Physics implication from a Z_3 symmetry of matter

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I show that breaking $B-L$ by one unit of this charge is suitable for neutrino mass generation through an inverse seesaw mechanism, stabilizing a dark matter candidate without supersymmetry, as well as solving the muon anomalous magnetic moment and the W mass deviation via dark field contributions. The new physics is governed by the residual Z_3 symmetry of $B-L$ isomorphic to the center of the color group, instead of the well-studied matter parity.

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I. INTRODUCTION

Of the exact conservations in physics, the conservation of baryon number minus lepton number, say $B-L$, is questionable. Of the fundamental dynamics in physics, the confinement of colors within hadrons that allows only hadronic states of types qqq, qq^*, and their conjugation/combination causes curiosity. Such behavior of hadrons indeed obeys an exact Z_3 symmetry that governs constituent quarks, independent of the colors. There is no necessary principle of the $B-L$ conservation as well as the Z_3 symmetry of quarks, since they directly result from the standard model gauge symmetry. Indeed, every interaction of the standard model separately preserves B and L such that $B-L$ is conserved and anomaly-free, thus quantum consistent, if right-handed neutrinos are simply imposed, while the Z_3 symmetry of quarks is accidentally conserved by the $SU(3)_C$ color group and never violated, because this Z_3 can be regarded, isomorphic to the center of the color group.

In contrast to electric and color charges, the excess of baryons over antibaryons of the universe suggests that $B-L$ would be broken. Furthermore, $B-L$ breaking is strongly implied by compelling neutrino mass mechanisms [1–9]. $B-L$ is likely to occur in the theories of left-right symmetry [10–12] and grand unification [13], but no such traditional theories manifestly explain the existence of the accident Z_3 symmetry of quarks, similarly to the standard model. I point out that such hidden features of the standard model naturally arise from a $U(1)_{B-L}$ gauge symmetry. It is noted that in a period the matter parity—a residual symmetry of $B-L$ transforming trivially on normal matter—has been found usefully in supersymmetry [14]. I argue that there is no matter parity at all. The Z_3 symmetry of quarks plays the role instead in which this Z_3 relates to $B-L$ as the smallest and unique residual symmetry of $B-L$ itself.

Consequently, this proposal leads to novel physical results for neutrino mass [15,16], dark matter [17–19], the muon anomalous magnetic moment [20], and the W mass deviation [21], without necessity of any left-right symmetry, grand unification, or supersymmetry. Namely, the neutrino mass generation is induced by an inverse seesaw mechanism due to the breaking of $B-L$ by one unit. The dark matter stability is ensured by the residual Z_3 symmetry of $B-L$, i.e., the Z_3 symmetry of quarks, while the muon magnetic moment and the W mass are contributed by the dark sector that contains the dark matter.

II. PROPOSAL OF THE MODEL

The full gauge symmetry is

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}. \tag{1}$$

Leptons and quarks transform under this symmetry as

$$l_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \end{pmatrix} \sim (1, 2, -1/2, -1), \tag{2}$$

$$\nu_{aR} \sim (1, 1, 0, -1), \qquad e_{aR} \sim (1, 1, -1, -1), \tag{3}$$

$$q_{aL} = \begin{pmatrix} u_{aL} \\ d_{aL} \end{pmatrix} \sim (3, 2, 1/6, 1/3), \tag{4}$$

$$u_{aR} \sim (3, 1, 2/3, 1/3), \qquad d_{aR} \sim (3, 1, -1/3, 1/3), \tag{5}$$

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where the subscript $a = 1, 2, 3$ is a family index, and the right-handed neutrinos ν_{aR} are included for $B-L$ anomaly cancelation, as usual. The gauge anomaly always vanishes if including any gauge-singlet chiral fermion (or sterile fermion), such as

$$N_{aL} \sim (1,1,0,0), \tag{6}$$

where three copies of the sterile fermion are proposed, corresponding to three families. Note that the gauge symmetry suppresses bare masses of $\nu_R \nu_R$ type, while it allows bare masses of such type for $N_L N_L$.

The gauge symmetry breaking proceeds through the usual Higgs doublet,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1,2,1/2,0), \tag{7}$$

and a scalar singlet,

$$\chi \sim (1,1,0,1), \tag{8}$$

that couples N_L to ν_R through $\bar{N}_L \nu_R \chi$ couplings. They have vacuum expectation values (VEVs),

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \qquad \langle \chi \rangle = \Lambda/\sqrt{2}, \tag{9}$$

such that $\Lambda \gg v = 246$ GeV for consistency with the standard model. The scheme of symmetry breaking is

$$\begin{gathered} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L} \\ \downarrow \Lambda \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes R \\ \downarrow v \\ SU(3)_C \otimes U(1)_Q \otimes R \end{gathered}$$

Here $Q = T_3 + Y$ combines the weak isospin and hypercharge, as usual, whereas $R = Z_3$ is the residual symmetry of $B-L$, explicitly derived below.

Notice that our theory does not conserve a matter parity, $M_P = (-1)^{3(B-L)+2s}$, since it is broken by Λ, in contrast to the usual theories of $B-L$, left-right symmetry, and $SO(10)$ unification. Intriguingly, the postulate of the $B-L$ gauge symmetry and its breaking by a single $B-L$ charge, i.e., $B-L=1$, reveal important results of neutrino mass, dark matter, muon $g-2$, and W mass deviation, presented in order.

III. NEUTRINO MASS GENERATION VIA INVERSE SEESAW

The relevant Lagrangian includes

$$\begin{aligned} \mathcal{L} &\supset h_{ab}\bar{l}_{aL}\tilde{\phi}\nu_{bR} + f_{ab}\bar{N}_{aL}\nu_{bR}\chi - \frac{1}{2}\mu_{ab}\bar{N}_{aL}N^c_{bL} + \text{H.c.} \\ &\supset -\frac{1}{2}(\bar{\nu}_{aL}\bar{\nu}^c_{aR}\bar{N}_{aL}) \begin{pmatrix} 0 & m_{ab} & 0 \\ m_{ba} & 0 & M_{ba} \\ 0 & M_{ab} & \mu_{ab} \end{pmatrix} \begin{pmatrix} \nu^c_{bL} \\ \nu_{bR} \\ N^c_{bL} \end{pmatrix} + \text{H.c.} \end{aligned}$$

Here $b = 1, 2, 3$ is a family index as a is, $\tilde{\phi} = i\sigma_2\phi^* \sim (1,2,-1/2,0)$, and a superscript ^{c} indicates charge conjugation. Additionally, the mass terms in second line are obtained by substituting the VEVs of scalars, in which $m_{ab} = -h_{ab}v/\sqrt{2}$ and $M_{ab} = -f_{ab}\Lambda/\sqrt{2}$ are Dirac mass matrices that couple ν_{aL} to ν_{bR} and N_{aL} to ν_{bR}, respectively, while μ_{ab} is a Majorana mass matrix that couples N_L's themselves, as given.

$m \ll M$ is naturally imposed, since $v \ll \Lambda$. Assuming $\mu \ll m \ll M$, the total mass matrix of neutrinos and sterile fermions takes a form of inverse seesaw [22–24]. Hence, the observed neutrino mass matrix is approximately given as $\mathcal{L} \supset -\frac{1}{2}\bar{\nu}_{aL}(m_\nu)_{ab}\nu^c_{bL} + \text{H.c.}$, where

$$m_\nu \simeq mM^{T,-1}\mu M^{-1}m^T \sim (v/\Lambda)^2\mu, \tag{10}$$

which is doubly suppressed by v/Λ, in contrast to the canonical seesaw recognized in the usual $U(1)_{B-L}$ model with $B-L$ breaking by two units instead. The neutrino masses take sub-eV values suitable to observation, say $m_\nu \sim 0.1$ eV [25], given that $\Lambda \sim 10$ TeV and $\mu \sim 1$ keV. Note that the mixing of ν_L with (ν^c_R, N_L) is suppressed by $mM^{-1} \ll 1$ and is thus neglected. The new fermions ν_R, N_L obtain a Dirac mass $\sim M$ at TeV scale.

The unique property of this seesaw setup is specified as follows. Besides giving the new gauge boson mass, the $B-L$ breaking VEV, i.e., Λ, is the largest scale in the inverse seesaw for neutrino masses. This is contrary to the conventional inverse seesaw in which $B-L$ is broken at a low scale, around keV, to induce a Majorana mass term; here, this symmetry is broken above the weak scale, giving rise to a Dirac mass term. Hence, the required smallness of such a Majorana mass, i.e., μ, is not related to the $B-L$ symmetry at all. It is noted that in the limit $\mu \to 0$, our theory contains a global lepton-like symmetry, i.e., $f \to e^{i\varphi}f$ for $f = l_L, \nu_R, e_R, N_L$, which has a nature distinct from the $B-L$ gauge symmetry. Hence, the small μ is due to this symmetry protection, i.e., naturally explained by a bigger theory via a large scale or loops. Our proposal also differs from the conventional inverse seesaw in that an unreasonable Majorana mass term for ν_R is suppressed by the $B-L$ gauge symmetry, while in the conventional theory such ν_R Majorana mass arises as it has an origin identical to the N_L Majorana mass.

The Λ scale as given is suitable to collider constraints on the $U(1)_{B-L}$ gauge boson, called Z'. Indeed, the LEPII studied processes $e^+e^- \to ff^c$ for $f = \mu, \tau$ contributed by Z', giving a bound $m_{Z'}/g_{B-L} > 6$ TeV [26]. Here g_{B-L} is the $U(1)_{B-L}$ coupling, and the Z' mass is $m_{Z'} = g_{B-L}\Lambda$. This translates to $\Lambda > 6$ TeV [27]. The LHC searched for dilepton signals through $pp \to ff^c$ contributed by Z', supplying a bound $m_{Z'} \sim 4$ TeV for Z' couplings identical to those of the Z boson [28]. This converts to $\Lambda \sim m_{Z'}/g \sim 6$ TeV, similar to the LEPII.

IV. RESIDUAL SYMMETRY AND RESULTANT DARK SECTOR

Note that Λ breaks only $U(1)_{B-L}$ down to R, whereas v that breaks the electroweak symmetry obviously conserves R. The residual symmetry R takes the form $R = e^{i\alpha(B-L)}$ since it is a $U(1)_{B-L}$ transformation. R conserves the vacuum Λ if $R\Lambda = e^{i\alpha(1)}\Lambda = \Lambda$, since Λ has $B-L=1$. It follows that $e^{i\alpha}=1$, or $\alpha = 2\pi k$, for k integer. Hence, I obtain $R = e^{i2\pi k(B-L)} = [w^{3(B-L)}]^k$, where $w \equiv e^{i2\pi/3}$ is the cube root of unity. The model fields transform under R as in Table I, where $B-L$ is supplied for convenience in reading. It is clear that $R=1$ for every field corresponds to the smallest value of $|k|=3$, except for the identity with $k=0$. Hence, the residual symmetry R is automorphic to

$$Z_3 = \{1, \mathcal{G}, \mathcal{G}^2\}, \tag{11}$$

where $\mathcal{G} \equiv w^{3(B-L)}$, and $\mathcal{G}^3 = 1$ for every field, as mentioned [29]. Obviously, the residual symmetry Z_3 is generated by $\mathcal{G}$, called *matter generator*, opposite to the matter parity studied in supersymmetry.

Z_3 has three irreducible representations $\underline{1}$, $\underline{1}'$, and $\underline{1}''$ according to $\mathcal{G} = 1$, w, and w^2, respectively. The field representations under Z_3 are given in Table II. It is clear that every field transforms trivially under Z_3 with $\mathcal{G}=1$, except for quarks. Quarks are in $\underline{1}'$ with $\mathcal{G}=w$, whereas antiquarks belong to $\underline{1}''$ with $\mathcal{G}=w^2$. Hence, the hidden Z_3 symmetry of quarks in the standard model can be interpreted to be the residual symmetry of $B-L$. In contrast to the hidden symmetry, the residual symmetry explicitly relates to $B-L$ that would lead to dark matter with an appropriate $B-L$ value. That said, a dark field possesses a $B-L$ charge such that the matter generator is nontrivial, i.e., $\mathcal{G} = w^{3(B-L)} \neq 1$. Combined with $\mathcal{G}^3 = 1$ that ensures the Z_3 symmetry, I obtain

$$B-L = \begin{bmatrix} -1/3 + k \\ -2/3 + k' \end{bmatrix} = \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{5}{3}, \cdots \tag{12}$$

for k, k' integer. This identification of dark field is independent of its spin. Additionally, the signs $\pm$ correspond to a dark field and its conjugation. Each dark field can pick up a $B-L$ charge only differing from either of the two basic charges, say $-1/3$ and $-2/3$, by an integer number, because of the cyclic property of Z_3. For such reasons, it is sufficient to introduce two dark fields with respect to the two basic charges, respectively; that is, a dark (Dirac) fermion and a dark vector transform under the gauge symmetry as

$$F \sim (1,1,0,-1/3), \qquad V = \begin{pmatrix} V^0 \\ V^- \end{pmatrix} \sim (1,2,-1/2,-2/3),$$

which couple to lepton doublets,

$$\mathcal{L} \supset x_a \bar{l}_{aL}\gamma^\mu F_L V_\mu + \text{H.c.}, \tag{13}$$

in order to make the model phenomenologically viable. The detailed reason of this choice (cf. [30]) comes from the muon $g-2$, presented below. Notice that V and F transform under Z_3 as $\underline{1}'$ and $\underline{1}''$, for $\mathcal{G} = w$ and w^2, respectively, as given in Table III.

Apart from the above couplings, V and F possess the Lagrangian terms [31],

$$\begin{aligned}\mathcal{L} \supset{}& \bar{F}(i\gamma^\mu D_\mu - m_F)F - \frac{1}{2}V^\dagger_{\mu\nu}V^{\mu\nu} + m_V^2 V^\dagger_\mu V^\mu \\ &+ i\kappa_1 V^\dagger_\mu A^{\mu\nu}V_\nu + i\kappa_2 V^\dagger_\mu B^{\mu\nu}V_\nu + i\kappa_3 V^\dagger_\mu C^{\mu\nu}V_\nu \\ &+ \alpha_1(V^\dagger_\mu V^\mu)(V^\dagger_\nu V^\nu) + \alpha_2(V^\dagger_\mu V^\nu)(V^\dagger_\nu V^\mu) \\ &+ \alpha_3(V^\dagger_\mu V^\nu)(V^{\dagger\mu}V_\nu) + \lambda_1(\chi^\dagger\chi)(V^\dagger_\mu V^\mu) \\ &+ \lambda_2(\phi^\dagger\phi)(V^\dagger_\mu V^\mu) + \lambda_3(\phi^\dagger V_\mu)(V^{\dagger\mu}\phi),\end{aligned} \tag{14}$$

where $V_{\mu\nu} \equiv D_\mu V_\nu - D_\nu V_\mu$, and $D_\mu = \partial_\mu + igT_jA_{j\mu} + ig_Y YB_\mu + ig_{B-L}(B-L)C_\mu$ is covariant derivative, in which

TABLE I. $B-L$ charge and R value of all fields, where l, q, N, and A define every lepton (including ν_R), quark, sterile fermion, and gauge boson, respectively.

Field	l	q	χ	$\{N,\phi,A\}$
$B-L$	-1	$1/3$	1	0
R	1	w^k	1	1

TABLE II. Matter generator and field representations under the residual symmetry Z_3.

Field	l	q	χ	$\{N,\phi,A\}$
$\mathcal{G}$	1	w	1	1
Z_3	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$

TABLE III. Dark field identification according to Z_3.

Dark-field	V	F
$\mathcal{G}$	w	w^2
Z_3	$\underline{1}'$	$\underline{1}''$

A_μ ($A_{\mu\nu}$), B_μ ($B_{\mu\nu}$), and C_μ ($C_{\mu\nu}$) denote gauge fields (field strengths) of $SU(2)_L$, $U(1)_Y$, and $U(1)_{B-L}$, respectively. (Omitting a small kinetic mixing between two $U(1)$ gauge fields, C is identical to Z', while A, B define W, Z, γ.) This theory preserves the Z_3 symmetry that acts on V, F, in contrast to that in [32–34]. After the symmetry breaking, the vector doublet is separated in mass, $m^2_{V^\pm} - m^2_{V^0} = \lambda_3 v^2/2$, proportional to the weak scale, small compared to V masses, $m^2_{V^\pm} = m^2_V + \lambda_1\Lambda^2/2 + (\lambda_2 + \lambda_3)v^2/2$ and $m^2_{V^0} = m^2_V + \lambda_1\Lambda^2/2 + \lambda_2 v^2/2$, at Λ scale. Dark vectors generically violate the unitarity of S-matrix. The unitarity condition for $\langle VV^\dagger|S|V_g V_g^\dagger\rangle$ with $V_g \in \{A, B, C\}$ constrains $\kappa_1 = g$, $\kappa_2 = -g_Y/2$, and $\kappa_3 = -2g_{B-L}/3$, whose coefficients correspond to the gauge charges of V under symmetry (1). This match of $\kappa_{1,2,3}$ to gauge couplings must be applied so that the theory works well up to the current energy of colliders at TeV, where the standard model is still good, in agreement with [32]. The unitarity condition for elements like $\langle VV^\dagger|S|VV^\dagger\rangle$ and $\langle VV|S|VV\rangle$ would relate $\alpha_{1,2,3}$ themselves, but since $\alpha_{1,2,3}$ are irrelevant to the processes studied in this work, I will not refer to them further. On the other hand, interactions in (13) also give rise to unitarity violations like $\langle VV^\dagger|S|ll^\dagger\rangle$. The unitarity is preserved, independent of x_a, by introducing either a new fermion or a new vector that appropriately couples to V, l. But, except for this role, the extra particle would not alter our results considered below, thus skipped.

It is noteworthy that because F and V are color neutral, the lightest field of them cannot decay to colored quarks, despite the fact that both the dark field and quarks transform nontrivially under Z_3. Indeed, since the lightest dark field is color neutral, it cannot decay to a single quark, due to color conservation. Further, if the dark field decays to a pair of quarks, the final state must take the form qq^c due to color conservation. Because qq^c is trivial under Z_3, while the dark field is nontrivial under Z_3, the decay of the dark field to qq^c is suppressed by Z_3 conservation. If the dark field decays to three kinds of quarks, the final state must take the form qqq due to color conservation. But, this state is trivial under Z_3, while the dark field is not. Hence, the decay of the dark field to qqq is suppressed by Z_3 conservation. Generically, if the dark field decays to a number of quarks, the final state must be composed of qq^c and/or qqq due to color conservation. But this final state is trivial under Z_3, hence suppressed by Z_3 conservation. In this case, the stability of the lightest dark field is preserved by the color charge conservation, in addition to Z_3. This stability mechanism differs from many extensions for dark matter, including supersymmetry.

V. DARK MATTER ABUNDANCE AND DETECTION

There are two candidates for dark matter, V^0 and F. For the case of V^0, it must be the lightest of dark fields, $m_{V^0} < m_F$ and $m_{V^0} < m_{V^\pm}$ [35]. Unfortunately, this vector

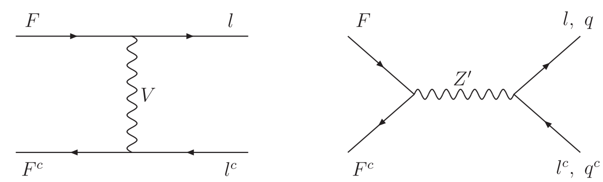

FIG. 1. Dark matter annihilation to normal matter.

candidate as a complex field belongs to a weak doublet interacting with the usual Z boson and is not separated in mass. The gauge interaction will induce a large scattering cross section of V^0 with nuclei by t-channel Z exchange in direct detection, which is already ruled out by experiments, analogously to the inert scalar doublet [36]. The model predicts the realistic dark matter to be a dark fermion, F [37]. This fermion candidate interacts with the usual particles via V and Z' portals. The annihilation processes of F to usual particles are described by Feynman diagrams in Fig. 1, where we define $l = \{\nu_a, e_a\}$ for usual leptons and $q = \{u_a, d_a\}$ for usual quarks.

As shown below for the muon $g-2$, the $V^\pm$ mass and x_2 coupling satisfy $|x_2|^2/4\pi m^2_{V^\pm} \sim (800\ \mathrm{GeV})^{-2}$. Hence, the t-channel diagram exchanged by V largely contributes to the annihilation cross section, unless m_F is much smaller than m_V, in agreement with [34]. I also assume $m_F \ll m_{Z'}$, besides the condition $m_F \ll m_V$. Further, because of $m_{V^0} \approx m_{V^\pm}$ and $m_{Z'} = g_{B-L}\Lambda$, the annihilation cross section that includes both V, Z' contributions as in Fig. 1 is approximated as

$$\langle\sigma v\rangle \simeq 1\ \mathrm{pb}\left(\frac{m_F}{6.5\ \mathrm{GeV}}\right)^2\left(\frac{800\ \mathrm{GeV}}{m_{V^\pm}}\right)^4\left[\left(\frac{\sum_a|x_a|^2}{4\pi}\right)^2 - \left(\frac{\sum_a|x_a|^2}{4\pi}\right)\frac{1}{6\pi}\frac{m^2_{V^\pm}}{\Lambda^2} + \frac{37}{432\pi^2}\frac{m^4_{V^\pm}}{\Lambda^4}\right]. \quad (15)$$

This result excludes annihilation to top quarks, similarly to annihilation to right-handed neutrinos, since the dark matter is radically lighter than such fields. It is clear that $(m_{V^\pm}/\Lambda)^2 \sim 10^{-2}(|x_2|^2/4\pi)$ for $\Lambda \sim 10$ TeV. Hence, the contributions of the $m_{V^\pm}/\Lambda$ terms, i.e., of the Z' boson, to the annihilation cross section are small. The expression in brackets is dominated by the first term due to the contribution of V. Taking $\sum_a |x_a|^2/4\pi \sim 1$ in perturbative limit and $m_{V^\pm} \sim 800$ GeV similar to the muon $g-2$ below, the dark matter gets a correct abundance, i.e., $\langle\sigma v\rangle \sim 1$ pb [25], if $m_F \sim 6.5$ GeV. Here I assume that there is no asymmetry in number density between a dark particle and a dark antiparticle.

In direct detection, the dark matter F scatters with quarks confined in nucleons exchanged by Z', described by the effective Lagrangian,

$$\mathcal{L}_{\rm eff} \supset \frac{g_{B-L}^2}{9m_{Z'}^2}(\bar{F}\gamma^\mu F)(\bar{q}\gamma_\mu q). \quad (16)$$

Therefore, the scattering cross section of F on a nucleon (p, n) is evaluated by

$$\sigma_{p,n} \simeq 3.7 \times 10^{-45}\left(\frac{10\ \text{TeV}}{\Lambda}\right)^4\ \text{cm}^2. \quad (17)$$

Given that $\Lambda = 10$ TeV, the model predicts $\sigma_{p,n} \simeq 3.7 \times 10^{-45}$ cm^2, in good agreement with the XENON1T experiment for dark matter mass at 6.5 GeV [38,39].

It is noted that the Z_3 symmetry allows only multi dark-particles produced at particle colliders. Monophoton events may be recognized at the LEPII experiment, recoiled against the missing energy carried by a pair of dark matter F, governed by the effective interactions

$$\mathcal{L}_{\rm eff} \supset \frac{|x_1|^2}{4m_{V^\pm}^2}(\bar{F}\gamma^\mu F)(\bar{e}\gamma_\mu e) + (AA) + (VA) + (AV), \quad (18)$$

which are derived directly from (13), with the aid of the Fierz identity. These vector and axial vector operators have been studied in [40], leading to a bound

$$m_{V^\pm} > \frac{|x_1|}{2} \times 470\ \text{GeV} \sim 800\ \text{GeV}, \quad (19)$$

according to $|x_1|^2/4\pi \sim 0.92$, as expected. This mass limit agrees with the relic density and direct detection, as well as the muon $g-2$ below.

Further, monojet signals may be generated at the LHC against large missing energy carried by a F pair, set by the effective interaction as in (16) because the Z' mediator for this process possesses a mass $m_{Z'} = g_{B-L}\Lambda$ radically heavier than the transferred momentum (<1 TeV), with an appropriate g_{B-L} value. Reference [41] has limited $g_{B-L}^2/9m_{Z'}^2 = (1/3\Lambda)^2 < (1/1.1\ \text{TeV})^2$, which is always satisfied for Λ at TeV. Indeed for $\Lambda \sim 10$ TeV, the monojet signature is negligible. Additionally, since the LHC is more energetic, a pair of dark vectors each with mass about 800 GeV may be produced as $pp \to VV^\dagger$ and followed by $V, V^\dagger$ decays to stable F dark matter, $V \to F^c l$ and $V^\dagger \to F l^c$, due to Z_3 conservation. Total cross section is $\sigma(pp \to VV^\dagger \to FF^c ll^c) = \sigma(pp \to VV^\dagger) \times \text{Br}(V \to F^c l) \times \text{Br}(V^\dagger \to F l^c)$, with the aid of narrow width approximation. The cross section $\sigma(pp \to VV^\dagger)$ is governed by γ, Z but not totally understood in this setup, since it violates unitarity similarly to the mentioned process $\langle VV^\dagger|S|ll^\dagger\rangle$, due to lack of UV completion. It is shown that relevant UV theory [42] only removes unphysical contributions arising from bad behavior of V at high energy, while does not significantly modify the cross section $\sigma(pp \to VV^\dagger)$ that comes from new fields living at UV regime >1 TeV (cf., e.g., [43]). Hence, $\sigma(pp \to VV^\dagger)$ is obtained by γ, Z contributions after removing the bad terms, given at quark level as $\sigma(qq^c \to VV^\dagger) \simeq (\pi\alpha^2/36E^2)(1 - m_V^2/E^2)^{3/2} [Q_q^2 Q_V^2 + Q_q Q_V v_q v_V / s_W^2 c_W^2 + (v_q^2 + a_q^2) v_V^2 / s_W^4 c_W^4]$, where the energy of quark obeys $E = \frac{1}{2}\sqrt{s} > m_V \gg m_Z$. I have defined $v_q = T_{3q} - 2s_W^2 Q_q$, $a_q = T_{3q}$, and $v_V = T_{3V} - s_W^2 Q_V$, where $Q_{q,V}$ $(T_{3q,V})$ are the electric charge (weak isospin) of q, V, respectively. Alternatively, this cross section can be derived, assuming the equivalence theorem $\sigma(qq^c \to VV^\dagger) \simeq \sigma(qq^c \to \Phi\Phi^\dagger)$, where Φ denotes the Goldstone boson doublet associated to V, which couples to γ, Z like V. At high energy V is identical to Φ that has quantum numbers as left-handed slepton doublet, i.e., $\sigma(qq^c \to VV^\dagger) \simeq \sigma(qq^c \to \tilde{l}_L \tilde{l}_L^\dagger)$. The ATLAS [44] and CMS [45] have studied a process for direct slepton production $pp \to \tilde{l}\tilde{l}^* \to ll^c \tilde{\chi}_1^0 \tilde{\chi}_1^0$ assuming $\text{Br}(\tilde{l} \to l\tilde{\chi}_1^0) \simeq 1$, where $\tilde{\chi}_1^0$ is the LSP dark matter, setting a bound for charged slepton mass at 700 GeV. Given that V significantly couples to ll^c product, i.e., $\text{Br}(V \to F^c l) \simeq 1$, the SUSY result applies to our case without change, i.e., $m_V > 700$ GeV. Hence, the equivalence theorem ensures high energy behavior of V as a well-studied slepton, predicting its mass limit, as expected.

VI. MUON $g-2$

The anomalous magnetic moment of muon, $a_\mu = \frac{1}{2}(g-2)_\mu$, in the standard model is now established, $a_\mu(\text{SM}) = 116591810(43) \times 10^{-11}$ [46]. The recent measurement of a_μ provides an exciting hint for the new physics [20], in which this new result combined with the old E821 result [47] gives a deviation,

$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}, \quad (20)$$

at 4.2σ from the standard model prediction. If this result is confirmed, many new physics approaches might be disfavored, since such deviation is larger than the electroweak contribution, say $a_\mu(\text{EW}) = 153.6(1.0) \times 10^{-11}$, and potentially in tension with those from the electroweak precision test and current colliders.

I suggest to solve this question by a contribution from the dark sector. That said, the presence of interactions in (13) contributes to the muon $g-2$ through a diagram given in Fig. 2. Assuming $m_\mu \ll m_F, m_{V^\pm}$, I obtain

$$\Delta a_\mu = \frac{|x_2|^2 m_\mu^2}{8\pi^2 m_{V^\pm}^2}\int_0^1 dt t \frac{t(1+t)m_{V^\pm}^2 + (1-t)(1-\frac{t}{2})m_F^2}{t m_{V^\pm}^2 + (1-t)m_F^2},$$

where x_2 couples F to the muon doublet of interest. The integral is of the order of 1, thus

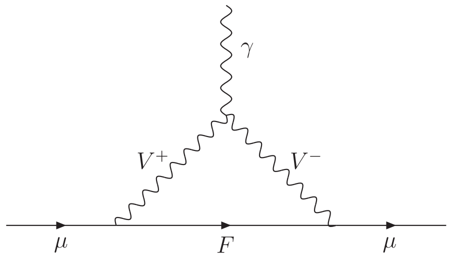

FIG. 2. Dark field contribution to the muon $g-2$.

$$\Delta a_\mu \sim 2.5\times 10^{-9}\left(\frac{|x_2|^2}{4\pi}\right)\left(\frac{800\ \mathrm{GeV}}{m_{V^\pm}}\right)^2. \tag{21}$$

Compared to the muon $g-2$ deviation in (20), it gives

$$m_{V^\pm} \sim 800\sqrt{\frac{|x_2|^2}{4\pi}}\ \mathrm{GeV}. \tag{22}$$

This prediction agrees with the dark matter constraint. The $V^\pm$ field gains $m_{V^\pm}\sim 800$ GeV for $|x_2|^2/4\pi\sim 1$.

VII. W MASS DEVIATION

The renormalized masses of W, Z in the on-shell scheme are related by $m_W^2(1-m_W^2/m_Z^2)=(\pi\alpha/\sqrt{2}G_F)(1+\Delta r)$, where $\Delta r=(\Delta r)^{\mathrm{SM}}+(\Delta r)^{\mathrm{NP}}$ presents quantum corrections due to the standard model and the new physics, respectively. The standard model predicts $m_W^{\mathrm{SM}}=80.357\pm 0.006$ GeV, extracted upon the precisely measured parameters (G_F,α,m_Z) and $(\Delta r)^{\mathrm{SM}}\simeq 0.038$ [48]. Given that the new physics arises as oblique contributions, one obtains $(\Delta r)^{\mathrm{NP}}=-(c_W^2/s_W^2)\Delta\rho$, where $\Delta\rho=\alpha(m_Z)T$ is the ρ-parameter deviation from the standard model related via the T-parameter. Recently, the CDF II collaboration has reported a novel result of W mass, $m_W=80.4335\pm 0.0094$ GeV, differing from the standard model prediction at 7σ [21]. This high precision measurement of W mass reveals an exciting hint for the new physics, implying $(\Delta r)^{\mathrm{NP}}\simeq -0.00489$. With $\alpha(m_Z)=1/128$ and $s_W^2=0.231$, it gives rise to $T\simeq 0.188$.

In the present model, the deviation of the measured W mass from the standard model expectation arises from a positive contribution of the non-degenerate vector doublet V to the T-parameter, evaluated by

$$T=\frac{3\alpha^{-1}(m_Z)}{16\pi^2v^2}\left[m_{V^\pm}^2+m_{V^0}^2-\frac{2m_{V^\pm}^2m_{V^0}^2}{m_{V^\pm}^2-m_{V^0}^2}\ln\frac{m_{V^\pm}^2}{m_{V^0}^2}\right],$$

where the coefficient 3 comes from three physical degrees of freedom of massive vectors [49]. I have included the contributions of V to W, Z self-energies arising from both gauge interactions of V and $\kappa_{1,2,3}$ couplings furnished by the unitarity constraint. The computation in [50] for T in 't Hooft-Feynman gauge coincides with the above result in the unitary gauge. Notice that gauge dependence similar to the standard model W, Z, γ contributions to T does not arise, since V is not a gauge field [51]. Because the vector mass splitting is small, i.e., $m_{V^\pm}^2-m_{V^0}^2=\lambda_3v^2/2\ll m_{V^0}^2$, I further approximate

$$T\simeq 0.188\frac{\lambda_3^2}{\pi}\left(\frac{783\ \mathrm{GeV}}{m_{V^0}}\right)^2. \tag{23}$$

This coincides with the measured value of W mass, i.e., $T\simeq 0.188$, given that

$$m_{V^0}\simeq 783\sqrt{\frac{\lambda_3^2}{\pi}}\ \mathrm{GeV}. \tag{24}$$

This mass is comparable to that of the charged dark vector, if λ_3 is similar in size to x_2.

VIII. CONCLUDING REMARKS

I have investigated a Z_3 symmetry of matter set by $\mathcal{G}=w^{3(B-L)}$ transformation, governing quarks as well as neutrino masses via inverse seesaw. This Z_3 yields two dark fields V, F as potential solutions to dark matter, muon $g-2$, and W mass deviation. Components of V gain a mass about 800 GeV, whereas F mass is at 6.5 GeV. Couplings of V with leptons and Higgs boson are near perturbative limit, $|x_2|^2/4\pi\sim 1$, $|x_1|^2/4\pi\lesssim 0.92$, and $\lambda_3^2/\pi\sim 1$ [52]. Such V also satisfies all other high energy collider bounds [32]. The present effective theory of V, F with predicted couplings reveals that the more fundamental theory may encounter either a Landau pole or a technicolor scheme above TeV [53].

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