Catastrophic dark matter particle capture

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In this paper we describe a new idea which may be relevant to the formation of galaxies via the infall of baryonic matter (BM) and dark matter (DM) onto a preexisting overdensity. While BM can under certain circumstances be captured by thermal processes, DM particles fly through a static overdensity without being captured. We propose a simple model for DM capture: if during the passage through it, the mass of the overdensity increases, then slow DM particles are captured by it, further increasing its mass, while faster particles slow down, transferring part of their energy to the galaxy. We estimate the minimum initial velocity of a particle required for a passage without capture through the center of the galaxy and derive a nonlinear equation describing the rate of galaxy mass increase. An analysis carried out using the ideas of catastrophes theory shows that if the increase in the mass of baryonic matter exceeds a certain threshold value, this can lead to a very intensive capture of dark matter. We speculate that this process may be associated on the one hand with the accretion of matter during the early stages of galaxy formation and, on the other hand, also later with the merger of galaxies. For the studied process to take place, the density of intergalactic DM must exceed some threshold value. Then the rate of increase in the mass of DM can be much higher than the one of baryonic matter. The capture sharply decreases after the DM density drops below the threshold value, e.g., due to the expansion of the Universe.

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I. INTRODUCTION

It is well established that every galaxy is surrounded by a dark halo. This halo consists of dark matter (DM) and its size is much larger than the visible size of the galaxy. The mass of the DM halo is the dominant contribution to the total mass of the galaxy. We assume that DM consists of some still unknown particles that are not affected by strong and electromagnetic interactions. They interact between themselves and with baryonic matter (BM) only gravitationally. In the present paper we neglect a possible very weak nongravitational interaction of dark matter with baryonic matter or with itself.

The formation of galaxies or more generally the large scale structure of the Universe (LSS) is a very important problem in cosmology. The basic picture is that galaxies have grown out of small initial fluctuations from inflation by gravitational instability. This idea is in good agreement with the small fluctuations observed in the temperature of the cosmic microwave background [1,2]. As long as perturbations are small, they can be studied with linear or higher order perturbation theory. But for the formation of galaxies, such an approach is not suitable for two reasons. First of all, the over densities of galaxies, $\rho_{\rm gal}/\rho_m \gtrsim 10^5$, are much larger than 1 so that perturbation theory cannot be expected to converge. Furthermore, we assume DM particles to be collisionless. In this case they should be described by Vlasov's equation in phase space. While this leads to a similar Jeans scale as the fluid approach, just replacing the sound speed by the velocity dispersion [3,4], nonlinear aspects are very different as soon a shell crossing becomes relevant which leads to singularities in the fluid approach.

For this reason, the nonlinear regime of cosmological structure formation is usually treated either via N-body simulations, which have, at present, achieved an impressive amount of detail [5–8] or by simple analytical models, e.g., spherical collapse or secondary infall models [9–13].

Even though these works are very important and have provided a rather clear picture of the formation of cosmological LSS, they have their intrinsic limitations: the best N-body simulations have a resolution of $10^{-11}M_\odot$, (see [13]) to $10^{12}M_\odot$, depending on the size of the region they want to simulate and on computational resources. Even $10^{-11}M_\odot$ is much larger than the mass of dark matter particles which have typically masses of elementary particles, $m \sim 100$ Gev$\simeq 10^{-45}M_\odot$. This is not quite true in the case of primordial

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black holes which have a window of possible masses between $10^{-16} M_\odot < m < 10^{-11} M_\odot$, but can also be heavier, see [14] for a review. In Ref. [13], the authors have shown that the halo density profiles are universal over the entire mass range and are well described by simple two-parameter fitting formulas. Nevertheless, at small scales they zoom in on regions with moderate over densities of at most 17 which is much less than the density of a typical galaxy. Furthermore, these simulations do not include baryonic physics which is crucial for the effect discussed in the present work.

Even though the above results are very important and promising, it is still not entirely clear that mass resolution is irrelevant. Naively, the process needed for the collapse of collisionless particles, dynamical friction, has a cross section which is proportional to m^2. The energy loss of a particle with mass m moving with the velocity v through the media with matter density ρ is given by [15]

$$\frac{dE}{dt} = -\frac{4\pi G^2 m^2 \rho}{v} \ln\left(\frac{b_{\max}}{b_{\min}}\right), \tag{1}$$

where G is the gravitational constant and b is the impact parameter of the collision. Taking into account that $E = (mv^2)/2$, we find that $dv/dt \propto m$. Might it be that due to the much smaller effect from dynamical friction, elementary particles might behave significantly different from "chunks of phase space" with a mass of $10^6 M_\odot$?

What concerns the analytical models, the problem is that assuming spherical symmetry one can "circumvent" Liouville's theorem which states that phase space volume is conserved under Hamiltonian evolution. In a spherically symmetric situation the phase space volume vanishes already in the initial condition since velocities have only 1 nonzero component. More physically: since all particles move radially, the total angular momentum vanishes and its conservation does not constrain the infall.

In this paper we consider secondary infall into a spherically symmetric overdensity. This problem has been addressed before, e.g., in Refs. [11,12], but only for radially infalling particles. Here, even though we assume a spherically symmetric gravitational potential, particles may fall in with arbitrary, nonzero impact parameter. Note, also that we assume DM and BM to interact only gravitationally. There are of course also models where DM and baryons interact with nongravitational forces, see [16] for arguments to favor this possibility, but we neglect such interactions in the present study.

If the gravitational potential remains constant, the particles will gain velocity during infall and will lose it again when climbing out of the potential, but they will not be captured. However, if the potential is growing during the infall, some particles with sufficiently low initial velocities can get captured.

In this paper we show an interesting new phenomenon: if the growth rate of the gravitational potential is sufficient, a catastrophe (in the mathematical sense [17]) happens, enhancing very significantly the capture rate. This may lead to the formation of heavily DM dominated objects like some dwarf galaxies or low surface brightness galaxies. The astrophysical significance of this new effect for galaxy formation still requires a more detailed study. In this paper we just derive and explain the effect of catastrophic DM capture and we illustrate it with some numerical examples.

Even though this effect may in principle be inside N-body simulations, we present here an entirely new and semi-analytic understanding of very enhanced dark matter capture as it can occur if there is significant baryonic accretion. Initially, N-body simulations of cold dark matter do not include velocity dispersion. However, once shell crossing has occurred, significant velocity dispersion is generated and our process can take place. For warm and hot dark matter, where velocity dispersion is relevant from the beginning, our process can occur as soon a baryons are accreted at a significant rate. For fuzzy dark matter however, which can be considered as a Bose-Einstein condensate, see, e.g., [18], our description of DM accretion is not adequate.

Independent numerical studies will be necessary to investigate the mechanism outlined here in more detail, in order to decide about its relevance for cosmological structure formation. Such simulations will need to capture the quite complicated baryonic physics to determine the accretion rate of baryonic matter. Here we study the total mass accretion as a function of the baryonic matter accretion which we treat as unknown external parameter.

The reminder of the paper is structured as follows: In the next section we discuss the basics of DM capture in a gravitational potential. In Sec. III, the main section of this article, we show that under certain conditions a fold catastrophe can build up leading to a jump in the dark matter capture rate. In Sec. IV we give some quantitative estimates which show that this may happen during galaxy formation and in Sec. V we summarize our findings and conclude.

II. CAPTURE OF DM PARTICLES

A. The capture velocity of DC particles

The velocities of DM particles increase as they enter from intergalactic space into the halo of a galaxy and decrease as they leave it. If during the flight the mass of the galaxy is increasing, then slow DM particles are captured by the galaxy further increasing its mass, while faster particles slow down, transferring some of their kinetic energy to the galaxy, reducing its gravitational binding energy. Let us consider this mechanism using a simple model that allows us to draw a number of qualitative conclusions.

We consider a spherically symmetric matter overdensity which we call "a galaxy." It may represent a dark halo with a visible galaxy inside. The first consists mainly of DM while the second consists mainly of BM. We assume that the total matter density ρ depends only on the radius, $\rho(r)$. This greatly simplifies the model, but contradicts the results of N-body simulations described, e.g., in Sec. 9.3.3 in [4]. Spherical symmetry does certainly not apply to BM in spiral galaxies. However, it is reasonable to assume that this simplification does not qualitatively change the behavior we now discuss. Within this approximation, a particle moving radially cannot be deflected in any direction. It is clear that the halo does not have a sharp boundary. This does not prevent us from using a reasonable estimate for its radius R. We neglect the DM density $\rho(r)$ at $r > R$. This radius which we assume to be constant is not a value like r_{200}, which changes as galactic halos form (see Chapter 9 of [4]). We choose R slightly larger than the maximum size of the halo at all stages of the evolution of the galaxy, at which it can be called a galaxy. Taking into account the ambiguous definition of this quantity, we will use various possible values of the halo radius R in numerical estimates, from underestimated to overestimated.

We assume DM particles to be nonrelativistic, so that we can use Newtonian mechanics. Naturally, nothing prevents us from taking into account the effects of special relativity, but this does not change the qualitative results of our model. We include Hubble expansion to account for the change in the density of dark matter particles in intergalactic space.

Let us consider a DM particle that approaches the overdensity with an initial speed v_0 at $r \gg R$. Then at the boundary of the halo the particle velocity is equal to $v(R) = \sqrt{v_0^2 + u^2}$ with $u^2 = 2GM/R$ and inside halo at a distance r from the center it is equal to

$$v(r) = \left(v_0^2 + u^2 + 8\pi G \int_r^R \frac{dx}{x^2} \int_0^x y^2 \rho(y) dy\right)^{1/2}. \qquad (2)$$

The total mass of the overdensity is

$$M = 4\pi \int_0^R y^2 \rho(y) dy. \qquad (3)$$

At the very center the particle velocity reaches a maximum value equal to $\sqrt{v_0^2 + \alpha u^2}$. The factor α is equal 1.5 for a constant density of matter inside the galaxy. It is easy to calculate it for any given density distribution, e.g., for Navarro-Frenk-White profile [19]. However, for a reasonable density distribution in which the density decreases with distance from the center, the value of α is not much larger than 1.5 and, for a rough estimate, we may set $\alpha \simeq 1$ and $v(r) \simeq v(R)$ at $r < R$.

If the gravitational field is static, the particle will leave the halo again with speed $v(R)$. Far away from the galaxy it will move again with speed v_0. There is no particle capture. However, capture is possible if the mass of the galaxy increases during the passage of the particle. Indeed, galaxies continue to grow also after their formation. This can happen by the accretion of the surrounding BM, by the capture of DM particles, and also by merging with other galaxies. As a result, the gravitational potential well formed by the galaxy becomes deeper, and the potential barrier surrounding it becomes higher. The DM particle may not have sufficient kinetic energy to leave the potential well and it can be captured.

We denote by M the total mass of the galaxy at the moment the DM particle enters ($|x_p| = R$), and by τ the time of flight of the particle through the galaxy. Then, at the time the particle leaves the galaxy, the mass of the galaxy will be equal to $M + \dot{M}\tau$, where $\dot{M}$ is the average rate of mass increase of the galaxy. The particle capture condition takes the form

$$v_0 \leq \sqrt{\frac{2G\dot{M}\tau}{R}}. \qquad (4)$$

It is reasonable to expect that the mass increase $\dot{M}\tau$ does not exceed the mass M. Therefore, the maximal initial speed of captured particles is less than u and we can estimate $v(R) \simeq u$ so that

$$\tau \simeq \frac{\ell}{u} \qquad (5)$$

for the time of flight of particles with a minimum initial velocity at which they are not captured by the galaxy. Here ℓ is the length of the path traveled inside the halo.

It may seem that these approximations are too crude, but they are not. We demonstrate this with an example. As is well known, the rotation curves of galaxies are perfectly flat if the density decreases like $\rho(r) = M/(4\pi R r^2)$ at $r < R$. In this extreme case, the density diverges at the center, and this density profile is usually modified at small radius to avoid this divergence. Let us, however, consider the unmodified profile. In the framework of classical mechanics, the velocity of a particle passing through the center is then equal to $v(r) = u\sqrt{\ln\frac{R}{r}}$ at $r < R$ if $v_0 \ll u$. (The factor α introduced below Eq. (3) is infinite in this case.) But we are interested in the time of flight, which is equal to $\tau = 2CR/u$ with $C = e\sqrt{\pi}\mathrm{erfc}(1) \simeq 0.76$. So, the estimate (5) deviates from the exact value of τ only by 25% even for this extreme density profile.

Let us denote the minimal initial velocity of a DM particle which is able to fly through the galaxy and escape from it by v_p (the subscript p indicates passage). For $v_0 < v_p$ the particle is captured by galaxy. If a particle flies through the center we denote its minimal initial velocity by v_{pc} (the subscript pc indicates passage through the center). Within the above approximations we obtain the estimates

$$v_p = \sqrt{\frac{G\dot{M}\ell}{Ru}} = \left(\frac{G\dot{M}^2\ell^2}{2RM}\right)^{1/4}, \tag{6}$$

$$v_{pc} = \sqrt{\frac{2G\dot{M}}{u}} = \left(\frac{2G\dot{M}^2 R}{M}\right)^{1/4}. \tag{7}$$

We can use the simplest estimate $\dot{M} \simeq M/T$, where T is the age of the galaxy and find with (7)

$$v_{pc} \simeq \sqrt{\frac{2GM\tau}{RT}} = u\sqrt{\frac{\tau}{T}}. \tag{8}$$

B. Evolution of the DM particle velocity

If a particle escapes from the galaxy, its velocity far from it, v_1, is smaller than the initial velocity v_0 due to the growth of the galaxy mass,

$$v_1^2 = v_0^2 - \frac{2G\dot{M}\tau}{R}. \tag{9}$$

Each particle reduces its speed as it passes through a growing galaxy, transferring the released kinetic energy to the DM and BM inside the galaxies. This deceleration mechanism is similar to the integrated Sachs–Wolfe effect. It works more efficiently in galaxy clusters and poorly in voids, simply because of the difference in the number of galaxies that a particle passes through in the same amount of time. Therefore, we expect the mean kinetic energy of DM particles to be higher in voids than in superclusters.

For fast particles with $v_0 \gg u$ we can set $v(R) \simeq v_0$. If such particles fly a path of length ℓ inside the galaxy, then $\tau \simeq \ell/v_0$ and

$$v_1 \simeq \sqrt{v_0^2 - \frac{2G\dot{M}\ell}{Rv_0}} \simeq v_0 - \frac{G\dot{M}\ell}{Rv_0^2}. \tag{10}$$

The faster the particle, the less is the loss of speed. Therefore, the initial velocity distribution of particles not only shifts in the direction of decreasing velocities, but this shifts depends on v_0, changing its velocity spectrum.

Let us estimate the rate of energy loss for a DM particle with mass m. During the time τ of passage through the galaxy, it transfers the energy $\frac{Gm\dot{M}\tau}{R}$ to it. In this case, the distance traveled is $\ell \simeq v(R)\tau$. The rate of energy loss per unit time and per unit path are

$$\dot{E} = \frac{Gm\dot{M}}{R}, \qquad \frac{dE}{d\ell} \simeq \frac{Gm\dot{M}}{Rv(R)}. \tag{11}$$

The trapped particles transfer all their kinetic energy and their mass to the galaxy. The fact that the motion of the particles in the nonstationary field of the contracting matter is an efficient mechanism for the dissipation their energy has also been mentioned in the past in connection with other problems. Consider, for example, Ref. [20].

III. CATASTROPHIC DM CAPTURE

A. The rate of increase in the galaxy mass

We have already mentioned various mechanisms for increasing the mass of galaxies. Let us denote the rate of galaxy mass increase due to DM particle capture by $\dot{M}_{\rm DM}$, the total rate of galaxy mass increase due to all effects by $\dot{M}$, and the rate of mass increase due to accretion of baryonic matter by $\dot{M}_b$. Obviously

$$\dot{M} = \dot{M}_{\rm DM} + \dot{M}_b. \tag{12}$$

We consider $\dot{M}_b$ as a given, external quantity that can change with time and is determined in part by nongravitational processes of cooling which we do not investigate here. Even though a part of the baryonic matter, e.g. stars, does behave like collisionless particles, we assume that some baryonic matter, like e.g. gas, is accreted via collisional processes and by emitting radiation. We do not want to study this in any detail but consider the baryon accretion rate as an external parameter $\dot{M}_b$. We now study $\dot{M}$ as a function of the baryon accretion rate, $\dot{M}_b$.

We consider particles in extragalactic space far from galaxies. We assume that their velocities are distributed isotropically in the reference frame of the galaxy, and the number of particles with velocities in the range from v_0 to $v_0 + dv_0$ in a unit volume is equal to $dN = f(v_0)dv_0$. The total density of DM particles in extragalactic space is $N = \int_0^\infty f(v_0)dv_0$. Then the number of particles flying in extragalactic space through an area dS into a solid angle $d\Omega$ in a time dt with velocities in the range from v_0 to $v_0 + dv_0$ is

$$dn = \frac{v_0}{4\pi}\cos(\phi)f(v_0)dv_0 dS d\Omega dt, \tag{13}$$

where ϕ is the angle between the particle velocity direction and the normal to the area.

Consider a sphere of radius $R_1 \gg R$, surrounding the galaxy and concentric to it. Its surface area is $4\pi R_1^2$. The number of particles passing through it in a time dt with velocities in the range from v_0 to $v_0 + dv$ is given by Eq. (13). The halo is reached by particles emitted into a solid angle $\Omega \simeq \pi R^2/R_1^2 \ll 1$. Let us define the function $k(v_0, \phi)$ which is equal to 1 if a particle with angle ϕ and velocity v_0 is trapped and 0 if it is not trapped.

With this we obtain for the rate of increase in the mass of the galaxy by DM particles capture

$$\dot{M}_{\rm DM} \simeq 2\pi R_1^2 m \int dv_0 \int d\phi \, \sin\phi \, \cos\phi f(v_0) v_0 k(v_0, \phi). \tag{14}$$

Let us evaluate this integral. Considering that our model is rather a toy-model, not very accurate estimates are applicable. In order for a particle to be captured by a galaxy, it must enter it. This happens if $0 \le \phi \le \arcsin(\beta)$ with $\beta = R/R_1 \ll 1$. We therefore can set $\sin\phi \simeq \phi$ and $\cos\phi \simeq 1$. Let us also introduce the variable $\xi = \phi/\beta$. If we neglect the curvature of the particle trajectory inside the galaxy (this does not significantly affect the path length for particles flying through), then the path length inside the galaxy is about

$$\ell = 2R\sqrt{1 - \phi^2/\beta^2} = 2R\sqrt{1 - \xi^2}. \tag{15}$$

Capture occurs and $k = 1$ if the condition (4) is met, that is, if

$$1 - \xi^2 > \frac{v_0^4(v_0^2 + u^2)}{4(G\dot{M})^2} \simeq \frac{v_0^4 u^2}{4(G\dot{M})^2} = \left(\frac{v_0}{v_{pc}}\right)^4. \tag{16}$$

(Remember that the initial velocity of captured particles, v_0, is much smaller than the escape velocity u). The particle is captured if the variable ξ, which is proportional to the angle of deviation of the particle velocity from the center of the halo ϕ, does not exceed the value

$$\xi_0^2(v_0) \simeq \max\left(0, 1 - \left(\frac{v_0}{v_{pc}}\right)^4\right). \tag{17}$$

A particle flying through the very center of the halo is captured if its initial velocity is less than v_{pc} given in (7). In an off-center passage, the particle is captured if

$$v_0 \le v_p = v_{pc}(1 - \xi^2)^{1/4}. \tag{18}$$

If this inequality is not satisfied, then there is no capture and $k = 0$. With this we can write

$$\begin{aligned} \dot{M}_{\rm DM} &\simeq 2\pi R^2 m \int_0^\infty f(v_0) v_0 dv_0 \int_0^{\xi_0} k(v_0, \xi) \xi d\xi \\ &= \pi R^2 m \int_0^\infty \xi_0^2 f(v_0) v_0 dv_0 \\ &= \pi R^2 m \int_0^{v_{pc}} \left(1 - \frac{v_0^4}{v_{pc}^4}\right) f(v_0) v_0 dv_0. \end{aligned} \tag{19}$$

The integral on the right hand side depends on $\dot{M}$ via v_{pc}, and this dependence is highly nonlinear. The combination of (12) and (19) determines $\dot{M}$ for a given $\dot{M}_b$ and a given velocity distribution $f(v_0)$. At $\dot{M}_b = 0$ there is a trivial solution $\dot{M} = 0$.

Assuming the form of the function f, we can obtain the dependence of $\dot{M}_{\rm DM}$ on $\dot{M}$. For example, if f is a simple Maxwell-Boltzmann distribution,

$$f(v) = \frac{4Nv^2}{\sqrt{\pi} v_{\max}^3} \exp\left[-\left(\frac{v}{v_{\max}}\right)^2\right], \qquad v_{\max} = \sqrt{\frac{2k\Theta}{m}} \tag{20}$$

with temperature Θ and maximum at $v = v_{\max}$, then from (19) one obtains

$$\dot{M}_{\rm DM} = 2\sqrt{\pi} R^2 m N v_{\max} P(v_{pc}^2/v_{\max}^2), \tag{21}$$

where the function P is given by

$$P(x) = 1 - 6x^{-2} + 2e^{-x}(1 + 3x^{-1} + 3x^{-2}). \tag{22}$$

Of course, we cannot really assume that DM particles obey a thermal distribution. However, interesting qualitative conclusions can be drawn from general considerations without detailed assumptions about the velocity distribution of the DM particles (more precisely, their phase space number density f or mass density mf). We just assume that the function $f(v_0) \ge 0$, that is it continuous, that it vanishes at $v_0 = 0$, reaches a maximum at some value $v_0 = v_{\max}$, and quickly decreases at high velocities, most likely exponentially in v_0^2. This function is proportional to the particle density N.

We consider the expansion of f in a Taylor series. It includes only even powers of v_0. As for the Maxwell distribution, the expansion starts with a quadratic term due to the three independent Cartesian velocity components:

$$f(v_0) = \sum_{i=1}^\infty a_i v_0^{2i} = N \sum_{i=1}^\infty \tilde{a}_i v_0^{2i}. \tag{23}$$

The quantities $\tilde{a}_i = a_i/N$ do not depend on N. So that

$$\begin{aligned} \dot{M}_{\rm DM} &\simeq \pi m R^2 \int_0^{v_{pc}} \left(1 - \frac{v_0^4}{v_{pc}^4}\right) \sum_{i=1}^\infty a_i v_0^{2i+1} dv_0 \\ &= \pi m R^2 \sum_{i=1}^\infty \frac{a_i}{(1+i)(3+i)} v_{pc}^{2i+2} \\ &= \dot{M} \sum_{i=1}^\infty b_i \dot{M}^i = \dot{M} N \sum_{i=1}^\infty \tilde{b}_i \dot{M}^i \quad \text{with} \end{aligned} \tag{24}$$

$$b_i = \pi m R^2 \left(\frac{2GR}{M}\right)^{(i+1)/2} \frac{a_i}{(1+i)(3+i)}. \tag{25}$$

Also here we have introduced $\tilde{b}_i = b_i/N$. These quantities help to explicitly extract the dependence on the particle density N, which varies significantly over the lifetime of galaxies. If DM particles are not formed and do not decay, then $N(z) = N(0)(1+z)^3$ and during the existence of a galaxy formed at $z = 10$, the density decreases by a factor 1000 due to the expansion of the Universe. However, due to the considered capture of DM particles, their number density in intergalactic space, N is reduced even further.

The quantities $\tilde{a}_i$ change with time due to changes in the velocity distribution, in particular, because of the processes under consideration. However, most likely, the most significant contribution to the time dependence of the parameters a_i is associated to the evolution of N. The same can be expected for the coefficients b_i, although in this case the (weak) dependence of $\tilde{b}_i$ on time gets additional contributions from the evolution of M.

Positivity of f for small velocities requires that $b_1 > 0$. We assume that $b_2 < 0$ like for the Maxwell-Boltzmann distribution.

Let us try to draw some conclusions based on these basic properties of the distribution function. We start with the case when captured DM particles have initial velocities much smaller than their mean velocity. So that they are described by the first term of the expansion of $f(v_0)$,

$$f(v_0) \simeq a_1 v_0^2. \tag{26}$$

Let us also assume that the considered galaxy is not very large and the time of passage of particles through it is much less than its age and than the age of the Universe. Then we can approximately set $a_1 = \text{const.}$ and $M = \text{const.}$ during the time of flight. In this case

$$\dot{M} - \dot{M}_b = \dot{M}_{\rm DM} \simeq b_1 \dot{M}^2 = \frac{\pi a_1 R^3 m G}{4M}\dot{M}^2,$$
$$\dot{M}_b = \dot{M} - b_1\dot{M}^2 = \frac{1}{4b_1} - b_1\left(\frac{1}{2b_1} - \dot{M}\right)^2. \tag{27}$$

Note that a_1 has the dimensions $[(\ell^2/t)^{-3}]$ so that b_1 has the dimensions $[t/m]$ and $b_1\dot{M}$ is dimensionless. For $\dot{M}_b = 0$ this gives us the equation

$$\dot{M}\left(1 - \frac{\pi}{4} a_1 R^3 m G \frac{\dot{M}}{M}\right) = 0. \tag{28}$$

Only the trivial solution, $\dot{M} = 0$ is physical. Requiring that the second factor vanishes gives us a rough estimate of characteristic time of mass accretion, $T = M/\dot{M}$, as $T \simeq \pi N \tilde{a}_1 G R^3 m/4$ in (8). This is clearly an unphysical solution with $T \propto N$. So that, T tends to 0 with $N = 0$, or, in other words, $\dot{M}_{\rm DM}$ grows indefinite when N tends to 0, i.e., when DM becomes less and less abundant, which is meaningless and is a consequence of our approximation which breaks down when $\dot{M}$ becomes large.

B. A jump in the particle capture rate

In order to understand this situation, we apply the methods of catastrophe theory. For this it is important to note that we consider $\dot{M}_b$ as an external parameter which is determined by accretion and baryonic cooling processes which we do not describe in our model. We want to study the increasing of total mass for a given $\dot{M}_b$.

Figure 1(a) shows the dependence $\dot{M}$ on $\dot{M}_b$ according to Eq. (27). This fold consists of two parts. The lower half of the parabola AB (in solid) corresponds to a stable solution. With an increase in the baryon mass growth rate $\dot{M}_b$, the DM particle capture rate $\dot{M}_{\rm DM}$ increases. It is described by (27) The upper half of the parabola BC (dashed) with a negative slope corresponds to an unstable solution. Two points A and C of intersection of the curve with the y-axis correspond to the two solutions of the equation (28). Of these, only the solution $\dot{M} = 0$ is stable.

At a nonzero matter accretion rate $\dot{M}_b$, particle capture begins. However, the rate of increase in the mass of the dark halo $\dot{M}_{\rm DM}$ is less than the rate of increase in baryonic matter

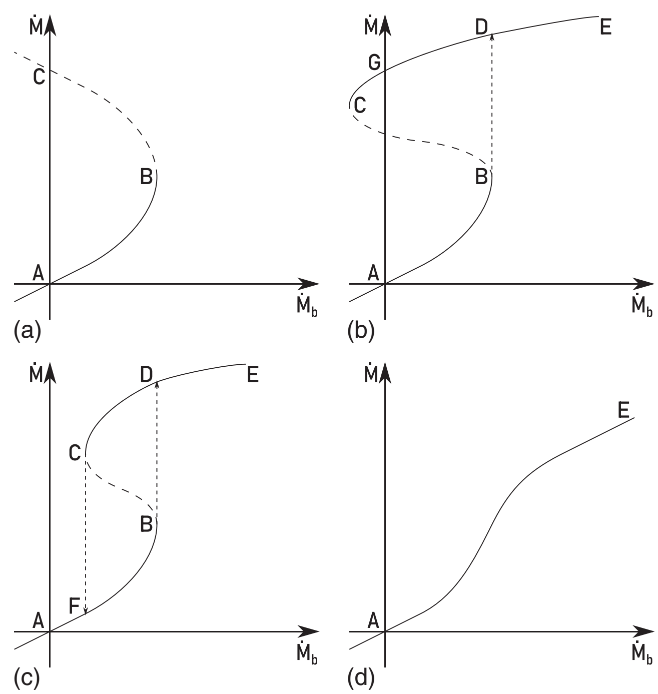

FIG. 1. Plots of $\dot{M}(\dot{M}_b)$ for various functions f and DM densities. Solid curves show stable branches, dashed curves show unstable ones, vertical dotted arrows show jumps in the state of the system. The four panels correspond to the following cases. Panel (a) depicts approximation (26) for low particle velocities. Panels (b) and (c) show two possibilities for density above threshold. Both are s-shaped. They differ in the position of the left boundary of the upper stable branch at point C. Panel (d) shows the case when the baryon accretion rate is below the threshold $\dot{M}_{bc}$ defined in Eq. (29) and the function becomes monotonically increasing.

$\dot{M}_b$. They become equal only at the top of the parabola. At point B we have

$$\dot{M}_b = \dot{M}_{\rm DM} = 1/(4b_1) \equiv \dot{M}_{bc}. \tag{29}$$

From this, two conclusions can be drawn. First, within the approximations made in our model, the capture of dark matter requires the accretion of ordinary matter. There is no DM capture without the baryonic matter accretion. Secondly, at the ratio of the mass growth of baryonic and dark matter described by the curve in Fig. 1(a), it is not possible to form a galaxy containing 85% dark matter.

The system (27) does not have a solution for $\dot{M}_b > 1/(4b_1)$. With a further $\dot{M}_b$ increase, the state of the system reaches the top of the parabola, after which a sudden regime change begins. and the rate of mass growth $\dot{M}$ and v_{pc} rapidly increases. Therefore, we cannot consider the particle velocities to be small and use the approximation (26). Equation (27) ceases to adequately describe the process. The accretion rate $\dot{M}_b = 1/(4N\tilde{b}_1)$, which is required to lose stability in point B, can be quite small at the time of galaxy formation, when N is very large.

To consider larger accretion rates we take into account the next term in the expansion (24) which leads to the equation

$$\dot{M}_{\rm DM} = \dot{M} - \dot{M}_b \simeq b_1\dot{M}^2 + b_2\dot{M}^3, \qquad b_2 < 0. \tag{30}$$

Figures 1(b) and 1(c) show the curves ABCDE which can be obtained in this case. They have a characteristic s-shaped form, typical for the fold catastrophe which is well known in catastrophe theory, see, e.g., [17]. It consists of three parts, two of which have a positive slope (AB and CDE) and are stable. BC with a negative slope is unstable. When the end of the stable part is reached, a jump from B to D to the second, upper stable branch occurs. The upper stable solution can lead to very significant DM accretion, since for it the ratio of the mass growth rates of DM and baryonic matter can be quite large. We call this branch the regime of *catastrophic DM capture*. In order to enter this regime, the baryon accretion rate must exceed the threshold value $\dot{M}_{bc} \simeq 1/(4N\tilde{b}_1)$ corresponding to a jump in the capture rate $\dot{M}_{\rm DM}$.

Note also that a kind of a hysteresis loop can occur in the situation shown in Fig. 1(c): if $\dot{M}_b$ is decreasing during the regime of catastrophic DM capture (upper branch), the state of the system on the graph shifts to the left along the upper stable branch. When it reaches the edge of the stable branch at point C, it jumps back to the lower branch at point F and leaves the regime of catastrophic DM capture. With a certain ratio of the coefficients b_1 and b_2 this formally happens at a negative value of $\dot{M}_b$ as is the case in panel 1(b). It is easy to calculate, that point C corresponds to a negative value of $\dot{M}_b$ if $N > -4\tilde{b}_2\tilde{b}_1^{-2}$ and a positive value of $\dot{M}_b$ for $N < -4\tilde{b}_2\tilde{b}_1^{-2}$.

Thus, depending on the evolution of $f(v_0)$, the value of $\dot{M}_{\rm DM}$ can remain in the catastrophic DM capture regime if in the past the galaxy had a value of $\dot{M}_b$ larger than the critical value and there was a jump, even if later $\dot{M}_b$ decreases or vanishes. As the particle density N decreases, the left boundary of the upper stable branch, i.e., the point C, crosses the y-axis and the system can return to the lower stable branch via the CF transition.

It is clear that we cannot in general restrict ourselves to a finite number of expansion terms in (24). Therefore we now study the qualitative form of the dependence of $\dot{M}$ on $\dot{M}_b$ without using the series expansion. We introduce the new variable $\eta = v_0/v_{pc}$. With (19) we obtain

$$\dot{M}_{\rm DM} \simeq \pi R^2 m v_{pc}^2 F, \tag{31}$$

$$F = \int_0^1 (1-\eta^4) f(\eta v_{pc}) \eta d\eta = \int_0^1 (1-\eta^4) f\left(\frac{\eta}{\eta_0} v_{\rm max}\right) \eta d\eta, \tag{32}$$

$$\eta_0 = \frac{v_{\rm max}}{v_{pc}}. \tag{33}$$

The integral F goes over a fixed interval. The integrand is the product of the function $\eta(1-\eta^4)$ that vanishes at both ends of the interval and an unknown function f whose properties we discussed above. The function f reaches its maximum at $v_0 = v_{\rm max}$, i.e., at $\eta = \eta_0$. For small v_{pc} we have $\eta_0 \gg 1$. The maximum of f lies outside the region of integration and the integral is proportional to $\eta_0^{-2} \propto v_{pc}^2$. As a result, we can approximate f by (26). At large v_{pc} we have $\eta_0 \ll 1$ and the maximum of f shifts to the lower boundary of the interval. The integral is approximately proportional to $\eta_0^2 \propto v_{pc}^{-2}$, which is compensated by the prefactor v_{pc}^2. Note that here "large" and "small" v_{pc} is considered with respect to $v_{\rm max}$. Hence the colder the DM, i.e., the smaller $v_{\rm max}$, the less baryonic matter accretion is required to be in the large v_{pc} regime.

A more accurate estimate for the large v_{pc} regime can be obtained directly from the expression (19), in which the upper limit of integration is replaced by infinity, which gives a negligible error in this case. As a result, we obtain the asymptotic expression

$$\dot{M}_{\rm DM} \simeq \pi R^2 \int_0^\infty \left(1 - \frac{v_0^4}{v_{pc}^4}\right) m f(v_0) v_0 dv_0 \simeq C_1 - C_2 v_{pc}^{-4} = C_1 - C_3 \dot{M}^{-2}, \tag{34}$$

$$C_1 = \pi m R^2 \int_0^\infty f(v_0) v_0 dv_0 > 0, \tag{35}$$

$$C_1 \propto N,$$
$$C_2 = \pi m R^2 \int_0^\infty f(v_0) v_0^5 dv_0 > 0, \tag{36}$$

$$C_2 \propto N, \tag{37}$$

$$C_3 = \frac{M}{2GR} C_2. \tag{38}$$

Approximating $C_2 \sim C_1 v_{\max}^4$, we find that at high accretion rate, $v_{pc} \gg v_{\max}$ we may neglect the second term in (34) and the DM capture rate is saturated. The dependence acquires the asymptotic form $\dot{M} \to C_1 + \dot{M}_b$. In Figs. 1(b) and 1(c), the slopes of the curves in the upper right corner are not drawn to scale.

Let us also consider the behavior of the function for intermediate values of η_0. The integral F in (32) is a function of the variable v_{pc}, that decreases at large and small values of the argument. Therefore, it reaches a maximum at a certain value of v_{pc} which we call v_{pcm}. We have $\frac{\partial F}{\partial v_{pc}} = 0$ for v_{pcm}. The value of F is proportional to $f(v_{\mathrm{pcm}})$, which, in turn, is proportional to the density of particles in intergalactic space N. The integral F is multiplied by the factor $v_{pc}^2 \propto \dot{M}$. Therefore, for $v_{pc} = v_{\mathrm{pcm}}$ we obtain a dependence of the form

$$\dot{M}_{\mathrm{DM}} \simeq Q(M,R) m N \dot{M} \tag{39}$$

where the function $Q(M,R)$ does not depend on $\dot{M}$. On the other hand, $\dot{M}_b = \dot{M} - \dot{M}_{\mathrm{DM}}$. For small $\dot{M}$ we have (27) with positive slope. For large $\dot{M}$ we have (34) also with positive slope. At the maximum of F we have (39) with the slope $1 - Q(M,R)mN$, which is negative if the DM particle density N exceeds some critical value. If this happens, we obtain an s-shaped curve $\dot{M}(\dot{M}_b)$ like in Fig. 1(b) and 1(c). We apply the theory of catastrophes and find that with a continuous increase of $\dot{M}_b$, a jump in $\dot{M}$ occurs and something like a hysteresis loop can appear. This shows that the appearance of the fold catastrophe is quite generic.

It is clear that the catastrophic capture regime must lie above the point with a negative derivative with respect to $\dot{M}_b$, which is attained at v_{pcm} for which F is maximal. Let us evaluate this maximum. The integrand in F is the product of two functions, each of which has a maximum. The function f reaches its maximum at $v_0 = v_{\max}$, i.e., at $\eta = \eta_0$. The maximum of the function $\eta(1-\eta^4)$ achieved at $\eta = \eta_1 \approx 0.7$. The integral is maximal if these two maxima roughly agree, hence $\eta_0 \approx \eta_1$. In this case, the speed v_{pc} for the upper stable branch is about $v_{\mathrm{pcm}} \simeq 1.4 v_{\max}$. A DM particle with an initial velocity $v_0 = v_{\max}$ is then captured by the galaxy if its trajectory passes at a distance less than $0.85R$ from the center. This means that a significant fraction of the DM particles is captured as they pass through the galaxy.

The ratio of the influx rates of dark and baryonic matter can be quite large. But the jump into the catastrophic capture regime is not possible if the rate of accretion of baryonic matter did not exceed some threshold value in the past or present.

The jump also requires the presence of DM with a density N exceeding a certain threshold value. Taking into account that N decreases with time both due to Hubble expansion and because of the capture of particles as discussed in this paper, it can be assumed that eventually the s-shaped curve has turned or will turn into the monotonic dependence shown in Fig. 1(d), where no catastrophe exists and the capture process is significantly weaker. This moment may lay in the past or in the future depending on the parameters of a given galaxy. With $\dot{M}$ also v_{pc} decreases significantly.

Let us also determine the positions of the points B and C in Figs. 1(b) and 1(c) which determine the baryon accretion rate at the entry into and the exit from the catastrophic DM capture regime. They are given by the condition $d\dot{M}_b/d\dot{M} = 0$. Taking into account (12) this yields $d\dot{M}_{\mathrm{DM}}/d\dot{M} = 1$. With (19) we can write this condition as

$$H(v_{pc}) \coloneqq N^{-1} v_{pc}^{-6} \int_0^{v_{pc}} v_0^5 f(v_0) dv_0$$
$$= \frac{1}{2\pi R^2 m N} \left(\frac{M}{2GR}\right)^{1/2}. \tag{40}$$

The function $H(v_{pc})$ tends to zero as $v_{pc} \to 0$ and as $v_{pc} \to \infty$. This means that it has a maximum at a certain $v_{pc} = v_1$. It can be estimated that $v_1 \approx v_{\max}$. We denote

$$N_1 = \frac{1}{2\pi R^2 m H(v_1)} \left(\frac{M}{2GR}\right)^{1/2}. \tag{41}$$

At $N < N_1$ we have no solution and the inflection points B and C do not exist. The value $N = N_1$ corresponds to the transition from s-shaped curve to the monotonic one. Assuming the generic shape of monotonic increase and decay for $H(v_{pc})$, at $N > N_1$ we have two solutions of Eq. (40). The solution with smaller v_{pc} corresponds to point B, one with larger v_{pc} to point C. For small v_{pc} we can use the approximation (26) and obtain the same estimate for coordinates of point B.

$$\dot{M}_b = \dot{M}_{bc} \simeq \frac{1}{4N\tilde{b}_1} = \frac{M}{\pi N R^3 m G \tilde{a}_1}. \tag{42}$$

Using the function H, we can show that more complex scenarios are possible in which the transition to the state of intense capture and/or exit from it can occur in two stages.

This is possible, e.g., in the case of the existence of two different types of DM particles or a bimodal velocity distribution $f(v)$ [21].

The point C corresponds to $\dot{M}_b = 0$ [crossing over to Fig. 1(b)], when, in addition to (40), the condition

$$v_{pc}^{-2}\int_0^{v_{pc}} v_0 f(v_0)dv_0 = \frac{3}{2\pi R^2 m}\left(\frac{M}{2GR}\right)^{1/2} \tag{43}$$

is satisfied. We can determine the values of N and $\dot{M}$ in this case by solving the system of equations (40) and (43).

Let us roughly estimate the ratio of the rates of increase in the mass of DM and BM immediately after the jump from B to D. At point B, these rates are approximately equal and, according to (42), they are inversely proportional to the DM mass density Nm. At point D, lying on the upper branch, the value of $\dot{M}_b$ is the same as at point B. The ratio $\dot{M}_{\mathrm{DM}}$ to $\dot{M}_b$ at point D is approximately equal to the ratio of $\dot{M}_{\mathrm{DM}}$ at points D and B, which is clearly greater than 1. We can use (34) and set $\dot{M}_{\mathrm{DM}} \simeq C_1 \propto Nm$. So, the ratio of the rates of increase in the mass of DM and BM at point D is proportional to N^2 and can be very large at the early stages of galaxy evolution.

Let us confirm these arguments with the example of the Maxwell-Boltzmann distribution and use Eq. (21). Since we are interested in the ratio of mass gain rates, we introduce the dimensionless variables

$$x = \gamma \dot{M}_b, \qquad y = \gamma \dot{M}, \qquad \gamma = \sqrt{\frac{2GR}{M}}v_{\max}^{-2}. \tag{44}$$

From (21) and (12) we obtain

$$x = y - \mu P(y) \tag{45}$$

$$\begin{aligned}\mu &= \frac{2R^2 mN}{v_{\max}}\sqrt{\frac{2\pi GR}{M}} \\ &\simeq 0.01\frac{mN}{0.3\rho_{c0}/h^2}\frac{100\ \mathrm{km/s}}{v_{\max}}\left(\frac{R}{0.3\ \mathrm{Mpc}}\right)^{5/2}\left(\frac{10^{12}M_\odot}{M}\right)^{1/2}\end{aligned} \tag{46}$$

where the function $P(y)$ is given in (22). In (45) the parameter dependence has been reduced to the single dimensionless parameter μ, which decreases with time, mainly due to the decrease in N. Figure 2 shows four curves corresponding to the values of this parameter equal (from left to right) 10, 7, 5, and 4. They are shown not schematically, as in Fig. 1, but accurately to scale.

At $\mu = 4$ the curve is monotonic like in Fig. 1(d) and this value is slightly below the critical value for the transition to the s-shaped curve. For $\mu = 5$ we have a curve similar to that shown in Fig. 1(c) and this value is slightly below the value of μ at which the left edge of the top branch intersects the y-axis. These two special values of μ are rather close, they differ only by a factor 1.25. For larger μ the curve has the form shown in Fig. 1(b). The ratio of the rates of mass gain of DM and BM after the jump to upper branch is approximately 11 at $\mu = 5$, 55 at $\mu = 7$, and 85 at $\mu = 10$. It increases rapidly with increasing $\mu \propto N$. As a result, the galaxy at the stage of intense capture accumulates a lot of dark matter.

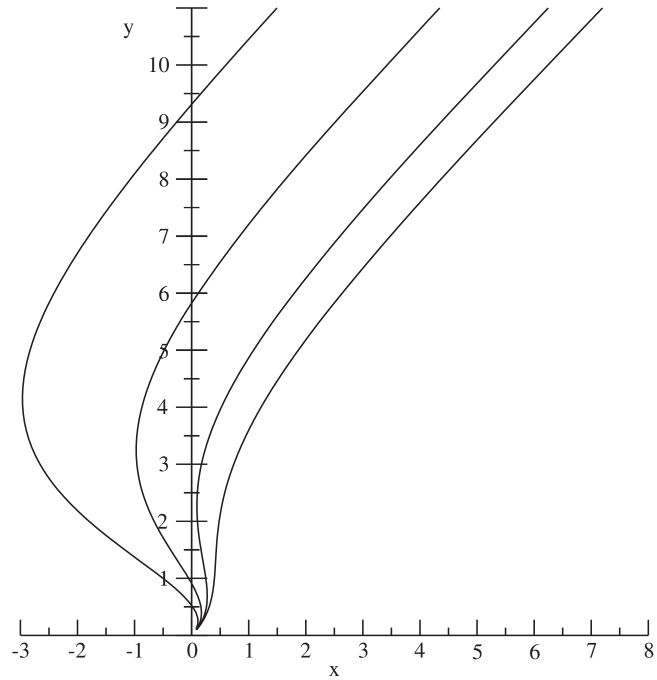

FIG. 2. Dependence of the quantities (44) proportional to the rates of increase in the total and baryonic masses of the galaxy for the case of the Maxwell distribution of DM particle velocities. The curves from left to right correspond to the different values of the parameter μ from (46) equal to 10, 7, 5, and 4.

C. Qualitative description of catastrophic DM capture

After analyzing the conclusions obtained from Eqs. (12) and (19) within the framework of our model, we can describe the process of accumulation of dark matter inside the halo of the galaxy.

As a result of the growth of small density fluctuations, regions of increased density emerge, in which galaxies can form. The surrounding matter, both baryonic and dark, begins to fall into them. The infalling BM cools and is captured. In the absence of accretion of baryonic matter, DM particles fly through protogalaxies and are not captured.

The situation changes with an increase in the mass of the baryonic component of the galaxy due to accretion, mergers of galaxies and other processes. A fraction of the DM particles flying into the galaxy at sufficiently low speeds is being captured. In Fig. 1(b), this is described by the section AB on the lower stable branch of the s-shaped curve. The mass of dark matter inside the galaxy begins to grow, but the rate of its increase is smaller than the rate of increase in the mass of BM.

If the rate of increase in the mass of the BM exceeds a certain threshold value, $\dot{M}_{bc}$, something similar to a phase transition occurs with a change in the state of the system. In Fig. 1(b), this corresponds to a sharp jump from B to D after reaching the right boundary of the bottom stable branch at point B. The simple estimate (42) determines the dependence of the relative critical growth rate of baryonic matter $\dot{M}_{bc}/M$ on the size of the galaxy R and on the mass density of dark matter in the intergalactic space Nm (during the formation of galaxies, this is simply the density of dark matter). The value of N decreases rapidly due to the expansion of the Universe, hence if the jump did not occur during the formation of the galaxy, then it will not occur at later times. The only exception might be the process of merging of galaxies, which can provide a transition to the upper branch due to a sharp temporary increase in the rate of baryonic mass growth, $\dot{M}_b$.

The critical baryonic mass growth rate required for the jump is smaller for objects with large R (galaxies and their clusters) than for objects with small R (stars, etc.). Therefore, dark halos form around galaxies, but not around stars.

After a jump to the upper stable branch, the state of the system corresponds to point D or its vicinity. Let us consider the case where the density N is high enough such that $-4b_2/b_1^2 < 1$ and the $\dot{M}(\dot{M}_b)$ curve has the shape 1(b). If $\dot{M}_b$ increases, the system shifts to the right, say, to point E. If $\dot{M}_b$ decreases, the system shifts to the left along the curve DG. On this curves, $\dot{M}_{\rm DM} \gg \dot{M}_b$. During this phase, galaxies can become DM dominated. Note that in case 1(b) the capture of DM particles continues even if the accretion of baryonic matter ceases at point G.

However, not only the value of $\dot{M}_b$, but also the curve itself changes with time. The dynamics of the change in the curve is associated primarily with the decrease of N. The change in the shape of the distribution of the velocities of one DM particle, say, the coefficients $\tilde{b}_i$, has a much weaker time dependence.

When the threshold value of the intergalactic DM density $N = -4b_2/b_1^2$ is reached, the left boundary of the upper stable branch crosses the y-axis. The curve takes the form shown in Fig. 1(c). At point C, the system can jump to point F on the lower branch. In this case, the mass of the dark halo practically stops growing.

Without knowledge of the DM velocity distribution, we cannot determine the density at which the BD jump occurred, so we do not know whether the curve is described by graph 1(b) or 1(c) at a given time. But the transition from curve 1b to curve 1c is inevitable. In the above description, we assume that it happened later than the jump from B to D.

If the state of the system has not descended to the lower branch and intensive capture of DM particles continues due to the high rate of accretion of baryonic matter $\dot{M}_b$, then with a further decrease in N, the $\dot{M}(\dot{M}_b)$ curve becomes monotonic as shown in Fig. 1(d) and represents a single stable branch at densities N below the next threshold value N_1 given in Eq. (41). This can be considered the end of the stage of intensive capture of DM particles. It is obvious that this transformation occurs later than the crossing of point C through the y-axis.

IV. SOME QUANTITATIVE ESTIMATES

For an estimation we use the parameters of our Galaxy. The Milky Way cannot be considered typical as there are many more dwarf galaxies in the Universe, but it is a good example of a large galaxy. We set $M \approx 10^{12} M_\odot \approx 2 \times 10^{42}$ kg and $R \approx 10^6$ ly $\approx 10^{22}$ m. The last estimate is based on the value $R = 292 \pm 61$ kpc [22] and is a rather large value. It is of the order of the average distance between galaxies and slightly less than half the distance to the Andromeda galaxy, M31. For this values the time of flight through the center of the Galaxy exceeds 2 million years even for an ultrarelativistic particle. But we are more interested in slow particles captured by the Galaxy. As mentioned above, their initial speed is less than $u = (2GM/R)^{1/2}$, and the speed of passage of the halo is approximately equal to $u = 170$ km/s. This gives an upper bound on the time-of-flight of a galaxy τ for the noncapture case as $\tau \le 4.5 \times 10^9$ years.

There are alternative estimation of R. Some of them one can find in the review article by [23] and in papers by [24,25]. If we choose the value $R = 200$ kpc $\approx 6.5 \times 10^5$ ly $\approx 6.2 \times 10^{21}$ m with the same estimate of M, we find $u \approx 210$ km/s, $\tau \le 10^9$ years. If we choose a lower estimate $R = 100$ kpc, then $u \approx 300$ km/s and $\tau \le 3.3 \times 10^8$ years.

These τ values are less or much less than the ages of the Universe and of the Galaxy for all estimates of R. This confirms the assumption underlying the model that during the passage of a particle that is not captured by the galaxy, the mass of the latter increases, but not by very much. It is clear that this is also true for dwarf galaxies with significantly smaller halo sizes.

We can estimate v_{pc} from (8), setting $\dot{M} = M/T$ with $T \simeq 1.3 \times 10^{10}$ years, which corresponds to galaxy formation at $z \simeq 5$ to 20. For the Milky Way we obtain $v_{pc} \approx 100$ km/s for $R = 300$ kpc, $v_{pc} \approx 60$ km/s for $R = 200$ kpc, and $v_{pc} \approx 50$ km/s for $R = 100$ kpc.

We are more interested in estimating the rate of halo mass increase due to DM capture. Let us assume that the Milky Way has not left the stage of intense capture and apply the formula (34), more precisely, its limit for large $\dot{M}$. Using the (35) with $\int_0^\infty f(v_0) v_0 dv_0 \approx N v_{\max}$, we find

$$\dot{M}_{\rm DM} \approx C_1 \approx \pi R^2 \rho_{\rm DM} v_{\max} \approx 0.08 \varkappa h^2 \left(\frac{R}{200\ \mathrm{kpc}}\right)^2 \frac{v_{\max}}{100\ \mathrm{km/s}} M_\odot\ \text{per year}. \qquad (47)$$

Here we have denoted the DM mass density in the intergalactic space as ρ_{DM}. In our estimation, we took into account that $Nm = \rho_{\mathrm{DM}} = 0.25\varkappa\rho_c$. It is less than the average density of dark matter in the Universe, which should be approximately 25% of the critical density ρ_c, determined by the Hubble constant $H_0 = h100\ \mathrm{km/s/Mpc}$. The coefficient $\varkappa < 1$ is introduced to account for this difference, which caused by the fact that part of the dark matter is accumulated in the halos of galaxies.

The product $\dot{M}_{\mathrm{DM}}$ times the age of the galaxy is much smaller than the DM mass in our Galaxy. The reason for this is that the rate of mass increase was significantly higher in the early stages of the capture of dark matter particles by the Galaxy. Let us estimate the mass of dark matter $M_{\mathrm{DM}}(z_0)$ captured from the time corresponding to the redshift z_0 to today. We assume that all this time there was an intense capture of particles and the mass gain is described by the Eq. (47). The mean density of dark matter in the Universe is proportional to $(1+z)^3$. We can set $\varkappa \simeq 1$ for the early stages of galaxy evolution which account for most of the captured DM.

Let us assume that the galaxy from the beginning of the considered period formed a gravitationally bound system and its dimensions did not increase due to Hubble expansion. It is difficult to estimate by how much $v_{\max}$ changes with the expansion of the Universe. On the one hand, the speed of a particle flying far from galaxies and not interacting with other particles remains almost unchanged. On the other hand, an analogy can be drawn with the cooling of an ideal gas during its adiabatic expansion. However, it is doubtful that DM particles would be in a state of thermal equilibrium.

Therefore, and for simplicity, we estimate $M_{\mathrm{DM}}(z_0)$, assuming that the values R and $v_{\max}$ to be approximately constant during the period under consideration and the change in the capture rate is to be determined mainly by the change in the DM density.

Let us denote the current capture rate as $\dot{M}_{\mathrm{DM}}(0)$ and apply the flat ΛCDM model. We obtain

$$\begin{aligned} M_{\mathrm{DM}}(z_0) &= \int \dot{M}_{\mathrm{DM}}(0)(1+z)^3 dt \\ &= W(z_0)\dot{M}_{\mathrm{DM}}(0)H_0^{-1} \end{aligned} \tag{48}$$

with

$$W(z_0) = \int_0^{z_0} \frac{(1+z)^2}{\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}} dz. \tag{49}$$

Here $\Omega_\Lambda \approx 0.7$ and $\Omega_m \approx 0.3$ are density parameters for the cosmological constant and matter. Thus, the mass of dark matter captured by the Galaxy from the moment $z = z_0$ corresponds to that which it would have captured during the time $W(z_0)H_0^{-1} = W(z_0) \times 10^{10}/h$ years if the current rate of capture was maintained. We can calculate $W(19) \approx 114$, $W(24) \approx 161$, $W(32.3) \approx 250$. We see that, according to our rough estimate, the dark matter that forms the dark halo of the Galaxy can be captured if the process of intense capture begins at $z \simeq 20$ or $z \simeq 30$.

In order to avoid misunderstanding, we emphasize once again that we do not assert that at the present time the Galaxy continues to actively capture DM particles and its state should be on the upper branch. Almost all DM was captured at the earliest stage of this process. It can be assumed that the process of moving to the lower branch (the fall from C to F in Fig. 1(c)) has already occurred. We do not know if active capture resumed temporarily during the capture of a single dwarf galaxy (see [26]). It may also be that some (or most) galaxies never had significant baryonic accretion and never underwent catastrophic DM capture and maintained their ratio of baryonic to dark matter from the initial time of formation.

There are galaxies in the Universe more massive than the Milky Way. When evaluating (47), we assumed that the capture rate is maximal. Differences in the mass of dark matter in galaxies can be related to the sizes of galaxies and the moment of the beginning and end of intense capture, i.e., the time of the jump to the upper branch and fall back to the lower branch.

The rate of matter capture is proportional to R^2. From the estimate (42) we can assume that for a larger proto-galaxy the jump to the upper stable branch occurred earlier than for a smaller one. The system can descend to the lower branch not before the point C crosses the y-axis and the curve takes the form shown in Fig. 1(c). But a significant accretion rate of BM allows it to remain on the upper branch for some time after that and to continue to accumulate dark matter at a significant rate. The accretion rate is clearly larger in a big massive galaxy, all other parameters being fixed. We know galaxies with estimates of R more than 300–400 kpc. This are, e.g., NGC 4889, NGC 4874, ESO 306-17 and others. It can be assumed that their large masses are associated, among other things, with a particularly effective capture of dark matter.

Another possibility is associated with the merger of two galaxies of comparable mass, continuous merging of galaxies in the cluster potential ("galactic cannibalism"), or early merging during cluster formation. An example of such a merger is the giant interacting elliptical galaxy ESO 146-5 (ESO 146-IG 005) in the center of the cluster Abell 3827. Its total mass is $(2.7 \pm 0.4) \times 10^{13} M_\odot$ within $37h^{-1}$ kpc according to [27]. This estimate was obtained from strong gravitational lensing. The total halo mass of ESO 146-5 is larger. It is perhaps the most massive galaxy in the nearby universe.

In conclusion, if a galaxy or a galaxy cluster is formed from a strong density perturbation and has a larger than average size and a high initial rate of baryonic mass increase $\dot{M}_b$, it will accumulate more DM.

V. CONCLUSIONS

We studied the capture of DM particles passing through a galaxy. It is associated with an increase in the mass of the galaxy, primarily its dark halo. The kinetic energy of a particle, which increases as it enters the halo and decreases while it leaves it again, may become insufficient for the particle to leave the halo if the galaxy mass increases sufficiently during the passage. This requires the combined action of two factors. One is an increase in the mass of the baryonic component of the galaxy, and the other is determined by the particle flux. Both have to be sufficiently large for significant capture of DM particles. Furthermore, both change in time, leading to quick changes in the DM capture rate. A capture occurs precisely by objects of the size of galaxies, but not by much smaller astronomical objects like stars.

As a result, the particle is captured and begins to move inside the gravitational potential well of the galaxy. The capture process can be described by catastrophe theory. DM accretion can jump from a moderate capture rate of order the baryonic mass growth to a much large value which we denote *catastrophic DM capture*. Its start may even be as early as the nonlinear growth of primordial density fluctuations during the Dark Ages. The ratio of the influx rates of dark and baryonic matter can be very significant during catastrophic DM capture which may explain the large observed DM to BM ratio in certain galaxies.

The growth rate of the mass of baryonic matter inside a galaxy, for example due to accretion and cooling or due to galaxy cannibalism, must exceed a certain threshold value to enter the catastrophic capture regime. Also, the density of DM particles in intergalactic space must exceed a certain threshold value in the catastrophic DM capture mode. Taking into account that the matter density decreases with time both due to the Hubble expansion and because of the capture of particles as discussed in this paper, it can be assumed that the capture process has either weakened significantly in the past, or will do so in the future.

Particles with sufficiently high initial velocity can fly through the galaxy, leaving it with a reduced speed due to the action of the mechanism under consideration. The higher the initial velocity, the smaller the loss of both velocity and energy of the particle. As a result, a general decrease in energy and a change in the velocity distribution of the particles occur. This process is more efficient if the galaxy is in a cluster rather than in a void.

A qualitative description of the formation of a dark halo around galaxies is given in Sec. III C. The process includes several transformations and changes in the state of the system. Particularly strong fluctuations lead to the appearance of large galaxies, often in clusters. Their size and high mass accretion rates ensure the capture of almost all DM particles that enter inside. Some quantitative estimates of the considered process in the Milky Way galaxy are presented in Sec. IV.

We believe that the approach proposed in this work, in particular the idea of a sharp transition to a regime of intense DM particle capture, can supplement our understanding of the formation of the dark halos of galaxies. Note also, that the velocity dispersion of DM is neglected in the initial conditions of N-body simulations where it is assumed that DM particle velocities are fixed exactly by the peculiar velocity field. Even if DM is expected to be cold so that velocity dispersion is probably small, our effect might help to lead to earlier galaxy formation and explain the surprising data of the JWST [28,29].

The work presented here is preliminary as we just show the main features studying a toy model. Within this toy model we can demonstrate the existence of a mode of intense DM particles capture with a catastrophic transition to this mode and back, focusing on the physical aspects of the process. On the other hand, the picture described in the article is certainly simplified. For simplicity we assume spherical symmetry of the halo and we neglect peculiar motion in a reference frame in which the distribution of DM particle velocities is isotropic. In addition we implicitly consider the capture of particles by an already sufficiently formed galaxy. However, during the formation of a galaxy from the initial overdensity, the density contrast and the DM accretion rate increase from small, linear initial conditions. For a more adequate treatment, a more detailed model will be needed, which considers the capture of dark matter during the growth of density fluctuation at its different stages, including nonlinear growth. A more realistic N-body simulation, including hydrodynamical effects of baryons is needed to show that catastrophic DM capture may be truly relevant for cosmological structure formation. Another possible continuation of this work is related to the statistical behavior of a system which is not in thermodynamic equilibrium. DM particles after passing through a growing galaxy are slowed down, lose part of their speed. It would be interesting to investigate the influence of this process on the particle velocity distribution by writing a Boltzmann transport equation for this process.

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