Multipole decomposition of tensor interactions of fermionic probes with composite particles and BSM signatures in nuclear reactions

Ayala Glick-Magid[1,2] and Doron Gazit[1,*]

[1]*Racah Institute of Physics, The Hebrew University, The Edmond J. Safra Campus, Givat Ram, Jerusalem 9190401, Israel*

[2]*Department of Physics, University of Washington, Seattle, Washington 98195, USA*

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A multipole decomposition of a cross section is a useful tool to simplify the analysis of reactions due to their symmetry properties. By using a new approach to decompose antisymmetric tensor-type interactions within the multipole analysis, we introduce a general mathematical formalism for working with tensor couplings. This allows us to present a general tensor nuclear response, which is particularly useful for ongoing β-decay experiments looking for physics beyond the Standard Model (BSM), as well as other exotic particle scatterings off nuclei, e.g., in dark matter direct detection experiments. Using this method, BSM operators identify with the known Standard Model operators, eliminating the need for calculations of additional matrix elements. We present in detail BSM expressions useful for β-decay experiments and give an exemplary application for ^{6}He β-decay, although the formalism is easily generalizable for calculating other exotic scattering reactions.

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I. INTRODUCTION

Tensor interactions have been investigated over the years with a focus on gravitational radiation, which introduces a coupling between symmetric tensors—a space-time metric and the stress-energy-momentum tensor (see, e.g., Ref. [1] and references therein). Recently, there was a renewed interest in tensor couplings, this time in the search for beyond the Standard Model (BSM) interactions, involving interactions with fermions, and therefore, introducing antisymmetric tensors (e.g., Ref. [2] and references therein).

A priori, when discussing the weak nuclear interaction of quarks and leptons, the most general Lorentz-invariant form of an interaction Hamiltonian can be written as a linear combination of the five bilinear covariants with specific symmetries, i.e., scalar (S), pseudoscalar (P), polar-vector (V), axial-vector (A), and tensor (T) [3]. However, it was shown experimentally, initially using β-decays, that the weak interaction between quarks and leptons has a $V-A$ structure, i.e., a polar-vector current and an axial-vector current, with the same amplitude and opposite signs [4].

In recent years, several experiments [5–11] have focused on β-decays again, but this time to find deviations from the $V-A$ structure of the Standard Model (SM). In particular, these experiments search for minute signatures of interactions with scalar and tensor symmetries. To identify such effects, it is necessary to determine the theoretical properties of transitions with these symmetries.

*doron.gazit@mail.huji.ac.il

The theoretical interest in understanding the qualitative behavior of transitions of esoteric character stems additionally from ongoing efforts to directly detect dark matter [12]. The existence of this material is currently inferred indirectly, as it provides an explanation for certain cosmological gravitational phenomena. Elucidating the nature of dark matter is one of the most pressing challenges in contemporary particle physics and astrophysics.

Among the candidates for dark matter are weakly interacting massive particles (WIMPs), such as the neutralino in supersymmetric extensions of the SM. This paradigm has spurred the development of detectors on Earth, searching for direct interactions of WIMPs from outer space, by measuring the recoil energy of WIMP scattering off nuclei on the detectors. The relevant momentum transfer in such reaction is $q \sim 100$ MeV [13], compared to the typical momentum transfer of β-decays, just a few MeV (1–4). These BSM particles might have many different kinds of couplings to matter, so the overall expression, including the tensor term, will be necessary to interpret the data from these detectors.

In the low energy regime of the weak interaction, one can assume the force-carrying exchange boson is heavy compared to the momentum transfer. This is particularly a reasonable assumption in β-decay where the momentum transfer is usually around a few MeV. The weak interaction Hamiltonian between nuclei and light particles is then presumed to be a multiplication of a nuclear current and a probe current of the same kind. Focusing on the tensor type, the interaction Hamiltonian is expressed in the Schrödinger picture as

$$\hat{H}_{\mathrm{w}}^{T}=\int d^{3}r j_{\mu\nu}(\vec{r})\mathcal{J}^{\mu\nu}(\vec{r}), \tag{1}$$

whereas $\mathcal{J}^{\mu\nu}(\vec{r})$ corresponds to the tensor hadron current and $j_{\mu\nu}(\vec{r})$ to the tensor probe current.

As opposed to the vector and axial weak interactions, which have been extensively studied within the SM, and to the scalar and pseudoscalar symmetries, which also have their own formalisms, both for the exotic weak interactions [13–15], and for dark matter [16,17], a complete study of cross sections of nuclei with tensor interactions has not been performed.

Here we develop a method of decomposing the tensor coupling within the multipole expansion. In the method we present, we do not restrict ourselves to the weak interaction between hadron and lepton currents, but only require a tensor coupling between antisymmetric tensors (i.e., consisting of fermions). This work can be viewed as complementary to previous works regarding symmetric tensor couplings for the case of gravitational radiation [1]. We then use this method to present a general mathematical formalism for the tensor type of interactions with nuclei, applicable to semileptonic interactions like β-decays, introducing a BSM complementary to the SM general framework introduced by Donnelly and Walecka in Refs. [18–23] for treating semileptonic processes in nuclei.

We note that, particularly in the early days of β-decay research, there have been several studies [3,24,25] that aimed to calculate the antisymmetric tensor coupling, during the mission to discover the symmetry nature of the weak nuclear current [4,5]. These have focused on Fermi and Gamow-Teller (allowed) decays and had explicit low momentum transfer approximations. Furthermore, a recent study, matching the quark-level effective field theory (EFT) to the nucleon-level, using pionless EFT, extends the expressions of the allowed β-decays to high order recoil corrections [26]. In addition, there are some neutrino and dark matter studies (e.g., Refs. [27–30]), which also deal with some tensor interactions, but their focus is either coherent scatterings or fully leptonic interactions, which do not require a multipole decomposition. There is, however, no general formula for nonvanishing momentum transfer of the tensor coupling, suitable for any β-decay transition, allowed and forbidden. In addition, the formalism we present here aims to reach a form suitable and easy for use for many-body nuclear calculations that separates easily SM corrections and BSM signatures.

The paper is organized as follows. In Sec. II, we present an approach for decomposing a generic coupling of antisymmetric tensor currents within the multipole expansion. This decomposition is suitable for any antisymmetric tensor probe. In the current work, we concentrate on the interaction of a tensor probe with a nucleus. Hence, in Sec. III we focus on the tensor nuclear single-nucleon current, construct it through our decomposition, and derive from it tensor multipole operators, along with other BSM multipole operators suitable for any semileptonic process (a derivation of scalar and pseudoscalar multipole operators is detailed in Appendix D). Then, in Sec. IV, we focus the discussion on and present the β-decay formalism, writing a general rate expression and reviewing how BSM signatures appear in β-decay observables relevant to contemporary experiments. Detailed expressions for the specific cases of allowed and forbidden transitions are provided in Appendix A. In Sec. V we give an exemplary application for ^{6}He β-decay, of current experimental interest. We summarize our findings and provide an outlook for future research in Sec. VI.

II. TENSOR MULTIPOLE DECOMPOSITION

Consider an interaction of a general tensor density of a composite object, e.g., a nucleus, $\mathcal{J}^{\mu\nu}$ with a CPT invariant (Lorentz invariant) probe $j_{\mu\nu}$, taking the form $\int d^{3}r j_{\mu\nu}(\vec{r})\mathcal{J}^{\mu\nu}(\vec{r})$. Assuming the probe has a plane wave character,[1] its general matrix element between its initial and final states can be written as $\langle f|j_{\mu\nu}(\vec{r})|i\rangle \equiv l_{\mu\nu}e^{-i\vec{q}\cdot\vec{r}}$, where $\vec{q}\equiv\vec{k}_{f}-\vec{k}_{i}$ is the momentum transfer between the final and initial probe states, and $l_{\mu\nu}$ depends on all the other physical properties of the probe (a detailed $l_{\mu\nu}$ for a lepton current can be found in Appendix B).

Typically, the multipole expansion is expressed as a sum of spherical harmonics. For the polar-vector and axial-vector weak interactions in the SM, the traditional way to perform the multipole expansion involves using vector spherical harmonics [21], which are an extension of scalar spherical harmonics. For a multipole expansion of a tensor coupling, we naturally turn to the notion of tensor spherical harmonics. The tensor spherical harmonics have been constructed and used in several works on general relativity problems [1]. Although they were defined in that field only for symmetric representations of ranks 0 and 2 (antisymmetric representations of rank 1 are of no relevance to gravitational wave theory), their completeness for rank 1 stems easily.

However, since rank 1 tensors are actually vectors, we suggest, instead, simplifying the tensor decomposition, taking advantage of its vector nature. For that, we suggest dismantling the antisymmetric tensors into vectorlike objects as follows: first, we decompose $l_{\mu\nu}$ into a temporal scalar l_{00}, two mixed spatial-temporal 3-vectors l_{0i} and l_{i0}, and a Euclidean (spatial-only) 3×3 tensor l_{ij}, where $i,j\in\{1,2,3\}$. Following its antisymmetric nature, we get that $l_{00}=0$ and $l_{i0}=-l_{0i}$. For convenience, we will define a vector $\vec{l}^{T'}$ such that

[1]If the probe does not have a plane wave character, one should expand it in plane waves, as in the case of a muon capture from an atomic orbital [21].

$$l_i^{T'} \equiv \sqrt{2} l_{0i}. \tag{2a}$$

Let us now focus on the remaining tensor, l_{ij}. It is a Cartesian tensor of the second rank and therefore can be decomposed into three irreducible spherical tensors of ranks 0, 1, and 2. These will be a scalar, which is the trace of the Cartesian tensor, a vector, which is the antisymmetric part of the Cartesian tensor, and a quadrupole spherical trace-free tensor, which is the remaining symmetric part of the Cartesian tensor. Using again the fact that $l_{\mu\nu}$ is antisymmetric, it follows that the symmetric scalar and quadrupole spherical tensors vanish, leaving us only with the reduced spherical tensor of rank 1, the spherical vector projector $\vec{l}^T \equiv [l_{ij}]^{(1)}$. This is a vector that its Cartesian components $i \in \{1, 2, 3\}$ are defined by

$$l_i^T \equiv -\frac{i}{\sqrt{2}} \epsilon_{ijk} l_{jk}, \tag{2b}$$

where ϵ_{ijk} is the 3-d Levi-Civita symbol (for the conventions used in this paper see Appendix G).

The same procedure is done for $\mathcal{J}^{\mu\nu}$, which is also antisymmetric, with the definitions of its spatial and spatial-temporal parts as was done to $l_{\mu\nu}$:

$$\mathcal{J}_i^T \equiv -\frac{i}{\sqrt{2}} \epsilon_{ijk} \mathcal{J}_{jk}, \tag{2c}$$

$$\mathcal{J}_i^{T'} \equiv \sqrt{2} \mathcal{J}_{0i}. \tag{2d}$$

We finally conclude the tensor decomposition into vectorlike objects, and get to write the tensor product $l_{\mu\nu}\mathcal{J}^{\mu\nu}$ as a sum of vector products, a product of the spatial vectorlike parts of the original tensors, and a product of the spatial-temporal vectorlike parts of the original tensors[2]:

$$\begin{aligned} l_{\mu\nu}\mathcal{J}^{\mu\nu}(\vec{r}) &= -[\vec{l}^T \cdot \vec{\mathcal{J}}^T(\vec{r}) + \vec{l}^{T'} \cdot \vec{\mathcal{J}}^{T'}(\vec{r})] \\ &= -\sum_{\lambda=-1}^{1} [l_\lambda^T \hat{e}_\lambda^+ \cdot \vec{\mathcal{J}}^T(\vec{r}) + l_\lambda^{T'} \hat{e}_\lambda^+ \cdot \vec{\mathcal{J}}^{T'}(\vec{r})]. \end{aligned} \tag{3}$$

We use the multipole decomposition of the circular polarization base unit vectors $\hat{e}_\lambda$, to present the tensor product using the spherical harmonics, and get the multipole expansion of the tensor interaction:

$$\begin{aligned} &\left\langle f \left| \int d^3 r j_{\mu\nu}(\vec{r}) \mathcal{J}^{\mu\nu}(\vec{r}) \right| i \right\rangle \\ &= -\sum_{J=0}^{\infty} \sqrt{4\pi(2J+1)} (-i)^J [l_3^T \langle f|\hat{L}_{J0}^T|i\rangle + l_3^{T'} \langle f|\hat{L}_{J0}^{T'}|i\rangle] \\ &\quad + \sum_{J=1}^{\infty} \sqrt{2\pi(2J+1)} (-i)^J \sum_{\lambda=\pm1} [l_\lambda^T \langle f|\hat{E}_{J,-\lambda}^T + \lambda \hat{M}_{J,-\lambda}^T|i\rangle \\ &\quad + l_\lambda^{T'} \langle f|\hat{E}_{J,-\lambda}^{T'} + \lambda \hat{M}_{J,-\lambda}^{T'}|i\rangle] \end{aligned} \tag{4}$$

[for $\mathcal{J}^{\mu\nu}$ a hadron current and $j_{\mu\nu}$ a lepton current, this is the matrix element of the tensor part of the weak interaction Hamiltonian described in Eq. (1), i.e., $\langle f|\hat{H}_\mathrm{w}^T|i\rangle$]. Note that while the matrix element on the left side of Eq. (4) is calculated between the initial and final states of the whole interaction, the matrix elements on the right side are calculated only between the initial and final states of the composite object (e.g., a nucleus), not the probe. Here the superscript T (T') denotes a multipole operator calculated with the spatial (spatial-temporal) vectorlike part of the original tensor, $\vec{\mathcal{J}}^T$ ($\vec{\mathcal{J}}^{T'}$). The Coulomb, longitudinal, electric, and magnetic multipole operators are defined by

$$\hat{C}_{JM}(q) \equiv \int d^3 r M_{JM}(q\vec{r}) \mathcal{J}_0(\vec{r}), \tag{5a}$$

$$\hat{L}_{JM}(q) \equiv \frac{i}{q} \int d^3 r \vec{\nabla} M_{JM}(q\vec{r}) \cdot \vec{\mathcal{J}}(\vec{r}), \tag{5b}$$

$$\hat{E}_{JM}(q) \equiv \frac{1}{q} \int d^3 r \left[\vec{\nabla} \times \vec{M}_{JJ1}^M(q\vec{r})\right] \cdot \vec{\mathcal{J}}(\vec{r}), \tag{5c}$$

$$\hat{M}_{JM}(q) \equiv \int d^3 r \vec{M}_{JJ1}^M(q\vec{r}) \cdot \vec{\mathcal{J}}(\vec{r}), \tag{5d}$$

where

$$M_{JM}(q\vec{r}) \equiv j_J(qr) Y_{JM}(\hat{r}), \tag{6a}$$

$$\vec{M}_{JL1}^M(q\vec{r}) \equiv j_L(qr) \vec{Y}_{JL1}^M(\hat{r}), \tag{6b}$$

with j_J the spherical Bessel functions, and Y_{JM} ($\vec{Y}_{JL1}^M$) the spherical harmonics (vector spherical harmonics).

Unlike the vector multipole expansion (see, e.g., Ref. [21]), the tensor multipole expansion presented in Eq. (4) does not contain the Coulomb multipole operator, $\hat{C}_{JM}$, which depends on the temporal part $\mathcal{J}_0$ (charge) of a 4-vector current $\mathcal{J}_\mu$. It perfectly makes sense, since the tensor is antisymmetric, and therefore, its pure temporal part, $\mathcal{J}_{00}$, vanishes. As will be presented in the following, the Coulomb multipole operator appears in expressions related to the scalar and pseudoscalar interactions (a detailed discussion about the scalar and pseudoscalar symmetries is presented in Appendix D).

[2]While the minus sign of $\vec{l}^{T'} \cdot \vec{\mathcal{J}}^{T'}$ comes from the metric, as the involved parts have only one spatial index, the minus sign before $\vec{l}^T \cdot \vec{\mathcal{J}}^T$ comes from the definitions of $\vec{l}^T$ and $\vec{\mathcal{J}}^T$.

III. BSM NUCLEAR MULTIPOLE OPERATORS

For the weak interaction, the multipole expansion of the matrix element of the tensor Hamiltonian [Eq. (1)], described in Eq. (4), depends on the multipole operators [Eq. (5)] calculated with the density of the tensor nuclear current. In the traditional nuclear physics picture, the nuclear current is constructed from the properties of free nucleons. In the case of experimental BSM searches, BSM signatures are most likely to be 10^{-3} at most [31]. Thus, we will ignore two-body (and above) currents, leading to a systematic additional uncertainty of $\epsilon_{\rm EFT} \sim 0.3$ in the nuclear model [32].

For dark matter searches, where the couplings to tensor sources need not be smaller than other couplings, the experiments aim at a discovery rather than measuring to high precision a specific coupling. Thus, lower accuracy is needed from the nuclear calculations, a fact that allows neglecting two-body tensor currents at least in the initial stage. Moreover, chiral perturbation theory with tensor sources suggests that two-body tensor currents are expected at higher order [33].

The general form of a single-nucleon matrix element of the tensor part of the charge-changing weak current, $\mathcal{J}_{\mu\nu}(\vec{r}) = \frac{1}{2}\bar{\phi}(\vec{r})\sigma_{\mu\nu}\phi'(\vec{r})$ (here $\phi^{(\prime)}$ are fields of up or down quarks), can be written as [34]

$$\begin{aligned}
&\langle \vec{p}', \sigma', \rho' | \mathcal{J}_{\mu\nu} | \vec{p}, \sigma, \rho \rangle \\
&\quad = \frac{1}{\Omega}\bar{u}(\vec{p}', \sigma')\eta^{+}_{\rho'}\frac{1}{2}[g_T(q^2)\sigma_{\mu\nu} + g_T^{(1)}(q^2)(q_\mu\gamma_\nu - q_\nu\gamma_\mu) \\
&\qquad + g_T^{(2)}(q^2)(q_\mu P_\nu - q_\nu P_\mu) + g_T^{(3)}(q^2)(\gamma_\mu \not{q}\gamma_\nu - \gamma_\nu \not{q}\gamma_\mu)] \\
&\qquad \times \tau^{\pm}\eta_\rho u(\vec{p}, \sigma),
\end{aligned} \tag{7}$$

with Ω a normalization volume,[3] $u(\vec{p}, \sigma)$ the Dirac spinor for a free nucleon of mass m_N and momentum p_μ, η_ρ a two-component Pauli isospinor, and $\tau^{\pm}$ the isospin raising and lowering operators (for conventions see Appendix G). Here, $P_\mu \equiv p_\mu + p'_\mu$, and $q_\mu \equiv p_\mu - p'_\mu$ is the momentum transfer, as before.

Lattice QCD calculations suggest that the tensor nuclear charge g_T has a magnitude similar to the magnitude of the SM axial-vector nuclear charge g_A [35]. The other tensor form factors $g_T^{(i)}(q^2)$ $(i \in \{1, 2, 3\})$ are smaller: in the nomenclature of Ref. [31] that we will use in the following, they are of the order of $\epsilon_{\rm recoil} \sim \frac{q}{m_N}$ (≈ 0.002 for an endpoint of ≈ 2 MeV) [34]. In addition, $g_T^{(3)}$ is a second-class current and therefore vanishes in the isospin $[{\rm SU}(2)_f]$ limit [36]. Although $g_T \sim g_A$, the tensor expression is suppressed by a coefficient of the effective theory, $\epsilon_T \propto (\frac{m_W}{\Lambda})^n$, which comes from the effective weak interaction Lagrangian, where m_W is the mass of the W boson, Λ represents the new physics scale, and $n \geq 2$. For the simplest BSM operator ($n = 2$), a TeV scale means $\epsilon_T \sim 10^{-3}$. New experiments, looking for BSM signatures, will have this 10^{-3} level of precision, making them sensitive to new physics at the TeV scale.

To obtain the vectorlike tensor multipole operators used in Eq. (4), we extract the tensor current density from Eq. (7), and separate it into its spatial and spatial-temporal vectorlike parts, respectively[4]:

$$\begin{aligned}
\vec{\mathcal{J}}^{T}(\vec{r}) = &-\frac{i}{\sqrt{2}}\sum_{j=1}^{A}\left(g_T + 2iE_0 g_T^{(3)}\right)\vec{\sigma}_j\delta^{(3)}(\vec{r} - \vec{r}_j)\tau_j^{\pm} \\
&+ \mathcal{O}(\epsilon_{\rm NR}^2),
\end{aligned} \tag{8a}$$

$$\begin{aligned}
\vec{\mathcal{J}}^{T'}(\vec{r}) = &\frac{1}{\sqrt{2}}\sum_{j=1}^{A}\bigg\{\left(ig_T^{(1)} - \frac{g_T}{2m_N}\right)\vec{\nabla}\delta^{(3)}(\vec{r} - \vec{r}_j) \\
&- \frac{g_T}{2m_N}\vec{\sigma}_j \times \{\vec{p}_j, \delta^{(3)}(\vec{r} - \vec{r}_j)\} \\
&+ \left(2g_T^{(3)} - \frac{E_0}{2m_N}g_T^{(1)}\right)\vec{\sigma}_j \times \vec{\nabla}\delta^{(3)}(\vec{r} - \vec{r}_j)\bigg\}\tau_j^{\pm} \\
&+ \mathcal{O}(\epsilon_{\rm NR}^2),
\end{aligned} \tag{8b}$$

where A is the mass number of the nucleus, $\vec{r}_j$ (τ_j^{+}) is the position vector (isospin-raising operator) of nucleon j, $\vec{\sigma}_j$ is the Pauli spin matrices vector associated with nucleon j, and $E_0 = q_0$ is the energy transfer. Here, we substituted the explicit form of Dirac spinors and used the nonrelativistic expansion to expand the currents in powers of $\epsilon_{\rm NR} \sim \frac{P_{\rm Fermi}}{m_N} \approx 0.2$ ($P_{\rm Fermi}$ is the Fermi momentum).

In the nuclear tensor current densities we obtained, one can see that the terms of the spatial-temporal current [Eq. (8b)] are suppressed by $\epsilon_{\rm NR}$ or $\epsilon_{\rm recoil}$. These suppressions are on top of the small tensor effective theory coefficient, so the spatial-temporal current does not appear in the BSM leading order. That leaves us with the spatial vectorlike tensor current. A closer look reveals that its leading order is the same as the leading order of the 3-vector spatial component of the SM axial-vector current density, i.e.,[5]

$$\vec{\mathcal{J}}^{T}(\vec{r}) = -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\vec{\mathcal{J}}^{A}(\vec{r}) + \mathcal{O}(\epsilon_{\rm NR}^2). \tag{9}$$

[3] We impose periodic boundary conditions on the large volume Ω and its dependence drops subsequently.

[4] We note that a similar nonrelativistic expansion was performed in Ref. [26], using matrix spin-J generators of the SO(3) Lie algebra. However, we believe that the simplicity of this technique, utilizing our vectorlike decomposition, makes it attractive.

[5] A more accurate form will include second-class currents: $\vec{\mathcal{J}}^{T}(\vec{r}) = -\frac{i}{\sqrt{2}}\frac{g_T + 2iE_0 g_T^{(3)}}{g_A - \frac{E_0}{2m_N}\tilde{g}_{T(A)}}\vec{\mathcal{J}}^{A}(\vec{r}) + \mathcal{O}(\epsilon_{\rm NR}^2)$, where $g_T^{(3)}$ and $\tilde{g}_{T(A)}$ are second-class current form factors which vanish in the isospin-symmetric limit [36] in addition to their expressions being suppressed by $\mathcal{O}(\epsilon_{\rm recoil})$. For more details, see Appendix E.

With these current densities, the multipole operators from Eq. (5) can be written as a sum of one-body operators. Equation (9) clearly shows that the spatial vectorlike tensor multipole operators are proportional to the spatial axial-vector multipole operators[6]:

$$\hat{O}_J^T(q) \approx -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{O}_J^A(q), \qquad \hat{O} \in \{\hat{L}, \hat{E}, \hat{M}\}, \tag{10}$$

where the superscript A denotes a multipole operator calculated with the axial-vector nuclear current $\mathcal{J}_\mu^A$ (not to be confused with the sum over the A nucleons). Equation (10) here is accurate to $\mathcal{O}(\epsilon_{qr}^J\epsilon_{\mathrm{NR}}^2)$ for $\hat{M}_J$, and to $\mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\mathrm{NR}}^2)$ for $\hat{E}_J$ and $\hat{L}_J$ [when $J>0$; for $\hat{L}_0$ it is $\mathcal{O}(\epsilon_{qr}\epsilon_{\mathrm{NR}}^2)$], with $\epsilon_{qr} \sim qR$ ($\approx 0.01A^{\frac{1}{3}}$ for an endpoint of ≈ 2 MeV. R is the radius of the nucleus).

This is a significant result that greatly simplifies the work with the tensor, allowing calculations of the BSM tensor interaction using only the well-known SM axial-vector multipole operators[7]:

$$\hat{L}_J^A(q) = ig_A\sum_{j=1}^A\left[\frac{1}{q}\vec{\nabla}M_J(q\vec{r}_j)\right]\cdot\vec{\sigma}\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\mathrm{NR}}^2), \tag{11a}$$

$$\hat{E}_J^A(q) = g_A\sum_{j=1}^A\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right]\cdot\vec{\sigma}_j\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\mathrm{NR}}^2), \tag{11b}$$

$$\hat{M}_J^A(q) = g_A\sum_{j=1}^A\vec{M}_{JJ1}(q\vec{r}_j)\cdot\vec{\sigma}_j\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^J\epsilon_{\mathrm{NR}}^2). \tag{11c}$$

The vectorlike spatial-temporal tensor current introduces new multipole operators:

$$\begin{aligned}\hat{L}_J^{T'}(q) = -\frac{1}{\sqrt{2}}\frac{q}{m_N}\sum_{j=1}^A\Bigg\{&(2m_N g_T^{(1)} + ig_T)M_J(q\vec{r}_j)\\ &+ g_T\left[\left(\frac{1}{q}\vec{\nabla}M_J(q\vec{r}_j)\right)\times\vec{\sigma}_j\right]\cdot\frac{1}{q}\vec{\nabla}\Bigg\}\tau_j^{\pm}\\ &+ \mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\mathrm{NR}}^2),\end{aligned} \tag{12a}$$

$$\begin{aligned}\hat{E}_J^{T'}(q) = \frac{1}{\sqrt{2}}\frac{q}{m_N}\sum_{j=1}^A\Bigg\{&ig_T\left[\left(\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right)\times\vec{\sigma}_j\right]\cdot\frac{1}{q}\vec{\nabla}\\ &+\left(\frac{i}{2}g_T + \frac{E_0}{2}g_T^{(1)} - 2m_N g_T^{(3)}\right)\vec{\sigma}_j\cdot\vec{M}_{JJ1}(q\vec{r}_j)\Bigg\}\tau_j^{\pm}\\ &+\mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\mathrm{NR}}^2),\end{aligned} \tag{12b}$$

$$\begin{aligned}\hat{M}_J^{T'}(q) = \frac{1}{\sqrt{2}}\frac{q}{m_N}\sum_{j=1}^A\Bigg\{&ig_T[\vec{M}_{JJ1}(q\vec{r}_j)\times\vec{\sigma}_j]\cdot\frac{1}{q}\vec{\nabla}\\ &+\left(\frac{i}{2}g_T + \frac{E_0}{2}g_T^{(1)} - 2m_N g_T^{(3)}\right)\vec{\sigma}_j\\ &\cdot\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right]\Bigg\}\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^J\epsilon_{\mathrm{NR}}^2),\end{aligned} \tag{12c}$$

but, as mentioned above, they do not appear in the BSM leading order. The BSM leading order is controlled only by the multipole operators $\hat{L}_J^T, \hat{E}_J^T, \hat{C}_J^P \propto \epsilon_{qr}^{J-1}$ or $\hat{M}_J^T, \hat{C}_J^S \propto \epsilon_{qr}^J$, depending on the parity of the transition in question. $\hat{C}_J^S$ ($\hat{C}_J^P$) is the Coulomb multipole operator when it is calculated with the BSM scalar (pseudoscalar) nuclear current. Similarly to the tensor leading order nuclear current, the form of the scalar leading order nuclear current is proportional to an SM nuclear current—the temporal part (charge) of the 4-vector polar-vector current: $\mathcal{J}^S(\vec{r}) = \frac{g_S}{g_V}\mathcal{J}_0^V(\vec{r}) + \mathcal{O}(\epsilon_{\mathrm{NR}}^2)$. Therefore, the scalar multipole operator is proportional to an SM multipole operator:

$$\hat{C}_J^S(q) = \frac{g_S}{g_V}\hat{C}_J^V(q) + \mathcal{O}(\epsilon_{qr}^J\epsilon_{\mathrm{NR}}^2), \tag{13}$$

where $\hat{C}_J^V$ is the polar-vector Coulomb multipole operator:

$$\hat{C}_J^V(q) = g_V\sum_{j=1}^A M_J(q\vec{r}_j)\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^J\epsilon_{\mathrm{NR}}^2). \tag{14}$$

Here g_V is the vector nuclear charge form factor, which, due to the conservation of the vector current, is 1 up to second-order corrections in isospin breaking [37,38]. The scalar nuclear charge $g_S = g_V\frac{M_n - M_p}{m_d - m_u} \approx 0.8$–$1.2$, where M_n (M_p) is the mass of the neutron (proton) and m_d (m_u) is the mass of the down (up) quark. Since this is a scalar, other multipole operators, associated with the vector type of the current, do not exist.

In order to complete the picture, let us introduce the last BSM multipole operator—the Coulomb multipole operator calculated with the pseudoscalar nuclear current. Like the vectorlike spatial-temporal tensor operators, the pseudoscalar multipole operator is suppressed by an additional small parameter, $\epsilon_{\mathrm{recoil}}$:

$$\hat{C}_J^P(q) = \frac{q}{2m_N}\frac{g_P}{g_A}\hat{L}_J^A(q) + \mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\mathrm{NR}}^2). \tag{15}$$

However, we will still consider it as contributing to the leading order of BSM, since the pseudoscalar charge, $g_P = g_A\frac{M_n + M_p}{m_d + m_u} = 349(9)$ [39], is two orders of magnitude greater than other, SM and BSM, nuclear charges ($\frac{q}{2m_N}\frac{g_P}{g_A} \approx 0.4$ for an endpoint of ≈ 2 MeV). For a full discussion and

[6] A more accurate form will include second-class currents: $\hat{O}_J^T(q) = -\frac{i}{\sqrt{2}}\frac{g_T + 2iE_0 g_T^{(3)}}{g_A - \frac{E_0}{2m_N}\tilde{g}_{T(A)}}\hat{O}_J^A(q) + \mathcal{O}(\epsilon_{\mathrm{NR}}^2) = -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}[1 + E_0(2i\frac{g_T^{(3)}}{g_T} + \frac{g_A}{g_T}\frac{\tilde{g}_{T(A)}}{2m_N})]\hat{O}_J^A(q) + \mathcal{O}(\epsilon_{\mathrm{NR}}^2)$.

[7] For a more accurate form including the second-class currents, see Appendix E.

derivation of the scalar and pseudoscalar multipole operators, refer to Appendix D.

In summary, we found that for their leading orders, the BSM multipole operators identify with the well-known SM multipole operators. In this way, BSM contributions can be calculated only from the SM phenomena, without calculating new matrix elements for BSM.

For this discussion to be complete, we must make note of another aspect of the BSM signatures in nuclear currents, the second-class currents. These currents do not add any new multipole operators but correct the existing SM polar-vector and axial-vector operators with some small contributions (those can be found in Appendix E).

IV. BSM SIGNATURES IN β-DECAY OBSERVABLES

The search for evidence of BSM interactions has been a focus of research in recent years, and accurate measurements of nuclear β-decays in various nuclei are used as a means of searching for these BSM interactions. In order to identify potential BSM signatures, it is necessary to compare those experimental measurements with theoretical predictions for the response of nuclei to interactions characterized by symmetries beyond the known SM interactions. Using the approach we developed for decomposing the tensor probe of the weak interaction within the multipole analysis, we are able to derive general expressions for calculating the decay rates of β-decays including BSM tensor couplings and to examine how BSM signatures appear in observables relevant to current experiments.

Nuclear beta minus (plus) decay is a weak reaction in which an atomic nucleus transforms into another by changing one of its nuclear neutrons (protons) into a proton (neutron), increasing (decreasing) its charge by one, and emitting an electron (positron) and an antineutrino (neutrino). Consider a $\beta^{\mp}$-decay process with p_μ (p'_μ) as the initial (final) nucleus momentum, k_μ (ν_μ) as the electron (neutrino) momentum, and $q_\mu \equiv p_\mu - p'_\mu = k_\mu + \nu_\mu$ as the momentum transfer. Its decay rate, which follows from the golden rule of Fermi, is [21]

$$\frac{d^5\omega}{dE\frac{d\hat{k}}{4\pi}\frac{d\hat{\nu}}{4\pi}} = \frac{4}{\pi^2}(E_0 - E)^2 kEF^{\mp}(Z_f, E)C_{\mathrm{corr}}\frac{1}{2J_i+1}\Theta(q, \vec{\beta}\cdot\hat{\nu}), \tag{16}$$

where the function

$$\Theta(q, \vec{\beta}\cdot\hat{\nu}) \equiv \frac{1}{4\pi}\frac{\Omega^2}{2}\sum_{\text{lepton spins}}\sum_{M_i}\sum_{M_f}|\langle f|\hat{H}_{\mathrm{w}}|i\rangle|^2 \tag{17}$$

is the part depending on the nuclear wave functions, represented here as the initial and final states. This is also the part that is affected by non-$V-A$ currents which may be included in the weak interaction Hamiltonian. We sum over final target states (spin projection M_f), and average over initial states (M_i). J_i is the total angular momentum of the initial nucleus, E_i (E_f) is the initial (final) energy of the nuclear system, while $E \equiv k_0$ is the electron energy, $\vec{\beta} \equiv \frac{\vec{k}}{E}$, and ν is the energy of the neutrino.

To Eq. (16), we have added some known corrections. The deformation of the lepton wave function, due to the long-range electromagnetic interaction with the nucleus, is taken into account in the Fermi function $F^{\mp}(Z_f, E)$ for a $\beta^{\mp}$-decay, where Z_f is the charge of the nucleus after the decay. Other corrections to the nuclear-independent part, such as radiative corrections, finite mass, and electrostatic finite size effects, as well as atomic and chemical effects, are represented by C_{corr}. In the literature, these corrections are assumed to be known and do not seem to limit experimental accuracy significantly (for more details see Refs. [40,41]).

Jackson, Treiman, and Wyld in their paper from 1957 [42], described the β-decay rate at its leading order, i.e., in allowed (Fermi and Gamow-Teller) transitions, as proportional to

$$d^5\omega \propto \xi\left(1 + a\vec{\beta}\cdot\hat{\nu} + b\frac{m_e}{E} + \ldots\right), \tag{18}$$

where a is the electron-neutrino angular correlation, and b is the Fierz interference term, both are observables that play a leading role in ongoing BSM searches. The angular correlation, a, can be extracted from measurements of the angle between the emitted leptons, and its value changes in the presence of BSM physics. The Fierz interference term, b, can be extracted from measurements of the energy spectrum of the electron. It does not exist in the SM leading order, but appears only when considering the full probe-nucleus interaction Hamiltonian, $\hat{H}_{\mathrm{w}} = \hat{H}_{\mathrm{w}}^{\mathrm{SM}} + \hat{H}_{\mathrm{w}}^{\mathrm{BSM}}$, which results in an interference term involving both SM and BSM currents.

Using the tensor multipole decomposition that we presented in the previous sections, we calculated $\Theta(q, \vec{\beta}\cdot\hat{\nu})$ with tensor currents for all the different transitions, extending these important observables also to forbidden transitions, which were previously unavailable in their complete form for BSM tensor symmetry. As we demonstrated in the previous section, the BSM multipole operators identify with the SM multipole operators. Therefore, we are able to present BSM expressions and forecasts using only SM nuclear matrix elements, that have been studied and known for many years (see, e.g., Refs. [43,44] for analytical expressions and Ref. [45] for *Mathematica* script for these nuclear matrix elements in the harmonic oscillator base). Considering the technical nature of this list of $\Theta(q, \vec{\beta}\cdot\hat{\nu})$ expressions, as well as the list of BSM signatures appearing in the mentioned observables, we present the explicit BSM contributions in Appendix A, in the hope that it will prove useful to the broad community interested in BSM searches involving β-decays.

V. SENSITIVITY TO BSM SIGNATURES IN ^{6}He AS AN EXEMPLARY NUCLEUS OF CURRENT EXPERIMENTAL INTEREST

^{6}He decays into ^{6}Li in a pure Gamow-Teller β-decay transition. This is a light nucleus, amendable to state-of-the-art *ab initio* calculations, and its half-life is about 1 second, making it ideal for experimental study using traps. For these reasons, it has a prominent role in several ongoing precision experiments (see Ref. [46]). We thus use it here as a case study to demonstrate the application of the theory presented here.

Identifying a BSM signal relies on correctly evaluating the theoretical prediction of the β-decay observables. We thus plot the ratio, $R(\mathrm{BSM/SM})$, of BSM signal to the size of associated nuclear structure-related SM corrections. If the ratio $R(\mathrm{BSM/SM})$ is of the order of 1, then the corrections should be calculated explicitly. The limit of theoretical uncertainty consequently occurs when the ratio $R(\mathrm{BSM/SM})$ is about the size of the theoretical uncertainty in calculating the SM corrections.

We focus on tensor couplings of the order of $|\frac{C_T}{C_A}| = |\frac{C'_T}{C_A}| \sim 10^{-3}$, corresponding to new physics at a few TeV scale. Figure 1 compares the ratio $R(\mathrm{BSM/SM})$ of tensor BSM signatures in the aforementioned β-decay observables, a and b, to the associated SM correction calculated for ^{6}He β-decay in Ref. [46].

As ^{6}He β-decay is a pure Gamow-Teller transition, we use Eq. (A10) from Appendix A and compare its Fierz term to the associated SM correction δ_b in the spectrum (see Ref. [46]), originating in nuclear structure corrections that has a spectral behavior similar to the Fierz term, i.e., $R^b(\mathrm{BSM/SM}) = |2\mathfrak{Re}\frac{C_A^* C_T + C_A'^* C'_T}{|C_A|^2+|C'_A|^2}/\delta_b|$. In Fig. 1, domains where current theory enables separation of the BSM signal from nuclear structure-related SM corrections appearing in Fierz term, i.e., domains where $R^b(\mathrm{BSM/SM}) > |(\delta_b\,\text{uncertainty})/\delta_b|$ are shown in white and light gray. As apparent in Fig. 1, a BSM Fierz signal is already identifiable for tensor couplings as small as $|\frac{C_T}{C_A}| \sim 10^{-4}$.

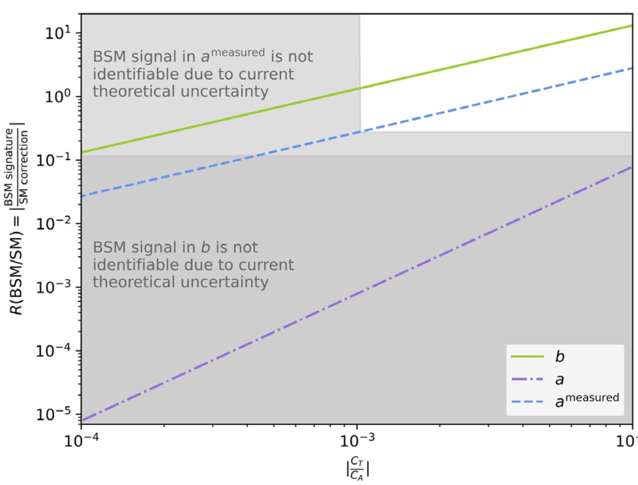

FIG. 1. The ratio $R(\mathrm{BSM/SM})$ of BSM signatures in β-decay observables to the associated SM correction calculated for ^{6}He β-decay in Ref. [46], for different values of the BSM coupling constant. For visualization simplicity, we assume $C'_A = C_A$ and $C'_T = C_T$. The solid green line is the ratio for Fierz term b. The dashed-dotted purple line is the ratio for the angular correlation a. The dashed blue line is the ratio for the measured value of the angular correlation, a^{measured}. In the white domain, considering the theoretical uncertainty from Ref. [46], separating the BSM signal from the SM corrections in both b and a^{measured} is possible. In the other domains, separation is limited by theoretical uncertainties.

In contrast, a similar approach for the angular correlation, i.e., $R^a(\mathrm{BSM/SM}) = |2\frac{|C_T|^2+|C'_T|^2}{|C_A|^2+|C'_A|^2}/\langle\tilde{\delta}_a\rangle|$ [see Eq. (A9) in Appendix A], where the angle brackets represent an average weighted by the spectrum, shows that the theory cannot identify the naive BSM signal of the angular correlation from the SM corrections even for $|\frac{C_T}{C_A}| \sim 10^{-2}$ (the dark gray domain in Fig. 1).

However, when taking into account the way that a is extracted from measurements, the spectral shape suggests that $a^{\mathrm{measured}} = \frac{\langle a\rangle}{1+b\langle\frac{m_e}{E}\rangle}$ [47], making the measured value of a sensitive also to the Fierz term, as specified in the following relation for Gamow-Teller and unique forbidden transitions:

$$a^{\mathrm{measured}} = -\frac{1}{2J+1}\left(1+\langle\tilde{\delta}_a^{J^{(-)^{J-1}}}\rangle - \delta_b^{J^{(-)^{J-1}}}\left\langle\frac{m_e}{E}\right\rangle \mp 2\mathfrak{Re}\frac{C_A^* C_T + C_A'^* C'_T}{|C_A|^2+|C'_A|^2}\left\langle\frac{m_e}{E}\right\rangle - 2\frac{|C_T|^2+|C'_T|^2}{|C_A|^2+|C'_A|^2}\right). \tag{19}$$

This results in a relative size of the BSM signal,

$$R^{a^{\mathrm{measured}}}(\mathrm{BSM/SM}) = \left|\frac{2\mathfrak{Re}\frac{C_A^* C_T + C_A'^* C'_T}{|C_A|^2+|C'_A|^2}\langle\frac{m_e}{E}\rangle + 2\frac{|C_T|^2+|C'_T|^2}{|C_A|^2+|C'_A|^2}}{\langle\tilde{\delta}_a\rangle - \delta_b\langle\frac{m_e}{E}\rangle}\right|, \tag{20}$$

which enables a separation between the BSM signal and SM corrections for $|\frac{C_T}{C_A}| = |\frac{C'_T}{C_A}| \sim 10^{-3}$, as shown in Fig. 1 (the white domain).

To understand the ability of experiments to identify these signals, we notice that the SM nuclear structure-related corrections are of the order of 10^{-3} for b and for a^{measured}. Thus, experimental accuracy of about 10^{-3} in the measurement of both these observables is needed, as is aimed in current and planned experimental campaigns.

A complete application of the theory presented, extracting both the angular correlation and Fierz term from measurements of the recoiled ion energy of the ^{23}Ne β-decay, and combining the theory and experiment sensitivities to present new bounds on BSM tensor coupling constants, can be found in Ref. [48].

The above discussion concentrated on allowed transitions, as they are the focus of many experimental campaigns. However, theoretical considerations that arise also from the formalism presented here, show that studying β-decays beyond allowed transitions can be of advantage. The spectrum of unique forbidden transitions contains an additional term [see Eq. (A8) in Appendix A],

$$\frac{J-1}{2J+1}\frac{E(E_0-E)}{q^2}[\beta^2-(\vec{\beta}\cdot\hat{\nu})^2] \times\left(1+\tilde{\delta}_{\beta^2}^{J^{(-)^{J-1}}}-2\frac{|C_T|^2+|C_T'|^2}{|C_A|^2+|C_A'|^2}\right), \quad (21)$$

which does not appear in Gamow-Teller transitions. This makes the energy spectrum of the electron sensitive to both angular correlation and Fierz term, as we detailed in Ref. [49] for unique first-forbidden transitions. Consequently, several experimental campaigns were initiated to measure unique first-forbidden transitions, such as ^{16}N at the SARAF accelerator, Israel [11,50], and ^{90}Sr at the Hebrew University, Israel. In addition, a new study at the Oak Ridge National Laboratory, Tennessee, USA, opens the door to many more possibilities [51].

VI. CONCLUSIONS AND OUTLOOK

In this paper, we introduced a general mathematical formalism for calculating the interaction of a Lorentz invariant probe with a nucleus. As we demonstrated, this general formalism is useful for various types of BSM physics analysis, from exotic interactions with standard particles to interactions with new particles, such as those expected in the astrophysical dark matter scenario, and thus completing the theoretical parts needed for analyzing ongoing and future experiments looking for BSM physics.

This paper results in three main findings: the first is the multipole expansion of tensor interactions with a composite particle, presented in Eq. (4). The second is that the needed approximation of the tensor nuclear current is the same as the leading order of the SM axial-vector current, as appears in Eq. (9). The third is BSM expressions for β-decay, useful for precision experiments searching for BSM signatures, displayed explicitly in Appendix A, with an exemplary application given in Sec. V. The latter shows the usefulness of the theoretical analysis we presented in the analysis of the potential of experimental campaigns in identifying BSM signals.

With the presented technique, BSM multipole operators are identical to those appearing in the SM to the required approximations, what greatly simplifies future calculations of BSM signatures. Consequently, in order to compute BSM contributions to semileptonic processes, such as β-decays, which are frequently used in BSM searches today, there is no need to compute any new nuclear matrix elements, but to use the well-established SM matrix elements, as shown in Eqs. (10), (13), and (15).

Moreover, the additional terms we found, which complete the ingredients for β-decays, are crucial for accurately identifying the expected size of the BSM effect, as we demonstrate for ^{6}He in Sec. V. This can assist in the design of future experiments to study BSM effects, as we pointed out in Ref. [49], where, supported by this formalism, we showed that the unique first-forbidden decay spectrum is more sensitive to BSM signatures. In light of that, experiments are underway at the SARAF accelerator, Israel, the Hebrew University, Israel, and the Oak Ridge National Laboratory, Tennessee, to measure the spectrum of unique first-forbidden β-decays [11,50,51].

Finally, we mention that recent studies have already used some of the results of this research, to describe the effect of BSM tensor interactions in processes other than β-decays and dark matter, such as neutrino scattering [52] and $\mu\to e$ conversion [53].

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APPENDIX A: BSM SIGNATURES IN β-DECAY OBSERVABLES

Here we present explicitly BSM contributions for the experimentally important case of β-decays, which is intensively used these days in BSM searches. The BSM contributions were calculated using the tensor multipole decomposition displayed in this paper. Detailed derivations can be found in Appendixes B (pure tensor terms), C (tensor Fierz interference term), and D (scalar and pseudoscalar terms, including their Fierz interference with tensor terms).

For presenting these contributions, we use the notation we developed in Ref. [31]. As outlined there, a decay rate of a $J_i^{\pi_i}\to J_f^{\pi_f}$ β-decay transition with J_i (J_f) and π_i (π_f), the initial (final) angular momentum and parity, will include all integer angular momentum changes that satisfy the selection rules $|J_i-J_f|\le J\le J_i+J_f$ and $\Delta\pi=\pi_i\cdot\pi_f$:

$$\Theta(q,\vec{\beta}\cdot\hat{\nu})=\sum_{J=|J_i-J_f|}^{J_i+J_f}\Theta^{J^{\Delta\pi}}(q,\vec{\beta}\cdot\hat{\nu}). \quad (A1)$$

[see Eq. (17) in the main text for the definition of $\Theta(q,\vec{\beta}\cdot\hat{\nu})$.] In the following, we will present the BSM contributions to each $\Theta^{J^{\Delta\pi}}$, divided, as in Ref. [31], into two parity types (for $1\le J$; the special cases $J^{\Delta\pi}=0^{\pm}$ will be discussed separately): $\Delta\pi=(-)^J$, presenting nonunique Jth forbidden transitions, and $\Delta\pi=(-)^{J-1}$, presenting Gamow-Teller ($J=1$) and unique ($J-1$)th forbidden transitions ($J>1$). We note that allowed transitions were recently studied comprehensively in Ref. [26].

1. $J^{\Delta\pi}=0^+$: Fermi transition

Having $J=0$, BSM contributions to the Fermi transition ($J^{\Delta\pi}=0^+$) come from the scalar multipole operator $\hat{C}_0^S$, which is proportional to the Fermi operator, $\hat{C}_0^S \approx \frac{g_S}{g_V} \hat{C}_0^V$ [see Eq. (13) in the main text for more details]:

$$\Theta^{0^+}(q,\vec{\beta}\cdot\hat{\nu}) = \frac{|C_V|^2+|C'_V|^2}{2|g_V|^2} |\langle\|\hat{C}_0^V\|\rangle|^2 \times \left(1+\delta_1^{0^+}+\frac{|C_S|^2+|C'_S|^2}{|C_V|^2+|C'_V|^2}\right) \times \left[1+\vec{\beta}\cdot\hat{\nu}\left(1+\tilde{\delta}_a^{0^+}-2\frac{|C_S|^2+|C'_S|^2}{|C_V|^2+|C'_V|^2}\right) + \frac{m_e}{E}\left(\delta_b^{0^+}\pm 2\mathfrak{Re}\frac{C_V C_S^* + C'_V C'^*_S}{|C_V|^2+|C'_V|^2}\right)\right], \quad \text{(A2)}$$

where $\pm$ are for $\beta^{\mp}$-decays, $\langle\|\hat{O}_J\|\rangle$ is a short notation for the reduced matrix element $\langle f\|\hat{O}_J\|i\rangle$ of a multipole operator $\hat{O}_J$ between the final and initial nuclear states, and $\delta_1^{0^+}$, $\tilde{\delta}_a^{0^+}$, and $\delta_b^{0^+}$ are the SM next-to-leading order (NLO) nuclear structure and recoil corrections discussed in Ref. [31]. There are two observables of interest here for the search for BSM signatures. The first is the electron-neutrino angular correlation,

$$a^{0^+} = 1+\tilde{\delta}_a^{0^+} - 2\frac{|C_S|^2+|C'_S|^2}{|C_V|^2+|C'_V|^2}, \quad \text{(A3)}$$

which is $a^{0^+}=1$ in the SM leading order. The second is the Fierz interference term,

$$b^{0^+} = \delta_b^{0^+} \pm 2\mathfrak{Re}\frac{C_V C_S^* + C'_V C'^*_S}{|C_V|^2+|C'_V|^2}, \quad \text{(A4)}$$

which vanishes in the SM leading order. These results recover the well-known Jackson, Treiman, and Wyld results [42] for allowed leading orders. In their formulation [Eq. (18) in the main text],

$$\xi^{0^+} = \frac{|\langle\|\hat{C}_0^V\|\rangle|^2}{|g_V|^2}[(|C_V|^2+|C'_V|^2)(1+\delta_1^{0^+})+|C_S|^2+|C'_S|^2], \quad \text{(A5a)}$$

$$a^{0^+}\xi^{0^+} = \frac{|\langle\|\hat{C}_0^V\|\rangle|^2}{|g_V|^2}[(|C_V|^2+|C'_V|^2)(1+\delta_a^{0^+}) - |C_S|^2-|C'_S|^2], \quad \text{(A5b)}$$

$$b^{0^+}\xi^{0^+} = \frac{|\langle\|\hat{C}_0^V\|\rangle|^2}{|g_V|^2}[(|C_V|^2+|C'_V|^2)\delta_b^{0^+} \pm 2\mathfrak{Re}(C_V C_S^* + C'_V C'^*_S)], \quad \text{(A5c)}$$

where $\frac{\langle\|\hat{C}_0^V\|\rangle}{g_V} = M_F$ is the Fermi matrix element used in their paper, and the NLO SM corrections $\delta_1^{0^+}$, $\delta_a^{0^+} = \tilde{\delta}_a^{0^+}+\delta_1^{0^+}$, and $\delta_b^{0^+}$ are higher order precision corrections not found in the Jackson, Treiman, and Wyld paper.

2. $J^{\Delta\pi}=0^-$: A nonunique first-forbidden transition

For the nonunique first-forbidden transition $J^{\Delta\pi}=0^-$, the nuclear structure expression includes BSM contributions from the tensor multipole operator $\hat{L}_0^T \approx -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{L}_0^A$ and the pseudoscalar multipole operator $\hat{C}_0^P \approx \frac{q}{2m_N}\frac{g_P}{g_A}\hat{L}_0^A$ [see Eqs. (10) and (15) in the main text] as follows:

$$\begin{aligned}
\Theta^{0^-}(q,\vec{\beta}\cdot\hat{\nu}) = \frac{|C_A|^2+|C'_A|^2}{2|g_A|^2}\Bigg\{ & |\langle\|\hat{C}_0^A\|\rangle|^2 + \left[1+\frac{|C_T|^2+|C'_T|^2}{|C_A|^2+|C'_A|^2}+\left(\frac{q}{m_N}\right)^2\frac{|C_P|^2+|C'_P|^2}{|C_A|^2+|C'_A|^2}\right. \\
& \left.\pm\frac{m_e}{m_N}\mathfrak{Re}\frac{C_A C_P^*+C'_A C'^*_P}{|C_A|^2+|C'_A|^2} \pm \frac{E_0-2E}{m_N}\mathfrak{Re}\frac{C_T C_P^*+C'_T C'^*_P}{|C_A|^2+|C'_A|^2}\right]|\langle\|\hat{L}_0^A\|\rangle|^2 \\
& - 2\mathfrak{Re}\left[\left(\frac{E_0}{q}\mp\frac{m_e}{q}\frac{C_A^*C_T+C'^*_A C'_T}{|C_A|^2+|C'_A|^2}\right)\langle\|\hat{L}_0^A\|\rangle\langle\|\hat{C}_0^A\|\rangle^*\right] \\
& +\vec{\beta}\cdot\hat{\nu}\left[|\langle\|\hat{C}_0^A\|\rangle|^2+\left(1-\frac{|C_T|^2+|C'_T|^2}{|C_A|^2+|C'_A|^2}-\left(\frac{q}{m_N}\right)^2\frac{|C_P|^2+|C'_P|^2}{|C_A|^2+|C'_A|^2}\right.\right. \\
& \left.\mp\frac{m_e}{m_N}\mathfrak{Re}\frac{C_A C_P^*+C'_A C'^*_P}{|C_A|^2+|C'_A|^2}\mp\frac{E_0-2E}{m_N}\mathfrak{Re}\frac{C_T C_P^*+C'_T C'^*_P}{|C_A|^2+|C'_A|^2}\right)|\langle\|\hat{L}_0^A\|\rangle|^2 \\
& \left. - 2\mathfrak{Re}\left[\left(\frac{E_0}{q}\pm\frac{m_e}{q}\frac{C_A^*C_T+C'^*_A C'_T}{|C_A|^2+|C'_A|^2}\right)\langle\|\hat{L}_0^A\|\rangle\langle\|\hat{C}_0^A\|\rangle^*\right]\right] \\
& +\frac{m_e}{E}2\mathfrak{Re}\left[\left(\frac{m_e}{q}\mp\frac{E_0}{q}\frac{C_A^*C_T+C'^*_A C'_T}{|C_A|^2+|C'_A|^2}\mp\frac{q}{m_N}\frac{C_A^*C_P+C'^*_A C'_P}{|C_A|^2+|C'_A|^2}\right)\langle\|\hat{L}_0^A\|\rangle\langle\|\hat{C}_0^A\|\rangle^*\right. \\
& \left.\pm\left(\frac{C_A^*C_T+C'^*_A C'_T}{|C_A|^2+|C'_A|^2}-\frac{E_0}{m_N}\frac{C_A^*C_P+C'^*_A C'_P}{|C_A|^2+|C'_A|^2}-\frac{m_e}{m_N}\frac{C_T^*C_P+C'^*_T C'_P}{|C_A|^2+|C'_A|^2}\right)|\langle\|\hat{L}_0^A\|\rangle|^2\right] \\
& +2\frac{E(E_0-E)}{q^2}\left[\beta^2-\left(\vec{\beta}\cdot\hat{\nu}\right)^2\right]\left(1-\frac{|C_T|^2+|C'_T|^2}{|C_A|^2+|C'_A|^2}\right)|\langle\|\hat{L}_0^A\|\rangle|^2\Bigg\} + \mathcal{O}(\epsilon_M). \quad \text{(A6)}
\end{aligned}$$

[$\epsilon_M \sim \frac{\Delta M}{M_{\rm min}}$, where $\Delta M \equiv M_i - M_f$, $M_{\rm min} \equiv \min(M_i, M_f)$, and M_i (M_f) is the mass of the initial (final) nucleus, presents SM recoiled nucleus corrections, which we will not give explicitly here, since they are relevant only for very light nuclei. For example, for the β-decay of ^{6}He, $\epsilon_M \sim 7\times 10^{-4}$ [31].] The BSM tensor and pseudoscalar signatures, $\frac{|C_T|^2+|C'_T|^2}{|C_A|^2+|C'_A|^2}$, $\frac{|C_P|^2+|C'_P|^2}{|C_A|^2+|C'_A|^2}$, $\frac{C_A^*C_T+C'^*_AC'_T}{|C_A|^2+|C'_A|^2}$, $\frac{C_A^*C_P+C'^*_AC'_P}{|C_A|^2+|C'_A|^2}$ and $\frac{C_T^*C_P+C'^*_TC'_P}{|C_A|^2+|C'_A|^2}$, in the observables of this nonunique first-forbidden transition, can be recognized similarly to those in the observables of allowed transitions. In the notion of Jackson, Treiman, and Wyld, these observables will be

$$\begin{aligned}\xi^{0^-} = \frac{1}{|g_A|^2}\Bigg\{&(|C_A|^2+|C'_A|^2)|\langle\|\hat{C}_0^A\|\rangle|^2\\&+\Bigg[|C_A|^2+|C'_A|^2+|C_T|^2+|C'_T|^2+\left(\frac{q}{m_N}\right)^2(|C_P|^2+|C'_P|^2)\\&\pm\frac{m_e}{m_N}\mathfrak{Re}(C_AC_P^*+C'_AC'^*_P)\pm\frac{E_0-2E}{m_N}\mathfrak{Re}(C_TC_P^*+C'_TC'^*_P)\Bigg]|\langle\|\hat{L}_0^A\|\rangle|^2\\&-2\mathfrak{Re}\left\{\left[\frac{E_0}{q}(|C_A|^2+|C'_A|^2)\mp\frac{m_e}{q}(C_A^*C_T+C'^*_AC'_T)\right]\langle\|\hat{L}_0^A\|\rangle\langle\|\hat{C}_0^A\|\rangle^*\right\}\Bigg\},\end{aligned} \tag{A7a}$$

$$\begin{aligned}a^{0^-}\xi^{0^-} = \frac{1}{|g_A|^2}\Bigg\{&(|C_A|^2+|C'_A|^2)|\langle\|\hat{C}_0^A\|\rangle|^2\\&+\Bigg[|C_A|^2+|C'_A|^2-|C_T|^2-|C'_T|^2-\left(\frac{q}{m_N}\right)^2(|C_P|^2+|C'_P|^2)\\&\mp\frac{m_e}{m_N}\mathfrak{Re}(C_AC_P^*+C'_AC'^*_P)\mp\frac{E_0-2E}{m_N}\mathfrak{Re}(C_TC_P^*+C'_TC'^*_P)\Bigg]|\langle\|\hat{L}_0^A\|\rangle|^2\\&-2\mathfrak{Re}\left\{\left[\frac{E_0}{q}(|C_A|^2+|C'_A|^2)\pm\frac{m_e}{q}(C_A^*C_T+C'^*_AC'_T)\right]\langle\|\hat{L}_0^A\|\rangle\langle\|\hat{C}_0^A\|\rangle^*\right\}\Bigg\},\end{aligned} \tag{A7b}$$

$$\begin{aligned}b^{0^-}\xi^{0^-} = \frac{1}{|g_A|^2}2\mathfrak{Re}\Bigg\{&\Bigg[\pm(C_A^*C_T+C'^*_AC'_T)\mp\frac{E_0}{m_N}(C_A^*C_P+C'^*_AC'_P)\\&\mp\frac{m_e}{m_N}(C_T^*C_P+C'^*_TC'_P)\Bigg]|\langle\|\hat{L}_0^A\|\rangle|^2+\Bigg[\frac{m_e}{q}(|C_A|^2+|C'_A|^2)\\&\mp\frac{E_0}{q}(C_A^*C_T+C'^*_AC'_T)\mp\frac{q}{m_N}(C_A^*C_P+C'^*_AC'_P)\Bigg]\langle\|\hat{L}_0^A\|\rangle\langle\|\hat{C}_0^A\|\rangle^*\Bigg\},\end{aligned} \tag{A7c}$$

where $\hat{C}_0^A \propto \epsilon_{\rm NR}$ and $\hat{L}_0^A \propto \epsilon_{qr}$ are the SM operators that dominate the $J^{\Delta\pi} = 0^-$ nonunique first-forbidden transition.

3. $\Delta\pi = (-)^{J-1}$: Gamow-Teller and unique forbidden transitions

In discussing the $\Theta^{J^{\Delta\pi}}$ expressions for J's greater than 0, we distinguish between transitions with two parity types: $\Delta\pi = (-)^J$, and $\Delta\pi = (-)^{J-1}$. $J^{(-)^J}$ angular momentum presents nonunique Jth forbidden transitions, while $J^{(-)^{J-1}}$ presents, for $J = 1$, the allowed Gamow-Teller transition, and for $J > 1$, unique $(J-1)$th forbidden transitions (we will refer to them together as unique transitions).

Starting with the unique transitions, their BSM contributions involve the tensor and pseudoscalar multipole operators, $\hat{L}_J^T \approx -\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{L}_J^A$ [Eq. (10) in the main text] and $\hat{C}_J^P \approx \frac{q}{2m_N}\frac{g_P}{g_A}\hat{L}_J^A$ [Eq. (15) in the main text]. A general expression that includes the BSM contributions along with the SM NLO corrections can be written as

$$\Theta^{J^{(-)^{J-1}}}(q,\vec{\beta}\cdot\hat{\nu}) = \frac{|C_A|^2+|C_A'|^2}{2|g_A|^2}|\langle\|\hat{L}_J^A\|\rangle|^2\frac{2J+1}{J}\Bigg\{1+\delta_1^{J^{(-)^{J-1}}}+\frac{|C_T|^2+|C_T'|^2}{|C_A|^2+|C_A'|^2}$$
$$+\frac{J}{2J+1}\left[\left(\frac{q}{m_N}\right)^2\frac{|C_P|^2+|C_P'|^2}{|C_A|^2+|C_A'|^2}\pm\frac{m_e}{m_N}\mathfrak{Re}\frac{C_A^*C_P+C_A'^*C_P'}{|C_A|^2+|C_A'|^2}\pm\frac{E_0-2E}{m_N}\mathfrak{Re}\frac{C_T^*C_P+C_T'^*C_P'}{|C_A|^2+|C_A'|^2}\right]\Bigg\}$$
$$\times\Bigg\{1-\frac{1}{2J+1}\vec{\beta}\cdot\hat{\nu}\Bigg[1+\tilde{\delta}_a^{J^{(-)^{J-1}}}-2\frac{|C_T|^2+|C_T'|^2}{|C_A|^2+|C_A'|^2}$$
$$+J\left(\left(\frac{q}{m_N}\right)^2\frac{|C_P|^2+|C_P'|^2}{|C_A|^2+|C_A'|^2}\pm\frac{m_e}{m_N}\mathfrak{Re}\frac{C_A^*C_P+C_A'^*C_P'}{|C_A|^2+|C_A'|^2}\pm\frac{E_0-2E}{m_N}\mathfrak{Re}\frac{C_T^*C_P+C_T'^*C_P'}{|C_A|^2+|C_A'|^2}\right)\Bigg]$$
$$+\frac{m_e}{E}\left[\delta_b^{J^{(-)^{J-1}}}\pm 2\mathfrak{Re}\frac{C_A^*C_T+C_A'^*C_T'}{|C_A|^2+|C_A'|^2}\mp\frac{J}{2J+1}\left(\frac{E}{m_N}\mathfrak{Re}\frac{C_A^*C_P+C_A'^*C_P'}{|C_A|^2+|C_A'|^2}+\frac{m_e}{q}\mathfrak{Re}\frac{C_T^*C_P+C_T'^*C_P'}{|C_A|^2+|C_A'|^2}\right)\right]$$
$$+\frac{J-1}{2J+1}\frac{E(E_0-E)}{q^2}\left[\beta^2-(\vec{\beta}\cdot\hat{\nu})^2\right]\left(1+\tilde{\delta}_{\beta^2}^{J^{(-)^{J-1}}}-2\frac{|C_T|^2+|C_T'|^2}{|C_A|^2+|C_A'|^2}\right)\Bigg\}+\mathcal{O}(\epsilon_M\epsilon_{qr}^{2J-2}), \tag{A8}$$

where the different $\delta^{J^{(-)^{J-1}}}$ are the NLO SM corrections described in Ref. [31] ($\epsilon_M\epsilon_{qr}^{2J-2}$ presents SM recoiled nucleus corrections, which we will not display here, since they are relevant only for very light nuclei [31]). Note that for the Gamow-Teller case ($J=1$), the term $\frac{J-1}{2J+1}(1+\tilde{\delta}_{\beta^2}^{J^{(-)^{J-1}}}-2\frac{|C_T|^2+|C_T'|^2}{|C_A|^2+|C_A'|^2})\frac{E(E_0-E)}{q^2}[\beta^2-(\vec{\beta}\cdot\hat{\nu})^2]$ does not exist. Instead, there is an NLO SM correction, $\tilde{\delta}^{1^+}_{\beta^2,(\beta\nu)^2}$ [31].

According to the $V-A$ structure of the weak interaction, the leading order of the electron-neutrino angular correlation should be $a^{J^{(-)^{J-1}}}=-\frac{1}{2J+1}$. As was already known for Gamow-Teller transitions but actually applies to any unique transition, when adding BSM tensor contributions, the angular correlation becomes

$$a^{J^{(-)^{J-1}}}=-\frac{1}{2J+1}\left(1+\tilde{\delta}_a^{J^{(-)^{J-1}}}-2\frac{|C_T|^2+|C_T'|^2}{|C_A|^2+|C_A'|^2}\right). \tag{A9}$$

Also the Fierz term which vanishes for the leading order in the $V-A$ structure as was already known for Gamow-Teller but actually applies to any unique transition, differs from zero when including tensor BSM contributions (here we also include a term with a similar spectral behavior that can be extracted from the NLO SM spectrum):

$$b^{J^{(-)^{J-1}}}=\delta_b^{J^{(-)^{J-1}}}\pm 2\mathfrak{Re}\frac{C_A^*C_T+C_A'^*C_T'}{|C_A|^2+|C_A'|^2}. \tag{A10}$$

However, pseudoscalar terms may also contribute to leading orders of BSM. Their inclusion along with the above tensor terms can be simplified using the notion of Jackson, Treiman, and Wyld:

$$\xi^{J^{(-)^{J-1}}}=\frac{|\langle\|\hat{L}_J^A\|\rangle|^2}{|g_A|^2}\Bigg\{(|C_A|^2+|C_A'|^2)(1+\delta_1^{J^{(-)^{J-1}}})+|C_T|^2+|C_T'|^2$$
$$+\frac{J}{2J+1}\left[\left(\frac{q}{m_N}\right)^2(|C_P|^2+|C_P'|^2)\pm\frac{m_e}{m_N}\mathfrak{Re}(C_A^*C_P+C_A'^*C_P')\pm\frac{E_0-2E}{m_N}\mathfrak{Re}(C_T^*C_P+C_T'^*C_P')\right]\Bigg\}, \tag{A11a}$$

$$a^{J^{(-)^{J-1}}}\xi^{J^{(-)^{J-1}}}=-\frac{1}{2J+1}\frac{|\langle\|\hat{L}_J^A\|\rangle|^2}{|g_A|^2}\Bigg\{(|C_A|^2+|C_A'|^2)(1+\delta_a^{J^{(-)^{J-1}}})-|C_T|^2-|C_T'|^2$$
$$+J\left[\left(\frac{q}{m_N}\right)^2(|C_P|^2+|C_P'|^2)\pm\frac{m_e}{m_N}\mathfrak{Re}(C_A^*C_P+C_A'^*C_P')\pm\frac{E_0-2E}{m_N}\mathfrak{Re}(C_T^*C_P+C_T'^*C_P')\right]\Bigg\}, \tag{A11b}$$

$$b^{J^{(-)^{J-1}}}\xi^{J^{(-)^{J-1}}}=\frac{|\langle\|\hat{L}_J^A\|\rangle|^2}{|g_A|^2}\Bigg\{(|C_A|^2+|C_A'|^2)\delta_b^{J^{(-)^{J-1}}}\pm 2\mathfrak{Re}(C_A^*C_T+C_A'^*C_T')$$
$$\mp\frac{J}{2J+1}\left[\frac{E}{m_N}\mathfrak{Re}(C_A^*C_P+C_A'^*C_P')+\frac{m_e}{q}\mathfrak{Re}(C_T^*C_P+C_T'^*C_P')\right]\Bigg\}. \tag{A11c}$$

By substituting $J=1$, one obtains the well-known Jackson, Treiman, and Wyld results for the leading order observables of the Gamow-Teller transition, where $\frac{\langle\|\hat{L}_1^A\|\rangle}{g_A}=M_{\mathrm{GT}}$ is the Gamow-Teller matrix element used in their paper. Here, again, the NLO SM corrections δ_1, $\delta_a=\tilde{\delta}_a+\delta_1$ and δ_b are higher order precision corrections not found in the Jackson, Treiman, and Wyld paper, and so are the terms containing the pseudoscalar coupling constants $C_P^{(\prime)}$.

A full allowed (mixed Gamow-Teller and Fermi) transition will be a sum of Eqs. (A2) and (A8) for $J=1$ (with the NLO SM correction $\tilde{\delta}^{1^+}_{\beta^2,(\beta\nu)^2}$ to replace the vanishing term, as explained above), where the full ξ presented in the Jackson, Treiman, and Wyld paper is the sum of Eqs. (A5a) and (A11a) for $J=1$, $a\xi$ is the sum of Eqs. (A5b) and (A11b) for $J=1$, and $b\xi$ is the sum of Eqs. (A5c) and (A11c) for $J=1$. All are in agreement with their paper.

4. $\Delta\pi=(-)^J$: Nonunique forbidden transitions

Finally, the case of nonunique Jth forbidden transitions, i.e., decays with angular momentum change J greater than 0, and parity change $\pi=(-)^J$, involves BSM contributions from the scalar and tensor multipole operators, $\hat{C}_J^S\approx\frac{g_S}{g_V}\hat{C}_J^V$ [Eq. (13) in the main text], and $\hat{M}_J^T\approx-\frac{i}{\sqrt{2}}\frac{g_T}{g_A}\hat{M}_J^A$ [Eq. (10) in the main text]. The $\Theta^{J^{(-)^J}}$ expression can be written as

$$\begin{aligned}
\Theta^{J^{(-)^J}}(q,\vec{\beta}\cdot\hat{\nu}) = &\left\{\frac{|C_V|^2+|C_V'|^2}{2|g_V|^2}\left[1+\frac{1}{J}\frac{E_0^2}{q^2}(1-(J+1)2\mathfrak{Re}\delta^{J^{(-)^J}})\right]+\frac{|C_S|^2+|C_S'|^2}{2|g_V|^2}\right.\\
&\left.\mp\frac{m_eE_0}{q^2}2\mathfrak{Re}\frac{C_VC_S^*+C_V'C_S'^*}{2|g_V|^2}\right\}|\langle\|\hat{C}_J^V\|\rangle|^2+\left(\frac{|C_A|^2+|C_A'|^2}{2|g_A|^2}+\frac{|C_T|^2+|C_T'|^2}{2|g_A|^2}\right)|\langle\|\hat{M}_J^A\|\rangle|^2\\
&\pm\sqrt{\frac{J+1}{J}}2\mathfrak{Re}\left[\frac{E_0}{q}\left(\frac{E_0-2E}{q}\frac{C_VC_A^*+C_V'C_A'^*}{2g_Vg_A^*}(1-\delta^{J^{(-)^J}})-\frac{m_e}{q}\frac{C_VC_T^*+C_V'C_T'^*}{2g_Vg_A^*}\right)\langle\|\hat{C}_J^V\|\rangle\langle\|\hat{M}_J^A\|\rangle^*\right]\\
&+\vec{\beta}\cdot\hat{\nu}\left\{\left[\left[1-\frac{2J+1}{J}\frac{E_0^2}{q^2}\left(1-\frac{J+1}{2J+1}2\mathfrak{Re}\delta^{J^{(-)^J}}\right)\right]\frac{|C_V|^2+|C_V'|^2}{2|g_V|^2}-\frac{|C_S|^2+|C_S'|^2}{2|g_V|^2}\right.\right.\\
&\left.\pm\frac{m_eE_0}{q^2}2\mathfrak{Re}\frac{C_VC_S^*+C_V'C_S'^*}{2|g_V|^2}\right]|\langle\|\hat{C}_J^V\|\rangle|^2+\left(-\frac{|C_A|^2+|C_A'|^2}{2|g_A|^2}+\frac{|C_T|^2+|C_T'|^2}{2|g_A|^2}\right)|\langle\|\hat{M}_J^A\|\rangle|^2\\
&\left.\mp\sqrt{\frac{J+1}{J}}2\mathfrak{Re}\left[\frac{E_0}{q}\left(\frac{E_0-2E}{q}\frac{C_VC_A^*+C_V'C_A'^*}{2g_Vg_A^*}(1-\delta^{J^{(-)^J}})-\frac{m_e}{q}\frac{C_VC_T^*+C_V'C_T'^*}{2g_Vg_A^*}\right)\langle\|\hat{C}_J^V\|\rangle\langle\|\hat{M}_J^A\|\rangle^*\right]\right\}\\
&+\frac{m_e}{E}\left\{\left[2\frac{m_eE_0}{q^2}\frac{|C_V|^2+|C_V'|^2}{2|g_V|^2}\pm\left(1+\frac{E_0^2}{q^2}\right)2\mathfrak{Re}\frac{C_VC_S^*+C_V'C_S'^*}{2|g_V|^2}\right]|\langle\|\hat{C}_J^V\|\rangle|^2\right.\\
&\mp2\mathfrak{Re}\frac{C_A^*C_T+C_A'^*C_T'}{2|g_A|^2}|\langle\|\hat{M}_J^A\|\rangle|^2\pm\sqrt{\frac{J+1}{J}}2\mathfrak{Re}\left[\left(\frac{m_eE_0}{q^2}\frac{C_VC_A^*+C_V'C_A'^*}{2g_Vg_A^*}(1-\delta^{J^{(-)^J}})\right.\right.\\
&\left.\left.\left.+\frac{E_0^2}{q^2}\frac{C_VC_T^*+C_V'C_T'^*}{2g_Vg_A^*}\right)\langle\|\hat{C}_J^V\|\rangle\langle\|\hat{M}_J^A\|\rangle^*\right]\right\}\\
&+\frac{E(E_0-E)}{q^2}[\beta^2-(\vec{\beta}\cdot\hat{\nu})^2]\left[\frac{J-1}{J}\frac{E_0^2}{q^2}\left(1+\frac{J+1}{J-1}2\mathfrak{Re}\delta^{J^{(-)^J}}\right)\frac{|C_V|^2+|C_V'|^2}{2|g_V|^2}|\langle\|\hat{C}_J^V\|\rangle|^2\right.\\
&\left.+\left(-\frac{|C_A|^2+|C_A'|^2}{2|g_A|^2}+\frac{|C_T|^2+|C_T'|^2}{2|g_A|^2}\right)|\langle\|\hat{M}_J^A\|\rangle|^2\right]+\mathcal{O}(\epsilon_M\epsilon_{qr}^{2J}),
\end{aligned}\tag{A12}$$

where $\delta^{J^{(-)^J}}$ is an NLO SM correction described in Ref. [31] ($\epsilon_M\epsilon_{qr}^{2J}$ presents SM recoiled nucleus corrections, which we will not display here, since they are relevant only for very light nuclei [31]). The multipole operators involved are $\hat{C}_J^V,\hat{M}_J^A\propto\epsilon_{qr}^J$, and the equivalents to the terms of Jackson, Treiman, and Wyld will be

$$\begin{aligned}\xi^{J^{(-)^J}} &= \Bigg\{\left[1+\frac{1}{J}\frac{E_0^2}{q^2}(1-(J+1)2\mathfrak{Re}\delta^{J^{(-)^J}})\right](|C_V|^2+|C_V'|^2)+(|C_S|^2+|C_S'|^2)\\&\quad\mp\frac{m_eE_0}{q^2}2\mathfrak{Re}(C_VC_S^*+C_V'C_S'^*)\Bigg\}\frac{|\langle\|\hat{C}_J^V\|\rangle|^2}{|g_V|^2}\\&\quad+(|C_A|^2+|C_A'|^2+|C_T|^2+|C_T'|^2)\frac{|\langle\|\hat{M}_J^A\|\rangle|^2}{|g_A|^2}\\&\quad\pm\sqrt{\frac{J+1}{J}}2\mathfrak{Re}\Bigg\{\left[\frac{E_0(E_0-2E)}{q^2}(C_VC_A^*+C_V'C_A'^*)(1-\delta^{J^{(-)^J}})\right.\\&\quad\left.-\frac{m_eE_0}{q^2}(C_VC_T^*+C_V'C_T'^*)\right]\frac{\langle\|\hat{C}_J^V\|\rangle\langle\|\hat{M}_J^A\|\rangle^*}{g_Vg_A^*}\Bigg\},\end{aligned}\tag{A13a}$$

$$\begin{aligned}a^{J^{(-)^J}}\xi^{J^{(-)^J}} &= \Bigg\{\left[1-\frac{2J+1}{J}\frac{E_0^2}{q^2}\left(1-\frac{J+1}{2J+1}2\mathfrak{Re}\delta^{J^{(-)^J}}\right)\right](|C_V|^2+|C_V'|^2)-(|C_S|^2+|C_S'|^2)\\&\quad\pm\frac{m_eE_0}{q^2}2\mathfrak{Re}(C_VC_S^*+C_V'C_S'^*)\Bigg\}\frac{|\langle\|\hat{C}_J^V\|\rangle|^2}{|g_V|^2}\\&\quad+(-|C_A|^2-|C_A'|^2+|C_T|^2+|C_T'|^2)\frac{|\langle\|\hat{M}_J^A\|\rangle|^2}{|g_A|^2}\\&\quad\mp\sqrt{\frac{J+1}{J}}2\mathfrak{Re}\Bigg\{\left[\frac{E_0(E_0-2E)}{q^2}(C_VC_A^*+C_V'C_A'^*)(1-\delta^{J^{(-)^J}})\right.\\&\quad\left.-\frac{m_eE_0}{q^2}(C_VC_T^*+C_V'C_T'^*)\right]\frac{\langle\|\hat{C}_J^V\|\rangle\langle\|\hat{M}_J^A\|\rangle^*}{g_Vg_A^*}\Bigg\},\end{aligned}\tag{A13b}$$

$$\begin{aligned}b^{J^{(-)^J}}\xi^{J^{(-)^J}} &= \left[2\frac{m_eE_0}{q^2}(|C_V|^2+|C_V'|^2)\pm\left(1+\frac{E_0^2}{q^2}\right)2\mathfrak{Re}(C_VC_S^*+C_V'C_S'^*)\right]\frac{|\langle\|\hat{C}_J^V\|\rangle|^2}{|g_V|^2}\\&\quad\mp 2\mathfrak{Re}(C_A^*C_T+C_A'^*C_T')\frac{|\langle\|\hat{M}_J^A\|\rangle|^2}{|g_A|^2}\\&\quad\pm\sqrt{\frac{J+1}{J}}2\mathfrak{Re}\Bigg\{\left[\frac{m_eE_0}{q^2}(C_VC_A^*+C_V'C_A'^*)(1-\delta^{J^{(-)^J}})\right.\\&\quad\left.+\frac{E_0^2}{q^2}(C_VC_T^*+C_V'C_T'^*)\right]\frac{\langle\|\hat{C}_J^V\|\rangle\langle\|\hat{M}_J^A\|\rangle^*}{g_Vg_A^*}\Bigg\}.\end{aligned}\tag{A13c}$$

APPENDIX B: TENSOR SEMILEPTONIC NUCLEAR PROCESS

Here we derive explicitly the pure tensor multipole expansion for semileptonic nuclear processes. We write the tensor lepton current in its most general form [24,25], adjusted to the nowadays convention [10]:

$$j_{\mu\nu}(\vec{r})=\bar{\psi}'(\vec{r})\sigma_{\mu\nu}(C_T-C_T'\gamma_5)\psi(\vec{r}),\tag{B1}$$

where $\psi^{(\prime)}$ are fermion fields, and γ_5 ($\sigma_{\mu\nu}$) is the fifth gamma matrix (the commutator of the gamma matrices) (see Appendix G for conventions). The coupling constants $C_{\mathrm{sym}}^{(\prime)}$ ($\mathrm{sym}\in\{S,P,V,A,T\}$) are real if time reversal invariance is preserved in the process, but this will not be assumed in the following. Assuming the leptons have a plane wave character (interaction with the nucleus will be inserted perturbatively), the general matrix element can be written as $\langle f|j_{\mu\nu}(\vec{r})|i\rangle\equiv l_{\mu\nu}e^{-i\vec{q}\cdot\vec{r}}$, where $\vec{q}\equiv\vec{k}_f-\vec{k}_i$ is the momentum transfer, and $l_{\mu\nu}$ is defined as $l_{\mu\nu}=\frac{1}{\Omega}\bar{l}'(\vec{k}')\sigma_{\mu\nu}(C_T-C_T'\gamma_5)l(\vec{k})$, where $l,l'\in\{u,v\}$.

Using Wigner-Eckart theorem [54], we distract from Eq. (4) in the main text, a general result for any semileptonic nuclear tensor process in terms of reduced matrix elements of the multipole operators:

$$\begin{aligned}\sum_{M_i}\sum_{M_f}|\langle f|\hat{H}^T_{\mathrm{w}}|i\rangle|^2 = 4\pi\Bigg\{&\sum_{J=0}^{\infty}\Bigg[l_3^T l_3^{T*}|\langle\|\hat{L}_J^T\|\rangle|^2 + l_3^{T'} l_3^{T'*}|\langle\|\hat{L}_J^{T'}\|\rangle|^2 + 2\Re\mathrm{e}(l_3^T l_3^{T'*}\langle\|\hat{L}_J^T\|\rangle\langle\|\hat{L}_J^{T'}\|\rangle^*)\Bigg]\\
&+\frac{1}{2}\sum_{J=1}^{\infty}\Bigg[(\vec{l}^T\cdot\vec{l}^{T*} - l_3^T l_3^{T*})(|\langle\|\hat{E}_J^T\|\rangle|^2 + |\langle\|\hat{M}_J^T\|\rangle|^2)\\
&+(\vec{l}^{T'}\cdot\vec{l}^{T'*} - l_3^{T'} l_3^{T'*})(|\langle\|\hat{E}_J^{T'}\|\rangle|^2 + |\langle\|\hat{M}_J^{T'}\|\rangle|^2)\\
&+2\Re\mathrm{e}[(\vec{l}^T\cdot\vec{l}^{T'*} - l_3^T l_3^{T'*})(\langle\|\hat{E}_J^T\|\rangle\langle\|\hat{E}_J^{T'}\|\rangle^* + \langle\|\hat{M}_J^T\|\rangle\langle\|\hat{M}_J^{T'}\|\rangle^*)]\\
&-2\Re\mathrm{e}[i(\vec{l}^T\times\vec{l}^{T*})_3\langle\|\hat{E}_J^T\|\rangle\langle\|\hat{M}_J^T\|\rangle^* + i(\vec{l}^{T'}\times\vec{l}^{T'*})_3\langle\|\hat{E}_J^{T'}\|\rangle\langle\|\hat{M}_J^{T'}\|\rangle^*\\
&+i(\vec{l}^T\times\vec{l}^{T'*})_3(\langle\|\hat{E}_J^T\|\rangle\langle\|\hat{M}_J^{T'}\|\rangle^* + \langle\|\hat{M}_J^T\|\rangle\langle\|\hat{E}_J^{T'}\|\rangle^*)]\Bigg]\Bigg\}.\end{aligned}\tag{B2}$$

Summing over the lepton spins and substituting the lepton traces (see Appendix F) produces a general expression for a semileptonic process of tensor symmetry between any two nuclear states. After taking into account also the parity selection rules, as well as the relation $\hat{E}_J = \sqrt{\frac{J+1}{J}}\hat{L}_J + \mathcal{O}((qR)^{J+1})$ for $J > 0$, where $\hat{L}_J$ is $\mathcal{O}((qR)^{J-1})$ [21], the summation over the lepton spins is reduced to [see Eq. (17) in the main text for the definition of $\Theta(q,\vec{\beta}\cdot\hat{\nu})$]:

$$\begin{aligned}\Theta(q,\vec{\beta}\cdot\hat{\nu}) \approx \frac{|C_T|^2+|C_T'|^2}{g_T^2}\Bigg\{&(1+\vec{\beta}\cdot\hat{\nu} - 2(\hat{\nu}\cdot\hat{q})(\vec{\beta}\cdot\hat{q}))(|\langle\|\hat{L}_0^T\|\rangle|^2 + |\langle\|\hat{L}_0^{T'}\|\rangle|^2)\\
&+\sum_{J=1}^{\infty}\Bigg[\frac{2J+1}{J}\left(1+\frac{J}{2J+1}\vec{\beta}\cdot\hat{\nu} - \frac{J-1}{2J+1}(\hat{\nu}\cdot\hat{q})(\vec{\beta}\cdot\hat{q})\right)(|\langle\|\hat{L}_J^T\|\rangle|^2 + |\langle\|\hat{L}_J^{T'}\|\rangle|^2)\\
&+(1+(\hat{\nu}\cdot\hat{q})(\vec{\beta}\cdot\hat{q}))(|\langle\|\hat{M}_J^T\|\rangle|^2 + |\langle\|\hat{M}_J^{T'}\|\rangle|^2)\\
&+\sqrt{\frac{J+1}{J}}\hat{q}\cdot(\hat{\nu}+\vec{\beta})2\Re\mathrm{e}(\langle\|\hat{L}_J^T\|\rangle\langle\|\hat{M}_J^{T'}\|\rangle^* + \langle\|\hat{M}_J^T\|\rangle\langle\|\hat{L}_J^{T'}\|\rangle^*)\Bigg]\Bigg\}.\end{aligned}\tag{B3}$$

Finally, using the connection from Eq. (10) in the main text, we find the leading order BSM expression:

$$\begin{aligned}\Theta(q,\vec{\beta}\cdot\hat{\nu}) \approx \frac{|C_T|^2+|C_T'|^2}{2g_A^2}\Bigg\{&(1+\vec{\beta}\cdot\hat{\nu} - 2(\hat{\nu}\cdot\hat{q})(\vec{\beta}\cdot\hat{q}))|\langle\|\hat{L}_0^A\|\rangle|^2\\
&+\sum_{J=1}^{\infty}\Bigg[\frac{2J+1}{J}\left(1+\frac{J}{2J+1}\vec{\beta}\cdot\hat{\nu} - \frac{J-1}{2J+1}(\hat{\nu}\cdot\hat{q})(\vec{\beta}\cdot\hat{q})\right)|\langle\|\hat{L}_J^A\|\rangle|^2(1+(\hat{\nu}\cdot\hat{q})(\vec{\beta}\cdot\hat{q}))|\langle\|\hat{M}_J^A\|\rangle|^2\Bigg]\Bigg\}.\end{aligned}\tag{B4}$$

This is a general result that holds for any β-decay transition, and, up to the signs in the lepton traces, for any semileptonic nuclear process (see Appendix F for more details), including different types of BSM physics. It yields the pure tensor terms presented in Appendix A (see Appendix C for interference terms).

APPENDIX C: TENSOR FIERZ TERM

To complete the discussion, the full probe-nucleus interaction Hamiltonian $\hat{H}_{\mathrm{w}} = \hat{H}_{\mathrm{w}}^{\mathrm{SM}} + \hat{H}_{\mathrm{w}}^{\mathrm{BSM}}$ should be considered. This leads to an interference term, known as the Fierz term, involving both SM and BSM Hamiltonians. The matrix element of the SM Hamiltonian $\hat{H}_{\mathrm{w}}^{\mathrm{SM}} = \hat{H}_{\mathrm{w}}^V + \hat{H}_{\mathrm{w}}^A$, where $\hat{H}_{\mathrm{w}}^{V(A)} = \int d^3r j_\mu^{V(A)}(\vec{r})\mathcal{J}^{V(A)\mu}(\vec{r})$ is its polar-vector (axial-vector) part, can be written as [21]

$$\begin{aligned}\langle f|\hat{H}_{\mathrm{w}}^{V(A)}|i\rangle &= \sum_{J=0}^{\infty}\sqrt{4\pi(2J+1)}\,(-i)^J[l_0^{V(A)}\langle f|\hat{C}_{J0}^{V(A)}|i\rangle - l_3^{V(A)}\langle f|\hat{L}_{J0}^{V(A)}|i\rangle] \\ &\quad + \sum_{J=1}^{\infty}\sqrt{2\pi(2J+1)}\,(-i)^J\sum_{\lambda=\pm1}l_\lambda^{V(A)}\langle f|\hat{E}_{J,-\lambda}^{V(A)} + \lambda\hat{M}_{J,-\lambda}^{V(A)}|i\rangle, \end{aligned} \tag{C1}$$

where the superscript $V(A)$ denotes a multipole operator [Eq. (5) in the main text] calculated with the polar-vector (axial-vector) nuclear current (described in Appendix E), and $l_\mu^V = \frac{1}{\Omega}\bar{l}'(\vec{k}')\gamma_\mu(C_V - C'_V\gamma_5)l(\vec{k})$ ($l_\mu^A = \frac{1}{\Omega}\bar{l}'(\vec{k}')\gamma_\mu(C'_A - C_A\gamma_5)l(\vec{k})$).

For the tensor BSM case, $\hat{H}_{\mathrm{w}} = \hat{H}_{\mathrm{w}}^{\mathrm{SM}} + \hat{H}_{\mathrm{w}}^T$, following both multipole expansions for $V - A$ [Eq. (C1)] and tensor couplings [Eq. (4) in the main text], a general interference term for any semileptonic nuclear process, involving both SM currents and BSM tensor currents, will be

$$\begin{aligned}\sum_{M_i}\sum_{M_f}2\mathfrak{Re}(\langle f|\hat{H}_{\mathrm{w}}^{V(A)}|i\rangle\langle f|\hat{H}_{\mathrm{w}}^T|i\rangle^*) &= 8\pi\mathfrak{Re}\Bigg\{\sum_{J=0}^{\infty}[l_3^{V(A)}l_3^{T*}\langle\|\hat{L}_J^{V(A)}\|\rangle\langle\|\hat{L}_J^T\|\rangle^* + l_3^{V(A)}l_3^{T'*}\langle\|\hat{L}_J^{V(A)}\|\rangle\langle\|\hat{L}_J^{T'}\|\rangle^* \\ &\quad - l_0^{V(A)}l_3^{T*}\langle\|\hat{C}_J^{V(A)}\|\rangle\langle\|\hat{L}_J^T\|\rangle^* - l_0^{V(A)}l_3^{T'*}\langle\|\hat{C}_J^{V(A)}\|\rangle\langle\|\hat{L}_J^{T'}\|\rangle^*] \\ &\quad + \frac{1}{2}\sum_{J=1}^{\infty}[(\vec{l}^{V(A)}\cdot\vec{l}^{T*} - l_3^{V(A)}\cdot l_3^{T*})(\langle\|\hat{E}_J^{V(A)}\|\rangle\langle\|\hat{E}_J^T\|\rangle^* + \langle\|\hat{M}_J^{V(A)}\|\rangle\langle\|\hat{M}_J^T\|\rangle^*) \\ &\quad + (\vec{l}^{V(A)}\cdot\vec{l}^{T'*} - l_3^{V(A)}l_3^{T'*})(\langle\|\hat{E}_J^{V(A)}\|\rangle\langle\|\hat{E}_J^{T'}\|\rangle^* + \langle\|\hat{M}_J^{V(A)}\|\rangle\langle\|\hat{M}_J^{T'}\|\rangle^*) \\ &\quad - i(\vec{l}^{V(A)}\times\vec{l}^{T*})_3(\langle\|\hat{E}_J^{V(A)}\|\rangle\langle\|\hat{M}_J^T\|\rangle^* + \langle\|\hat{M}_J^{V(A)}\|\rangle\langle\|\hat{E}_J^T\|\rangle^*) \\ &\quad - i(\vec{l}^{V(A)}\times\vec{l}^{T'*})_3(\langle\|\hat{E}_J^{V(A)}\|\rangle\langle\|\hat{M}_J^{T'}\|\rangle^* + \langle\|\hat{M}_J^{V(A)}\|\rangle\langle\|\hat{E}_J^{T'}\|\rangle^*)]\Bigg\}. \end{aligned} \tag{C2}$$

Summing over the lepton spins, substituting the lepton traces (Appendix F), and considering parity selection rules lead to the Fierz vector-tensor (axial-tensor) interference term. After taking into account parity selection rules, as well as the relation $\hat{E}_J \approx \sqrt{\frac{J+1}{J}}\hat{L}_J$ (see Appendix B), one stays with the full tensor Fierz term:

$$\begin{aligned}\Theta(q,\vec{\beta}\cdot\hat{\nu})^{VT,AT} &\approx \mp\frac{m_e}{E}\sqrt{2}\mathfrak{Re}\Bigg\{i\Bigg[\frac{C_AC_T^* + C'_AC'^*_T}{g_Ag_T^*}\langle\|\hat{L}_0^A\|\rangle\langle\|\hat{L}_0^T\|\rangle^* + \frac{C_VC_T^* + C'_VC'^*_T}{g_Vg_T^*}\langle\|\hat{L}_0^V\|\rangle\langle\|\hat{L}_0^{T'}\|\rangle^* \\ &\quad - \frac{C_AC_T^* + C'_AC'^*_T}{g_Ag_T^*}(\hat{\nu}\cdot\hat{q})\langle\|\hat{C}_0^A\|\rangle\langle\|\hat{L}_0^T\|\rangle^* - \frac{C_VC_T^* + C'_VC'^*_T}{g_Vg_T^*}(\hat{\nu}\cdot\hat{q})\langle\|\hat{C}_0^V\|\rangle\langle\|\hat{L}_0^{T'}\|\rangle^*\Bigg] \\ &\quad + i\sum_{J=1}^{\infty}\Bigg[\frac{C_AC_T^* + C'_AC'^*_T}{g_Ag_T^*}\frac{2J+1}{J}\langle\|\hat{L}_J^A\|\rangle\langle\|\hat{L}_J^T\|\rangle^* + \frac{C_VC_T^* + C'_VC'^*_T}{g_Vg_T^*}\frac{2J+1}{J}\langle\|\hat{L}_J^V\|\rangle\langle\|\hat{L}_J^{T'}\|\rangle^* \\ &\quad - \frac{C_AC_T^* + C'_AC'^*_T}{g_Ag_T^*}(\hat{\nu}\cdot\hat{q})\langle\|\hat{C}_J^A\|\rangle\langle\|\hat{L}_J^T\|\rangle^* - \frac{C_VC_T^* + C'_VC'^*_T}{g_Vg_T^*}(\hat{\nu}\cdot\hat{q})\langle\|\hat{C}_J^V\|\rangle\langle\|\hat{L}_J^{T'}\|\rangle^* \\ &\quad + \frac{C_AC_T^* + C'_AC'^*_T}{g_Ag_T^*}\langle\|\hat{M}_J^A\|\rangle\langle\|\hat{M}_J^T\|\rangle^* + \frac{C_VC_T^* + C'_VC'^*_T}{g_Vg_T^*}\langle\|\hat{M}_J^V\|\rangle\langle\|\hat{M}_J^{T'}\|\rangle^* \\ &\quad + \sqrt{\frac{J+1}{J}}(\hat{\nu}\cdot\hat{q})\frac{C_VC_T^* + C'_VC'^*_T}{g_Vg_T^*}(\langle\|\hat{L}_J^V\|\rangle\langle\|\hat{M}_J^T\|\rangle^* + \langle\|\hat{M}_J^V\|\rangle\langle\|\hat{L}_J^T\|\rangle^*) \\ &\quad + \sqrt{\frac{J+1}{J}}(\hat{\nu}\cdot\hat{q})\frac{C_AC_T^* + C'_AC'^*_T}{g_Ag_T^*}(\langle\|\hat{L}_J^A\|\rangle\langle\|\hat{M}_J^{T'}\|\rangle^* + \langle\|\hat{M}_J^A\|\rangle\langle\|\hat{L}_J^{T'}\|\rangle^*)\Bigg]\Bigg\}, \end{aligned} \tag{C3}$$

or its BSM leading orders [using Eq. (10) in the main text]:

$$\Theta(q,\vec{\beta}\cdot\hat{\nu})^{VT,AT}\approx\pm\frac{m_e}{E}2\mathfrak{Re}\left\{\frac{C_A^*C_T+C_A'^*C_T'}{2|g_A|^2}[|\langle\|\hat{L}_0^A\|\rangle|^2-(\hat{\nu}\cdot\hat{q})\langle\|\hat{C}_0^A\|\rangle^*\langle\|\hat{L}_0^A\|\rangle]\right.$$
$$+\sum_{J=1}^{\infty}\left[\frac{C_A^*C_T+C_A'^*C_T'}{2|g_A|^2}\left(\frac{2J+1}{J}|\langle\|\hat{L}_J^A\|\rangle|^2-(\hat{\nu}\cdot\hat{q})\langle\|\hat{C}_J^A\|\rangle^*\langle\|\hat{L}_J^A\|\rangle+|\langle\|\hat{M}_J^A\|\rangle|^2\right)\right.$$
$$\left.\left.+\sqrt{\frac{J+1}{J}}(\hat{\nu}\cdot\hat{q})\frac{C_V^*C_T+C_V'^*C_T'}{2|g_V|^2}\langle\|\hat{L}_J^V\|\rangle^*\langle\|\hat{M}_J^A\|\rangle\right]\right\}. \quad \text{(C4)}$$

This yields the tensor Fierz terms presented in Appendix A. Note that a complete BSM discussion, affecting the full Fierz term, will include also scalar and pseudoscalar terms, described in Appendix D.

APPENDIX D: SCALAR AND PSEUDOSCALAR MULTIPOLE DECOMPOSITION

Here we derive the scalar and pseudoscalar terms within the multipole decomposition, including their interference terms with the SM Hamiltonian and the BSM tensor Hamiltonian. We start from the scalar Hamiltonian, $\hat{H}_{\rm w}^S=\int d^3r j^S(\vec{r})\mathcal{J}^S(\vec{r})$, with the scalar lepton current $j^S(\vec{r})=\bar{\psi}'(\vec{r})(C_S-C_S'\gamma_5)\psi(\vec{r})$ [24,25]. Its matrix element can be written using the multipole expansion:

$$\langle f|\hat{H}_{\rm w}^S|i\rangle=\sum_{J=0}^{\infty}\sqrt{4\pi(2J+1)}\,(-i)^J l^S\langle f|\hat{C}_J^S|i\rangle, \quad \text{(D1)}$$

where $l^S=\frac{1}{\Omega}\bar{l}'(\vec{k}')(C_S-C_S'\gamma_5)l(\vec{k})$, yielding the term

$$\sum_{M_i}\sum_{M_f}|\langle f|\hat{H}_{\rm w}^S|i\rangle|^2=4\pi\sum_{J=0}^{\infty}l^S l^{S*}|\langle\|\hat{C}_J^S\|\rangle|^2, \quad \text{(D2)}$$

as well as the Fierz interference terms,

$$\sum_{M_i}\sum_{M_f}2\mathfrak{Re}(\langle f|\hat{H}_{\rm w}^{V(A)}|i\rangle\langle f|\hat{H}_{\rm w}^S|i\rangle^*)$$
$$=8\pi\sum_{J=0}^{\infty}\mathfrak{Re}[l_0^{V(A)}l^{S*}\langle\|\hat{C}_J^{V(A)}\|\rangle\langle\|\hat{C}_J^S\|\rangle^*$$
$$-l_3^{V(A)}l^{S*}\langle\|\hat{L}_J^{V(A)}\|\rangle\langle\|\hat{C}_J^S\|\rangle^*], \quad \text{(D3)}$$

$$\sum_{M_i}\sum_{M_f}2\mathfrak{Re}(\langle f|\hat{H}_{\rm w}^T|i\rangle\langle f|\hat{H}_{\rm w}^S|i\rangle^*)$$
$$=8\pi\sum_{J=0}^{\infty}\mathfrak{Re}[-l_3^T l^{S*}\langle\|\hat{L}_J^T\|\rangle\langle\|\hat{C}_J^S\|\rangle^*$$
$$-l_3^{T'}l^{S*}\langle\|\hat{L}_J^{T'}\|\rangle\langle\|\hat{C}_J^S\|\rangle^*], \quad \text{(D4)}$$

where $\hat{C}_J^S$ is the Coulomb multipole operator, defined in Eq. (5) in the main text, calculated with the scalar nuclear current which will be described following. The pseudoscalar coupling, originating from the pseudoscalar lepton current $j^P=\bar{\psi}'(\vec{r})(C_P\gamma_5-C_P')\psi(\vec{r})$, will have the same expansion, only with $\hat{H}_{\rm w}^P$, $\hat{C}_J^P$ and $l^P=\frac{1}{\Omega}\bar{l}'(\vec{k}')(C_P\gamma_5-C_P')l(\vec{k})$, instead of $\hat{H}_{\rm w}^S$, $\hat{C}_J^S$, and l^S. Note that although it is possible to calculate a scalar-pseudoscalar interference term, according to parity selection rules, there will not be any transition that will involve this kind of term. Summing over the lepton spins, substituting the lepton traces (Appendix F), and considering parity selection rules, lead to the scalar and pseudoscalar expression:

$$\Theta(q,\vec{\beta}\cdot\hat{\nu})^{S,P}=\sum_{J=0}^{\infty}\left\{(1-\vec{\beta}\cdot\hat{\nu})\left[\frac{|C_S|^2+|C_S'|^2}{2|g_S|^2}|\langle\|\hat{C}_J^S\|\rangle|^2+\frac{|C_P|^2+|C_P'|^2}{2|g_P|^2}|\langle\|\hat{C}_J^P\|\rangle|^2\right]\right.$$
$$\pm2\frac{m_e}{E}\mathfrak{Re}\left[\frac{C_VC_S^*+C_V'C_S'^*}{2g_Vg_S^*}\langle\|\hat{C}_J^V\|\rangle\langle\|\hat{C}_J^S\|\rangle^*-\frac{C_AC_P^*+C_A'C_P'^*}{2g_Ag_P^*}\langle\|\hat{C}_J^A\|\rangle\langle\|\hat{C}_J^P\|\rangle^*\right]$$
$$\pm2\frac{m_e}{E}(\hat{\nu}\cdot\hat{q})\mathfrak{Re}\left[\frac{C_VC_S^*+C_V'C_S'^*}{2g_Vg_S^*}\langle\|\hat{L}_J^V\|\rangle\langle\|\hat{C}_J^S\|\rangle^*-\frac{C_AC_P^*+C_A'C_P'^*}{2g_Ag_P^*}\langle\|\hat{L}_J^A\|\rangle\langle\|\hat{C}_J^P\|\rangle^*\right]$$
$$\left.\pm2\sqrt{2}\hat{q}\cdot(\hat{\nu}-\vec{\beta})\mathfrak{Re}\left[i\frac{C_TC_S^*+C_T'C_S'^*}{2g_Tg_S^*}\langle\|\hat{L}_J^{T'}\|\rangle\langle\|\hat{C}_J^S\|\rangle^*+i\frac{C_TC_P^*+C_T'C_P'^*}{2g_Tg_P^*}\langle\|\hat{L}_J^T\|\rangle\langle\|\hat{C}_J^P\|\rangle^*\right]\right\}. \quad \text{(D5)}$$

In order to complete this discussion, one needs to calculate the scalar and pseudoscalar multipole operators. The general forms of the single-nucleon matrix element of the scalar and pseudoscalar parts of the charge-changing weak current are [34]

$$\langle \vec{p}', \sigma', \rho' | \mathcal{J}^S | \vec{p}, \sigma, \rho \rangle = \frac{1}{\Omega} g_S(q^2) \bar{u}(\vec{p}', \sigma') \eta^{+}_{\rho'} \tau^{\pm} \eta_\rho u(\vec{p}, \sigma) + \mathcal{O}(\epsilon^2_{\mathrm{recoil}}), \tag{D6a}$$

$$\langle \vec{p}', \sigma', \rho' | \mathcal{J}^P | \vec{p}, \sigma, \rho \rangle = \frac{1}{\Omega} g_P(q^2) \bar{u}(\vec{p}', \sigma') \eta^{+}_{\rho'} \gamma_5 \tau^{\pm} \eta_\rho u(\vec{p}, \sigma) + \mathcal{O}(\epsilon^2_{\mathrm{recoil}}). \tag{D6b}$$

Expanding the needed matrix elements in the inverse mass (following what we did to the tensor matrix element in Sec. III), one obtains the following nonrelativistic expansion for the scalar and pseudoscalar nuclear current densities[8]:

$$\mathcal{J}^S(\vec{r}) = g_S \sum_{j=1}^{A} \tau^{\pm}_j \delta^{(3)}(\vec{r} - \vec{r}_j) + \mathcal{O}(\epsilon^2_{\mathrm{NR}}) = \frac{g_S}{g_V} \mathcal{J}^V_0(\vec{r}) + \mathcal{O}(\epsilon^2_{\mathrm{NR}}, \epsilon_{\mathrm{recoil}}), \tag{D7a}$$

$$\mathcal{J}^P(\vec{r}) = -\frac{i}{2m_N} g_P \sum_{j=1}^{A} \vec{\nabla} \delta^{(3)}(\vec{r} - \vec{r}_j) \cdot \vec{\sigma}_j \tau^{\pm}_j + \mathcal{O}(\epsilon^2_{\mathrm{NR}}), \tag{D7b}$$

where the scalar current is proportional to the temporal part (charge) of the 4-vector polar-vector current $\mathcal{J}^V_0$. The multipole operators [Eq. (5) in the main text], calculated with the scalar and pseudoscalar symmetry contributions to the weak nuclear current, can be written as a sum of one-body operators:

$$\hat{C}^S_J(q) = \frac{g_S}{g_V} \hat{C}^V_J(q) + \mathcal{O}(\epsilon^J_{qr} \epsilon^2_{\mathrm{NR}}, \epsilon^J_{qr} \epsilon_{\mathrm{recoil}}), \tag{D8a}$$

$$\hat{C}^P_J(q) = \frac{iq}{2m_N} g_P \sum_{j=1}^{A} \left[\frac{1}{q} \vec{\nabla} M_J(q\vec{r}_j) \right] \cdot \vec{\sigma}_j \tau^{\pm}_j + \mathcal{O}(\epsilon^J_{qr} \epsilon^2_{\mathrm{NR}}) = \frac{q}{2m_N} \frac{g_P}{g_A} \hat{L}^A_J(q) + \mathcal{O}(\epsilon^{J-1}_{qr} \epsilon^2_{\mathrm{NR}}, \epsilon^J_{qr} \epsilon_{\mathrm{recoil}}), \tag{D8b}$$

where $\hat{C}^V_J$ is the SM polar-vector Coulomb multipole operator [Eq. (14) in the main text], and $\hat{L}^A_J$ is the SM axial-vector longitudinal multipole operator [Eq. (11a) in the main text]. Unlike Eq. (D8a), originating from the relation between the scalar and polar-vector charges, Eq. (D8b) is an accidental relation between the multipoles.

Despite the fact that the pseudoscalar multipole is suppressed by $\epsilon_{\mathrm{recoil}}$, we consider its contribution a BSM leading order since the pseudoscalar charge $g_P = g_A \frac{M_n + M_p}{m_d + m_u} = 349(9)$ [39] is two orders of magnitude larger than g_A. Therefore, the scalar and pseudoscalar contributions to the BSM leading order are as follows:

$$\begin{aligned} \Theta(q, \vec{\beta} \cdot \hat{\nu})^{S,P} \approx \sum_{J=0}^{\infty} \Bigg\{ & (1 - \vec{\beta} \cdot \hat{\nu}) \left[\frac{|C_S|^2 + |C'_S|^2}{2|g_V|^2} |\langle \| \hat{C}^V_J \| \rangle|^2 + \left(\frac{q}{2m_N} \right)^2 \frac{|C_P|^2 + |C'_P|^2}{2|g_A|^2} |\langle \| \hat{L}^A_J \| \rangle|^2 \right] \\ & \pm 2 \frac{m_e}{E} \mathfrak{Re} \left[\frac{C_V C^*_S + C'_V C'^*_S}{2|g_V|^2} |\langle \| \hat{C}^V_J \| \rangle|^2 - \frac{q}{2m_N} \frac{C_A C^*_P + C'_A C'^*_P}{2|g_A|^2} \langle \| \hat{C}^A_J \| \rangle \langle \| \hat{L}^A_J \| \rangle^* \right] \\ & \pm 2 \frac{m_e}{E} (\hat{\nu} \cdot \hat{q}) \mathfrak{Re} \left[\frac{C_V C^*_S + C'_V C'^*_S}{2|g_V|^2} \langle \| \hat{L}^V_J \| \rangle \langle \| \hat{C}^V_J \| \rangle^* - \frac{q}{2m_N} \frac{C_A C^*_P + C'_A C'^*_P}{2|g_A|^2} |\langle \| \hat{L}^A_J \| \rangle|^2 \right] \\ & \pm 2 \hat{q} \cdot (\hat{\nu} - \vec{\beta}) \mathfrak{Re} \left[\frac{q}{2m_N} \frac{C_T C^*_P + C'_T C'^*_P}{2|g_A|^2} |\langle \| \hat{L}^A_J \| \rangle|^2 \right] \Bigg\}. \end{aligned} \tag{D9}$$

This yields the scalar and pseudoscalar terms (including their interference Fierz terms) presented in Appendix A.

APPENDIX E: SECOND-CLASS MULTIPOLE OPERATORS

The general forms of the single-nucleon matrix elements of the vector and axial parts of the charge-changing weak current are (respectively) [34]

$$\langle \vec{p}', \sigma', \rho' | \mathcal{J}^V_\mu(0) | \vec{p}, \sigma, \rho \rangle = \frac{1}{\Omega} \bar{u}(\vec{p}', \sigma') \eta^{+}_{\rho'} \left[g_V(q^2) \gamma_\mu - i \frac{\tilde{g}_{T(V)}(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu + \frac{\tilde{g}_S(q^2)}{2m_N} q_\mu \right] \times \tau^{\pm} \eta_\rho u(\vec{p}, \sigma), \tag{E1a}$$

$$\langle \vec{p}', \sigma', \rho' | \mathcal{J}^A_\mu(0) | \vec{p}, \sigma, \rho \rangle = \frac{1}{\Omega} \bar{u}(\vec{p}', \sigma') \eta^{+}_{\rho'} \left[g_A(q^2) \gamma_\mu - i \frac{\tilde{g}_{T(A)}(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu + \frac{\tilde{g}_P(q^2)}{2m_N} q_\mu \right] \times \gamma_5 \tau^{\pm} \eta_\rho u(\vec{p}, \sigma). \tag{E1b}$$

In the SM, $g_V = 1$ up to second-order corrections in isospin breaking [37,38] as a result of the conservation of the vector

[8]A more accurate form will include second-class currents: $\mathcal{J}^S(\vec{r}) = \frac{g_S}{g_V + \frac{E_0}{2m_N} \tilde{g}_S} \mathcal{J}^V_0(\vec{r}) + \mathcal{O}(\epsilon^2_{\mathrm{NR}})$, where the second-class current form factor $\tilde{g}_S$ is suppressed by $\mathcal{O}(\epsilon_{\mathrm{recoil}})$.

current, and $g_A \approx 1.276 g_V$ [55,56]. The induced charges, $\tilde{g}_{T(V)}$, $\tilde{g}_S$, $\tilde{g}_{T(A)}$, and $\tilde{g}_P$ (not to be confused with the actual BSM charges, g_S, g_P, and g_T, which appear in the scalar, pseudoscalar, and tensor currents), contribute to the currents at the recoil order ϵ_{recoil} [34]. $\tilde{g}_{T(A)}$ is known also as the weak magnetism, and $\tilde{g}_P \sim -(\frac{2m_N}{m_\pi})^2 g_A$. $\tilde{g}_S$ and $\tilde{g}_{T(A)}$, known as second-class currents, do not exist in the SM, $\tilde{g}_S$ due to current conservation, and $\tilde{g}_{T(A)}$ because of G-parity considerations [36].

Substituting the explicit form of Dirac spinors and making a nonrelativistic expansion, leads to the explicit expressions for the multipole operators calculated with the vector and axial currents:

$$\hat{C}_J^{V[2c]}(q) = \left(g_V + \frac{E_0}{2m_N}\tilde{g}_S\right)\sum_{j=1}^{A} M_J(q\vec{r}_j)\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^J \epsilon_{\text{NR}}^2), \tag{E2a}$$

$$\hat{L}_J^{V[2c]}(q) = -\frac{q}{2m_N}\sum_{j=1}^{A}\left\{(g_V - \tilde{g}_S)M_J(q\vec{r}_j) - 2g_V\left[\frac{1}{q}\vec{\nabla}M_J(q\vec{r}_j)\right]\cdot\frac{1}{q}\vec{\nabla}\right\}\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\text{NR}}^2), \tag{E2b}$$

$$\hat{E}_J^{V[2c]}(q) = \frac{q}{m_N}\sum_{j=1}^{A}\left\{-ig_V\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right]\cdot\frac{1}{q}\vec{\nabla} + \frac{g_V + \tilde{g}_{T(V)}}{2}\vec{M}_{JJ1}(q\vec{r}_j)\cdot\vec{\sigma}_j\right\}\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\text{NR}}^2), \tag{E2c}$$

$$\hat{M}_J^{V[2c]}(q) = -\frac{iq}{m_N}\sum_{j=1}^{A}\left\{g_V\vec{M}_{JJ1}(q\vec{r}_j)\cdot\frac{1}{q}\vec{\nabla} + i\frac{g_V + \tilde{g}_{T(V)}}{2}\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right]\cdot\vec{\sigma}_j\right\}\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^J\epsilon_{\text{NR}}^2), \tag{E2d}$$

$$\hat{C}_J^{A[2c]}(q) = -\frac{iq}{m_N}\sum_{j=1}^{A}\left\{g_A M_J(q\vec{r}_j)\vec{\sigma}_j\cdot\frac{1}{q}\vec{\nabla} + \frac{1}{2}\left(g_A - \frac{E_0}{2m_N}\tilde{g}_P + \tilde{g}_{T(A)}\right)\left[\frac{1}{q}\vec{\nabla}M_J(q\vec{r})\right]\cdot\vec{\sigma}_j\right\}\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^J\epsilon_{\text{NR}}^2), \tag{E2e}$$

$$\hat{L}_J^{A[2c]}(q) = i\left[g_A + \left(\frac{q}{2m_N}\right)^2\tilde{g}_P - \frac{E_0}{2m_N}\tilde{g}_{T(A)}\right]\times\sum_{j=1}^{A}\left[\frac{1}{q}\vec{\nabla}M_J(q\vec{r}_j)\right]\cdot\vec{\sigma}_j\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\text{NR}}^2), \tag{E2f}$$

$$\hat{E}_J^{A[2c]}(q) = \left[g_A + \left(\frac{q}{2m_N}\right)^2\tilde{g}_P - \frac{E_0}{2m_N}\tilde{g}_{T(A)}\right]\times\sum_{j=1}^{A}\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right]\cdot\vec{\sigma}_j\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^{J-1}\epsilon_{\text{NR}}^2), \tag{E2g}$$

$$\hat{M}_J^{A[2c]}(q) = \left[g_A + \left(\frac{q}{2m_N}\right)^2\tilde{g}_P - \frac{E_0}{2m_N}\tilde{g}_{T(A)}\right]\times\sum_{j=1}^{A}\vec{M}_{JJ1}(q\vec{r}_j)\cdot\vec{\sigma}_j\tau_j^{\pm} + \mathcal{O}(\epsilon_{qr}^J\epsilon_{\text{NR}}^2), \tag{E2h}$$

where the superscript $V[2\text{c}]$ ($A[2\text{c}]$) indicates that the multipole operators are calculated using the vector (axial) current including its second-class part. M_J and $\vec{M}_{JL1}$ are defined in Eq. (6) in the main text. One can recognize that these multipole operators that include the second-class currents are actually the SM multipole operators with small changes:

$$\hat{C}_J^{V[2c]}(q) = \frac{g_V + \frac{E_0}{2m_N}\tilde{g}_S}{g_V}\hat{C}_J^V(q), \tag{E3a}$$

$$\hat{L}_J^{V[2c]}(q) = \hat{L}_J^V(q) + \frac{q}{2m_N}\frac{\tilde{g}_S}{g_V}\hat{C}_J^V(q), \tag{E3b}$$

$$\hat{C}_J^{A[2c]}(q) = \hat{C}_J^A(q) - \frac{q}{2m_N}\frac{\tilde{g}_{T(A)}}{g_A}\hat{L}_J^A(q), \tag{E3c}$$

$$\hat{O}_J^{A[2c]}(q) = \frac{g_A + (\frac{q}{2m_N})^2\tilde{g}_P - \frac{E_0}{2m_N}\tilde{g}_{T(A)}}{g_A + (\frac{q}{2m_N})^2\tilde{g}_P}\hat{O}_J^A(q), \quad \hat{O} \in \{\hat{L}, \hat{E}, \hat{M}\}. \tag{E3d}$$

The multipole operators $\hat{E}_J^V$ and $\hat{M}_J^V$ stay with no change for their leading orders when including second-class currents.

APPENDIX F: BSM LEPTON TRACES

As lepton traces produce the coefficients of the multipole expansion, they are essential for any specific calculation. Here we present BSM lepton traces for β-decays. Although they were derived here for β-decays, they should be the same for other semileptonic weak nuclear processes involving a neutrino or an antineutrino (as neutrino/antineutrino reaction and charged lepton capture), except for the sign, which might be different.

Note that the symmetry coefficients appearing in the lepton traces are the nucleon-level coefficients $C_{\text{sym}} \sim g_{\text{sym}} \cdot \epsilon_{\text{sym}}$. Since the quark-level nuclear currents already contain the g_{sym} form factors, when coming to use the lepton traces with the quark-level multipole operators we discuss in Sec. III, there is a need to make a small adjustment: a simple replacement of the following obtained lepton coefficients $C_{\text{sym}}^{(\prime)}$, with adjusted coefficients $\frac{C_{\text{sym}}^{(\prime)}}{g_{\text{sym}}}$, would satisfy this need.

Let us start with the tensor lepton traces:

$$\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_{\mu\nu} l^*_{\rho\sigma} = \frac{1}{2}\mathrm{Tr}\left[\sigma_{\mu\nu}(C_T - C'_T\gamma_5)\left(\frac{\gamma_\alpha \nu^\alpha}{2\nu}\right)(C^*_T + C'^*_T\gamma_5)\sigma_{\rho\sigma}\left(\frac{\gamma_\beta k^\beta + m_e}{2E}\right)\right]$$
$$= \frac{|C_T|^2 + |C'_T|^2}{2}\left[(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\nu}g_{\sigma\mu})\frac{\nu^\alpha k_\alpha}{\nu E} + g_{\sigma\mu}\left(\frac{\nu_\rho k_\nu}{\nu E} + \frac{\nu_\nu k_\rho}{\nu E}\right)\right.$$
$$\left. - g_{\sigma\nu}\left(\frac{\nu_\rho k_\mu}{\nu E} + \frac{\nu_\mu k_\rho}{\nu E}\right) + g_{\rho\nu}\left(\frac{\nu_\sigma k_\mu}{\nu E} + \frac{\nu_\mu k_\sigma}{\nu E}\right) - g_{\rho\mu}\left(\frac{\nu_\sigma k_\nu}{\nu E} + \frac{\nu_\nu k_\sigma}{\nu E}\right)\right]$$
$$+ i\frac{C_T C'^*_T + C'_T C^*_T}{2}\epsilon_{\mu\nu\gamma\delta}\left[g^\gamma_\rho g^\delta_\sigma \frac{\nu^\alpha k_\alpha}{\nu E} + g^\gamma_\sigma\left(\frac{\nu_\rho k^\delta}{\nu E} + \frac{\nu^\delta k_\rho}{\nu E}\right) + g^\delta_\rho\left(\frac{\nu_\sigma k^\gamma}{\nu E} + \frac{\nu^\gamma k_\sigma}{\nu E}\right)\right], \tag{F1}$$

where $g_{\mu\nu}$ is the metric and $\epsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric Levi-Civita tensor (see Appendix G for their conventions). Using the definitions of $l_i^{T^{(\prime)}}$ from Eq. (2) in the main text, we find that (notice that $\hat{q}$ is the third direction $\hat{z} \equiv \frac{\vec{q}}{|q|}$, and that one can write $\hat{q} = \frac{\vec{k}+\vec{\nu}}{q} = \frac{E\vec{\beta}+(E_0-E)\hat{\nu}}{q}$ to obtain recoil terms):

$$\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_3^T l_3^{T*} = (|C_T|^2 + |C'_T|^2)[1 + \vec{\beta}\cdot\hat{\nu} - 2(\hat{\nu}\cdot\hat{q})(\vec{\beta}\cdot\hat{q})], \tag{F2a}$$

$$\frac{1}{2}\frac{\Omega^2}{2}\sum_{\text{lepton spins}} (\vec{l}^T\cdot\vec{l}^{T*} - l_3^T l_3^{T*}) = (|C_T|^2 + |C'_T|^2)[1 + (\hat{\nu}\cdot\hat{q})(\vec{\beta}\cdot\hat{q})], \tag{F2b}$$

$$-\frac{i}{2}\frac{\Omega^2}{2}\sum_{\text{lepton spins}} (\vec{l}^T\times\vec{l}^{T*})_3 = (C_T C'^*_T + C'_T C^*_T)\hat{q}\cdot(\hat{\nu}+\vec{\beta}). \tag{F2c}$$

Replacing $l_i^T l_j^{T*}$ with $l_i^{T'} l_j^{T'*}$ produces the exact same expressions. However, replacing $l_i^T l_j^{T*}$ with $l_i^T l_j^{T'*}$ produces the same lepton terms, but with different symmetry coefficients—wherever the unmixed trace $l_i^T l_j^{T*}$ produces $|C_T|^2 + |C'_T|^2$, the mixed trace $l_i^T l_j^{T'*}$ will produce $C_T C'^*_T + C'_T C^*_T$, and vice versa.

Second, the mixed lepton traces for the tensor Fierz term, will be

$$\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l^A_\mu l^*_{\rho\sigma} = \frac{1}{2}\mathrm{Tr}\left[\gamma_\mu(C'_A - C_A\gamma_5)\left(\frac{\gamma_\alpha\nu^\alpha}{2\nu}\right)\right.$$
$$\left.\times(C^*_T + C'^*_T\gamma_5)\sigma_{\rho\sigma}\left(\frac{\gamma_\beta k^\beta \pm m_e}{2E}\right)\right]$$
$$= \pm i\frac{m_e}{E}\frac{C'_A C^*_T + C_A C'^*_T}{2}\left(g_{\mu\sigma}\frac{\nu_\rho}{\nu} - g_{\mu\rho}\frac{\nu_\sigma}{\nu}\right)$$
$$\pm\frac{m_e}{E}\frac{C_A C^*_T + C'_A C'^*_T}{2}\epsilon_{\mu\alpha\rho\sigma}\frac{\nu^\alpha}{\nu}, \tag{F3}$$

$$\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_3^A l_3^{T*} = \mp\frac{i}{\sqrt{2}}\frac{m_e}{E}(C_A C^*_T + C'_A C'^*_T), \tag{F4a}$$

$$-\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_0^A l_3^{T*} = \pm\frac{i}{\sqrt{2}}\frac{m_e}{E}(C_A C^*_T + C'_A C'^*_T)(\hat{\nu}\cdot\hat{q}), \tag{F4b}$$

$$\frac{1}{2}\frac{\Omega^2}{2}\sum_{\text{lepton spins}} (\vec{l}^A\cdot\vec{l}^{T*} - l_3^A l_3^{T*}) = \mp\frac{i}{\sqrt{2}}\frac{m_e}{E}(C_A C^*_T + C'_A C'^*_T), \tag{F4c}$$

$$-\frac{i}{2}\frac{\Omega^2}{2}\sum_{\text{lepton spins}} (\vec{l}^A\times\vec{l}^{T*})_3 = \mp\frac{i}{\sqrt{2}}\frac{m_e}{E}(C'_A C^*_T + C_A C'^*_T)(\hat{\nu}\cdot\hat{q}), \tag{F4d}$$

where $\pm$ are for $\beta^\mp$-decays. Replacing $l_i^A l_j^{T*}$ with $l_i^A l_j^{T'*}$ produces the same lepton terms, but with different symmetry coefficients: $C_A C^*_T + C'_A C'^*_T$ should be replaced with $C'_A C^*_T + C_A C'^*_T$, and vice versa. The interference with the polar-vector current will have the same expressions, only with the superscript V, C'_V, and C_V instead of the superscript A, C_A, and C'_A, respectively (note the replacement of $^{(\prime)}$).

Last, the scalar and their interference traces will be

$$\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l^S l^{S*} = \frac{|C_S|^2 + |C'_S|^2}{2}(1 - \vec{\beta}\cdot\hat{\nu}), \tag{F5a}$$

$$\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_0^V l^{S*} = \pm\frac{C_V C^*_S + C'_V C'^*_S}{2}\frac{m_e}{E}, \tag{F5b}$$

$$\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_0^A l^{S*} = \pm\frac{C'_A C^*_S + C_A C'^*_S}{2}\frac{m_e}{E}, \tag{F5c}$$

$$-\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_3^V l^{S*} = \pm\frac{C_V C_S^* + C_V' C_S'^*}{2}(\hat{\nu}\cdot\hat{q})\frac{m_e}{E}, \tag{F5d}$$

$$-\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_3^A l^{S*} = \pm\frac{C_A' C_S^* + C_A C_S'^*}{2}(\hat{\nu}\cdot\hat{q})\frac{m_e}{E}, \tag{F5e}$$

$$-\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_3^T l^{S*} = \mp i\sqrt{2}\frac{C_T' C_S^* + C_T C_S'^*}{2}\hat{q}\cdot(\hat{\nu}-\vec{\beta}), \tag{F5f}$$

$$-\frac{\Omega^2}{2}\sum_{\text{lepton spins}} l_3^{T'} l^{S*} = \pm i\sqrt{2}\frac{C_T C_S^* + C_T' C_S'^*}{2}\hat{q}\cdot(\hat{\nu}-\vec{\beta}). \tag{F5g}$$

Pseudoscalar traces will be similar, replacing the coefficients C_S and C_S' with the coefficients $-C_P'$ and $-C_P$, respectfully.

APPENDIX G: CONVENTIONS

In this paper we use the nowadays convention for the gamma matrices (see, e.g., Ref. [57]):

$$\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu}, \qquad \sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu,\gamma_\nu],$$
$$\gamma^5 = \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad \gamma_5\sigma_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma}, \tag{G1}$$

where $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ is the Minkowski metric, and ϵ_{ijk} [$\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}$] is the Levi-Civita symbol [tensor], which is 1 if (i,j,k) [(μ,ν,ρ,σ)] is an even permutation of (1,2,3) [(0,1,2,3)], -1 if it is an odd permutation, and 0 if any index is repeated. Some of the calculations make use of the Dirac representation of the gamma matrices:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \vec{\gamma} = \gamma^0\begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \tag{G2}$$

with the Pauli matrices $\vec{\sigma}$.

The conventions we use for a Dirac spinor of a free particle of mass m, momentum p_μ, and energy $E = \sqrt{p^2+m^2}$ is

$$u(\vec{p},\sigma) = \sqrt{\frac{E+m}{2E}}\begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}\chi_\sigma, \tag{G3a}$$

where χ_σ is a two-component Pauli spinor for a spin up and down along the $\hat{p}$ axis, and the normalization is

$$u^+u = 1, \qquad \sum_{\text{lepton spins}} u(\vec{p})\bar{u}(\vec{p}) = \frac{\gamma_\mu p^\mu + m}{2E}, \qquad \bar{u} \equiv u^+\gamma_0. \tag{G3b}$$

The isospin raising and lowering operators we use are defined as

$$\tau^\pm \equiv \mp\frac{1}{2}(\tau_x \pm i\tau_y). \tag{G4}$$

The circular polarization base unit vectors we use are

$$\hat{e}_{\pm 1} \equiv \mp\frac{1}{\sqrt{2}}(\hat{x}\pm i\hat{y}), \qquad \hat{e}_0 \equiv \hat{z} \equiv \frac{\vec{q}}{|q|}, \tag{G5}$$

where we choose the $\hat{z}$ axis to be the direction of the momentum transfer $\vec{q}$, and the multipole decomposition of this base is [21,54]

$$\hat{e}_\lambda^+ e^{-i\vec{q}\cdot\vec{r}} = \begin{cases} \frac{i}{q}\sum_{J=0}^{\infty}\sqrt{4\pi(2J+1)}\,(-i)^J\nabla[j_J(qr)Y_{J0}(\hat{r})] & \lambda = 0 \\ -\sum_{J=1}^{\infty}\sqrt{2\pi(2J+1)}\,(-i)^J\{\lambda j_J(qr)\vec{Y}_{JJ1}^{-\lambda}(\hat{r}) + \frac{1}{q}\nabla\times[j_J(qr)\vec{Y}_{JJ1}^{-\lambda}(\hat{r})]\} & \lambda\in\{\pm 1\} \end{cases}, \tag{G6}$$

with j_J the spherical Bessel functions, Y_{JM} the spherical harmonics, and $\vec{Y}_{JL1}^M$ the vector spherical harmonics defined by the relation [54]

$$\vec{Y}_{JL1}^M(\hat{r}) \equiv \sum_{\mu=-L}^{L}\sum_{\lambda=-1}^{1}\langle L\mu 1\lambda|JM\rangle Y_{L\mu}(\hat{r})\hat{e}_\lambda, \tag{G7}$$

where $\langle j_1 m_1 j_2 m_2|JM\rangle$ are the Clebsch-Gordan coefficients.

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