Orbital angular momentum entanglement in noninertial reference frame

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The orbital angular momentum (OAM) of photons has shown the ability to generate high-dimensional entanglement. Here, we examine a novel kind of entanglement degradation caused by the Unruh effect. By paraxially quantizing the light field, we derive the mode functions in both inertial and noninertial reference frames and demonstrate that, from the perspective of an accelerating observer, empty inertial space may seem to be full of OAM particles. They may inundate the high-dimensional entangled OAM photons and cause them to decay. Additionally, we investigate high-dimensional entanglement in the vicinity of a Schwarzschild black hole. Our results suggest that higher-dimensional entanglement will suffer from greater adverse impacts in both instances.

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I. INTRODUCTION

Quantum entanglement is one of the most nonclassical physical phenomena. It has shown tremendous potential in the fields of computation, information, and communication [1–9]. Among the well-known applications are quantum teleportation, quantum cryptography, and superdense coding. Quantum entanglement has been realized with photons, electrons, molecules, and the different degrees of freedom associated with them. Take the photon, for example; entanglement has been manifested by its position, momentum, polarization, and energy. The discovery of the orbital angular momentum (OAM) of photons has revealed additional degrees of freedom for entanglement. In contrast to polarization, which is described in a two-dimensional space, the OAM of light may take on well-defined values of $l\hbar$, where $l = 0, \pm 1, \pm 2, \ldots$, that span an infinite-dimensional Hilbert space [1,10–21]. There has been a surge of interest in researching high-dimensional OAM entanglement during recent years [1,12,15].

Regrettably, quantum entanglement has a highly complicated structure and is very sensitive to its surroundings [22,23]. In particular, entanglement decay may occur when the system interacts with its environment. In this paper, we explore another source of entanglement deterioration that is not related to the environment, but rather to the fact that particle definitions vary across different reference frames [24–31]. This was first predicted by Hawking radiation, and then the same prediction was made by the Unruh effect [32–36]. According to the Unruh effect, a noninertial observer moving through the Minkowski vacuum may observe a thermal spectrum of particles. Moreover, a recent study found that if an observer has a rotational vortex structure in the transverse dimensions and carries a well-defined OAM, it can absorb or emit Rindler particles with the same OAM when interacting with the background thermal bath [37]. These particles exhibit no hidden connections, and they obscure the original entangled particles. As a result, an accelerating observer may detect entanglement degradation. This kind of decay has been investigated for a variety of objects, including fermions and bosons. However, to the best of our knowledge, a related research for photons with OAM has not been conducted.

In this paper, we study the quantization of the paraxial light field in Minkowski and Rindler spacetimes by using the Laguerre-Gaussian (LG) modes, which in cylindrical coordinates are given by

$$\mathrm{LG}_{l,p}(\mathbf{x};k) = \frac{\mathcal{N}}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|}\mathrm{L}_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right) \times \exp\left(\frac{-r^2}{w^2(z)} + il\phi + \frac{ikr^2}{2R(z)} + i\Phi(z)\right), \quad (1)$$

where $\mathcal{N} = \sqrt{\frac{2p!}{\pi(|l|+p)!}}$ is the normalization constant, $r = \sqrt{x^2+y^2}$, $\phi = \arctan(y/x)$, $w(z) = w_0\sqrt{1+(z/z_R)^2}$ with w_0 being the waist radius and $z_R = kw_0^2/2$ being the Rayleigh range, $R(z) = z[1+(z_R/z)^2]$ is the curvature radius of the wave fronts, $\Phi(z) = -(|l|+2p+1)\arctan(z/z_R)$ is the Gouy phase, k is the wave number, and $\mathrm{L}_p^{|l|}(x)$ are the associated Laguerre polynomials. The radial index $p = 0, 1, 2, \ldots$ indicates the number of radial nodes of the mode, while the azimuthal index $l = 0, \pm 1, \pm 2, \ldots$ corresponds to the topological charge. It can be shown that the azimuthal index l represents the amount of orbital angular momentum carried by each photon in that mode. We demonstrate that a vacuum space

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for an inertial observer may be saturated with uncorrelated Rindler OAM photons for an accelerating observer. In light of this finding, we study the deterioration of high-dimensional OAM entanglements in Rindler spacetime. Finally, we extend our findings by demonstrating that entanglements shared by a free-falling observer and a stationary observer are impacted by a black hole.

Unless otherwise specified, geometrized units with $c = G = 1$ are used. We use the metric signature $(-,+,+,+)$. All greek indices run in $\{0,1,2,3\}$. The following approximations are made for simplicity. First, the paraxial approximation is applied so the paraxial quantization is valid; second, the degrees of freedom associated with polarization will be ignored, since these degrees of freedom are unrelated to those associated with OAM in our study.

This paper is structured as follows. In Sec. II a paraxial quantization procedure in Minkowski spacetime is reviewed. Section III extends this procedure to Rindler spacetime and finds the corresponding mode functions and annihilation operators. In Sec. IV, by constructing another set of mode functions in Minkowski spacetime, the Bogoliubov transformation between these two spacetimes is derived. In Sec. V, high-dimensional OAM entanglements in Rindler spacetime are explored to show that the system loses its coherence and entanglement when the acceleration is increasing. Moreover, higher-dimensional system will be affected more severely. Section VI generalizes this result to the Schwarzschild metric. Last, in Sec. VII, our results are summarized and discussed.

II. QUANTIZATION OF PHOTONS WITH OAM IN MINKOWSKI SPACETIME

Here, we review the paraxial quantization procedure introduced in Ref. [11]. For now, the polarization degrees of freedom will be omitted, so the light field $\phi(x^\mu)$ satisfies the massless Klein-Gordon equation, $\Box\phi(x^\mu) = 0$. By solving this equation, one may find that its normalized mode function with four-wave-vector $k^\mu = (\omega_k, \mathbf{k})$, where ω_k is the frequency, $\mathbf{k}$ is the wave vector, and $\omega_k = |\mathbf{k}|$, is given by

$$u_{\mathbf{k}}(x^\mu) = (16\pi^3\omega_k)^{-1/2} e^{ik_\mu x^\mu}, \tag{2}$$

and the general solution can be written as

$$\phi(x^\mu) = \int_{\mathbf{k}} [\alpha(\mathbf{k}) u_{\mathbf{k}}(x^\mu) + \text{c.c.}]. \tag{3}$$

We introduce a positive function

$$f(\mathbf{k}) = \frac{k_z + \sqrt{k_z^2 + 2q^2}}{2}, \tag{4}$$

where $\mathbf{q} = (k_x, k_y)$, and $q = |\mathbf{q}|$. Then in the paraxial approximation, by utilizing $\int_0^\infty dk\delta[k - f(\mathbf{k})]e^{ik(z-t)}/e^{ik(z-t)} = 1$, one can find

$$\phi(x^\mu) = \int_0^\infty dk \int d^2\mathbf{q} k^{-1/2}[\alpha(\mathbf{q}, k) e^{ik(z-t)} e^{i(\mathbf{q}\cdot\mathbf{r}_\perp - k\vartheta^2 z)} + \text{c.c.}], \tag{5}$$

where $\mathbf{r}_\perp = (x, y)$, and $\vartheta = q/\sqrt{2k^2}$ is the paraxiality parameter. Since the LG modes satisfy the relation $e^{i(\mathbf{q}\cdot\mathbf{r}_\perp - k\vartheta^2 z)} = \sum_{l,p} \mathcal{LG}^*_{l,p}(\mathbf{q}) \mathrm{LG}_{l,p}(\mathbf{x}; k)$, where $\mathcal{LG}_{l,p}(\mathbf{q})$ is the Fourier-transformed LG modes, the field can be expressed as

$$\phi(x^\mu) = \sum_{l,p} \int_0^\infty dk[\alpha_{l,p}(k) f_{k,l,p}(x^\mu) + \text{c.c.}], \tag{6}$$

where $f_{k,l,p}(x^\mu) = (4\pi k)^{-1/2} e^{ik(z-t)} \mathrm{LG}_{l,p}(\mathbf{x}; k)$ are the paraxial mode functions and $\alpha_{l,p}(k) = \int d^2\mathbf{q} \mathcal{LG}^*_{l,p}(\mathbf{q}) \alpha(\mathbf{q}, k)$. By using the inner product defined by [24] $(\phi_1, \phi_2) = -i \int_\Sigma (\phi_1 \nabla_\mu \phi_2^* - \phi_2^* \nabla_\mu \phi_1) n^\mu \sqrt{\gamma} d^{n-1}x$, where Σ is a spacelike hypersurface with induced metric γ_{ij} and unit normal vector n_μ, one can verify that the paraxial mode functions $f_{k,l,p}(x^\mu)$ are orthonormal, i.e.,

$$(f_{k,l,p}(x^\mu), f_{k',l',p'}(x^\mu)) = \delta(k - k')\delta_{ll'}\delta_{pp'}. \tag{7}$$

By using the conventional field quantization formalism, one may identify $\alpha_{l,p}(k)$ as the LG mode annihilation operator $a_{k,l,p}$, so the quantized version of the OAM field is

$$\phi(x^\mu) = \sum_{l,p} \int_0^\infty dk[a_{k,l,p} f_{k,l,p}(x^\mu) + a^\dagger_{k,l,p} f^*_{k,l,p}(x^\mu)]. \tag{8}$$

In particular, the paraxial OAM operator is given by $L_z = \sum_{l,p} \hbar l \int_0^\infty dk a^\dagger_{k,l,p} a_{k,l,p}$. As will be shown later, the azimuthal number l plays an important role, so from now on, we will use only one index l to represent a pair of OAM numbers (l, p), and let $-l$ denote $(-l, p)$, i.e., the radial index remains unaltered.

III. QUANTIZATION OF PHOTONS WITH OAM IN RINDLER SPACETIME

In the context of special relativity, an observer, called the Rindler observer, moving at a uniform acceleration of magnitude α in the z direction has the trajectory

$$t(\tau) = \alpha^{-1} \sinh(\alpha\tau), \qquad z(\tau) = \alpha^{-1} \cosh(\alpha\tau), \tag{9}$$

where τ is the proper time for the observer. We introduce new coordinates (η, ξ), adapted by [24]

$$t = a^{-1}e^{a\xi}\sinh(a\eta), \quad z = a^{-1}e^{a\xi}\cosh(a\eta) \quad (z > |t|), \tag{10}$$

with ranges $-\infty < \eta, \xi < +\infty$ and where a is a parameter. These coordinates expand region I in Rindler space. Similarly, coordinates in region IV can be defined by

$$t = -a^{-1}e^{a\xi}\sinh(a\eta), \quad z = -a^{-1}e^{a\xi}\cosh(a\eta) \quad (z < -|t|). \tag{11}$$

Both regions have the same metric, $ds^2 = e^{2a\xi}(-d\eta^2 + d\xi^2) + dx^2 + dy^2$. The spacetime diagram is shown in Fig. 1. The hyperbolas are trajectories for uniformly accelerating particles. For an observer with $a = \alpha$, its trajectory is $\eta = \tau$, $\xi = 0$.

Since the Rindler metric does not depend on coordinates x and y, we can decouple these coordinates and reserve them for the transverse components. The Klein-Gordon equation now reads

$$\Box\psi(x'^{\mu}) = e^{-2a\xi}(-\partial_\eta^2 + \partial_\xi^2)\psi(x'^{\mu}) = 0, \tag{12}$$

where $x'^{\mu} = (\eta, x, y, \xi)$. By switching coordinates $t \to \eta$, $z \to \xi$, we can easily derive the normalized mode functions in Rindler coordinates,

$$g_{k,l}^{(\mathrm{I})}(x'^{\mu}) = (4\pi k)^{-1/2}e^{ik(\xi-\eta)}\mathrm{LG}_l(x, y), \tag{13}$$

$$g_{k,l}^{(\mathrm{IV})}(x'^{\mu}) = (4\pi k)^{-1/2}e^{ik(\xi+\eta)}\mathrm{LG}_l(x, y), \tag{14}$$

where $k > 0$, $g_{k,l}^{(\mathrm{I})}(x'^{\mu})$ and $g_{k,l}^{(\mathrm{IV})}(x'^{\mu})$ are defined in Rindler region I and IV, respectively, and we have dropped the dependence of k and ξ in LG modes because for a given frequency k and coordinate ξ, their orthonormal properties do not rely on them. One can verify that $g_{k,l}^{(\mathrm{I})}(x'^{\mu})$ and $g_{k,l}^{(\mathrm{IV})}(x'^{\mu})$ satisfy the orthonormal property,

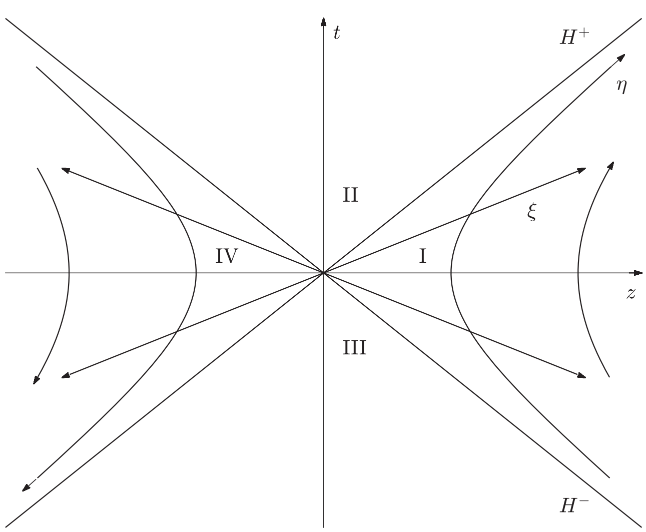

FIG. 1. Spacetime diagram for Minkowski spacetime in Rindler coordinates.

$$(g_{k,l}^{(\mathrm{S})}, g_{k',l'}^{(\mathrm{S}')}) = \delta(k - k')\delta_{ll'}\delta_{pp'}\delta_{\mathrm{SS}'}, \tag{15}$$

where S = I or IV.

Their corresponding annihilation operators are labeled by $b_{k,l}^{(\mathrm{I})}$ and $b_{k,l}^{(\mathrm{IV})}$, respectively. Hence, the quantized field can be described by

$$\psi(x'^{\mu}) = \sum_l \int_0^\infty dk[b_{k,l}^{(\mathrm{I})}g_{k,l}^{(\mathrm{I})}(x'^{\mu}) + b_{k,l}^{(\mathrm{IV})}g_{k,l}^{(\mathrm{IV})}(x'^{\mu}) + \mathrm{H.c.}]. \tag{16}$$

The operator $b_{k,l}^{(\mathrm{I})\dagger}$ will create an OAM photon moving in Rindler region I, while the photon created by $b_{k,l}^{(\mathrm{IV})\dagger}$ moves in region IV. Although these two particles are correlated, they are in different Rindler regions, and hence they may never access the information carried by their counterpart.

IV. THE BOGOLIUBOV TRANSFORMATION

We want to find the following Bogoliubov transformation, which can be used to describe the modes in Minkowski spacetime by those in Rindler spacetime,

$$\begin{aligned} f_{k,l}(x^{\mu}) = \sum_j \int_0^\infty dk'[&\alpha_{k,k',l,j}^{(\mathrm{I})}g_{k',j}^{(\mathrm{I})}(x'^{\mu}) + \alpha_{k,k',l,j}^{(\mathrm{IV})}g_{k',j}^{(\mathrm{IV})}(x'^{\mu}) \\ &+\beta_{k,k',l,j}^{(\mathrm{I})*}g_{k',j}^{(\mathrm{I})*}(x'^{\mu}) + \beta_{k,k',l,j}^{(\mathrm{IV})*}g_{k',j}^{(\mathrm{IV})*}(x'^{\mu})]. \end{aligned} \tag{17}$$

This Bogoliubov transformation is difficult to find. Instead, we adopt the conventional procedure [24] where one may use the fact that

$$e^{a(\xi\pm\eta)} = \begin{cases} a(\pm t + z) & \mathrm{I}, \\ -a(\pm t + z) & \mathrm{IV}, \end{cases} \tag{18}$$

along with the LG mode orthonormal properties $\int d^2\mathbf{x}_\perp \mathrm{LG}_m(\mathbf{x}; k)\mathrm{LG}_n^*(\mathbf{x}; k) = \delta_{m,n}$ and $\int d^2\mathbf{x}_\perp \mathrm{LG}_m^*(\mathbf{x}; k)$ $\mathrm{LG}_n^*(\mathbf{x}; k) = \delta_{m,-n}$, where $(-n)$ means that only the sign of the azimuthal number is flipped. Two new sets of normalized modes can be constructed as

$$\begin{aligned} h_{k,l}^{(\mathrm{I})}(x'^{\mu}) &= \alpha[\beta g_{k,l}^{(\mathrm{I})}(x'^{\mu}) + \beta^{-1}g_{-k,-l}^{(\mathrm{IV})*}(x'^{\mu})] \\ &\propto (z - t)^{ik/a}\mathrm{LG}_l(x, y), \end{aligned} \tag{19}$$

$$\begin{aligned} h_{k,l}^{(\mathrm{IV})}(x'^{\mu}) &= \alpha[\beta g_{k,l}^{(\mathrm{IV})}(x'^{\mu}) + \beta^{-1}g_{-k,-l}^{(\mathrm{I})*}(x'^{\mu})] \\ &\propto (-z - t)^{ik/a}\mathrm{LG}_l(x, y), \end{aligned} \tag{20}$$

where $\alpha = 1/\sqrt{2\sinh(\pi k/a)}$ and $\beta = e^{\pi k/2a}$. Their corresponding annihilation operators are labeled by $c_{k,l}^{(\mathrm{I})}$ and $c_{k,l}^{(\mathrm{IV})}$, respectively. Similarly, their corresponding creation operators describe particles moving in Rindler region I and IV, respectively. Note that these modes are not monochromatic wave packages. Instead, they are a polychromatic combination of them. However, they are sufficient to study the Bogoliubov transformation since they have well-defined OAM indices l, which we are most interested in. The Bogoliubov transformation tells us that the Rindler operators and Minkowski operators are related by

$$b_{k,l}^{(\mathrm{I})} = \alpha[\beta c_{k,l}^{(\mathrm{I})} + \beta^{-1} c_{-k,-l}^{(\mathrm{IV})\dagger}], \tag{21}$$

$$b_{k,l}^{(\mathrm{IV})} = \alpha[\beta c_{k,l}^{(\mathrm{IV})} + \beta^{-1} c_{-k,-l}^{(\mathrm{I})\dagger}]. \tag{22}$$

We introduce a new Dirac notation $|n,k,l\rangle_{\mathrm{R}}$ in the occupation number representation, which means that there are n photons with frequency k, and an OAM index l in region R. R = I, IV, or M stands for Rindler region I, IV, or the Minkowski spacetime, respectively. Particles defined in Rindler spacetime will be named Rindler particles. By using Eqs. (21) and (22), the Minkowski vacuum state can be shown to be

$$\begin{aligned}|0\rangle_{\mathrm{M}} &= \prod_{i,j}(1-\beta_i^{-4})^{1/2}\sum_n \beta_i^{-2n}|n,k_i,l_j\rangle_{\mathrm{I}}|n,-k_i,-l_j\rangle_{\mathrm{IV}} \\ &= \prod_{i,j} F_{ij},\end{aligned} \tag{23}$$

where $\prod$ represents direct products. We see that one photon with frequency k_i and OAM index l_j in one Rindler region will be accompanied by one with the same frequency and inverse OAM index in the other Rindler region, so in total the orbital angular momentum is conserved. Similarly, a one-photon state that propagates in Minkowski spacetime can be written as

$$\begin{aligned}|1,k_i,l_j\rangle_{\mathrm{M}} &= \alpha_i\beta_i(1-\beta_i^{-4})^{1/2}\sum_n \beta_i^{-2n}\sqrt{n+1}|n+1,k_i,l_j\rangle_{\mathrm{I}} \\ &\quad \times |n,-k_i,-l_j\rangle_{\mathrm{IV}} \prod_{i'\neq i, j'\neq j} F_{i'j'}.\end{aligned} \tag{24}$$

From these transformations, we can see that Rindler particles are linked via two distinct Rindler regions. Due to the spacelike nature of the two regions, an observer in one region may never acquire information from the other region, resulting in the loss of connections between those Rindler particles. As a result, particles in one region are anticipated to be uncorrelated. For a Minkowski vacuum state, the expected number of particles of frequency k and OAM index l observed by a Rindler observer in region I is $\langle 0|_M b_{k,l}^{(\mathrm{I})\dagger} b_{k,l}^{(\mathrm{I})}|0\rangle_M = (e^{2\pi k/a}-1)^{-1}\delta(0)$. Therefore, these particles exhibit a Planck spectrum with temperature $T = a/2\pi$.

V. HIGH-DIMENSIONAL OAM ENTANGLEMENTS IN RINDLER SPACETIME

We now consider a high-dimensional entangled state. Initially, the first photon is carried by the observer Alice, while the other one is stored by Bob and the system is written as

$$|\psi\rangle = D^{-1/2}\sum_{l=-M}^{M}|1,k,-l\rangle_{\mathrm{A}}|1,k,l\rangle_{\mathrm{B}}, \tag{25}$$

where $D = 2M+1$ is the dimension of the system. Let Bob send their photon to a Rindler observer Rob in region I. For brevity, we introduce another Dirac notation $|n_{-M}, n_{-M+1}, \ldots, n_{M-1}, n_M\rangle_{\mathrm{R}}$ meaning that there are n_l photons in the OAM state l. Note that we have omitted the frequency dependence in it because under the single-mode approximation, states in region I all share the frequency k, while those in region IV have frequency $-k$. Then, the state can be written as

$$\begin{aligned}|\psi\rangle &= \frac{\alpha\beta(1-\beta^{-4})^{D/2}}{\sqrt{D}}\sum_{l=-M}^{M}\sum_{\mathbf{n}}\beta^{-2N}\sqrt{n_l+1}|-l\rangle_{\mathrm{A}} \\ &\quad \times |n_{-M},\ldots,n_l+1,\ldots,n_M\rangle_{\mathrm{R}}|n_{-M},\ldots,n_M\rangle_{\bar{\mathrm{R}}},\end{aligned} \tag{26}$$

where $\bar{\mathrm{R}}$ is an imaginary Rindler observer in region IV, $\mathbf{n} = (n_{-M}, n_{-M+1}, \ldots, n_{M-1}, n_M)$ whose elements can take integers from zero to infinity, and $N = \sum n_i$. The density operator will be $\rho = |\psi\rangle\langle\psi|$. Since observer R can never access information received by observer $\bar{\mathrm{R}}$, we can partially trace the $\bar{\mathrm{R}}$ part out. Calculations show that the normalized partial density operator for Alice and Rob is

$$\begin{aligned}\rho^{\mathrm{AR}} &= \frac{(1-\beta^{-4})^{D+1}}{D}\sum_{l,j=-M}^{M}\sum_{\mathbf{n}}\beta^{-4N}\sqrt{(n_l+1)(n_j+1)} \\ &\quad \times |-l\rangle_{\mathrm{A}}|n_{-M},\ldots,n_l+1,\ldots,n_M\rangle_{\mathrm{R}} \\ &\quad \times \langle -j|_{\mathrm{A}}\langle n_{-M},\ldots,n_j+1,\ldots,n_M|_{\mathrm{R}}.\end{aligned} \tag{27}$$

To test its coherence, we calculate its purity [38–40] as

$$\mathcal{P} = \mathrm{tr}[(\rho^{\mathrm{AR}})^2] = \frac{(1-\beta^{-4})^{2(D+1)}(\beta^{-8}+D)}{D(1-\beta^{-8})^{D+2}}. \tag{28}$$

We use the negativity to quantify its entanglement, defined by [5,41,42] $\mathcal{N} = -\sum_{\sigma_i<0}\sigma_i$, where σ_i are eigenvalues of the partial transpose density. The partial transpose density, with respect to the second photon, is

$$\rho_{\mathrm{PT}}^{\mathrm{AR}} = \frac{(1-\beta^{-4})^{D+1}}{D} \sum_{l,j=-M}^{M} \sum_{\mathbf{n}} \beta^{-4N} \sqrt{(n_l+1)(n_j+1)} \times |-l\rangle_{\mathrm{A}} |n_{-M}, \ldots, n_j+1, \ldots, n_M\rangle_{\mathrm{R}} \times \langle -j|_{\mathrm{A}} \langle n_{-M}, \ldots, n_l+1, \ldots, n_M|_{\mathrm{R}}. \tag{29}$$

It is difficult to find its eigenvalues. Instead, we notice that the factor β^{-4N} will suppress states with $N > 0$, and as stated before, all Rindler particles are uncorrelated. Therefore, we will approximately calculate its eigenvalues by setting $\mathbf{n} = 0$. Then, the matrix $\rho_{\mathrm{PT}}^{\prime\mathrm{AR}} = D^{-1}(1-\beta^{-4})^{D+1} \sum_{l,j=-M}^{M} |-l\rangle_{\mathrm{A}} |j\rangle_{\mathrm{R}} \langle -j|_{\mathrm{A}} \langle l|_{\mathrm{R}}$ has eigenvalues $(1-\beta^{-4})^{D+1}/D$ with degeneracy $D(D+1)/2$, and $-(1-\beta^{-4})^{D+1}/D$ with degeneracy $D(D-1)/2$. Therefore, its negativity is approximated by

$$\mathcal{N} = \frac{(D-1)(1-\beta^{-4})^{D+1}}{2}. \tag{30}$$

We define the decay acceleration a_{D} to be the acceleration of the observer at which the negativity of the system drops to $1/e$ of the initial value, i.e., $\mathcal{N}(a_{\mathrm{D}}) = \frac{1}{e}\mathcal{N}(a=0)$. For the system under consideration, its decay acceleration is

$$a_{\mathrm{D}} = -\frac{2\pi k}{\ln\left[1-e^{-1/(D+1)}\right]}. \tag{31}$$

Figures 2 and 3 depict the purities and negativities for different dimensional systems, respectively. Also, the decay accelerations, in units of kc^2, where the speed of light c is temporarily restored, are 4.165, 2.934, 2.487, 2.241, 2.080 for $D = 3, 7, 11, 15, 19$, respectively, showing that higher-dimensional entanglement will suffer more deterioration caused by the Rindler particles. This is because the structure of a higher-dimensional entanglement is more complicated and is covered up by more Rindler OAM photons. Note that this kind of entanglement deterioration occurs as a result of the disparate definitions of particles in different kinds of reference frames. As stated by the Unruh effects, an observer traveling in the Minkowski vacuum with uniform acceleration sees a thermal spectrum of particles. Here, two entangled OAM photons are flooded by a large number of Rindler OAM photons from the view of an accelerating observer. Because the observer cannot tell the difference between such particles and the original entangled ones, detection will be impaired.

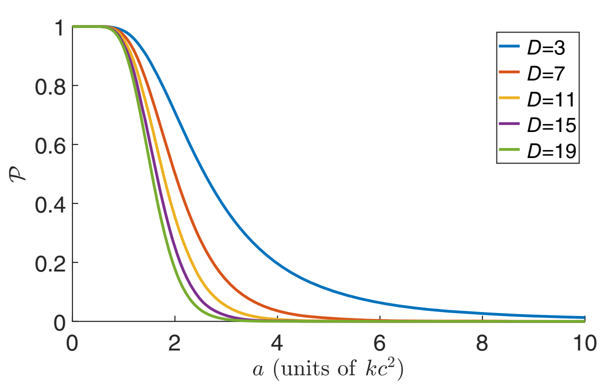

FIG. 2. Purities for different dimensional entanglements.

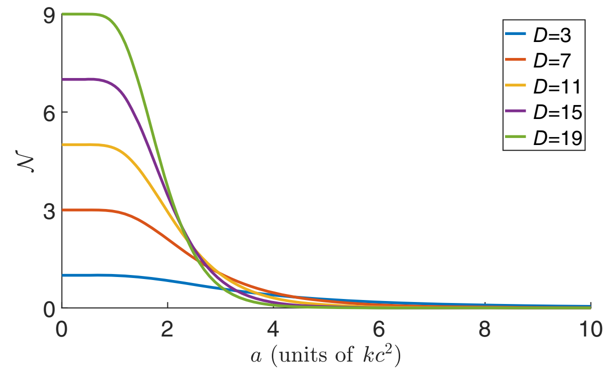

FIG. 3. Negativities for different dimensional entanglements.

VI. HIGH-DIMENSIONAL OAM ENTANGLEMENT DEGRADATION NEAR A SCHWARZSCHILD BLACK HOLE

Last, we would like to briefly generalize our result to spacetime near a Schwarzschild black hole. In the Kruskal coordinates, i.e.,

$$T = [r/(2M)-1]^{1/2} e^{r/(4M)} \sinh(t/(4M)), \tag{32}$$

$$R = [r/(2M)-1]^{1/2} e^{r/(4M)} \cosh(t/(4M)), \tag{33}$$

the spacetime diagram for the Schwarzschild metric is displayed in Fig. 4. Note that the hyperbolas are trajectories

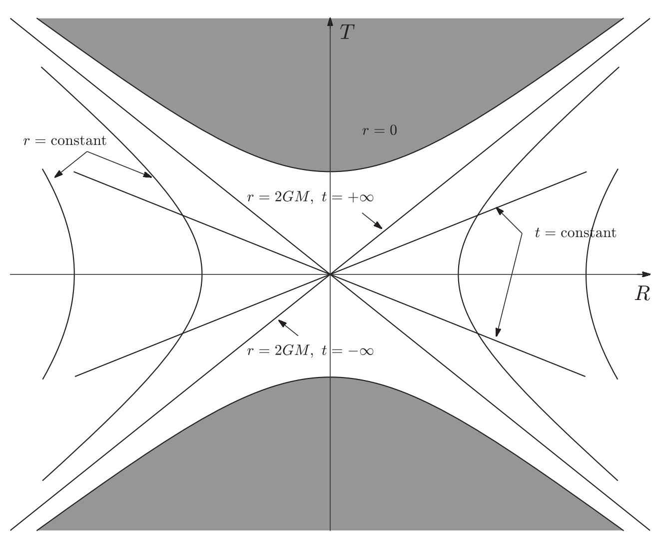

FIG. 4. Spacetime diagram for the Schwarzschild metric in Kruskal coordinates.

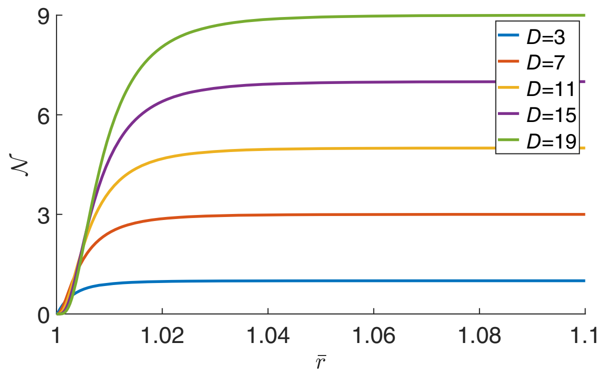

FIG. 5. Negativities for different radii of a stationary observer near the horizon. When the observer is far away from the horizon, the degradation will fade quickly.

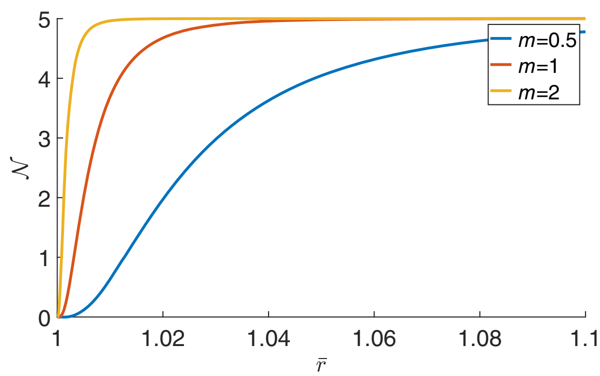

FIG. 6. Negativities for different masses of black holes. The entanglement dimension is set to 11. $m = 1$ is one solar mass. Less mass black hole will emit stronger Hawking radiation, resulting higher decay in entanglement.

for orbits with fixed radii. The similarities between Rindler and Schwarzschild spacetime show that the effective acceleration for a stationary observer with a fixed radius r near a Schwarzschild black hole is [26] $a = \kappa f^{-1/2} = [4m\sqrt{1-1/\bar{r}}]^{-1}$, where $f = 1–2m/r$, $\kappa = 1/(4m)$ is the surface gravity, $\bar{r} = r/R_s$, and $R_s = 2m$ is the Schwarzschild radius. This result is valid only when the Rindler approximation, $(r - R_s)/R_s \ll 1$, holds. Now let Rob be the stationary observer with radius r, and Alice be freely falling into the black hole. They share a high-dimensional bipartite system whose entanglement will be computed via the negativity.

The results for various observer radii and black hole masses are shown in Figs. 5 and 6. The mass of the black hole in Fig. 5 is set to one solar mass $m_\odot = 2.95 \times 10^3$ m. In Fig. 6, The entanglement dimension is set to 11 with $M = 5$ and the black hole masses are set to 0.5, 1, and 2 solar masses. First, we see that deterioration occurs only in a limited region around the black hole horizon. Thus, in the future, if researchers want to transmit OAM data near a black hole, they may wish to maintain a reasonable distance from the horizon. Second, Fig. 6 demonstrates that the deterioration diminishes rapidly as the mass of the black hole increases, which is compatible with Hawking radiation.

VII. CONCLUSION

In this paper, we studied the paraxial quantization of light in Minkowski and Rindler spacetimes. We determined the Bogoliubov transformation between them by building the appropriate mode functions in Minkowski spacetime. Then, we saw that an empty region for an inertial observer may become saturated with uncorrelated Rindler OAM particles as the observer accelerates. We investigated high-dimensional OAM entanglements in Rindler spacetime using this finding. We saw that the Rindler OAM particles have adverse effects on the entanglements, resulting in decoherence and entanglement deterioration. Additionally, a higher-dimensional entanglement will be more severely impacted. Finally, we extended our results to the Schwarzschild spacetime near the horizon, which bears some resemblance to Rindler spacetime. The findings demonstrate that as a stationary observer approaches the black hole horizon, the OAM entanglements shared by a free-falling observer are broken. Additionally, as anticipated by Hawking radiation, a black hole with less mass would emit more OAM particles, resulting in a greater deterioration of OAM entanglement.

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