Dirac sea effects on heavy quarkonia decay widths in magnetized matter: A field theoretical model of composite hadrons

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We study the partial decay widths of charmonium (bottomonium) states to $D\bar{D}(B\bar{B})$ mesons in magnetized (nuclear) matter using a field theoretical model of composite hadrons with quark (and antiquark) constituents. These are computed from the mass modifications of the decaying and produced mesons within a chiral effective model, including the nucleon Dirac sea effects. The mass modifications of the open charm (bottom) mesons are calculated from their interactions with the nucleons and the scalar mesons, whereas the mass shift of the heavy quarkonium state is obtained from the medium change of a scalar dilaton field, χ, which mimics the gluon condensates of QCD. The Dirac sea contributions are observed to lead to a rise (drop) in the quark condensates as the magnetic field is increased; an effect called the (inverse) magnetic catalysis. These effects are observed to be significant, and the anomalous magnetic moments (AMMs) of the nucleons are observed to play an important role. For $\rho_B = 0$, there is observed to be magnetic catalysis (MC) without and with AMMs, whereas, for $\rho_B = \rho_0$, the inverse magnetic catalysis is observed when the AMMs are taken into account, contrary to MC, when the AMMs are ignored. In the presence of a magnetic field, there are also mixings of spin 0 (pseudoscalar) and spin 1 (vector) states (PV mixing), which modify the masses of these mesons. The magnetic field effects on the heavy quarkonium decay widths should have observable consequences on the production the heavy flavor mesons, which are created in the early stage of ultrarelativistic peripheral heavy ion collisions, at RHIC and LHC, when the produced magnetic fields can still be extremely large.

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I. INTRODUCTION

The study of the in-medium properties of the heavy flavor mesons [1], in particular in the presence of strong magnetic fields, has been a topic of intense research due to its relevance in relativistic heavy ion collision experiments. The heavy flavor mesons are created at the early stage when the magnetic fields resulting from ultrarelativistic peripheral heavy ion collisions, estimated to be huge [2], can still be extremely large. The heavy quarkonium states and the open heavy flavor mesons have been studied extensively in the literature using the potential models [3–13], the QCD sum rule approach [14–31], the coupled channel approach [32–38], the quark meson coupling (QMC) model [39–47] as well as using a chiral effective model [48–55]. Studies of heavy quarkonium states ($\bar{Q}Q$ bound states, $Q = c, b$) in presence of a gluon field, assuming the distance between Q and $\bar{Q}$ to be small as compared to the scale of the gluonic fluctuations, show that the mass modifications of these states arise from the medium modification of the scalar gluon condensate in the leading order [56–58]. A study of the mass modifications of the charmonium states due to the gluon condensates as well as $\bar{D}D$ meson loop [59] showed that the dominant contributions are due to the medium modifications of the gluon condensates. In a chiral effective model, the in-medium masses of the heavy quarkonium (charmonium and bottomonium) have been computed from the medium change of a scalar dilaton field [50,51,55], which simulates the gluon condensates of QCD within the effective hadronic model.

The chiral effective model, in the original version with three flavors of quarks [SU(3) model] [60–63], has been used extensively in the literature, for the study of finite nuclei [61], strange hadronic matter [62], light vector mesons [63], strange pseudoscalar mesons, e.g., the kaons and antikaons [64–67] in isospin asymmetric hadronic matter as well as for the study of bulk matter of neutron

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stars [68]. Within the QCD sum rule framework, the light vector mesons [69,70] as well as the heavy quarkonium states [16–18] in (magnetized) hadronic matter have been studied, using the medium changes of the light quark condensates and gluon condensates calculated within the chiral SU(3) model. Using the in-medium masses of the heavy flavor mesons in the (magnetized) hadronic matter, calculated within the chiral effective model, the partial decay widths of the heavy quarkonium states to the open heavy flavor mesons have been studied in (magnetized) hadronic medium [51,71], using a light quark-antiquark pair creation model [72], namely the $^3P_0$ model [73–76] as well as using a field theoretical model for composite hadrons with quark (and antiquark) constituents [77–81]. The effects of magnetic field on the masses of the heavy flavor mesons have been studied in Refs. [82–89], and it is observed that the spin-magnetic field interaction leads to mixing between the pseudoscalar meson and the longitudinal component of the vector meson (PV mixing). This results in a dominant rise (drop) in the mass of the longitudinal component of the vector meson (pseudoscalar) meson for the heavy quarkonia (charmonia and bottomonia) states as well as for open charm (bottom) mesons [84–89]. In the presence of a magnetic field, the studies of the effects of Dirac sea (DS) in the quark matter sector [90–93] within the Nambu-Jona-Lasinio model [94–96] are observed to lead to enhancement of the light quark condensates with increase in the magnetic field; an effect called the magnetic catalysis (MC). The opposite trend of the light quark condensates with magentic field, namely the inverse magnetic catalysis (IMC) is observed in some lattice QCD calculations [97], where the critical temperature, T_c is seen to decrease with increase in the magnetic field. For the nuclear matter, the effects of Dirac sea (DS) have been studied using the Walecka model as well as an extended linear sigma model in Ref. [98]. These are observed to lead to magnetic catalysis (MC) effect for zero temperature and zero density, which is observed as a rise in the effective nucleon mass with the increase in magnetic field. In Ref. [99], the contributions of Dirac sea of the nucleons to the self-energies of the nucleons have been studied in the Walecka model by summing over the scalar (σ) and vector (ω) tadpole diagrams, in a weak magnetic field approximation of the fermion propagator. At zero density, the effects of the Dirac sea are seen to lead to magnetic catalysis (MC) effect at zero temperature [99]. When the anomalous magnetic moments (AMMs) of the nucleons are taken into account, at a finite density and zero temperature, there is observed to be a drop in the effective nucleon mass with increase in the magnetic field. This behavior with the magnetic field is observed when the temperature is raised from zero to nonzero values, up to the critical temperature, T_c, when the nucleon mass has a sudden drop, corresponding to the vacuum to nuclear matter phase transition. The decrease in T_c with increase in value of B is identified with the inverse magnetic catalysis (IMC) [99].

In the present work, the partial decay widths of the charmonium (bottomonium) states to open heavy flavor mesons, $D\bar{D}(B\bar{B})$ are studied in magnetized (nuclear) matter using a field theoretical model of composite hadrons. As the matter created in ultrarelativiistic peripheral heavy ion collisions is dilute, we study the partial decay widths of the lowest quarkonium states in the charm and bottom sectors, $\psi(3770)$ and $\Upsilon(4S)$ (which decay to $D\bar{D}$ and $B\bar{B}$ in vacuum). These are investigated for $\rho_B = 0$ as well as for $\rho_B = \rho_0$, the nuclear matter saturation density, for symmetric as well as asymmetric nuclear matter in the presence of an external magnetic field. The study of effects of temperature on the open charm and charmonium masses (and hence on the charmonium decay widths) [50,51] have been observed to be marginal for small densities (up to ρ_0). Within the chiral effective model, the mass shift of the heavy quarkonium states and the open heavy flavor mesons arise from the medium modifications of the dilaton field and the scalar fields, which have marginal modifications due to temperature, and hence, the temperature effects on the quarkonium decay widths (due to mass modification of these mesons) are not taken into account in the present study. The magnetic effects are the most dominant effects for the (dilute) matter resulting from ultrarelativistic peripheral collisons, which include the contributions from the magnetized Dirac sea of nucleons as well as PV mixing, in addition to the Landau level contributions for the charged hadrons. In the chiral effective model, the effects of the Dirac sea are incorporated to the nucleon propagator, through summation of scalar (σ, ζ, and δ) and vector (ω and ρ) tadpole diagrams. When the anomalous magnetic moments (AMMs) of the nucleons are not taken into account, for zero density as well as for $\rho_B = \rho_0$, magnetic catalysis (MC) is observed. However, when the AMMs of nucleons are considered, for $\rho_B = \rho_0$ (both for symmetric and asymmetric nuclear matter), inverse magnetic catalysis (IMC) is observed; i.e., the quark condensate is observed to be reduced with rise in the magnetic field.

The outline of the paper is as follows. In Sec. II, we describe briefly the chiral effective model used to calculate the masses of the charmonium (bottomonium) and open charm (bottom) mesons, accounting for the effects of the Dirac sea for the nucleons. The PV mixing effects are also taken into account which modify the masses of the heavy quarkonium states as well as open heavy flavor mesons. In Sec. III, the computations of the decay widths of $\psi(3770) \to D\bar{D}$ and $\Upsilon(4S) \to B\bar{B}$ using the field theoretical model of composite hadrons are briefly described, and, the salient features of the model are presented in the Appendix. The results of the partial decay widths in magnetized (nuclear) matter are discussed in Sec. IV, and the summary of the present work are given in Sec. V.

II. MASS MODIFICATIONS OF CHARM AND BOTTOM MESONS

We describe briefly the chiral effective model used to study the open charm (bottom) mesons and the charmonium (bottomonium) states in magnetized nuclear matter. The model is a generalization of a chiral SU(3) model, based on a nonlinear realization of chiral symmetry and the breaking of scale invariance of QCD. The scale symmetry breaking is incorporated through a scalar dilaton field (which mimics the scalar gluon condensate), and the mass modifications of the heavy quarkonium states are obtained from medium modifications of the dilaton field. The in-medium masses of the open heavy (charm and bottom) flavor mesons are obtained by generalizing the chiral SU(3) model to include the interactions of the open charm and bottom mesons with the light hadrons.

In the presence of a magnetic field, the Lagrangian for SU(3) model has the form [100],

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \sum_W \mathcal{L}_{\rm BW} + \mathcal{L}_{\rm vec} + \mathcal{L}_0 + \mathcal{L}_{\rm scalebreak} + \mathcal{L}_{\rm SB} + \mathcal{L}^{B\gamma}_{\rm mag}, \quad (1)$$

where $\mathcal{L}_{\rm kin}$ refers to the kinetic energy terms of the baryons and the mesons, $\mathcal{L}_{\rm BW}$ is the baryon-meson interaction term, $\mathcal{L}_{\rm vec}$ describes the dynamical mass generation of the vector mesons via couplings to the scalar mesons and contain additionally quartic self-interactions of the vector fields, $\mathcal{L}_0$ contains the meson-meson interaction terms, $\mathcal{L}_{\rm scalebreak}$ is the scale invariance breaking term and $\mathcal{L}_{\rm SB}$ describes the explicit chiral symmetry breaking. The kinetic energy terms are given as

$$\begin{aligned}\mathcal{L}_{\rm kin} = {} & i{\rm Tr}\bar{B}\gamma_\mu D^\mu B + \frac{1}{2}{\rm Tr}D_\mu X D^\mu X \\ & + {\rm Tr}(u_\mu X u^\mu X + X u_\mu u^\mu X) + \frac{1}{2}{\rm Tr}D_\mu Y D^\mu Y \\ & + \frac{1}{2}D_\mu\chi D^\mu\chi - \frac{1}{4}{\rm Tr}(\tilde{V}_{\mu\nu}\tilde{V}^{\mu\nu}) - \frac{1}{4}{\rm Tr}(\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu}) \\ & - \frac{1}{4}{\rm Tr}(F_{\mu\nu}F^{\mu\nu}),\end{aligned} \quad (2)$$

where B is the baryon octet, X is the scalar meson multiplet, Y is the pseudoscalar chiral singlet, χ is the scalar dilaton field, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\mathcal{A}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ are the field strength tensors of the vector meson multiplet, V^μ, the axial vector meson multiplet, $\mathcal{A}^\mu$, and the photon field, A^μ. In Eq. (2),

$$u_\mu = -\frac{i}{4}[(u^\dagger(\partial_\mu u) - (\partial_\mu u^\dagger)u) - (u(\partial_\mu u^\dagger) - (\partial_\mu u)u^\dagger)], \quad (3)$$

and the covariant derivative of a field $\Phi(\equiv B, X, Y, \chi)$ reads $D_\mu\Phi = \partial_\mu\Phi + [\Gamma_\mu, \Phi]$, with

$$\Gamma_\mu = -\frac{i}{4}[(u^\dagger(\partial_\mu u) - (\partial_\mu u^\dagger)u) + (u(\partial_\mu u^\dagger) - (\partial_\mu u)u^\dagger)], \quad (4)$$

where $u = \exp\left(\frac{i}{\sigma_0}P\gamma_5\right)$, where $P = \pi^a\lambda^a$, with π^a and λ^a, $i = 1, \ldots 8$, as the pseudoscalar mesons and the Gell-Mann matrices. The interaction of the baryons with the meson, W (scalar, pesudoscalar, vector, axialvector meson) is given as

$$\begin{aligned}\mathcal{L}_{\rm BW} = {} & -\sqrt{2}g_8^W(\alpha_W[\bar{B}OBW]_F + (1-\alpha_W)[\bar{B}OBW]_D) \\ & - \frac{g_1^W}{\sqrt{3}}{\rm Tr}(\bar{B}OBW){\rm tr}(W),\end{aligned} \quad (5)$$

where, the F-type (antisymmetric) and D-type (symmetric) couplings are defined as $[\bar{B}OBW]_F = {\rm Tr}(\bar{B}OWB - \bar{B}OBW)$ and $[\bar{B}OBW]_D = {\rm Tr}(\bar{B}OWB + \bar{B}OBW) - \frac{2}{3}{\rm Tr}(\bar{B}OB){\rm Tr}(W)$. In Eq. (5), $(W, O) \equiv (X, 1), (u, \gamma^5), (V, \gamma^\mu)$, and $(\mathcal{A}, \gamma^\mu\gamma^5)$ for the interactions of the baryons with the scalar, the pseudoscalar, the vector, and the axial-vector mesons, respectively.

The Lagrangian for the vector meson interaction is written as

$$\begin{aligned}\mathcal{L}_{\rm vec} = {} & \frac{m_V^2}{2}\frac{\chi^2}{\chi_0^2}{\rm Tr}(V_\mu V^\mu) + \frac{\mu}{4}{\rm Tr}(V_{\mu\nu}V^{\mu\nu}X^2) \\ & + \frac{\lambda_V}{12}({\rm Tr}(V^{\mu\nu}))^2 + 2(g_4)^4{\rm Tr}(V_\mu V^\mu)^2.\end{aligned} \quad (6)$$

The masses of ω, ρ, and ϕ are fitted from m_V, μ, and λ_V. The Lagrangian describing the interaction for the scalar mesons, X, and pseudoscalar singlet, Y, is given as [61]

$$\mathcal{L}_0 = -\frac{1}{2}k_0\chi^2 I_2 + k_1(I_2)^2 + k_2 I_4 + 2k_3\chi I_3, \quad (7)$$

with $I_2 = {\rm Tr}(X+iY)^2$, $I_3 = \det(X+iY)$, and $I_4 = {\rm Tr}(X+iY)^4$. In the above, χ is the scalar dilaton field that is introduced in order to mimic the QCD trace anomaly, i.e., the nonvanishing energy-momentum tensor,

$$\theta^\mu_\mu = (\beta_{\rm QCD}/2g)\langle G^a_{\mu\nu}G^{\mu\nu a}\rangle + \sum_i m_i\bar{q}_i q_i, \quad (8)$$

where $G^a_{\mu\nu}$ is the gluon field tensor and the second term in the trace accounts for the finite quark masses, with m_i as the current quark mass for the quark of flavor, $i = u, d, s$. The scale breaking and the explicit chiral symmetry breaking terms are given as [60,61]

$$\mathcal{L}_{\rm scalebreak} = -\frac{1}{4}\chi^4 \ln\frac{\chi^4}{\chi_0^4} + \frac{d}{3}\chi^4 \ln\left(\left(\frac{I_3}{\det\langle X\rangle_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right), \quad (9)$$

$$\mathcal{L}_{\rm SB} = {\rm Tr}A_p(u(X+iY)u + u^\dagger(X-iY)u^\dagger), \quad (10)$$

with $A_p = 1/\sqrt{2} m_\pi^2 f_\pi \mathrm{diag}(1, 1, \frac{2m_K^2 f_K}{m_\pi^2 f_\pi} - 1)$; here, m_π and m_K are the masses of the pion and K meson, and f_π and f_K, their decay widths.

In the present investigation, we use the mean field approximation, where all the meson fields are treated as classical fields. In this approximation, only the scalar and the vector fields contribute to the baryon-meson interaction, $\mathcal{L}_{\mathrm{BW}}$, since for all the other mesons, the expectation values are zero. The various terms of the Lagrangian density in the mean field approximation are given as

$$\mathcal{L}_{BX} + \mathcal{L}_{BV} = -\sum_i \bar{\psi}_i [g_{i\omega}\gamma_0 \omega + g_{i\phi}\gamma_0\phi + m_i^*]\psi_i \tag{11}$$

$$\mathcal{L}_{\mathrm{vec}} = \frac{1}{2}\frac{\chi^2}{\chi_0^2}(m_\omega^2\omega^2 + m_\rho^2\omega^2 + m_\phi^2\omega^2) + g_4^4(\omega^4 + 2\phi^4 + 6\omega^2\rho^2 + \rho^4) \tag{12}$$

$$\mathcal{L}_0 = -\frac{1}{2}k_0\chi^2(\sigma^2 + \zeta^2 + \delta^2) + k_1(\sigma^2 + \zeta^2 + \delta^2)^2 + k_2\left(\frac{\sigma^4}{2} + \frac{\delta^4}{2} + \zeta^4\right) + k_3\chi(\sigma^2 - \delta^2)\zeta - k_4\chi^4 \tag{13}$$

$$\mathcal{L}_{\mathrm{scalebreak}} = -\frac{1}{4}\chi^4 \ln\frac{\chi^4}{\chi_0^4} + \frac{d}{3}\chi^4 \ln\left(\frac{(\sigma^2 - \delta^2)\zeta}{\sigma_0^2\zeta_0}\left(\frac{\chi}{\chi_0}\right)^3\right). \tag{14}$$

The baryon-scalar meson interactions generate the baryon masses and the parameters corresponding to these interactions are adjusted so as to obtain the baryon masses as their experimentally measured vacuum values. In Eq. (11), the effective mass of the baryon of type i ($i = p, n, \Lambda, \Sigma^{\pm,0}, \Xi^{0,-}$) is given as

$$m_i^* = -g_{\sigma i}\sigma - g_{\zeta i}\zeta - g_{\delta i}\delta, \tag{15}$$

which is calculated from the values of the scalar fields in the magnetized medium, and the masses with the vacuum values of the scalar fields correspond to the experimentally measured vacuum values of the baryons.

The explicit chiral symmetry breaking term is given as

$$\begin{aligned}\mathcal{L}_{\mathrm{SB}} &= \mathrm{Tr}\left[\mathrm{diag}\left(-\frac{1}{2}m_\pi^2 f_\pi(\sigma + \delta), -\frac{1}{2}m_\pi^2 f_\pi(\sigma - \delta), \left(\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right)\right] \\ &= -\left[m_\pi^2 f_\pi \sigma + \left(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right].\end{aligned} \tag{16}$$

In the above, the matrix, whose trace gives the Lagrangian density corresponding to the explicit chiral symmetry breaking in the chiral SU(3) model, has been explicitly written down. Comparing the above term with the explicit chiral symmetry breaking term of the Lagrangian density in QCD given as

$$\mathcal{L}_{\mathrm{SB}}^{\mathrm{QCD}} = -\mathrm{Tr}[\mathrm{diag}(m_u\bar{u}u, m_d\bar{d}d, m_s\bar{s}s)], \tag{17}$$

one obtains the nonstrange quark condensates ($\langle\bar{u}u\rangle$ and $\langle\bar{d}d\rangle$) and the strange quark condensate ($\langle\bar{s}s\rangle$) to be related to the scalar fields, σ, δ, and ζ as

$$m_u\langle\bar{u}u\rangle = \frac{1}{2}m_\pi^2 f_\pi(\sigma + \delta); \quad m_d\langle\bar{d}d\rangle = \frac{1}{2}m_\pi^2 f_\pi(\sigma - \delta);$$
$$m_s\langle\bar{s}s\rangle = \left(\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta. \tag{18}$$

It might be noted here that with the choice for A_p in the explicit symmetry breaking term as given by Eq. (10), together with the constraints $\sigma_0 = -f_\pi$, $\zeta_0 = -\frac{1}{\sqrt{2}}(2f_K - f_\pi)$ assure that the PCAC relations of the pion and kaon are fulfilled. Using one loop QCD β function $\beta_{\mathrm{QCD}}(g) = -\frac{11N_c g^3}{48\pi^2}(1 - \frac{2N_f}{11N_c})$, with $N_c = 3$, the number of colors and N_f as the number of quark flavor, in the trace of energy momentum tensor in QCD given by Eq. (8) and equating with θ_μ^μ of the chiral model,

$$\theta_\mu^\mu = \chi\frac{\partial\mathcal{L}}{\partial\chi} - 4\mathcal{L} = (1 - d)\chi^4, \tag{19}$$

the scalar gluon condensate gets related to the dilaton fleld as [51]

$$\left\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^a G^{\mu\nu a}\right\rangle = \frac{24}{(33 - 2N_f)}(1 - d)\chi^4, \tag{20}$$

in the limiting situation of massless quarks in the energy momentum tensor of QCD given by Eq. (8).

The term $\mathcal{L}_{\mathrm{mag}}^{B\gamma}$ in the Lagrangian given by Eq. (1), describes the interaction of the baryons with the electromagnetic field and is given as [101–103]

$$\mathcal{L}_{\mathrm{mag}}^{B\gamma} = -\bar{\psi}_i q_i \gamma_\mu A^\mu \psi_i - \frac{1}{4}\kappa_i \mu_N \bar{\psi}_i \sigma^{\mu\nu} F_{\mu\nu}\psi_i, \tag{21}$$

where, ψ_i corresponds to the ith baryon. The tensorial interaction of baryons with the electromagnetic field given by the second term in the above equation is related to the anomalous magnetic moments of the baryons. We choose the magnetic field to be uniform and along the z axis and take the vector potential to be $A^\mu = (0, 0, Bx, 0)$. The number and scalar densities of the proton have contributions from the Landau energy levels and the neutrons have contributions to their number and scalar densities due to the anomalous magnetic moment, in the presence of a magnetic field [101,102]. The expressions for the number and scalar densities of the proton in the presence of a uniform magnetic field (chosen to be along z direction) and accounting for the anomalous magnetic moments for the nucleons are given as [104–106]

$$\rho_p = \frac{eB}{4\pi^2}\left[\sum_{\nu=0}^{\nu_{\max}^{(S=1)}} k^{(p)}_{f,\nu,1} + \sum_{\nu=1}^{\nu_{(\max)}^{(S=-1)}} k^{(p)}_{f,\nu,-1}\right] \tag{22}$$

and

$$\begin{aligned}\rho_p^s = \frac{eBm_p^*}{2\pi^2}&\left[\sum_{\nu=0}^{\nu_{\max}^{(S=1)}} \frac{\sqrt{m_p^{*\,2}+2eB\nu}+\Delta_p}{\sqrt{m_p^{*\,2}+2eB\nu}}\right. \\ &\times \ln\left|\frac{k^{(p)}_{f,\nu,1}+E_f^{(p)}}{\sqrt{m_p^{*\,2}+2eB\nu}+\Delta_p}\right| \\ &\left.+\sum_{\nu=1}^{\nu_{\max}^{(S=-1)}} \frac{\sqrt{m_p^{*\,2}+2eB\nu}-\Delta_p}{\sqrt{m_p^{*\,2}+2eB\nu}} \ln\left|\frac{k^{(p)}_{f,\nu,-1}+E_f^{(p)}}{\sqrt{m_p^{*\,2}+2eB\nu}-\Delta_p}\right|\right] \\ &+\rho_p^{s\,(\mathrm{DS})},\end{aligned} \tag{23}$$

where $k^{(p)}_{f,\nu,\pm1}$ are the Fermi momenta of protons for the Landau level, ν for the spin index, $S=\pm1$, i.e., for spin up and spin down projections for the proton. These Fermi momenta are related to the Fermi energy of the proton as

$$k^{(p)}_{f,\nu,S} = \sqrt{E_f^{(p)2} - \left(\sqrt{m_p^{*\,2}+2eB\nu}+S\Delta_p\right)^2}. \tag{24}$$

The number density and the scalar density of neutrons are given as

$$\begin{aligned}\rho_n = \frac{1}{4\pi^2}\sum_{S=\pm1}&\left\{\frac{2}{3}k^{(n)3}_{f,S} + S\Delta_n\left[(m_n^*+S\Delta_n)k^{(n)}_{f,S}\right.\right. \\ &\left.\left.+E_f^{(n)2}\left(\sin^{-1}\left(\frac{m_n^*+S\Delta_n}{E_f^{(n)}}\right)-\frac{\pi}{2}\right)\right]\right\}\end{aligned} \tag{25}$$

and

$$\begin{aligned}\rho_n^s = \frac{m_n^*}{4\pi^2}\sum_{S=\pm1}&\left[k^{(n)}_{f,S}E_f^{(n)} - (m_n^*+S\Delta_n)^2\ln\left|\frac{k^{(n)}_{f,S}+E_f^{(n)}}{m_n^*+S\Delta_n}\right|\right] \\ &+\rho_n^{s\,(\mathrm{DS})}.\end{aligned} \tag{26}$$

The Fermi momentum, $k^{(n)}_{f,S}$ for the neutron with spin projection, S [$S=\pm1$ for the up (down) spin projection] is related to the Fermi energy for the neutron, $E_f^{(n)}$ as

$$k^{(n)}_{f,S} = \sqrt{E_f^{(n)2} - (m_n^*+S\Delta_n)^2}. \tag{27}$$

In the Eqs. (22)–(27), the parameter Δ_i is related to the anomalous magnetic moment for the nucleon, i ($i=p,n$) as $\Delta_i = -\frac{1}{2}\kappa_i\mu_N B$, where κ_i occurring in the second term in the Lagrangian density given by Eq. (21), is the value of the gyromagnetic ratio of the nucleon corresponding to the anomalous magnetic moment of the nucleon. In the present study of magnetized (nuclear) matter, the meson fields are treated as classical in the mean field approximation, and nucleons as quantum fields and the self energies of the nucleons include the contributions from the Dirac sea. In addition to using the mean field approximation, where the meson fields are replaced by their expectation values, we also use the approximations that $\bar{\psi}_i\psi_j = \delta_{ij}\langle\bar{\psi}_i\psi_i\rangle \equiv \delta_{ij}\rho_i^s$ and $\bar{\psi}_i\gamma^\mu\psi_j = \delta_{ij}\delta^{\mu0}\langle\bar{\psi}_i\gamma^0\psi_i\rangle \equiv \delta_{ij}\delta^{\mu0}\rho_i$, where ρ_i^s and ρ_i are the scalar and number density of baryon of species i (corresponding to neutron and proton in the present investigation). Using the scalar densities of the nucleons in the presence of magnetic field, the values of the scalar fields, σ, ζ, and δ are obtained by solving their coupled equations of motion, for given values of the baryon density, isospin asymmetry parameter and magnetic field. The last terms in Eqs. (23) and (26) correspond to the contributions of the Dirac sea for the scalar densities of proton and neutron. The magnetized Dirac sea contribution to the nucleon self-energy has been calculated by summing over the tadpole diagrams arising due to the interaction of the nucleons with the scalar field σ within the Walecka model in the weak magnetic field approximation [99]. Generalizing to include the interactions of the nucleons to the strange ζ and the nonstrange isovector δ scalar fields as well, in addition to the interaction with the nonstrange σ field, for the chiral effective model used in the present investigation, the contribution due to the magnetized Dirac sea to the self-energy of the ith nucleon ($i=p,n$) is given as

$$\begin{aligned}\Sigma_i = \sum_{\alpha=\sigma,\zeta,\delta}\frac{g_{\alpha i}{}^2}{4\pi^2 m_\alpha^2}&\left[\frac{(q_iB)^2}{3m_i^*} + \{\Delta_iB)^2m_i^* + (|q_i|B)(\Delta_iB)\}\right. \\ &\left.\times\left\{\frac{1}{2}+2\ln\left(\frac{m_i^*}{m_i}\right)\right\}\right],\end{aligned} \tag{28}$$

where q_i is the charge and $\Delta_i = -\frac{1}{2}\kappa_i\mu_N B$ is related to the anomalous magnetic moment of the baryon i (p and n in the present investigation).

The interactions of the $D(\bar{D})$ and $B(\bar{B})$ mesons with the baryons and the scalar mesons are obtained by genralizing the chiral SU(3) model to the charm and bottom sectors [48–51,53]. For the chiral SU(3) model, the baryon as well as meson octets can be written in terms of the 3×3 Gell-Mann matrices, as $\Phi \sim \lambda_a\phi^a$, $\Phi = B, P, X, V_\mu, A_\mu$. However, when the model is generalized to SU(4) to include the charm hadrons, the meson multiplets (being 15-plet) can be expressed as 4×4 Gell-Mann matrices (λ_a, $a=1,\ldots15$), but the baryon multiplet, being a 20-plet, can not be written as a square matrix of the same order as meson multiplets. When chiral SU(3) model is generalized to the charm (and bottom) sectors, the baryons are represented by the tensor, B^{ijk}, which is antisymmetric in the first two indices. The baryon-pseudoscalar meson interaction term (the Weinberg-Tomozawa term) is then written as

$$\mathcal{L}_{\rm WT} = -\frac{1}{2}[\bar{B}_{ijk}\gamma^{\mu}((\Gamma_{\mu})_l{}^k B^{ijl} + 2(\Gamma_{\mu})_l{}^j B^{ilk})]. \quad (29)$$

For the nuclear matter as considered in the present study, the relevant entries of the baryon tensor are $B^{121} = -B^{211}$ and $B^{122} = -B^{212}$, corresponding to p and n, respectively. The masses of the open charm (bottom) mesons are obtained from the interaction Lagrangian,

$$\mathcal{L}_{\rm int} = \mathcal{L}_{\rm WT} + \mathcal{L}_{\rm SME} + \mathcal{L}_{\rm 1strange} + \mathcal{L}_{d1} + \mathcal{L}_{d_2}, \quad (30)$$

where the first term is the Weinberg-Tomozawa term, $\mathcal{L}_{\rm SME}$ is the scalar exchange term, and $\mathcal{L}_{\rm 1strange}$, $\mathcal{L}_{d_1}$ and $\mathcal{L}_{d_2}$ are the range terms. The scalar meson exchange term is obtained from explicit symmetry breaking term given by (10), with the generalizations: $A_p = 1/\sqrt{2}m_\pi^2 f_\pi \mathrm{diag}(1, 1, \frac{2m_K^2 f_K}{m_\pi^2 f_\pi} - 1, \frac{2m_D^2 f_D}{m_\pi^2 f_\pi} - 1)$ and $A_p = 1/\sqrt{2}m_\pi^2 f_\pi \mathrm{diag}(1, 1, \frac{2m_K^2 f_K}{m_\pi^2 f_\pi} - 1, \frac{2m_D^2 f_D}{m_\pi^2 f_\pi} - 1, \frac{2m_B^2 f_B}{m_\pi^2 f_\pi} - 1)$, and the scalar meson multiplet for the SU(4) and SU(5) cases is given as $X = \mathrm{diag}(\frac{(\sigma-\delta)}{\sqrt{2}}, \frac{(\sigma+\delta)}{\sqrt{2}}, \zeta, \zeta_c)$ and $X = \mathrm{diag}\left(\frac{(\sigma-\delta)}{\sqrt{2}}, \frac{(\sigma+\delta)}{\sqrt{2}}, \zeta, \zeta_c, \zeta_b\right)$, respectively, where $\zeta_c \sim \langle \bar{c}c \rangle$ and $\zeta_b \sim \langle \bar{b}b \rangle$. The range terms are obtained from the interaction terms [51],

$$\mathcal{L}_{\rm 1st\,range} = \mathrm{Tr}(u_\mu X u^\mu X + X u_\mu u^\mu X), \quad (31)$$

$$\mathcal{L}_{d_1} = \frac{d_1}{4}(\bar{B}_{ijk}B^{ijk}(u_\mu)_l{}^m(u^\mu)_m{}^l), \quad (32)$$

and

$$\mathcal{L}_{d_2} = \frac{d_2}{2}[\bar{B}_{ijk}(u^\mu)_l{}^m((u^\mu)_m{}^k B^{ijl} + 2(u^\mu)_m{}^j B^{ilk})]. \quad (33)$$

In the above equations, u occurring in the expressions of u^μ and Γ^μ given by Eqs. (3) and (4) is given as, $u = \exp(\frac{i}{\sigma_0}\lambda_a \pi^a \gamma^5)$, where λ_a are the 4×4 (5×5) Gell-Mann matrices with $a = 1, \ldots 15$ ($a = 1, \ldots 24$) for the generalization to the case of SU(4) [SU(5)] model. The masses of the open charm ($D^{\pm}, D^0, \bar{D}^0$) and the open bottom ($B^{\pm}, B^0, \bar{B}^0$) mesons in magnetized (nuclear) matter are modified due to their interactions with the nucleons and the scalar fields [101,102]. The in-medium masses are obtained by solving their dispersion relations, which are obtained from the Fourier transformations of their equations of motion. These are given as

$$-\omega^2 + \vec{k}^2 + m^2_{F(\bar{F})} - \Pi_{F(\bar{F})}(\omega, |\vec{k}|) = 0, \quad (34)$$

where $\Pi_{F(\bar{F})}$, denotes the self energy of the meson $F(\equiv D, B)$, $\bar{F}(\equiv \bar{D}, \bar{B})$ in the medium. Explicitly, the self energies for the D and $\bar{D}$ are given as [101]

$$\begin{aligned}\Pi_D(\omega, |\vec{k}|) = {}& \frac{1}{4f_D^2}[3(\rho_p + \rho_n) \pm (\rho_p - \rho_n))]\omega \\ & + \frac{m_D^2}{2f_D}(\sigma' + \sqrt{2}\zeta_c' \pm \delta') \\ & + \left[-\frac{1}{f_D}(\sigma' + \sqrt{2}\zeta_c' \pm \delta') + \frac{d_1}{2f_D^2}(\rho_p^s + \rho_n^s) \right. \\ & \left. + \frac{d_2}{4f_D^2}((\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s))\right](\omega^2 - \vec{k}^2),\end{aligned} \quad (35)$$

and

$$\begin{aligned}\Pi_{\bar{D}}(\omega, |\vec{k}|) = {}& -\frac{1}{4f_D^2}[3(\rho_p + \rho_n) \pm (\rho_p - \rho_n)]\omega \\ & + \frac{m_D^2}{2f_D}(\sigma' + \sqrt{2}\zeta_c' \pm \delta') \\ & + \left[-\frac{1}{f_D}(\sigma' + \sqrt{2}\zeta_c' \pm \delta') + \frac{d_1}{2f_D^2}(\rho_p^s + \rho_n^s) \right. \\ & \left. + \frac{d_2}{4f_D^2}((\rho_p^s + \rho_n^s) \pm (\rho_n^s - \rho_n^s))\right](\omega^2 - \vec{k}^2),\end{aligned} \quad (36)$$

where the $\pm$ signs refer to the D^0 and D^+, respectively, in Eq. (35) and to the $\bar{D}^0$ and D^-, respectively, in Eq. (36). For the B meson doublet (B^+,B^0) and $\bar{B}$ meson doublet (B^-, $\bar{B}^0$), the self energies are given by [102]

$$\begin{aligned}\Pi_B(\omega, |\vec{k}|) = {}& -\frac{1}{4f_B^2}[3(\rho_p + \rho_n) \pm (\rho_p - \rho_n)]\omega \\ & + \frac{m_B^2}{2f_B}(\sigma' + \sqrt{2}\zeta_b' \pm \delta') \\ & + \left[-\frac{1}{f_B}(\sigma' + \sqrt{2}\zeta_b' \pm \delta') + \frac{d_1}{2f_B^2}(\rho_s^p + \rho_s^n) \right. \\ & \left. + \frac{d_2}{4f_B^2}(3(\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s))\right](\omega^2 - \vec{k}^2),\end{aligned} \quad (37)$$

and

$$\begin{aligned}\Pi_{\bar{B}}(\omega, |\vec{k}|) = {}& \frac{1}{4f_B^2}[3(\rho_p + \rho_n) \pm (\rho_p - \rho_n)]\omega \\ & + \frac{m_B^2}{2f_B}(\sigma' + \sqrt{2}\zeta_b' \pm \delta') \\ & + \left[-\frac{1}{f_B}(\sigma' + \sqrt{2}\zeta_b' \pm \delta') + \frac{d_1}{2f_B^2}(\rho_s^p + \rho_s^n) \right. \\ & \left. + \frac{d_2}{4f_B^2}(3(\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s)\right](\omega^2 - \vec{k}^2),\end{aligned} \quad (38)$$

where the $\pm$ signs refer to the B^+ and B^0, respectively, in Eq. (37) and to the B^- and $\bar{B}^0$ mesons, respectively, in

Eq. (38). The terms in the self-energies refer to the leading Weinberg-Tomozawa term and the subleading terms (the scalar exchange term and the range terms) in chiral perturbation expansion. The parameters d_1 and d_2 are fitted from the KN scattering lengths [51]. In Eqs. (35)–(38), $\sigma'(=\sigma-\sigma_0)$, $\zeta_b'(=\zeta_b-\zeta_{b0})$, and $\delta'(=\delta-\delta_0)$ are the fluctuations of σ, ζ_b, and δ, from their vacuum expectation values.

The masses are given as $m^*_{F(\bar{F})}=\omega(|\vec{k}|=0)$, which depend (through the self energies) on the values of the scalar fields (σ, ζ, and δ) as well as the number and scalar densities of the nucleons. In the presence of a magnetic field, the lowest Landau level (LLL) contributions are taken into account for the charged $D^{\pm}(B^{\pm})$ mesons. The effective masses of the open charm and bottom mesons are thus given as

$$m^{\rm eff}_{D^{\pm}}=\sqrt{{m^*_{D^{\pm}}}^2+eB},\qquad m^{\rm eff}_{D^0(\bar{D^0})}=m^*_{D^0(\bar{D^0})},$$
$$m^{\rm eff}_{B^{\pm}}=\sqrt{{m^*_{B^{\pm}}}^2+eB},\qquad m^{\rm eff}_{B^0(\bar{B^0})}=m^*_{B^0(\bar{B^0})},\tag{39}$$

where $m^*_{F(\bar{F})}$ is the mass of the open charm (bottom) meson obtained as solution of the dispersion relation given by Eq. (34).

The mass shift of the heavy quarkonium states arises from the medium modification of the scalar gluon condensate in the leading order and is given as [56–59]

$$\Delta m_{\Psi(\Upsilon)}=\frac{1}{18}\int d|\mathbf{k}|^2\left\langle\left|\frac{\partial\psi(\mathbf{k})}{\partial\mathbf{k}}\right|^2\right\rangle\frac{|\mathbf{k}|}{|\mathbf{k}|^2/m_{c(b)}+\epsilon}\times\left(\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{\mu\nu a}\right\rangle-\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{\mu\nu a}\right\rangle_0\right),\tag{40}$$

which, using Eq. (20), gives the mass shift of the heavy quarkonium state as [50,51]

$$\Delta m_{\Psi(\Upsilon)}=\frac{4}{81}(1-d)\int d|\mathbf{k}|^2\left\langle\left|\frac{\partial\psi(\mathbf{k})}{\partial\mathbf{k}}\right|^2\right\rangle\frac{|\mathbf{k}|}{|\mathbf{k}|^2/m_{c(b)}+\epsilon}\times(\chi^4-{\chi_0}^4),\tag{41}$$

where

$$\left\langle\left|\frac{\partial\psi(\mathbf{k})}{\partial\mathbf{k}}\right|^2\right\rangle=\frac{1}{4\pi}\int\left|\frac{\partial\psi(\mathbf{k})}{\partial\mathbf{k}}\right|^2 d\Omega.\tag{42}$$

In Eq. (41), d is a parameter introduced in the scale breaking term in the Lagrangian, χ and χ_0 are the values of the dilaton field in the magnetized medium and in vacuum, respectively. The wave functions of the quarkonium states, $\psi(\mathbf{k})$, are assumed to be harmonic oscillator wave functions, $m_{c(b)}$ is the mass of the charm (bottom) quark, and $\epsilon=2m_{c(b)}-m_{\psi(\Upsilon)}$ is the binding energy of the charmonium (bottomonium) state of mass, $m_{\psi(\Upsilon)}$. It might be noted here that the leading order mass formula [given by Eq. (40)] was derived using the binding of the heavy quark and antiquark in the heavy quarkonium state to be Coulombic. This is a good approximation for the ground state but not realistic for the excited states [59] as the mass shift formula contains derivatives of the wave function, which measure the dipole size of the system. The wave functions for the charmonium and bottomonium states are assumed to be harmonic oscillator type, with the strengths of the potential determined from the rms radii of the quarkonium states. The mass shifts of the heavy quarkonium states are thus obtained from the values of the dilaton field, χ [using Eq. (41)].

The Dirac sea contributions are included in the scalar densities of the nucleons, which occur in the equations of motion of the scalar fields, σ, ζ and δ. For given values of the baryon density, ρ_B, the isospin asymmetry parameter, $\eta=(\rho_n-\rho_p)/(2\rho_B)$ (with ρ_n and ρ_p as the neutron and proton number densities), the magnetic field, B (chosen to be along z direction), the fields (σ, ζ, δ, and χ) are solved from their coupled equations of motion. Within the chiral effective model, the masses of the open charm and bottom mesons are given by Eq. (39), which are obtained from the solutions of the dispersion relations given by Eq. (34) for $|\vec{k}|=0$, with additional Landau level contributions for the charged mesons.

A. Pseudoscalar-vector meson (PV) mixing

In the presence of a magnetic field, there is mixing between the spin 0 (pseudoscalar) meson and spin 1 (vector) mesons, which modifies the masses of these mesons [78,79,84–88,107]. The PV mixing leads to a drop (rise) in the mass of the pseudoscalar (longitudinal component of the vector) meson. The mass modifications have been studied using a phenomenological Lagrangian density of the form [86–88],

$$\mathcal{L}_{\rm PV\gamma}=\frac{g_{\rm PV}}{m_{av}}e\tilde{F}_{\mu\nu}(\partial^\mu P)V^\nu,\tag{43}$$

for the heavy quarkonia [78,81,85–87], the open charm mesons [79], and the strange (K and $\bar{K}$) mesons [107]. In Eq. (43), $m_{av}=(m_V+m_P)/2$, m_P and m_V are the masses for the pseudoscalar and vector charmonium states, and $\tilde{F}_{\mu\nu}$ is the dual electromagnetic field. In Eq. (43), the coupling parameter $g_{\rm PV}$ is fitted from the observed value of the radiative decay width, $\Gamma(V\to P+\gamma)$. Assuming the spatial momenta of the heavy quarkonia to be zero, there is observed to be mixing between the pseudoscalar and the longitudinal component of the vector field from their equations of motion obtained with the phenomenological $PV\gamma$ interaction given by Eq. (43). The physical masses of the pseudoscalar and the longitudinal component of the vector mesons, including the mixing effects, obtained by solving their equations of motion, are given as [86–88]

$$m^{(\mathrm{PV})}_{P,V^{\parallel}} = \frac{1}{2}\left(M_+^2 + \frac{c_{\mathrm{PV}}^2}{m_{av}^2} \mp \sqrt{M_-^4 + \frac{2c_{\mathrm{PV}}^2 M_+^2}{m_{av}^2} + \frac{c_{\mathrm{PV}}^4}{m_{av}^4}}\right), \quad (44)$$

where $M_+^2 = m_P^2 + m_V^2$, $M_-^2 = m_V^2 - m_P^2$, and $c_{\mathrm{PV}} = g_{\mathrm{PV}} eB$. The effective Lagrangian term given by Eq. (43) has been observed to lead to the mass modifications of the longitudinal J/ψ and η_c due to the presence of the magnetic field, which agree extremely well with a study of these charmonium states using a QCD sum rule approach incorporating the mixing effects [85,86].

The PV mixing effects for the open charm mesons (due to $D - D^*$ and $\bar{D} - \bar{D}^*$ mixings) [79], in addition to the mixing of the charmonium states (due to $J/\psi - \eta_c$, $\psi' - \eta_c'$ and $\psi(3770) - \eta_c'$ mixings) [78,79], as calculated using the phenomenological Lagrangian given by Eq. (43), have been observed to lead to an appreciable drop (rise) in the mass of the pseudoscalar (longitudinal component of the vector) meson. These were observed to modify the partial decay width of $\psi(3770) \to D\bar{D}$ [78,79], with the modifications being much more dominant due to the PV mixing in the open charm ($D - D^*$ and $\bar{D} - \bar{D}^*$) mesons.

For the bottom sector, due to a lack of data on radiative decays, $V \to P\gamma$, the modifications in the masses of the pseudoscalar and vector mesons ($Q_1\bar{Q}_2$ bound states) due to the PV mixing effects have been estimated from the mixing of spin with the external magnetic field [81], using the Hamiltonian [84,88],

$$H_{\mathrm{spin-mixing}} = -\sum_{i=1}^{2} \boldsymbol{\mu}_i \cdot \mathbf{B}, \quad (45)$$

which describes the interaction of the magnetic moments of the quark (antiquark) with the external magnetic field. In the above, $\boldsymbol{\mu}_i = g|e|q_i\mathbf{S_i}/(2m_i)$ is the magnetic moment of the ith particle, g is the Lande g factor [taken to be 2 (−2) for the quark (antiquark)], q_i, $\mathbf{S_i}$, m_i are the electric charge (in units of the magnitude of the electronic charge, $|e|$), spin, and mass of the ith particle [86,88]. This interaction leads to a drop (increase) of the mass of the pseudoscalar (longitudinal component of the vector meson) given as [84]

$$\Delta M^{\mathrm{PV}} = \frac{\Delta E}{2}((1+\Delta^2)^{1/2} - 1), \quad (46)$$

where $\Delta = 2g|eB|((q_1/m_1) - (q_2/m_2))/\Delta E$, $\Delta E = m_V - m_P$ is the difference in the masses of the pseudoscalar and vector mesons. It was observed in Ref. [81] that the partial decay widths $\Upsilon(4S) \to B\bar{B}$ in the presence of an external magnetic field, calculated using a field theoretical model of composite hadrons, has significantly larger contributions from the PV mixng effects from the open bottom mesons ($B - B^*$ and $\bar{B} - \bar{B}^*$ mixings) as compared to the mixing of the bottomonium states, $\Upsilon(4S)$ and $\eta_b(4S)$. As we shall see later, the inclusion of the Dirac sea contributions are observed to lead to significant modifications to the meson masses, especially when the AMMs of the nucleons are taken into consideration, which, in turn, has significant effects on the partial decay widths of the charmonium (bottomonium) states to the open charm (bottom) mesons. In the following section, we shall briefly describe the field theoretical model used to calculate the heavy quarkonium partial decay widths [77–81].

III. PARTIAL DECAY WIDTHS OF CHARMONIUM (BOTTOMONIUM) STATE TO $D\bar{D}(B\bar{B})$

In this section, we briefly describe the field theoretical model of composite hadrons [108–110] used to study the partial decay widths of the vector heavy quarkonium states to open heavy flavor mesons in magnetized (nuclear) matter, specifically, the decay widths of the charmonium state $\psi(3770)$ and the bottomonium state $\Upsilon(4S)$, which are the lowest states which decay to $D\bar{D}$ and $B\bar{B}$ in vacuum. As the matter produced in the noncentral ultrarelativistic heavy ion collisions (where strong magnetic fields are created) is dilute, the quarkonium decay widths are studied for vacuum ($\rho_B = 0$) and for $\rho_B = \rho_0$ in the presence of a magnetic field. The model used for the calculation of the decay widths describes the hadrons as comprising of quark (and antiquark) constituents. The constituent quark field operators of the hadron in motion are constructed from the constituent quark field operators of the hadron at rest, by a Lorentz boosting. Similar to the MIT bag model [111], where the quarks (antiquarks) occupy specific energy levels inside the hadron, it is assumed in the present model for the composite hadrons that the quark (antiquark) constituents carry fractions of the mass (energy) of the hadron at rest (in motion) [108,109]. With explicit constructions of the charmonium (bottomonium) state and the open charm (bottom) mesons, the decay width of the heavy quarkonium state to open heavy flavor mesons is calculated using the light quark antiquark pair creation term of the free Dirac Hamiltonian for constituent quark field [77]. The salient features of the field theoretic model for composite hadrons are presented in the Appendix.

The relevant part of the quark pair creation term is through the $q\bar{q}(q = u, d)$ creation for decay of the charmonium (bottomonium) state, Ψ (Υ), to the final state, $D\bar{D}$ ($B\bar{B}$). The pair creation term is given as

$$\mathcal{H}_{q^\dagger\tilde{q}}(\mathbf{x}, t=0) = Q_q^{(p)}(\mathbf{x})^\dagger(-i\boldsymbol{\alpha}\cdot\nabla + \beta M_q)\tilde{Q}_q^{(p')}(\mathbf{x}), \quad (47)$$

where M_q is the constituent mass of the light quark (antiquark). The subscript q of the field operators in Eq. (47) refers to the fact that the light antiquark, $\bar{q}$, and light quark, q, are the constituents of the $D(B)$ and $\bar{D}(\bar{B})$ mesons with momenta $\mathbf{p}$ and $\mathbf{p}'$, respectively, in the final state of the decay of the charmonium (bottomonium) state, $\Psi(3770)(\Upsilon(4S))$.

Assuming the initial and final state mesons to be bound by a harmonic oscillator potential, the explicit constructions for the vector quarkonium states $\psi(3770)$ (corresponding to 1D state) and $\Upsilon(4S)$, at rest (with spin projection m) are given as [77,80,112]

$$|\psi^m(3770)(\mathbf{0})\rangle = \frac{1}{4\sqrt{3\pi}}\int d\mathbf{k}\, u_{\psi(1D)}(\mathbf{k}) c_r{}^i(\mathbf{k})^\dagger \times u_r(\boldsymbol{\sigma}^m - 3(\boldsymbol{\sigma}\cdot\hat{k})\hat{k}^m)\tilde{c}_s{}^i(-\mathbf{k})v_s|vac\rangle, \tag{48}$$

with

$$u_{\psi(1D)}(\mathbf{k}) = \left(\frac{16}{15}\right)^{1/2}\pi^{-1/4}(R^2_{\psi(1D)})^{7/4}\mathbf{k}^2 \times \exp\left(-\frac{1}{2}R_{\psi(1D)}{}^2\mathbf{k}^2\right), \tag{49}$$

and

$$|\Upsilon^m(4S)(\vec{0})\rangle = \int d\mathbf{k}_1 b_r^i(\mathbf{k}_1)^\dagger u_r^\dagger u_{\Upsilon(4S)}(\mathbf{k}_1) \times \sigma^m \tilde{b}_s{}^i v_s(-\mathbf{k}_1)|vac\rangle, \tag{50}$$

with

$$u_{\Upsilon(4S)}(\mathbf{k}_1) = -\frac{1}{\sqrt{6}}\frac{\sqrt{35}}{4}\left(\frac{R^2_{\Upsilon(4S)}}{\pi}\right)^{3/4}\left(1 - 2R^2_{\Upsilon(4S)}\mathbf{k}_1^2 + \frac{4}{5}R^4_{\Upsilon(4S)}\mathbf{k}_1^4 - \frac{8}{105}R^6_{\Upsilon(4S)}\mathbf{k}_1^6\right) \times \exp\left[-\frac{1}{2}R^2_{\Upsilon(4S)}\mathbf{k}_1^2\right]. \tag{51}$$

In Eqs. (48) and (50), $c_r^{i\,\dagger}(b_r^{i\,\dagger})$ creates a charm (bottom) quark of spin r and color i, $\tilde{c}_s^i(\tilde{b}_s^i)$ creates a charm (bottom) antiquark of spin s and color i, $S^m \equiv \frac{1}{2}\sigma^m$ gives the spin projection of the charm (bottom) quark (antiquark), and u_r and v_s are the two component spinors for the quark and antiquark. The value of the harmonic oscillator strength for the charmonium state $\psi(3770)$ is fixed from its rms radius, $r_{\rm rms} = 1$ fm, to be $R^{-1}_{\psi(3770)} = 370$ MeV [51,59], and for the bottomonium state, $\Upsilon(4S)$, it is fixed from the value of the leptonic decay width $[\Upsilon(4S) \to e^+e^-]$ of 0.272 keV to be $R^{-1}_{\Upsilon(4S)}$ as 638.6 MeV [55,80].

The states for the open charm and bottom mesons $(F \equiv D, B, \bar{F} \equiv \bar{D}, \bar{B})$ with finite momenta are constructed in terms of the constituent quark field operators, obtained from the quark field operators of these mesons at rest through a Lorentz boosting [110]. These are given as

$$|F(\mathbf{p})\rangle = \frac{1}{\sqrt{6}}\left(\frac{R_F^2}{\pi}\right)^{3/4}\int d\mathbf{k}\,\exp\left(-\frac{R_F^2\mathbf{k}^2}{2}\right) \times Q_r{}^i(\mathbf{k}+\lambda_2\mathbf{p})^\dagger u_r^\dagger \tilde{q}_s{}^i(-\mathbf{k}+\lambda_1\mathbf{p})v_s d\mathbf{k}, \tag{52}$$

$$|\bar{F}(\mathbf{p}')\rangle = \frac{1}{\sqrt{6}}\left(\frac{R_F^2}{\pi}\right)^{3/4}\int d\mathbf{k}\,\exp\left(-\frac{R_F{}^2\mathbf{k}^2}{2}\right) \times q_r{}^i(\mathbf{k}+\lambda_1\mathbf{p}')^\dagger u_r^\dagger \tilde{Q}_s{}^i(-\mathbf{k}+\lambda_2\mathbf{p}')v_s d\mathbf{k}, \tag{53}$$

where, for the heavy charm quark, $Q \equiv c$, $q = (d, u)$ correspond to the states (D^+, D^-) and $(D^0, \bar{D}^0)$, respectively, and, for heavy bottom quark, $Q \equiv b$, $q = (u, d)$ correspond to the open bottom mesons (B^-, B^+) and $(\bar{B}^0, B^0)$, respectively. In Eqs. (52) and (53), λ_1 and λ_2 are the fractions of the mass (energy) of the open charm (bottom) meson at rest (in motion), carried by the constituent light $[q = (d, u)]$ antiquark (quark) and the constituent heavy charm (bottom) quark (antiquark), with $\lambda_1 + \lambda_2 = 1$. The values of λ_1 and λ_2 are calculated by assuming the binding energy of the hadron as shared by the quark (antquark) to be inversely proportional to the quark (antiquark) mass [77,80,109]. Taking the constituent masses of the u and d quarks to be same $(M_u = M_d = M_q)$, the energies of $q(\bar{q})$, $(q = u,\ d)$, and $\bar{Q}(Q)$, with $Q = (c, b)$ in $\bar{F}(F)$ meson are then given as [109]

$$\omega_1 = M_q + \frac{\mu}{M_q}\times BE \quad \text{and} \quad \omega_2 = M_Q + \frac{\mu}{M_Q}\times BE, \tag{54}$$

with μ is the reduced mass of the light heavy, $Q\bar{q}(q\bar{Q})$ system, given as $1/\mu = 1/M_Q + 1/M_q$ with M_Q and M_q as the constituent masses of the heavy (Q) quark and light (q) quark, respectively, BE is the binding energy, $BE = m_{F(\bar{F})} - M_Q - M_q$, and, $\lambda_i = \frac{\omega_i}{m_{F(\bar{F})}}$, $i = 1, 2$ are the energies carried by the light quark (antiquark) and heavy antiquark (quark). The motivation for the assumption that the contributions from the quark (antiquark) to the binding energy of the hadron to be inversely proportional to the mass of the quark (antiquark) as in Eq. (54) is as follows. In fact, in general, the contributions to the binding energy of the bound state composed of particles of 1 and 2, with masses m_1 and m_2, are assumed to be given as μ/m_i, $i = 1$, 2, multiplied by the binding energy of the bound state, where μ is the reduced mass of the system, calculated from $1/\mu = 1/m_1 + 1/m_2$. In other words, the contributions from the particles to binding energy are inversely proportional to their masses, and the total binding energy is the sum of the individual contributions, i.e., $BE = ((\mu/m_1) + (\mu/m_2)) \times BE = BE$, as it should be. The reason for making this assumption comes from the example of hydrogen atom, which is the bound state of the proton and the electron. As the mass of proton is much larger as compared to the mass of the electron, the binding energy contribution from the electron is $\frac{\mu}{m_e} \times BE \simeq BE$ of hydrogen atom, and the contribution from the proton is $\frac{\mu}{m_p} \times BE$, which is negligible as compared to the total binding energy of hydrogen atom, since $m_p \gg m_e$. With this assumption, the binding energies of the heavy-light mesons, e.g., D and $\bar{D}$ mesons as well as for B and $\bar{B}$ mesons, mostly arise from the contribution from the light quark (antiquark).

The decay width of the quarkonium state, M, for the decay process $M \to F\bar{F}$, with $(M, F, \bar{F}) \equiv (\psi(3770), D, \bar{D})$, $(\Upsilon(4S), B, \bar{B})$, is calculated from the matrix element of the light quark-antiquark pair creation part of the free Dirac Hamiltonian, between the intial quarkonium state and the final state mesons for the reaction $M \to F(\mathbf{p})\bar{F}(\mathbf{p}')$ as given by

$$\langle F(\mathbf{p})|\langle \bar{F}(\mathbf{p}')| \int \mathcal{H}_{q^\dagger \tilde{q}}(\mathbf{x}, t=0) d\mathbf{x} |M^m(\vec{0})\rangle = \delta(\mathbf{p}+\mathbf{p}') A_M(|\mathbf{p}|) p_m, \tag{55}$$

where the expression for $A_M(|\mathbf{p}|)$ is written in the Appendix. The decay width is calculated to be

$$\Gamma(M \to F(\mathbf{p})\bar{F}(-\mathbf{p})) = \gamma_\psi^2 \frac{8\pi^2}{3} |\mathbf{p}|^3 \frac{p_F^0(|\mathbf{p}|) p_{\bar{F}}^0(|\mathbf{p}|)}{m_M} A_M(|\mathbf{p}|)^2, \tag{56}$$

with $p^0_{F(\bar{F})}(|\mathbf{p}|) = (m^2_{F(\bar{F})} + |\mathbf{p}|^2)^{1/2}$, and, $|\mathbf{p}|$, the magnitude of the momentum of the outgoing $F(\bar{F})$ meson is given as

$$|\mathbf{p}| = \left(\frac{{m_M}^2}{4} - \frac{{m_F}^2 + {m_{\bar{F}}}^2}{2} + \frac{({m_F}^2 - {m_{\bar{F}}}^2)^2}{4{m_M}^2} \right)^{1/2}. \tag{57}$$

In the above, the masses of the $F(\bar{F})$ and heavy quarkonium state are the in-medium masses in the magnetized nuclear matter calculated in the chiral effective model, with additional contributions from lowest Ladau levels for the charged open charm (bottom) mesons, as given by Eqs. (39) and (41). The parameter, γ_M, in the expression for the quarkonium decay width, is a measure of the coupling strength for the creation of the light quark antiquark pair, to produce the $F\bar{F}$ final state. This parameter is adjusted to reproduce the vacuum decay widths of $\psi(3770)$ to D^+D^- and $D^0\bar{D}^0$ [77] for the charm sector and $\Upsilon(4S) \to B^+B^-$ and $\Upsilon(4S) \to B^0\bar{B}^0$ [80] for the bottom sector.

When we include the PV mixing effect, the expression for the decay width is modified to

$$\begin{aligned}\Gamma^{\rm PV}(M \to F(\mathbf{p})\bar{F}(-\mathbf{p})) &= \gamma_M^2 \frac{8\pi^2}{3} \left[\left(\frac{2}{3} |\mathbf{p}|^3 \frac{p_F^0(|\mathbf{p}|) p_{\bar{F}}^0(|\mathbf{p}|)}{m_M} A_M(|\mathbf{p}|)^2 \right) \right. \\ &\quad \left. + \left(\frac{1}{3} |\tilde{\mathbf{p}}|^3 \frac{p_F^0(|\tilde{\mathbf{p}}|) p_{\bar{F}}^0(|\tilde{\mathbf{p}}|)}{m_M^{\rm PV}} A^M(|\tilde{\mathbf{p}}|)^2 \right) \right].\end{aligned} \tag{58}$$

where, $\tilde{\mathbf{p}}$ is the expression of $\mathbf{p}$ [given by Eq. (57)], with $m_M \to \left(m_M^{PV}\right)$. In Eq. (58), the first term corresponds to the transverse polarizations for the quarkonium state, M, whose masses remain unaffected by the mixing of the pseudoscalar and vector charmonium states. The second term in (58) corresponds to the longitudinal component, whose mass is modified due to the mixing with the pseudoscalar meson in the presence of the magnetic field.

IV. RESULTS AND DISCUSSIONS

We discuss the results obtained due to the effects of Dirac sea contributions for the nucleons and the PV mixing on the decay widths of charmonium state, $\psi(3770) \to D\bar{D}$ as well as $\Upsilon(4S) \to B\bar{B}$, in magnetized isospin asymmetric nuclear matter. The decay widths are calculated using a field theoretical model of composite hadrons for $\psi(3770)$ and $\Upsilon(4S)$, the lowest quarkonium states, which decay to $D\bar{D}$ and $\bar{B}B$ in vacuum. As the created matter produced in peripheral ultrarelativistic heavy ion collision experiments, e.g., at RHIC, BNL, and at LHC, CERN, is extremely dilute, we study the effects of the magnetic field on the quarkonium partial decay widths at zero density and at $\rho_B = \rho_0$, for symmetric as well as asymmetric magnetized nuclear matter. In magnetized nuclear matter, the medium modifications of the quarkonia decay widths are obtained from the mass modifications of the initial (quarkonia states) and the final (open charm and bottom mesons) calculated using a chiral effective model [from Eqs. (39) and (41)], including the effects of Dirac sea of the nucleons, with additional LLL contributions for the charged $D^\pm(B^\pm)$ mesons, which further undergo mass modifications due to pseudoscalar meson-vector meson (PV) mixing in the presence of a magnetic field, [given by Eqs. (44) and (46) for the charm and bottom sectors]. As has already been mentioned, the open charm and bottom meson masses are obtained from interactions with the nucleons and scalar mesons (σ, ζ, and δ) and mass shifts of the quarkonium states are obtained from the modifications of a scalar dilaton field, χ, which mimics the gluon condensates of QCD in the chiral effective model. The scalar fields and the dilaton field are solved from their coupled equations of motion, for given values of the baryon density, ρ_B, isospin asymmetry parameter, η, and the magnetic field, B. In the present study, the AMMs of the nucleons are considered, which are observed to be important for the mass modifications, especially, when the Dirac sea effects are taken into account.

There is observed to be enhancement of the quark condensates [calculated from the scalar fields σ and ζ using Eq. (18)] with increase in the magnetic field, due to Dirac sea contributions for zero density as well as for $\rho_B = \rho_0$ (when AMMs of nucleons are not considered) both for symmetric ($\eta = 0$) and asymmetric (with $\eta = 0.5$) nuclear matter, an effect called magnetic catalysis (MC). However, when the AMMs of nucleons are taken into account, there is observed to be inverse magnetic catalysis (IMC) for $\rho_B = \rho_0$, both for symmetric as well as asymmetric (with $\eta = 0.5$) nuclear matter in presence of a magnetic field. The Dirac sea contributions have appreciable effects on the meson masses and hence, on the decay widths of $\psi(3770) \to D\bar{D}$ and $\Upsilon(4S) \to B\bar{B}$. The quarkonium decay widths in magnetized (nuclear) matter were studied using a field theoretical model of composite hadrons [78], including the effects of the mixing of the charmonium (bottomonium) states [$\psi(3770) - \eta_c(2S)$ [$\Upsilon(4S) - \eta_b(4S)$] mixings] [78,81] as well as the PV mixing of the open charm (bottom) mesons [$D(B) - D^*(B^*)$ and $\bar{D}(\bar{B}) - \bar{D}^*(\bar{B}^*)$ mixings] [79,81], in addition to the Landau level contributions for the charged $D^{\pm}(B^{\pm})$ mesons. The Dirac sea contributions to the self energies of the nucleons are observed to lead to important modifications on the decay widths, which were not considered in Refs. [78,79,81] for the mass modifications of the initial and final state mesons, hence on the quarkonia decay widths.

Including the effects of the Dirac sea of the nucleons, the masses of the open charm [113], the bottom meson mesons [114], and the heavy quarkonia states [115] have been studied in magnetized (nuclear) matter. The inclusion (exclusion) of the AMMs of nucleons give rise to the IMC (MC) for $\rho_B = \rho_0$, which lead to very different behaviors for the masses of the quarkonium states $\psi(1D)$ and $\Upsilon(4S)$, with a drop (increase) in the mass, with an increase in the magnetic field, when the PV effects are not taken into account [115]. For the open heavy flavor mesons, there is observed to be a monotonic increase with magnetic field when the AMMs are not taken into account, whereas there is obserevd to be an initial increase followed by a drop in these masses when the magnetic field is further increased, and the behavior remains similar when the PV mixing effects are also taken into account [113]. The decay width of the quarkonium state $\psi(1D)$ [$\Upsilon(4S)$] (decaying at rest) to $D\bar{D}$ ($B\bar{B}$) depends on the magnitude of the momentum of the outgoing open heavy flavor mesons, $|\mathbf{p}|$, given by Eq. (57) in terms of the in-medium masses of the quarkonium state and the open heavy flavor mesons. The dependence of the quarkonium decay width on $|\mathbf{p}|$ is through a polynomial term multiplied by an exponential term, as can be seen from the expression of the decay width given by Eq. (58), in which the expression $A^M[|\mathbf{p}|$ given by Eq. (A12)] is in the form of an exponential as well as polynomials, T_i^M, whose explicit expressions are written down in the Appendix. As we shall see there is observed to be a significant difference in the decay width of $\Upsilon(4S) \to B\bar{B}$, for $\rho_B = \rho_0$, for both symmetric and asymmetric nuclear matter, when the AMMs are taken into account, as compared to when these are ignored. This is due to the different behaviors of the masses of the quarkonium and open charm (bottom) mesons, due to the different behaviors of the scalar fields, corresponding to (inverse) magnetic catalysis, in the presence (absence) of the AMMs of the nucleons. The effects of the Dirac sea contributions are seen to be more significant for the $\Upsilon(4S) \to B\bar{B}$, with observation of nodes at high values of the magnetic field, for both the charged and neutral $B\bar{B}$ final state decay widths.

In Fig. 1, we plot the decay widths of $\psi(3770) \to D\bar{D}$ for $\rho_B = 0$ including the Dirac sea (DS) contributions for the nucleons as well as effects from the PV mixing in the presence of a magnetic field. In Fig. 1(a) shows the decay widths of (I) $\psi(3770) \to D^+D^-$, (II) $\psi(3770) \to D^0\bar{D}^0$, and sum of these subchannels, in the absence of the PV mixing of the charmonium states as well as open charm mesons. In the absence of the DS contributions, for $\rho_B = 0$, the masses of the charmonium and the neutral open charm mesons remain at their vacuum values, but the masses of

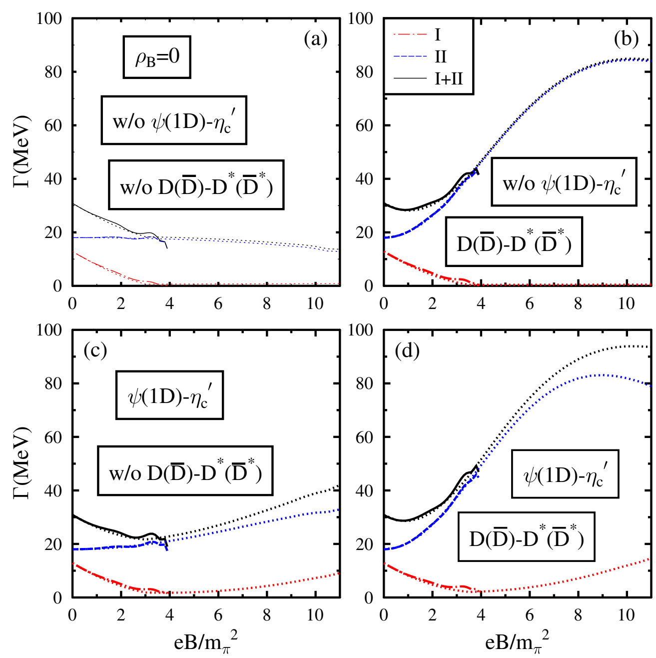

FIG. 1. Decay widths of (I) $\psi(1D) \to D^+D^-$, (II) $\psi(1D) \to D^0\bar{D}^0$, and (III) the sum of these two channels (I) and (II), as functions of eB/m_π^2, for $\rho_B = 0$ with the AMMs of nucleons taken into account. The effects due to the Dirac sea (DS) contributions are included. The effects of the $D - D^*$ ($\bar{D} - \bar{D}^*$) mixing on these decay widths are shown in (b) and (d), without and with the additional effect from $\psi(1D) - \eta_c'$ mixing, respectively. The results are compared with the cases when the AMMs of nucleons are not considered (shown as dotted lines).

charged mesons $D^{\pm}$ have positive shifts in the presence of a magnetic field due to the lowest Landau level (LLL) contributions. Hence, when the Dirac sea effects are neglected, the decay width with the $D^0\bar{D}^0$ final state stays at its vacuum value, whereas the decay width of $\psi(3770) \to D^+D^-$ decreases with increase in the magnetic field (due to the increase in the masses of the charged $D^{\pm}$ mesons) and becomes zero at and larger than a certain value of magnetic field (when the decay is no longer kinematically possible). In the presence of the Dirac sea contributions, but when the PV mixing effects are not taken into account, the masses of the neutral open charm mesons and charmonium states are observed to have negligible dependence on the magnetic field [113,115]. However, there is increase in the masses of the charged $D^{\pm}$ mesons due to the lowest Landau level (LLL) contributions, which leads to a drop in the decay width for the charged open charm meson pair final state in the presence of a magnetic field, whereas, the decay width of charmonium to neutral $D\bar{D}$ is observed to drop marginally with increase in the magnetic field, in the absence of PV mixing, as can be seen from panel (a) in Fig. 1. The contributions due to PV mixing have been observed to be significant in Refs. [78,79]. The Dirac sea contributions are taken into account using the summation of the tadpole diagrams, using weak field approximation for the nucleon propagator [99], and, in the presence of AMMs of the nucleons, the solutions do not exist for the scalar fields for $eB \geq 4m_\pi^2$, for $\rho_B = 0$ in this approximation. The effects of AMMs on the charmonium decay widths, for this range of magnetic field where the solutions for the scalar fields and hence the masses of the open and hidden charm mesons exist, are observed to be quite small, as compared to the case when the AMMs are neglected (shown as the dotted lines). The mixings of the $D - D^*$ and $\bar{D} - \bar{D}^*$ mesons lead to drop in the masses of the open charm pseudoscalar mesons, and this is observed as a significant enhenacement of the decay width in the neutral $D\bar{D}$ channel, as can be observed in panel (b) in Fig. 1. However, the $D(\bar{D}) - D^*(\bar{D}^*)$ mixings are not observed to affect the decay channel with D^+D^- final state, the reason for this is due to the fact that the PV effects on the masses of the charged $D^{\pm}$ mesons become appreciable for higher values of magnetic fields ($eB \geq 3m_\pi^2$) [113], and, for these values of the magnetic field, the decay to the charged $D\bar{D}$ is no longer kinematically possible, due to the positive Landau level contributions leading to increase in the masses of the charged $D^{\pm}$ mesons. In the presence of the $\psi(1D) - \eta_c(2S)$ mixing (which leads to an increase in the mass of the longitudinal component of $\psi(1D)$), but without accounting for the mixing in the open charm meson sectors, there is observed to be a rise in the decay widths for both the sub channels for high values of magnetic field, as can be seen from panel (c) in Fig. 1. When both the mixings (for the charmonium as well as open charm mesons) are considered, there is observed to be significant rise in the charmonium decay width to the neutral $D\bar{D}$, as well as, an increase for the charged $D\bar{D}$ channel at higher values of the magnetic field, as can be seen in panel (d) of Fig. 1.

The decay widths of $\psi(1D) \to D\bar{D}$, along with the decay widths for the subchannels (I) $\psi(1D) \to D^+D^-$ and (II) $\psi(1D) \to D^0\bar{D}^0$, are shown for $\rho_B = \rho_0$, accounting for the Dirac sea contributions to the scalar densities of the nucleons as well as with the PV mixing effects from the charmonium states [$\psi(1D) - \eta_c'$ mxing]. These are shown without and with the PV effects for the open charm ($D(\bar{D}) - D^*(\bar{D}^*)$ mixing) mesons, for symmetric ($\eta = 0$) nuclear matter, in Figs. 2 and 3, respectively. When the AMMs of the nucleons are considered, the Dirac sea contributions are observed to modify the decay width of charmonium to the neutral $D\bar{D}$ appreciably at high magnetic fields, for $\eta = 0$ in the absence of PV mixing of open charm mesons. However, the additional PV mixing for the charmonium states [$\psi(1D) - \eta_c'$ mixing], is observed to only modify the decay widths marginally, as can be seen from panels (b) and (d) of Fig. 2. There is observed to be significant rise in the decay widths when the PV mixing in open charm sectors, is taken into account, as can be seen

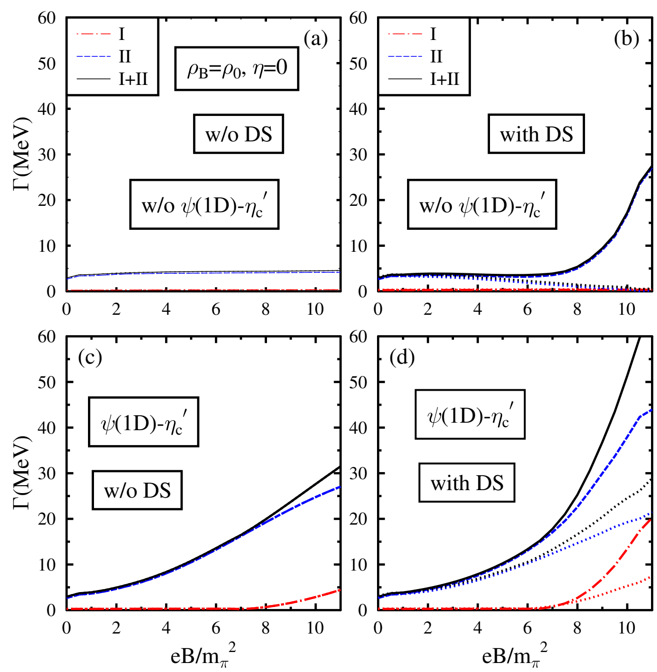

FIG. 2. Decay widths of (I) $\psi(1D) \to D^+D^-$, (II) $\psi(1D) \to D^0\bar{D}^0$, and (III) the sum of these two channels (I) and (II), as functions of eB/m_π^2, for $\rho_B = \rho_0$ and $\eta = 0$ with the AMMs of nucleons taken into account. The effects due to the Dirac sea (DS) contributions are shown in (b) and (d), without and with the $\psi(1D) - \eta_c'$ mixing, respectively. The results are compared with the cases when the AMMs of nucleons are not considered (shown as dotted lines).

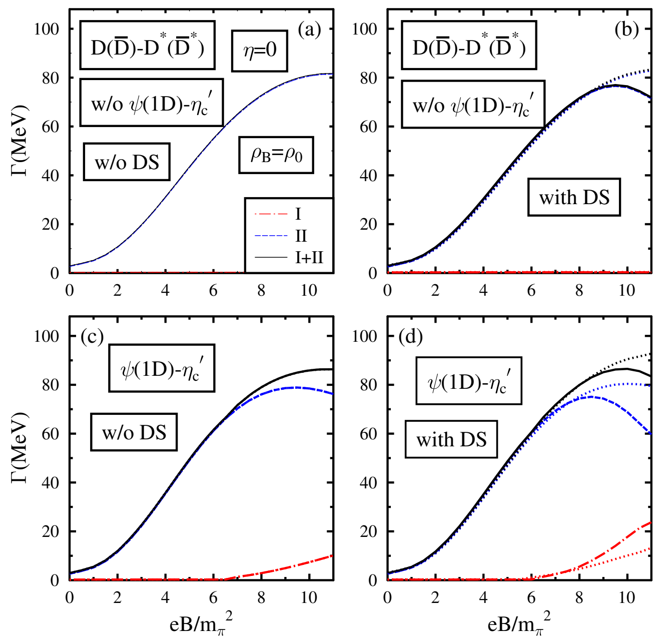

FIG. 3. Same as Fig. 2, with additional mass modifications of the open charm mesons from $D - D^*$ and $\bar{D} - \bar{D}^*$ mixing effects.

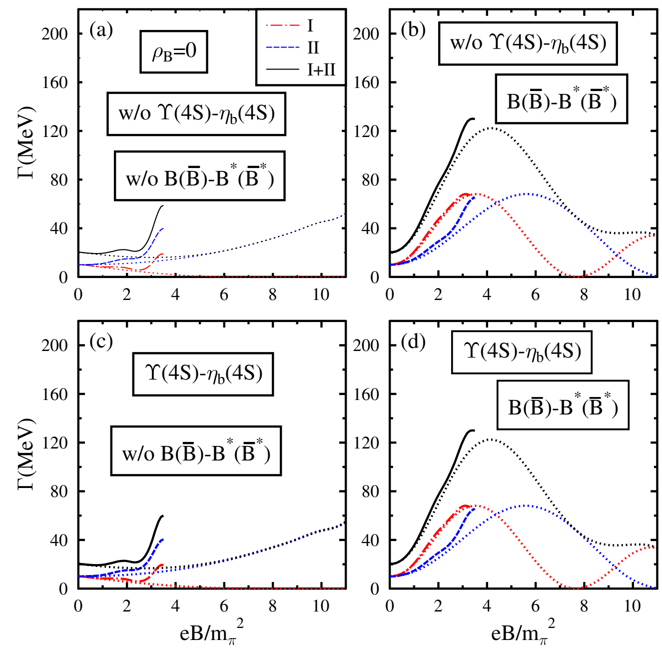

FIG. 4. Decay widths of (I) $\Upsilon(4S) \to B^+B^-$, (II) $\Upsilon(4S) \to B^0\bar{B}^0$, and (III) the sum of these two channels (I) and (II), as functions of eB/m_π^2, for $\rho_B = 0$ with the AMMs of nucleons taken into account. The effects due to the Dirac sea (DS) contributions are included. The effects of the $B - B^*$ ($\bar{B} - \bar{B}^*$) mixing on these decay widths are shown in (b) and (d), without and with the additional effect from $\Upsilon(4S) - \eta_b(4S)$ mixing, respectively. The results are compared with the cases when the AMMs of nucleons are not considered (shown as dotted lines).

from Fig. 3. The effects of the isospin asymmetry is observed to be much less dominant as compared to the effects due to the Dirac sea contributions and the PV mixing effects. The AMMs of the nucleons however do play an important role, and the Dirac contribution effects lead to inverse magnetic catalysis (IMC) when the AMMs are considered, whereas there is observed to be magnetic catalysis (MC) when the AMMs are neglected. Due to the opposite behavior of the scalar fields (proportional to the light quark condensates), the behaviors of the open charm mesons are quite different without and with the inclusion of the AMMs of the nucleons, at $\rho_B = \rho_0$, for symmetric as well as asymmetric nuclear matter in the presence of a magnetic field.

In Fig. 4, the decay widths of $\Upsilon(4S) \to B\bar{B}$, along with the subchannels corresponding to the final states (I) charged and (II) neutral $B\bar{B}$ are shown for $\rho_B = 0$, taking into account the Dirac sea contributions. In Fig. 4(a), in the absence of the PV mixings for the bottomonium states ($\Upsilon(4S) - \eta_b(4S)$) as well as for the open bottom mesons ($B - B^*$ and $\bar{B} - \bar{B}^*$), due to the positive contributions to $B^\pm$ masses from Landau levels, one observes a drop in the width of the decay to B^+B^- final state with increase in the magnetic field, which becomes (and remains) zero for $eB \geq 5m_\pi^2$. On the other hand, the decay width for the neutral $B\bar{B}$ final state shows a steady increase with the magnetic field, reaching a value of 45.16 MeV at $eB = 10m_\pi^2$ from the vacuum value of around 10 MeV. There is observed to be a significant increase in the decay widths, more dominant for the B^+B^- final state, due to the PV mixing in the $B(\bar{B}) - B^*(\bar{B}^*)$ mesons, as can be seen in Fig. 4(b). With further rise in the magnetic field, there is observed to be a drop in the decay widths of both the subchannels, reaching zero value (corresponding to the nodes), for the values of eB of around 7.5 and 11 m_π^2 for the subchannels (I) and (II), respectively. Similar behaviors of the decay widths are observed when the PV mixng in the bottomonium sector is also taken into account [shown in Fig. 4(d)]. However, the PV mixing effects in the open bottom sector are observed to be much more appreciable as compared to the PV mixing effect in the bottomonium sector, as can be seen in panels (c) and (d) of Fig. 4. The observation of the nodes (vanishing of the decay widths) arises due to the dependence of the decay widths [given by Eq. (58)] on the magnitude of the momentum of the outgoing $B(\bar{B})$ meson [given by Eq. (57)] as a polynomial term multiplied by a gaussian contribution, and the node occurs when the polynomial part becomes zero. The nodes arise from taking into consideration the internal structure of the mesons in terms of the quark and antiquark constituents [51,72,78,81]. On the other hand, a phenomenological interaction, $\mathcal{L}_{\text{int}} \sim \Upsilon^\mu(\bar{B}(\partial_\mu B) - (\partial_\mu \bar{B})B)$, without accounting for the internal structure of the mesons, leads to the decay widths, which increase monotonically with increase in $|\mathbf{p}|$.

In Fig. 5, the decay widths are shown for $\rho_B = \rho_0$ for symmetric nuclear matter, without accounting for the mixings in the open bottom sector. In Figs. 5(a) and 5(c), these are plotted for the cases of without and with $\Upsilon(4S) - \eta_b(4S)$ mixing, and, without accounting for the Dirac sea effects. The decay widths are observed to have significant contributions with inclusion of Dirac sea effects, when the AMMs of the nucleons are taken into account, as can be seen from Fig. 5(b) and 5(d), respectively. There is observed to be an initial rise and then a drop and vanishing of the decay widths at around $eB = 10m_\pi^2$, for both the subchannels [(I) $\Upsilon(4S) \to B^+B^-$ and (II) $\Upsilon(4S) \to B^0\bar{B}^0$], when the AMMs are taken into account. As the magnetic field is further increased, there is observed to be increase in these decay widths. As can be observed in panels (b) and (d) in Fig. 5, including the Dirac sea effects, when the AMMs of the nucleons are ignored (shown as dotted lines), there is observed to be a drop in the decay width (I) $\Upsilon(4S) \to B^+B^-$, which becomes zero for $eB \sim 6m_\pi^2$, whereas the decay width for the neutral $B\bar{B}$ final state shows a steady, but slow decrease with rise in the magnetic field, without and with the $\Upsilon(4S) - \eta_b(4S)$ mixing effect taken into consideration. The effects on the decay widths of $\Upsilon(4S) \to B\bar{B}$ from the PV mixing of the bottomonium states are observed to be marginal as compared to the effects from Dirac sea contributions, as can be seen from Fig. 5. In the presence of Dirac sea effects and AMMs of nucleons, when the $\Upsilon(4S) - \eta_b(4S)$ mixing is also taken into account, there is observed to be a nonsmooth behavior of the decay widths in both charged and neutral $D\bar{D}$ channels at around $eB \sim 7m_\pi^2$ [as can be seen in panel (d) of Fig. 5]. This behavior of the decay widths (which depend on $|\mathbf{p}|$) arises from dependence of the mass of the bottomonium state, $\Upsilon(4S)$ (hence of $|\mathbf{p}|$) with the magnetic field, which is observed to be nonsmooth at around this value of eB [115].

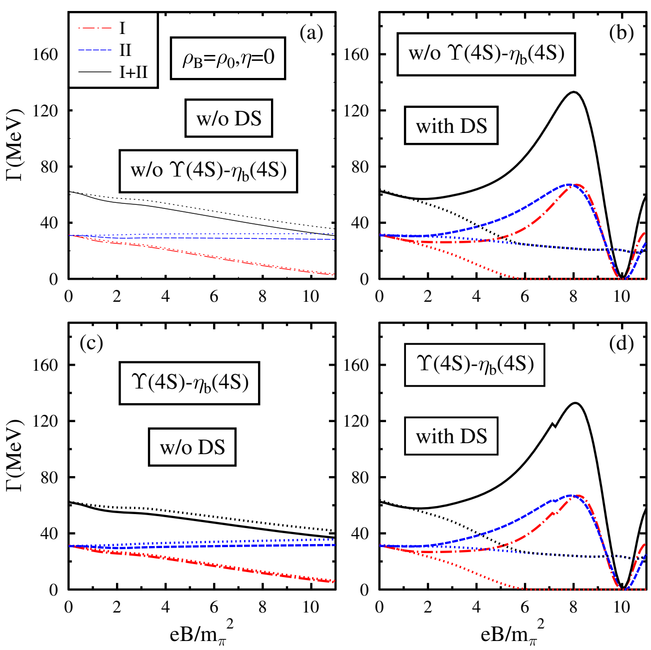

FIG. 5. Decay widths of (I) $\Upsilon(4S) \to B^+B^-$, (II) $\Upsilon(4S) \to B^0\bar{B}^0$, and (III) the sum of these two channels (I) and (II), as functions of eB/m_π^2, for $\rho_B = \rho_0$ and $\eta = 0$ with the AMMs of nucleons taken into account. The effects due to the Dirac sea (DS) contributions are shown in (b) and (d), without and with the $\Upsilon(4S) - \eta_b(4S)$ mixing, respectively. The results are compared with the cases when the AMMs of nucleons are not considered (shown as dotted lines).

In Fig. 6, the decay widths are shown accounting for the $B(\bar{B}) - B^*(\bar{B}^*)$ mixings. In the absence of DS effects, there is observed to be appreciable effect due to these mixings which are observed to lead to only marginal modifications, when the $\Upsilon(4S) - \eta_b(4S)$ mixing is also considered [see Figs. 6(b) and 6(d) as compared to Figs. 6(a) and 6(c)]. There is observed to be a node in the decay width for the B^+B^- final state at around $eB \sim 7m_\pi^2$ in the absence of DS effects, without and with the PV mixing effects taken into account, as can be seen from Figs. 6(a) and 6(c). In the presence of DS effects, the initial rise is followed by a drop leading to vanishing of the decay width and again an increase as the magnetic field is further increased. The nodes are observed for values of eB around 6.3 (8.2) and 9 (10.5) m_π^2, for the charged (neutral) $B\bar{B}$ final states when the AMMs of the nucleons are taken into account. The DS effects are observed to be much larger at higher values of the magnetic fields, when the AMMs are considered.

In Fig. 7, the decay widths are shown for $\rho_B = \rho_0$ for asymmetric (with $\eta = 0.5$) nuclear matter, accounting for the mixings in the bottomonium as well as the open bottom

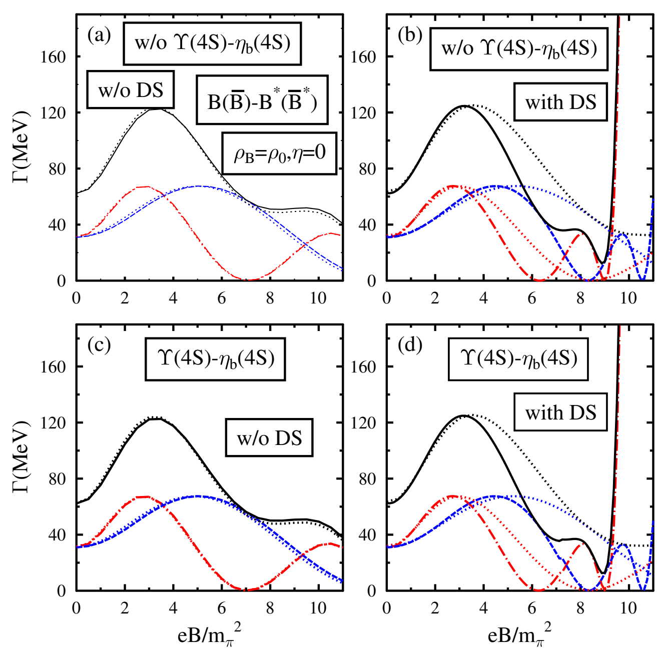

FIG. 6. Same as Fig. 5 with additional mass modifications of the open bottom mesons from $B - B^*$ and $\bar{B} - \bar{B}^*$ mixing effects.

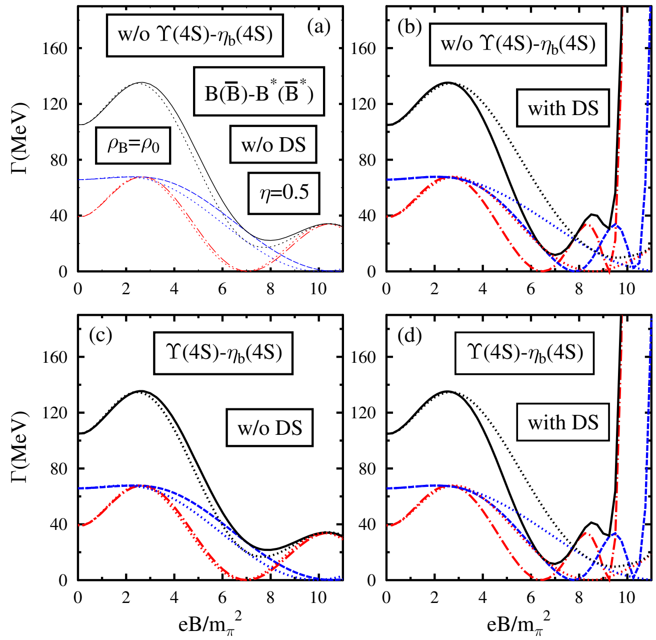

FIG. 7. Same as Fig. 6, with $\eta = 0.5$.

sector. In Figs. 7(a) and 7(c), these are plotted without DS effects, which are observed to show similar behavior as for the symmetric nuclear matter shown in Fig. 6; however, the values for $eB = 0$ are much higher for the asymmetric nuclear matter as compared to the symmetric matter. In the absence of the DS contributions, for zero magnetic field [80], the different mass modifications of the bottomonium state and open bottom mesons (and hence of values of $\mathbf{p}|$), in the asymmetric and symmetric nuclear matter, lead to the difference in the decay widths of $\Upsilon(4S) \to B\bar{B}$. The effects from the PV mixings for the open bottom mesons are observed to dominate over the effects due to the mixing in the bottomonium sector, both in the symmetric and asymmetric nuclear matter.

The magnetic field effects considered on the decay width of the charmonium (bottomonium) state $\psi(1D) \to D\bar{D}$ ($\Upsilon(4S) \to B\bar{B}$) in the present work, are due to the Dirac sea effects of the nucleons, the effects from $\psi(1D) - \eta_c'$ ($\Upsilon(4S) - \eta_b(4S)$), $D - D^*$ ($B - B^*$), and $\bar{D} - \bar{D}^*$ ($\bar{B} - \bar{B}^*$) mixings and Landau level contributions for the charged, $D^\pm(B^\pm)$ mesons. The Dirac contributions are observed to lead to significant modifications to the quarkonium decay widths. The decay of $\Psi(3770)$ to the $D^0\bar{D}^0$ is observed to have much larger contribution from the Dirac sea effects as compared to the decay width for the charged D^+D^- final state. The effects of the Dirac sea contributions are observed to be more significant for the $\Upsilon(4S) \to B\bar{B}$ (as compared to the decay width of $\psi(1D) \to D\bar{D}$). With Dirac sea effects, there is observed to be a significant difference in the decay width of $\Upsilon(4S) \to B\bar{B}$ in magnetized nuclear matter, for $\rho_B = \rho_0$, for the cases of ignoring (including) the AMMs of the nucleons, when the (inverse) magnetic catalysis is observed. The strong magnetic field created at the early stage should have observable consequences on the production of the hidden and open charm mesons arising from ultrarelativistic heavy ion collision experiments.

V. SUMMARY

To summarize, we have studied the decay widths of the charmonium states $\psi(1D)$ to $D\bar{D}$ and of the upsilon state $\Upsilon(4S) \to B\bar{B}$ in magnetized (nuclear) matter, accounting for the Dirac sea contributions for the self-energies of the nucleons within a chiral effective model. The open charm (bottom) mesons are calculated from their interactions with the nucleons and the scalar mesons, whereas the quarkonium masses are calculated within a chiral effective model from the medium change of a scalar dilaton field, which mimics the gluon condensates of QCD. There is observed to be magnetic catalysis effect, i.e., enhancement of the quark condensates (given in terms of the scalar fields) with rise in magnetic field, for $\rho_B = 0$, for both the cases of accounting and ignoring the AMMs of the nucleons. However, for $\rho_B = \rho_0$, there is observed to be inverse magnetic catalysis (IMC) when the AMMs of the nucleons are taken into account. The effects from PV mixing [$\psi(1D) - \eta_c'$, $D - D^*$ and $\bar{D} - \bar{D}^*$ mixings for the charm sector and $\Upsilon(4S) - \eta_b(4S)$, $B - B^*$ and $\bar{B} - \bar{B}^*$ mixings for the bottom sector] in the presence of the magnetic field are also taken into account, in addition to the Landau contributions for the charged open charm (bottom) mesons. The effects of the Dirac sea as well as PV mixings are observed to be quite significant on the heavy quarkonium decay widths. These should have observable consequences on the production of heavy quarkonium states and open heavy flavor mesons, as these are created at the early stage of the noncentral ultrarelativistic heavy ion collision experiments, when the magnetic field can be still be large.

ACKNOWLEDGMENTS

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APPENDIX: MODEL FOR COMPOSITE HADRONS

The model describes hadrons comprising of quark (and antiquark) constituents. The field operator for a constituent quark for a hadron at rest at time, $t = 0$, is written as

$$\psi(\mathbf{x}, t = 0) = (2\pi)^{-3/2} \int [U(\mathbf{k}) u_r q_r(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{x}) + V(\mathbf{k}) v_s \tilde{q}_s(\mathbf{k}) \exp(-i\mathbf{k}\cdot\mathbf{x})] d\mathbf{k} \equiv Q(\mathbf{x}) + \tilde{Q}(\mathbf{x}), \tag{A1}$$

where $U(\mathbf{k})$ and $V(\mathbf{k})$ are given as

$$U(\mathbf{k}) = \begin{pmatrix} f(|\mathbf{k}|) \\ \boldsymbol{\sigma}\cdot\mathbf{k}g(|\mathbf{k}|) \end{pmatrix}, \quad V(\mathbf{k}) = \begin{pmatrix} \boldsymbol{\sigma}\cdot\mathbf{k}g(|\mathbf{k}|) \\ f(|\mathbf{k}|) \end{pmatrix}. \quad \text{(A2)}$$

The functions $f(|\mathbf{k}|)$ and $g(|\mathbf{k}|)$ satisfy the constraint [108], $f^2 + g^2\mathbf{k}^2 = 1$, as obtained from the equal time anticommutation relation for the four-component Dirac field operators. These functions, for the case of free Dirac field of mass M, are given as

$$f(|\mathbf{k}|) = \left(\frac{k_0 + M}{2k_0}\right)^{1/2}, \quad g(|\mathbf{k}|) = \left(\frac{1}{2k_0(k_0+M)}\right)^{1/2}, \quad \text{(A3)}$$

where $k_0 = (|\mathbf{k}|^2 + M^2)^{1/2}$. In the above, M is the constituent quark/antiquark mass. In Eq. (A1), u_r and v_s are the two component spinors for the quark and antiquark, respectively, satisfying the relations $u_r^\dagger u_s = v_r^\dagger v_s = \delta_{rs}$. The operator $q_r(\mathbf{k})$ annihilates a quark with spin r and momentum $\mathbf{k}$, whereas $\tilde{q}_s(\mathbf{k})$ creates an antiquark with spin s and momentum $\mathbf{k}$, and these operators satisfy the usual anticommutation relations,

$$\{q_r(\mathbf{k}), q_s(\mathbf{k}')^\dagger\} = \{\tilde{q}_r(\mathbf{k}), \tilde{q}_s(\mathbf{k}')^\dagger\} = \delta_{rs}\delta(\mathbf{k}-\mathbf{k}'). \quad \text{(A4)}$$

The field operator for the constituent quark of hadron with finite momentum is obtained by Lorentz boosting the field operator of the constituent quark of hadron at rest, which requires the time dependence of the quark field operators. Similar to the MIT bag model [111], where the quarks (antiquarks) occupy specific energy levels inside the hadron, it is assumed in the present model for the composite hadrons that the quark/antiquark constituents carry fractions of the mass (energy) of the hadron at rest (in motion) [108,109]. The time dependence for the ith quark(antiquark) of a hadron of mass m_H at rest is given as

$$Q_i(x) = Q_i(\mathbf{x})e^{-i\lambda_i m_H t}, \quad \tilde{Q}_i(x) = \tilde{Q}_i(\mathbf{x})e^{i\lambda_i m_H t}, \quad \text{(A5)}$$

where λ_i is the fraction of the energy (mass) of the hadron carried by the quark (antiquark), with $\sum_i \lambda_i = 1$. For a hadron in motion with four momentum p, the field operators for quark annihilation and antiquark creation, for $t = 0$, are obtained by Lorentz boosting the field operator of the hadron at rest, and are given as [110]

$$Q^{(p)}(\mathbf{x}, t) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} S(L(p))U(\mathbf{k})Q(\mathbf{k}+\lambda\mathbf{p}) \times \exp[i(\mathbf{k}+\lambda\mathbf{p})\cdot\mathbf{x} - i\lambda p^0 t] \quad \text{(A6)}$$

and

$$\tilde{Q}^{(p)}(\mathbf{x}, t) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} S(L(p))V(-\mathbf{k})\tilde{Q}(-\mathbf{k}+\lambda\mathbf{p}) \times \exp[-i(-\mathbf{k}+\lambda\mathbf{p})\cdot\mathbf{x} + i\lambda p^0 t]. \quad \text{(A7)}$$

In the above, λ is the fraction of the energy of the hadron, carried by the constituent quark (antiquark). In Eqs. (A6) and (A7), $L(p)$ is the Lorentz transformation matrix, which yields the hadron at finite four-momentum p from the hadron at rest, and is given as [109]

$$L_{\mu 0} = L_{0\mu} = \frac{p^\mu}{m_H}; \quad L_{ij} = \delta_{ij} + \frac{p^i p^j}{m_H(p^0 + m_H)}, \quad \text{(A8)}$$

where $\mu = 0, 1, 2, 3$ and $i = 1, 2, 3$, and the Lorentz boosting factor $S(L(p))$ is given as

$$S(L(p)) = \left[\frac{(p^0 + m_H)}{2m_H}\right]^{1/2} + \left[\frac{1}{2m_H(p^0 + m_H)}\right]^{1/2} \vec{\alpha}\cdot\vec{p}, \quad \text{(A9)}$$

where $\vec{\alpha} = \left(\begin{smallmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{smallmatrix}\right)$, are the Dirac matrices. The Lorentz transformations used to obtain the constituent quark and antiquark operators for hadron at rest to hadron with momentum, p, as given by Eqs. (A6) and (A7) have the effect of addition of the hadron fractional momentum, $\lambda\mathbf{p}$, as a translation to the constituent quark (antiquark) momentum, $\mathbf{k}(-\mathbf{k})$ [110]. This is similar to the quasipotential approach, where the Lorentz transformation plays the role of a translation [116]. Using the composite model picture with Lorentz transformations as considered in the present work, the various properties of hadrons, e.g., charge radii of the proton and pion, the nucleon magnetic moments [108,109] have been studied.

The pair creation term of the Dirac Hamiltonian density,

$$\mathcal{H}_{Q^\dagger\tilde{Q}}(x) = Q(x)^\dagger(-i\boldsymbol{\alpha}\cdot\nabla + \beta M)\tilde{Q}(x), \quad \text{(A10)}$$

is used to describe the decay of the heavy charmonium (bottomonium) state, M at rest to open heavy flavor mesons $F(\mathbf{p})$ and $\bar{F}(\mathbf{p}')$. The operators for the light ($q = u, d$) quark and antiquark creation in the above term, thus belong to different hadrons, F and $\bar{F}$ with four-momenta p and p', respectively. The light quark pair creation term of the Hamiltonian density is used to describe the decay of a heavy charmonium state ($\bar{Q}Q$), $Q = b, c$ to $D(B)$ and $\bar{D}(\bar{B})$ states, which are bound states of $Q\bar{q}$ and $\bar{Q}q$ respectively, with light (u, d) quark antiquark pair creation. We evaluate the matrix element of the quark-antiquark pair creation part of the Hamiltonian, between the initial charmonium (bottomonium) state and the final state, $F\bar{F}$, $F \equiv (D, B)$, using explicit constructions for the initial and final state mesons,

$$\langle F(\mathbf{p})|\langle\bar{F}(\mathbf{p}')|\int\mathcal{H}_{q^\dagger\tilde{q}}(\mathbf{x},t=0)d\mathbf{x}|M_m(\vec{0})\rangle = \delta(\mathbf{p}+\mathbf{p}')A^{(M)}(|\mathbf{p}|)p_m. \quad \text{(A11)}$$

With $\langle f|S|i\rangle = \delta_4(P_f - P_i)M_{fi}$, we have $M_{fi} = 2\pi(-iA^M(|\mathbf{p}|)p_m$ for evaluation of the matrix element of the quark-antiquark pair creation part of the Hamiltonian, between the initial charmonium state and the final state $F\bar{F}$, $F=(D,B)$ state as given by Eq. (A11), As the $D(B)$ and $\bar{D}(\bar{B})$ mesons are nonrelativistic, we shall assume S(L(p)) and S(L(p')) to be unity. We shall also take the approximate forms (with a small momentum expansion) for the functions $f(|\mathbf{k}|)$ and $g(|\mathbf{k}|)$ of the field operator as given by $g(|\mathbf{k}|) = 1/(2k_0(k_0+M))^{1/2} \simeq 1/(2M)$, and $f(|\mathbf{k}|) = (1-g^2\mathbf{k}^2)^{1/2} \approx 1-((g^2\mathbf{k}^2)/2)$ [77].

The expression for the decay width of $M\to F\bar{F}$ is obtained as given by Eq. (56). The expression for $A^M(|\mathbf{p}|)$ in the decay width is given as

$$A^M(|\mathbf{p}|) = 6c_M\exp[(a_M {b_M}^2 - R_F^2\lambda_2^2)\mathbf{p}^2] \cdot\left(\frac{\pi}{a_M}\right)^{3/2}\left[T_0^M + T_1^M\frac{3}{2a_M} + T_2^M\frac{15}{4a_M^2} + T_3^M\frac{105}{8a_M^3} + T_4^M\frac{105\times 9}{16a_M^4}\right], \quad \text{(A12)}$$

where a_M, b_M are given as [77] $a_M = \frac{1}{2}R_M^2 + R_F^2$; $b_M = R_F^2\lambda_2/a_M$, and c_M, for $M\equiv\psi(3770)$, and $M\equiv\Upsilon(4S)$ are given as

$$c_{\psi(3770)} = \frac{1}{4\sqrt{3\pi}}\left(\frac{16}{15}\right)^{1/2}\cdot\pi^{-1/4}\cdot(R^2_{\psi(3770)})^{7/4}\cdot\frac{1}{6}\cdot\left(\frac{R_D^2}{\pi}\right)^{3/2}$$

and

$$c_{\Upsilon(4S)} = \frac{1}{6\sqrt{6}}\left(\frac{\sqrt{35}}{4}\right)\left(\frac{R^2_{\Upsilon(4S)}}{\pi}\right)^{3/4}\cdot\left(\frac{R_B^2}{\pi}\right)^{3/2},$$

respectively. In the above expressions, R_M and R_F refer to the strengths of the harmonic oscillator wave functions for the charmonium state, $\psi(3770)$ [bottomonium state $\Upsilon(4S)$], and the $F(\bar{F})$, $F=D$, B mesons.

The expressions for T_i^M for $M\equiv(\Psi(3770), \Upsilon(4S)$, are given as

$$\begin{aligned}
T_0^{\psi(3770)} &= 2b^2_{\psi(3770)}(1-\lambda_2)\mathbf{p}^2 + 2b^2_{\psi(3770)}g^2(\mathbf{p}^2)^2(b_{\psi(3770)}-\lambda_2)((3/2)b^2_{\psi(3770)} - (2+\lambda_2)b_{\psi(3770)} + 2\lambda_2 - (1/2)\lambda_2^2),\\
T_1^{\psi(3770)} &= g^2\mathbf{p}^2[14{b_{\psi(3770)}}^3 - b^2_{\psi(3770)}((32/3)+(37/3)\lambda_2) + b_{\psi(3770)}((28/3)\lambda_2 - (1/3)\lambda_2^2)],\\
T_2^{\psi(3770)} &= g^2[7b_{\psi(3770)} - (2/3)\lambda_2 - (4/3)],\\
T_3^M &= 0, \qquad T_4^M = 0.
\end{aligned} \quad \text{(A13)}$$

$$\begin{aligned}
T_0^{\Upsilon(4S)} &= \frac{1}{2}(b_{\Upsilon(4S)}-1)(b_{\Upsilon(4S)}-\lambda_2)(3b_{\Upsilon(4S)}+\lambda_2-4)g^2|\mathbf{p}|^2\\
&\quad\times\left(1 - 2R^2_{\Upsilon(4S)}b^2_{\Upsilon(4S)}|\mathbf{p}|^2 + \frac{4}{5}R^4_{\Upsilon(4S)}b^4_{\Upsilon(4S)}|\mathbf{p}|^4 - \frac{8}{105}R^6_{\Upsilon(4S)}b^6_{\Upsilon(4S)}|\mathbf{p}|^6\right)\\
T_1^{\Upsilon(4S)} &= \frac{g^2}{6}(9(b_{\Upsilon(4S)}-1) - 2(3b_{\Upsilon(4S)}-\lambda_2-2))\\
&\quad + \frac{g^2|\mathbf{p}|^2R^2_{\Upsilon(4S)}}{3}[(-5b_{\Upsilon(4S)}+3)(3b_{\Upsilon(4S)}+\lambda_2-4)(b_{\Upsilon(4S)}-\lambda_2)\\
&\quad - 9b^2_{\Upsilon(4S)}(b_{\Upsilon(4S)}-1) + 2b_{\Upsilon(4S)}(3b_{\Upsilon(4S)}-\lambda_2-2)(3b_{\Upsilon(4S)}-2)]\\
&\quad + \frac{4g^2|\mathbf{p}|^4R^4_{\Upsilon(4S)}b^2_{\Upsilon(4S)}}{15}\left[(7b_{\Upsilon(4S)}-5)(3b_{\Upsilon(4S)}+\lambda_2-4)(b_{\Upsilon(4S)}-\lambda_2)\right.\\
&\quad \left. + \frac{9}{2}(b_{\Upsilon(4S)}-1)b^2_{\Upsilon(4S)} - b_{\Upsilon(4S)}(5b_{\Upsilon(4S)}-4)(3b_{\Upsilon(4S)}-\lambda_2-2)\right]\\
&\quad - \frac{8g^2|\mathbf{p}|^6R^6_{\Upsilon(4S)}b^4_{\Upsilon(4S)}}{105}\left[\frac{1}{2}(9b_{\Upsilon(4S)}-7)(3b_{\Upsilon(4S)}+\lambda_2-4)(b_{\Upsilon(4S)}-\lambda_2)\right.\\
&\quad \left. + \frac{3}{2}b^2_{\Upsilon(4S)}(b_{\Upsilon(4S)}-1) - \frac{1}{3}b_{\Upsilon(4S)}(3b_{\Upsilon(4S)}-\lambda_2-2)(7b_{\Upsilon(4S)}-6)\right]
\end{aligned}$$

$$
\begin{aligned}
T_2^{\Upsilon(4S)} &= \frac{1}{3} g^2 R^2_{\Upsilon(4S)} (-9 b_{\Upsilon(4S)} - 2\lambda_2 + 5) \\
&\quad + \frac{4}{5} g^2 R^4_{\Upsilon(4S)} |\mathbf{p}|^2 \bigg[b^2_{\Upsilon(4S)} (7 b_{\Upsilon(4S)} - 5) + \frac{1}{6} (3 b_{\Upsilon(4S)} + \lambda_2 - 4)(b_{\Upsilon(4S)} - \lambda_2)(7 b_{\Upsilon(4S)} - 3) \\
&\quad - \frac{2}{15} b_{\Upsilon(4S)} (3 b_{\Upsilon(4S)} - \lambda_2 - 2)(21 b_{\Upsilon(4S)} - 10) \bigg] \\
&\quad + \frac{4}{5} g^2 R^6_{\Upsilon(4S)} |\mathbf{p}|^4 b^2_{\Upsilon(4S)} \bigg[-\frac{1}{7} b^2_{\Upsilon(4S)} (9 b_{\Upsilon(4S)} - 7) - \frac{4}{15} b_{\Upsilon(4S)} (b_{\Upsilon(4S)} - \lambda_2)(3 b_{\Upsilon(4S)} + \lambda_2 - 4) \\
&\quad - \frac{1}{3} (b_{\Upsilon(4S)} - 1)(b_{\Upsilon(4S)} - \lambda_2)(3 b_{\Upsilon(4S)} + \lambda_2 - 4) + \frac{2}{105} b_{\Upsilon(4S)} (3 b_{\Upsilon(4S)} - \lambda_2 - 2)(45 b_{\Upsilon(4S)} - 28) \bigg], \\
T_3^{\Upsilon(4S)} &= \frac{2g^2}{15} R^4_{\Upsilon(4S)} (15 b_{\Upsilon(4S)} + 2\lambda_2 - 5) \\
&\quad + \frac{4}{5} g^2 R^6_{\Upsilon(4S)} |\mathbf{p}|^2 \bigg[-\frac{4}{5} b^3_{\Upsilon(4S)} - (b_{\Upsilon(4S)} - 1) b^2_{\Upsilon(4S)} - \frac{2}{21} b_{\Upsilon(4S)} (b_{\Upsilon(4S)} - \lambda_2)(3 b_{\Upsilon(4S)} + \lambda_2 - 4) \\
&\quad - \frac{1}{21} (b_{\Upsilon(4S)} - 1)(b_{\Upsilon(4S)} - \lambda_2)(3 b_{\Upsilon(4S)} + \lambda_2 - 4)) + \frac{2}{105} b_{\Upsilon(4S)} (3 b_{\Upsilon(4S)} - \lambda_2 - 2)(27 b_{\Upsilon(4S)} - 10) \bigg], \\
T_4^{\Upsilon(4S)} &= -\frac{4 g^2 R^6_{\Upsilon(4S)}}{35 \times 9} (21 b_{\Upsilon(4S)} + 2\lambda_2 - 5).
\end{aligned} \tag{A14}
$$

In the expressions for the decay widths of the $\psi(3770)(\Upsilon(4S))$ state, decaying to $D\bar{D}(B\bar{B})$, the parameter, γ_M is introduced, which refers to the production strength of $D\bar{D}(B\bar{B})$ from decay of $\Psi(3770)(\Upsilon(4S))$ through light quark pair creation. This parameter is chosen so as to reproduce the vacuum decay widths for the decay channels $M \to F^+F^-$ and $M \to F^0\bar{F}^0$.

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