation [3].

The above-mentioned swampland conjectures are concretely a broad range of assumptions about the conditions required to admit local gauge symmetries and at least one Planck mass particle so to account for the weakness of gravity. One must also require that high-order terms in the effective action do not admit superluminal propagation (see Ref. [4] for a review). To our knowledge, there is no assumption, among this set of requirements, concerning the strong equivalence principle and implying that the gravity theory is necessarily general relativity.

Thus, it is natural to ask if the swampland conjectures hold for alternative theories of gravity in the context of which single-field inflation can take place. This is the case of gravity theories with nonminimal coupling between matter and curvature [5], where inflationary solutions can be found [6].

In order to be more specific about the conditions to be met, let us review the swampland conjectures relevant for our discussion. These conjectures impose some constraints on scalar fields emerging at low energy, generically denoted by ϕ [7,8], namely,

$$\frac{\Delta\phi}{M_{\rm P}} < c_1, \tag{1}$$

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$$M_{\rm P}\frac{|\partial_\phi V|}{V} > c_2, \tag{2}$$

where $\Delta\phi$ is the range of variation of the field, $M_{\rm P} \equiv M_{\rm Pl}/\sqrt{8\pi}$ is the reduced Planck's mass, $V(\phi)$ is the scalar field potential, c_1 and c_2 are constants of order 1, and we have used the notation $\partial_\phi V \equiv \partial V/\partial\phi$. It has been further argued that one should consider the more refined condition [9–11]

$$M_{\rm P}^2\frac{\partial^2_{\phi\phi}V}{V} < -c_3, \tag{3}$$

where c_3 is also a constant of order 1 and $\partial^2_{\phi\phi}V \equiv \partial^2 V/\partial\phi^2$.

Conditions given by Eqs. (2) and (3) can, in principle, be compared with the onset conditions of single-field inflation, which require that the parameters for the inflaton field [12],

$$\epsilon = \frac{M_{\rm P}^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2, \tag{4}$$

and

$$\eta = M_{\rm P}^2\frac{\partial^2_{\phi\phi}V}{V}, \tag{5}$$

satisfy the slow-roll requirements $\epsilon \ll 1$ and $|\eta| \ll 1$ at the onset of inflation, so that at the end of inflation $\epsilon \sim |\eta| \sim 1$.

These last requirements are consistent with constraints arising from the Cosmic Microwave Background (CMB) data [12] (see Ref. [13] for a detailed discussion),

$$\epsilon < 0.0044, \tag{6}$$

and

$$\eta = -0.015 \pm 0.006, \tag{7}$$

whose values, clearly, do not match the requirements on c_2 and c_3.

Actually, it can be shown that the incompatibility remains for whatever number of scalar fields drives inflation, provided their kinetic energy terms are canonical [13]. However, it is possible to reconcile the swampland conjectures with observations in the context of warm inflationary models [14,15] in the regime of strong dissipation for one [16,17] or more scalar fields [13].

In what follows we shall consider the situation in the context of a nonminimally coupled matter-curvature gravity theory in a single-field inflationary setup to be specified below. Thus, in the next section, we shall detail the alternative gravity theory in consideration and the associated inflationary model. We shall see that despite the similarities between the slow-roll parameters in the nonminimal coupled model and warm inflation, it is not possible, in the context of the former, to satisfy the swampland conjectures. Finally, in Sec. III, we present our conclusions.

II. THEORIES OF GRAVITY WITH NONMINIMAL MATTER-CURVATURE COUPLING

String theory itself does give origin to more complex gravitational theories than general relativity. Effective models of string theory exhibit corrections to general relativity that include, for instance, high-order curvature terms and curvature terms coupled with derivatives of the dilaton field (see, for instance, Refs. [18–22]).

However, independently from string and quantum gravity considerations, alternative theories of gravity are motivated as possible routes for addressing cosmological and astrophysical phenomena, such as the accelerated expansion of the Universe and the flattening of the rotation curves of galaxies, instead of resorting to dark energy and dark matter. Well studied models include $f(R)$ gravity [23,24], where the scalar curvature R in the Einstein–Hilbert action is replaced by a more general function, $f(R)$. A further possibility to generalize general relativity is to nonminimally couple matter and curvature, substituting the Einstein–Hilbert action by a more general form involving two functions of curvature, $f_1(R)$ and $f_2(R)$ [5]. The function $f_1(R)$ has a role analogous to $f(R)$ gravity theory, and the function $f_2(R)$ multiplies the matter Lagrangian density giving rise to a nonminimal coupling between matter and geometry. This possibility has been extensively studied in the context of dark matter [25], dark energy [26], inflation [6], energy density fluctuations [27], gravitational waves [28], and the cosmic virial theorem [29]. This model has also been examined with the Newton–Schrödinger approach [30,31].

Analytic extensions at $R = 0$ of functions $f_1(R)$, $f_2(R)$ were also considered and constraints to the resulting nonminimally coupled gravity model have been computed through perturbations to the perihelion precession of Mercury's orbit [32].

It turns out that nonminimally coupled gravity modifies the gravitational attraction by introducing both a fifth force of the Yukawa type and an extra force which depends on the spatial gradient of the Ricci scalar R. While the Yukawa force is typical also of $f(R)$ gravity, the existence of the extra force is specific to nonminimally coupled gravity [5,33], and it is an effect of the nonminimal coupling that induces a nonvanishing covariant derivative of the energy-momentum tensor. The arising Yukawa contribution can give origin to static solutions even though in the absence of pressure [31]. The Yukawa contribution was also examined in the context of experiments in deep ocean [34] and through the Cassini radiometric experiment [35].

A. Action, field equations and main features

In the present work we consider theories of gravity with an action functional of the form [5]

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}f_1(R) + f_2(R)\mathcal{L}\right], \qquad (8)$$

where $f_i(R)$ (with $i = 1, 2$) are functions of the Ricci scalar curvature R, $\mathcal{L}$ is the Lagrangian density of matter, and g is the metric determinant. The Einstein–Hilbert action of general relativity is recovered by taking $f_1(R) = R$ and $f_2(R) = 1$.

The variation of the action functional with respect to the metric $g_{\mu\nu}$ yields the field equations

$$\left(F_1 + \frac{2F_2\mathcal{L}}{M_{\rm P}^2}\right)G_{\mu\nu} = \frac{f_2}{M_{\rm P}^2}T_{\mu\nu} + \Delta_{\mu\nu}\left(F_1 + \frac{2F_2\mathcal{L}}{M_{\rm P}^2}\right) + \frac{1}{2}g_{\mu\nu}\left(f_1 - F_1 R - \frac{2F_2\mathcal{L}}{M_{\rm P}^2}R\right), \qquad (9)$$

where $G_{\mu\nu}$ is the Einstein tensor, $F_i = \partial f_i/\partial R$ $(i = 1, 2)$, and $\Delta_{\mu\nu} \equiv \nabla_\mu\nabla_\mu - g_{\mu\nu}\nabla_\alpha\nabla^\alpha$.

A relevant feature of nonminimally coupled gravity is that the energy-momentum tensor of matter is not covariantly conserved. Indeed, applying the Bianchi identities to Eq. (9), one obtains that

$$\nabla^\mu T_{\mu\nu} = \frac{F_2}{f_2}(\mathcal{L}g_{\mu\nu} - T_{\mu\nu})\nabla^\mu R, \qquad (10)$$

meaning that the nonminimal coupling cannot be "gauged away" by a convenient conformal transformation, being thus a distinctive feature of the nonminimal model discussed here.

B. Inflation in the nonminimally coupled theory

As widely discussed, within general relativity the swampland conjectures and the slow-roll conditions cannot be matched for single-field cold inflation if adiabatic scalar fluctuations are due to inflaton fluctuations (see, for instance, Refs. [36,37]). In fact, as pointed out in Ref. [37], there are scenarios where adiabatic cosmological perturbations are due to additional degrees of freedom, hence the consistency relation, which relates the scalar-tensor ratio with the slow-roll parameter ϵ, is no longer valid and, consequently, the second swampland condition, given by Eq. (2), does not hold. Some of these scenarios include the curvaton mechanism [38–41], the modulated decay scenario [42,43], or the cases when, at the end of inflation, the inflaton field acquires some critical value that depends on an additional scalar field [44,45]. However, throughout this work we shall assume that adiabatic curvature perturbations are due only to the inflaton.

Given that the incompatibility of the swampland conjectures with the observations has been an object of critique from the authors of Ref. [46] and that multifield inflationary models show no contradiction with the CMB features [47], it was logical to ask whether the swampland conjectures would hold for many fields. In fact, multifield cosmological models open interesting perspectives, for instance, for unification of dark matter and dark energy [48–51]. Two-field inflationary models were first considered in the context of $N = 1$ supergravity [52], and their dynamics were scrutinized in Refs. [53,54] for a broader class of models. In a broad context and in string theory, two-field inflationary with different mass scales and an interaction term were considered in Refs. [55–57]. In the context of the swampland conjectures, two-field inflationary models were examined in Refs. [58,59], where in Ref. [58] noncanonical kinetic energy terms have been considered. More recently, it has been shown that multifield inflation cannot be made compatible with the swampland conjectures without a significant amount of dissipation [13].

In this paper, we will consider single-field cold inflation within the nonminimally coupled theory of gravity defined by the action functional (8).

Assuming the Friedmann–Lemaître–Robertson–Walker metric,

$$ds^2 = -dt^2 + a^2(t)dx^2, \qquad (11)$$

from Eqs. (9) and (10) we obtain

$$H^2 = \frac{1}{6F}\left[\frac{2f_2\rho}{M_{\rm P}^2} - 6H\dot{F} - f_1 + FR\right], \qquad (12)$$

$$-2(2\dot{H} + 3H^2)F = \frac{2f_2 p}{M_{\rm P}^2} + 2\ddot{F} + 6H\dot{F} + f_1 - FR, \qquad (13)$$

$$\dot{\rho} + 3H(\rho + p) = -\frac{F_2}{f_2}(\rho + p)\dot{R}, \qquad (14)$$

where $a(t)$ is the scale factor, $H = \dot{a}/a$ is the Hubble parameter, an overdot denotes a derivative with respect to time t, and we have introduced the notation $F \equiv F_1 + 2F_2 p/M_{\rm P}^2$. In the above equations, we have also assumed that matter is represented by an homogeneous scalar field ϕ with a Lagrangian density $\mathcal{L} = p$, for which pressure and energy density are given by $p = \dot{\phi}^2/2 - V(\phi)$ and $\rho = \dot{\phi}^2/2 + V(\phi)$, respectively.

In what follows, we consider theories for which the pure gravitational sector of the action has the Einstein–Hilbert form; more specifically, we choose $f_1(R) = R$, implying $F_1 = 1$.

With this assumption, Eqs. (12) and (13) become

$$H^2 = \frac{1}{3(M_{\rm P}^4 - 4G^2)}[\rho f_2(M_{\rm P}^2 - G) - 3pf_2 G - 6H(M_{\rm P}^2 + 2G)\dot{G} - 6G\ddot{G}], \qquad (15)$$

and

$$\dot{H} = -\frac{f_2(\rho + p) + 2\ddot{G}}{2(M_P^2 + 2G)}, \tag{16}$$

where the notation $G \equiv pF_2$ was introduced.

Let us now choose

$$f_2(R) = 1 + \alpha\left(\frac{R}{6M_P^2}\right)^3, \tag{17}$$

where α is a positive dimensionless parameter that sets the scale of the nonminimal coupling, which is not necessarily the Planck scale. The cubic choice is the simplest power-law type function which renders nontrivial solutions for the Friedmann equation. In fact, for a linear monomial no real solutions for that equation are found, and for the quadratic scenario the standard solution in general relativity is surprisingly retrieved; as for cubic and higher monomials, the behavior is similar among each choice, up to small numerical factors [6]. Furthermore, we assume that inflation is quasiexponential, i.e., $V \gg \dot{\phi}^2$, implying $\rho \simeq -p \simeq V$.

Under these assumptions, and taking into account that

$$R = 6(\dot{H} + 2H^2), \tag{18}$$

we obtain

$$f_2 = 1 + \frac{\alpha}{M_P^6}(8H^6 + 12H^4\dot{H} + 6H^2\dot{H}^2 + \dot{H}^3), \tag{19}$$

$$G = \frac{\alpha}{M_P^6}\left(2H^4 p + 2H^2\dot{H}p + \frac{\dot{H}^2}{2}p\right), \tag{20}$$

$$\dot{G} = \frac{\alpha}{M_P^6}\left(8H^3\dot{H}p + 4H\dot{H}^2 p + 2H^4\dot{p} + 2H^2\dot{H}\,\dot{p} + \frac{\dot{H}^2}{2}\dot{p} + 2H^2\ddot{H}p + \dot{H}\,\ddot{H}\,p\right), \tag{21}$$

$$\ddot{G} = \frac{\alpha}{M_P^6}(24H^2\dot{H}^2 p + 4\dot{H}^3 p + 16H^3\dot{H}\,\dot{p} + 8H\dot{H}^2\dot{p} + 8H^3\ddot{H}p + 12H\dot{H}\,\ddot{H}\,p + 4H^2\ddot{H}\,\dot{p} + 2\dot{H}\,\ddot{H}\,\dot{p} + \ddot{H}^2 p + 2H^4\ddot{p} + 2H^2\dot{H}\,\ddot{p} + \frac{\dot{H}^2}{2}\ddot{p} + 2H^2H^{(3)}p + \dot{H}H^{(3)}p), \tag{22}$$

where $H^{(3)}$ denotes the third derivative of H with respect to time t.

Equation (15) can now be written as

$$3\left(M_P^2 - \frac{4\alpha H^4}{M_P^6}p\right)\left(M_P^2 + \frac{4\alpha H^4}{M_P^6}p\right) \simeq \rho\left(1 + \frac{8\alpha H^6}{M_P^6}\right)\left(M_P^2 - \frac{2\alpha H^4}{M_P^6}p\right) - 3p^2\left(1 + \frac{8\alpha H^6}{M_P^6}\right)\frac{2\alpha H^4}{M_P^6}, \tag{23}$$

or for $M_P^8 - 4\alpha H^4 V \neq 0$,

$$4\alpha V H^6 + 3M_P^8 H^2 - M_P^6 V \simeq 0, \tag{24}$$

where we have taken only the first two terms of f_2 and the first term of G in Eqs. (19) and (20), respectively; all the other terms in these equations, as well as the terms of $\dot{G}$ and $\ddot{G}$ in Eqs. (21) and (22), were neglected since they contain time derivatives of H and p.

For the sake of simplicity, let us now introduce the dimensionless variable

$$\bar{V} = \frac{V}{M_P^4}. \tag{25}$$

Then, Eq. (24) becomes

$$4\alpha\bar{V}H^6 + 3M_P^4 H^2 - M_P^6\bar{V} \simeq 0, \tag{26}$$

yielding the solution

$$H^2 \simeq \frac{-1 + \left(\sqrt{\alpha\bar{V}^3} + \sqrt{1 + \alpha\bar{V}^3}\right)^{2/3}}{2\sqrt{\alpha\bar{V}}\left(\sqrt{\alpha\bar{V}^3} + \sqrt{1 + \alpha\bar{V}^3}\right)^{1/3}} M_P^2. \tag{27}$$

Taking into account that the energy scale of inflation, defined as $E_{\mathtt{inf}} = V^{1/4}$, is much smaller than the reduced Planck mass, the right-hand side of Eq. (27) can be expanded in power series of $\bar{V} \ll 1$, yielding

$$H^2 \simeq \frac{M_P^2}{3}\bar{V}\left(1 - \frac{4}{27}\alpha\bar{V}^3\right), \tag{28}$$

where the first term on the right-hand side is the general relativity term, and the second one is a correction due the presence of a nonminimal coupling between matter and curvature.

Let us now turn to Eq. (16). It can be written as

$$2\left(M_P^2 + \frac{4\alpha H^4}{M_P^6}p\right)\dot{H} \simeq -\left(1 + \frac{8\alpha H^6}{M_P^6}\right)(\rho + p), \tag{29}$$

or, equivalently,

$$\dot{H} \simeq -\frac{(M_P^6 + 8\alpha H^6)\dot{\phi}^2}{2(M_P^8 - 4\alpha M_P^4 H^4 \bar{V})}, \tag{30}$$

where we have used $\rho + p = \dot{\phi}^2$ and taken only the first two terms of f_2 and the first term of G in Eqs. (19) and (20), respectively; all the other terms in these equations, as well as the terms of $\ddot{G}$ in Eq. (22), were neglected since they contain time derivatives of H and p.

Using H^2 given by Eq. (28) and expanding in power series of $\bar{V}$, Eq. (30) yields

$$\dot{H} \simeq -\frac{\dot{\phi}^2}{2M_P^2}\left(1 + \frac{20}{27}\alpha\bar{V}^3\right), \tag{31}$$

where, again, the first term on the right-hand side is the general relativity term, and the second is a correction due to the direct matter-curvature coupling.

Note that, if in Eq. (29) we had also taken for f_2 the term proportional to $\dot{H}$ and for $\ddot{G}$ the term proportional to $\dot{H}\,\dot{p}$, then we would have obtained in the right-hand side of Eq. (31) an extra term proportional to $\dot{\phi}^4\bar{V}^2$; however, since $\dot{\phi}^2/M_P^4 \ll \bar{V}$, this extra term can be neglected in comparison with the term proportional to $\dot{\phi}^2\bar{V}^3$.

Now, taking the time derivative of Eq. (28) and using Eq. (31) to eliminate $\dot{H}$, we obtain

$$\partial_\phi V \simeq -3H\dot{\phi}\left(1 + \frac{20}{27}\alpha\bar{V}^3\right)\left(1 - \frac{16}{27}\alpha\bar{V}^3\right)^{-1}, \tag{32}$$

or expanding in power series of $\bar{V}$,

$$\partial_\phi V \simeq -3H\dot{\phi}\left(1 + \frac{4}{3}\alpha\bar{V}^3\right). \tag{33}$$

Taking a second time derivative, Eq. (33) becomes

$$\partial^2_{\phi\phi} V \simeq 3H^2\left(1 + \frac{4}{3}\alpha\bar{V}^3\right)\left(-\frac{\dot{H}}{H^2} - \frac{\ddot{\phi}}{\dot{\phi}H}\right), \tag{34}$$

where we have neglected terms proportional to $\dot{\phi}^2\bar{V}^2$.

Using Eqs. (28), (31), (33), and (34), the quantities $\dot{H}/H^2$ and $\ddot{\phi}/(\dot{\phi}H)$ can be expressed as

$$\frac{\dot{H}}{H^2} \simeq \epsilon\left(1 - \frac{44}{27}\alpha\bar{V}^3\right), \tag{35}$$

and

$$\frac{\ddot{\phi}}{\dot{\phi}H} \simeq \epsilon\left(1 - \frac{44}{27}\alpha\bar{V}^3\right) - \eta\left(1 - \frac{32}{27}\alpha\bar{V}^3\right), \tag{36}$$

where the slow-roll parameters ϵ and η are given by Eqs. (4) and (5).

Now, taking into account that in the slow-roll inflationary regime $|\dot{H}|/H^2 \ll 1$ and $|\ddot{\phi}/(\dot{\phi}H)| \ll 1$, we conclude that

$$\epsilon \ll 1 + \frac{44}{27}\alpha\bar{V}^3 \quad \text{and} \quad |\eta| \ll 1 + \frac{32}{27}\alpha\bar{V}^3. \tag{37}$$

Since $\bar{V} \equiv V/M_P^4 \ll 1$ and assuming for naturalness that $\alpha = \mathcal{O}(1)$, we conclude that in the nonminimally coupled theory of gravity under consideration, the slow-roll parameters satisfy the conditions $\epsilon \ll 1$ and $|\eta| \ll 1$. Because these parameters are related to the constants c_2 and c_3 arising within the de Sitter swampland conjectures [see Eqs. (1) and (2)] through the relations

$$c_2^2 < 2\epsilon \quad \text{and} \quad c_3 < |\eta|, \tag{38}$$

we arrive at the conclusion that $c_2 \ll 1$ and $c_3 \ll 1$ during a quasiexponential inflationary period.

Thus, we clearly see that the swampland conjectures cannot be met for inflation in the context of theories of gravity with nonminimally coupled matter and curvature.

III. DISCUSSION AND CONCLUSIONS

In this work, in the context of the nonminimally coupled matter-curvature theory of gravity, we have considered the compatibility of the slow-roll conditions of inflation and the de Sitter swampland conjectures.

Despite the specificities of the nonminimally coupled theory and the fact that it can lead to an inflationary regime, which differs from the one in general relativity for the choice of the $f_2(R)$-function such as in Eq. (17), we find that under quite general conditions the requirements for the inflaton potential are still very much controlled by the slow-roll conditions. Even though it is conceivable that the free parameter α introduced in Eq. (17), which sets the impact of the nonminimal coupling, could be greater than 1, it cannot overcome the typical scale of the inflaton potential and its smallness in comparison with the Planck scale. Of course, for naturalness reasons, we assume that $\alpha = \mathcal{O}(1)$. Thus, we conclude that the de Sitter swampland conditions cannot be met in the context of gravity theories with nonminimal coupling between matter and curvature (recall that, as mentioned above, we are assuming that adiabatic curvature perturbations are only due to the inflaton scalar field). We expect these conclusions to hold for any number of inflaton fields as the nonminimal coupling function $f_2(R)$ acts on a linear matter Lagrangian, in contrast with $f(R, \mathcal{L})$ theories, for instance, where nonlinear matter Lagrangian terms could appear. Furthermore, we expect that the warm inflation scenario results [13,60] would hold within these alternative theories of gravity, as the effect of the nonminimal coupling, proportional to the slow-roll parameter in the equation of motion for the inflaton [6], is not expected to dominate over the warm inflation term.

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