Simple rules for evanescent operators in one-loop basis transformations

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Basis transformations often involve Fierz and other relations that are only valid in $D = 4$ dimensions. In general D spacetime dimensions, however, evanescent operators have to be introduced in order to preserve such identities. Such evanescent operators contribute to one-loop basis transformations as well as to two-loop renormalization group running. We present a simple procedure on how to systematically change basis at the one-loop level by obtaining shifts due to evanescent operators. As an example we apply this method to derive the one-loop basis transformation from the Buras, Misiak and Urban basis useful for next-to-leading order QCD calculations, to the Jenkins, Manohar and Stoffer basis used in the matching to the standard model effective theory.

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I. INTRODUCTION

In recent years a lot of progress has been made concerning next-to-leading order (NLO) analyses, which involve one-loop matching calculations as well as solving two-loop renormalization group equations (RGEs) for the Wilson coefficients of effective field theories. For an up to date review see [1]. For instance, concerning the standard model effective theory (SMEFT), the full matching from the SMEFT onto the weak effective theory (WET) valid below the electroweak (EW) scale is known at tree level [2] and since recently also at the one-loop level [3,4]. Furthermore, the one-loop RGEs in the SMEFT [5–7] and in the WET [8,9] are known.

In the process of performing a NLO analysis, it is often necessary to perform one-loop transformations between different operator bases, since, for instance, anomalous dimension matrices (ADMs) are known only in a particular basis, whereas the matching conditions are given in a different one. This is, for example, the case for the WET, where the two-loop ADMs are known in the Buras, Misiak and Urban (BMU) basis [10] as elaborated on recently in [11]. On the other hand, the one-loop matching from SMEFT onto WET is given in the Jenkins, Manohar and Stoffer (JMS) basis defined in [2].[1]

Generally, to translate the results from one basis to another one at NLO, one-loop basis transformations have to be taken into account. In this respect particular care has to be taken if the tree-level [leading order (LO)] transformations involve Dirac space identities, which are only valid in four spacetime dimensions. Examples include Fierz identities or identities involving gamma matrices. When using dimensional regularization, where the divergent loop integrals are continued to D dimensions, such identities cannot be used directly, but need to be generalized by introducing evanescent (EV) operators [10]. Such evanescent operators vanish in four dimensions to conserve the original identities but are nonzero in D dimensions. They are therefore formally speaking proportional to $\epsilon = (4 - D)/2$, which implies that they give nonzero contributions when inserted into divergent loop diagrams. These are exactly the contributions that enter basis transformations at the one-loop order. In this article we discuss a simple procedure on how to obtain these contributions by computing one-loop corrections resulting from the presence of the evanescent operators.[2] To this end the evanescent operators are simply defined as the difference between operators and their transformed versions using $D = 4$ identities. The resulting contributions will manifest themselves in shifts in the corresponding Wilson coefficients of the initial operators.

Having a simple algorithm at hand to perform NLO basis changes is important when performing one-loop matching calculations or computing two-loop ADMs. In this work we explain the underlying formal framework and provide such an algorithm, based on Greek projections [13], which facilitates the aforementioned calculations. In this manner this procedure is an important ingredient in the pursuit of a complete NLO SMEFT analysis.

[1]We follow here the `WCxf` convention defined in [12].

[2]A more formal procedure on how to perform NLO basis changes can be found in [10].

TABLE I. Nonleptonic $\Delta F = 1$ operators (baryon and lepton number conserving) in the JMS basis [2]. Note that $Q_{uddu}^{V1,LR}$ and $Q_{uddu}^{V8,LR}$ have Hermitian conjugates. The same holds for the operators $(\bar{L}R)(\bar{L}R)$. This choice of basis eliminates all operators with Dirac structures $\sigma^{\mu\nu}$. The class of operators $(\bar{L}R)(\bar{R}L)$ + H.c. does not contain nonleptonic operators, but only semileptonic ones.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$	
$[O_{dd}^{V,LL}]_{prst}$	$(\bar{d}_L^p\gamma_\mu d_L^r)(\bar{d}_L^s\gamma^\mu d_L^t)$	$[O_{dd}^{V,RR}]_{prst}$	$(\bar{d}_R^p\gamma_\mu d_R^r)(\bar{d}_R^s\gamma^\mu d_R^t)$
$[O_{ud}^{V1,LL}]_{prst}$	$(\bar{u}_L^p\gamma_\mu u_L^r)(\bar{d}_L^s\gamma^\mu d_L^t)$	$[O_{ud}^{V1,RR}]_{prst}$	$(\bar{u}_R^p\gamma_\mu u_R^r)(\bar{d}_R^s\gamma^\mu d_R^t)$
$[O_{ud}^{V8,LL}]_{prst}$	$(\bar{u}_L^p\gamma_\mu T^A u_L^r)(\bar{d}_L^s\gamma^\mu T^A d_L^t)$	$[O_{ud}^{V8,RR}]_{prst}$	$(\bar{u}_R^p\gamma_\mu T^A u_R^r)(\bar{d}_R^s\gamma^\mu T^A d_R^t)$

$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{L}R)$ + H.c.	
$[O_{dd}^{V1,LR}]_{prst}$	$(\bar{d}_L^p\gamma_\mu d_L^r)(\bar{d}_R^s\gamma^\mu d_R^t)$	$[O_{dd}^{S1,RR}]_{prst}$	$(\bar{d}_L^p d_R^r)(\bar{d}_L^s d_R^t)$
$[O_{dd}^{V8,LR}]_{prst}$	$(\bar{d}_L^p\gamma_\mu T^A d_L^r)(\bar{d}_R^s\gamma^\mu T^A d_R^t)$	$[O_{dd}^{S8,RR}]_{prst}$	$(\bar{d}_L^p T^A d_R^r)(\bar{d}_L^s T^A d_R^t)$
$[O_{ud}^{V1,LR}]_{prst}$	$(\bar{u}_L^p\gamma_\mu u_L^r)(\bar{d}_R^s\gamma^\mu d_R^t)$	$[O_{ud}^{S1,RR}]_{prst}$	$(\bar{u}_L^p u_R^r)(\bar{d}_L^s d_R^t)$
$[O_{ud}^{V8,LR}]_{prst}$	$(\bar{u}_L^p\gamma_\mu T^A u_L^r)(\bar{d}_R^s\gamma^\mu T^A d_R^t)$	$[O_{ud}^{S8,RR}]_{prst}$	$(\bar{u}_L^p T^A u_R^r)(\bar{d}_L^s T^A d_R^t)$
$[O_{du}^{V1,LR}]_{prst}$	$(\bar{d}_L^p\gamma_\mu d_L^r)(\bar{u}_R^s\gamma^\mu u_R^t)$	$[O_{uddu}^{S1,RR}]_{prst}$	$(\bar{u}_L^p d_R^r)(\bar{d}_L^s u_R^t)$
$[O_{du}^{V8,LR}]_{prst}$	$(\bar{d}_L^p\gamma_\mu T^A d_L^r)(\bar{u}_R^s\gamma^\mu T^A u_R^t)$	$[O_{uddu}^{S8,RR}]_{prst}$	$(\bar{u}_L^p T^A d_R^r)(\bar{d}_L^s T^A u_R^t)$
$[O_{uddu}^{V1,LR}]_{prst}$	$(\bar{u}_L^p\gamma_\mu d_L^r)(\bar{d}_R^s\gamma^\mu u_R^t)$ + H.c.		
$[O_{uddu}^{V8,LR}]_{prst}$	$(\bar{u}_L^p\gamma_\mu T^A d_L^r)(\bar{d}_R^s\gamma^\mu T^A u_R^t)$ + H.c.		

The rest of the article is organized as follows: In Sec. II we outline the general procedure on how to compute one-loop basis transformations between two operator bases. In Sec. III we show an explicit example by performing a NLO change from the BMU to the JMS basis. In Sec. IV we define and calculate the EV operators that in turn give us the transformation matrix between BMU and JMS bases at the one-loop level. Finally we conclude in Sec. V. Additional material used in the calculations is collected in the appendixes.

II. PROCEDURE

In this section we discuss the full NLO basis transformation between general operator bases. However, to make the subsequent sections more transparent we will dub the two operator bases JMS (see Table I for the list of operators considered in this work) and BMU. We start with the simple LO basis transformation which will also set the notation used. Then, in the second subsection we discuss the issue of evanescent operators that become relevant when performing basis changes at the one-loop level.

A. Tree-level transformation

Let us consider the two bases

$$\vec{\mathcal{O}}_{\rm BMU} = \{Q_1, Q_2, ..., Q_N\}, \quad \vec{\mathcal{O}}_{\rm JMS} = \{O_1, O_2, ..., O_N\}, \tag{1}$$

containing N operators each. At tree level each operator is given by a linear combination of operators from the other basis:

$$\vec{\mathcal{O}}_{\rm JMS} = \hat{R}^{(0)}\vec{\mathcal{O}}_{\rm BMU}, \tag{2}$$

where the $N \times N$ matrix $\hat{R}^{(0)}$ denotes the linear transformation between the two bases. The superindex "(0)" denotes the tree-level transformation in anticipation of the one-loop transformation discussed in the next subsection. The matrix $\hat{R}^{(0)}$ is obtained by applying identities such as Fierz relations and gamma matrix identities to the operators on the left-hand side (LHS) of Eq. (2). It is independent of the renormalization scale and only contains numerical factors.[3]

As seen in Eq. (2) the BMU operator basis is transformed via $\hat{R}^{(0)}$ into the JMS basis. This transformation is useful because the hadronic matrix elements of operators are usually calculated in the BMU basis and this transformation allows one to obtain them in the JMS basis. However, the Wilson coefficients are nowadays calculated in the JMS basis, since the SMEFT matching results are only available in that particular basis. Therefore, for Wilson coefficients the transformation from JMS to BMU is more useful, which reads

$$\vec{C}^{(0)}_{\rm BMU} = (\hat{R}^{(0)})^T\vec{C}^{(0)}_{\rm JMS}; \tag{3}$$

that is, $(\hat{R}^{(0)})^T$ is involved.

Since $\hat{R}^{(0)}$ is invertible by definition, one can easily express the BMU operators in terms of JMS ones and the JMS Wilson coefficients in terms of BMU ones. For our

[3]This is true up to possible normalization factors, which can contain coupling constants or other parameters that depend on the renormalization scale.

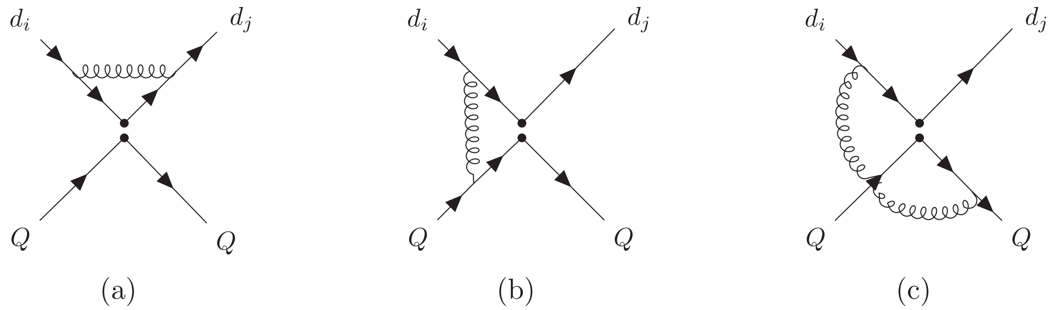

FIG. 1. The QCD current-current insertions at one-loop.

purposes, however, the transformations in Eqs. (2) and (3) are more convenient.

It should be stressed that although the procedure outlined below is general and can be used for any basis transformation, the explicit values of various coefficients are given in our paper in the NDR-$\overline{\mathrm{MS}}$ scheme as defined in [14] with evanescent operators entering two-loop calculations defined by the so-called Greek method. The details in the context of WET and SMEFT are discussed in Appendix E of [15].

B. One-loop transformation

At the one-loop level special care has to be taken when identities have been used in the LO transformation that are only valid in $D = 4$ spacetime dimensions. Therefore, for all the lines in $R^{(0)}$ that were obtained using $D = 4$ relations, evanescent operators have to be introduced to generalize these identities. In the case of the BMU → JMS transformation we proceed as follows:

Step 1:

We perform a Fierz transformation on every operator in the BMU basis Q_i and denote the result of this transformation by $\tilde{Q}_i$ given generally by

$$\tilde{Q}_i = \sum_k \omega_{ik} Q_k, \tag{4}$$

with operators Q_k belonging to the BMU basis and coefficients ω_{ik} determined through the Fierz identities collected in Appendix B. It should be emphasized that the relation above and analogous relations below should be interpreted as effective contributions from this operator in one-loop diagrams. This means that the calculations performed in two bases, in our case BMU and JMS, will give the same results for physical observables when this transformation is taken into account at all stages, therefore also when calculating anomalous dimensions. These relations and analogous relations involving evanescent operators can also be used for the calculations of two-loop anomalous dimensions of operators because there the evanescent operators contributing to two-loop anomalous dimensions appear in one-loop subdiagrams [14]. To go to the next order in perturbation theory the shifts should include corrections of order $\mathcal{O}(\alpha_s^2)$ that can be obtained through insertions of the operators into two-loop diagrams.

Step 2:

We insert Q_i and $\tilde{Q}_i$ defined by (4) into current-current and QCD penguin diagrams of Figs. 1 and 2, respectively. Because of the presence of evanescent operators that are simply defined by

$$Q_i = \tilde{Q}_i + \mathrm{EV}_i, \tag{5}$$

the insertion of Q_i and $\tilde{Q}_i$ into one-loop diagrams will generally differ at $\mathcal{O}(\alpha_s)$, leading to the result

$$Q_i = \tilde{Q}_i + \frac{\alpha_s}{4\pi} \sum_r \tilde{\omega}_{ir} Q_r, \tag{6}$$

where the operators in the sum can again be written in the BMU basis but are generally different from the ones in the definition of $\tilde{Q}_i$ in (4) and also $\omega_{ik} \neq \tilde{\omega}_{ir}$. Note that this can easily be done at the one-loop level because in transforming the operators entering these corrections $D = 4$ Fierz identities can be used as will be explained below. Expressing the shift in terms of BMU operators

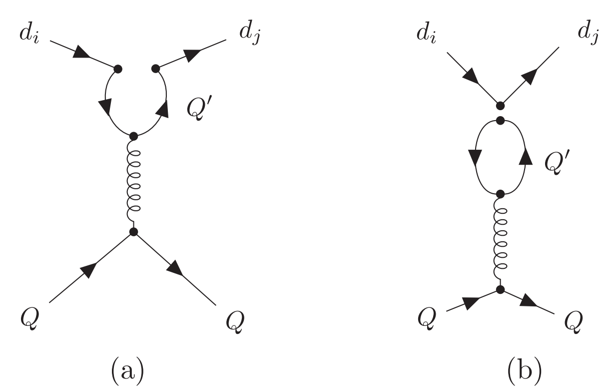

FIG. 2. The QCD-penguin insertions with open-type (left) and closed-type (right) fermion loops.

allows one to use these results for transformations of the BMU basis to any basis, not just the JMS one.

To compute the α_s corrections resulting from the evanescent operator EV_i in (5), one simply inserts the difference $Q_i - \tilde{Q}_i$ into the relevant one-loop diagrams. Since the evanescent operator is formally $\mathcal{O}(\epsilon)$, only the divergent pieces of the loop integrals will contribute and consequently the finite pieces can be discarded in the calculation.

The relation between operators in the JMS basis and the Fierz transformed operators in BMU are further discussed in Sec. III B. In particular, the tree-level relations are given in Eqs. (46)–(49), and the one-loop relations in Eqs. (50)–(52).

Step 3:

Having the relations in (6) and inspecting which Fierz transformations had to be performed to find the LO matrix $\hat{R}^{(0)}$ we can generalize the basis change matrix $\hat{R}^{(0)}$ to the one-loop order, as follows[4]:

$$\vec{\mathcal{O}}_{\rm JMS} = \hat{R}\vec{\mathcal{O}}_{\rm BMU}, \qquad \hat{R} = \hat{R}^{(0)} + \frac{\alpha_s}{4\pi}\hat{R}^{(1)}, \tag{7}$$

where $\hat{R}^{(1)}$ is the one-loop basis transformation matrix resulting exclusively from the evanescent operators.

When computing the one-loop corrections to the difference $Q_i - \tilde{Q}_i$, new Dirac structures can appear, which are not present in the original basis, in our case the BMU basis. To reduce these structures to the ones in $\vec{\mathcal{O}}_{\rm BMU}$ it is essential to use the same projections as in the calculations of two-loop anomalous dimensions of the operators. Only then are the evanescent operators entering the two-loop calculations the same as the ones used in the one-loop matching between WET and SMEFT, and in particular they correspond to the ones in the basis change. This prescription guarantees that the renormalization scheme dependence of two-loop matrix elements can be canceled by the one present in one-loop contributions so that the physical amplitudes are renormalization scheme independent.

These issues have been discussed in the context of the NLO QCD calculations of Wilson coefficients at length in [14,16,17] and the summary can be found in Sec. 5.2.9 of [1]. There the so-called Greek projection [13], properly generalized to include evanescent operators in [14], has been discussed in detail. As pointed out in [17] this is not the only way to include evanescent operators but, in fact, the simplest one. It has been used in all two-loop calculations performed by the second author and will be used in the following but this time in the context of basis change. It should be kept in mind that this procedure defines the evanescent operators in the NDR-$\overline{\rm MS}$ scheme combined with the Greek projections as used in [14]. While giving the same results it is much simpler than the formal method presented in [10] and used recently in [11].

The NLO transformation in terms of the Wilson coefficients is given as follows:

$$\vec{\mathcal{C}}_{\rm BMU} = \hat{R}^T\vec{\mathcal{C}}_{\rm JMS}. \tag{8}$$

Writing

$$\vec{\mathcal{C}}_{\rm JMS} = \vec{\mathcal{C}}^{(0)}_{\rm JMS} + \frac{\alpha_s}{4\pi}\vec{\mathcal{C}}^{(1)}_{\rm JMS}, \qquad \vec{\mathcal{C}}_{\rm BMU} = \vec{\mathcal{C}}^{(0)}_{\rm BMU} + \frac{\alpha_s}{4\pi}\vec{\mathcal{C}}^{(1)}_{\rm BMU}, \tag{9}$$

the transformation for Wilson coefficients reads

$$\begin{aligned}\vec{\mathcal{C}}^{(0)}_{\rm BMU} &= (\hat{R}^{(0)})^T\vec{\mathcal{C}}^{(0)}_{\rm JMS},\\ \vec{\mathcal{C}}^{(1)}_{\rm BMU} &= (\hat{R}^{(0)})^T\vec{\mathcal{C}}^{(1)}_{\rm JMS} + (\hat{R}^{(1)})^T\vec{\mathcal{C}}^{(0)}_{\rm JMS}.\end{aligned} \tag{10}$$

A few remarks concerning the generality of our results are in order: The procedure laid out in this article is not limited to the BMU and JMS basis, but is valid for any pair of operator bases that are related via Fierz transformations. The full set of relevant evanescent operators is generated by applying Greek identities or any other identities to reduce the resulting Dirac structures from the one-loop calculation. This choice of Dirac reduction, together with the chosen finite counterterms and the treatment of γ_5 fixes the renormalization scheme [17]. In this article we have chosen the NDR-$\overline{\rm MS}$ scheme in combination with the Greek identities. Choosing a different renormalization scheme would change the entries of the rotation matrix $R^{(1)}$ in Eq. (7), but is related via a trivial change of scheme to our results [18,19]. Furthermore, the procedure is independent of possible group theory relations between the two bases, since these are still valid in $D \neq 4$ spacetime dimensions, and therefore do not obtain any one-loop shifts from evanescent structures. Finally, we note that the operator shifts presented in this paper can be interpreted as one-loop corrections to the original Fierz identities. This has been shown in two recent publications [20,21], in which the shifts for all possible four-Fermi operators together with the contributions from dipole operators have been taken into account. A similar procedure in the SMEFT has been employed in [22].

C. How to use this procedure

Having the results in (6) to be presented in the next section, our goal will be to find the matrix $\hat{R}$. To this end comparing the BMU and JMS bases one has to find those operators or groups of them for which a Fierz transformation on operators in the BMU basis has to be performed in order to obtain the operators in the JMS basis with order α_s corrections taken into account.

[4]In the case JMS $\leftrightarrow$ BMU we will focus on QCD corrections. The relation in Eq. (7) can easily be generalized to include other one-loop corrections.

Generally the matrix $\hat{R}^{(0)}$ will have a block structure so that operators in a given block can be separately considered from other blocks. In this context the following three cases arise:

(i) If no Fierz transformations are required in a given block, the corresponding matrix $\hat{R}^{(1)}$ will vanish and tree-level results will be valid also at one-loop. One can use the corresponding block in $\hat{R}^{(0)}$ in that case.

(ii) If Fierz transformations in a given block are required but the contributions of evanescent operators will vanish, the corresponding block in the tree-level matrix $\hat{R}^{(0)}$ will again represent the corresponding block in the full $\hat{R}$.

(iii) Finally, in certain blocks the necessity of performing Fierz transformations will introduce evanescent operators which will contribute to $\hat{R}^{(1)}$.

In the next section we will present the three step procedure outlined in this section in explicit terms.

III. BMU TO JMS TRANSLATION AT ONE-LOOP

A. Basic method

As outlined above, the transformation of the BMU basis to the JMS basis requires Fierz transformations on some of the BMU operators. This generates then additional contributions to the one-loop matching performed within the JMS basis. To find these one-loop contributions, one has to insert the difference $Q_i - \tilde{Q}_i$ into current-current and penguin diagrams of Figs. 1 and 2, respectively. The details of the calculation for the one-loop insertions are given in Appendix C.

With all BMU operators listed in Appendix A the following items turn out:

(i) For Q_k with $k = 1$–18 these additional contributions come only from penguin insertions and moreover only for a few among these operators listed below.

(ii) Fierz transformations on $Q_1^{\mathrm{SLR},Q}$ and $Q_2^{\mathrm{SLR},Q}$ do not generate any evanescent contributions, and consequently in this case $D = 4$ identities can be used.

(iii) For $Q_k^{\mathrm{SRR},Q}$ with $k = 1$–4 and $Q_l^{\mathrm{SRR},D}$ with $l = 1, 2$ the contributions come only from current-current operators and involve all operators considered. However, all these operators do not contribute to K and B decays being forbidden within SMEFT. For completeness we list these contributions below because they could be useful for charm physics.

In the case of the SM operators Q_k with $k = 1$–10 all the contributions from Fierz transformations for LL (left-left) operators can be obtained from two properties:

$$Q_1 = \tilde{Q}_1, \qquad Q_2 = \tilde{Q}_2 + \frac{1}{3}\frac{\alpha_s}{4\pi}P, \tag{11}$$

with

$$P = Q_4 + Q_6 - \frac{1}{3}(Q_3 + Q_5). \tag{12}$$

We find

$$Q_3 = \tilde{Q}_3 + \frac{2}{3}\frac{\alpha_s}{4\pi}P, \qquad Q_4 = \tilde{Q}_4 - \frac{N_f}{3}\frac{\alpha_s}{4\pi}P, \qquad Q_5 = \tilde{Q}_5, \qquad Q_6 = \tilde{Q}_6, \tag{13}$$

$$Q_7 = \tilde{Q}_7, \qquad Q_8 = \tilde{Q}_8, \qquad Q_9 = \tilde{Q}_9 - \frac{1}{3}\frac{\alpha_s}{4\pi}P, \qquad Q_{10} = \tilde{Q}_{10} - \frac{1}{3}\left(N_u - \frac{N_d}{2}\right)\frac{\alpha_s}{4\pi}P. \tag{14}$$

We observe that the Fierz transformations on the VLR (Vector Left-Right) operators Q_k with $k = 5$–8 do not bring any contributions from evanescent operators.

In the case of the new physics (NP) operators Q_k with $k = 11$–18 only the Fierz transformation on Q_{11} brings a contribution from evanescent operators so that

$$Q_{11} = \tilde{Q}_{11} + \frac{2}{3}\frac{\alpha_s}{4\pi}P, \qquad Q_k = \tilde{Q}_k, \qquad k = 12 - 18. \tag{15}$$

The corresponding results for $Q_{1,2,3,4}^{\mathrm{SRR},D}$ with $D = d_i$ or d_j operators can be obtained by using the results of [23], in particular the results in Eqs. (29)–(32) of that paper. In this case the $\tilde{Q}_k^{\mathrm{SRR},D}$ operators with $k = 1$–4 are given as follows:

$$\begin{aligned} \tilde{Q}_1^{\mathrm{SRR},D} &= -\frac{1}{2}Q_2^{\mathrm{SRR},D} + \frac{1}{8}Q_4^{\mathrm{SRR},D}, \\ \tilde{Q}_2^{\mathrm{SRR},D} &= -\frac{1}{2}Q_1^{\mathrm{SRR},D} + \frac{1}{8}Q_3^{\mathrm{SRR},D}, \end{aligned} \tag{16}$$

$$\begin{aligned} \tilde{Q}_3^{\mathrm{SRR},D} &= 6Q_2^{\mathrm{SRR},D} + \frac{1}{2}Q_4^{\mathrm{SRR},D}, \\ \tilde{Q}_4^{\mathrm{SRR},D} &= 6Q_1^{\mathrm{SRR},D} + \frac{1}{2}Q_3^{\mathrm{SRR},D}. \end{aligned} \tag{17}$$

As this time only current-current diagrams are involved, the flavor structure relative to the one considered in [23] does not matter, and the full calculation of the matrix elements of $\tilde{Q}_i$ operators can readily be performed in no time using results of [23]. The shifts caused by evanescent operators involve four operators but those with $k = 1,\ 3$ can be eliminated[5] using

$$Q_3^{\mathrm{SRR},D} = 6Q_2^{\mathrm{SRR},D} + \frac{1}{2}Q_4^{\mathrm{SRR},D} + \text{Fierz ev} \tag{18}$$

$$Q_1^{\mathrm{SRR},D} = -\frac{1}{2}Q_2^{\mathrm{SRR},D} + \frac{1}{8}Q_4^{\mathrm{SRR},D} + \text{Fierz ev}, \tag{19}$$

so that the shifts depend only on $Q_{2,4}^{\mathrm{SRR},D}$. Note that in the BMU basis $Q_{2,4}^{\mathrm{SRR},D}$ are denoted as $Q_{1,2}^{\mathrm{SRR},i}$ or $Q_{1,2}^{\mathrm{SRR},j}$ in Ref. [11]; see Appendix A for definitions.

We find then

$$Q_1^{\mathrm{SRR},D} = \tilde{Q}_1^{\mathrm{SRR,D}} + \frac{\alpha_s}{4\pi}\sum_{k=2,4} A_k Q_k^{\mathrm{SRR,D}}, \tag{20}$$

$$Q_2^{\mathrm{SRR},D} = \tilde{Q}_2^{\mathrm{SRR,D}} + \frac{\alpha_s}{4\pi}\sum_{k=2,4} B_k Q_k^{\mathrm{SRR,D}}, \tag{21}$$

$$Q_3^{\mathrm{SRR},D} = \tilde{Q}_3^{\mathrm{SRR,D}} + \frac{\alpha_s}{4\pi}\sum_{k=2,4} C_k Q_k^{\mathrm{SRR,D}}, \tag{22}$$

$$Q_4^{\mathrm{SRR},D} = \tilde{Q}_4^{\mathrm{SRR,D}} + \frac{\alpha_s}{4\pi}\sum_{k=2,4} D_k Q_k^{\mathrm{SRR,D}}, \tag{23}$$

with the coefficients A_k, B_k, C_k, D_k given as follows[6]:

$$A_2 = \frac{1}{2} + \frac{5}{N_c} - \frac{7N_c}{4} = -\frac{37}{12}, \qquad A_4 = -\frac{1}{2} + \frac{1}{4N_c} - \frac{N_c}{16} = -\frac{29}{48}; \tag{24}$$

$$B_2 = -\frac{17}{4} - \frac{1}{N_c} = -\frac{55}{12}, \qquad B_4 = -\frac{3}{16} + \frac{3}{4N_c} - \frac{N_c}{8} = -\frac{5}{16}; \tag{25}$$

$$C_2 = 36 + \frac{28}{N_c} - 7N_c = \frac{73}{3}, \qquad C_4 = -\frac{1}{2} - \frac{5}{N_c} + \frac{3N_c}{4} = \frac{1}{12}; \tag{26}$$

$$D_2 = -21 - \frac{44}{N_c} + 14N_c = \frac{19}{3}, \qquad D_4 = \frac{13}{4} + \frac{1}{N_c} = \frac{43}{12}. \tag{27}$$

The same procedure can be applied to $Q_{1,2,3,4}^{\mathrm{SRR},Q}$ with $Q = u_k, d_k \neq d_i, d_j$ defined in Eq. (A13). The rules for the shifts read

$$Q_1^{\mathrm{SRR},Q} = \tilde{Q}_1^{\mathrm{SRR,Q}} + \frac{\alpha_s}{4\pi}\sum_{k=1,2,3,4} a_k Q_k^{\mathrm{SRR,Q}}, \tag{28}$$

$$Q_2^{\mathrm{SRR},Q} = \tilde{Q}_2^{\mathrm{SRR,Q}} + \frac{\alpha_s}{4\pi}\sum_{k=1,2,3,4} b_k Q_k^{\mathrm{SRR,Q}}, \tag{29}$$

$$Q_3^{\mathrm{SRR},Q} = \tilde{Q}_3^{\mathrm{SRR,Q}} + \frac{\alpha_s}{4\pi}\sum_{k=1,2,3,4} c_k Q_k^{\mathrm{SRR,Q}}, \tag{30}$$

$$Q_4^{\mathrm{SRR},Q} = \tilde{Q}_4^{\mathrm{SRR,Q}} + \frac{\alpha_s}{4\pi}\sum_{k=1,2,3,4} d_k Q_k^{\mathrm{SRR,Q}}. \tag{31}$$

Here the flavor structure of the tilde operators $\tilde{Q}_{1,2,3,4}^{\mathrm{SRR},Q}$ [see (16) and (17) for the definition with D replaced by Q] is $(\bar{d}_j\Gamma Q)(\bar{Q}\Gamma d_i)$ and the BMU operators $Q_{1,2,3,4}^{\mathrm{SRR},Q}$ have the form $(\bar{d}_j\Gamma d_i)(\bar{Q}\Gamma Q)$.

The coefficients a_k, b_k, c_k, d_k are given as follows:

$$a_1 = \frac{N_c}{2} - \frac{1}{N_c} = \frac{7}{6}, \qquad a_2 = \frac{1}{2}, \tag{32}$$

$$a_3 = -\frac{N_c}{4} + \frac{3}{4N_c} = -\frac{1}{2}, \qquad a_4 = -\frac{1}{2}; \tag{33}$$

$$b_1 = 1, \qquad b_2 = -\frac{1}{N_c} = -\frac{1}{3}, \tag{34}$$

$$b_3 = -\frac{5}{8}, \qquad b_4 = -\frac{N_c}{8} + \frac{3}{4N_c} = -\frac{1}{8}; \tag{35}$$

$$c_1 = 8N_c - \frac{44}{N_c} = \frac{28}{3}, \qquad c_2 = 36, \tag{36}$$

$$c_3 = -\frac{N_c}{2} + \frac{1}{N_c} = -\frac{7}{6}, \qquad c_4 = -\frac{1}{2}; \tag{37}$$

$$d_1 = 30, \qquad d_2 = 14N_c - \frac{44}{N_c} = \frac{82}{3}, \tag{38}$$

$$d_3 = -1, \qquad d_4 = \frac{1}{N_c} = \frac{1}{3}. \tag{39}$$

With these rules we can find the matrix $\hat{R}$ as defined in (7).

B. Transformation matrices at one-loop

In this section we present our final result for the transformation matrices between the BMU and JMS bases at the one-loop level. The details of the calculation are given in Sec. IV. The calculation can be split into the three disconnected sectors VLL, VLR, and SRR (Scalar Left-Left), which denote the $\gamma_\mu P_L \otimes \gamma^\mu P_L$, $\gamma_\mu P_L \otimes \gamma^\mu P_R$, and $P_R \otimes P_R$ Dirac structures of the involved four-Fermi operators, respectively. For the BMU operator we use the following reference ordering:

$$\mathrm{VLL}: \{Q_1, Q_2, Q_3, Q_4, Q_9, Q_{10}, Q_{11}, Q_{14}\}, \tag{40}$$

$$\mathrm{VLR}: \{Q_5, Q_6, Q_7, Q_8, Q_{12}, Q_{13}, Q_{15}, \ldots, Q_{24}\}, \tag{41}$$

$$\mathrm{SRR}: \{Q_{25}, \ldots, Q_{40}\}. \tag{42}$$

[5] Equivalently one could also eliminate any other two operators but here we follow the conventions used in Ref. [11].

[6] N_c is the number of colors with $N_c = 3$ in the final results.

For the JMS basis we use the ordering

$$\begin{aligned}\mathrm{VLL}\colon\{&[O_{ud}^{V1,LL}]_{11ji},[O_{ud}^{V8,LL}]_{11ji},[O_{ud}^{V1,LL}]_{22ji},[O_{ud}^{V8,LL}]_{22ji},\\&[O_{dd}^{V,LL}]_{jikk},[O_{dd}^{V,LL}]_{jkki},[O_{dd}^{V,LL}]_{jiii},[O_{dd}^{V,LL}]_{jijj}\},\end{aligned} \tag{43}$$

$$\begin{aligned}\mathrm{VLR}\colon\{&[O_{du}^{V1,LR}]_{ji11},[O_{du}^{V8,LR}]_{ji11},[O_{du}^{V1,LR}]_{ji22},[O_{du}^{V8,LR}]_{ji22},\\&[O_{dd}^{V1,LR}]_{jikk},[O_{dd}^{V8,LR}]_{jikk},[O_{dd}^{V1,LR}]_{jiii},[O_{dd}^{V8,LR}]_{jiii},\\&[O_{dd}^{V1,LR}]_{jijj},[O_{dd}^{V8,LR}]_{jijj},[O_{uddu}^{V1,LR}]^{\dagger}_{1ji1},[O_{uddu}^{V8,LR}]^{\dagger}_{1ji1},\\&[O_{uddu}^{V1,LR}]^{\dagger}_{2ji2},[O_{uddu}^{V8,LR}]^{\dagger}_{2ji2},[O_{dd}^{V1,LR}]_{jkki},[O_{dd}^{V8,LR}]_{jkki}\},\end{aligned} \tag{44}$$

$$\begin{aligned}\mathrm{SRR}\colon\{&[O_{dd}^{S1,RR}]_{jiii},[O_{dd}^{S8,RR}]_{jiii},[O_{dd}^{S1,RR}]_{jijj},[O_{dd}^{S8,RR}]_{jijj},\\&[O_{ud}^{S1,RR}]_{11ji},[O_{ud}^{S8,RR}]_{11ji},[O_{uddu}^{S1,RR}]_{1ij1},[O_{uddu}^{S8,RR}]_{1ij1},\\&[O_{ud}^{S1,RR}]_{22ji},[O_{ud}^{S8,RR}]_{22ji},[O_{uddu}^{S1,RR}]_{2ij2},[O_{uddu}^{S8,RR}]_{2ij2},\\&[O_{dd}^{S1,RR}]_{jikk},[O_{dd}^{S8,RR}]_{jikk},[O_{dd}^{S1,RR}]_{jkki},[O_{dd}^{S8,RR}]_{jkki}\}.\end{aligned} \tag{45}$$

At tree level the transformation matrix $\hat{R}^{(0)}$ reads

$$\hat{R}^{(0)}=\begin{pmatrix}\hat{R}^{(0)}_{\mathrm{VLL}} & 0_{8\times16} & 0_{8\times16}\\ 0_{16\times8} & \hat{R}^{(0)}_{\mathrm{VLR}} & 0_{16\times16}\\ 0_{16\times8} & 0_{16\times16} & \hat{R}^{(0)}_{\mathrm{SRR}}\end{pmatrix}, \tag{46}$$

$$\hat{R}^{(0)}_{\mathrm{VLL}}=\begin{pmatrix}1&0&0&0&0&0&0&0\\-\frac{1}{6}&\frac{1}{2}&0&0&0&0&0&0\\-1&0&\frac{1}{3}&0&\frac{2}{3}&0&0&0\\\frac{1}{6}&-\frac{1}{2}&-\frac{1}{18}&\frac{1}{6}&-\frac{1}{9}&\frac{1}{3}&0&0\\0&0&\frac{2}{3}&0&-\frac{2}{3}&0&-1&0\\0&0&0&\frac{2}{3}&0&-\frac{2}{3}&-1&0\\0&0&0&0&0&0&\frac{1}{2}&\frac{1}{2}\\0&0&0&0&0&0&\frac{1}{2}&-\frac{1}{2}\end{pmatrix},\qquad \hat{R}^{(0)}_{\mathrm{SRR}}=\begin{pmatrix}\hat{A}^{(0)}_{\mathrm{SRR}}&0_{2\times2}&0_{2\times4}&0_{2\times4}&0_{2\times4}\\0_{2\times2}&\hat{A}^{(0)}_{\mathrm{SRR}}&0_{2\times4}&0_{2\times4}&0_{2\times4}\\0_{4\times2}&0_{4\times2}&\hat{B}^{(0)}_{\mathrm{SRR}}&0_{4\times4}&0_{4\times4}\\0_{4\times2}&0_{4\times2}&0_{4\times4}&\hat{B}^{(0)}_{\mathrm{SRR}}&0_{4\times4}\\0_{4\times2}&0_{4\times2}&0_{4\times4}&0_{4\times4}&\hat{B}^{(0)}_{\mathrm{SRR}}\end{pmatrix}, \tag{47}$$

$$\hat{A}^{(0)}_{\mathrm{SRR}}=\begin{pmatrix}1&0\\-\frac{5}{12}&\frac{1}{16}\end{pmatrix},\qquad \hat{B}^{(0)}_{\mathrm{SRR}}=\begin{pmatrix}0&1&0&0\\\frac{1}{2}&-\frac{1}{6}&0&0\\-\frac{1}{2}&0&\frac{1}{8}&0\\\frac{1}{12}&-\frac{1}{4}&-\frac{1}{48}&\frac{1}{16}\end{pmatrix}, \tag{48}$$

$$\hat{R}^{(0)}_{\rm VLR} = \begin{pmatrix}
\frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{36} & \frac{1}{12} & -\frac{1}{18} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{4} & -\frac{1}{12} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{36} & \frac{1}{12} & -\frac{1}{18} & \frac{1}{6} & 0 & 0 & 0 & 0 & -\frac{1}{4} & \frac{1}{12} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2}{3} & 0 & -\frac{2}{3} & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{9} & \frac{1}{3} & \frac{1}{9} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{4} & -\frac{1}{12} & \frac{1}{4} & -\frac{1}{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{4} & -\frac{1}{12} & -\frac{1}{4} & \frac{1}{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -1
\end{pmatrix}. \tag{49}$$

At the one-loop level the corrections due to the EV operators are given by the matrix $\hat{R}^{(1)}$

$$\hat{R}^{(1)} = \begin{pmatrix} \hat{R}^{(1)}_{\rm VLL} & R^{(1)}_{8\times 16} & 0_{8\times 16} \\ 0_{16\times 8} & \hat{R}^{(1)}_{\rm VLR} & 0_{16\times 16} \\ 0_{16\times 8} & 0_{16\times 16} & \hat{R}^{(1)}_{\rm SRR} \end{pmatrix}, \tag{50}$$

$$\hat{R}^{(1)}_{\rm VLL} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{18} & -\frac{1}{6} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{18} & \frac{1}{6} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{3} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \qquad
\hat{R}^{(1)}_{8\times 16} = \begin{pmatrix}
0 & 0 & \cdots & 0 \\
\frac{1}{18} & -\frac{1}{6} & \cdots & 0 \\
0 & 0 & \ldots & 0 \\
-\frac{1}{18} & \frac{1}{6} & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
-\frac{1}{3} & 1 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0
\end{pmatrix}, \qquad
\hat{R}^{(1)}_{\rm VLR} = 0_{16\times 16}, \tag{51}$$

$$\hat{R}^{(1)}_{\rm SRR} = \begin{pmatrix}
\hat{A}^{(1)}_{\rm SRR} & 0_{2\times 2} & 0_{2\times 4} & 0_{2\times 4} & 0_{2\times 4} \\
0_{2\times 2} & \hat{A}^{(1)}_{\rm SRR} & 0_{2\times 4} & 0_{2\times 4} & 0_{2\times 4} \\
0_{4\times 2} & 0_{4\times 2} & \hat{B}^{(1)}_{\rm SRR} & 0_{4\times 4} & 0_{4\times 4} \\
0_{4\times 2} & 0_{4\times 2} & 0_{4\times 4} & \hat{B}^{(1)}_{\rm SRR} & 0_{4\times 4} \\
0_{4\times 2} & 0_{4\times 2} & 0_{4\times 4} & 0_{4\times 4} & \hat{B}^{(1)}_{\rm SRR}
\end{pmatrix}, \qquad
\begin{aligned}
\hat{A}^{(1)}_{\rm SRR} &= \begin{pmatrix} 0 & 0 \\ -\frac{37}{24} & -\frac{29}{96} \end{pmatrix}, \\
\hat{B}^{(1)}_{\rm SRR} &= \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-\frac{7}{12} & -\frac{17}{4} & -\frac{5}{48} & -\frac{3}{16} \\
-\frac{55}{36} & -\frac{13}{12} & -\frac{11}{144} & -\frac{1}{48}
\end{pmatrix}.
\end{aligned} \tag{52}$$

Here, we have used $N_f = 5$, $N_d = 3$, and $N_u = 2$.

IV. EVANESCENT OPERATORS

In this section, we define and calculate the EV operators, which in turn gives us the matrix $\hat{R}^{(1)}$.

A. Definition of EV operators in the VLL sector

At the one-loop level, we need to add an EV contribution to a tree-level basis transformation rule if it involves a Fierz transformation. On the other hand, we do not need an EV contribution for the cases in which no Fierz is required for the tree-level basis change. In the VLL (Vector Left-Left) sector, there are total eight independent operators where only five of them involve Fierz relation (B3) for the change of basis. The corresponding EV operators $E_I^{\mathrm{VLL}}, I = 1\text{–}5$ are defined by the following relations:

$$[O_{ud}^{V1,LL}]_{11ji} \overset{\mathcal{F}}{=} Q_1 + E_1^{\mathrm{VLL}}, \tag{53}$$

$$[O_{ud}^{V8,LL}]_{11ji} \overset{\mathcal{F}}{=} -\frac{1}{6}Q_1 + \frac{1}{2}Q_2 + E_2^{\mathrm{VLL}}, \tag{54}$$

$$[O_{ud}^{V1,LL}]_{22ji} \overset{\mathcal{F}}{=} -Q_1 + \frac{1}{3}Q_3 + \frac{2}{3}Q_9 + E_3^{\mathrm{VLL}}, \tag{55}$$

$$[O_{ud}^{V8,LL}]_{22ji} \overset{\mathcal{F}}{=} \frac{1}{6}Q_1 - \frac{1}{2}Q_2 - \frac{1}{18}Q_3 + \frac{1}{6}Q_4 - \frac{1}{9}Q_9 + \frac{1}{3}Q_{10} + E_4^{\mathrm{VLL}}, \tag{56}$$

$$[O_{dd}^{V,LL}]_{jikk} = \frac{2}{3}Q_3 - \frac{2}{3}Q_9 - Q_{11}, \tag{57}$$

$$[O_{dd}^{V,LL}]_{jkki} \overset{\mathcal{F}}{=} \frac{2}{3}Q_4 - \frac{2}{3}Q_{10} - Q_{11} + E_5^{\mathrm{VLL}}, \tag{58}$$

$$[O_{dd}^{V,LL}]_{jiii} = \frac{1}{2}Q_{11} + \frac{1}{2}Q_{14}, \tag{59}$$

$$[O_{dd}^{V,LL}]_{jijj} = \frac{1}{2}Q_{11} - \frac{1}{2}Q_{14}. \tag{60}$$

Here $\mathcal{F}$ indicates that the Fierz identity (B3) is needed for the change of basis.

B. Calculation of the EV operators in the VLL sectors

Now we are in position to use the rules presented in Sec. III. A to obtain E_1^{VLL}–E_5^{VLL}, which contribute to $\hat{R}^{(1)}$. To use the rules of Sec. III. A, first we need to express the JMS operators on the LHS in terms of the $\tilde{Q}_I$ operators. In general, there are three categories of operators as discussed in Sec. II C.

1. Operators requiring no Fierz

Since the following set of operators in the VLL sector do not require Fierz transformations for the basis change

$$[O_{dd}^{V,LL}]_{jikk}, [O_{dd}^{V,LL}]_{jiii}, [O_{dd}^{V,LL}]_{jijj}, \tag{61}$$

there are no EV operator contributions at the one-loop level basis transformation given by (57), (59), and (60). Hence, the corresponding entries in the matrix $\hat{R}^{(1)}$ vanish.

2. Operators requiring Fierz but no EV shifts

There are two operators in the JMS basis that require Fierz transformation but the EV contributions still vanish. The tree-level transformations for these operators are given by (53) and (55). To see this, one has to express the JMS operators on the LHS in terms of the $\tilde{Q}_I$ operators, and doing so we obtain

$$E_1^{\mathrm{VLL}} = [O_{ud}^{V1,LL}]_{11ji} - Q_1 = \tilde{Q}_1 - Q_1 = 0, \tag{62}$$

$$E_3^{\mathrm{VLL}} = [O_{ud}^{V1,LL}]_{22ji} - \left(-Q_1 + \frac{1}{3}Q_3 + \frac{2}{3}Q_9\right) = \tilde{Q}_1 - Q_1 = 0. \tag{63}$$

Here the shift in $\tilde{Q}_1 - Q_1$ is given by rule (11).

3. Operators requiring Fierz and EV shifts

Finally, we turn to the cases for which Fierz transformation at the tree level as well as the EV contributions at the one-loop level are necessary for the basis transformation. The tree-level transformations can be read from Eqs. (54), (56), and (58). The EV operators are then given by

$$E_2^{\mathrm{VLL}} = [O_{ud}^{V8,LL}]_{11ji} - \left(-\frac{1}{6}Q_1 + \frac{1}{2}Q_2\right) = \frac{1}{6}(Q_1 - \tilde{Q}_1) - \frac{1}{2}(Q_2 - \tilde{Q}_2) = -\frac{1}{6}\frac{\alpha_s}{4\pi}P, \tag{64}$$

$$E_4^{\mathrm{VLL}} = [O_{ud}^{V8,LL}]_{22ji} - \left(\frac{1}{6}Q_1 - \frac{1}{2}Q_2 - \frac{1}{18}Q_3 + \frac{1}{6}Q_4 - \frac{1}{9}Q_9 + \frac{1}{3}Q_{10}\right) = \frac{1}{6}(\tilde{Q}_1 - Q_1) + \frac{1}{2}(Q_2 - \tilde{Q}_2) = \frac{1}{6}\frac{\alpha_s}{4\pi}P, \tag{65}$$

$$E_5^{\mathrm{VLL}} = [O_{dd}^{V,LL}]_{jkki} - \left(\frac{2}{3}Q_4 - \frac{2}{3}Q_{10} - Q_{11}\right) = \frac{2}{3}(\tilde{Q}_4 - Q_4) - \frac{2}{3}(\tilde{Q}_{10} - Q_{10}) = \frac{2N_f + N_d - 2N_u}{9}\frac{\alpha_s}{4\pi}P. \tag{66}$$

Here the one-loop shifts are given by the rules in (11), (13), (14), and (15), respectively.

C. Definition of EV operators in the SRR sector

In this case, in addition to the color identity (B7), we need the Fierz relations given in (B1) and (B5).[7] The one-loop basis transformations including the EV operators E_I^{SRR} read

[7]Note that here we have used the definition $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. Also we define the operators $Q_2^{\mathrm{SRR},i}$ and $Q_3^{\mathrm{SRR},Q}$, $Q_4^{\mathrm{SRR},Q}$ with an additional negative sign as compared to Ref. [11].

$$[O_{dd}^{S1,RR}]_{jiii} = Q_2^{\mathrm{SRR},i}, \tag{67}$$

$$[O_{dd}^{S8,RR}]_{jiii} \stackrel{\mathcal{F}}{=} -\frac{5}{12}Q_2^{\mathrm{SRR},i} + \frac{1}{16}Q_4^{\mathrm{SRR},i} + E^{\mathrm{SRR,i}}. \tag{68}$$

Similar relations hold for $jiii \to jijj$. We note that a separate evanescent operator $E^{\mathrm{SRR,j}}$ is needed for this relation.

For the SRR,Q operators we find

$$[O_{ud}^{S1,RR}]_{11ji} = Q_2^{\mathrm{SRR},u}, \tag{69}$$

$$[O_{ud}^{S8,RR}]_{11ji} = \frac{1}{2}Q_1^{\mathrm{SRR},u} - \frac{1}{6}Q_2^{\mathrm{SRR},u}, \tag{70}$$

$$[O_{uddu}^{S1,RR}]_{1ij1} \stackrel{\mathcal{F}}{=} -\frac{1}{2}Q_1^{\mathrm{SRR},u} + \frac{1}{8}Q_3^{\mathrm{SRR},u} + E_1^{\mathrm{SRR,u}}, \tag{71}$$

$$[O_{uddu}^{S8,RR}]_{1ij1} \stackrel{\mathcal{F}}{=} \frac{1}{12}Q_1^{\mathrm{SRR},u} - \frac{1}{4}Q_2^{\mathrm{SRR},u} - \frac{1}{48}Q_3^{\mathrm{SRR},u} + \frac{1}{16}Q_4^{\mathrm{SRR},u} + E_2^{\mathrm{SRR,u}}. \tag{72}$$

Similar relations hold for $1 \to 2$ and $u \to c$ on the LHS and RHS, respectively. Finally,

$$[O_{dd}^{S1,RR}]_{jikk} = Q_2^{\mathrm{SRR},d_k}, \tag{73}$$

$$[O_{dd}^{S8,RR}]_{jikk} = \frac{1}{2}Q_1^{\mathrm{SRR},d_k} - \frac{1}{6}Q_2^{\mathrm{SRR},d_k}, \tag{74}$$

$$[O_{dd}^{S1,RR}]_{jkki} \stackrel{\mathcal{F}}{=} -\frac{1}{2}Q_1^{\mathrm{SRR},d_k} + \frac{1}{8}Q_3^{\mathrm{SRR},d_k} + E_1^{\mathrm{SRR},d_k}, \tag{75}$$

$$[O_{dd}^{S8,RR}]_{jkki} \stackrel{\mathcal{F}}{=} \frac{1}{12}Q_1^{\mathrm{SRR},d_k} - \frac{1}{4}Q_2^{\mathrm{SRR},d_k} - \frac{1}{48}Q_3^{\mathrm{SRR},d_k} + \frac{1}{16}Q_4^{\mathrm{SRR},d_k} + E_2^{\mathrm{SRR},d_k}. \tag{76}$$

D. Calculation of EV operators in the SRR sector

1. Operators requiring no Fierz

In the SRR sector, the following operators do not require Fierz transformations for the basis change at the tree level:

$$[O_{dd}^{S1,RR}]_{jiii},\quad [O_{dd}^{S1,RR}]_{jijj},\quad [O_{ud}^{S1,RR}]_{11ji},\quad [O_{ud}^{S8,RR}]_{11ji},\quad [O_{ud}^{S1,RR}]_{22ji},$$
$$[O_{ud}^{S8,RR}]_{22ji},\quad [O_{dd}^{S1,RR}]_{jikk},\quad [O_{ud}^{S8,RR}]_{jikk}. \tag{77}$$

Therefore, for the basis change at the one-loop level no EV contributions are required.

2. Operators requiring Fierz but no EV shifts

In this sector there are no such operators that require Fierz without having nonvanishing EV shifts at the one-loop level.

3. Operators requiring Fierz and EV shifts

The SRR operators that require the Fierz relation and nonvanishing EV operators for the basis transformation are given by (68), (71), (72), (75), and (76). The EV operators are then given by

$$\begin{aligned} E^{\mathrm{SRR,i}} &= [O_{dd}^{S8,RR}]_{jiii} - \left(-\frac{5}{12}Q_2^{\mathrm{SRR},i} + \frac{1}{16}Q_4^{\mathrm{SRR},i}\right) = -\frac{1}{2}(\tilde{Q}_1^{\mathrm{SRR},i} - Q_1^{\mathrm{SRR},i}) \\ &= -\frac{\alpha_s}{4\pi}\left(\frac{37}{24}Q_2^{\mathrm{SRR},i} + \frac{29}{96}Q_4^{\mathrm{SRR},i}\right), \end{aligned} \tag{78}$$

$$\begin{aligned} E_1^{\mathrm{SRR,u}} &= [O_{uddu}^{S1,RR}]_{1ij1} - \left(-\frac{1}{2}Q_1^{\mathrm{SRR},u} + \frac{1}{8}Q_3^{\mathrm{SRR},u}\right) \\ &= -\frac{1}{2}(\tilde{Q}_1^{\mathrm{SRR},u} - Q_1^{\mathrm{SRR},u}) + \frac{1}{8}(\tilde{Q}_3^{\mathrm{SRR},u} - Q_3^{\mathrm{SRR},u}) \\ &= \frac{\alpha_s}{4\pi}\sum_{k=1,2,3,4} p_k Q_k^{\mathrm{SRR,u}}, \end{aligned} \tag{79}$$

$$\begin{aligned} E_2^{\mathrm{SRR,u}} &= [O_{uddu}^{S8,RR}]_{1ij1} - \left(\frac{1}{12}Q_1^{\mathrm{SRR},u} - \frac{1}{4}Q_2^{\mathrm{SRR},u} - \frac{1}{48}Q_3^{\mathrm{SRR},u} + \frac{1}{16}Q_4^{\mathrm{SRR},u}\right) \\ &= \frac{1}{12}(\tilde{Q}_1^{\mathrm{SRR},u} - Q_1^{\mathrm{SRR},u}) - \frac{1}{4}(\tilde{Q}_2^{\mathrm{SRR},u} - Q_2^{\mathrm{SRR},u}) - \frac{1}{48}(\tilde{Q}_3^{\mathrm{SRR},u} - Q_3^{\mathrm{SRR},u}) + \frac{1}{16}(\tilde{Q}_4^{\mathrm{SRR},u} - Q_4^{\mathrm{SRR},u}) \\ &= \frac{\alpha_s}{4\pi}\sum_{k=1,2,3,4} q_k Q_k^{\mathrm{SRR,u}}. \end{aligned} \tag{80}$$

The coefficients p_k and q_k are found to be

$$p_1 = -\frac{7}{12}, \qquad p_2 = -\frac{17}{4}, \qquad p_3 = -\frac{5}{48}, \qquad p_4 = -\frac{3}{16}, \tag{81}$$

$$q_1 = -\frac{55}{36}, \qquad q_2 = -\frac{13}{12}, \qquad q_3 = -\frac{11}{144}, \qquad q_4 = -\frac{1}{48}. \tag{82}$$

The evanescent operators $E^{\mathrm{SRR,j}}$ and $E_{1,2}^{\mathrm{SRR,c}}, E_{1,2}^{\mathrm{SRR},d_k}$ follow from the corresponding $E^{\mathrm{SRR,i}}$ and $E_{1,2}^{\mathrm{SRR,u}}$ by the corresponding flavor replacements.

In the above calculation, the shifts $Q_1^{\mathrm{SRR},D} - \tilde{Q}_1^{\mathrm{SRR},D}$ and $Q_I^{\mathrm{SRR},Q} - \tilde{Q}_I^{\mathrm{SRR},Q}$ for $Q = u$ or d_k are obtained using the rules (20) and (28)–(31), respectively. It is worth noting at the one-loop QCD only current-current insertions are involved for the SRR operators in obtaining these rules. Therefore, the flavor structure of the operators is immaterial. For instance, the operators $[O_{uddu}^{S1,RR}]_{1ij1}$ and $Q_2^{\mathrm{SRR,u}}$ having the same color and Lorentz structures can be treated on the same footing even though they have different flavor structures.

E. Definition of EV operators in the VLR sector

In this subsection we turn our attention to the VLR sector and define the corresponding evanescent operators. Using the Fierz relation in Eq. (B4) as well as color relations one finds

$$\begin{aligned}
[O_{du}^{V1,LR}]_{ji11} &\stackrel{\mathcal{F}}{=} \frac{1}{6}Q_5 + \frac{1}{3}Q_7 + \frac{1}{2}Q_{18} + E_1^{\mathrm{VLR}},\\
[O_{du}^{V8,LR}]_{ji11} &\stackrel{\mathcal{F}}{=} -\frac{1}{36}Q_5 + \frac{1}{12}Q_6 - \frac{1}{18}Q_7 + \frac{1}{6}Q_8 + \frac{1}{4}Q_{17} - \frac{1}{12}Q_{18} + E_2^{\mathrm{VLR}},\\
[O_{du}^{V1,LR}]_{ji22} &\stackrel{\mathcal{F}}{=} \frac{1}{6}Q_5 + \frac{1}{3}Q_7 - \frac{1}{2}Q_{18} + E_3^{\mathrm{VLR}},\\
[O_{du}^{V8,LR}]_{ji22} &\stackrel{\mathcal{F}}{=} -\frac{1}{36}Q_5 + \frac{1}{12}Q_6 - \frac{1}{18}Q_7 + \frac{1}{6}Q_8 - \frac{1}{4}Q_{17} + \frac{1}{12}Q_{18} + E_4^{\mathrm{VLR}},\\
[O_{dd}^{V1,LR}]_{jikk} &= \frac{2}{3}Q_5 - \frac{2}{3}Q_7 - Q_{13},\\
[O_{dd}^{V8,LR}]_{jikk} &\stackrel{\mathcal{F}}{=} -\frac{1}{9}Q_5 + \frac{1}{3}Q_6 + \frac{1}{9}Q_7 - \frac{1}{3}Q_8 - \frac{1}{2}Q_{12} + \frac{1}{6}Q_{13} + E_5^{\mathrm{VLR}},\\
[O_{dd}^{V1,LR}]_{jiii} &= \frac{1}{2}Q_{13} + \frac{1}{2}Q_{16},\\
[O_{dd}^{V8,LR}]_{jiii} &= \frac{1}{4}Q_{12} - \frac{1}{12}Q_{13} + \frac{1}{4}Q_{15} - \frac{1}{12}Q_{16},\\
[O_{dd}^{V1,LR}]_{jijj} &= \frac{1}{2}Q_{13} - \frac{1}{2}Q_{16},\\
[O_{dd}^{V8,LR}]_{jijj} &= \frac{1}{4}Q_{12} - \frac{1}{12}Q_{13} - \frac{1}{4}Q_{15} + \frac{1}{12}Q_{16},\\
[O_{uddu}^{V1,LR}]_{1ji1}^{\dagger} &= -2Q_{19},\\
[O_{uddu}^{V8,LR}]_{1ji1}^{\dagger} &= \frac{1}{3}Q_{19} - Q_{20},\\
[O_{uddu}^{V1,LR}]_{2ji2}^{\dagger} &= -2Q_{21},\\
[O_{uddu}^{V8,LR}]_{2ji2}^{\dagger} &= \frac{1}{3}Q_{21} - Q_{22},\\
[O_{dd}^{V1,LR}]_{jkki} &= -2Q_{23},\\
[O_{dd}^{V8,LR}]_{jkki} &= \frac{1}{3}Q_{23} - Q_{24}.
\end{aligned} \tag{83}$$

In the VLR sector, the following set operators do not require Fierz transformations for the basis change at the tree level

$$[O_{dd}^{V1,LR}]_{jikk}, \quad [O_{dd}^{V1,LR}]_{jiii}, \quad [O_{dd}^{V8,LR}]_{jiii}, \quad [O_{dd}^{V1,LR}]_{jijj}, \quad [O_{dd}^{V8,LR}]_{jijj}, \quad [O_{uddu}^{V1,LR}]^{\dagger}_{1ji1},$$
$$[O_{uddu}^{V8,LR}]^{\dagger}_{1ji1}, \quad [O_{uddu}^{V1,LR}]^{\dagger}_{2ji2}, \quad [O_{uddu}^{V8,LR}]^{\dagger}_{2ji2}, \quad [O_{dd}^{V1,LR}]_{jkki}, \quad [O_{dd}^{V8,LR}]_{jkki}. \tag{84}$$

Therefore, for the basis change at the one-loop level no EV contributions are required. The rest of the operators in the VLR sector requiring Fierz are

$$[O_{du}^{V1,LR}]_{ji11}, \quad [O_{du}^{V8,LR}]_{ji11}, \quad [O_{du}^{V1,LR}]_{ji22}, \quad [O_{du}^{V8,LR}]_{ji22}, \quad [O_{dd}^{V8,LR}]_{jikk}. \tag{85}$$

However, as discussed in Sec. III A the corresponding EV vanish:

$$E_1^{\rm VLR} = E_2^{\rm VLR} = E_3^{\rm VLR} = E_4^{\rm VLR} = E_5^{\rm VLR} = 0. \tag{86}$$

V. CONCLUSIONS

We have presented a simple recipe to perform one-loop basis transformations involving evanescent operators. The procedure consists of computing the commutator of a one-loop (L) correction using dimensional regularization and a Fierz ($\mathcal{F}$) transformation of a given operator Q, which in all generality is nonvanishing:

$$[L, \mathcal{F}]Q \neq 0. \tag{87}$$

The presented method has already been used successfully in several contexts such as NLO basis transformations [11,15,24], one-loop matching calculations [25], as well as in several two-loop calculations [10]. But it has not been presented in any detail and, in particular, in this generality in the literature so far. The present paper should help to clarify possible issues involving evanescent operators. In the coming years one-loop matching and two-loop running effects will become more important in NP analyses than they are now.

We illustrated the outlined procedure by computing explicitly the complete one-loop basis change from the BMU to the JMS basis at $\mathcal{O}(\alpha_s)$, and this example should allow the reader to perform the transformation between different bases. In this context our method will serve as a simple tool to perform one-loop basis transformations. One particular example would be the basis change to the CMM (Chetyrkin, Misiak and Munz) basis [18], which is most suited for multiloop computations.

Since the one-loop basis change consists of a series of simple algebraic manipulations, it would be interesting to automate this procedure. After having computed all one-loop corrections to the operators in question, a simple algorithm might be included in codes such as ABC-EFT [26].

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APPENDIX A: $\Delta F = 1$ BMU BASIS FOR $N_f = 5$

In this appendix we collect the full set of BMU operators.

We start the list with the vector operators, where the first ten operators are the well-known SM operators $Q_1 - Q_{10}$:

$$Q_1 = Q_1^{\rm VLL,u} = (\bar{d}_j^\alpha \gamma_\mu P_L u^\beta)(\bar{u}^\beta \gamma^\mu P_L d_i^\alpha),$$
$$Q_2 = Q_2^{\rm VLL,u} = (\bar{d}_j^\alpha \gamma_\mu P_L u^\alpha)(\bar{u}^\beta \gamma^\mu P_L d_i^\beta), \tag{A1}$$

$$Q_3 = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)\sum_q (\bar{q}^\beta \gamma^\mu P_L q^\beta), \qquad Q_4 = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta)\sum_q (\bar{q}^\beta \gamma^\mu P_L q^\alpha),$$
$$Q_5 = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)\sum_q (\bar{q}^\beta \gamma^\mu P_R q^\beta), \qquad Q_6 = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta)\sum_q (\bar{q}^\beta \gamma^\mu P_R q^\alpha), \tag{A2}$$

$$Q_7 = \frac{3}{2}(\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)\sum_q Q_q (\bar{q}^\beta \gamma^\mu P_R q^\beta), \qquad Q_8 = \frac{3}{2}(\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta)\sum_q Q_q (\bar{q}^\beta \gamma^\mu P_R q^\alpha),$$
$$Q_9 = \frac{3}{2}(\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)\sum_q Q_q (\bar{q}^\beta \gamma^\mu P_L q^\beta), \qquad Q_{10} = \frac{3}{2}(\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta)\sum_q Q_q (\bar{q}^\beta \gamma^\mu P_L q^\alpha). \tag{A3}$$

The NP vector operators in the BMU basis are given by

$$\begin{aligned} Q_{11} &= Q_1^{\mathrm{VLL},i+j} = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)[(\bar{d}_i^\beta \gamma^\mu P_L d_i^\beta) + (\bar{d}_j^\beta \gamma^\mu P_L d_j^\beta)], \\ Q_{12} &= Q_1^{\mathrm{VLR},i+j} = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta)[(\bar{d}_i^\beta \gamma^\mu P_R d_i^\alpha) + (\bar{d}_j^\beta \gamma^\mu P_R d_j^\alpha)], \\ Q_{13} &= Q_2^{\mathrm{VLR},i+j} = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)[(\bar{d}_i^\beta \gamma^\mu P_R d_i^\beta) + (\bar{d}_j^\beta \gamma^\mu P_R d_j^\beta)], \end{aligned} \tag{A4}$$

$$\begin{aligned} Q_{14} &= Q_1^{\mathrm{VLL},i-j} = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)[(\bar{d}_i^\beta \gamma^\mu P_L d_i^\beta) - (\bar{d}_j^\beta \gamma^\mu P_L d_j^\beta)], \\ Q_{15} &= Q_1^{\mathrm{VLR},i-j} = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta)[(\bar{d}_i^\beta \gamma^\mu P_R d_i^\alpha) - (\bar{d}_j^\beta \gamma^\mu P_R d_j^\alpha)], \\ Q_{16} &= Q_2^{\mathrm{VLR},i-j} = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)[(\bar{d}_i^\beta \gamma^\mu P_R d_i^\beta) - (\bar{d}_j^\beta \gamma^\mu P_R d_j^\beta)], \end{aligned} \tag{A5}$$

$$\begin{aligned} Q_{17} &= Q_1^{\mathrm{VLR},u-c} = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\beta)[(\bar{u}^\beta \gamma^\mu P_R u^\alpha) - (\bar{c}^\beta \gamma^\mu P_R c^\alpha)], \\ Q_{18} &= Q_2^{\mathrm{VLR},u-c} = (\bar{d}_j^\alpha \gamma_\mu P_L d_i^\alpha)[(\bar{u}^\beta \gamma^\mu P_R u^\beta) - (\bar{c}^\beta \gamma^\mu P_R c^\beta)]. \end{aligned} \tag{A6}$$

Finally, we introduce the scalar sector of the BMU basis. In the SRL sector we use the structures

$$Q_1^{\mathrm{SRL},Q} = (\bar{d}_j^\alpha P_R d_i^\beta)(\bar{Q}^\beta P_L Q^\alpha), \qquad Q_2^{\mathrm{SRL},Q} = (\bar{d}_j^\alpha P_R d_i^\alpha)(\bar{Q}^\beta P_L Q^\beta), \tag{A7}$$

which define the operators

$$\begin{aligned} &(Q_{19}, Q_{20}) = (Q_1^{\mathrm{SRL},u}, Q_2^{\mathrm{SRL},u}), \\ &(Q_{21}, Q_{22}) = (Q_1^{\mathrm{SRL},c}, Q_2^{\mathrm{SRL},c}), \qquad (Q_{23}, Q_{24}) = (Q_1^{\mathrm{SRL},d_k}, Q_2^{\mathrm{SRL},d_k}). \end{aligned} \tag{A8}$$

In the SRR sector with three equal quarks we introduce for completeness the redundant structures

$$Q_1^{\mathrm{SRR},i} = (\bar{d}_j^\alpha P_R d_i^\beta)(\bar{d}_i^\beta P_R d_i^\alpha), \qquad Q_3^{\mathrm{SRR},i} = -(\bar{d}_j^\alpha \sigma_{\mu\nu} P_R d_i^\beta)(\bar{d}_i^\beta \sigma^{\mu\nu} P_R d_i^\alpha), \tag{A9}$$

together with the operators

$$Q_{25} = Q_2^{\mathrm{SRR},i} = (\bar{d}_j^\alpha P_R d_i^\alpha)(\bar{d}_i^\beta P_R d_i^\beta), \qquad Q_{26} = Q_4^{\mathrm{SRR},i} = -(\bar{d}_j^\alpha \sigma_{\mu\nu} P_R d_i^\alpha)(\bar{d}_i^\beta \sigma^{\mu\nu} P_R d_i^\beta), \tag{A10}$$

and similar for the SRR, j sector

$$Q_{27} = Q_2^{\mathrm{SRR},j} = (\bar{d}_j^\alpha P_R d_i^\alpha)(\bar{d}_j^\beta P_R d_j^\beta), \qquad Q_{28} = Q_4^{\mathrm{SRR},j} = -(\bar{d}_j^\alpha \sigma_{\mu\nu} P_R d_i^\alpha)(\bar{d}_j^\beta \sigma^{\mu\nu} P_R d_j^\beta), \tag{A11}$$

together with

$$Q_1^{\mathrm{SRR},j} = (\bar{d}_j^\alpha P_R d_i^\beta)(\bar{d}_j^\beta P_R d_j^\alpha), \qquad Q_3^{\mathrm{SRR},j} = -(\bar{d}_j^\alpha \sigma_{\mu\nu} P_R d_i^\beta)(\bar{d}_j^\beta \sigma^{\mu\nu} P_R d_j^\alpha). \tag{A12}$$

Note that we choose the operators Q_{26} and Q_{28} with an opposite sign, compared to the basis in [11]. For the SRR sector with four different quarks we define the structures

$$\begin{aligned} Q_1^{\mathrm{SRR},Q} &= (\bar{d}_j^\alpha P_R d_i^\beta)(\bar{Q}^\beta P_R Q^\alpha), \qquad Q_3^{\mathrm{SRR},Q} = -(\bar{d}_j^\alpha \sigma_{\mu\nu} P_R d_i^\beta)(\bar{Q}^\beta \sigma^{\mu\nu} P_R Q^\alpha), \\ Q_2^{\mathrm{SRR},Q} &= (\bar{d}_j^\alpha P_R d_i^\alpha)(\bar{Q}^\beta P_R Q^\beta), \qquad Q_4^{\mathrm{SRR},Q} = -(\bar{d}_j^\alpha \sigma_{\mu\nu} P_R d_i^\alpha)(\bar{Q}^\beta \sigma^{\mu\nu} P_R Q^\beta), \end{aligned} \tag{A13}$$

where the tensor structures have again opposite signs compared to the convention adopted in [11]. With these definitions we define the operators Q_{29}–Q_{40}

$$(Q_{29}, Q_{30}, Q_{31}, Q_{32}) = (Q_1^{\mathrm{SRR},u}, Q_2^{\mathrm{SRR},u}, Q_3^{\mathrm{SRR},u}, Q_4^{\mathrm{SRR},u}),$$
$$(Q_{33}, Q_{34}, Q_{35}, Q_{36}) = (Q_1^{\mathrm{SRR},c}, Q_2^{\mathrm{SRR},c}, Q_3^{\mathrm{SRR},c}, Q_4^{\mathrm{SRR},c}),$$
$$(Q_{37}, Q_{38}, Q_{39}, Q_{40}) = (Q_1^{\mathrm{SRR},d_k}, Q_2^{\mathrm{SRR},d_k}, Q_3^{\mathrm{SRR},d_k}, Q_4^{\mathrm{SRR},d_k}). \quad \text{(A14)}$$

Finally, as far as the chirality-flipped operators are concerned, their numbering in the BMU basis is given by

$$Q_{40+i} = Q_i[P_L \leftrightarrow P_R]; \quad \text{(A15)}$$

i.e., they are found by interchanging $P_L \leftrightarrow P_R$ in the "nonflipped" operators.

APPENDIX B: FIERZ IDENTITIES

In the process of transforming from one operator basis to another one requires the Fierz identities [27] that allow one to transfer a given chain of spinors into another one. We list here the usual Fierz identities valid in $D = 4$ dimensions that we used in our analysis.

All Fierz identities used are of the type $(12)(34) \to (14)(32)$ in which the exchange of fermion fields $2 \leftrightarrow 4$ (or equivalently $1 \leftrightarrow 3$) takes place. In the formulas below P_A and P_B stand for the usual projectors $P_{L,R}$ but in a given relation $P_A \neq P_B$. This means that if $P_A = P_L$, then $P_B = P_R$, and vice versa.

We have then

$$(\bar\psi_1 P_A \psi_2)(\bar\psi_3 P_A \psi_4) = -\frac{1}{2}(\bar\psi_1 P_A \psi_4)(\bar\psi_3 P_A \psi_2) - \frac{1}{8}(\bar\psi_1 \sigma_{\mu\nu} P_A \psi_4)(\bar\psi_3 \sigma^{\mu\nu} P_A \psi_2), \quad \text{(B1)}$$

$$(\bar\psi_1 P_A \psi_2)(\bar\psi_3 P_B \psi_4) = -\frac{1}{2}(\bar\psi_1 \gamma_\mu P_B \psi_4)(\bar\psi_3 \gamma^\mu P_A \psi_2), \quad \text{(B2)}$$

$$(\bar\psi_1 \gamma_\mu P_A \psi_2)(\bar\psi_3 \gamma^\mu P_A \psi_4) = (\bar\psi_1 \gamma_\mu P_A \psi_4)(\bar\psi_3 \gamma^\mu P_A \psi_2), \quad \text{(B3)}$$

$$(\bar\psi_1 \gamma_\mu P_A \psi_2)(\bar\psi_3 \gamma^\mu P_B \psi_4) = -2(\bar\psi_1 P_B \psi_4)(\bar\psi_3 P_A \psi_2), \quad \text{(B4)}$$

$$(\bar\psi_1 \sigma_{\mu\nu} P_A \psi_2)(\bar\psi_3 \sigma^{\mu\nu} P_A \psi_4) = -6(\bar\psi_1 P_A \psi_4)(\bar\psi_3 P_A \psi_2) + \frac{1}{2}(\bar\psi_1 \sigma_{\mu\nu} P_A \psi_4)(\bar\psi_3 \sigma^{\mu\nu} P_A \psi_2), \quad \text{(B5)}$$

$$(\bar\psi_1 \sigma_{\mu\nu} P_A \psi_2)(\bar\psi_3 \sigma^{\mu\nu} P_B \psi_4) = 0. \quad \text{(B6)}$$

For more Fierz identities involving charge conjugated fields see Appendix A. 3 in [1].

Apart from this we also need the color identity

$$T^A \otimes T^A = \frac{1}{2}\left(\tilde{\mathbb{1}} - \frac{1}{N_c}\mathbb{1}\right). \quad \text{(B7)}$$

APPENDIX C: MASTER FORMULAS FOR ONE-LOOP OPERATOR INSERTIONS

In this section, we present master formulas for the one-loop operator insertions. These can be used to obtain the shifts given in Sec. III. A. Consider a four-fermion operator

$$(\bar q_1 \hat V_1 \Gamma_1 q_2)(\bar q_3 \hat V_2 \Gamma_2 q_4), \quad \text{(C1)}$$

where $\hat V_{1,2}$ and $\Gamma_{1,2}$ represent the color and Dirac structures. There are two types of penguin insertions: an *open penguin* and the *closed penguin*. In the next two subsections we evaluate the corresponding amplitudes.

1. Open penguin insertion

The open penguin insertion of the operator (C1) gives

$$P_{op} = W_\lambda(-i g_s T^b \gamma_{\lambda'})\left(\frac{-i g^{\lambda\lambda'}}{q^2}\right), \quad \text{(C2)}$$

$$W_\lambda = i^2 \hat V_1 (-i g_s T^a) \hat V_2 I^{\mu\nu} T^\lambda_{\mu\nu}, \quad \text{(C3)}$$

where $T^\lambda_{\mu\nu} = \Gamma_1 \gamma_\nu \gamma_\lambda \gamma_\mu \Gamma_2$ and

$$I^{\mu\nu} = \int \frac{d^D k}{(2\pi)^D} \frac{k^\nu (k-q)^\mu}{k^2 (k-q)^2} = -\frac{i}{16\pi^2}\frac{1}{\varepsilon}\left(\frac{1}{6}q_\mu q_\nu + \frac{1}{12}q^2 g_{\mu\nu}\right) - \frac{i}{16\pi^2}\left(\frac{5}{18}q_\mu q_\nu + \frac{2}{9}q^2 g_{\mu\nu}\right). \tag{C4}$$

Therefore, the finite and infinite parts of P_{op} can be written as

$$P_{op}(\text{infinite}) = -C_{op}\frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\left(\frac{1}{6}\frac{q_\mu q_\nu}{q^2} + \frac{1}{12}g_{\mu\nu}\right)\Gamma_1 \gamma_\mu \gamma_\lambda \gamma_\nu \Gamma_2 \otimes \gamma^\lambda, \tag{C5}$$

$$P_{op}(\text{finite}) = -C_{op}\frac{\alpha_s}{4\pi}\left(\frac{5}{18}\frac{q_\mu q_\nu}{q^2} + \frac{2}{9}g_{\mu\nu}\right)\Gamma_1 \gamma_\mu \gamma_\lambda \gamma_\nu \Gamma_2 \otimes \gamma^\lambda. \tag{C6}$$

Here $C_{op} = \hat{V}_1 T^b \hat{V}_2 \otimes T^b$.

2. Closed penguin insertion

The closed penguin insertion of the operator (C1) gives

$$P_{cl} = W_\lambda (-i g_s T^b \gamma_{\lambda'})\left(\frac{-i g^{\lambda\lambda'}}{q^2}\right), \tag{C7}$$

$$W_\lambda = (-1)(i^2)(-i g_s)\mathrm{Tr}(\hat{V}_1 T^b)\hat{V}_2 \mathrm{Tr}(\Gamma_1 \gamma_\mu \gamma_\lambda \gamma_\nu)\Gamma_2 I^{\mu\nu}. \tag{C8}$$

Here the $I^{\mu\nu}$ is given by (C4).

Therefore, the finite and infinite parts of P_{cl} can be written as

$$P_{cl}(\text{infinite}) = C_{cl}\frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\left(\frac{1}{6}\frac{q_\mu q_\nu}{q^2} + \frac{1}{12}g_{\mu\nu}\right)\mathrm{Tr}(\Gamma_1 \gamma_\mu \gamma_\lambda \gamma_\nu)\Gamma_2 \otimes \gamma^\lambda, \tag{C9}$$

$$P_{cl}(\text{finite}) = C_{cl}\frac{\alpha_s}{4\pi}\left(\frac{5}{18}\frac{q_\mu q_\nu}{q^2} + \frac{2}{9}g_{\mu\nu}\right)\mathrm{Tr}(\Gamma_1 \gamma_\mu \gamma_\lambda \gamma_\nu)\Gamma_2 \otimes \gamma^\lambda. \tag{C10}$$

Here $C_{cl} = \mathrm{Tr}(\hat{V}_1 T^b)\hat{V}_2 \otimes T^b$.

3. Special Cases

In Tables II and III we give the finite and singular parts for the penguin operators insertions with various Dirac structures.

TABLE II. Finite and infinite parts of the open penguin insertion. $C_{op} = \hat{V}_1 T^b \hat{V}_2 \otimes T^b$.

Dirac structure	P_{op} (infinite)	P_{op} (finite)
$\Gamma_1 = \gamma_\rho P_L$, $\Gamma_2 = \gamma^\rho P_L$	$C_{op}\frac{\alpha_s}{4\pi}\gamma_\lambda P_L \otimes \gamma^\lambda$	$-C_{op}\frac{\alpha_s}{4\pi}\frac{13}{9}\gamma_\lambda P_L \otimes \gamma^\lambda$
$\Gamma_1 = \gamma_\rho P_L$, $\Gamma_2 = \gamma^\rho P_R$	0	0
$\Gamma_1 = P_L$, $\Gamma_2 = P_L$	0	0
$\Gamma_1 = P_L$, $\Gamma_2 = P_R$	$-C_{op}\frac{\alpha_s}{4\pi}\frac{1}{6}\gamma_\lambda P_R \otimes \gamma^\lambda$	$C_{op}\frac{\alpha_s}{4\pi}\frac{13}{8}\gamma_\lambda P_R \otimes \gamma^\lambda$
$\Gamma_1 = \sigma_{\alpha\beta} P_L$, $\Gamma_2 = \sigma^{\alpha\beta} P_L$	0	0

TABLE III. Finite and infinite parts of the closed penguin insertion. $C_{cl} = \mathrm{Tr}(\hat{V}_1 T^b)\hat{V}_2 \otimes T^b$.

Dirac structure	P_{cl} (finite)	P_{cl} (infinite)
$\Gamma_1 = \gamma_\rho P_L$, $\Gamma_2 = \gamma^\rho P_L$	$-C_{cl}\frac{\alpha_s}{4\pi}\frac{13}{9}\gamma_\lambda P_L \otimes \gamma^\lambda$	$C_{cl}\frac{\alpha_s}{4\pi}\frac{1}{3}\gamma_\lambda P_L \otimes \gamma^\lambda$
$\Gamma_1 = \gamma_\rho P_L$, $\Gamma_2 = \gamma^\rho P_R$	$-C_{cl}\frac{\alpha_s}{4\pi}\frac{13}{9}\gamma_\lambda P_R \otimes \gamma^\lambda$	$C_{cl}\frac{\alpha_s}{4\pi}\frac{1}{3}\gamma_\lambda P_R \otimes \gamma^\lambda$
$\Gamma_1 = P_L$, $\Gamma_2 = P_L$	0	0
$\Gamma_1 = P_L$, $\Gamma_2 = P_R$	0	0
$\Gamma_1 = \sigma_{\alpha\beta} P_L$, $\Gamma_2 = \sigma^{\alpha\beta} P_L$	0	0

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