# Arrangement for the $K_2^*$ meson family

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Two observed structures with  $M = 1868 \pm 8^{+40}_{-57}$  MeV and  $M = 2073 \pm 94^{+245}_{-240}$  MeV are the same states  $[K_2^*(1980)]$  in Particle Data Group. In this paper, analysis of the mass spectrum and calculation of the strong decay for  $K_2^*$  mesons support the assignment of  $2^3P_2$  and  $1^3F_2$  as the low- and high-mass states for  $K_2^*(1980)$ . This analysis reveals very important criteria for the assignment of the observed  $K_2^*(1980)$ , and experimental findings for this assignment are suggested. Additionally, some partial decay widths are predicted based on the high excitations of the  $K_2^*$  family. This study is crucial for establishing and searching for the higher excitations in the future.

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### I. INTRODUCTION

The  $K_2^*$  meson family is a crucial component of the kaon family. There are two members in this family of  $K_2^*$  mesons:  $K_2^*(1430)$  and  $K_2^*(1980)$ .  $K_2^*(1430)$  is well established as the ground state of the  $K_2^*$  meson family with a  $1^3P_2$ assignment.  $K_2^*(1980)$  is now listed in Particle Data Group (PDG) [1] with an average mass and width of  $1995_{-50}^{+60}$  MeV and  $349_{-30}^{+50}$  MeV. The existence of  $K_2^*(1980)$  as a  $2^3P_2$  or a  $1^3F_2$  state has aroused our attention. The  $K_2^*(1980)$  meson was reported by the LASS Collaboration in 1987 and 1989 in  $K^- p \rightarrow \bar{K}^0 \pi^+ \pi^- n$ and  $K^- p \to \bar{K}^0 \pi^- p$  processes, whose mass is  $1973 \pm 8 \pm$ 25 MeV and corresponding width is  $373 \pm 33 \pm 60$  MeV in 1989, the LASS Collaboration gave the resonance parameters of  $K_2^*(1980)$  with  $M = 1978 \pm 40$  MeV and  $\Gamma = 398 \pm 47$  MeV, respectively) [1,2]. This meson is likely to be the candidate for the  $2^{3}P_{2}$  state or  $1^{3}F_{2}$  state.

Recently, the BESIII Collaboration observed  $K_2^*(1980)$ in the  $K\pi$  channel in the process  $J/\psi \to K^+ K^- \pi^0$ . They obtained two solutions when fitting the experimental data,  $M = 1817 \pm 11$  MeV and  $\Gamma = 312 \pm 28$  MeV, or M =  $1868 \pm 8^{+40}_{-51}$  MeV and  $\Gamma = 272 \pm 24^{+50}_{-15}$  MeV [3]. The mass of this meson is approximately 250 MeV lower than the value of  $2073 \pm 94^{+245}_{-240}$  MeV detected by the LHCb Collaboration [4]. The resonances for  $K_2^*(1980)$  were later provided by the LHCb Collaboration. They regarded  $K_2^*(1980)$  as the  $2^3P_2$  state, where  $J^P = 2^+$ , the mass is of  $1988 \pm 22^{+194}_{-101}$  MeV, and the width is of  $318 \pm$   $82^{+481}_{-101}$  MeV [5]. Recent observations of  $K_2^*(1980)$  by the BESIII Collaboration by means of partial-wave analysis of  $\psi(3686) \rightarrow K^+K^-\eta$  also gave the resonance parameters  $M = 2046^{+17+67}_{-16-15}$  MeV and  $\Gamma = 408^{+38+72}_{-34-44}$  MeV [6]. Do these structures belong to the same state?

We contrast the  $a_2$  and  $K_2$  families in Fig. 1 to find the solution to this query. In the  $a_2$  family, the mass difference between the  $1^3P_2$  and  $2^3P_2$  states is 388 MeV.



FIG. 1. Mass gap comparison between  $K_2^*$  and  $a_2$  family. The left part of the figure is the mass gap of  $a_2$  family and the right part of the figure is the mass gap of  $K_2^*$  family.

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Additionally, there is a mass gap of 713 MeV between the  $1^{3}P_{2}(a_{2}(1320))$  and  $1^{3}F_{2}(a_{2}(2030))$  states. If we put this mass gap into the  $K_{2}$  family, and use 1427.3 MeV as the mass of the  $1^{3}P_{2}(K_{2}^{*}(1430))$  state, then the mass of the  $2^{3}P_{2}$  state will be 1815.3 MeV, which is extremely close to the experimental value  $M = 1817 \pm 11$  MeV (or solution two,  $1868 \pm 8^{+40}_{-57}$  MeV) [1]. The mass of the  $1^{3}F_{2}$  state will be 2140.3 MeV, consistent with  $2073 \pm 94^{+245}_{-240}$  MeV [1]. Following this analysis, the structure with  $M = 1817 \pm 11$  MeV (or the second solution  $1868 \pm 8^{+40}_{-57}$  MeV, which, in this work, we name  $K_{2}^{*}(1870)$ ) [1] should be the  $2^{3}P_{2}$  state and the state with  $M = 2073 \pm 94^{+245}_{-240}$  MeV [1] [which, in this work, we name  $K_{2}^{*}(2070)$ ] could be the  $K_{2}^{*}(1^{3}F_{2})$  state.

To test the proposed assignment of  $K_2^*(1870)$  and  $K_2^*(2070)$ , in the following, we determine the mass and decay width for  $K_2^*(2^3P_2)$  and  $K_2^*(1^3F_2)$  via the mass spectrum and two-body strong decay of  $K_2^*$ . At the same time, the property of the higher excited  $K_2^*$  states will be investigated.

Godfrey and Isgur proposed the GI model for describing relativistic meson spectra in 1985 [7]. The screened effect was demonstrated by the lattice calculations [8–11], which stimulated the study of the quark-quark interaction in the works [12–14]. References [15,16] discussed the relation between the coupled-channel effect with intermediate meson-meson loops and the screened effect. Song et al. developed the modified GI (MGI) model taking into account the screened effect in the GI model [17,18]. In this work, we study the mass spectra for  $K_2^*$  mesons more preferably with the help of MGI model considering this phenomenological screened potential. In fact, the interaction energy obtained by lattice-field theoretic computations is not a potential; rather, it reveals the adiabatic crossing of two potentials, one for a heavy quarkantiquark system, and one for a meson-meson system [8–11]. Indeed, integrating out the continuum channel gives rise to a short-distance correction to the quarkantiquark interaction, not a long-distance one. Thus, there is no reason to believe that the total energy of kaon system goes to a maximum value of 3.72 GeV when  $r \to \infty$  [19].

The spatial wave functions obtained by the modified GI model can be taken as input when we study the  $K_2^*$  family's Okubo-Zweig-Iizuka (OZI)-allowed two-body strong decays adopting the quark-pair creation (QPC) model, which was proposed in Ref. [20] and extensively applied to studies of other hadrons in Refs. [21–47].

This paper is organized as follows. After the Introduction, in Sec. II, we explain the modified Godfrey-Isgur model and the QPC model. In Sec. III, we adopt the modified Godfrey-Isgur model by including the screened effect to study the mass spectra obtained for the  $K_2^*$  family. We further obtain the structural information for the observed  $K_2^*$  via making a comparison between theoretical

and experimental results. We present a detailed study of the OZI-allowed two-body strong decays of the discussed kaons. The paper ends with a conclusion.

# II. PHENOMENOLOGICAL ANALYSIS OF K<sup>\*</sup><sub>2</sub> MESONS

In this work, the modified GI quark model is utilized to calculate the mass spectrum and wave functions for the  $K_2^*$  meson family. We also investigated the twobody strong decay of the  $K_2^*$  meson family with the QPC model. In the following, these models will be illustrated in detail.

## A. Brief review of the MGI and QPC models

## 1. MGI model

Godfrey and Isgur proposed the GI model for describing relativistic meson spectra with great success, exactly for low-lying mesons [7]. For excited states, the screened potential must be taken into account for coupled-channel effect [48–50].

The interaction between the quark and antiquark is depicted by the Hamiltonian of the potential model, including the kinetic energy and effective potential,

$$\tilde{H} = \sqrt{m_1^2 + \mathbf{p}^2} + \sqrt{m_2^2 + \mathbf{p}^2} + \tilde{V}_{\text{eff}}(\mathbf{p}, \mathbf{r}), \quad (1)$$

where  $m_1$  and  $m_2$  denote the mass of the quark and antiquark, respectively, and the effective potential  $\tilde{V}_{eff}$ contains two ingredients, a short-range  $\gamma^{\mu} \otimes \gamma_{\mu}$  onegluon-exchange interaction and a 1  $\otimes$  1 linear confinement interaction. The meaning of the tilde will be explained later.

In the nonrelativistic limit, the effective potential has a familiar format [7,51]:

$$V_{\rm eff}(r) = H^{\rm conf} + H^{\rm hyp} + H^{\rm so}, \qquad (2)$$

with

$$H^{\text{conf}} = \left[ -\frac{3}{4} (br+c) + \frac{\alpha_s(r)}{r} \right] (\boldsymbol{F}_1 \cdot \boldsymbol{F}_2)$$
  
=  $S(r) + G(r),$  (3)

$$H^{\text{hyp}} = -\frac{\alpha_s(r)}{m_1 m_2} \left[ \frac{8\pi}{3} S_1 \cdot S_2 \delta^3(r) + \frac{1}{r^3} \left( \frac{3S_1 \cdot rS_2 \cdot r}{r^2} - S_1 \cdot S_2 \right) \right] (F_1 \cdot F_2), \quad (4)$$

$$H^{\rm so} = H^{\rm so(cm)} + H^{\rm so(tp)},\tag{5}$$

where  $H^{\text{conf}}$  includes the spin-independent linear confinement piece S(r) and Coulomb-like potential from onegluon-exchange G(r).  $H^{\text{hyp}}$  denotes the color-hyperfine interaction consisting of tensor and contact terms.  $H^{SO}$  is the spin-orbit interaction with

$$H^{\rm so(cm)} = \frac{-\alpha_s(r)}{r^3} \left(\frac{1}{m_1} + \frac{1}{m_2}\right) \left(\frac{S_1}{m_1} + \frac{S_2}{m_2}\right) \cdot L(F_1 \cdot F_2),$$
(6)

which is caused by one-gluon exchange and

$$H^{\rm so(tp)} = -\frac{1}{2r} \frac{\partial H^{\rm conf}}{\partial r} \left( \frac{S_1}{m_1^2} + \frac{S_2}{m_2^2} \right) \cdot L, \tag{7}$$

which is the Thomas precession term.

For the above formulas,  $S_1/S_2$  indicates the spin of the quark/antiquark and L is the orbital momentum between the two particles. F is relevant to the Gell-Mann matrix, i.e.,  $F_1 = \lambda_1/2$  and  $F_2 = -\lambda_2^*/2$ , and for a meson,  $\langle F_1 \cdot F_2 \rangle = -4/3$ .

Now the relativistic effects of distinguishing influence must be considered especially in meson systems, which are embedded in two different ways. First, based on the nonlocal interactions and new **r** dependence, a smearing function is introduced for a meson  $q\bar{q}$ :

$$\rho(\mathbf{r} - \mathbf{r}') = \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2(\mathbf{r} - \mathbf{r}')^2},\tag{8}$$

which is applied to S(r) and G(r) to obtain smeared potentials  $\tilde{S}(r)$  and  $\tilde{G}(r)$  by

$$\tilde{f}(r) = \int d^3 \mathbf{r}' \rho(\mathbf{r} - \mathbf{r}') f(r'), \qquad (9)$$

with

$$\sigma_{12}^2 = \sigma_0^2 \left[ \frac{1}{2} + \frac{1}{2} \left( \frac{4m_1m_2}{(m_1 + m_2)^2} \right)^4 \right] + s^2 \left( \frac{2m_1m_2}{m_1 + m_2} \right)^2.$$
(10)

Second, owing to relativistic effects, a general potential should rely on the center of mass of the interacting quarks. Momentum-dependent factors that will be unity in the nonrelativistic limit are applied as

$$\tilde{G}(r) \to \left(1 + \frac{p^2}{E_1 E_2}\right)^{1/2} \tilde{G}(r) \left(1 + \frac{p^2}{E_1 E_2}\right)^{1/2},$$
 (11)

and

$$\frac{\tilde{V}_i(r)}{m_1 m_2} \to \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon_i} \frac{\tilde{V}_i(r)}{m_1 m_2} \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon_i}, \quad (12)$$

where  $\tilde{V}_i(r)$  represents the contact, tensor, vector spinorbit, and scalar spin-orbit terms, and  $\epsilon_i$  is the relevant modification parameter. The screened effect can be introduced by the transformation  $br + c \rightarrow \frac{b(1-e^{-\mu r})}{\mu} + c$ , where  $\mu$  is the screened parameter whose particular value is given by Ref. [19]. The modified confinement potential also requires similar relativistic correction, which has been mentioned in the GI model. Then, we further write

$$\tilde{V}^{\text{scr}}(r) = \int d^3 r' \rho(\mathbf{r} - \mathbf{r}') \frac{b(1 - e^{-\mu r'})}{\mu}$$
  
=  $\frac{b}{\mu r} \left[ r + e^{\frac{\mu^2}{4\sigma^2} + \mu r} \frac{\mu + 2r\sigma^2}{2\sigma^2} \left( \frac{1}{\sqrt{\pi}} \int_0^{\frac{\mu + 2r\sigma^2}{2\sigma}} e^{-x^2} dx - \frac{1}{2} \right) - e^{\frac{\mu^2}{4\sigma^2} - \mu r} \frac{\mu - 2r\sigma^2}{2\sigma^2} \left( \frac{1}{\sqrt{\pi}} \int_0^{\frac{\mu - 2r\sigma^2}{2\sigma}} e^{-x^2} dx - \frac{1}{2} \right) \right].$  (13)

The mass spectrum and the wave function for  $K_2^*$  mesons can be obtained by solving eigenvalue and eigenvector of the  $\tilde{H}$  in Eq. (1) with the simple harmonic oscillator (SHO) baseexpanding method. In configuration and momentum space, SHO wave functions have explicit forms, respectively,

$$\Psi_{nLM_L}(\mathbf{r}) = R_{nL}(r,\beta)Y_{LM_L}(\Omega_r),$$
  
$$\Psi_{nLM_L}(\mathbf{p}) = R_{nL}(p,\beta)Y_{LM_L}(\Omega_p),$$
 (14)

with

$$R_{nL}(r,\beta) = \beta^{3/2} \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}} (\beta r)^L e^{\frac{-r^2 \beta^2}{2}} \times L_n^{L+1/2} (\beta^2 r^2),$$
(15)

$$R_{nL}(p,\beta) = \frac{(-1)^n (-i)^L}{\beta^{3/2}} e^{-\frac{p^2}{2\beta^2}} \sqrt{\frac{2n!}{\Gamma(n+L+3/2)}} \left(\frac{p}{\beta}\right)^L \times L_n^{L+1/2} \left(\frac{p^2}{\beta^2}\right),$$
(16)

where  $Y_{LM_L}(\Omega)$  is the spherical harmonic function, and  $L_{n-1}^{L+1/2}(x)$  is the associated Laguerre polynomial.

### 2. QPC model

The QPC model was first proposed by Micu [20], and was further developed by the Orsay group [21,52–55]. It was widely applied to the OZI-allowed two-body strong decay of hadrons in Refs. [22,23,26,28,30,32–37,40–43,45,46,56–61].

The decay process  $A \rightarrow B + C$  can be expressed as follows:

$$\langle BC|\mathcal{T}|A\rangle = \delta^3(\mathbf{P}_B + \mathbf{P}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}},\qquad(17)$$

where  $\mathbf{P}_{B(C)}$  is the three-momentum for a meson B(C) in the rest frame of a meson A. The superscript  $M_{J_i}$ 

TABLE I. Spectrum for the  $K_2^*$  meson family, where Exp. represent the experimental data [65]. Unit of mass is MeV.

State	Ref. [19]	Ref. [63]	GI [7]	Ebert [64]	Exp.
$1^{3}P_{2}$	1450	1432	1409	1424	$1427 \pm 1.5$
$2^{3}P_{2}$	1906	1870	1924	1896	$1868\pm8^{+40}_{-51}$
$3^{3}P_{2}$	2274	2198	2370		
$4^{3}P_{2}$	2570	2438	2756		
$1^{3}F_{2}$	2200	2092	2168	1.964	$2073 \pm 94^{+245}_{-240}$
$2^{3}F_{2}$	2415	2356	2565		
$3^3F_2$	2682	2552	2917		

(i = A, B, C) denotes an orbital magnetic momentum. The transition operator T is introduced to describe a quarkantiquark pair creation from vacuum, which has the quantum number  $J^{PC} = 0^{++}$ , T can be written as

$$\mathcal{T} = -3\gamma \sum_{m} \langle 1m; 1-m|00\rangle \int d\mathbf{p}_{3} d\mathbf{p}_{4} \delta^{3}(\mathbf{p}_{3}+\mathbf{p}_{4}) \\ \times \mathcal{Y}_{1m} \left(\frac{\mathbf{p}_{3}-\mathbf{p}_{4}}{2}\right) \chi^{34}_{1,-m} \phi^{34}_{0}(\omega^{34}_{0})_{ij} b^{\dagger}_{3i}(\mathbf{p}_{3}) d^{\dagger}_{4j}(\mathbf{p}_{4}), \quad (18)$$

which is the transition operator and it can describe the creation of a quark-antiquark pair from vacuum, where the quark and antiquark are denoted by the indices 3 and 4,



FIG. 2. Our theoretical results and the resonance parameters of  $K_2^*(1980)$  measured by different experiments collected in PDG [1]. Here, the green lines with triangle and disk denote the central value of mass and decay width of  $K_2^*(1980)$  measured by the BESIII Collaboration, respectively [3,4]. The green line with rose pentagon denotes the central value of decay-branching ratio of  $K_2^*(1980)$  measured by the LASS Collaboration [2]. The purplish blue lines are our calculation results.

TABLE II. Allowed partial strong decay widths of  $2^{3}P_{2}$  and  $1^{3}F_{2}$  state.

Decay channels	$2^{3}P_{2}$	$1^{3}F_{2}$
Kb <sub>1</sub>	8.15-26.3	118–145
$K_1\pi$	12.4-33.8	104-152
$Ka_1$	4.3-13.8	61.8-79.8
$K^* ho$	55.8-77.8	24.1-131
$Kh_1$	5.51-10.8	39.5-47.5
$K_2^*\pi$	15.2-28.3	18.9-34.4
Ka <sub>2</sub>	0-25.9	14-32.4
$Kf_1$	0.23-3.2	15.7-25.1
$K^*\omega$	18.7-24.9	7.64-42.3
Κρ	31.6-33	19.1-21.5
$K^{*}(892)\pi$	18.1-21.8	16.8-20.1
Κπ	0.0117-1.17	13.9-19.5
$K(1460)\pi$	5.15-21.7	9.26-16.6
$Kf_2(1270)$	1.29-12.4	6.03-11.2
Kω	10.5-11	6.43-7.19
$K\pi(1300)$	0-10.5	3.02-9.17
$K^{*}(1410)\pi$	15.2-46.7	3.37-5.7
$K^*\eta$	3.93-5.02	3.79-4.47
Κη	0.361-0.801	3.77-4.4
$K\eta'$	0.632-0.818	2.31-3.27
$K'_1\pi$	12.5-16.9	0.113-0.498
$K_1 \rho$	0	0-149
$K_1\eta$	0	24.3-37.9
$K\eta_2$	0	0-62.8
$K_1\omega$	0	0-48.1
$K^*h_1$	0	0-34.2
$K^*b_1$	0	0-61.4
$K^*a_1$	0	0-33.7
$K_2^*\eta$	0	0-4.99
$K^*\eta'$	0	0.319-2.45
$K_3^*(1780)\pi$	0	0.0372-7.77
$K^{*}(1680)\pi$	0	0.0562-3.73

respectively. The parameter  $\gamma$  depicts the strength of the creation of  $q\bar{q}$  from vacuum.  $\mathcal{Y}_{\ell m}(\mathbf{p}) = |\mathbf{p}|^{\ell} Y_{\ell m}(\mathbf{p})$  are the solid harmonics.  $\chi$ ,  $\phi$ , and  $\omega$  denote the spin, flavor, and color wave functions, respectively, which can be treated separately. The subindices *i* and *j* denote the color of a  $q\bar{q}$  pair.

The decay amplitude can be expressed in another form by the Jacob-Wick formula [62]:

$$\mathcal{M}^{JL}(\mathbf{P}) = \frac{\sqrt{4\pi(2L+1)}}{2J_A + 1} \sum_{M_{J_B}M_{J_C}} \langle L0; JM_{J_A} | J_A M_{J_A} \rangle$$
$$\times \langle J_B M_{J_B}; J_C M_{J_C} | J_A M_{J_A} \rangle \mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}.$$
(19)

Then, the general decay width will be

$$\Gamma = \frac{\pi}{4} \frac{|\mathbf{P}|}{m_A^2} \sum_{J,L} |\mathcal{M}^{JL}(\mathbf{P})|^2, \qquad (20)$$

where  $m_A$  is the mass of an initial state A. In our calculation, the spatial wave functions for the mesons are given in Ref. [19]. The value of  $\gamma$  is 11.6.

### **B.** Mass spectrum analysis

In Table I, we give the mass spectrum for the  $K_2^*$  mesons by using different models. We can see from our previous work (Ref. [63]) that the ground state of  $K_2^*(1P)$  has a mass of 1432 MeV, which is close to the result obtained from the experimental data [1]. For the highly excited states of the P-wave  $K_2^*$  mesons,  $K_2^*(2P)$ ,  $K_2^*(3P)$ , and  $K_2^*(4P)$  have masses of 1870, 2198, and 2438 MeV, respectively, which are smaller than those reported in Ref. [7]. For the F-wave  $K_2^*$ ,  $K_2^*(1F)$  is predicted to have a mass of 2092 MeV, which is similar to the value of  $2073 \pm 94^{+245}_{-240}$  obtained from the LHCb data in Ref. [4].  $K_2^*(2F)$  has a mass of 2356 MeV and  $K_2^*(3F)$  has a mass of 2552 MeV, which are both smaller than that reported in Ref. [7]. Additionally, the results of  $K_2^*(1P)$ ,  $K_2^*(2P)$ , and  $K_2^*(1F)$  in Ref. [64] are also close to the experimental data.

## III. TWO-BODY STRONG DECAY ANALYSIS

# A. The $2^{3}P_{2}$ and $1^{3}F_{2}$ states of $K_{2}^{*}$

When we use the experimental values  $1868 \pm 8^{+40}_{-57}$  MeV and  $2073 \pm 94$  MeV [here, we do not take into account the large systematic error in  $K_2^*(2070)$ ] as inputs for the mass of  $2^{3}P_{2}$  and  $1^{3}F_{2}$  states. The QPC model provides an effective approach for determining the decay widths for the  $2^{3}P_{2}$  and  $1^{3}F_{2}$  states, which have values of  $285 \pm 45$  and  $855 \pm 225$  MeV, respectively. To clearly compare our prediction with the resonance parameters of  $K_2^*(1980)$ measured by different experiments collected in PDG [1], we present the total width and decay-branching ratio for  $K_2^*(1870)$  and  $K_2^*(2070)$  with the variation of the mass. A comparison of our theoretical result for the total width of  $K_2^*(1870)$  with experimental data is shown by the lower diagram in Fig. 2; note that our result is very close to the BESIII experimental data [3] marked by the green line with triangle in Fig. 2. Based on the decay-branching ratio of  $\frac{K\rho}{K^*\pi}$ , when  $K_2^*(1870)$  was regarded as a  $2^3P_2$  state, our result



FIG. 3. *M* dependence of the calculated decay widths of  $3^3P_2$  state.

of  $\frac{\Gamma_{K\rho}}{\Gamma_{K^*\pi}} = 1.62^{+0.11}_{-0.15}$  conforms well to the experimental value,  $\frac{\Gamma_{K\rho}}{\Gamma_{K^*\pi}} = 1.49 \pm 0.24 \pm 0.09$  [2]. Thus, explaining  $K_2^*(1870)$  as a  $2^3P_2$  state is further tested through the decay-branching ratio of  $\frac{K\rho}{K^*\pi}$ . In the upper diagram of Fig. 2, when we treat  $K_2^*(2070)$  as the 1F state, the ratio of  $\frac{\Gamma_{K\rho}}{\Gamma_{V^*}}$  is approximately 1.05-1.13. The width has an overlap with the LHCb data  $\Gamma_{K_2^*(2070)} = 678 \pm 311^{+559}_{-1153}$  MeV in the M =(1979-2167) MeV range  $(M_{K_2^*(2070)} = 2073 \pm 94^{+245}_{-240}$  MeV [4]). The total error +640 MeV is comparable with the center width of 678 MeV, and another total error -1194 MeV is nearly two times this center width value, which is the largest experimental error for the experimental data obtained for the K meson family in PDG [1]. Note that the mass and width of the  $K^*(2^+)2^3P_2$  state are fitted using the quantum numbers  $n^{2\hat{S}+1}L_J = 2^3P_2$  in Ref. [4]. Additionally, we also note that PDG edition 2022 does not adopt these LHCb data [66]; perhaps this "nonadoption" can be attributed to the large error. Therefore, we suggest that experimenters add the  $K_2^*(1^3F_2)$  state in the fit scheme reported in [4], which makes it possible to reduce the experimental error of  $K_2^*$ .

We have a nice description of  $K_2^*(1870)$  for the mass spectrum and decay behaviors under the assignment of  $K_2^*$  $(2^3P_2)$ . The assignment of  $K_2^*$   $(1^3F_2)$  for  $K_2^*(2070)$ requires more experimental support. We hope our theoretical result can help to establish this  $K_2^*(1F)$  state.

The two-body decay information for the  $2^{3}P_{2}$  and  $1^{3}F_{2}$  states is shown in Table II.  $Kb_{1}$ ,  $K_{1}\pi$ ,  $Ka_{1}$ , and  $K^{*}\rho$  all make important contributions to the  $2^{3}P_{2}$  and  $1^{3}F_{2}$  states. The decay modes  $K_{1}\rho$ ,  $K_{1}\eta$ ,  $K_{1}\eta_{2}$ , and  $K_{1}\omega$  are predicted to be dominant to  $1^{3}F_{2}$  state, but has no contribution to  $2^{3}P_{2}$  state.  $Kh_{1}$ ,  $K_{2}^{*}\pi$ ,  $Ka_{2}$ ,  $Kf_{1}$ ,  $K^{*}\omega$ ,  $K\rho$ , and  $K^{*}\pi$  have visible contribution to the total width of  $2^{3}P_{2}$  and  $1^{3}F_{2}$  state.  $K^{*}\eta'$ ,  $K_{3}^{*}(1780)\pi$ , and  $K^{*}(1680)\pi$  have very small widths in the final states of the  $2^{3}P_{2}$  and  $1^{3}F_{2}$  states. The SPEC experiment found an indication of a decay channel for  $K_{2}^{*}(1980)$ :  $Kf_{2}(1270)$  in 2003 [67]. The PDG considers only  $K\rho$ ,  $K^{*}\pi$ ,  $Kf_{2}(1270)$ ,  $K^{*}\phi$  modes for



FIG. 4. *M* dependence of the calculated decay widths of  $4^{3}P_{2}$  state.

" $K_2^*(1980)(2P)$ " state, and  $K_2^*(1980) \rightarrow K\eta$  was observed for the first time by Chen *et al.* in the  $D_0 \rightarrow K^-\pi^+$  decays [68]. The width of the  $Kf_2(1270)$  channel is predicted to be 1.29–12.4 and 6.03–11.2 MeV for the  $2^3P_2$  and  $1^3F_2$ states, respectively. The predicted ordering of two widths  $K\rho > K^*\pi$  is in agreement with experiment [2], and the predicted and observed decay-branching ratios are roughly consistent with each other.

The largest channels for the  $1^{3}F_{2}$  state are predicted to be  $K_{1\rho}$ ,  $Kb_{1}$ ,  $K^{*}\rho$ ,  $K_{1}\pi$ , and  $Ka_{1}$ , with branching fractions of 1, 7, 3, 7, and 4%, respectively, for M = 2070 MeV. Two of these channels are larger than 5%:  $Kb_{1}$  and  $K_{1}\pi$ . Note that some decay channels have a strong dependence on the change in the mass, such as  $K^{*}(1680)\pi$ ,  $K^{*}\eta'$ ,  $K(1460)\eta$ ,  $K\eta(1295)$ ,  $Kf_{1}(1420)$ , and  $K^{*}(1410)\eta$ . Observation of these channels like  $K\rho$ ,  $K^{*}\pi$  and  $K\pi$  can provide useful information about the nature of the  $K_{2}^{*}(1980)$  meson.

# **B.** Predicted $K_2^*(3P)$ and $K_2^*(4P)$ states

When further discussing the decay behavior of the  $3^3P_2$ state of the  $K_2^*$  meson family, we can estimate the total decay width of  $K_2^*(3P)$  to be (360–540) MeV and the mass to be (2200–2276) MeV. The predicted main decay channels of  $K_2^*(3P)$  include  $K^*(1410)\rho$ ,  $K^*\rho$ ,  $K\rho$ ,  $K^*\pi$ ,  $K\rho(1450)$ , and the  $K^*(1410)\rho$  channel has a regnant position. The  $K_1\pi$ ,  $K_2^*\eta$ ,  $K\eta$ ,  $K_1\eta$ , and  $K(1460)\eta$  channels make very small contributions to the total decay, and they are not sensitive to the change in the mass. More details can be found in Fig. 3.

The calculated total decay width for  $K_2^*(4P)$  is (225– 430) MeV when taking M = (2436-2566) MeV. It is evident from Fig. 4 that when searching for these decay modes, we find that only a few channels have branching fractions larger than a few percent.  $K^*(1410)\rho$ ,  $K\rho(1450)$ ,  $K_1\rho$ ,  $Ka_2(1700)$ ,  $K^*\pi$ ,  $K_2^*\pi$ , and  $K^*a_2$  are the main decay modes of the  $K_2^*(4P)$ , which have branching ratios of 0.04– 0.10, 0.04–0.06, 0.03–0.06, 0.03–0.05, 0.04, 0.03–0.04, and 0.03–0.07, respectively.

# C. Predicted $K_2^*(2F)$ and $K_2^*(3F)$ states

The predicted mass for the  $2^{3}F_{2}$  state in the  $K_{2}^{*}$  meson family is 2415 MeV. Our result (Fig. 5) shows that when we take the mass to range from 2355 to 2565 MeV, the



FIG. 5. *M* dependence of the calculated decay widths of  $2^{3}F_{2}$  state.



FIG. 6. *M* dependence of the calculated decay widths of  $3^3F_2$  state.

total width is  $\Gamma_{K_2^*(2^3F_2)} = 580 \pm 80$  MeV. The largest decay channel  $K_1\pi$  does not change significantly in the mass of M = (2355-2565) MeV, and its branch ratio is approximately 0.12. The important decay channels are  $Kb_1$ ,  $K^*(1410)\rho$ ,  $K^*a_1$ ,  $K_1\rho$ ,  $Ka_1$ , and  $K^*\rho$ . In addition,  $Kh_1$ ,  $K\pi$ ,  $K^*(1410)\omega$ ,  $K_1\eta$ ,  $K^*\pi$ ,  $K\rho$ , and  $K(1460)\rho$  also make certain contributions.  $K\omega$ ,  $K(1460)\omega$ ,  $Kf_2$ ,  $K^*\eta$ ,  $K\omega(1420)$ , and  $K(1630)\eta$  make small contributions. We consider that the predicted behavior of  $K_2^*(2F)$  will be helpful for the experimental search for  $K_2^*(2F)$  state.

As we can observe from Fig. 6, compared to the predicted  $K_2^*(1F)$  state, the predicted  $K_2^*(2F)$  state and  $K_2^*(3F)$  state have more decay channels. The obtained mass is  $M_{3^3F_2} = 2624 \pm 58$  MeV. The corresponding total decay width is approximately  $\Gamma_{3^3F_2} = 370 \pm 120$  MeV. Note that the important decays are again distributed over several modes, and the larger decay modes are  $K_1\pi$ ,  $K^*\rho$ ,  $Kb_1$ ,  $K_1\rho$ ,  $K^*a_1$ ,  $K^*\rho(1450)$ ,  $K\pi$ , and  $K\pi(1300)$ .  $Kf_2$ ,  $K^*(1410)\omega$ ,  $K\omega(1420)$ ,  $K_1\pi(1300)$ , and  $K(1460)\pi$  contribute very little to the total decay width of  $K_2^*(3F)$ .

#### IV. CONCLUSION

The observed  $K_2^*(1870)$  and  $K_2^*(2070)$  are first described as  $2^{3}P_{2}$  and  $1^{3}F_{2}$  states, respectively. By analyzing the mass spectra obtained for the P-wave and F-wave  $K_2^*$  meson family and calculating the two-body strong decay for these two states, we find that our predicted results for  $K_2^*(1870)$  are consistent with existing experimental findings. Our results about  $K_2^*(2070)$  have a large overlap with existing experimental findings. Our theoretical results show that,  $K_2^*(1870)$ can be regard as a  $2^{3}P_{2}$  state based on comparison with the experimental data.  $K_2^*(2070)$  is likely to be a  $1^3F_2$  state. The  $1^{3}F_{2}$  state can have a relatively large width of  $855\pm$ 225 MeV, and the ratio of  $\frac{\Gamma_{K\rho}}{\Gamma_{K^*\pi}}$  is 1.05–1.13. Because of our explanations of  $K_2^*(1870)$  and  $K_2^*(2070)$ , the spectroscopy for the P-wave and F-wave  $K_2^*$  mesons becomes abundant. Additionally, we predict the decay behaviors for the  $K_2^*(2P)$  and  $K_2^*(1F)$  states and the decay widths of channels such as  $K\rho$ ,  $K^*\pi$ , and  $Kf_2(1270)$  are calculated. These findings are expected to be revealed in future experiments. In addition to the  $2^{3}P_{2}$  and  $1^{3}F_{2}$  strange mesons, the decay behaviors for the other higher excited  $K_2^*$  mesons is also predicted in the present work. The masses and widths for these predicted states provide some basic information that will help in the search for these strange mesons in future experiments.

In addition, we hope that the resonance parameter (total width) for  $K_2^*(2070)$  can be fitted again by the experimental group considering the quantum numbers  $n^{2S+1}L_J = 1^3F_2$  for  $K_2^*$ , which will provide a powerful criterion for testing information to further confirm the  $K_2^*(2070)$  state.

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