Integrating out heavy scalars with modified equations of motion: Matching computation of dimension-eight SMEFT coefficients

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The shift in focus towards searches for physics beyond the Standard Model employing model-independent effective field theory methods necessitates a rigorous approach to matching to guarantee the validity of the obtained results and constraints. The limits on the leading dimension-six effective field theory effects can be rather inaccurate for LHC searches that suffer from large uncertainties while exploring an extensive energy range. Similarly, precise measurements can, in principle, test the subleading effects of the operator expansion. In this work, we present an algorithmic approach to automatize matching computations for dimension-eight operators for generic scalar extensions with proper implementation of equations of motion. We devise a step-by-step procedure to obtain the dimension-eight Wilson coefficients in a nonredundant basis to arrive at complete matching results. We apply this formalism to a range of scalar extensions of the Standard Model and provide tree-level and loop-suppressed results. Finally, we discuss the relevance of the dimension-eight operators for a range of phenomenological analyses, particularly focusing on Higgs and electroweak physics.

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I. INTRODUCTION

Searches for physics beyond the Standard Model (BSM) chiefly performed at the Large Hadron Collider (LHC) have, so far, not revealed any significant deviation from the Standard Model (SM) predictions. This is puzzling, on the one hand, given the SM's plethora of known flaws and shortcomings. On the other hand, these findings have motivated the application of model-independent techniques employing effective field theory (EFT) [1] to LHC data. The EFT approach breaks away from the assumption of concrete model-dependent correlations, thus opening up the possibility of revealing new (and perhaps noncanonical) BSM interactions through a holistic approach to data correlation interpretation. The inherent assumption of such an approach is that there is a significant mass gap between the BSM spectrum and the (process-dependent) characteristic energy scale at which the LHC operates to "integrate out" BSM states to obtain a low energy effective description that is determined by the SM's particle and symmetry content.

Efforts to apply EFT to the multiscale processes of the LHC environment have received considerable interest recently, reaching from theory-led proof-of-principle fits to LHC data [2–7] (with a history of almost a decade) over the adoption of these techniques by the LHC experiments (e.g., [8,9] for recent examples), to perturbative improvements of the formalism [10–21]. In doing so, most attention has been devoted to SMEFT at dimension-six level [22]

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i. \tag{1.1}$$

While EFT is a formidable tool to put correlations at the forefront of BSM searches, the significant energy coverage of the LHC can lead to blurred sensitivity estimates even in instances when Eq. (1.1) is a sufficiently accurate expansion. When pushing the cutoff scale Λ to large values, the experimental sensitivity to deviations from the SM can be

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too small to yield perturbatively meaningful or relevant constraints when matched to concrete UV scenarios (see, e.g., [23]). In contrast, dimension-eight contributions can be sizeable when the new physics cutoff Λ is comparably low in the case of more significant BSM signals at the LHC. To understand the ramifications for concrete UV models, it is then important to (i) have a flexible approach to mapping out the dimension-eight interactions and (ii) gauge the importance of dimension-eight contributions relative to those of dimension-six to quantitatively assess the error of the (potential) dimension-eight truncation.

A common bottleneck in constructing EFT interactions is removing redundancies. This is historically evidenced by the emergence of the so-called Warsaw basis [22]. Equations of motion are typically considered in eliminating redundant operators. Still, they are not identical to general (nonlinear) field redefinitions, which are the actual redundant parameters of the field theory [24–28]. When truncating a given operator dimension, this can be viewed as a scheme dependence not too unfamiliar from renormalizable theories, however, with less controlled side effects when the new physics scale is comparably low. Additional operator structures need to be included to elevate classical equations of motion to field redefinitions [28,29] to achieve a consistent classification at the dimension-eight level.

In this work, we devise a generic approach to this issue that enables us to provide a complete framework to match any dimension-eight structure that emerges in the process of integrating-out a heavy non-SM scalar and obtaining the form of the Wilson coefficients (WCs). Along the way of systematically reorganizing the operators into a nonredundant basis, resembling the one discussed in Refs. [30,31], we show that removing the higher-derivative operators produced at the dimension-six level itself can induce a non-negligible effect on dimension-eight matching coefficients along with the direct contribution to the same which can be computed following the familiar methodologies of the covariant derivative expansion [32,33] of the path integral [34–36] or the diagrammatic approach [37,38]. Finally, it is worth mentioning that, even though the one-loop effective action at dimension eight is yet to be formulated, it is possible to receive equally suppressed, loop-induced corrections from the dimension-six coefficients computed precisely at one loop. These can present themselves as the leading order contributions for the WCs, which generally appear at one-loop.

This paper is organized as follows: in Sec. II, we discuss the implementation of the Higgs field equation of motion and study its equivalence with field redefinitions. This gives rise to the desired dimension-eight operator structures after removing redundancies (Sec. II C). Our approach is tested and validated against available results for the real triplet scalar extension in Sec. III. In Sec. IV, the matching coefficients are presented explicitly considering a range of scalar extensions of the SM. Finally, the significance of the dimension-eight operators is analyzed based on observables in a model-dependent manner in Sec. V. We conclude in Sec. VI.

II. COMPLETE MATCHING AT DIMENSION EIGHT

We start by studying the structures of the higher-dimensional operators that can arise from heavy scalar extensions of the SM generically once the heavy field (Φ) is integrated out. The most generalized structure of the renormalized Lagrangian involving heavy scalars can be written as [32,39]

$$\mathcal{L}[\Phi] \supset \Phi^\dagger(P^2 - m^2 - U(x))\Phi + (\Phi^\dagger B(x) + \text{H.c.}) + \frac{1}{4}\lambda_\Phi(\Phi^\dagger\Phi)^2. \tag{2.1}$$

Here, $U(x)$ and $B(x)$ contain the interactions that are quadratic and linear in Φ, respectively, and only involve the lighter degrees of freedom. Once Φ is integrated out, we obtain a tower of operators that can be arranged according to their canonical dimension. It is important to note that the operators generated in this process might not be independent. Depending on phenomenological considerations, several sets of operators are defined in the literature. A set of dimension-six operators was prescribed in Ref. [40]. It was improved by systematically removing the redundant structures and promoting it to form a complete nonredundant basis in Ref. [22],[1] popularly known as the Warsaw basis. There is another set of operators known as the Green's set [42–44], which is overcomplete. The operators here are independent under the Fierz identities and integration by parts but otherwise redundant on account of equations of motion.[2] This source of redundancy contributes to higher dimensional operators. In this paper, we use the *Mathematica* package CODEX [45] to generate WCs of the operators in the SILH set [46,47] up to one loop, including the relevant redundant terms. Since we are interested in the corrections to the dimension-eight coefficients resulting from the dimension-six redundant structures, we recast the SILH operators into Green's setlike structures[3] to single out redundant and nonredundant operators using the following equations:

[1]A minimal set of four-fermionic operators is also constructed in Ref. [41].

[2]Here we are being ambiguous about the use of the equation of motion or field redefinition in removing the redundancies, see Refs. [24,29] for more details.

[3]We call it "Green's setlike structures" because we differ in some redundant operator structures as defined in Ref. [42], see Appendix A for more details.

$$
\begin{aligned}
\mathcal{Q}_H &\equiv \frac{1}{2}\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H) = -\frac{1}{2}(H^\dagger H)\Box(H^\dagger H),\\
\mathcal{Q}_T &\equiv \frac{1}{2}[H^\dagger \overleftrightarrow{\mathcal{D}}^\mu H][H^\dagger \overleftrightarrow{\mathcal{D}}^\mu H] = -2(\mathcal{D}_\mu H^\dagger H)(H^\dagger \mathcal{D}_\mu H) - \frac{1}{2}(H^\dagger H)\Box(H^\dagger H),\\
\mathcal{Q}_R &\equiv (H^\dagger H)(\mathcal{D}_\mu H^\dagger D^\mu H) = \frac{1}{2}[(H^\dagger H)\Box(H^\dagger H) - \boldsymbol{(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)}],\\
\mathcal{Q}_\mathcal{D} &\equiv \mathcal{D}^2 H^\dagger D^2 H = \boldsymbol{-\frac{1}{2}(Y_{pq}(\bar\psi_p\psi_q)\mathcal{D}^2 H + \mathrm{H.c.}) - \lambda'(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)},\\
\mathcal{Q}_{2W} &\equiv -\frac{1}{2}(\mathcal{D}_\mu W^I_{\mu\nu})^2\\
&= \boldsymbol{-\frac{g_W^2}{32}Y^{-1}_{pq}((\bar\psi_p\psi_q)\mathcal{D}^2 H + \mathrm{H.c.})} - \frac{ig_W^2}{4}(\bar\psi_p\gamma^\mu\tau^I\psi_p)(H^\dagger i\overleftrightarrow{\mathcal{D}}^I_\mu H) + \frac{g_W^2}{4}\lambda'^2 Y^{-1}_{pq}Y^{-1}_{qp}(H^\dagger H)^3\\
&\quad + \frac{g_W^2}{8}(\mathcal{D}^\mu H^\dagger H)(H^\dagger \mathcal{D}_\mu H) - \frac{g_W^2}{16}\mathcal{Q}_R + \boldsymbol{\frac{g_W^2}{32}(1 + 2\lambda' Y^{-1}_{pq}Y^{-1}_{qp})(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)},\\
\mathcal{Q}_{2B} &\equiv -\frac{1}{2}(\partial_\mu B_{\mu\nu})^2\\
&= \boldsymbol{-\frac{g_Y^2}{4}Y^{-1}_{pq}((\bar\psi_p\psi_q)\mathcal{D}^2 H + \mathrm{H.c.})} - ig_Y^2(\bar\psi_p\gamma^\mu\psi_p)(H^\dagger i\overleftrightarrow{\mathcal{D}}_\mu H) + g_Y^2\mathcal{Q}_R + 2g_Y^2\lambda'^2 Y^{-1}_{pq}Y^{-1}_{qp}(H^\dagger H)^3\\
&\quad + \boldsymbol{\frac{g_Y^2}{2}(1 + \lambda' Y^{-1}_{pq}Y^{-1}_{qp})(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)},\\
\mathcal{Q}_{2G} &\equiv -\frac{1}{2}(\mathcal{D}_\mu G^a_{\mu\nu})^2\\
&= \boldsymbol{-\frac{g_G^2}{3}Y^{-1}_{pq}((\bar\psi_p\psi_q)\mathcal{D}^2 H + \mathrm{H.c.})} + \frac{4g_G^2}{3}\lambda'^2 Y^{-1}_{pq}Y^{-1}_{qp}(H^\dagger H)^3 + \boldsymbol{\frac{2g_G^2}{3}\lambda' Y^{-1}_{pq}Y^{-1}_{qp}(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)},\\
\mathcal{Q}_W &\equiv ig_W(H^\dagger\tau^I\overleftrightarrow{\mathcal{D}}^\mu H)\mathcal{D}^\nu W^I_{\mu\nu}\\
&= \frac{ig_W^2}{2}(H^\dagger i\overleftrightarrow{\mathcal{D}}^I_\mu H)(\bar\psi\gamma^\mu\tau^I\psi) - \frac{g_W^2}{8}(\mathcal{D}_\mu H^\dagger H)(H^\dagger \mathcal{D}^\mu H) + \frac{g_W^2}{16}\mathcal{Q}_R - \boldsymbol{\frac{g_W^2}{32}(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)},\\
\mathcal{Q}_B &\equiv ig_Y(H^\dagger\overleftrightarrow{\mathcal{D}}^\mu H)\partial^\nu B_{\mu\nu} = ig_Y^2(H^\dagger\overleftrightarrow{\mathcal{D}}^\mu H)(\bar\psi\gamma^\mu\psi) - 2g_Y^2\mathcal{Q}_R - \boldsymbol{g_Y^2(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)},\\
\mathcal{Q}_{WW} &\equiv g_W^2(H^\dagger H)W^I_{\mu\nu}W^{I,\mu\nu},\\
\mathcal{Q}_{BB} &\equiv g_Y^2(H^\dagger H)B_{\mu\nu}B^{\mu\nu},\\
\mathcal{Q}_{WB} &\equiv 2g_W g_Y(H^\dagger\tau^I H)W^I_{\mu\nu}B^{\mu\nu},\\
\mathcal{Q}_{GG} &\equiv g_G^2(H^\dagger H)G^a_{\mu\nu}G^{a\mu\nu},
\end{aligned}
\tag{2.2}
$$

where Y_{pq} denotes the SM Yukawa coupling matrix, $\{p,q\} \in (1,2,3)$ are the flavor indices. We denote the Wilson coefficients of the SILH operators as $\mathcal{C}_i$ with i labeling the operators in Eqs. (2.2). Taking into account all the H-involved structures that can appear from a scalar extension of the SM, one can write the following:

$$
\begin{aligned}
\mathcal{L} = \mathcal{L}^{(4)}_{\mathrm{SM}} &+ \tilde\lambda(H^\dagger H)^2 + \zeta^{(6)}_1(H^\dagger H)^3 + \zeta^{(6)}_2(H^\dagger H)\Box(H^\dagger H) + \zeta^{(6)}_3(\mathcal{D}_\mu H^\dagger H)(H^\dagger \mathcal{D}_\mu H)\\
&+ \zeta^{(6)}_4(H^\dagger H)(B_{\mu\nu}B^{\mu\nu}) + \zeta^{(6)}_5(H^\dagger H)(W^I_{\mu\nu}W^{I\mu\nu}) + \zeta^{(6)}_6(H^\dagger\tau^I H)(B_{\mu\nu}W^{I\mu\nu})\\
&+ \zeta^{(6)}_7(H^\dagger H)(G^a_{\mu\nu}G^{a\mu\nu}) + \zeta^{(6)}_{8,1}(H^\dagger i\overleftrightarrow{\mathcal{D}}_\mu H)(\bar\psi\gamma^\mu\psi) + \zeta^{(6)}_{8,2}(H^\dagger i\overleftrightarrow{\mathcal{D}}^I_\mu H)(\bar\psi\tau^I\gamma^\mu\psi)\\
&+ \boldsymbol{\xi^{(6)}_1(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H) + \xi^{(6)}_2[(\bar\psi\psi)\mathcal{D}^2 H + \mathrm{H.c.}]}.
\end{aligned}
\tag{2.3}
$$

We highlight the redundant terms in Eq. (2.3) in bold font; they need to be removed. The coefficients of the Green's setlike structures in Eq. (2.3) can be expressed in terms of SILH coefficients through the relations given in Table I.

We now discuss our approach to properly implement the Higgs equation of motion (EOM) to compute the corrections to dimension-eight coefficients.

TABLE I. Translation of SILH coefficients into the Green's setlike form. Here, Y_{pq} denotes the SM Yuakawa coupling, $\{p,q\} \in (1,2,3)$ are the flavor indices.

Coefficients of Green's setlike operators	Relation in terms of SILH coefficients
$\zeta_1^{(6)}$	$[\mathcal{C}_6 + \frac{g_W^2}{4}\lambda'^2 Y_{pq}^{-1}Y_{qp}^{-1}\mathcal{C}_{2W} + 2g_Y^2\lambda'^2 Y_{pq}^{-1}Y_{qp}^{-1}\mathcal{C}_{2B} + \frac{4g_G^2}{3}\lambda'^2 Y_{pq}^{-1}Y_{qp}^{-1}\mathcal{C}_{2G}]$
$\zeta_2^{(6)}$	$[-\frac{1}{2}\mathcal{C}_H - \frac{1}{2}\mathcal{C}_T + \frac{1}{2}\mathcal{C}_R - \frac{g_W^2}{32}\mathcal{C}_{2W} + \frac{g_Y^2}{2}\mathcal{C}_{2B} + \frac{g_W^2}{32}\mathcal{C}_W - g_Y^2\mathcal{C}_B]$
$\zeta_3^{(6)}$	$[-2\mathcal{C}_T + \frac{g_W^2}{8}\mathcal{C}_{2W} - \frac{g_W^2}{8}\mathcal{C}_W]$
$\zeta_4^{(6)}$	$g_Y^2\mathcal{C}_{BB}$
$\zeta_5^{(6)}$	$g_W^2\mathcal{C}_{WW}$
$\zeta_6^{(6)}$	$2g_W g_Y\mathcal{C}_{WB}$
$\zeta_7^{(6)}$	$g_G{}^2\mathcal{C}_{GG}$
$\zeta_{8,1}^{(6)}$	$[-ig_Y^2\mathcal{C}_{2B} + ig_Y^2\mathcal{C}_B]$
$\zeta_{8,2}^{(6)}$	$[-\frac{ig_W^2}{4}\mathcal{C}_{2W} + \frac{ig_W^2}{2}\mathcal{C}_W]$
$\xi_1^{(6)}$	$[-\frac{1}{2}\mathcal{C}_R + \frac{g_W^2}{32}(1+2\lambda' Y_{pq}^{-1}Y_{qp}^{-1})\mathcal{C}_{2W} + \frac{g_Y^2}{2}(1+\lambda' Y_{pq}^{-1}Y_{qp}^{-1})\mathcal{C}_{2B} + \frac{2g_G^2}{3}\lambda' Y_{pq}^{-1}Y_{qp}^{-1}\mathcal{C}_{2G} - \frac{g_W^2}{16}\mathcal{C}_W] - \lambda'\mathcal{C}_{\mathcal{D}}$
$\xi_2^{(6)}$	$-Y_{pq}^{-1}[\frac{g_W^2}{32}\mathcal{C}_{2W} + \frac{g_Y^2}{4}\mathcal{C}_{2B} + \frac{g_G^2}{3}\mathcal{C}_{2G}] - \frac{1}{2}Y_{pq}\mathcal{C}_{\mathcal{D}}$

A. Removing redundancies: Field redefinition and Higgs field EOM

In quantum field theory (QFT), the experimentally observable quantities are related to the S-matrix elements, which remain invariant under field redefinition. Naively, this can be inferred from the fact that when calculating correlation functions using the path integral formalism, the field is just an integration variable. The correlation functions and S-matrix elements can be connected by the Lehmann-Symanzik-Zimmermann (LSZ)-reduction formula [48,49]. In the case of a renormalizable Lagrangian, we exploit this freedom and rewrite the Lagrangian in the canonical form. In an effective theory, we can perform nonlinear field redefinitions due to the presence of higher dimensional operators. This invariance gives rise to a rule to remove redundant terms from the effective Lagrangian and leads to the construction of "on-shell" effective theory [24].

One way of removing the redundancies with higher derivatives of operators involving the Higgs field H is to redefine the field in a perturbative manner [24,28,29,50]. For example, to remove the term $\boldsymbol{\xi_1^{(6)}(H^\dagger H)(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)}$ in Eq. (2.3), we can use the redefinition $H \to H + \xi_1^{(6)}(H^\dagger H)H$, in which case the redundancy at the level of $\mathcal{O}(\xi_1^{(6)})$ will be removed. Subsequently, it will give rise to higher-dimensional operator at $\mathcal{O}((\xi_1^{(6)})^2)$.

Now the same outcome can be achieved by employing the EOM judiciously. We compute the classical EOM for the Higgs considering all possible structures up to dimension-six from Eq. (2.3)

$$\begin{aligned}
\mathcal{D}^2 H = {} & \underline{-2\lambda'(H^\dagger H)H - \mathcal{Y}} + 3\zeta_1^{(6)}(H^\dagger H)^2 H + \zeta_3^{(6)}(\mathcal{D}_\mu H^\dagger H)\mathcal{D}^\mu H \\
& + \xi_1^{(6)}(\mathcal{D}^2 H^\dagger H + H^\dagger \mathcal{D}^2 H)H + \xi_1^{(6)}(H^\dagger H)\mathcal{D}^2 H - \zeta_3^{(6)}\mathcal{D}_\mu[(H^\dagger \mathcal{D}_\mu H)H] \\
& + 2\zeta_2^{(6)} H\Box(H^\dagger H) + \xi_1^{(6)}\mathcal{D}^2[(H^\dagger H)H] + \zeta_4^{(6)} H(B_{\mu\nu}B^{\mu\nu}) \\
& + \zeta_5^{(6)} H(W^I_{\mu\nu}W^{I,\mu\nu}) + \zeta_6^{(6)}(\tau^I H)(B_{\mu\nu}W^{I,\mu\nu}) + \zeta_7^{(6)} H(G^a_{\mu\nu}G^{a\mu\nu}) \\
& + \boxed{i\zeta_{8,1}^{(6)}\mathcal{D}_\mu[H(\bar\psi\gamma^\mu\psi)]} + i\zeta_{8,1}^{(6)}(\mathcal{D}_\mu H)(\bar\psi\gamma^\mu\psi) + \boxed{i\zeta_{8,2}^{(6)}\mathcal{D}^I_\mu[H(\bar\psi\gamma^\mu\tau^I\psi)]} \\
& + i\zeta_{8,2}^{(6)}(\mathcal{D}^I_\mu H)(\bar\psi\gamma^\mu\tau^I\psi) + \boxed{\xi_2^{(6)}\mathcal{D}^2(\bar\psi\psi)}.
\end{aligned} \tag{2.4}$$

Here, $\lambda' = \lambda - \tilde\lambda$ with λ being the SM Higgs quartic coupling in the renormalizable Lagrangian. $\tilde\lambda$ is the direct contribution to the former, obtained from integrating out the heavy field as shown in Eq. (2.3). The underlined part on the right-hand side of the Eq. (2.4) denotes the contribution from the renormalizable part of the Lagrangian ($\mathcal{L}_{\rm SM}^{(4)}$), which is considered as the first-order term in the EOM. The remainder arises from the effective operators at dimension-six and is considered second-order terms [49]. We can think of Eq. (2.4) as some special field redefinition

TABLE II. Contributions to the dimension-eight operators from the dimension-six structures after implementing the EOM. Here $Y_{\rm SM}$ denotes the SM Yukawa coupling. We have suppressed the flavor indices without any loss of generality.

Operator	Wilson coefficients	Operator	Wilson coefficients
$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$	$(\xi^{(6)}_1)^2+8\zeta^{(6)}_2\xi^{(6)}_1-\zeta^{(6)}_3\xi^{(6)}_1$	$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}$	$4\zeta^{(6)}_3\xi^{(6)}_1$
$\mathcal{O}^{(8)}_{\psi^2H^4\mathcal{D},1}$	$i\zeta^{(6)}_{8,1}\xi^{(6)}_1$	$\mathcal{O}^{(8)}_{\psi^2H^4\mathcal{D},2}$	$i\zeta^{(6)}_{8,2}\xi^{(6)}_1$
$\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,1}$	$i\zeta^{(6)}_{8,1}\xi^{(6)}_2$	$\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,2}$	$i\zeta^{(6)}_{8,2}\xi^{(6)}_2$
$\mathcal{O}^{(8)}_{H^4B^2}$	$2\zeta^{(6)}_4\xi^{(6)}_1$	$\mathcal{O}^{(8)}_{H^4W^2}$	$2\zeta^{(6)}_5\xi^{(6)}_1$
$\mathcal{O}^{(8)}_{H^4WB}$	$2\zeta^{(6)}_6\xi^{(6)}_1$	$\mathcal{O}^{(8)}_{H^4G^2}$	$2\zeta^{(6)}_7\xi^{(6)}_1$
$\mathcal{O}^{(8)}_{\psi^2B^2H}$	$\zeta^{(6)}_4\xi^{(6)}_2$	$\mathcal{O}^{(8)}_{\psi^2W^2H}$	$\zeta^{(6)}_5\xi^{(6)}_2$
$\mathcal{O}^{(8)}_{\psi^2WBH}$	$\zeta^{(6)}_6\xi^{(6)}_2$	$\mathcal{O}^{(8)}_{\psi^2G^2H}$	$\zeta^{(6)}_7\xi^{(6)}_2$
$\mathcal{O}^{(8)}_{\psi^2H^5}$	$(-4(\xi^{(6)}_1)^2-4\zeta^{(6)}_2\xi^{(6)}_1+\frac{1}{2}\zeta^{(6)}_3\xi^{(6)}_1)Y_{\rm SM}+3\zeta^{(6)}_1\xi^{(6)}_2$ $-12\lambda'\xi^{(6)}_1\xi^{(6)}_2+2\lambda'\xi^{(6)}_2\zeta^{(6)}_3-8\lambda'\xi^{(6)}_2\zeta^{(6)}_2$	$\mathcal{O}^{(8)}_H$	$6\zeta^{(6)}_1\xi^{(6)}_1-6(\xi^{(6)}_1)^2\lambda'-4\lambda'(4(\xi^{(6)}_1)^2$ $+4\zeta^{(6)}_2\xi^{(6)}_1-\frac{1}{2}\zeta^{(6)}_3\xi^{(6)}_1)$
$\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,1}$	$+4\xi^{(6)}_2\zeta^{(6)}_2+2\xi^{(6)}_1\xi^{(6)}_2$	$\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,2}$	$-\xi^{(6)}_2\zeta^{(6)}_3$
$\mathcal{O}^{(8)}_{\psi^4H^2,1}$	$(-4\xi^{(6)}_1\xi^{(6)}_2-4\zeta^{(6)}_2\xi^{(6)}_2+2\zeta^{(6)}_3\xi^{(6)}_2)Y_{\rm SM}$	$\mathcal{O}^{(8)}_{\psi^4H^2,2}$	$(-3\xi^{(6)}_1\xi^{(6)}_2-2\xi^{(6)}_2\zeta^{(6)}_2)Y_{\rm SM}$

and directly employ the first-order terms to remove the redundancies in Eq. (2.3), but this will not generate any higher-dimensional structures. This is the common practice to obtain the complete basis at dimension six. Working with the second-order terms is a nontrivial task, substituting it directly into Eq. (2.3) to obtain a contribution to higher dimension operators could lead to incompleteness as pointed out in [29]: The missing contributions can be encapsulated by including a term, $(1/2)(\xi^{(6)}_1(H^\dagger H))^2\delta^2\mathcal{L}/\delta H^\dagger\delta H$. Calculating this term from Eq. (2.3) we obtain the following contribution:

$$\begin{aligned}\mathcal{L}'&=(\xi^{(6)}_1)^2\mathcal{D}_\mu[(H^\dagger H)H^\dagger]\mathcal{D}^\mu[(H^\dagger H)H]-6(\xi^{(6)}_1)^2\lambda'(H^\dagger H)^4\\&=-2(\xi^{(6)}_1)^2\lambda'(H^\dagger H)^4+(\xi^{(6)}_1)^2(H^\dagger H)^2(H^\dagger\mathcal{Y}+\mathcal{Y}^\dagger H)\\&\quad-(\xi^{(6)}_1)^2(H^\dagger H)^2(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H).\end{aligned}\tag{2.5}$$

Since our primary concern is the redundant operators involving the Higgs field, the boxed structures in the Eq. (2.4) containing the derivative of fermion fields can be reduced to other structures by applying the first-order fermionic EOM.

We are now ready to implement the methodology discussed above to compute the dimension-eight coefficients from dimension-six operators.

B. Impact of dimension-six structures on dimension-eight coefficients

It is a common practice to employ the first-order EOM, i.e., the classical equation of motion obtained from the renormalizable Lagrangian, to transform the operators from one basis to another at a given mass dimension. Here we extend this strategy to generate higher-order terms in the effective Lagrangian. The contribution to the WCs arising from the EOM substitution considering second-order terms will be important. In Table II we provide the contribution to dimension-eight operators coming from the dimension-six Lagrangian. The operator structures are shown in Appendix A.

As the process of integrating out heavy fields becomes more complicated at higher operator dimensions, our method of generating WCs from lower-dimensional ones becomes economical. Following the expressions shown in Table II, one can quickly work out the dimension-eight contribution without explicitly performing the matching at that order. In the following subsection, we compute the complete basis at dimension eight.

C. Removing redundancies at dimension eight

We consider all (redundant and nonredundant) structures that only involve H and its derivatives at dimension-eight that can arise directly after integrating out at tree level.

$$\begin{aligned}\mathcal{L}^{(8)}_{\rm eff}&=\zeta^{(8)}_1(H^\dagger H)^4+\zeta^{(8)}_2(H^\dagger H)(H^\dagger\mathcal{D}_\mu H\mathcal{D}^\mu H^\dagger H)+\zeta^{(8)}_3(H^\dagger H)^2(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H)+\zeta^{(8)}_4(\mathcal{D}_\mu H^\dagger\mathcal{D}^\nu H)(\mathcal{D}^\nu H^\dagger\mathcal{D}_\mu H)\\&\quad+\zeta^{(8)}_5(\mathcal{D}_\mu H^\dagger\mathcal{D}^\nu H)(\mathcal{D}^\mu H^\dagger\mathcal{D}_\nu H)+\zeta^{(8)}_6(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H)(\mathcal{D}_\nu H^\dagger\mathcal{D}^\nu H)+\boldsymbol{\xi^{(8)}_1(H^\dagger H)^2(\mathcal{D}^2H^\dagger H+H^\dagger\mathcal{D}^2H)}\\&\quad+\boldsymbol{\xi^{(8)}_2(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H)(\mathcal{D}^2H^\dagger H+H^\dagger\mathcal{D}^2H)+\xi^{(8)}_3[(\mathcal{D}_\mu H^\dagger H)(\mathcal{D}^2H^\dagger\mathcal{D}^\mu H)+\mathrm{H.c.}]}\\&\quad+\boldsymbol{\xi^{(8)}_4[(\mathcal{D}^2H^\dagger H)(\mathcal{D}^2H^\dagger H)+\mathrm{H.c.}]+\xi^{(8)}_5[(\mathcal{D}_\mu H^\dagger H)(\mathcal{D}_\mu H^\dagger\mathcal{D}^2H)+\mathrm{H.c.}]}\\&\quad+\boldsymbol{\xi^{(8)}_6(\mathcal{D}^2H^\dagger\mathcal{D}^2H)(H^\dagger H)+\xi^{(8)}_7(H^\dagger\mathcal{D}^2H)(\mathcal{D}^2H^\dagger H)}.\end{aligned}\tag{2.6}$$

TABLE III. Matching contributions to the nonredundant dimension-eight operators from the dimension-eight structures after implementing the EOM. Here Y_{SM} denotes the SM Yukawa coupling. We have suppressed the flavor indices without any loss of generality and continued throughout the rest of the paper.

Operator	Wilson coefficients	Operator	Wilson coefficients
$\mathcal{O}_H^{(8)}$	$\zeta_1^{(8)} - 4\lambda'\xi_1^{(8)} + 8\lambda'^2\xi_4^{(8)} + 4\lambda'^2\xi_6^{(8)} + 4\lambda'^2\xi_7^{(8)}$	$\mathcal{O}_{\psi^2H^5}^{(8)}$	$(-\xi_1^{(8)} + 4\lambda'\xi_4^{(8)} + 2\lambda'\xi_6^{(8)} + 2\lambda'\xi_7^{(8)})Y_{\mathrm{SM}}$
$\mathcal{O}_{H^6\mathcal{D}^2,1}^{(8)}$	$\zeta_3^{(8)} - 4\lambda'\xi_2^{(8)}$	$\mathcal{O}_{H^6\mathcal{D}^2,2}^{(8)}$	$\zeta_2^{(8)} - 4\lambda'\xi_3^{(8)} - 4\lambda'\xi_5^{(8)}$
$\mathcal{O}_{\psi^2H^3\mathcal{D}^2,1}^{(8)}$	$(-\xi_2^{(8)} - \xi_3^{(8)})Y_{\mathrm{SM}}$	$\mathcal{O}_{\psi^2H^3\mathcal{D}^2,2}^{(8)}$	$(-\xi_5^{(8)})Y_{\mathrm{SM}}$
$\mathcal{O}_{\psi^4H^2,1}^{(8)}$	$(\xi_6^{(8)} + \xi_7^{(8)})Y_{\mathrm{SM}}^2$	$\mathcal{O}_{\psi^4H^2,2}^{(8)}$	$(\xi_4^{(8)})Y_{\mathrm{SM}}^2$

The redundant structures, written in bold in Eq. (2.6), can be expressed in terms of the nonredundant basis structures in the following manner[4]:

$$
\begin{aligned}
\xi_1^{(8)}(H^\dagger H)^2(\mathcal{D}^2H^\dagger H + H^\dagger\mathcal{D}^2H) &= -4\lambda'\xi_1^{(8)}(H^\dagger H)^4 - \xi_1^{(8)}[(H^\dagger H)^2(\mathcal{Y}^\dagger H) + \text{H.c.}],\\
\xi_2^{(8)}(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H)(\mathcal{D}^2H^\dagger H + H^\dagger\mathcal{D}^2H) &= -4\lambda'\xi_2^{(8)}(H^\dagger H)^2(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H) - \xi_2^{(8)}[(H^\dagger\mathcal{Y})(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H) + \text{H.c.}],\\
\xi_3^{(8)}[(\mathcal{D}_\mu H^\dagger H)(\mathcal{D}^2H^\dagger\mathcal{D}^\mu H) + \text{H.c.}] &= -4\lambda'\xi_3^{(8)}(H^\dagger H)(H^\dagger\mathcal{D}_\mu H)(\mathcal{D}^\mu H^\dagger H) - \xi_3^{(8)}[(H^\dagger\mathcal{Y})(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H) + \text{H.c.}],\\
\xi_4^{(8)}[(\mathcal{D}^2H^\dagger H)(\mathcal{D}^2H^\dagger H) + \text{H.c.}] &= 8\lambda'^2\xi_4^{(8)}(H^\dagger H)^4 + 4\lambda'\xi_4^{(8)}[(H^\dagger H)^2(\mathcal{Y}^\dagger H) + \text{H.c.}] + \xi_4^{(8)}[(\mathcal{Y}^\dagger H)(\mathcal{Y}^\dagger H) + \text{H.c.}],\\
\xi_5^{(8)}[(\mathcal{D}_\mu H^\dagger H)(\mathcal{D}_\mu H^\dagger\mathcal{D}^2H) + \text{H.c.}] &= -4\lambda'\xi_5^{(8)}(H^\dagger H)(H^\dagger\mathcal{D}_\mu H)(\mathcal{D}^\mu H^\dagger H) - \xi_5^{(8)}[(\mathcal{D}_\mu H^\dagger H)(\mathcal{D}^\mu H^\dagger\mathcal{Y}) + \text{H.c.}],\\
\xi_6^{(8)}(\mathcal{D}^2H^\dagger\mathcal{D}^2H)(H^\dagger H) &= 4\lambda'^2\xi_6^{(8)}(H^\dagger H)^4 + 2\lambda'\xi_6^{(8)}[(H^\dagger H)^2(\mathcal{Y}^\dagger H) + \text{H.c.}] + \xi_6^{(8)}(H^\dagger H)(\mathcal{Y}^\dagger\mathcal{Y}),\\
\xi_7^{(8)}(H^\dagger\mathcal{D}^2H)(\mathcal{D}^2H^\dagger H) &= 4\lambda'^2\xi_7^{(8)}(H^\dagger H)^4 + 2\lambda'\xi_7^{(8)}[(H^\dagger H)^2(\mathcal{Y}^\dagger H) + \text{H.c.}] + \xi_7^{(8)}(H^\dagger H)(\mathcal{Y}^\dagger\mathcal{Y}).
\end{aligned}
\tag{2.7}
$$

In Table III, we present the coefficients in the nonredundant basis.

Before cross-checking the proposed method in the next section, we summarize our framework in the flowchart depicted in Fig. 1. This work considers SM extensions of only a single heavy scalar. CODEX [45] has been used to generate the operators and the WCs in the SILH set up to one loop at dimension six. We compute the EOM (only for the Higgs field), including contributions from dimension-six operators; we substitute the EOM in the redundant structures. The first-order terms transform the redundant structure of dimension six to nonredundant structures, while the second-order terms generate dimension-eight operators. To compensate for the missing contribution that renders the EOM equivalent to a field redefinition, we need to add a term proportional to the second-order derivative of the effective action. Furthermore, we calculate dimension-eight operators by integrating out the heavy field at the tree level, which gives rise to the leading effects at dimension eight. We then substitute the first-order terms in the EOM to convert them into a complete basis and combine all these contributions to obtain the complete matching result.

[4]These operators can be related to the structures shown in Ref. [44].

The following section applies this to reproduce the known results for the real triplet extension of the SM to validate our methodology.

III. CROSS-VALIDATION OF THE METHOD

To cross-check our approach, we first turn to the example case of a real triplet scalar (Φ) SM extension. The BSM part of the Lagrangian reads

$$
\begin{aligned}
\mathcal{L}_\Phi = \frac{1}{2}(\mathcal{D}_\mu\Phi^a)(\mathcal{D}^\mu\Phi^a) - \frac{1}{2}m_\Phi^2\Phi^a\Phi^a + 2kH^\dagger\tau^aH\Phi^a\\
-\eta(H^\dagger H)\Phi^a\Phi^a - \frac{1}{4}\lambda_\Phi(\Phi^a\Phi^a)^2.
\end{aligned}
\tag{3.1}
$$

Integrating out the heavy scalar leads to some correction to the renormalizable term $(H^\dagger H)^2$ as discussed in Eq. (2.3), the coefficient $\tilde{\lambda}$ for this case is $\tilde{\lambda} = k^2/(2m_\Phi^2)$.

The SILH dimension-six coefficients have been tabulated in Table IV. The one-loop contribution to the matching can be categorized into two different classes: the contribution arising from integrating out scalars from purely heavy loops has been highlighted in blue, and terms from heavy-light mixed loops are shown in red.

We compute the Green's setlike coefficients first following the relations provided in Table I and derive the corrections to

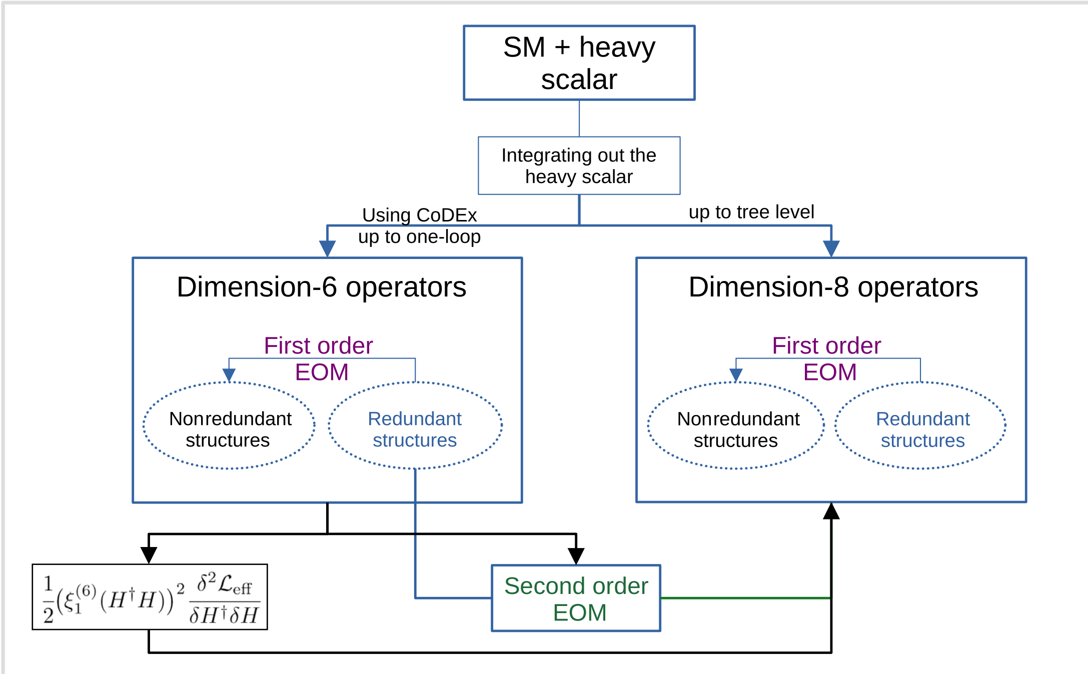

FIG. 1. Flow chart depicting the algorithmic approach considered to compute matching coefficients for both dimension-six and dimension-eight operators. Here, "First order EOM" and "Second order EOM" are formulated from the renormalizable and the dimension-six parts of the SM Lagrangian, respectively.

dimension-eight from dimension-six structures as shown in Table II. These contributions are computed considering only the tree-level part of the SILH coefficients. Thus, they are on the same footing as the direct dimension-eight contributions; they are tabulated separately in Table V. The direct contributions at dimension eight after "integrating out" can be captured by a total of seven coefficients as specified in Eq. (2.6). The values for the coefficients are

TABLE IV. Dimension-six SILH Wilson coefficients relevant for integrating out the real-triplet scalar of Eq. (3.1) at one loop. The terms within braces ($\{\}$) denote the contribution from pure heavy loops, whereas the brackets ($[]$) mark the contribution from light-heavy mixed loops.

SILH operator	Wilson coefficients	SILH operator	Wilson coefficients
$\mathcal{O}_6$	$-\frac{\eta k^2}{m_\Phi^4}-\{\frac{\eta^3}{8m_\Phi^2\pi^2}-\frac{5\eta k^2\lambda_\Phi}{8m_\Phi^4\pi^2}\}+[\frac{13\eta^2k^2}{8m_\Phi^4\pi^2}+\frac{47\eta k^4}{16m_\Phi^6\pi^2}+\frac{19k^6}{16m_\Phi^8\pi^2}-\frac{2\eta k^2\lambda}{m_\Phi^4\pi^2}-\frac{2k^4\lambda}{m_\Phi^6\pi^2}+\frac{11k^2\lambda^2}{16m_\Phi^4\pi^2}-\frac{5k^4\lambda_\Phi}{16m_\Phi^6\pi^2}]$	$\mathcal{O}_H$	$\{\frac{\eta^2}{16m_\Phi^2\pi^2}\}-[\frac{3\eta k^2}{8m_\Phi^4\pi^2}-\frac{9k^4}{32m_\Phi^6\pi^2}+\frac{5k^2\lambda}{16m_\Phi^4\pi^2}]$
$\mathcal{O}_R$	$\frac{2k^2}{m_\Phi^4}+\{\frac{5k^2\lambda_\Phi}{4m_\Phi^4\pi^2}\}-[\frac{21\eta k^2}{16m_\Phi^4\pi^2}-\frac{21k^4}{32m_\Phi^6\pi^2}+\frac{25k^2\lambda}{32m_\Phi^4\pi}]$	$\mathcal{O}_T$	$\frac{k^2}{m_\Phi^4}+\{\frac{5k^2\lambda_\Phi}{8m_\Phi^4\pi^2}\}-[\frac{\eta k^2}{2m_\Phi^4\pi^2}+\frac{k^4}{32m_\Phi^6\pi^2}+\frac{3k^2\lambda}{32m_\Phi^4\pi^2}]$
$\mathcal{O}_{WW}$	$\{\frac{\eta}{96m_\Phi^2\pi^2}\}+[\frac{25k^2}{768m_\Phi^4\pi^2}]$	$\mathcal{O}_{2W}$	$\{\frac{g_W^2}{480m_\Phi^2\pi^2}\}$
$\mathcal{O}_{WB}$	$[-\frac{k^2}{128m_\Phi^4\pi^2}]$	$\mathcal{O}_{BB}$	$[\frac{3k^2}{256m_\Phi^4\pi^2}]$
$\mathcal{O}_W$	$[-\frac{k^2}{288m_\Phi^4\pi^2}]$	$\mathcal{O}_B$	$[-\frac{7k^2}{96m_\Phi^4\pi^2}]$

TABLE V. Contributions to dimension-eight operators from dimension-six structures when integrating out the heavy real triplet scalar of Eq. (3.1). Here only the tree-level matching of the dimension-six structures have been considered while computing the results.

Operator	Wilson coefficients	Operator	Wilson coefficients
$\mathcal{O}_H^{(8)}$	$\frac{5k^6}{m_\Phi^{10}}+\frac{6k^4\eta}{m_\Phi^8}-\frac{10k^4\lambda}{m_\Phi^8}$	$\mathcal{O}_{\psi^2H^5}^{(8)}$	$-\frac{k^4}{m_\Phi^8}Y_{\rm SM}$
$\mathcal{O}_{H^6\mathcal{D}^2,1}^{(8)}$	$-\frac{5k^4}{m_\Phi^8}$	$\mathcal{O}_{H^6\mathcal{D}^2,2}^{(8)}$	$\frac{8k^4}{m_\Phi^8}$

TABLE VI. Contributions to dimension-eight operators from dimension-eight structures when integrating out the heavy real triplet scalar of Eq. (3.1).

Operator	Wilson coefficients	Operator	Wilson coefficients
$\mathcal{O}_{H}^{(8)}$	$\frac{2k^6}{m_\Phi^{10}}+\frac{4\eta k^4}{m_\Phi^8}-\frac{8\lambda k^4}{m_\Phi^8}-\frac{k^4\lambda_\Phi}{4m_\Phi^8}+\frac{2\eta^2k^2}{m_\Phi^6}-\frac{8\eta\lambda k^2}{m_\Phi^6}+\frac{8\lambda^2k^2}{m_\Phi^6}$	$\mathcal{O}_{\psi^2H^5}^{(8)}$	$(-\frac{2k^4}{m_\Phi^8}-\frac{2\eta k^2}{m_\Phi^6}+\frac{4\lambda k^2}{m_\Phi^6})Y_{\mathrm{SM}}$
$\mathcal{O}_{H^6\mathcal{D}^2,1}^{(8)}$	$-\frac{4k^4}{m_\Phi^8}-\frac{4\eta k^2}{m_\Phi^6}+\frac{8\lambda k^2}{m_\Phi^6}$	$\mathcal{O}_{H^6\mathcal{D}^2,2}^{(8)}$	$\frac{8k^4}{m_\Phi^8}+\frac{8\eta k^2}{m_\Phi^6}-\frac{16\lambda k^2}{m_\Phi^6}$

TABLE VII. Total tree-level matching of the dimension-eight coefficients for the real-triplet scalar extension of SM of Eq. (3.1).

Operator	Wilson coefficients	Operator	Wilson coefficients
$\mathcal{O}_{H}^{(8)}$	$\frac{7k^6}{m_\Phi^{10}}+\frac{2k^2(2\lambda-\eta)^2}{m_\Phi^6}+\frac{(40\eta-72\lambda-\lambda_\Phi)k^4}{4m_\Phi^8}$	$\mathcal{O}_{H^6\mathcal{D}^2,1}^{(8)}$	$-\frac{k^4}{m_\Phi^8}$
$\tilde{\mathcal{O}}_{H^6\mathcal{D}^2,2}^{(8)}$	$\frac{(4\eta-8\lambda)k^2}{m_\Phi^6}+\frac{8k^4}{m_\Phi^8}$	$\mathcal{O}_{\psi^2H^5}^{(8)}$	$Y_{\mathrm{SM}}(\frac{-3k^4-2m^2k^2(\eta-2\lambda)}{m_\Phi^8})$

$$
\zeta_1^{(8)}=\left(\frac{2\eta^2k^2}{m_\Phi^6}-\frac{k^4}{4m_\Phi^8}\lambda_\Phi\right),\quad \zeta_2^{(8)}=\frac{8\eta k^2}{m_\Phi^6},\quad \zeta_3^{(8)}=-\frac{4\eta k^2}{m_\Phi^6},
$$
$$
\zeta_4^{(8)}=\frac{4k^2}{m_\Phi^6},\quad \zeta_5^{(8)}=0,\quad \zeta_6^{(8)}=-\frac{2k^2}{m_\Phi^6},\quad \xi_1^{(8)}=\frac{2\eta k^2}{m_\Phi^6},
$$
$$
\xi_2^{(8)}=-\frac{2k^2}{m_\Phi^6},\quad \xi_3^{(8)}=\frac{4k^2}{m_\Phi^6},\quad \xi_4^{(8)}=\frac{k^2}{2m_\Phi^6},\quad \xi_5^{(8)}=0,
$$
$$
\xi_6^{(8)}=\frac{2k^2}{m_\Phi^6},\quad \xi_7^{(8)}=-\frac{k^2}{m_\Phi^6}. \tag{3.2}
$$

In Table VI, we show the direct contributions to dimension-eight structures removing redundancies at dimension-eight following the relations given in Table III. Lastly, in Table VII, we provide the complete tree-level matching results at dimension eight. Here, for comparison, we focus exclusively on those operators whose coefficients were previously derived in Ref. [50] employing the field redefinition of the Higgs field. We can connect the structure of $\tilde{\mathcal{O}}_{H^6\mathcal{D}^2,2}^{(8)}$ given in Ref. [50] with $\mathcal{O}_{H^6\mathcal{D}^2,1}^{(8)}$ and $\mathcal{O}_{H^6\mathcal{D}^2,2}^{(8)}$ (see Appendix A for the explicit structures of the operators), in the following way:

$$
\begin{aligned}
&(H^\dagger H)(H^\dagger\sigma^I H)(\mathcal{D}_\mu H^\dagger\sigma^I\mathcal{D}^\mu H)\\
&\quad=2(H^\dagger H)(\mathcal{D}_\mu H^\dagger H)(H^\dagger\mathcal{D}^\mu H)\\
&\qquad-(H^\dagger H)^2(\mathcal{D}_\mu H^\dagger\mathcal{D}^\mu H).
\end{aligned} \tag{3.3}
$$

These results are in agreement with the expressions provided for the tree-level matching of the dimension-eight coefficients in Table 10 of Ref. [50]. The remaining structures that arise after integrating out the heavy triplet scalar at dimension-eight, mainly at tree-level, including two- and four-fermionic operators are shown in Table VIII.

TABLE VIII. Dimension-eight matching coefficients for the real-triplet scalar extension of SM, Eq. (3.1).

$\mathcal{O}_{\psi^4H^2,1}^{(8)}$	$\frac{k^2}{m_\Phi^6}Y_{\mathrm{SM}}^2$		$\mathcal{O}_{\psi^4H^2,2}^{(8)}$		$\frac{k^2}{2m_\Phi^6}Y_{\mathrm{SM}}^2$
$\mathcal{O}_{\psi^2H^3\mathcal{D}^2,1}^{(8)}$	$-\frac{2k^2}{m_\Phi^6}Y_{\mathrm{SM}}$	$\mathcal{O}_{H^4\mathcal{D}^4,1}^{(8)}$	$\frac{4k^2}{m_\Phi^6}$	$\mathcal{O}_{H^4\mathcal{D}^4,3}^{(8)}$	$-\frac{2k^2}{m_\Phi^6}$

IV. EXAMPLE MODELS

This section applies the formalism described above to several example models to generate dimension-eight operators. We use CODEX [45] to obtain the operators and associated WCs in the SILH set at dimension six up to one loop, which we tabulate for each model. The coefficients are passed through Eqs. (2.2) to (2.5) that yield the contribution of dimension-six operators to dimension-eight operators. For clarity, we only present the leading contribution from the dimension-six tree-level generated operators and the direct integrated-out contribution at dimension eight. Subleading (yet non-negligible) corrections to the coefficients that arise from loop-generated operators can be obtained accordingly,[5] and the complete list of contributions can be obtained from a *Mathematica* notebook [53]. The models (apart from the leptoquark one) we discuss below are chosen as they generate operators at the tree level (see, e.g., the discussion in Refs. [54,55]). We can classify the contributions to WCs into the following two categories:

(1) Tree-level contributions: In this category, we only consider the contribution from those WCs generated at the tree level at dimension six. When the EOM is applied, they contribute on a par with the tree-level generated dimension-eight operators. Their combined effects are then considered to be the leading order contributions at dimension eight. We will mainly focus on this type of contribution and tabulate results for each model. It should be noted

[5]In Refs. [51,52] some of the dimension-eight operators up to one-loop order for a few models have been computed.

that these operators also receive subleading loop-induced contributions. The complete expressions for the coefficients can be found in the *Mathematica* notebook [53].

(2) Loop-induced and/or higher order contributions: If the redundant dimension-six operators are generated at one-loop level or the coefficients introduced through the application of the EOM appear with loop-level contributions or both, the WCs contain $(1/16\pi^2)$ or $(1/16\pi^2)^2$ suppressions, depending on interference between tree and loop parts. We will not list these types of contributions here, except for the leptoquark model for demonstration purposes. However, as mentioned above, the provided *Mathematica* notebook [53] contains all contributions, and interested readers are referred to the here documented results.

A. Complex triplet scalar

The SM can be extended with an electroweak complex triplet scalar (Δ) to explain the generation of neutrino masses through type-II seesaw mechanism [56,57]. This model also offers interesting collider signatures comprising rare lepton number and flavor violating processes [58]. In addition to contemplating the phenomenological significance of this model, a consistent effort has been made in the recent past to explore the effective theory of such an extension, see Refs. [59–61]. The BSM part of the Lagrangian reads

$$\begin{aligned}\mathcal{L}_\Delta \supset (\mathcal{D}_\mu\Delta^\dagger)(\mathcal{D}^\mu\Delta) - m_\Delta^2(\Delta^\dagger\Delta) - [\lambda_\Delta H^\dagger\sigma^I\tilde{H}\Delta^\dagger + \text{H.c.}]\\ -\lambda_1(\Delta^\dagger\Delta)^2 - \lambda_2(\Delta^\dagger T^I\Delta)(\Delta^\dagger T^I\Delta) - \lambda_3(H^\dagger H)(\Delta^\dagger\Delta)\\ -\lambda_4(H^\dagger\sigma^I H)(\Delta^\dagger T^I\Delta)\end{aligned} \tag{4.1}$$

(we neglect the interaction with the fermion fields in the following). As the Lagrangian contains a linear interaction for the field Δ, the renormalizable structure $(H^\dagger H)^2$ gets an extra contribution proportional to the coupling $\tilde{\lambda} = 2\lambda_\Delta^2/m_\Delta^2$.

Table IX contains the complete matching at one-loop order for the dimension-six SILH coefficients. After the tree-level integrating we obtain the following WCs at dimension-eight:

$$\begin{aligned}\zeta_1^{(8)} &= \left(\frac{2(\lambda_3-\lambda_4)^2\lambda_\Delta^2}{m_\Delta^6} - \frac{8(\lambda_1+\lambda_2)\lambda_\Delta^4}{m_\Delta^8}\right), \quad \zeta_2^{(8)} = \frac{8(\lambda_3-\lambda_4)\lambda_\Delta^2}{m_\Delta^6},\\ \zeta_3^{(8)} &= \frac{4(\lambda_3-\lambda_4)\lambda_\Delta^2}{m_\Delta^6}, \quad \zeta_4^{(8)} = \frac{8\lambda_\Delta^2}{m_\Delta^6}, \quad \xi_1^{(8)} = -\frac{2(\lambda_3-\lambda_4)\lambda_\Delta^2}{m_\Delta^6},\\ \xi_3^{(8)} &= \frac{8\lambda_\Delta^2}{m_\Delta^6}, \quad \xi_6^{(8)} = \frac{4\lambda_\Delta^2}{m_\Delta^6}, \quad \xi_7^{(8)} = \frac{4\lambda_\Delta^2}{m_\Delta^6}.\end{aligned} \tag{4.2}$$

As mentioned before, the total contribution to the dimension-eight operators is categorized into two categories depending on how they contribute. Below we write down the operators in their respective categories.

(1) Tree-level contribution: $\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}, \mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}, \mathcal{O}^{(8)}_{H}, \mathcal{O}^{(8)}_{\psi^2H^5}, \mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,1}, \mathcal{O}^{(8)}_{\psi^4H^2,1}$. The WCs corresponding to these operators are listed in Table X.

(2) Loop-induced and/or higher order contribution: $\mathcal{O}^{(8)}_{\psi^2H^4\mathcal{D},1}$, $\mathcal{O}^{(8)}_{\psi^2H^4\mathcal{D},2}$, $\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,1}$, $\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,2}$, $\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,2}$, $\mathcal{O}^{(8)}_{H^4X^2}$, $\mathcal{O}^{(8)}_{\psi^2X^2H}$, $\mathcal{O}^{(8)}_{\psi^4H^2,2}$. The WCs corresponding to these operators are listed in the *Mathematica* notebook [53].

B. General two-Higgs doublet model

One of the simplest and well-motivated extensions of the SM Higgs sector is the inclusion of an additional $SU(2)_\mathrm{L}$

TABLE IX. WCs of dimension-six SMEFT operators in the SILH set after integrating out the complex triplet scalar Eq. (4.1). The terms within braces ({}) denote the contribution from pure heavy loops, whereas the brackets ([]) mark the contribution from light-heavy mixed loops. We only use the uncolored coefficients for further calculation here. The complete calculation can be found in the provided *Mathematica* notebook [53].

SILH operator	Wilson coefficients	SILH operator	Wilson coefficients
$\mathcal{O}_6$	$\frac{\lambda_4\lambda_\Delta^2}{2m_\Delta^4} - \frac{2\lambda_3\lambda_\Delta^2}{m_\Delta^4} - \{\frac{\lambda_3^3}{32\pi^2m_\Delta^2} - \frac{\lambda_3\lambda_4^2}{64\pi^2m_\Delta^2} - \frac{2\lambda_1\lambda_3\lambda_\Delta^2}{\pi^2m_\Delta^4} - \frac{\lambda_2\lambda_3\lambda_\Delta^2}{\pi^2m_\Delta^4} - \frac{\lambda_1\lambda_4\lambda_\Delta^2}{2\pi^2m_\Delta^4} - \frac{\lambda_2\lambda_4\lambda_\Delta^2}{4\pi^2m_\Delta^4}\}$ $+[\frac{13\lambda_3^2\lambda_\Delta^2}{8\pi^2m_\Delta^4} - \frac{\lambda_3\lambda_4\lambda_\Delta^2}{2\pi^2m_\Delta^4} - \frac{8\lambda_3\lambda\lambda_\Delta^2}{\pi^2m_\Delta^4} + \frac{3\lambda_4^2\lambda_\Delta^2}{64\pi^2m_\Delta^4} - \frac{5\lambda_1\lambda_\Delta^4}{\pi^2m_\Delta^6} - \frac{5\lambda_2\lambda_\Delta^4}{\pi^2m_\Delta^6} + \frac{24\lambda_3\lambda_\Delta^4}{\pi^2m_\Delta^6}$ $+\frac{7\lambda_4\lambda\lambda\Delta^2}{4\pi^2m_\Delta^4} + \frac{11\lambda^2\lambda_\Delta^2}{\pi^2m_\Delta^4} - \frac{41\lambda^4\lambda_\Delta^4}{8\pi^2m_\Delta^6} - \frac{62\lambda\lambda_\Delta^4}{\pi^2m_\Delta^6} + \frac{74\lambda_\Delta^6}{\pi^2m_\Delta^8}]$	$\mathcal{O}_H$	$\frac{2\lambda_\Delta^2}{m_\Delta^4} + \{\frac{\lambda_3^2}{32\pi^2m_\Delta^2}\} - [\frac{3\lambda_3\lambda_\Delta^2}{4\pi^2m_\Delta^4} + \frac{\lambda_4\lambda_\Delta^2}{8\pi^2m_\Delta^4} + \frac{23\lambda\lambda_\Delta^2}{8\pi^2m_\Delta^4} - \frac{11\lambda_\Delta^4}{2\pi^2m_\Delta^6}]$
		$\mathcal{O}_T$	$-\frac{2\lambda_\Delta^2}{m_\Delta^4} - \{\frac{2\lambda_1\lambda_\Delta^2}{\pi^2m_\Delta^4} - \frac{\lambda_2\lambda_\Delta^2}{\pi^2m_\Delta^4} + \frac{\lambda_4^2}{768\pi^2m_\Delta^2}\}$ $+[\frac{\lambda_3\lambda_\Delta^2}{2\pi^2m_\Delta^4} - \frac{11\lambda_4\lambda_\Delta^2}{24\pi^2m_\Delta^4} - \frac{3\lambda\lambda_\Delta^2}{8\pi^2m_\Delta^4} + \frac{4\lambda_\Delta^4}{3\pi^2m_\Delta^6}]$
$\mathcal{O}_R$	$\frac{4\lambda_\Delta^2}{m_\Delta^4} + \{\frac{4\lambda_1\lambda_\Delta^2}{\pi^2m_\Delta^4} + \frac{2\lambda_2\lambda_\Delta^2}{\pi^2m_\Delta^4} + \frac{\lambda_4^2}{384\pi^2m_\Delta^2}\} - [\frac{13\lambda_3\lambda_\Delta^2}{8\pi^2m_\Delta^4} + \frac{\lambda_4\lambda_\Delta^2}{6\pi^2m_\Delta^4} + \frac{11\lambda\lambda_\Delta^2}{2\pi^2m_\Delta^4} - \frac{37\lambda_\Delta^4}{3\pi^2m_\Delta^6}]$	$\mathcal{O}_W$	$-[\frac{\lambda_\Delta^2}{72\pi^2m_\Delta^4}]$
		$\mathcal{O}_B$	$[\frac{11\lambda_\Delta^2}{24\pi^2m_\Delta^4}]$
$\mathcal{O}_D$	$[\frac{\lambda_\Delta^2}{8\pi^2m_\Delta^4}]$	$\mathcal{O}_{WW}$	$\{\frac{\lambda_3}{96\pi^2m_\Delta^2}\} + [\frac{25\lambda_\Delta^2}{192\pi^2m_\Delta^4}]$
$\mathcal{O}_{WB}$	$\{\frac{\lambda_4}{384\pi^2m_\Delta^2}\} - [\frac{13\lambda_\Delta^2}{96\pi^2m_\Delta^4}]$	$\mathcal{O}_{BB}$	$\{\frac{\lambda_3}{64\pi^2m_\Delta^2}\} + [\frac{11\lambda_\Delta^2}{64\pi^2m_\Delta^4}]$
$\mathcal{O}_{2W}$	$\{\frac{g_W^2}{240\pi^2m_\Delta^2}\}$	$\mathcal{O}_{2B}$	$\{\frac{g_Y^2}{160\pi^2m_\Delta^2}\}$

TABLE X. Total dimension-eight tree-level contribution after integrating out the complex triplet scalar of Eq. (4.1). Complete results at one loop with additional operators are presented in the *Mathematica* notebook [53].

Operator	Wilson coefficients	Operator	Wilson coefficients
$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$	$\frac{8(\lambda_3-\lambda_4)\lambda_\Delta^2}{m_\Delta^6}-\frac{20\lambda_\Delta^4}{m_\Delta^8}$	$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}$	$-\frac{8(4\lambda-\lambda_3+\lambda_4)\lambda_\Delta^2}{m_\Delta^6}+\frac{32\lambda_\Delta^4}{m_\Delta^8}$
$\mathcal{O}^{(8)}_{H}$	$\frac{2(16\lambda^2+4\lambda(\lambda_3-\lambda_4)+(\lambda_3-\lambda_4)^2)\lambda_\Delta^2}{m_\Delta^6}-\frac{8(21\lambda+\lambda_1+\lambda_2-\lambda_3+\lambda_4)\lambda_\Delta^4}{m_\Delta^8}+\frac{208\lambda_\Delta^6}{m_\Delta^{10}}$	$\mathcal{O}^{(8)}_{\psi^2H^5}$	$(\frac{2(8\lambda+\lambda_3-\lambda_4)\lambda_\Delta^2}{m_\Delta^6}-\frac{36\lambda_\Delta^4}{m_\Delta^8})Y_{\rm SM}$
		$\mathcal{O}^{(8)}_{H^4\mathcal{D}^4,1}$	$\frac{8\lambda_\Delta^2}{m_\Delta^6}$
$\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,1}$	$-\frac{8\lambda_\Delta^2}{m_\Delta^6}Y_{\rm SM}$	$\mathcal{O}^{(8)}_{\psi^4H^2,1}$	$\frac{8\lambda_\Delta^2}{m_\Delta^6}Y_{\rm SM}^2$

scalar doublet ($\mathcal{H}$) with hypercharge $\mathrm{Y}=1/2$, the well-known two-Higgs doublet model [62,63]. Many UV complete theories contain a two-Higgs doublet model in their minimal versions. This model has also been well discussed within SMEFT framework by integrating out the additional heavy Higgs doublet leading to dimension-six effective operators at one loop [64–66]. The relevant part of the BSM Lagrangian reads

$$\begin{aligned}\mathcal{L}_\mathcal{H} \supset\, &(\mathcal{D}_\mu\mathcal{H}^\dagger)(\mathcal{D}^\mu\mathcal{H}) - m_\mathcal{H}^2\mathcal{H}^\dagger\mathcal{H} - \frac{\lambda_\mathcal{H}}{4}(\mathcal{H}^\dagger\mathcal{H})^2\\ &+(\eta_H(\tilde{H}^\dagger\tilde{H})+\eta_\mathcal{H}(\mathcal{H}^\dagger\mathcal{H}))(\tilde{H}^\dagger\mathcal{H}+\mathcal{H}^\dagger\tilde{H})\\ &-\lambda_1(\tilde{H}^\dagger\tilde{H})(\mathcal{H}^\dagger\mathcal{H})^2-\lambda_2(\mathcal{H}^\dagger\tilde{H})(\tilde{H}^\dagger\mathcal{H})\\ &-\lambda_3[(\tilde{H}^\dagger\mathcal{H})^2+(\mathcal{H}^\dagger\tilde{H})^2]. \end{aligned} \tag{4.3}$$

The Wilson coefficients of dimension-six operators in the SILH set are presented in Table XI. After integrating out at tree-level the nonzero dimension-eight coefficients are given by

$$\zeta_1^{(8)} = -\frac{\eta_H^2}{m_\mathcal{H}^4}(\lambda_1+\lambda_2+2\lambda_3); \qquad \zeta_3^{(8)} = -\frac{\eta_H^2}{m_\mathcal{H}^4};$$
$$\xi_1^{(8)} = -\frac{\eta_H^2}{m_\mathcal{H}^4}. \tag{4.4}$$

We split the operators into our categories:

(1) Tree-level contribution: $\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}, \mathcal{O}^{(8)}_{H}, \mathcal{O}^{(8)}_{\psi^2H^5}$. The WCs corresponding to these operators are listed in Table XII.

(2) Loop-induced and/or higher order contribution: $\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}$, $\mathcal{O}^{(8)}_{\psi^2H^4\mathcal{D},1}$, $\mathcal{O}^{(8)}_{\psi^2H^4\mathcal{D},2}$, $\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,1}$, $\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,2}$, $\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,1}$, $\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,2}$, $\mathcal{O}^{(8)}_{H^4X^2}$, $\mathcal{O}^{(8)}_{\psi^2X^2H}$, $\mathcal{O}^{(8)}_{\psi^4H^2,1}$, $\mathcal{O}^{(8)}_{\psi^4H^2,2}$.

TABLE XI. WCs of dimension-six SMEFT operators in the SILH set after integrating out an additional Higgs doublet, Eq. (4.3). The terms within braces ({}) denote the contribution from pure heavy loops, whereas the brackets ([]) mark the contribution from light-heavy mixed loops. We only use the uncolored coefficients for further calculation here. The complete calculation can be found in the *Mathematica* notebook [53].

SILH operator	Wilson coefficients	SILH operator	Wilson coefficients
$\mathcal{O}_6$	$\frac{\eta_H^2}{m_\mathcal{H}^2}+\{\frac{3\eta_H^2\lambda_\mathcal{H}}{32m_\mathcal{H}^2\pi^2}+\frac{3\eta_H\eta_\mathcal{H}\lambda_2}{8m_\mathcal{H}^2\pi^2}-\frac{\lambda_1^3}{48m_\mathcal{H}^2\pi^2}+\frac{3\eta_H\eta_\mathcal{H}\lambda_2}{8m_\mathcal{H}^2\pi^2}$ $-\frac{\lambda_1^2\lambda_2}{32m_\mathcal{H}^2\pi^2}-\frac{\lambda_2^3}{96m_\mathcal{H}^2\pi^2}-\frac{\lambda_1\lambda_3^2}{8m_\mathcal{H}^2\pi^2}-\frac{\lambda_2\lambda_3^2}{8m_\mathcal{H}^2\pi^2}\}$ $+[\frac{15\eta_H^2\lambda}{8m_\mathcal{H}^2\pi^2}-\frac{3\eta_H^2\lambda_1}{4m_\mathcal{H}^2\pi^2}-\frac{13\eta_H^2\lambda_2}{16\mathcal{H}^2\pi^2}-\frac{7\eta_H^2\lambda_3}{4\mathcal{H}^2\pi^2}]$	$\mathcal{O}_H$ $\mathcal{O}_{2B}$	$\{-\frac{3\eta_H\eta_\mathcal{H}}{8\pi^2m_\mathcal{H}^2}+\frac{\lambda_1^2}{48\pi^2m_\mathcal{H}^2}+\frac{\lambda_1\lambda_2}{48\pi^2m_\mathcal{H}^2}+\frac{\lambda_2^2}{192\pi^2m_\mathcal{H}^2}+\frac{\lambda_3^2}{48\pi^2m_\mathcal{H}^2}\}+[\frac{5\eta_H^2}{16\pi^2m_\mathcal{H}^2}]$ $\{\frac{g_W^2}{960m_\mathcal{H}^2\pi^2}\}$
$\mathcal{O}_R$	$-\{\frac{3\eta_H\eta_\mathcal{H}}{8m_\mathcal{H}^2\pi^2}+\frac{\lambda_2^2}{96m_\mathcal{H}^2\pi^2}+\frac{\lambda_3^2}{24m_\mathcal{H}^2\pi^2}\}+[\frac{\eta_H^2}{8m_\mathcal{H}^2\pi^2}]$	$\mathcal{O}_T$	$\{\frac{\lambda_2^2}{192m_\mathcal{H}^2\pi^2}-\frac{\lambda_3^2}{48m_\mathcal{H}^2\pi^2}\}$
$\mathcal{O}_{WW}$	$\{\frac{\lambda_1}{384m_\mathcal{H}^2\pi^2}+\frac{\lambda_2}{768m_\mathcal{H}^2\pi^2}\}$	$\mathcal{O}_{BB}$	$\{\frac{\lambda_1}{384m_\mathcal{H}^2\pi^2}+\frac{\lambda_2}{768m_\mathcal{H}^2\pi^2}\}$
$\mathcal{O}_{WB}$	$\{\frac{\lambda_2}{384m_\mathcal{H}^2\pi^2}\}$	$\mathcal{O}_{2W}$	$\{\frac{g_W^2}{960m_\mathcal{H}^2\pi^2}\}$

TABLE XII. Total dimension-eight tree-level contribution after integrating out additional an Higgs Doublet, Eq. (4.3). Complete contributions at one-loop including additional operators are presented in the *Mathematica* notebook [53].

Operator	Wilson coefficients	Operator	Wilson coefficients	Operator	Wilson coefficients
$\mathcal{O}^{(8)}_{H}$	$\frac{\eta_H^2(4\lambda-\lambda_1-\lambda_2-2\lambda_3)}{m_\mathcal{H}^4}$	$\mathcal{O}^{(8)}_{\psi^2H^5}$	$\frac{\eta_H^2}{m_\mathcal{H}^4}Y_{\rm SM}$	$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$	$-\frac{\eta_H^2}{m_\mathcal{H}^4}$

The WCs corresponding to these operators are provided in the *Mathematica* notebook [53].

C. Complex quartet scalar (hypercharge Y = 3/2)

To generate neutrino masses through higher dimensional operators, an $SU(2)_{\mathrm{L}}$ quartet (Σ) with hypercharge $\mathrm{Y} = 3/2$ can be added to the SM [67–69]. Focusing on this part of the BSM Lagrangian,

$$\begin{aligned}\mathcal{L}_\Sigma \supset\, &(\mathcal{D}_\mu\Sigma^\dagger)(\mathcal{D}^\mu\Sigma) - M_\Sigma^2\Sigma^\dagger\Sigma + (\eta_\Sigma\Sigma^\dagger_{jkl}H^jH^kH^l + \mathrm{H.c.})\\ &- k_{\Sigma_1}(H^\dagger H)(\Sigma^\dagger\Sigma) - k_{\Sigma_2}(H^\dagger_m H^n)(\Sigma^\dagger_{jkn}\Sigma^{jkm})\\ &- \lambda_{\Sigma_1}(\Sigma^\dagger\Sigma)^2 - \lambda_{\Sigma_2}(\Sigma^\dagger T^a\Sigma)^2,\end{aligned} \tag{4.5}$$

we can integrate out the heavy scalar and match to the dimension-six SMEFT operators (see also [30,65,66]). After matching we obtain the effective operators and associated WCs in terms of the UV parameters, shown in Table XIII. Tree-level matching generates the following dimension-eight operators coefficients:

$$\begin{aligned}\zeta_1^{(8)} &= -\frac{|\eta_\Sigma|^2(k_{\Sigma_1}+k_{\Sigma_2})}{M_\Sigma^4}, \qquad \zeta_2^{(8)} = -\frac{6|\eta_\Sigma|^2}{M_\Sigma^4},\\ \zeta_3^{(8)} &= -\frac{6|\eta_\Sigma|^2}{M_\Sigma^4}.\end{aligned} \tag{4.6}$$

Again, we split the operators into the two categories:

(1) Tree-level contribution: $\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}, \mathcal{O}^{(8)}_{H}, \mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}$. The WCs corresponding to these operators are listed in Table XIV.

(2) Loop-induced and/or higher order contribution: $\mathcal{O}^{(8)}_{\psi^2H^5}$, $\mathcal{O}^{(8)}_{\psi^2H^4\mathcal{D},1}$, $\mathcal{O}^{(8)}_{\psi^2H^4\mathcal{D},2}$, $\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,1}$, $\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,2}$, $\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,1}$, $\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,2}$, $\mathcal{O}^{(8)}_{H^4X^2}$, $\mathcal{O}^{(8)}_{\psi^2X^2H}$, $\mathcal{O}^{(8)}_{\psi^4H^2,1}$, $\mathcal{O}^{(8)}_{\psi^4H^2,2}$. The WCs corresponding to these operators are shown in the *Mathematica* notebook [53].

D. Real singlet scalar model

The addition of a real singlet scalar to the SM is motivated by a range of SM shortcomings, related to dark matter, baryogenesis, and the electroweak hierarchy problem [70–72]. This model has been discussed extensively within the EFT framework through a complete one-loop matching to the SMEFT up to dimension six [36,38,59,65,66,73]. Here, we systematically extend these results. The Lagrangian involving the real singlet scalar field ($\mathcal{S}$) is given by

$$\begin{aligned}\mathcal{L}_\mathcal{S} \supset\, &\frac{1}{2}(\partial_\mu\mathcal{S})^2 - \frac{1}{2}M_\mathcal{S}^2\mathcal{S}^2 - \eta_\mathcal{S}(H^\dagger H)\mathcal{S}\\ &- k_\mathcal{S}(H^\dagger H)\mathcal{S}^2 - \frac{1}{4!}\lambda_\mathcal{S}\mathcal{S}^4.\end{aligned} \tag{4.7}$$

After integrating out the heavy field $\mathcal{S}$, we obtain an additional contribution to the quartic coupling of Higgs: $\tilde{\lambda} = -\eta_\mathcal{S}^2/M_\mathcal{S}^2$, along with the dimension-six SILH set operators as shown in Table XV. Since no redundant operator at dimension six is generated at tree level, there is no tree-level contribution to dimension-eight operators from dimension six. Thus the dominant contribution arises solely from removing redundancies at dimension-eight

TABLE XIII. WCs of dimension-six SMEFT operators in the SILH set after integrating out the quartet scalar of Eq. (4.5). The terms within braces ({}) denote the contribution from pure heavy loops, whereas the brackets ([]) mark the contribution from light-heavy mixed loops. We only use the uncolored coefficients for further calculation here. The complete calculation can be found in the *Mathematica* notebook [53].

SILH operator	Wilson coefficients	SILH operator	Wilson coefficients
$\mathcal{O}_6$	$\frac{\eta_\Sigma^2}{M_\Sigma^2} - \{\frac{k_{\Sigma_1}^2 k_{\Sigma_2}}{16\pi^2M_\Sigma^2} - \frac{k_{\Sigma_1}^3}{24\pi^2M_\Sigma^2} - \frac{7k_{\Sigma_1}k_{\Sigma_2}^2}{144\pi^2M_\Sigma^2}\} - [\frac{9\eta_\Sigma^2k_{\Sigma_1}}{8\pi^2M_\Sigma^2}]$ $-\{\frac{k_{\Sigma_2}^3}{72\pi^2M_\Sigma^2} + \frac{5\eta_\Sigma^2\lambda_{\Sigma_1}}{8\pi^2M_\Sigma^2} + \frac{15\eta_\Sigma^2\lambda_{\Sigma_2}}{32\pi^2M_\Sigma^2}\} - [\frac{19\eta_\Sigma^2k_{\Sigma_2}}{16\pi^2M_\Sigma^2}]$	$\mathcal{O}_H$	$\{\frac{k_{\Sigma_1}^2}{24\pi^2M_\Sigma^2} + \frac{k_{\Sigma_1}k_{\Sigma_2}}{24\pi^2M_\Sigma^2} + \frac{k_{\Sigma_2}^2}{96\pi^2M_\Sigma^2}\} + [\frac{3\eta_\Sigma^2}{8\pi^2M_\Sigma^2}]$
		$\mathcal{O}_{2B}$	$\{\frac{3g_Y^2}{160\pi^2M_\Sigma^2}\}$
$\mathcal{O}_R$	$\{\frac{5k_{\Sigma_2}^2}{432\pi^2M_\Sigma^2}\} + [\frac{3\eta_\Sigma^2}{4\pi^2M_\Sigma^2}]$	$\mathcal{O}_T$	$\{\frac{5k_{\Sigma_2}^2}{864\pi^2M_\Sigma^2}\} - [\frac{3\eta_\Sigma^2}{8\pi^2M_\Sigma^2}]$
$\mathcal{O}_{WW}$	$\{\frac{5k_{\Sigma_1}}{192\pi^2M_\Sigma^2} + \frac{5k_{\Sigma_2}}{384\pi^2M_\Sigma^2}\}$	$\mathcal{O}_{BB}$	$\{\frac{3k_{\Sigma_1}}{64\pi^2M_\Sigma^2} + \frac{3k_{\Sigma_2}}{128\pi^2M_\Sigma^2}\}$
$\mathcal{O}_{WB}$	$\{\frac{5k_{\Sigma_2}}{192\pi^2M_\Sigma^2}\}$	$\mathcal{O}_{2W}$	$\{\frac{g_W^2}{96\pi^2M_\Sigma^2}\}$

TABLE XIV. Total dimension-eight tree-level contribution after integrating out the quartet scalar of Eq. (4.5). Complete one-loop results including additional operators are available from the *Mathematica* notebook [53].

Operator	Wilson coefficients	Operator	Wilson coefficients	Operator	Wilson coefficients
$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$	$-\frac{6\eta_\Sigma^2}{m_\Sigma^4}$	$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}$	$-\frac{6\eta_\Sigma^2}{m_\Sigma^4}$	$\mathcal{O}^{(8)}_{H}$	$-\frac{\eta_\Sigma^2k_{\Sigma1}}{m_\Sigma^4} - \frac{\eta_\Sigma^2k_{\Sigma2}}{m_\Sigma^4}$

TABLE XV. WCs of dimension-six SMEFT operators in the SILH set after integrating out the real singlet scalar of Eq. (4.7). The terms within braces ({}) denote the contribution from pure heavy loops, whereas the brackets ([]) mark the contribution from light-heavy mixed loops.

$\mathcal{O}_6$	$-\frac{\eta_S^2 k_S}{M_S^4} - \{\frac{\eta_S^2 k_S \lambda_S}{16\pi^2 M_S^4} - \frac{k_S^3}{24\pi^2 M_S^2}\}$			$\mathcal{O}_H$	$\frac{\eta_S^2}{M_S^4} + \{\frac{\eta_S^2 \lambda_S}{16\pi^2 M_S^4} + \frac{k_S^2}{48\pi^2 M_S^2}\}$		
	$+[\frac{11\eta_S^2 k_S^2}{8\pi^2 M_S^4} + \frac{37\eta_S^4 k_S}{16\pi^2 M_S^6} - \frac{3\eta_S^4 \lambda}{2\pi^2 M_S^6} + \frac{43\eta_S^6}{48\pi^2 M_S^8}$				$-[\frac{17\eta S^2 k_S}{24\pi^2 M_S^4} + \frac{9\eta_S^2 \lambda}{32\pi^2 M_S^4} - \frac{5\eta_S^4}{12\pi^2 M_S^6}]$		
	$-\frac{3\eta_S^2 k_S \lambda}{2\pi^2 M_S^4} + \frac{9\eta_S^2 \lambda^2}{16\pi^2 M_S^4} - \frac{\eta_S^4 \lambda_S}{32\pi^2 M_S^6}]$			$\mathcal{O}_W$	$[-\frac{7\eta_S^2}{288\pi^2 M_S^4}]$	$\mathcal{O}_B$	$[-\frac{7\eta_S^2}{288\pi^2 M_S^4}]$
$\mathcal{O}_D$	$[\frac{\eta_S^2}{96\pi^2 M_S^4}]$	$\mathcal{O}_{WB}$	$[\frac{\eta_S^2}{128\pi^2 M_S^4}]$	$\mathcal{O}_{WW}$	$[\frac{\eta_S^2}{256\pi^2 M_S^4}]$	$\mathcal{O}_{BB}$	$[\frac{\eta_S^2}{256\pi^2 M_S^4}]$

level itself. The nonzero coefficients of dimension-eight operators generated via integrating out are

$$\zeta_1^{(8)} = \frac{2\eta_S^2 k_S^2}{M_S^6} - \frac{\lambda_S \eta_S^4}{24 M_S^8}, \quad \zeta_3^{(8)} = \frac{4\eta_S^2 k_S}{M_S^6}, \quad \zeta_6^{(8)} = \frac{2\eta_S^2}{M_S^6},$$
$$\xi_1^{(8)} = \frac{2\eta_S^2 k_S}{M_S^6}, \quad \xi_2^{(8)} = \frac{2\eta_S^2 k_S}{M_S^6}, \quad \xi_4^{(8)} = \frac{\eta_S^2 k_S}{2M_S^6}, \quad \xi_7^{(8)} = \frac{\eta_S^2 k_S}{M_S^6}. \tag{4.8}$$

We use Eq. (2.7) to remove the redundancies from the above equation and rewrite them in the complete basis of Table III; the coefficients of nonredundant SMEFT dimension-eight operators are shown in Table XVI. Expressed in the categories detailed above we arrive at

1. (1) Tree-level contribution: $\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$, $\mathcal{O}^{(8)}_{H}$, $\mathcal{O}^{(8)}_{\psi^2 H^3\mathcal{D}^2,1}$, $\mathcal{O}^{(8)}_{\psi^4 H^2,1}$, $\mathcal{O}^{(8)}_{\psi^2 H^5}$, $\mathcal{O}^{(8)}_{\psi^4 H^2,2}$. The WCs corresponding to these operators are listed in Table XIV.
2. (2) Loop-induced and/or higher order contribution: There is no redundancy at the dimension-six level, and no loop induced operators can be generated by the equation of motion. We can generate dimension-eight operators at one-loop-level itself by integrating out the heavy degree of freedom. This is beyond the scope of this paper, and we will leave this for future work.

E. Scalar leptoquark

Next, we consider the BSM model where the SM is extended by a scalar leptoquark, having quantum numbers $(3, 2, 1/6)$ under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. This scenario has recently received lots of attention as it can potentially address observed anomalies in B-meson decays [74,75]. This model has also been analyzed within the EFT framework, see Refs. [59,65]. We focus on the scalar interaction part of the Lagrangian for our discussion, which reads

$$\mathcal{L}_\Theta \supset (\mathcal{D}_\mu \Theta^\dagger)(\mathcal{D}^\mu \Theta) - M_\Theta^2 (\Theta^\dagger \Theta) - \eta_{\Theta_1} (\Theta^\dagger \Theta)(H^\dagger H)$$
$$- \eta_{\Theta_2} (\Theta^\dagger \tau^I \Theta)(H^\dagger \tau^I H) - \lambda_{\Theta_1} (\Theta^\dagger \Theta)^2$$
$$- \lambda_{\Theta_2} (\Theta^\dagger \tau^I \Theta)(\Theta^\dagger \tau^I \Theta). \tag{4.9}$$

No linear coupling of the Higgs field is present, and we do not obtain effective operators at tree level. Note that although dimension-eight one-loop-level operators are beyond the scope of this work, we can still capture contributions to dimension-eight operators using our formalism. Table XVII shows the one-loop generated operators and the corresponding WCs. Table XVIII contains the WCs contribution coming from the loop induced operators at dimension six only. The two operators quoted there is not an exhaustive list but serves the purpose of demonstrating that we can still obtain nonzero WCs from dimension-six operators without dimension-eight one-loop-level matching. Categorizing the WCs as above we find

1. (1) Tree-level contribution: No tree-level contribution.
2. (2) Loop-induced and/or higher order contribution: There is only loop induced contribution in this case. Table XVIII captures only a subset of operators which gets contribution from the lower dimension operators. The others are $\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}$, $\mathcal{O}^{(8)}_{\psi^2 H^5}$, $\mathcal{O}^{(8)}_{\psi^2 H^3\mathcal{D}^2,1}$, $\mathcal{O}^{(8)}_{\psi^2 H^3\mathcal{D}^2,2}$, $\mathcal{O}^{(8)}_{\psi^4 H^2,1}$, $\mathcal{O}^{(8)}_{\psi^4 H^2,2}$, $\mathcal{O}^{(8)}_{\psi^2 H^4\mathcal{D},1}$, $\mathcal{O}^{(8)}_{\psi^2 H^4\mathcal{D},2}$,

TABLE XVI. Nonredundant SMEFT dimension-eight operators and their corresponding coefficients after integrating out the real singlet scalar of Eq. (4.7).

$\mathcal{O}^{(8)}_{H}$	$\frac{2\eta_S^2 k_S^2}{m_S^6} - \frac{8\eta_S^2 k_S \lambda}{m_S^6} + \frac{8\eta_S^2 k_S \lambda^2}{m_S^6} + \frac{16\eta_S^4 k_S \lambda}{m_S^8} - \frac{\eta_S^4 \lambda_S}{24 m_S^8} + \frac{8\eta_S^6 k_S}{m_S^{10}}$			$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$	$\frac{4\eta_S^2 k_S}{m_S^6} - \frac{8\lambda\eta_S^2 k_S}{m_S^6}$
				$\mathcal{O}^{(8)}_{H^4\mathcal{D}^4,3}$	$\frac{2\eta_S^2}{m_S^6}$
				$\mathcal{O}^{(8)}_{\psi^2 H^5}$	$-\frac{2\eta_S^2 k_S Y_{\rm SM}(1-2\lambda)}{m_S^6} + \frac{4\eta_S^4 k_S Y_{\rm SM}}{m_S^8}$
$\mathcal{O}^{(8)}_{\psi^2 H^3\mathcal{D}^2,1}$	$-\frac{2\eta_S^2 k_S Y_{\rm SM}}{m_S^6}$	$\mathcal{O}^{(8)}_{\psi^4 H^2,2}$	$\frac{\eta_S^2 k_S Y_{\rm SM}^2}{2m_S^6}$	$\mathcal{O}^{(8)}_{\psi^4 H^2,1}$	$\frac{\eta_S^2 k_S Y_{\rm SM}^2}{m_S^6}$

TABLE XVII. WCs of dimension-six SMEFT operators in the SILH set for the scalar leptoquark of Eq. (4.9). The terms within braces ({}) denote the contribution from pure heavy loops.

SILH operator	Wilson coef.	SILH operator	Wilson coef.	SILH operator	Wilson coef.
$\mathcal{O}_6$	$\{-\frac{\eta_{\Theta_1}^3}{16\pi^2 M_\Theta^2}-\frac{3\eta_{\Theta_1}\eta_{\Theta_2}^2}{256\pi^2 M_\Theta^2}\}$	$\mathcal{O}_H$	$\{\frac{\eta_{\Theta_1}^2}{16\pi^2 M_\Theta^2}\}$	$\mathcal{O}_R$	$\{\frac{\eta_{\Theta_2}^2}{128\pi^2 M_\Theta^2}\}$
$\mathcal{O}_T$	$\{\frac{\eta_{\Theta_2}^2}{256\pi^2 M_\Theta^2}\}$	$\mathcal{O}_{WW}$	$\{\frac{\eta_{\Theta_1}}{128\pi^2 M_\Theta^2}\}$	$\mathcal{O}_{BB}$	$\{\frac{\eta_{\Theta_1}}{1152\pi^2 M_\Theta^2}\}$
$\mathcal{O}_{WB}$	$\{\frac{\eta_{\Theta_2}}{768\pi^2 M_\Theta^2}\}$	$\mathcal{O}_{2W}$	$\{\frac{g_W^2}{320\pi^2 M_\Theta^2}\}$	$\mathcal{O}_{2B}$	$\{\frac{g_Y^2}{2880\pi^2 M_\Theta^2}\}$

TABLE XVIII. Contribution to dimension-eight operators from dimension-six operators. This table does not capture the full contribution. Full results are available from our *Mathematica* notebook [53].

Operator	Wilson coefficients
$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$	$\frac{g_W^4 g_Y^4\lambda^2}{14745600\pi^4 m_\Theta^4 Y_{\rm SM}^2}+\frac{g_W^4 g_Y^4\lambda}{7372800\pi^4 m_\Theta^4 Y_{\rm SM}}+\frac{g_W^4 g_Y^4}{3686400\pi^4 m_\Theta^4}-\frac{\eta_{\Theta1}^2 g_W^4}{20480\pi^4 m_{\Theta1}^4}+\frac{\eta_{\Theta2}^2 g_W^4}{131072\pi^4 m_\Theta^4}-\frac{\eta_{\Theta1}^2 g_W^4\lambda}{20480\pi^4 m_\Theta^4 Y_{\rm SM}}+\frac{\eta_{\Theta2}^2 g_W^4\lambda}{327680\pi^4 m_\Theta^4 Y_{\rm SM}}$ $+\frac{g_W^8\lambda^2}{26214400\pi^4 m_\Theta^4 Y_{\rm SM}^2}-\frac{g_W^8\lambda}{6553600\pi^4 m_\Theta^4 Y_{\rm SM}}-\frac{g_W^8}{5242880\pi^4 m_\Theta^4}-\frac{\eta_{\Theta2}^2 g_Y^4}{184320\pi^4 m_\Theta^4}-\frac{\eta_{\Theta1}^2 g_Y^4\lambda}{23040\pi^4 m_\Theta^4 Y_{\rm SM}}+\frac{\eta_{\Theta2}^2 g_Y^4\lambda}{368640\pi^4 m_\Theta^4 Y_{\rm SM}}$ $+\frac{g_Y^8\lambda^2}{33177600\pi^4 m_\Theta^4 Y_{\rm SM}^2}+\frac{g_Y^8\lambda}{4147200\pi^4 m_\Theta^4 Y_{\rm SM}}+\frac{\eta_{\Theta1}^2\eta_{\Theta2}^2}{1024\pi^4 m_\Theta^4}-\frac{5\eta_{\Theta2}^4}{65536\pi^4 m_\Theta^4}$
$\mathcal{O}^{(8)}_{H}$	$-\frac{g_W^4 g_Y^4\lambda}{1843200\pi^4 m_\Theta^4}-\frac{g_W^4 g_Y^4\lambda^2}{1228800\pi^4 m_\Theta^4 Y_{\rm SM}}-\frac{11 g_W^4 g_Y^4\lambda^3}{7372800\pi^4 m_\Theta^4 Y_{\rm SM}^2}+\frac{\eta_\Theta^2 g_W^4\lambda}{10240\pi^4 m_\Theta^4}-\frac{3\eta_{\Theta1}^3 g_W^4}{40960\pi^4 m_\Theta^4}-\frac{9\eta_{\Theta1}\eta_{\Theta2}^2 g_W^4}{655360\pi^4 m_\Theta^4}+\frac{\eta_{\Theta2}^2 g_W^4\lambda}{65536\pi^4 m_\Theta^4}-\frac{g_W^8\lambda}{2621440\pi^4 m_\Theta^4}+\frac{\eta_{\Theta1}^2 g_W^4\lambda^2}{10240\pi^4 m_\Theta^4 Y_{\rm SM}}$ $-\frac{3\eta_{\Theta1}^3 g_W^4\lambda}{40960\pi^4 m_\Theta^4 Y_{\rm SM}}-\frac{9\eta_{\Theta1}\eta_{\Theta2}^2 g_W^4\lambda}{655360\pi^4 m_\Theta^4 Y_{\rm SM}}+\frac{7\eta_{\Theta2}^2 g_W^4\lambda^2}{163840\pi^4 m_\Theta^4 Y_{\rm SM}}-\frac{7 g_W^8\lambda^2}{3276800\pi^4 m_\Theta^4 Y_{\rm SM}}-\frac{23 g_W^8\lambda^3}{13107200\pi^4 m_\Theta^4 Y_{\rm SM}^2}+\frac{\eta_{\Theta2}^2 g_Y^4\lambda}{92160\pi^4 m_\Theta^4}+\frac{\eta_{\Theta1}^2 g_Y^4\lambda^2}{11520\pi^4 m_\Theta^4 Y_{\rm SM}}-\frac{\eta_{\Theta1}^3 g_Y^4\lambda}{15360\pi^4 m_\Theta^4 Y_{\rm SM}}$ $-\frac{\eta_{\Theta1}\eta_{\Theta2}^2 g_Y^4\lambda}{81920\pi^4 m_\Theta^4 Y_{\rm SM}}+\frac{\eta_{\Theta2}^2 g_Y^4\lambda^2}{184320\pi^4 m_\Theta^4 Y_{\rm SM}}-\frac{g_Y^8\lambda^2}{2073600\pi^4 m_{\Theta1}^4 Y_{\rm SM}}+\frac{g_Y^8\lambda^3}{16588800\pi^4 m_{\Theta1}^4 Y_{\rm SM}^2}-\frac{\eta_{\Theta1}^2\eta_{\Theta2}^2\lambda}{512\pi^4 m_\Theta^4}+\frac{3\eta_{\Theta1}^3\eta_{\Theta2}^2}{2048\pi^4 m_{\Theta1}^4}+\frac{9\eta_{\Theta1}\eta_{\Theta2}^4}{32768\pi^4 m_\Theta^4}-\frac{5\eta_{\Theta2}^4\lambda}{32768\pi^4 m_\Theta^4}$

$\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,1}$, $\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,2}$, $\mathcal{O}^{(8)}_{H^4X^2}$, $\mathcal{O}^{(8)}_{\psi^2X^2H}$. The full contribution can be accessed from the *Mathematica* notebook [53].

V. IMPACT OF DIMENSION-EIGHT OPERATORS ON BSM SCENARIOS

Given the plethora of data available after LHC run-II and run-III, we broadly classify the above UV theories by investigating their low-energy phenomenology in this section, emphasizing the relevance of the dimension-eight operators.

Different observables and precision measurements provide strong discriminators between UV scenarios when using matched EFT results. In this sense, the dimension-eight effects provide quantitatively crucial additional information. To gain a qualitative understanding of UV discrimination employing the results above, we consider three categories of experimental observables for guidance: (i) electroweak precision observables (EWPOs), (ii) Higgs signal strength (HSS) measurements, and (iii) vector boson scattering (VBS) measurements. We analyze the cases discussed in Sec. IV, reviewing the interplay of (i)–(iii) as shown in Fig. 2.

The characteristics of different models can be analyzed by adjudging their responses to the following questions. First, one needs to note which effective operators emerge from each model under consideration. As the observables can be parametrized in terms of the effective operators, the observable-model correspondence can be set up directly. Based on that, one can classify different UV models by

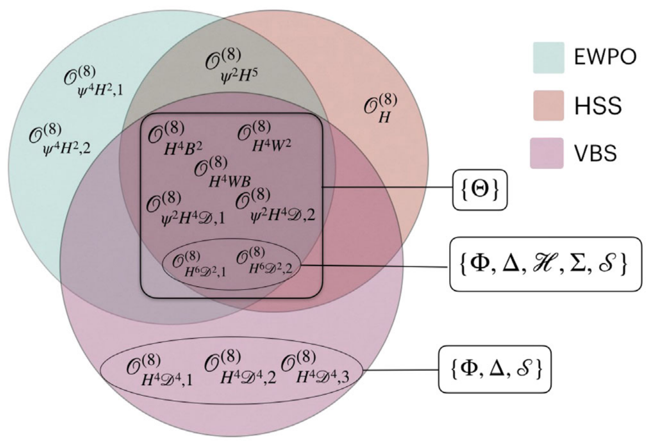

FIG. 2. Interplay of different observables for the categorization of complete models based on their sensitivity towards specific observable(s). $\{\Phi, \Delta, \mathcal{H}, \Sigma, \mathcal{S}\}$ produce any one or both from the set $\{\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}, \mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}\}$ at tree level. They contribute to all observables and are therefore severely constrained by EWPO. $\{\mathcal{O}^{(1)}_{H^4\mathcal{D}^4}, \mathcal{O}^{(2)}_{H^4\mathcal{D}^4}, \mathcal{O}^{(3)}_{H^4\mathcal{D}^4}\}$ are mainly constrained with VBS data, and filter out $\{\mathcal{H}, \Sigma\}$, which do not produce these operators. We highlight the operators produced by $\{\Theta\}$ that contribute to all observables with a box, these are loop suppressed.

carefully scrutinizing the overlapping sets of operators contributing to a set of observables. The second question relates to the order of the perturbative expansion at which the operators are being produced. That will give a hint of their possible sensitivity towards the observables. Keeping all these points in mind, we have clubbed those models that show degenerate sensitivity and prepared the different classes. Each class contains degenerate models with respect to their response to that particular observable in consideration.

The first question points to the need for new measurements with distinct features to correctly classify a wide range of complete models. The second one emphasizes the ability of existing measurements to constrain the underlying UV parameter spaces determines the measurements' BSM UV sensitivity.

Such an observable-based categorization has been studied recently (see, e.g., [76]) for dimension-six operators up to one loop, mainly in the Warsaw basis. Bringing dimension-eight operators into the picture helps resolve model degeneracies at the dimension-six level even without introducing new measurements. However, since the number of independent structures increases rapidly beyond dimension six, we confine our discussion only to the structures emerging for the six scalar extensions discussed in Secs. III and IV.

A. Electroweak precision observables

The precise measurements of the electroweak observables naturally calls for improvements on the theoretical side. This can be achieved by performing theoretical computations at next-to-leading order (NLO) and by extending the effective series expansion. As described in Refs. [19,50,77], we organize the operators below into lists based on how they contribute to EWPOs:

$$\begin{aligned}\text{Dimension-six LO:}\{&\mathcal{O}_{H\mathcal{D}},\mathcal{O}_{HWB},\mathcal{O}_{He},\mathcal{O}_{Hu},\mathcal{O}_{Hd},\mathcal{O}^{(1)}_{Hq},\\&\mathcal{O}^{(3)}_{Hq},\mathcal{O}^{(1)}_{Hl},\mathcal{O}^{(3)}_{Hl},\mathcal{O}_{ll}\};\end{aligned}\tag{5.1}$$

$$\begin{aligned}\text{Dimension-six NLO:}\{&\mathcal{O}_{\Box},\mathcal{O}_{HB},\mathcal{O}_{HW},\mathcal{O}_{W},\mathcal{O}_{uB},\mathcal{O}_{uW},\\&\mathcal{O}_{ed},\mathcal{O}_{ee},\mathcal{O}_{eu},\mathcal{O}_{lu},\mathcal{O}_{ld},\mathcal{O}_{le},\mathcal{O}^{(1)}_{lq},\\&\mathcal{O}^{(3)}_{lq},\mathcal{O}_{qe},\mathcal{O}^{(1)}_{qd},\mathcal{O}^{(3)}_{qq},\mathcal{O}^{(1)}_{qq},\mathcal{O}^{(1)}_{qu},\\&\mathcal{O}^{(1)}_{ud},\mathcal{O}_{uu},\mathcal{O}_{dd}\};\end{aligned}\tag{5.2}$$

$$\begin{aligned}\text{Dimension-eight LO:}\{&\mathcal{O}_{\psi^2H^5},\mathcal{O}^{(1)}_{H\mathcal{D},2},\mathcal{O}^{(2)}_{H\mathcal{D},2},\mathcal{O}^{(1)}_{\psi^2H^4\mathcal{D}},\\&\mathcal{O}^{(2)}_{\psi^2H^4\mathcal{D}},\mathcal{O}_{H^4WB},\mathcal{O}^{(1)}_{\psi^4H^2},\mathcal{O}^{(2)}_{\psi^4H^2}\}.\end{aligned}\tag{5.3}$$

Contrary to the Φ and Δ extensions, wherein $\{\mathcal{O}_{H\mathcal{D}}\}$ and $\{\mathcal{O}_{H\mathcal{D}},\mathcal{O}_{ll}\}$, respectively, are produced at tree level (see, e.g., Refs. [66,76]) and therefore provide the dominant contribution to the observable, other operators in Eq. (5.1) are generally sourced by heavy one-loop insertions. Their subleading contributions are comparable to the ones resulting from the operators produced at the tree-level and contribute to the observable at NLO [noted in Eq. (5.2)]. This implies that the BSM parameter space of such an extension is less sensitive to EWPOs than other observables when only dimension six is considered, which includes the models $\{\mathcal{H},\Sigma,\mathcal{S}\}$. The situation can be remedied by taking dimension-eight operators shown in Eq. (5.3) into account. In this case, if these operators appear at the tree level, the constraints can be improved significantly. Hence based on sensitivity towards EWPO, we can divide all the models into two broad categories:

$$\begin{aligned}&\text{Class A:}\{\Phi,\Delta,\mathcal{H},\Sigma,\mathcal{S}\},\\&\text{Class B:}\{\Theta\}.\end{aligned}$$

B. Higgs signal strength measurements

HSSs are inherently connected to the interplay of fundamental mass generation in the SM and electroweak symmetry breaking. Therefore, the analysis of the HSSs is a relevant discriminator in the space of Higgs sector extensions. Certain dimension-eight operators imply non-negligible effects when constraining the BSM parameter space through HSS measurements, for instance, when the new physics occurs at a relatively low scale or if new couplings occur at tree level after BSM states have been integrated out. According to Refs. [66,78,79], the operators that affect the HSS measurements are listed below:

$$\begin{aligned}\text{Dimension six:}\{&\mathcal{O}_{H},\mathcal{O}_{H\Box},\mathcal{O}_{H\mathcal{D}},\mathcal{O}_{HB},\mathcal{O}_{HW},\mathcal{O}_{HWB},\mathcal{O}_{eH},\\&\mathcal{O}_{uH},\mathcal{O}_{dH},\mathcal{O}_{He},\mathcal{O}_{Hu},\mathcal{O}_{Hd},\mathcal{O}^{(1)}_{Hq},\mathcal{O}^{(3)}_{Hq},\\&\mathcal{O}^{(1)}_{Hl},\mathcal{O}^{(3)}_{Hl}\};\end{aligned}\tag{5.4}$$

$$\begin{aligned}\text{Dimension eight:}\{&\mathcal{O}^{(8)}_{H},\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1},\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2},\mathcal{O}_{\psi^2H^5},\mathcal{O}_{H^4B^2},\\&\mathcal{O}_{H^4W^2},\mathcal{O}_{H^4WB}\}.\end{aligned}\tag{5.5}$$

Since the models $\{\Phi,\Delta,\mathcal{H},\mathcal{S}\}$ produce subsets of these operators $\{\mathcal{O}_{H},\mathcal{O}_{H\Box},\mathcal{O}_{H\mathcal{D}},\mathcal{O}_{uH},\mathcal{O}_{dH},\mathcal{O}_{eH}\}$ at tree level, while for $\{\Sigma,\Theta\}$ they are generated at one loop (see Ref. [76]), these models are seemingly less sensitive to HSS measurements at dimension-six. Following a similar approach as the one described in Sec. VA, we can infer that the impact of the operators given in Eq. (5.5) should be considered to properly explore the parameter space of $\{\Sigma,\Theta\}$. The fact that Σ generates $\mathcal{O}^{(8)}_{H}$, $\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$, and $\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}$ at tree level, as shown in Table XIV, leads to similar *a priori* sensitivity of Higgs measurements as for $\{\Phi, \Delta, \mathcal{H}, \mathcal{S}\}$. HSSs therefore discriminate:

$$\text{Class A}: \{\Phi, \Delta, \mathcal{H}, \Sigma, \mathcal{S}\},$$
$$\text{Class B}: \{\Theta\}.$$

C. Vector boson scattering measurements

Vector boson scattering measurements have been very crucial in the study of the electroweak sector, particularly in constraining anomalous gauge couplings, which have been discussed in detail in SMEFT at dimension six [80–83]. Individual bounds on dimension-eight couplings have been derived from VBS data as well [84–86]. At dimension-six, a total of nine operators contribute to the modification of observables through gauge-self couplings ($\{\mathcal{O}_W\}$), gauge-Higgs couplings ($\{\mathcal{O}_{H\mathcal{D}}, \mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{HWB}\}$), and fermion-gauge couplings ($\{\mathcal{O}^{(1)}_{Hl}, \mathcal{O}^{(3)}_{Hl}, \mathcal{O}^{(1)}_{Hq}, \mathcal{O}^{(3)}_{Hq}\}$). Among these, $\mathcal{O}_{H\mathcal{D}}$ and the two-fermionic operators are mainly constrained from EWPO observables [87], while the operators $\{\mathcal{O}_W, \mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{HWB}\}$ remain to be constrained by VBS measurements [88]. These are typically produced at the one-loop level (see Ref. [76]). For a low enough cutoff scale, these can be comparable to dimension-eight tree-level contributions:

$$\text{Dimension six}: \{\mathcal{O}_W, \mathcal{O}_{HB}, \mathcal{O}_{HW}, \mathcal{O}_{HWB}\}; \tag{5.6}$$

$$\text{Dimension eight}: \{\mathcal{O}^{(1)}_{H\mathcal{D},2}, \mathcal{O}^{(2)}_{H\mathcal{D},2}, \mathcal{O}^{(1)}_{H^4\mathcal{D}^4}, \mathcal{O}^{(2)}_{H^4\mathcal{D}^4}, \mathcal{O}^{(3)}_{H^4\mathcal{D}^4}\}. \tag{5.7}$$

We note that, $\{\mathcal{O}^{(1)}_{H\mathcal{D},2}, \mathcal{O}^{(2)}_{H\mathcal{D},2}\}$ contribute to the electroweak sector through the modification of gauge boson masses (see Appendix D of Ref. [79] for more details). Thus they are mostly constrained by EWPOs. Models that produce the full or a subset of the rest of the mentioned dimension-eight operators (i.e., $\{\Phi, \Delta, \mathcal{S}\}$) can be efficiently constrained by VBS measurements:

$$\text{Class-A}: \{\Phi, \Delta, \mathcal{S}\},$$
$$\text{Class-B}: \{\mathcal{H}, \Sigma, \Theta\}. \tag{5.8}$$

VI. CONCLUSIONS

The indirect search for new physics using EFT, while providing an ingenious way to uncover the physics that might lie just beyond our reach, faces several critical challenges when tracing constraints to possible complete and renormalizable UV scenarios. Moreover, since one encounters new signatures at dimension eight that may unravel the microscopic nature of new interactions, including their effects, can become vital when looking for new physics in a model-independent way. However, performing a global analysis of the entire parameter space of dimensions six and eight SMEFT is unrealistic. Broad model-dependent correlations can then help to hone the sensitivity to new interactions. This requires a transparent and effective way to perform matching to new physics scenarios beyond dimension six. In this work, we have explored these two issues in detail.

We present an easy-to-implement approach to compute the dimension-eight matching coefficients, capturing loop effects consistently. Furthermore, the method employs EOMs instead of the traditional field-redefinition formalism. The “missing piece” of the EOM that elevates it to a similar footing with field redefinition is included in a model-independent manner. It must be stressed that, while removing the redundant structures at dimension six, we obtain one-loop or two-loop equivalent contributions to dimension-eight structures due to the interference among the pieces generated at the tree and one-loop order.

We have applied this method to six different scalar extensions of the SM at one-loop order of the dimension-six coefficients considering both heavy-heavy and heavy-light loop propagators, validating our approach against results documented in the literature. Finally, we have clarified the relevance of dimension-eight operators for classifying UV-complete models given Higgs and electroweak measurements and VBS data.

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APPENDIX A: RELEVANT OPERATOR STRUCTURES

Here we discuss the operator structure that differs from the Green’s set as defined in Ref. [42]. At dimension six there are four structures in the $\Phi^4 D^2$ operator class. The operators are the following:

$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H), \qquad \mathcal{O}_{H\mathcal{D}} = |H^\dagger D_\mu H|^2,$$
$$\mathcal{O}'_{\mathbf{H}\mathcal{D}} = (\mathbf{H}^\dagger \mathbf{H})(\mathcal{D}^\mu \mathbf{H}^\dagger D_\mu \mathbf{H}),$$
$$\mathcal{O}''_{\mathbf{H}\mathcal{D}} = (\mathbf{H}^\dagger \mathbf{H})\mathcal{D}^\mu(\mathbf{H}^\dagger \mathbf{i}\overleftrightarrow{\mathcal{D}}_\mu \mathbf{H}). \tag{A1}$$

Among the above structures, the first two $(\mathcal{O}_{H\Box}, \mathcal{O}_{H\mathcal{D}})$ are considered to be the independent and part of the complete Warsaw basis. We can ignore the last one, $\mathcal{O}''_{H\mathcal{D}}$, which is

TABLE XIX. Dimension-six operator structures in the SILH set.

Operator	Operator structure	Operator	Operator structure
$\mathcal{Q}_H$	$\frac{1}{2}\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H)$	$\mathcal{Q}_6$	$\lvert H\rvert^6$
$\mathcal{Q}_R$	$(H^\dagger H)(\mathcal{D}_\mu H^\dagger D^\mu H)$	$\mathcal{Q}_T$	$\frac{1}{2}[H^\dagger \overleftrightarrow{\mathcal{D}}^\mu H][H^\dagger \overleftrightarrow{\mathcal{D}}^\mu H]$
$\mathcal{Q}_\mathcal{D}$	$(\mathcal{D}^2 H^\dagger)(\mathcal{D}^2 H)$	$\mathcal{Q}_{2W}$	$-\frac{1}{2}(\mathcal{D}_\mu W^I_{\mu\nu})^2$
$\mathcal{Q}_{2B}$	$-\frac{1}{2}(\partial_\mu B_{\mu\nu})^2$	$\mathcal{Q}_W$	$ig_W(H^\dagger \tau^a \overleftrightarrow{\mathcal{D}}^\mu H)\mathcal{D}^\nu W^a_{\mu\nu}$
$\mathcal{Q}_B$	$ig_Y(H^\dagger \overleftrightarrow{\mathcal{D}}^\mu H)\partial^\nu B_{\mu\nu}$	$\mathcal{Q}_{WW}$	$g_W^2(H^\dagger H)W^a_{\mu\nu}W^{a,\mu\nu}$
$\mathcal{Q}_{BB}$	$g_Y^2(H^\dagger H)B_{\mu\nu}B^{\mu\nu}$	$\mathcal{Q}_{WB}$	$2g_W g_Y(H^\dagger \tau^a H)W^a_{\mu\nu}B^{\mu\nu}$

TABLE XX. Structures of the dimension-eight operators in the nonredundant basis discussed throughout this work. ψ denotes any SM fermion ($\psi \in \{q,u,d,l,e\}$).

Operator	Operator structure	Operator	Operator structure
$\mathcal{O}^{(8)}_{H}$	$(H^\dagger H)^4$	$\mathcal{O}^{(8)}_{\psi^2 H^5}$	$(H^\dagger H)(\bar\psi_i\psi_j H)$
$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,1}$	$(H^\dagger H)^2(\mathcal{D}_\mu H^\dagger \mathcal{D}^\mu H)$	$\mathcal{O}^{(8)}_{H^6\mathcal{D}^2,2}$	$(H^\dagger H)(H^\dagger \mathcal{D}_\mu H \mathcal{D}^\mu H^\dagger H)$
$\mathcal{O}^{(8)}_{\psi^2 H^4\mathcal{D},1}$	$i(H^\dagger H)(\psi_i\gamma_\mu\psi_j)(H^\dagger \overleftrightarrow{\mathcal{D}}_\mu H)$	$\mathcal{O}^{(8)}_{\psi^2 H^4\mathcal{D},2}$	$i(H^\dagger H)(\psi_i\tau^I\gamma_\mu\psi_j)(H^\dagger \overleftrightarrow{\mathcal{D}}^I_\mu H)$
$\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,1}$	$i(\bar\psi_i\gamma^\mu\psi_j)[(\bar\psi_k\psi_l)\mathcal{D}_\mu H]$	$\mathcal{O}^{(8)}_{\psi^4\mathcal{D}H,2}$	$i(\bar\psi_i\gamma^\mu\tau^I\psi_j)[(\bar\psi_k\psi_l)\tau^I\mathcal{D}_\mu H]$
$\mathcal{O}^{(8)}_{H^4B^2}$	$(H^\dagger H)^2 B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(8)}_{H^4W^2}$	$(H^\dagger H)^2 W^I_{\mu\nu}W^{\mu\nu,I}$
$\mathcal{O}^{(8)}_{H^4WB}$	$(H^\dagger H)(H^\dagger\tau^I H)W^I_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(8)}_{\psi^2 WBH}$	$(\bar\psi_i\psi_j)\tau^I H W^I_{\mu\nu}B^{\mu\nu}$
$\mathcal{O}^{(8)}_{\psi^2B^2H}$	$(\bar\psi_i\psi_j)HB_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(8)}_{\psi^2W^2H}$	$(\bar\psi_i\psi_j)HW^I_{\mu\nu}W^{\mu\nu,I}$
$\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,1}$	$(\bar\psi_i\psi_j H)(\mathcal{D}_\mu H^\dagger \mathcal{D}^\mu H)$	$\mathcal{O}^{(8)}_{\psi^2H^3\mathcal{D}^2,2}$	$(H^\dagger \mathcal{D}_\mu H)(\bar\psi_i\psi_j\mathcal{D}^\mu H)$
$\mathcal{O}^{(8)}_{\psi^4H^2,1}$	$(H^\dagger H)(\bar\psi_i\psi_j\bar\psi_k\psi_l)$	$\mathcal{O}^{(8)}_{\psi^4H^2,2}$	$(\bar\psi_i\psi_j H)(\tilde H^\dagger\bar\psi_k\psi_l)$

CP violating and does not appear in our analysis. The redundant operator $\mathcal{O}'_{H\mathcal{D}}$ is the important structure for our analysis. In order to remove this redundancy, we derive the contribution to higher dimension, i.e., dimension eight in our case. Instead of using this exact structure, we use the following relation to convert it to a suitable form

$$(\mathbf{H}^\dagger\mathbf{H})(\mathcal{D}_\mu\mathbf{H}^\dagger\mathbf{D}^\mu\mathbf{H}) = \frac{1}{2}[(H^\dagger H)\Box(H^\dagger H) - (\mathbf{H}^\dagger\mathbf{H})(\mathcal{D}^2\mathbf{H}^\dagger\mathbf{H} + \mathbf{H}^\dagger\mathcal{D}^2\mathbf{H})]. \tag{A2}$$

Since we are replacing one redundant structure with another that is related to the former by the integration by parts, we term it a Green's set-like structure. We collect all relevant dimension-six operators in Table XIX. Dimension eight can be found in Table XX.

APPENDIX B: RENORMALIZABLE SM LANGRANGIAN AND EOM

The renormalizable SM Lagrangian is

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ (\mathcal{D}_\mu H^\dagger)(\mathcal{D}^\mu H) + \sum_{\psi=q,u,d,l,e}\bar\psi\, i\not{D}\psi - \lambda(H^\dagger H)^2 \\ &- [Y_d H^{\dagger j}\bar d q_j + Y_u \tilde H^{\dagger j}\bar u q_j + Y_e H^{\dagger j}\bar e l_j + \mathrm{H.c.}].\end{aligned} \tag{B1}$$

Here, we have ignored the (negative) mass term for the Higgs field which is not relevant for our analysis. We can calculate the EOMs for various fields using Eq. (B1). These EOMs constitute the first-order approximation of the full EOM, and they can be used to transform one set to another at a given mass dimension. The first-order EOM for Higgs field is [13,28]

$$\mathcal{D}^2 H_k + 2\lambda(H^\dagger H)H_k + \mathcal{Y}_k = 0,$$
$$\text{where } \mathcal{Y}_k = Y_u^\dagger \bar{q}^j u \epsilon_{jk} + Y_d \bar{d} q_k + Y_e \bar{e} l_k, \tag{B2}$$

and the EOMs for the fermions are [13,28]

$$i\not{D} q_j = Y_u^\dagger u \tilde{H}_j + Y_d^\dagger d H_j, \qquad i\not{D} d = Y_d q_j H^{\dagger j},$$
$$i\not{D} u = Y_u q_j \tilde{H}^{\dagger j}, \qquad i\not{D} l_j = Y_e^\dagger e H_j,$$
$$i\not{D} e = Y_e l_j H^{\dagger j}, \tag{B3}$$

with Yukawa couplings $Y_i \equiv Y_{\mathrm{SM},i}$. The EOMs for the gauge fields are [13,28]

$$[\mathcal{D}^\alpha, G_{\alpha\beta}]^A = g_G \sum_{\psi=u,d,q} \bar{\psi}\, T^A \gamma_\beta \psi,$$
$$[\mathcal{D}^\alpha, W_{\alpha\beta}]^I = g_W \left(\frac{1}{2} \bar{q} \tau^I \gamma_\beta q + \frac{1}{2} \bar{l} \tau^I \gamma_\beta l + \frac{1}{2} H^\dagger i \overleftrightarrow{\mathcal{D}}^I_\beta H \right),$$
$$[\mathcal{D}^\alpha, B_{\alpha\beta}] = g_Y \left(\sum_{\psi=u,d,q,e,l} \bar{\psi}\, \mathrm{y}_i \gamma_\beta \psi + H^\dagger i \overleftrightarrow{\mathcal{D}}_\beta H \right), \tag{B4}$$

where y_i denotes the $U(1)_\mathrm{Y}$ hypercharges of the fermions. We have also used the following the notation [13,22]

$$H^\dagger i \overleftrightarrow{\mathcal{D}}_\beta H = i H^\dagger (\mathcal{D}_\beta H) - i (\mathcal{D}_\beta H^\dagger) H,$$
$$H^\dagger i \overleftrightarrow{\mathcal{D}}^I_\beta H = i H^\dagger \tau^I (\mathcal{D}_\beta H) - i (\mathcal{D}_\beta H^\dagger) \tau^I H, \tag{B5}$$

to write the operators in the well-known compact forms.

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