

Evolution of charm-meson ratios in an expanding hadron gas

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We study the time evolution of the numbers of charm mesons after the kinetic freeze-out of the hadron gas produced by a central heavy-ion collision. The $\pi D^* \rightarrow \pi D^*$ reaction rates have t -channel singularities that give contributions inversely proportional to the thermal width of the D . The ratio of the D^0 and D^+ production rates can differ significantly from those predicted using the measured D^* branching fractions.

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I. INTRODUCTION

A remarkable aspect of the pseudoscalar and vector charm mesons D and D^* is that the $D^* - D$ mass splittings are all very close to the pion mass m_π . A previously unrecognized consequence is that there are charm-meson reactions with t -channel singularities. A t -channel singularity is a divergence in the rate for a reaction in which an unstable particle decays and one of its decay products is scattered. The singularity arises if the exchanged particle can be on shell. The existence of t -channel singularities was first pointed out by Peierls in 1961 in the context of πN^* scattering [1]. An example in the Standard Model of particle physics is the reaction $\mu^+ \mu^- \rightarrow W^+ e^- \bar{\nu}_e$, which can proceed through exchange of ν_μ . Melnikov and Serbo solved the divergence problem by taking into account the finite transverse sizes of the colliding μ^+ and μ^- beams [2]. A general discussion of t -channel singularities was presented by Grzadkowski *et al.* [3]. They pointed out that in a thermal medium, a t -channel singularity is regularized by the thermal width of the exchanged particle.

The simplest charm-meson reactions with a t -channel singularity are $\pi D^* \rightarrow \pi D^*$, which can proceed through the decay $D^* \rightarrow D\pi$ followed by the inverse decay $\pi D \rightarrow D^*$.

The t -channel singularity arises because the exchanged D can be on shell. The cross section diverges when the square of the center-of-mass energy s is in a narrow interval near the threshold. In the case of elastic scattering, the t -channel singularity region is

$$2M_*^2 - M^2 + 2m^2 < s < (M_*^2 - m^2)^2/M^2, \quad (1)$$

where M_* , M , and m are the masses of D^* , D , and π . The interval in \sqrt{s} is largest for the reaction $\pi^0 D^{*0} \rightarrow \pi^0 D^{*0}$, extending from 6.1 MeV to 8.1 MeV above the threshold $M_* + m = 2141.8$ MeV.

An obvious question is whether t -channel singularities in charm-meson reactions have any observable consequences. One possibility is that t -channel singularities could modify the ratios of charm mesons produced in a high-energy collision. We denote the numbers of D^0 , D^+ , D^{*0} , and D^{*+} by N_0 , N_+ , N_{*0} , and N_{*+} . The observed numbers of D^0 and D^+ can be predicted in terms of the numbers $(N_a)_0$ and $(N_{*a})_0$ before D^* decays and the measured branching fraction, $B_{+0} = (67.6 \pm 0.5)\%$ for $D^{*+} \rightarrow D^0 \pi^+$:

$$N_0 = (N_0)_0 + (N_{*0})_0 + B_{+0}(N_{*+})_0, \quad (2a)$$

$$N_+ = (N_+)_0 + 0 + (1 - B_{+0})(N_{*+})_0, \quad (2b)$$

where the last two terms come from D^{*0} and D^{*+} decays, respectively. These simple relations have been assumed in all previous analyses of charm-meson production. We will show that Eqs. (2a) and (2b) can be modified by t -channel singularities.

One situation in which the effects of t -channel singularities may be observable is in the production of charm mesons from ultrarelativistic heavy-ion collisions. A sufficiently

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central heavy-ion collision is believed to produce a region of quark-gluon plasma (QGP) that makes a transition to a hadron resonance gas (HRG). In the HRG, t -channel singularities are regularized by the thermal widths of the hadrons. The most divergent term in a reaction rate is replaced by a term inversely proportional to the thermal width of the exchanged hadron.

In this paper, we study the effects of t -channel singularities in charm-meson reactions in the expansion of the hadron gas after kinetic freeze-out. The restriction to after kinetic freeze-out offers many simplifications. The only hadrons in the HRG that remain are the most stable ones with lifetimes $c\tau \gg 1$ fm. The temperature is low enough that the important interactions can be described by an appropriate effective field theory. The only relevant charm mesons are D and D^* . The decays of D^* occur at much later times ($c\tau > 2000$ fm). The most abundant hadrons by far are pions, so the dominant contribution to the thermal width of a charm meson comes from the forward scattering of pions and is proportional to the decreasing pion number density.

II. CHARM MESONS

We denote the masses of D^+ , D^0 , D^{*+} , and D^{*0} by M_+ , M_0 , M_{*+} , and M_{*0} . We denote the masses of π^+ and π^0 by m_{π^+} and m_{π^0} (or collectively by m_π). The decay width of D^{*+} is measured: $\Gamma_{*+} = 83.4 \pm 1.8$ keV. The decay width of D^{*0} can be predicted using Lorentz invariance, chiral symmetry, isospin symmetry, and the measured D^* branching fractions: $\Gamma_{*0} = 55.4 \pm 1.5$ keV. The radiative decay rates of D^{*0} and D^{*+} are $\Gamma_{*0,\gamma} = 19.6 \pm 0.7$ keV and $\Gamma_{*+,\gamma} = 1.3 \pm 0.3$ keV.

Pions that are not ultrarelativistic can be described by chiral effective field theory (χ EFT). The self-interactions of pions in χ EFT at leading order (LO) are determined by the pion decay constant $f_\pi = 131.7$ MeV. Interactions between charm mesons and pions can be described using heavy hadron χ EFT (HH χ EFT) [4–6]. At LO, the interactions are determined by f_π and a dimensionless coupling constant $g_\pi = 0.5204 \pm 0.0059$.

In heavy-ion collisions, the QGP can have a substantial effect on the production of charm hadrons [7]. A simple model for particle production in heavy-ion collisions is the statistical hadronization model (SHM) [8]. According to the SHM, light hadrons are produced during hadronization, with the QGP and the HRG in thermal and chemical equilibrium. The application of the SHM to charm hadron production is sometimes called SHMc [9]. Charm quarks are primarily created in the hard collisions of the heavy ions. They are assumed to be in thermal equilibrium with the QGP, but they remain out of chemical equilibrium, since the charm-quark mass is much larger than the temperature T . Conservation of charm-quark number determines the charm-quark fugacity as a function of T . According to

SHMc, charm hadrons are produced during hadronization, with the charm quarks and charm hadrons in thermal equilibrium with the same fugacity g_c . As the HRG expands and cools, the total number of charm mesons remains essentially constant. After chemical freeze-out, the abundances of more stable charm hadrons are increased by the decays of higher charm-hadron resonances.

For Pb-Pb collisions at nucleon-nucleon center-of-mass energy $\sqrt{s_{NN}} = 5.02$ TeV, the charm-quark fugacity at hadronization has been determined to be $g_c = 29.6 \pm 5.2$ [9]. The predictions of SHMc for the multiplicities of charm hadrons are given in Ref. [9]. The predicted multiplicities dN/dy at midrapidity ($|y| < \frac{1}{2}$) from collisions in the centrality range 0%–10% are 6.42 for D^0 , 2.84 for D^+ , and 2.52 for D^{*+} , with errors that come predominantly from g_c . We estimate the multiplicity of D^{*0} by assuming that D^{*0} and D^{*+} are in thermal equilibrium at kinetic freeze-out with temperature $T_F = 115$ MeV: $N_{*0}/N_{*+} = (M_{*0}/M_{*+})^2 K_2(M_{*0}/T_F)/K_2(M_{*+}/T_F)$. The estimated multiplicity for D^{*0} is 2.59. We obtain plausible multiplicities before D^* decays by using Eqs. (2) with $B_{+0} = 0.677$:

$$(dN_0/dy)_0 = 2.12, \quad (dN_+/dy)_0 = 2.03, \quad (3a)$$

$$(dN_{*0}/dy)_0 = 2.59, \quad (dN_{*+}/dy)_0 = 2.52. \quad (3b)$$

Given these multiplicities, the ratio N_0/N_+ of the D^0 and D^+ multiplicities is predicted by Eqs. (2) to increase from 1.044 before D^* decays to 2.256 ± 0.014 at the detector, where the error bar is from B_{+0} only.

III. EXPANDING HADRON GAS

A simple model for the system produced by a sufficiently central heavy-ion collision at a proper time τ after the collision is a homogeneous system with volume $V(\tau)$ in thermal equilibrium at a temperature $T(\tau)$. Parametrizations of $V(\tau)$ and $T(\tau)$ based on the boost-invariant longitudinal expansion proposed by Bjorken [10] and an accelerated transverse expansion were presented in Ref. [11]. The system continues to expand and cool while remaining in thermal equilibrium until kinetic freeze-out. After kinetic freeze-out at proper time τ_F , the system continues to expand, but its temperature remains fixed at T_F . A simple model for the volume $V(\tau)$ for $\tau > \tau_F$ is continued longitudinal expansion at the speed of light and transverse expansion at the same speed v_F as at kinetic freeze-out:

$$V(\tau) = \pi[R_F + v_F(\tau - \tau_F)]^2 c\tau, \quad (4)$$

where R_F is the transverse radius at kinetic freeze-out. The parameters can be determined by fitting the output of a simplified hydrodynamic model [11]. The parameters for Pb-Pb collisions at 5.02 TeV are $\tau_F = 21.5$ fm/ c ,

$R_F = 24.0$ fm, and $v_F = 1.00 c$ [12]. The kinetic freeze-out temperature is $T_F = 115$ MeV.

The number density for pions at kinetic freeze-out $\mathbf{n}_\pi(\tau_F)$ can be estimated by assuming they are an ideal gas of relativistic bosons in thermal and chemical equilibrium at temperature T_F . At later times, $\mathbf{n}_\pi(\tau)$ is decreased by the expansion of the system: $\mathbf{n}_\pi(\tau) = [V(\tau_F)/V(\tau)]\mathbf{n}_\pi(\tau_F)$. The multiplicities of π^+ and π^- produced by Pb-Pb collisions at the LHC have been measured by the ALICE Collaboration at $\sqrt{s_{NN}} = 5.02$ TeV [13]. The common pion multiplicity for π^+ , π^- , and π^0 in the centrality range 10–0% is $dN_\pi/dy = 769 \pm 34$. We can use the pion multiplicity and the predicted charm-meson multiplicities in Eqs. (3) to estimate the charm-meson number densities at times τ before D^* decays:

$$\mathbf{n}_{D^{(*)}}(\tau) = [(dN_{D^{(*)}}/dy)/(dN_\pi/dy)]_0 \mathbf{n}_\pi(\tau). \quad (5)$$

IV. THERMAL MASSES AND WIDTHS

At kinetic freeze-out, the pion momentum distribution \mathbf{f}_π is a Bose-Einstein distribution: $\mathbf{f}_\pi^{(\text{eq})}(\omega_q) = 1/(e^{\omega_q/T_F} - 1)$, where $\omega_q = \sqrt{m_\pi^2 + q^2}$ and $T_F = 115$ MeV. The pion number density is $\mathbf{n}_\pi^{(\text{eq})} = \int \mathbf{f}_\pi^{(\text{eq})}(\omega_q) d^3q/(2\pi)^3$. We assume the subsequent expansion of the pion gas is isothermal, so the pion momentum distribution for $\tau > \tau_F$ is $\mathbf{f}_\pi(\omega_q) = (\mathbf{n}_\pi(\tau)/\mathbf{n}_\pi^{(\text{eq})})\mathbf{f}_\pi^{(\text{eq})}(\omega_q)$. We assume that D^+ , D^0 , D^{*+} , and D^{*0} all have relativistic Boltzmann distributions with the same temperature T_F .

When a particle propagates through a medium, its properties are modified by the interactions with the medium. The thermal mass shift and thermal width of a particle can be obtained by evaluating its in-medium self-energy on the mass shell and at zero three-momentum. The pion mass shift and thermal width after kinetic freeze-out can be calculated using χ EFT at LO. The pion mass shift is [14]

$$\delta m_\pi = (m_\pi/2f_\pi^2)\mathbf{n}_\pi\langle 1/\omega_q \rangle, \quad (6)$$

where the angular brackets represent the thermal average over the pion momentum \mathbf{q} . The pion thermal width is 0 at this order. For pions with the equilibrium number density $\mathbf{n}_\pi^{(\text{eq})}$ at $T_F = 115$ MeV, the mass shift is $\delta m_\pi = 1.550$ MeV. The thermal mass shift and thermal width of a charm meson can be calculated using HH χ EFT at LO. The D and D^* mass shifts are insensitive to isospin splittings:

$$\delta M = (3g_\pi^2/2f_\pi^2)\mathbf{n}_\pi\Delta\langle 1/\omega_q \rangle, \quad \delta M_* = -\delta M/3, \quad (7)$$

where $\Delta = 141.3$ MeV is the average of the four $M_* - M$ mass splittings. For pions with $\mathbf{n}_\pi^{(\text{eq})}$ and $T_F = 115$ MeV, the D thermal mass shift is $\delta M = 1.296$ MeV. The thermal widths of D and D^* are sensitive to isospin splittings only

through the rates $\Gamma_{*b,d}$ for the decays $D^{*b} \rightarrow D^d\pi$. The decay rates in the vacuum are

$$\Gamma_{**+,+} = \frac{g_\pi^2}{12\pi f_\pi^2} [(M_{**+} - M_+)^2 - m_{\pi 0}^2]^{3/2}, \quad (8a)$$

$$\Gamma_{**+,0} = \frac{g_\pi^2}{6\pi f_\pi^2} [(M_{**+} - M_0)^2 - m_{\pi+}^2]^{3/2}, \quad (8b)$$

$$\Gamma_{*0,0} = \frac{g_\pi^2}{12\pi f_\pi^2} [(M_{*0} - M_0)^2 - m_{\pi 0}^2]^{3/2}, \quad (8c)$$

and $\Gamma_{*0,+} = 0$. In the hadron gas, these rates are modified by the thermal mass shifts in Eqs. (6) and (7). The thermal mass shifts decrease the available phase space, so the decay $D^{*0} \rightarrow D^+\pi^-$ remains kinematically forbidden for $T = T_F$ and $\mathbf{n}_\pi < \mathbf{n}_\pi^{(\text{eq})}$. The interaction widths of D^a and D^{*a} in the hadron gas are

$$\Gamma_a = 3\mathbf{f}_\pi(\Delta) \sum_c \Gamma_{*c,a}, \quad (9a)$$

$$\Gamma_{*a} = [1 + \mathbf{f}_\pi(\Delta)] \sum_c \Gamma_{*a,c} + \Gamma_{*a,\gamma}, \quad (9b)$$

where $\Gamma_{*a,\gamma}$ is the radiative decay rate of D^{*a} . The thermal contribution has a factor $\mathbf{f}_\pi(\Delta)$, which is proportional to \mathbf{n}_π .

V. REACTION RATES

The reaction rates $\langle v\sigma_{\pi a,*b} \rangle$ in HH χ EFT at LO for the inverse decay $\pi D^a \rightarrow D^{*b}$, averaged over the three pion isospins, are

$$\langle v\sigma_{\pi+,*+} \rangle = [\mathbf{f}_\pi(\Delta)/\mathbf{n}_\pi]\Gamma_{**+,+}, \quad (10a)$$

$$\langle v\sigma_{\pi 0,*+} \rangle = [\mathbf{f}_\pi(\Delta)/\mathbf{n}_\pi]\Gamma_{**+,0}, \quad (10b)$$

$$\langle v\sigma_{\pi 0,*0} \rangle = [\mathbf{f}_\pi(\Delta)/\mathbf{n}_\pi]\Gamma_{*0,0}, \quad (10c)$$

and $\langle v\sigma_{\pi+,*0} \rangle = 0$. Note that the factor $\mathbf{f}_\pi(\Delta)/\mathbf{n}_\pi = \mathbf{f}_\pi^{(\text{eq})}(\Delta)/\mathbf{n}_\pi^{(\text{eq})}$ does not depend on the pion number density, so the dependence on \mathbf{n}_π enters only through the mass shifts in $\Gamma_{*a,b}$ given in Eqs. (8). The rates for the reactions $\pi D^* \leftrightarrow \pi D$, which change the charm-meson spin, can be simplified by taking the limit $M_* - M \rightarrow m_\pi$. The reaction rates $\langle v\sigma_{\pi^*a,\pi b} \rangle$ for $\pi D^{*a} \rightarrow \pi D^b$ and $\langle v\sigma_{\pi a,\pi^*b} \rangle$ for $\pi D^a \rightarrow \pi D^{*b}$ at T_F , averaged over initial and final pion isospins, are

$$\langle v\sigma_{\pi^*0,\pi 0} \rangle = \langle v\sigma_{\pi^*+, \pi+} \rangle = 0.2446g_\pi^4 m_\pi^2 / f_\pi^4, \quad (11a)$$

$$\langle v\sigma_{\pi^*0,\pi+} \rangle = \langle v\sigma_{\pi^*+, \pi 0} \rangle = 0.0056g_\pi^4 m_\pi^2 / f_\pi^4, \quad (11b)$$

$$\langle v\sigma_{\pi 0,\pi^*0} \rangle = \langle v\sigma_{\pi+, \pi^*+} \rangle = 0.2181g_\pi^4 m_\pi^2 / f_\pi^4, \quad (11c)$$

$$\langle v\sigma_{\pi 0, \pi^{*+}} \rangle = \langle v\sigma_{\pi^+, \pi^{*0}} \rangle = 0.0049 g_\pi^4 m_\pi^2 / f_\pi^4. \quad (11d)$$

The numerical factors depend only on m_π/T_F . We have used $m_\pi = (m_{\pi 0} + m_{\pi^+})/2$ and $T_F = 115$ MeV.

The reaction $\pi D^a \rightarrow \pi D^b$ has a resonance from an intermediate D^{*c} in the s channel. Its reaction rate $\langle v\sigma_{\pi a, \pi b} \rangle$ can be approximated by the sum of the nonresonant contribution defined by the limit $M_* - M \rightarrow m_\pi$ and the resonant contribution proportional to $1/\Gamma_{*c}$. The reaction $\pi D^{*a} \rightarrow \pi D^{*b}$ has a t -channel singularity from an intermediate D^c . Its reaction rate $\langle v\sigma_{\pi^{*a}, \pi^{*b}} \rangle$ can be approximated by the sum of the nonsingular contribution defined by the limit $M_* - M \rightarrow m_\pi$ and the most singular contribution proportional to $1/\Gamma_c$. The rates at T_F for the reactions that change the charm-meson flavor, averaged over initial and final pion isospins, are

$$\begin{aligned} \langle v\sigma_{\pi 0, \pi^+} \rangle = \langle v\sigma_{\pi^+, \pi 0} \rangle &= (1.0007 + 0.3336 g_\pi^4) \frac{m_\pi^2}{f_\pi^4} \\ &+ \frac{\mathfrak{f}_\pi(\Delta) \Gamma_{*+,0} \Gamma_{*+,+}}{\mathbf{n}_\pi \Gamma_{*+}}, \end{aligned} \quad (12a)$$

$$\begin{aligned} \langle v\sigma_{\pi^{*0}, \pi^{*+}} \rangle = \langle v\sigma_{\pi^{*+}, \pi^{*0}} \rangle &= (1.0007 + 0.3086 g_\pi^4) \frac{m_\pi^2}{f_\pi^4} \\ &+ \frac{\mathfrak{f}_\pi(\Delta) \Gamma_{*,0} \Gamma_{*+,0}}{\mathbf{n}_\pi \Gamma_0}. \end{aligned} \quad (12b)$$

The dependence on \mathbf{n}_π enters through the mass shifts in $\Gamma_{*a,b}$ and the factors of $1/\Gamma_{*+}$ and $1/\Gamma_0$. The resonance term in Eq. (12a) is about 3 orders of magnitude smaller than the nonresonant term for $n_\pi < n_\pi^{(\text{eq})}$. The t -channel

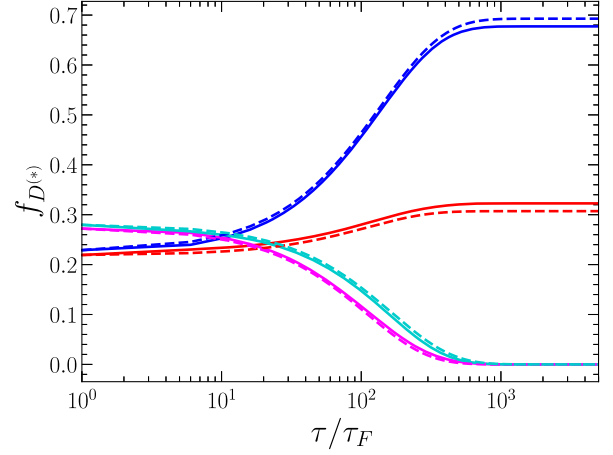


FIG. 1. Proper-time evolution of the charm-meson fractions f_{D^0} (blue), f_{D^+} (red), $f_{D^{*0}}$ (cyan), and $f_{D^{*+}}$ (magenta) from solving Eqs. (13a) and (13b) (solid curves), and from solving Eqs. (13a) and (13b) with only the D^* decay terms (dashed curves). The four fractions add up to 1.

singularity term in Eq. (12b) is larger than the nonsingular term when $n_\pi < 10^{-3} n_\pi^{(\text{eq})}$.

VI. EVOLUTION EQUATIONS

The time-evolution equation for a charm meson in the expanding hadron gas is most conveniently expressed as an equation for the ratio of its number density to the pion number density n_π ; this removes the effect of the increasing volume of the hadron gas. The evolution equations for D^a and D^{*a} are

$$\begin{aligned} \mathbf{n}_\pi \frac{d}{d\tau} \left(\frac{\mathbf{n}_{D^a}}{\mathbf{n}_\pi} \right) &= [1 + \mathfrak{f}_\pi(\Delta)] \sum_b \Gamma_{*b,a} \mathbf{n}_{D^{*b}} + \Gamma_{*a,\gamma} \mathbf{n}_{D^{*a}} - 3 \sum_b \langle v\sigma_{\pi a, *b} \rangle \mathbf{n}_{D^a} \mathbf{n}_\pi \\ &+ 3 \sum_{b \neq a} \langle v\sigma_{\pi b, \pi a} \rangle (\mathbf{n}_{D^b} - \mathbf{n}_{D^a}) \mathbf{n}_\pi + 3 \sum_b (\langle v\sigma_{\pi^* b, \pi a} \rangle \mathbf{n}_{D^{*b}} - \langle v\sigma_{\pi a, \pi^* b} \rangle \mathbf{n}_{D^a}) \mathbf{n}_\pi + \dots, \end{aligned} \quad (13a)$$

$$\begin{aligned} \mathbf{n}_\pi \frac{d}{d\tau} \left(\frac{\mathbf{n}_{D^{*a}}}{\mathbf{n}_\pi} \right) &= 3 \sum_b \langle v\sigma_{\pi b \rightarrow *a} \rangle \mathbf{n}_{D^b} \mathbf{n}_\pi - \left([1 + \mathfrak{f}_\pi(\Delta)] \sum_b \Gamma_{*a,b} + \Gamma_{*a,\gamma} \right) \mathbf{n}_{D^{*a}} \\ &+ 3 \sum_{b \neq a} (\langle v\sigma_{\pi b, \pi^* a} \rangle \mathbf{n}_{D^b} - \langle v\sigma_{\pi^* a, \pi b} \rangle \mathbf{n}_{D^{*a}}) \mathbf{n}_\pi + 3 \sum_{b \neq a} \langle v\sigma_{\pi^* b, \pi^* a} \rangle (\mathbf{n}_{D^{*b}} - \mathbf{n}_{D^{*a}}) \mathbf{n}_\pi + \dots. \end{aligned} \quad (13b)$$

The ratio of the total charm-meson density $\mathbf{n}_{D^0} + \mathbf{n}_{D^+} + \mathbf{n}_{D^{*0}} + \mathbf{n}_{D^{*+}}$ and \mathbf{n}_π remains constant in accordance with charm-quark conservation.

To illustrate the evolution of the charm-meson densities, we use Eq. (5), with the multiplicities inferred from SHMC in Eqs. (3), to provide the initial conditions on $\mathbf{n}_{D^{(*)}}/\mathbf{n}_\pi$. If the initial time τ is identified with the kinetic freeze-out time τ_F , the initial number-density ratios for D^0 , D^+ , D^{*0} ,

and D^{*+} are 0.00276, 0.00264, 0.00337, and 0.00328, respectively. We define charm-meson fractions by $f_{D^{(*)}} = \mathbf{n}_{D^{(*)}}/(\mathbf{n}_{D^0} + \mathbf{n}_{D^+} + \mathbf{n}_{D^{*0}} + \mathbf{n}_{D^{*+}})$. The solutions to the evolution equations in Eqs. (13) are shown in Fig. 1. The fractions $f_{D^{*0}}$ and $f_{D^{*+}}$ decrease exponentially to 0 on timescales comparable to the D^* lifetimes. The ratio of the multiplicities of D^0 and D^+ is predicted to increase from 1.044 at kinetic freeze-out to 2.100 at the detector.

If the t -channel singularity term in Eq. (12b) is omitted, the ratio increases instead to 2.168. The solutions to Eqs. (13) including only the D^* decay terms with the vacuum values of $\Gamma_{*a,b}$ are also shown in Fig. 1.

At times τ large enough that $n_\pi \ll \Gamma_* f_\pi^4 / m_\pi^2$, the only terms in Eqs. (13) that survive are one-body terms with only a single factor of \mathbf{n}_D or \mathbf{n}_{D^*} and no factors of \mathbf{n}_π . There are one-body terms from the decays $D^* \rightarrow D\pi$ and $D^* \rightarrow D\gamma$. There are additional one-body terms from the t -channel singularities in $\pi D^* \rightarrow \pi D^*$. The limiting behavior of the reaction rate in Eq. (12b) as $\mathbf{n}_\pi \rightarrow 0$ is

$$\langle v\sigma_{\pi^*0 \rightarrow \pi^*+} \rangle \rightarrow \frac{1}{3\mathbf{n}_\pi} \frac{\Gamma_{*0,0}\Gamma_{*+,0}}{\Gamma_{*0,0} + \Gamma_{*+,0}}. \quad (14)$$

The factor of $1/\mathbf{n}_\pi$ in the reaction rate cancels the explicit factor of \mathbf{n}_π in the evolution equation in Eq. (13b). If we only keep the one-body terms with the vacuum values of $\Gamma_{*a,b}$, the evolution equations can be solved analytically. The resulting predictions for the numbers of D^0 and D^+ are

$$N_0 = (N_0)_0 + \left(1 - \frac{(1 - B_{+0})\Gamma_{*+}\gamma}{\Gamma_{*+}\Gamma_{*0} + (\Gamma_{*+} + \Gamma_{*0})\gamma}\right)(N_{*0})_0 \\ + \left(B_{+0} + \frac{(1 - B_{+0})\Gamma_{*0}\gamma}{\Gamma_{*+}\Gamma_{*0} + (\Gamma_{*+} + \Gamma_{*0})\gamma}\right)(N_{*+})_0, \quad (15a)$$

$$N_+ = (N_+)_0 + \frac{(1 - B_{+0})\Gamma_{*+}\gamma}{\Gamma_{*+}\Gamma_{*0} + (\Gamma_{*+} + \Gamma_{*0})\gamma}(N_{*0})_0 \\ + \left(1 - B_{+0} - \frac{(1 - B_{+0})\Gamma_{*0}\gamma}{\Gamma_{*+}\Gamma_{*0} + (\Gamma_{*+} + \Gamma_{*0})\gamma}\right)(N_{*+})_0, \quad (15b)$$

where $1/\gamma = 1/(B_{00}\Gamma_{*0}) + 1/(B_{+0}\Gamma_{*+})$, and $B_{00} = (64.7 \pm 0.9)\%$ is the branching fraction for $D^{*0} \rightarrow D^0\pi^0$. The coefficients in Eqs. (15) depend only on B_{+0} , B_{00} , and Γ_{*0}/Γ_{*+} . If the multiplicities before D^* decays are those inferred from SHMc in Eqs. (3), the ratio N_0/N_+ is predicted to increase from 1.044 to 2.177 ± 0.016 at the detector, where the error bar is from B_{+0} , B_{00} , Γ_{*0} , and Γ_{*+} only. The difference between this ratio and the naive prediction after Eqs. (3), which ignores t -channel singularities, is -0.079 ± 0.006 , which differs from zero by about 13 standard deviations.

VII. CONCLUSIONS

We have studied the evolution of charm mesons after the kinetic freeze-out of an expanding hadron gas produced by a central heavy-ion collision. We have shown that the t -channel singularities in charm-meson reactions can have observable consequences. Their contributions to the reaction rates for $\pi D^* \rightarrow \pi D^*$ are inversely proportional to the thermal width of an exchanged D , which is proportional to \mathbf{n}_π because the hadron gas consists predominantly of

pions. The corresponding terms in the evolution equation for charm-meson number densities are effectively one-body terms like those from charm-meson decays. The conventional predictions for charm-meson ratios in Eqs. (2) are replaced by those in Eqs. (15).

There are other charm-meson reactions with t -channel singularities including $\pi D^* \leftrightarrow \pi\pi D$. The reaction $\pi D^* \rightarrow \pi\pi D$ has a pion t -channel singularity from the decay $D^* \rightarrow D\pi$ followed by the scattering $\pi\pi \rightarrow \pi\pi$. Its reaction rate is higher order in the HH χ EFT expansion than that for $\pi D^* \rightarrow \pi D^*$, but preliminary results indicate that pion t -channel singularities are more important than D -meson t -channel singularities.

There have been previous studies of the effects of a thermal hadronic medium on charm mesons [15–18]. In these studies, t -channel singularities have been completely overlooked. It might be worthwhile to look for other aspects of the thermal physics of charm mesons in which the effects of t -channel singularities are significant. One such aspect is the production of the exotic heavy hadrons $X(3872)$ and $T_{cc}^+(3875)$. Their tiny binding energies relative to a charm-meson-pair threshold imply that they are loosely bound charm-meson molecules. In previous studies of the production of charm-meson molecules, it has been assumed that they are produced before kinetic freeze-out [19–25]. It is possible that there could be significant effects on their production from t -channel singularities even after kinetic decoupling.

The problem of t -channel singularities is an unavoidable aspect of reactions involving unstable particles. Unstable particles are ubiquitous in hadronic physics. In the Standard Model of particle physics, the weak bosons and the Higgs are unstable particles. Most models of physics beyond the Standard Model have unstable particles. We have identified a simple aspect of charm-meson physics in which the effects of t -channel singularities are significant. This provides encouragement to look for other effects of t -channel singularities in hadronic, nuclear, and particle physics.

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