Comment on "Scrutinizing $\pi\pi$ scattering in light of recent lattice phase shifts"

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In a recent paper by Gao *et al.* [Phys. Rev. D **105**, 094002 (2022).], S-wave $\pi\pi$ scattering phase shifts obtained in a lattice-QCD calculation are analyzed using dispersive S-matrix methods. We question the reliability of the conclusion from this analysis that, for a pion mass of 391 MeV, the lattice phases favor the presence of both a σ-meson bound state and a nearby virtual state. Our main criticism concerns the neglect of the S-wave $K\bar{K}$ channel, which was considered alongside additional $s\bar{s}$ interpolating fields in the lattice computation used by the authors of Gao *et al.* and also in typical coupled-channel models. As an illustration, some results from such a recent model are presented as well. Concluding remarks concern possible improvements of the analysis in Gao *et al.* as well as further model tests.

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We comment on Ref. [1], concerning a two-pole description of S-wave ($IJ = 00$) pion-pion phase shifts as determined in the lattice calculation [2] of the Hadron Spectrum Collaboration (HSC) for a hypothetical pion mass of 391 MeV. The authors of Ref. [1] use dispersive S-matrix techniques with crossing-symmetry constraints to model and simultaneosly fit $\pi\pi$ phases in the channels $IJ = 00$, 20, 11 and for two different pion masses (391 MeV and 236 MeV) as computed in Ref. [2], and other papers by the HSC.

Here we focus on the conclusion in [1] that the S-wave $\pi\pi$ phase shifts mentioned above favor an S-matrix description in terms of both a bound-state (BS) and a relatively nearby virtual-state (VS) pole, instead of only a BS pole as reported in [2]. In the following we shall argue why the analysis of this particular case in [1] is unreliable. The main reason is the neglect of the $IJ = 00$ $K\bar{K}$ channel, which inevitably couples to the $IJ = 00$ $\pi\pi$ system and affects the σ resonance. In [2] both $\pi\pi$ and $K\bar{K}$ two-meson interpolating fields were included, besides the single-meson operators $u\bar{u} + d\bar{d}$ and $s\bar{s}$. Note that the inclusion of $s\bar{s}$ interpolators is crucial to describe the $f_0(980)$ resonance and the sudden jump of the S-wave $\pi\pi$ phases through 180° in the real situation with the actual pion mass. Therefore, the analysis carried out in [1] is not based on the same degrees of freedom as in the lattice simulation of [2].

So let us first consider the physical S-wave $\pi\pi$ phase shifts and their description in the coupled-channel and fully unitary quark-meson model of Ref. [3]. In this paper, the dynamically generated resonance pole of the σ on the second Riemann sheet was actually accompanied by 4095 other poles, i.e., one on each of the other Riemann sheets, owing to a total of twelve included meson-meson channels. Such a large number of two-meson channels was taken into account [3] in order to be able to predict S-wave $\pi\pi$ phases up to 1.3 GeV, as well as an additional and also complete scalar-meson nonet in the energy region 1.3–1.5 GeV. Nevertheless, all these channels couple to the two bare $^3P_0$ $u\bar{u} + d\bar{d}$ and $s\bar{s}$ channels for the coupled $f_0(500)$–$f_0(980)$ system. Near the σ resonance, the $\pi\pi$ scattering amplitude is well described [3] by the dynamically generated pole. In the lattice simulation of [2], with $m_\pi = 391$ MeV, this corresponds to the pole of a weakly bound state. All the other poles are too far away from the physical region to be very relevant there. A simple yet fully unitary toy model in Ref. [4] qualitatively shows the contribution of more distant poles to the total amplitude.

Returning to [1], Fig. 8 of the paper depicts typical S-wave subthreshold pole trajectories in the complex-energy (E) plane. After both poles hit the real axis, they can either end up as representing a pair of VSs or one VS and one true BS. The analysis in [1] leads to the latter possibility. However, the existence of a pair of two poles on the real E axis, viz. a VS pole, and either another VS pole or a BS

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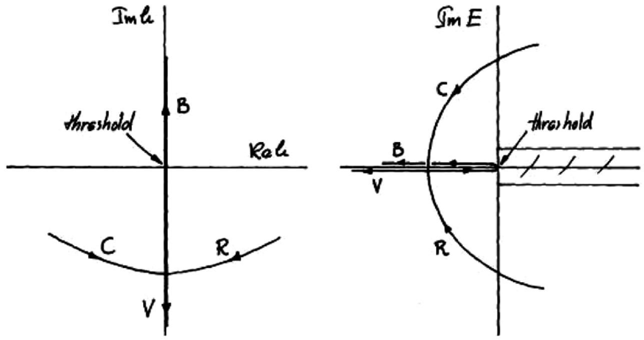

FIG. 1. S-matrix pole trajectories in the complex k and E planes, with $E = \sqrt{s} = 2\sqrt{k^2 + m_\pi^2}$ (reprinted Fig. 4.1 of Ref. [5]).

pole, has been known for decades in a single-channel study; see e.g. Figs. 4.1 (reproduced here in Fig. 1) and 4.2 of Ref. [5]. Hence, why the authors stress that "*the description of the* σ *at* $m_\pi = 391$ MeV *as a pair of bound and virtual poles is a novel finding in our study*" is not clear to us. For S-wave $K\pi$ and $K\bar{K}$ scattering we have shown [6] in detail the pole movements of the BS and VS poles as a function of the overall coupling (Figs. 6 and 7 of Ref. [6]). Similarly, the scalar $D_{s0}^*(2317)$ meson below the DK threshold is described in Ref. [7] and the axial-vector $c\bar{c}$ state $\chi_{c1}(3872)$ slightly below the S-wave $D^0\bar{D}^{\star 0}$ threshold in Ref. [8]. Threshold-mass variations were studied in Ref. [9].

In our recent modeling [10] of f_0 resonances for the physical $m_\pi = 139.57$ MeV and with the seventeen S- and D-wave meson-meson channels $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, $\rho\rho$, $\omega\omega$, $K^\star\bar{K}^\star$, $\phi\phi$, $f_0(500)f_0(500)$, $f_0(980)f_0(980)$, $K_0^\star(700)\bar{K}_0^\star(700)$, and $a_0(980)a_0(980)$, coupled to the two bare P-wave $u\bar{u} + d\bar{d}$ and $s\bar{s}$ channels as in Ref. [3], we found a dynamically generated resonance pole at $(455 - i232)$ MeV on the second Riemann sheet and no VS. This resulted from a fit to experimental S-wave $\pi\pi$ phase shifts up to 1.6 GeV. The corresponding $f_0(500)$ pole trajectories as a function of the overall model coupling λ are shown (also see Ref. [10]) in Fig. 2, with the physical $f_0(500)$ pole marked with an open circle, for $\lambda \approx 3.56$. The figure inset shows in detail how an S-wave resonance pole moves below the lowest threshold and splits into a pair of VS poles when hitting the real axis, in agreement with Ref. [1] and Ref. [5]. Taking a much larger overall coupling, somewhere between 6.0 and 6.5, we obtain a BS and a VS pole at 0.16 MeV and 62 MeV below the $\pi\pi$ threshold, respectively. However, for $m_\pi = 391$ MeV and with the fitted value of the overall coupling, we find a bound state at 760 MeV, compatible with the lattice result of 758 MeV in Ref. [2], but no nearby VS pole. This is to be contrasted to the dispersive analysis in Ref. [1] for the same $m_\pi = 391$ MeV, which extracted BS and VS poles at 1 MeV and 73 MeV below threshold, respectively.

One might question the trustworthiness of our model predictions in Refs. [3,10], owing to the lack of imposed crossing-symmetry constraints, despite the remarkably good predictions for the σ pole in both cases. An explanation may be provided by duality, as remarked in Ref. [11] (also see the pioneering articles in Ref. [12]):

> … the well-known dual model result for $q\bar{q}$ resonances, that a sum of s-channel resonances also describes t- and u-channel phenomena.

Note that the model calculation referred to in Ref. [11] only includes one (bare) s-channel state in a unitarized approach, whereas the unitarized models in Refs. [3,10] contain an infinite tower of such states.

To conclude, we do not question the technical rigor of the dispersive analysis in Ref. [1]. However, we hope to have made it clear that a reliable quantitative extraction of possible BS and VS poles from scattering data, be they experimental or resulting from lattice simulations, require the consideration of all nearby resonances and inelastic

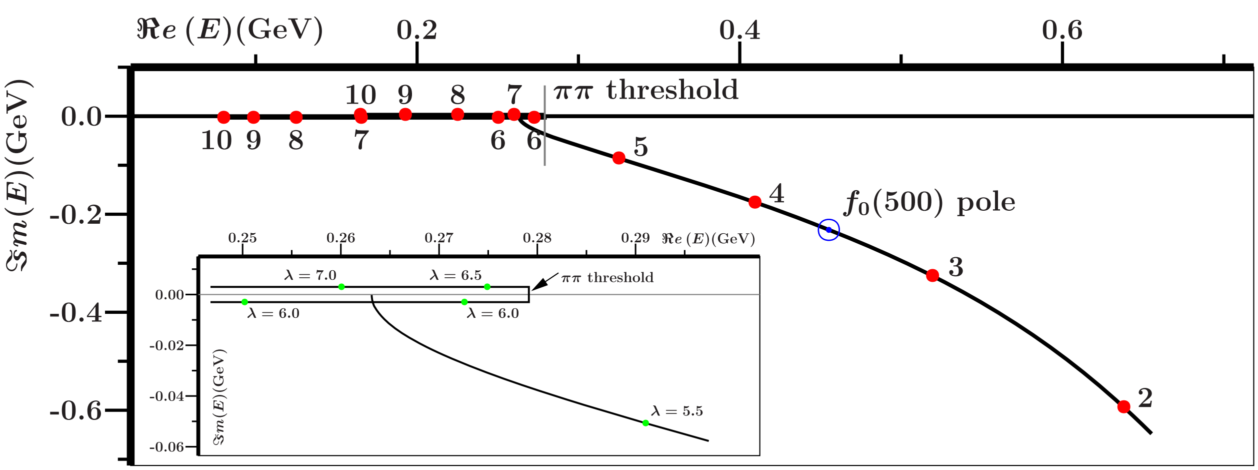

FIG. 2. $f_0(500)$ pole trajectory as a function of λ. The open circle corresponds to the fitted λ value. The inset shows details of (virtual) bound states, for clarity depicted slightly (below) above the real axis. Figure is reprinted from Fig. 2 of Ref. [10].

two-meson channels. In the particular case of S-wave $\pi\pi$ phase shifts, inclusion of the $f_0(980)$ resonance, which strongly affects [13] the phases around 1 GeV, as well as the $K\bar{K}$ threshold at about 990 MeV is indispensable. Taking a pion mass of 391 MeV and so generating the σ as a weakly bound state does not mean that the influence of the $K\bar{K}$ channel is negligible. Perhaps even more importantly and as already mentioned above, at the quark level the lattice calculation in [2] also included $s\bar{s}$ interpolators besides $u\bar{u} + d\bar{d}$, inevitably influencing the resulting $\pi\pi$ phases through the employed coupled-channel analysis. Moreover, our general experience with VS poles in multichannel models is that they are much more sensitive to small changes than BS poles. Finally, the $IJ = 00$ $\pi\pi$ lattice phases of [2] have sizable statistical error bars, so that any quantitative conclusion from a fit to those and other lattice data would already require a lot of caution.

The very lattice results of [2] appear to confirm the single-pole scenario, by having extracted a BS (and no nearby VS) very close to our result for $m_\pi = 391$ MeV in the multichannel model of Ref. [10], with the same $u\bar{u} + d\bar{d}$, $s\bar{s}$, $\pi\pi$, and $K\bar{K}$ degrees of freedom. We do not know whether a coupled-channel generalization of the dispersive methods in Ref. [1] so as to include besides $\pi\pi$ also the $K\bar{K}$ channel is feasible, but it would certainly be a topic of interest. For instance, in Ref. [14] a three-channel S-matrix parametrization with imposed crossing-symmetry constraints was used to analyze P-wave $\pi\pi$ scattering data and determine excited vector ρ resonances. Furthermore, we plan to do a comparative study in a simplified version of the model employed in Ref. [10], which would even allow to explore the behavior of bound-state and virtual σ poles as a continuous function of the pion mass.

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Correction: The byline footnote for the first author was set up incorrectly during the production process and has been remedied.