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(Received 14 October 2022; accepted 13 February 2023; published 8 March 2023)

We study the non-Abelian $SU(2)_D$ extension of the $U(1)_D$ Stueckelberg portal, which plays the role of mediator between the Standard Model (SM) and dark sector. This portal is specified by the Stueckelberg mechanism for generation of dark gauge boson masses. The proposed $U(1)_D \otimes SU(2)_D$ Stueckelberg portal has a connection with SM matter fields, in analogy with the familon model. We derive bounds on the couplings of dark portal bosons and SM particles, which govern diagonal and nondiagonal flavor transitions of quarks and leptons.

DOI: [10.1103/PhysRevD.107.055006](https://doi.org/10.1103/PhysRevD.107.055006)**I. INTRODUCTION**

The Standard Model (SM) of particle physics is a unified gauge theory of strong and electroweak interactions, which allows one to perform a precise description and explanation of most data extracted at worldwide facilities wherein there are signals of new physics, which cannot be explained within the SM and, therefore, require its extensions. These include, in particular, the muon anomalous magnetic moment with last measurements at Fermilab [1], where the current deviation of the SM prediction and experimental data lies at the $\sim 4.2\sigma$ level [2]. Besides this, there are further unresolved puzzles, such as the strong CP problem and rare meson decays [3–7], flavor nonuniversality [8–10], the $b-s$ quark anomaly [11], neutrino mass generation, etc. These puzzles initiated many efforts for SM extensions and new physics searches.

The existence of dark matter (DM) is required by a wide spectrum of gravitational, astrophysical, and cosmological phenomena. DM significantly contributes to the mass of the Universe. However, its precise nature is not yet known. The search for DM created an idea of portals between SM and

DM particles. In this respect, the main popular portals are Higgs [12], axion [13], axionlike particles [14–18], vector [9,19], and sterile neutrino [20] ones. Recently, we proposed the $U(1)_D$ dark gauge-invariant vector portal between particles of the SM and dark sector (DS) [21], where the mass of dark photon A' is generated via the Stueckelberg mechanism (see also Refs. [22,23]). Additional scalar field σ occurring in that approach is an unphysical degree of freedom, which plays the role of ghost field. There is a popular idea of interaction of gauge field and SM fermions based on the ansatz of a familon (or flavons) in the literature [16,24,25]. Such an interaction mechanism between dark photon and SM fermions leads to a rich phenomenology including novel information on couplings preserving and violating lepton symmetries, e.g., lepton flavor violation (LFV).

Phenomenological studies of the dark photon [26] have been performed using different scenarios and particle content (see, e.g., Refs. [9,27]). In particular, the dark photon could interact not only with a QED photon-induced so-called kinetic mixing term [26], but also with a leptonic pair including neutrinos (so-called Z' boson). Additional vector gauge bosons have been extensively studied and searched during three decades [28]. Especially, very promising studies have been performed at CERN, Fermilab, and other experimental facilities [29–38]. One should stress that phenomenological models considered before were not limited by simple kinetic mixing between SM and new

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gauge bosons [39–42]. Indeed, other possible scenarios have been also considered.

The main motivation of present paper is an extension of the Abelian $U(1)_D$ Stueckelberg portal [21] by adding the non-Abelian $SU(2)_D$ sector, because it gives an opportunity to study the processes with charged dark gauge bosons and currents. In particular, inclusion of additional particles from the $SU(2)_D$ portal could give a chance to understand existing deviations between SM and experiments. In particular, we introduce the $SU(2)_D$ triplet of dark gauge bosons (DGBs). Such an extension is natural and has the additional benefit of opening a window for couplings between SM and DS particles via charged currents. Note that the Abelian $U(1)_D$ Stueckelberg portal is based on the coupling of a dark photon with neutral current formed by SM fermions, while inclusion of the $SU(2)_D$ DGBs interacting with both charged and neutral currents can be performed in analogy to the weak sector of SM. Thereby, all properties of the Stueckelberg portal such as LFV are implemented. Besides, the LFV effect [43], such $SU(2)_D$ extension induces lepton number violation.

Discussion of the dark fermion sector, which is closely related to charge dark gauge bosons W'^{\pm} , is beyond the scope of present paper. By the way, we note that it is a very interesting topic (see, e.g., recent discussion in Ref. [44]). Instead, we focus on phenomenological aspects of the $SU(2)_D$ vector Stueckelberg portal and derivation of bounds on its couplings with SM fermions based on current deviations between the SM and data. In our study, we intend to consider both LFV and non-LFV lepton decays as a tool for such analysis. We also mention that the new W' bosons have been intensively studied in different theoretical approaches beyond the SM. In particular, the W' bosons have been proposed by several theoretical approaches based on gauge extensions of the Standard Model [28,45], extradimensional [46–48], technicolor [49,50], and composite Higgs models [51,52] models. Experimental searches of the W' bosons have been performed at LHC by the ATLAS and CMS Collaborations [53–56] based on the production of the W' in proton-proton collisions, which led to the constraint of the W' mass in the range 0.15–6 TeV. In our approach, we assume that the dark W' boson masses do not exceed the TeV region.

The paper is organized as follows. In Sec. II, we define the Lagrangian for the non-Abelian extension of the dark sector. In Sec. III, we estimate bounds on the couplings of the charged DGBs imposed by various muon decay processes, with $\mu \rightarrow e\nu_i\bar{\nu}_j$ in Sec. III A and $\mu \rightarrow e\gamma$ in Sec. III B. The latter is relevant for the muon $g-2$ anomaly. Finally, in Sec. IV, we present our conclusions.

II. NON-ABELIAN EXTENSION OF THE DARK SECTOR

In this section, we discuss an extension of the Abelian dark Stueckelberg portal to the non-Abelian case. In

particular, we propose the existence of three additional DGBs and three additional DSBs associated with the $SU(2)_D$ group, which are required by gauge-invariant principle. Our formalism is based of effective SM + DS Lagrangian $\mathcal{L}_{\text{SM+DS}}$, which after spontaneous breaking of electroweak symmetry to electromagnetic group $U(1)_Y \otimes SU(2)_L \rightarrow U(1)_{\text{em}}$ is manifestly gauge invariant under the product of electromagnetic group and groups of DS and $U(1)_{\text{em}} \otimes U(1)_D \otimes SU(2)_D$. First, we specify the fields of our Lagrangian. For convenience, in this paper, we use the notations introduced before for the Abelian case [21] and introduce additional ones relevant for the non-Abelian dark sector sector. The SM sector contains fundamental fermions—left q_L^{im} and right (U_R^i, D_R^i) quarks, left and right leptons (ℓ_L^{im}, ℓ_R^i fields), gauge fields (weak gauge bosons W^{\pm}, Z^0 , and photon A), and scalar Higgs field H . Left doublets and right singlets of quarks and leptons are specified as $q_L^{1m} = (u_L, d_L)$, $q_L^{2m} = (c_L, s_L)$, $q_L^{3m} = (t_L, b_L)$, $U_R^1 = u_R$, $U_R^2 = c_R$, $U_R^3 = t_R$, $D_R^1 = u_R$, $D_R^2 = c_R$, $D_R^3 = b_R$, $L^{1m} = (\nu_{eL}, e_L)$, $L^{2m} = (\nu_{\mu L}, \mu_L)$, $L^{3m} = (\nu_{\tau L}, \tau_L)$, $R^1 = e_R$, $R^2 = \mu_R$, and $R^3 = \tau_R$. The indices $i, j = 1, 2, 3$ number the fermion generations, while m, n denote the $SU(2)$ weak isospin and dark indices, respectively.

The DS sector contains singlet dark fermion χ , gauge bosons (singlet A' and triplet \mathbf{W}'), and scalars (singlet σ and triplet \mathbf{S}). The matrices \mathbf{S} and \mathbf{W}' are specified as

$$\mathbf{S} = \begin{pmatrix} S^0/\sqrt{2} & S^+ \\ S^- & -S^0/\sqrt{2} \end{pmatrix},$$

$$\mathbf{W}' = \begin{pmatrix} W'^0/\sqrt{2} & W'^+ \\ W'^- & -W'^0/\sqrt{2} \end{pmatrix}. \quad (1)$$

Triples of gauge bosons W' and scalars S have electric charge. In particular, W'^{\pm} and S^{\pm} have charges $Q = \pm 1$, respectively, while W'^0, S^0 , and dark fermions are electrically neutral.

Now we define the covariant derivatives acting on fermions and scalars. In the case of the SM sector, the change will be in adding of the terms containing dark gauge fields in the covariant derivatives acting on the left and right fermions:

$$(iD^L)_{\mu}^{mn} \rightarrow (iD^L)_{\mu}^{mn} - \frac{g_{W'}}{2} \mathbf{W}'_{\mu}{}^{mn} - g_{A'} \delta^{mn} A'_{\mu},$$

$$(iD^R)_{\mu} \rightarrow (iD^R)_{\mu} - g_{A'} A'_{\mu}, \quad (2)$$

where $g_{W'}$ and $g_{A'}$ are the gauge couplings associated with $SU(2)_D$ and $U(1)_D$ groups, respectively. In the case of the DS, the covariant derivatives acting on scalar fields ($D_{\mu}\sigma$) and ($D_{\mu}\mathbf{S}$) and fermions are defined, respectively, as

$$\begin{aligned} (D_\mu \sigma) &= \partial_\mu \sigma - M_{A'} A'_\mu, \\ (D_\mu \mathbf{S})^{mn} &= \partial_\mu \mathbf{S}^{mn} - M_{W'} \mathbf{W}'_\mu{}^{mn} \end{aligned} \quad (3)$$

and

$$(iD_\chi)_\mu = i\partial_\mu - g_{A'} A'_\mu. \quad (4)$$

Note that the covariant derivatives acting on dark scalars ($D_\mu \sigma$) and ($D_\mu \mathbf{S}$) include dark gauge bosons (singlet A') and triplet (W'^{\pm}, W'^0) having finite masses $M_{A'}$ and $M_{W'}$, respectively. The covariant derivatives ($D_\mu \sigma$) and ($D_\mu \mathbf{S}$) therefore contain DS gauge boson masses, generated via the Stueckelberg mechanism [23,57]. The $U(1)_D$ gauge boson A' is called the dark photon, and it has the mass $M_{A'}$. The scalar Stueckelberg fields σ and \mathbf{S} play the role of supplementary Goldstone bosons generating masses of dark photon and dark triplet gauge bosons \mathbf{W}' . Such an idea was considered before for the new Z' boson in Ref. [58] (see also discussion in Ref. [59]). DGBs acquire the masses in a manifestly gauge-invariant form. Finite masses of scalars violate $U(1)_D \otimes SU(2)_D$ symmetry (for a review, see Ref. [23]). Extension of the Stueckelberg mechanism on non-Abelian field leads to known problems of renormalizability and unitarity. Critical remarks and efforts to resolve these problems have been discussed in detail in Ref. [23]. Note that, due to the scale of new physics, Λ is much larger than the scale of SM ($\Lambda \gg \Lambda_{\text{SM}}$);

thus, we do not go to higher loops and restrict ourselves to one-loop approximation. Therefore, the problems mentioned above are not so critical for our purposes. Note that, for convenience, we write all terms in Lagrangian involving the \mathbf{S} triplet in a simplified form, which follows from Ref. [60], where this field was derived using adjoint representation of the $SU(2)_D$ gauge group

$$U(x) = \exp \left[-ig_{W'} \frac{\mathbf{S}(x)}{2M_{W'}} \right]. \quad (5)$$

In this approach,

$$\partial_\mu \mathbf{S}(x) = \frac{2M_{W'}}{g_{W'}} i\partial_\mu U(x) U^{-1}(x). \quad (6)$$

We propose that the Stueckelberg mechanism [23,57] for generating masses of gauge fields is extended to the group $U(1)_D \otimes SU(2)_D$.

The stress tensors for dark gauge bosons are defined as

$$\begin{aligned} A'_{\mu\nu} &= \partial_\mu A'_\nu - \partial_\nu A'_\mu, \\ \mathbf{W}'_{\mu\nu} &= \partial_\mu \mathbf{W}'_\nu - \partial_\nu \mathbf{W}'_\mu + \frac{ig_{W'}}{2} [\mathbf{W}'_\mu, \mathbf{W}'_\nu]. \end{aligned} \quad (7)$$

The $U(1)_D \otimes SU(2)_D$ gauge transformations of dark fermions, scalars, and gauge bosons are specified as

$$\begin{aligned} A'_\mu &\rightarrow A'_\mu + \frac{i}{g_{A'}} \partial_\mu \Omega_{A'} \Omega_{A'}^{-1}, & A'_{\mu\nu} &\rightarrow A'_{\mu\nu}, \\ \mathbf{W}'_\mu &\rightarrow \Omega_{W'} \mathbf{W}'_\mu \Omega_{W'}^{-1} + \frac{2i}{g_{W'}} \partial_\mu \Omega_{W'} \Omega_{W'}^{-1}, & \mathbf{W}'_{\mu\nu} &\rightarrow \Omega_{W'} \mathbf{W}'_{\mu\nu} \Omega_{W'}^{-1}, \\ \partial_\mu \sigma &\rightarrow \partial_\mu \sigma + \frac{iM_{A'}}{g_{A'}} \partial_\mu \Omega_{A'} \Omega_{A'}^{-1}, & (D_\mu \sigma) &\rightarrow (D_\mu \sigma), \\ U(x) &\rightarrow \Omega_{W'}(x) U(x), & \partial_\mu \mathbf{S} &\rightarrow \Omega_{W'} \partial_\mu \mathbf{S} \Omega_{W'}^{-1} + \frac{2iM_{W'}}{g_{W'}} \partial_\mu \Omega_{W'} \Omega_{W'}^{-1}, \\ (D_\mu \mathbf{S}) &\rightarrow \Omega_{W'} (D_\mu \mathbf{S}) \Omega_{W'}^{-1}, & \text{Tr}[(D_\mu \mathbf{S})(D_\mu \mathbf{S})] &\rightarrow \text{Tr}[(D_\mu \mathbf{S})(D_\mu \mathbf{S})], \\ \chi &\rightarrow \Omega_{A'} \chi, & i\mathcal{D}_\chi \chi &\rightarrow \Omega_{A'} i\mathcal{D}_\chi \chi, \end{aligned} \quad (8)$$

where

$$\Omega_{W'}(x) = \exp \left[\frac{i}{2} \vec{\theta}_{W'}(x) \vec{\tau} \right], \quad \Omega_{A'}(x) = \exp[i\theta_{A'}(x)] \quad (9)$$

are the matrices of the fundamental $SU(2)_D$ and $U(1)_D$ transformations.

Now we specify effective Lagrangian of our approach $\mathcal{L}_{\text{SM+DS}}$ combining SM and DS:

$$\mathcal{L}_{\text{SM+DS}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DS}} + \Delta\mathcal{L}. \quad (10)$$

This Lagrangian is by construction a low-energy Lagrangian, which is an extrapolation of new physics Lagrangian including the DS sector to the SM scale $\Lambda_{\text{SM}} \simeq M_{W^\pm/Z^0} \simeq 100$ GeV. Here, \mathcal{L}_{SM} is the term describing dynamics of the SM sector including the coupling of the SM fermions with DS gauge fields via extension of covariant derivatives introduced above, \mathcal{L}_{DS} is the term describing dynamics of the DS including the coupling of the dark fermions with SM gauge fields via extension of covariant derivatives introduced above, and two terms describing the coupling of SM and DS-GB mixing term

\mathcal{L}_{mix} and a term describing additional coupling of DSBs with SM fields $\Delta\mathcal{L}$ allowing a violation of lepton flavor and number violation and violation of the Glashow-Iliopoulos-Maiani (GIM) mechanism [61] in the quark sector.

Next, we specify the terms \mathcal{L}_{DS} and \mathcal{L}_{int} . The DS Lagrangian \mathcal{L}_{DS} is given by

$$\begin{aligned}\mathcal{L}_{\text{DS}} = & -\frac{1}{4}A'_{\mu\nu}A'^{\mu\nu} - \frac{1}{4}\mathbf{W}'_{\mu\nu}\mathbf{W}'^{\mu\nu} + \frac{1}{2}(D_\mu\sigma)(D^\mu\sigma) \\ & + \frac{1}{2}\text{Tr}[(D_\mu\mathbf{S})(D^\mu\mathbf{S})] + \bar{\chi}(i\not{D}_\chi - m_\chi)\chi \\ & - \frac{1}{2\xi_{W'}}\text{Tr}(\partial_\mu\mathbf{W}'^\mu + \xi_{W'}M_{W'}\mathbf{S})^2 \\ & - \frac{1}{2\xi_{A'}}(\partial_\mu A'^\mu + \xi_{A'}M_{A'}\sigma)^2.\end{aligned}\quad (11)$$

One should stress that quantization of the dark gauge field is required to add the gauge-fixing term into the dark $U(1)_D$ and $SU(2)_D$ sectors [62] (last two terms in \mathcal{L}_{DS}), where $\xi_{W'(A')}$ is an arbitrary gauge parameter corresponding to the $W'(A')$ gauge boson. It provides ‘‘decoupling’’ of gauge bosons and corresponding scalars particles with vanishing mixed terms:

$$\begin{aligned}\mathcal{L}_{\text{DS}} = & -\frac{1}{4}A'_{\mu\nu}A'^{\mu\nu} - \frac{1}{4}\mathbf{W}'_{\mu\nu}\mathbf{W}'^{\mu\nu} + \frac{1}{2}\text{Tr}(\partial_\mu\mathbf{S}\partial^\mu\mathbf{S}) \\ & + \frac{M_{W'}^2}{2}\text{Tr}(\mathbf{W}_\mu\mathbf{W}^\mu) + \frac{M_{A'}^2}{2}A'_\mu A'^\mu \\ & - \frac{1}{2\xi_{W'}}\text{Tr}(\partial_\mu\mathbf{W}'^\mu + \xi_{W'}M_{W'}\mathbf{S})^2 \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2\xi_{A'}}(\partial_\mu A'^\mu)^2 - \frac{\xi_{A'}}{2}M_{A'}^2\sigma^2 \\ & + \bar{\chi}_L(i\not{D}_\chi^L - m_\chi)\chi_L + \bar{\chi}_R(i\not{D}_\chi^R - m_\chi)\chi_R.\end{aligned}\quad (12)$$

We note that the masses of the σ and triplet \mathbf{S} are proportional to the gauge parameter ξ_i , signaling that these fields are unphysical. In the gauge we are using, the dark boson propagator takes the form

$$D^{\mu\nu}(k; \xi_i) = \frac{1}{k^2 - M^2} \left[g^{\mu\nu} - \frac{k^\mu k^\nu (1 - \xi_i)}{k^2 - \xi_i M^2} \right], \quad (13)$$

where M is the mass of the dark gauge boson.

The interaction $\Delta\mathcal{L}$ Lagrangian is constructed by analogy with the familon model proposed in Ref. [24] for the hypothetical familon field. The scalar fields are unphysical and are to be switched off by the choice of gauge fixing:

$$\begin{aligned}\Delta\mathcal{L} = & \frac{1}{\Lambda}(D_\mu\sigma)\sum_{ij}\left[\bar{q}_L^i c_\sigma^{ij}\gamma^\mu q_L^j + \bar{U}_R^i c_{U\sigma}^{ij}\gamma^\mu U_R^j\right. \\ & \left. + \bar{D}_R^i c_{D\sigma}^{ij}\gamma^\mu D_R^j + \bar{L}^i d_{L\sigma}^{ij}\gamma^\mu L^j + \bar{R}^i d_{R\sigma}^{ij}\gamma^\mu R^j\right] \\ & + \frac{1}{\Lambda}(D_\mu\mathbf{S})^{mn}\sum_{ij}\left[\bar{q}_L^{im} c_S^{ij}\gamma^\mu q_L^{jn} + \bar{L}^{im} d_S^{ij}\gamma^\mu L^{jn}\right],\end{aligned}\quad (14)$$

where Λ is the scale of new physics. Here, c^{ij} and d^{ij} are the 3×3 Hermitian matrices containing the couplings of dark scalars with the SM fermions and include effects of lepton flavor and number violation and violation of the GIM mechanism [61] in the quark sector. The parameter Λ is the characteristic scale of this effective operator, defining when it opens up in terms of renormalizable interactions of an UV completion.

As was pointed out in Ref. [14], after spontaneous breaking of electroweak symmetry in the SM, one should diagonalize the fermion mass matrices by means of unitary transformations

$$\begin{aligned}Y_U = (V_L^q)^\dagger y_U W_R^U, \quad Y_D = (V_L^q)^\dagger y_D W_R^D, \\ Y_\ell = (V_L^\ell)^\dagger y_\ell W_R^\ell.\end{aligned}\quad (15)$$

Here, V_L^q and V_L^ℓ are the transformation matrices acting on the left-handed quarks and leptons, respectively, W_R^U , W_R^D , and W_R^ℓ are the transformation matrices acting on right singlets, and y^{ij} are the $3 \otimes 3$ Yukawa matrices of couplings between two scalar doublets and SM fermions before spontaneous breaking of electroweak symmetry. The matrices V_L^q , V_L^ℓ , W_R^U , and W_R^D rotate the couplings of dark scalars with the SM fermions as

$$\begin{aligned}c_\sigma & \rightarrow C_\sigma = (V_L^q)^\dagger c_\sigma V_L^q, \\ c_{U\sigma} & \rightarrow C_{U\sigma} = (W_R^U)^\dagger c_{U\sigma} W_R^U, \\ c_{D\sigma} & \rightarrow C_{D\sigma} = (W_R^D)^\dagger c_{D\sigma} W_R^D, \\ d_{L\sigma} & \rightarrow D_{L\sigma} = (V_L^q)^\dagger c_{L\sigma} V_L^q, \\ d_{R\sigma} & \rightarrow D_{R\sigma} = (W_R^D)^\dagger d_{R\sigma} W_R^U, \\ c_S & \rightarrow C_S = (V_L^q)^\dagger c_S V_L^q, \\ d_S & \rightarrow D_S = (V_L^\ell)^\dagger d_S V_L^\ell.\end{aligned}\quad (16)$$

Note that the resulting coupling of the SM fermions with DS fields is contributed by three terms via minimal substitution of the covariant derivatives acting on the SM fermions and via additional effective Lagrangian (14). The first term does not mix the SM generations, does not violate certain symmetries (like lepton flavor and lepton number), and preserves the GIM mechanism:

$$\mathcal{L}_{\text{int}; 1} = g_{A'} \sum_i \bar{\psi}_i \gamma^\mu A'_\mu \psi_i + \frac{g_{W'}}{2} \sum_i \bar{\psi}_i^m \gamma^\mu \mathbf{W}'^{\mu mn} (1 - \gamma_5) \psi_i^n. \quad (17)$$

After the substitution of the covariant derivatives ($\partial\sigma$) and ($\partial\mathbf{S}$), we find that the gauge-invariant operator (14) additionally generates dimension-4 interactions of the dark photon and \mathbf{W}' bosons with the SM fermions ψ , in the form

$$\begin{aligned} \mathcal{L}_{\text{int}; 2} = & \sum_{ij} \bar{\psi}_i \gamma^\mu A'_\mu (g_{ij}^V + g_{ij}^A \gamma_5) \psi_j \\ & + \sum_{ij} g_{ij}^{VA} \bar{\psi}_i^m \gamma^\mu \mathbf{W}'^{\mu mn} (1 - \gamma_5) \psi_j^n, \end{aligned} \quad (18)$$

where vector g^V and axial-vector g^A dimensionless couplings are defined as

$$g_{ij}^V = \frac{m_{A'}}{\Lambda} v_{ij}, \quad g_{ij}^A = \frac{m_{A'}}{\Lambda} a_{ij}, \quad (19)$$

$$v_{ij} = \frac{1}{2} (D_{R\sigma}^{ij} + D_{L\sigma}^{ij})_{ij}, \quad a_{ij} = \frac{1}{2} (D_{R\sigma}^{ij} - D_{L\sigma}^{ij})_{ij}. \quad (20)$$

Dimensionless couplings of W' with leptons have the simple form $g_{ij}^{VA} = D_S^{ij} M_{W'}/\Lambda$. Scalar nonphysical fields can be switched off from consideration in the case of unitary gauge ($\xi \rightarrow \infty$), which corresponds to the limit of infinitely large masses of scalars or vanishing of their contribution to physical processes. Therefore, the full interaction Lagrangian of the SM fermions with DGBs reads

$$\begin{aligned} \mathcal{L}_{\text{int}} = \mathcal{L}_{\text{int}; 1} + \mathcal{L}_{\text{int}; 2} = & \sum_{ij} \bar{\psi}_i \gamma^\mu A'_\mu (G_{ij}^V + G_{ij}^A \gamma_5) \psi_j \\ & + \sum_{ij} G_{ij}^{VA} \bar{\psi}_i^m \gamma^\mu \mathbf{W}'^{\mu mn} (1 - \gamma_5) \psi_j^n, \end{aligned} \quad (21)$$

where

$$G_{ij}^V = g_{ij}^V + \delta_{ij} g_{A'}, \quad G_{ij}^A \equiv g_{ij}^A, \quad G_{ij}^{VA} = g_{ij}^{VA} + \delta_{ij} \frac{g_{W'}}{2}. \quad (22)$$

III. BOUNDS ON W' COUPLINGS WITH STANDARD MODEL PARTICLES

In this section, we discuss opportunities to estimate bounds on the couplings of the charged dark gauge boson W' with the SM particles. On the one hand, we base on data extracted from precise measurements, and, on the other hand, we can involve rare decays in our analysis. First, we

estimate diagonal couplings G_{ii}^{VA} , which give an additional contribution to the SM processes. Hereinafter, we will concentrate on nondiagonal couplings G_{ij}^{VA} ($i \neq j$), which can be responsible for a contribution to LFV processes.

A. Dominant and LFV $\mu \rightarrow e\nu_i\bar{\nu}_j$ decays

To derive a limit for the G_{ii}^{VA} coupling, we use one of the most precise measurements in particle physics—decay rate $\mu \rightarrow e\nu_\mu\bar{\nu}_e$ process. This decay gives a very accurate determination of the Fermi constant G_F^μ :

$$G_F^\mu = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} \quad (23)$$

at the level of 0.5 ppm [63–65].

It is clear that the diagonal coupling of dark gauge boson W' gives an additional contribution to the $\mu \rightarrow e\nu_\mu\bar{\nu}_e$ decay rate. In our benchmark scenario, we make a conjecture that there is an ambiguity for the precise determination of the Fermi constant G_F . In particular, this constant can be constrained using different data (see, e.g., Ref. [66]). In particular, G_F can be determined from analysis of kaon or τ decays using unitarity of the Cabibbo-Kobayashi-Maskawa matrix. Alternatively, one can use the global electroweak (EW) fit. Obtained values have a small deviation from the value of the G_F extracted from weak muon decay. Indeed, such uncertainty is a gap, which constrains the possible contribution of the W' to the $\mu \rightarrow e\nu_\mu\bar{\nu}_e$ muon decay and provides bounds on its diagonal or nondiagonal couplings in dependence on a type of dark vector boson.

For calculation of decay widths, we use the well-known formula

$$d\Gamma = \frac{4\pi^2}{2m} |M|^2 d\Phi_n, \quad (24)$$

where m is the mass of decaying particle, $|M|^2$ is the square of amplitude of a process, which defines its dynamics, and $d\Phi_n$ is an element of n -body phase space.

The muon decay rate $\mu \rightarrow e\nu_\mu\bar{\nu}_e$ in the SM is determined by

$$\begin{aligned} \Gamma(\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e) = & \left(\frac{\sqrt{2} g_{\text{EW}}^2}{8m_W^2} \right)^2 \frac{m_\mu^5}{192\pi^3} (1 + \Delta q) \\ = & \frac{(G_F^\mu)^2 m_\mu^5}{192\pi^3} (1 + \Delta q), \end{aligned} \quad (25)$$

where the quantity Δq includes the phase space, QED, and hadronic radioactive corrections (see Ref. [66]).

The square of the $\mu \rightarrow e\nu_\mu\bar{\nu}_e$ decay amplitude, taking into account of the mixing of the SM W and dark W' bosons, is given by

$$|M|^2 = - \frac{(s+t)(s+t-m_\mu^2)(g_{EW}^2(t-m_{W'}^2) + (G^{VA})^2(t-m_W^2))^2}{(m_{W'}^2-t)^2(t-m_W^2)^2}, \quad (26)$$

where the Mandelstam kinematical variables are defined as $s = m_\mu^2 - 2(p_\mu p_e)$, $t = m_\mu^2 - 2(p_\mu p_{\nu_1})$, and $u = m_\mu^2 - 2(p_\mu p_{\nu_2})$ [67] and g_{EW} is electroweak coupling. We obtain an expression for the decay width as a function of parameters of the new vector boson (mass of $m_{W'}$ and couplings with leptons G_{ij}^{VA}).

Using the difference between data of the global EW fit presented in Ref. [66] $G_F^{EW}|_{full} = 1.16716(39) \times 10^{-5} \text{ GeV}^{-2}$ and G_F^μ from $\mu \rightarrow e\nu_\mu\bar{\nu}_e$ decay, we can derive limits for the couplings of W' with the SM particles using the muon width as an input data parameter. Bounds on the coupling ratio G_{ij}^{VA}/g_{EW} as a function of the W' mass are shown in Fig. 1. These bounds in the case of the universality scenario of the coupling of the dark W' boson with quarks and leptons can be compared with limits extracted by the CMS Collaboration [56] (see Fig. 1). In this scenario at $m_{W'} \geq 0.5 \text{ TeV}$, the CMS limits are more stronger and our bounds established from study of the famous muon decay complement in this range of the dark boson mass. In the case of the nonuniversal coupling of dark boson W' with quark and leptons (general scenarios), our limits deduced from the $\mu \rightarrow e\nu_\mu\bar{\nu}_e^c$ decay cannot be directly compared with the CMS constraints. The difference between values of the G_F constant defined from different experimental data gives strong limits on diagonal couplings of the dark charge boson W'^{\pm} with the SM fermions.

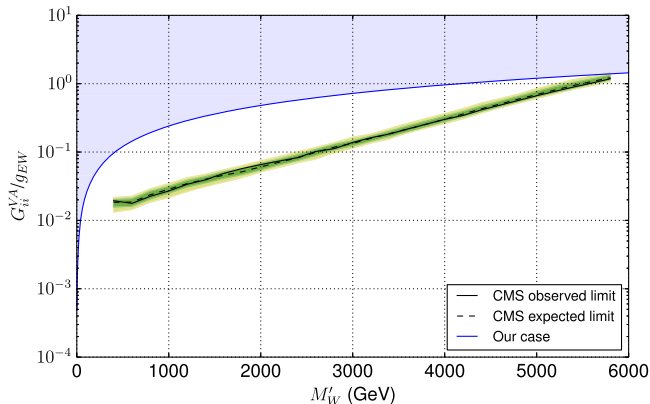


FIG. 1. Bounds on the ratio G_{ij}^{VA}/g_{EW} from possible contribution to the $\mu \rightarrow e\nu\nu$ decay rate (dirty blue line) as a function of $m_{W'}$ in the range 2 GeV–6 TeV. Black solid and dashed lines are CMS observed and expected limit, respectively, with one standard deviation and two standard deviations (green and yellow areas) [56]. These two limits can be compared only in the universality case of diagonal coupling dark boson with quarks and leptons.

We also want to mention a landscape of study, a possible dark charge W' boson. The existed investigation is based on studies of different process [56,68] and different models [40,41,69]. As output of such studies, different bounds on the masses, couplings, and other parameters of additional W' and Z' bosons have been proposed in the literature. Sometimes different bounds from different studies cannot be compared directly. Here, we stress that novel W' and Z' bosons have been proposed in the literature before in different extensions of the SM at TeV scales, including extradimensional models, technicolor, and composite Higgs (for review see, e.g., [70]). Also, the existence of these bosons has been searched at the LHC. Some constraints of the W' and Z' bosons masses in the TeV region have been performed in dependence of their coupling strengths with the SM fermions. On the other hand, searches of light W' and Z' are also attracted a lot of interest for resolving existing puzzles and anomalies. In particular, recently the ATLAS Collaboration [68] set upper limits on the Z' production cross section times the decay branching fraction of the $pp \rightarrow Z'\mu^+\mu^-\mu^+\mu^-$ process, varying from 0.31 to 4.3 fb at 95% C.L., in a Z' mass range 5–81 GeV, from which the coupling strength of the Z' to muons above 0.003 to 0.2 (depending on the Z' mass) are excluded in the same mass range.

Here, we suppose scenarios that the W' is heavier than the τ lepton. It is necessary to forbid invisible decays of leptons into neutrinos and dark bosons, which can further decay into light dark fermions. On the other hand, W' should be heavier than the SM W^\pm/Z bosons, as otherwise one could see the W' pair creation in the final state of physical reactions, e.g., in the e^+e^- annihilation or hadron-hadron collisions. Masses of dark fermions interacting with dark charge vector current should be larger than masses of the SM leptons. We see that the $U(1)_D$ portal can be connected with sub-GeV particles, whereas dark mediators from the $SU(2)_D$ sector are weak interactive massive particles, which have larger masses in comparison with one of the SM weak bosons.

In the universality scenario of interaction W' with all types of the SM leptons, we have bound on G_{ii}^{VA} less than $10^{-1} - 3 \times 10^{-4}$ in the following range of the W' mass: 2 GeV–1 TeV. Such limits correspond to the G_{ij}^{VA} non-diagonal coupling in the case when the corresponding decay is induced by exchange of intermediate neutral dark boson W'^0 . In the case of a neutral boson, we have less constraints as in the case of the charge partners W'^{\pm} . Therefore, the range of the neutral dark boson mass in Fig. 1 is extended to a region of smaller values of mass. Moreover, we will further show that these bounds are the

strongest ones that can be obtained from pure LFV decay $\mu^- \rightarrow e^- \bar{\nu}_\mu \nu_e$. Two LFV processes will be considered later in our manuscript.

Because of the structure of our interaction (21), we establish the same limits for the nondiagonal couplings of both neutral and charged dark bosons with leptons with specific flavor and mass. In case of nondiagonal couplings, we do not have restrictions on the mass of neutral boson from charge conservation as it occurs for charged dark bosons. It is different from a general scenario with a dark Z' boson, where we can describe interaction with leptons and neutrino independently; we consider neutral W'^0 component in couple with charged W'^{\pm} bosons. In our approach, dark photon A' plays the role of neutral dark boson Z' [21]. Therefore, due to possible interference of A' and W'^0 , one can establish new limits on couplings with leptons and neutrinos, which could be different from those derived in Ref. [21].

Next, we can calculate decay width of the W' to \bar{l}_i pair by analogy to the similar decay in SM:

$$\Gamma(W' \rightarrow e^- \bar{\nu}_e) \simeq \frac{(G_{e\nu_e}^{VA})^2 m_{W'}}{48\pi}, \quad (27)$$

and as an example for $m_{W'} = 200$ GeV we have $\Gamma(W' \rightarrow e^- \bar{\nu}_e) = 6.5 \times 10^{-4}$ MeV. It is by ~ 6 orders less than we have for weak W boson decay width.

Nondiagonal interaction couplings of the W' dark boson can be constrained from LFV processes, e.g., from muon LFV decay $\mu^- \rightarrow e^- \bar{\nu}_\mu \nu_e$. The best upper limit on the branching of this decay is 1.2% [71]. We note that this existing limit on the $\mu^- \rightarrow e^- \bar{\nu}_\mu \nu_e$ decay rate is not very sensitive and feeble competitive with many other experiments which limit most new physics scenarios. Contrariwise, we stress that this LFV decay gives limitation directly on one of the nondiagonal components, i.e., on coupling $G_{\mu e}^{VA}$. Bounds on this coupling are presented in Fig. 2. Because the interaction couplings in our model for

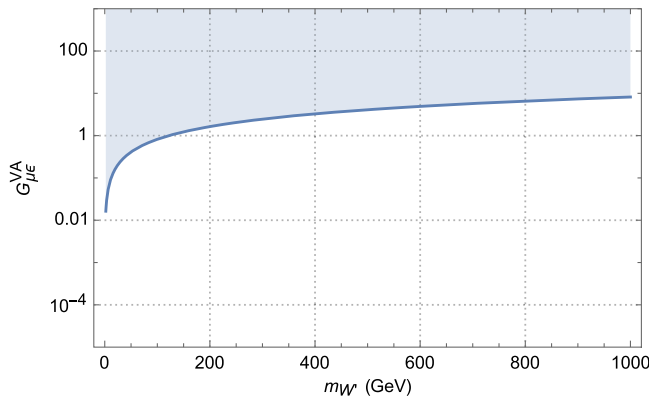


FIG. 2. Bounds on the $\mu - e$ nondiagonal lepton flavor coupling from existing LFV data $\mu^- \rightarrow \nu_e \bar{\nu}_\mu e^-$ [71]. The shaded area is a closed band for dark boson couplings.

charged and neutral W' are universal, we see that famous weak muon decay $\mu \rightarrow e \nu_\mu \bar{\nu}_e$ gives more stronger limits on the coupling constants in comparison with limits derived from LFV decay, e.g., from the $\mu^- \rightarrow e^- \bar{\nu}_\mu \nu_e$ decay mode.

B. LFV muon decay $\mu \rightarrow e \gamma$

New dark vector bosons can potentially explain LFV effects. In this respect, $\mu \rightarrow e \gamma$ decay is a good laboratory for the study of such effects. Feynman diagrams contributing to this process taking into account A' and W'^{\pm} bosons are presented in Fig. 3. Note that the diagram with a pair of intermediate charged bosons W'^{\pm} has an additional suppression factor in comparison with the dark photon A' exchange.

Existing experimental limits for the branchings of the LFV lepton decays $l_i \rightarrow l_k \gamma$ are [67]

$$\begin{aligned} \text{Br}(\mu \rightarrow e \gamma) &< 4.2 \times 10^{-13}, \\ \text{Br}(\tau \rightarrow e \gamma) &< 3.3 \times 10^{-8}, \\ \text{Br}(\tau \rightarrow \mu \gamma) &< 4.2 \times 10^{-8}. \end{aligned} \quad (28)$$

The general matrix element of this LFV process can be parametrized as

$$\begin{aligned} iM_{ik} &= ie e^\mu(q) \bar{u}_k(p_2, m_k) \left[\frac{i}{2m_i} \sigma_{\mu\nu} q^\nu F_M + \frac{i}{2m_i} \sigma_{\mu\nu} q^\nu \gamma_5 F_D \right] \\ &\times u_i(p_1, m_i), \end{aligned} \quad (29)$$

where the square of matrix element is

$$|M_{ik}|^2 = m_i^2 \left[1 - \frac{m_k^2}{m_i^2} \right]^2 (|F_M|^2 + |F_D|^2) \quad (30)$$

and the decay width is given by

$$\begin{aligned} \Gamma(l_i \rightarrow l_k \gamma) &= \frac{1}{2m_i} \int \frac{d^3 p_2 d^3 q}{4E_2 E_q (2\pi)^6} (2\pi)^4 \delta^{(4)} \\ &\times (p_1 - p_2 - q) |M_{ik}|^2 \\ &= \frac{\alpha}{2} m_i (|F_M|^2 + |F_D|^2). \end{aligned} \quad (31)$$

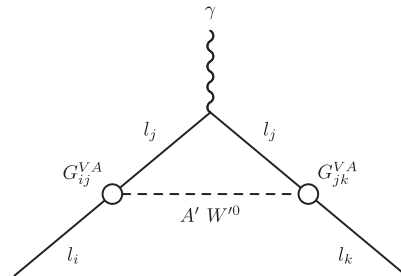


FIG. 3. Feynman diagrams with a contribution of dark photon A' and charge dark boson W' to the LFV process $l_i \rightarrow l_k \gamma$.

Here, F_M and F_D are the magnetic and dipole form factors, respectively, and $\alpha = e^2/(4\pi) = 1/137.036$ is the fine-structure constant. More explicitly, when the A' propagates in the loop (right-hand diagram in Fig. 3), for a general $l_i \rightarrow l_k \gamma$ with an i or k lepton in the loop, we obtain

$$\begin{aligned} F_M &= \frac{1}{16\pi^2} [(G_{ik}^V G_{ii}^V + G_{ik}^A G_{ii}^A) h_2^V(x_\mu) \\ &\quad + (G_{ik}^V G_{kk}^V + G_{ik}^A G_{kk}^A) h_3^V(x_\mu)], \\ F_D &= \frac{1}{16\pi^2} [(G_{ik}^V G_{ii}^A + G_{ik}^A G_{ii}^V) h_2^V(x_\mu) \\ &\quad + (G_{ik}^V G_{kk}^A + G_{ik}^A G_{kk}^V) h_3^V(x_\mu)], \end{aligned} \quad (32)$$

whereas for $\mu \rightarrow e\gamma$, with the τ lepton propagating in the loop and with double LFV coupling, we have

$$\begin{aligned} F_M &= \frac{1}{16\pi^2} \left(\frac{m_\mu}{m_\tau} \right) [G_{\mu\tau}^V G_{e\tau}^V h_1^V(x_\tau) - G_{\mu\tau}^A G_{e\tau}^A h_1^V(x_\tau)], \\ F_D &= \frac{1}{16\pi^2} \left(\frac{m_\mu}{m_\tau} \right) [G_{\mu\tau}^V G_{e\tau}^A h_1^V(x_\tau) - G_{\mu\tau}^A G_{e\tau}^V h_1^V(x_\tau)], \end{aligned} \quad (33)$$

where $x_i = m_{A'}^2/m_i^2$. Expressions for the loop functions $h_i^V(x_i)$ in the approximation $m_e \ll m_\mu \ll m_\tau$ are shown in the Appendix.

The matrix element corresponding to the loop LFV diagram induced by the coupling of a neutral W' boson with a photon and contributing to the $l_i \rightarrow l_k \gamma$ process is specified by two form factors:

$$\begin{aligned} F_M &= \frac{1}{16\pi^2} [(G_{ik}^{VA} G_{ii}^{VA}) h^{W^0}(x_\mu) + 2(G_{ik}^{VA} G_{kk}^{VA}) h_3^V(x_\mu)], \\ F_D &= \frac{1}{16\pi^2} [(G_{ik}^{VA} G_{ii}^{VA}) h^{W^0}(x_\mu) + 2(G_{ik}^{VA} G_{kk}^{VA}) h_3^V(x_\mu)], \end{aligned} \quad (34)$$

where $x = m_{W^0}^2/m_i^2$. Double LFV contributions are neglected here.

Dependence on the masses of dark bosons A' and W' is presented in Fig. 4. The peaks in Fig. 4 are connected with the behavior of the loop integrals $h_i(x)$ near the point $x = 1$ located in the vicinity of the vector boson production threshold. To resolve this problem, one needs to include in the analysis the finite width $\Gamma_{A'} \sim \tau_{A'}^{-1} \sim G_{ij}^2$ of the dark vector boson in the Breit-Wigner propagator. The latter is dominated by the decay width of the A' to the leptonic pair.

We make conservative estimate for coupling production $G_{ll}^{VA} G_{e\mu}^{VA}$ in proposition that diagonal couplings G_{ll}^{VA} are equal to each other. Corresponding bounds are shown in Fig. 4. It means that bounds on $G_{ll}^{VA} G_{e\mu}^{VA}$ from $\mu \rightarrow e\gamma$ LFV decay for neutral vector bosons are strict for a light mass boson. Limits on the A' couplings correspond to the fact that the bounds for the neutral dark boson W^0 are divided

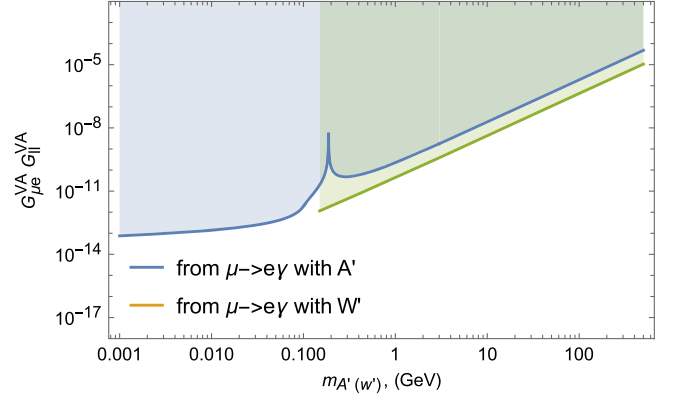


FIG. 4. Bounds to production $\mu - e$ nondiagonal lepton flavor coupling and diagonal coupling from existing FV data $\mu^- \rightarrow e^- \gamma$ [67]. The shaded area is a closed band for dark boson couplings (blue for dark photon and only G^V contribution and green for W^0 neutral dark boson due to full contribution from vector and axial-vector parts).

by a factor of 2, because they have the same mechanism of interaction with SM matter governed by both vector and axial couplings. Further discussion of bounds on couplings A' dark photon has been done in Ref. [21] (here, we note that in Ref. [21] in the plot of the curve for limit on the LFV coupling derived from the decay $\mu \rightarrow e\gamma$ the factor proportional to 10^{-6} was lost). With taking into account with factor the limits on the LFV couplings of the dark gauge bosons with SM fermions are consistent in this paper and Ref. [21]. At the same time, it is important to stress a constantly increasing research interest to study sub-GeV dark candidates which are the main goal in running and planning experiments for searching dark matter at a fixed target regime such as NA64 SPS at CERN [33–37,72–75], M³ [76], and LDMX [77]. LFV coupling between dark boson and SM particles can explain existing experimental anomalies, wherein different models provide different limits on mass and couplings of the Z' boson [78–80].

IV. CONCLUSION

We have proposed a phenomenological Lagrangian approach that combines the SM and DM sectors based on the Stueckelberg mechanism for generation of masses of dark $U(1)_D$ and $SU(2)_D$ gauge bosons due to the presence of dark scalar Stueckelberg fields. These scalar fields play the role of Goldstone bosons. A novel part consists in adding of non-Abelian part to the dark vector $U(1)_D$ portal. We derived bounds on diagonal and nondiagonal interaction couplings between $SU(2)_D$ dark gauge bosons and the SM leptons. In particular, we established limits on the couplings using data from canonical weak muon decay $\mu \rightarrow e\nu_\mu\bar{\nu}_e$ supposing that some correction to its decay rate is possible and given the ambiguity in the definition of value of G_F . Additionally, we have used the phenomenology of

lepton flavor-violating processes to derive limits on the W' couplings. In this paper, we concentrated on the gauge boson sector of our approach. It would be interesting to extend our formalism on the dark fermion sector and consider applications on other rare lepton decays.

ACKNOWLEDGMENTS

This work was funded by Bundesministerium für Bildung und Forschung (Germany) “Verbundprojekt 05P2021 (ErUM-FSP T01)—Run 3 von ALICE am LHC: Perturbative Berechnungen von Wirkungsquerschnitten für ALICE” (Förderkennzeichen: 05P21VTCAA), by Agencia Nacional de Investigación y Desarrollo Programa de Investigación Asociativa/APOYO AFB180002 and AFB220004 (Chile), by Fondo Nacional de Desarrollo Científico y Tecnológico (Chile) under Grant No. 1191103, and by ANID-Millennium Program-ICN2019_044 (Chile). The work of A. S. Z. is

supported by a grant of The Association of Young Scientists and Specialists of Joint Institute for Nuclear Research (22-302-02). The work was supported by the Foundation for the Advancement of Theoretical Physics and Mathematics “BASIS.”

APPENDIX: LOOP FUNCTIONS USED IN CALCULATION OF $\mu \rightarrow e\gamma$ LFV DECAY

In this appendix, we present analytical expressions for the loop integrals occurring in the amplitude of the LFV decays $l_i \rightarrow l_k\gamma$ for different new particle channels and leptons propagating in the loop, used in Eqs. (32)–(34). All results for the form factors have been numerically and analytically cross-checked using the *Mathematica* packages Package-X [81], FeynHelpers [82], and FeynCalc [83–85].

For dark heavy neutral boson W' , the loop integrals read

$$h^{W^0}(x) = 8 \left(\text{Li}_2(1-x) - \text{Li}_2 \left(\frac{2}{2-x+\sqrt{(x-4)x}} \right) + \text{Li}_2 \left(\frac{2}{x+\sqrt{(x-4)x}} \right) \right) + 2x(2x^2-9x+9) \frac{\log(x)}{x-1} + 2 \left(-4x + \frac{1}{x} + 8 \right) + 4 \log^2 \left(\frac{x+\sqrt{(x-4)x}-2}{x+\sqrt{(x-4)x}} \right) - 4\sqrt{(x-4)x}(2x-3) \log \left(\frac{x+\sqrt{(x-4)x}}{2\sqrt{x}} \right). \quad (\text{A1})$$

For the A' dark boson, the loop integrals are given by

$$h_1^V(x) = -\frac{(4x^3 - 3x^2 - 6x^2 \ln(x) - 1)}{x(1-x)^3}, \quad (\text{A2})$$

$$h_2^V(x) = 2 \left(2\text{Li}_2(1-x) - 2\text{Li}_2 \left(\frac{2}{-x+\sqrt{(x-4)x}+2} \right) + 2\text{Li}_2 \left(\frac{2}{x+\sqrt{(x-4)x}} \right) - 2x + 2 \log(x) \right) + \log^2 \left(\frac{x+\sqrt{(x-4)x}}{2x} \right) + \frac{(x+1)((x-4)x+2) \log(x)}{x-1} - 2x\sqrt{(x-4)x} \log \left(\frac{\sqrt{x}+\sqrt{(x-4)}}{2} \right) + 1, \quad (\text{A3})$$

$$h_3^V(x) = -4x + 4(x-1)^2 \ln \left(\frac{x}{x-1} \right) + 6. \quad (\text{A4})$$

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