Predictions of superexotic heavy mesons from $K^*B^{(*)}B^*$ interactions

M. Bayar [*]
Department of Physics, Kocaeli University, 41380 Izmit, Turkey

N. Ikeno [†]
Department of Agricultural, Life and Environmental Sciences, Tottori University, Tottori 680-8551, Japan and Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA

L. Roca [‡]
Departamento de Física, Universidad de Murcia, E-30100 Murcia, Spain

(Received 30 January 2023; accepted 15 March 2023; published 28 March 2023)

We make a theoretical study of the three-body system composed of $\bar{K}^*\bar{B}^*\bar{B}^*$ and $\bar{K}^*\bar{B}\bar{B}^*$ to look for possible bound states, which could be associated to mesonic resonances of very exotic nature, containing open strange and double-bottom flavors. The three-body interaction is evaluated by using the fixed center approach to the Faddeev equations where the $\bar{B}^*\bar{B}^*$ or $\bar{B}\bar{B}^*$ is bound forming an $I(J^P) = 0(1^+)$ state, as it was found in previous works, and the third particle, the $\bar{K}^*$, of much smaller mass, interacts with the components of the cluster. We obtain bound states for all the channels considered: spin $J = 0$, 1, and 2, all of them with isospin $I = 1/2$ and negative parity.

DOI: 10.1103/PhysRevD.107.054042

I. INTRODUCTION

The meson spectrum in the heavy flavor sector has gained a renewed impetus in the last two decades thanks to a significant increase of experimental results (see Ref. [1,2] for reviews). Of special interest and repercussion has been the proliferation of exotic states, which cannot be explained as ordinary $q\bar{q}$ mesons, like the hidden heavy flavor XYZ resonances, with theoretical interpretations ranging from tetraquarks to molecular states [2–6]. Even more challenging has been the recent discovery of non-$q\bar{q}$ open flavor mesons like the $X_0(2900)$ [7,8] with an open charm and strange flavor, which is undoubtedly exotic since it contains at least a c and an s quark and then needs at least two other antiquarks to form a color singlet. The theoretical interpretations of the $X_0(2900)$ range from the picture of tetraquarks [9–12] to a molecular structure [13–22] or even a kinematic triangle singularity [23,24]. Especially, sound has been the recent discovery of the manifestly exotic open double charm $T_{cc}(3875)$ [25,26] also with a natural interpretation as a molecular D^*D state [27–31]. The success of the molecular interpretation has triggered the search for possible bound states for other exotic heavy flavor combinations like open double bottom from B^*B, B^*B^*, $B_s^*B^*$ interaction [32]; open c and b from $\bar{B}D$, $\bar{B}^*D$, $\bar{B}D^*$, $\bar{B}\bar{D}^*$ [33]; and open b and s flavors from BK, B^*K, BK^*, B^*K^* interaction [34]. Most of these molecular interpretations are based on the implementation of unitarity to the amplitudes obtained from extensions of the lowest order chiral Lagrangians from where, in many cases, bound states and resonances appear dynamically without the need to include them as explicit degrees of freedom (see [35] for a classical early review and [36–38] for recent reviews of results in the heavy sector).

A natural and challenging step forward is to consider the extension to three-body systems. In the last decade dozens of works have found many states theoretically, even with open or hidden heavy flavors (see Ref. [39] for a review and list of references). The three-body system allows for even the possibility to have super exotic mesons with three open flavors like for instance bbb, which was found to bind in the $BB^*B^* - B^*B^*B^*$ interaction in [40] or ccs, where bound states were found for several total spins in the $D^*D^*\bar{K}^*$ system in [41]. These states would require at least three extra antiquarks to get the color singlet, then would correspond to hexaquarks in the standard quark picture. In the present work we take advantage of the findings of this latter work and extend the formalism to the $\bar{K}^*\bar{B}^*\bar{B}^*$ interaction to look for possible bound states which would have open bbs flavors. The $\bar{K}^*\bar{B}^*\bar{B}^*$ system is very

[*] melahat.bayar@kocaeli.edu.tr
[†] ikeno@tottori-u.ac.jp
[‡] luisroca@um.es

interesting since it is significantly different from the $\bar{K}B^*B^*$ [42] and $\bar{K}^{(*)}B^{(*)}\bar{B}^{(*)}$ [43] systems with hidden bottom flavors. On the other hand, in Ref. [32], it was also found that the B^*B system also forms a bound state with a binding energy similar to that of the B^*B^* case. Therefore we can also expect to obtain bound states for the $\bar{K}^*\bar{B}\bar{B}^*$ system, with the same quantum numbers as for the $\bar{K}^*\bar{B}^*\bar{B}^*$ case, and thus we also evaluate it in the present paper.

The standard way to tackle the three-body scattering problem has traditionally been to try to solve the Faddeev equations [44] implementing approximate methods, due to the practical impossibility of solving them exactly. This is indeed a well-known problem in nuclear and hadron physics like in the three-nucleon interaction [45,46], systems involving baryons and mesons [47–50], or three-meson interaction [51–53]. However, when two of the three particles are strongly correlated among themselves, and the third particle is lighter than the other particles [54], the Faddeev equations can be strongly simplified, and one can make use of a formalism called fixed center approximation (FCA) to the Faddeev equations [55–59]. The FCA have been successfully applied to dozens of three-body systems (see Table 1 in Ref. [39] for a list of different works). It is worth mentioning here that in related problems the FCA has been compared to the variational method, and similar results have been found. This is the case of the $D\bar{D}K$ system studied in [60] with the variational method and in [61] with the FCA, or the case of the $D^*D^*D^*$ system studied in [62] with the variational method and in [63] with the FCA.

In the present work we will study the $\bar{K}^*\bar{B}^*\bar{B}^*$ and $\bar{K}^*\bar{B}\bar{B}^*$ within the FCA, because in a previous work [32] it was found that the $\bar{B}^*\bar{B}^*$ in $I(J^P) = 0(1^+)$ was bound with a binding energy of about 40 MeV. In addition, the $\bar{K}^*\bar{B}^*$ was also found to be strongly attractive in [34] for all possible spins in $I = 0$. Then we can expect with confidence that the three-body $\bar{K}^*\bar{B}^*\bar{B}^*$ will present bound states. It adds even more confidence the fact that bound states were found in [41] in the $D^*D^*\bar{K}^*$ system, where analogously to our case the D^*D^* is bound [17] and the $D^*\bar{K}^*$ is also attractive. Advancing some results, we find three-body states for all three spin channels, $J = 0$, 1, and 2.

II. FORMALISM

A. Three-body scattering

The FCA to the Faddeev equations is an effective way to evaluate the three-body scattering when two of the particles form a bound state, which will be called cluster, and it is not excited in the intermediate states [54]. If the third particle is much lighter than the constituents of the cluster it is unlikely to have enough available energy to excite it. Let us explain first the formalism for the $\bar{K}^*\bar{B}^*\bar{B}^*$ case, for which we are in this situation. Indeed in Ref. [32] it was obtained, among other states, that the $\bar{B}^*\bar{B}^*$ system in $I(J^P) = 0(1^+)$ was bound with about 40 MeV [64], and the third particle

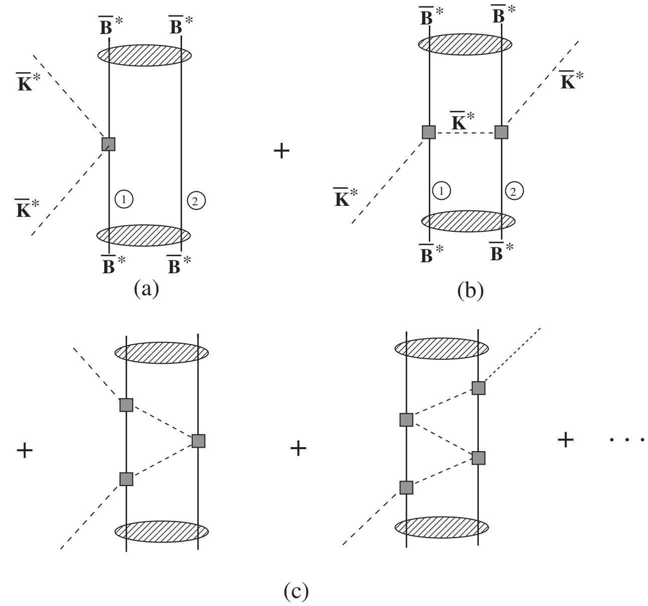

FIG. 1. Representation of the fixed center approximation to the Faddeev equations for the interaction of a $\bar{K}^*$ meson with a $\bar{B}^*\bar{B}^*$ bound state. (a) The single scattering contribution. (b) The double scattering contribution. (b) + (c) The multiple scattering contribution.

of the three-body system is a $\bar{K}^*$, which is much lighter than the $\bar{B}^*$ making up the cluster. The projectile, $\bar{K}^*$, rescatters repeatedly with each component of the cluster. This is depicted diagrammatically in Fig. 1, and the total three-body scattering amplitude, T, can then be formally written as a system of coupled equations

$$\begin{aligned} T_1 &= t_1 + t_1 G_0 T_2 \\ T_2 &= t_2 + t_2 G_0 T_1 \\ T &= T_1 + T_2 \end{aligned} \tag{1}$$

where the two partition functions, T_i, account for all the diagrams starting with the interaction of the $\bar{K}^*$ with the ith $\bar{B}^*$ particle in the cluster. In the present case, since the two particles in the clusters are the same, we have $t_1 = t_2$, $T_1 = T_2$, and, hence Eq. (1) decouples. Figure 1(a) represents the single scattering, t_1, and Fig. 1(b) represents the double scattering contribution, $t_1 G_0 t_2$. The infinite sum in Fig. 1(c) represents the rest of the rescattering to get the full T_1 amplitude in the FCA. In Eq. (1), G_0 stands for the Green function representing the exchange of a $\bar{K}^*$ between the $\bar{B}^*$ mesons inside the compound system, and is represented by a dashed line in Fig. 1, which is given by [65–67]

$$G_0(q^0) = \frac{1}{2M_c}\int \frac{d^3\vec{q}}{(2\pi)^3}\frac{F(\vec{q})}{(q^0)^2 - \omega_{\bar{K}^*}^2(\vec{q}) + i\epsilon}, \tag{2}$$

with $\omega_{K^*}(\vec{q}) = \sqrt{|\vec{q}|^2 + m_{K^*}^2}$. In Eq. (2), q^0 is the energy carried by the $\bar{K}^*$ meson between the components of the cluster, given by

$$q^0 = \frac{1}{2M_c}(s - m_{K^*}^2 - M_c^2), \tag{3}$$

and M_c is the mass of the $\bar{B}^*\bar{B}^*$ bound state (cluster) from Ref. [32], the value of which is explained in the Results section.

The form factor $F(\vec{q})$ in Eq. (2) encodes the information about the $\bar{B}^*\bar{B}^*$ bound state, which is related to the cluster wave function, $\Psi_c(\vec{r})$, by means of a Fourier transform [65,68],

$$F(\vec{q}) = \int d^3\vec{r} e^{-i\vec{q}\cdot\vec{r}} \Psi_c^2(\vec{r}). \tag{4}$$

The form factor can be derived in a similar way as was done in [65,68] and gives

$$F(\vec{q}) = \frac{1}{N}\int_\Omega d^3\vec{q}' \frac{1}{M_c - 2\omega_{\bar{B}^*}(\vec{q}')} \frac{1}{M_c - 2\omega_{\bar{B}^*}(\vec{q}-\vec{q}')}, \tag{5}$$

where Ω specifies the conditions $|\vec{q}'| < q_{\max}$ and $|\vec{q}-\vec{q}'| < q_{\max}$. The normalization factor N in Eq. (5) guarantees that $F(\vec{q}=0) = 1$, and thus it is given by

$$N = \int_{|\vec{q}'|<q_{\max}} d^3\vec{q}' \left(\frac{1}{M_c - 2\omega_{\bar{B}^*}(\vec{q}')}\right)^2. \tag{6}$$

In the Results section we discuss the value used for the cutoff $q_{\max}$ in the three-momentum integration. In Fig. 2 we show the form factor as a function of the modulus of the momentum for $q_{\max} = 420$ MeV, and in Fig. 3 the real and imaginary parts of the G_0 function, which close to threshold resemble very much the typical shape of the two meson loop function, in this case the $\bar{K}^*$ and the meson made of two $\bar{B}^*$.

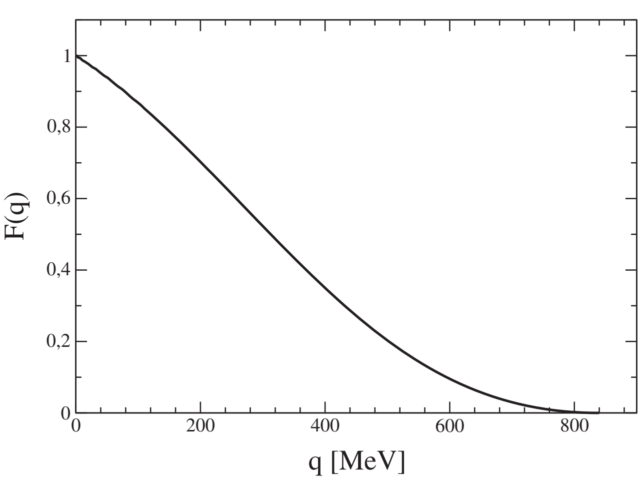

FIG. 2. Form factor of the $\bar{B}^*\bar{B}^*$ bound state.

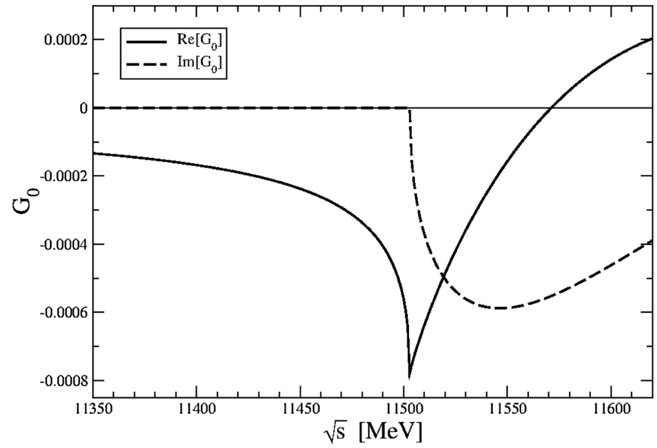

FIG. 3. Real and imaginary parts of the G_0 function, Eq. (2).

Finally, there is an important issue regarding the normalization of the amplitudes that one has to take into account when mixing, in the same expression, Eq. (1), three-body amplitudes, T, with two-body ones. Using the Mandl-Shaw [69] normalization for the mesonic fields, the S matrix for the single-scattering contribution can be written as

$$S^{(1)} = S_1^{(1)} + S_2^{(1)}, \tag{7}$$

with

$$S_i^{(1)} = -it_{Ab_i}\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_{p_i}}}\frac{1}{\sqrt{2\omega_{p_i'}}}\frac{1}{\sqrt{2\omega_k}}\frac{1}{\sqrt{2\omega_{k'}}} \times (2\pi)^4\delta(k + k_c - k' - k_c'), \tag{8}$$

where t_{Ab_i} are the single-scattering two-body amplitudes, $\mathcal{V}$ is an irrelevant normalization volume, $\omega_p = \sqrt{p^2 + m^2}$ is the on shell energy of a particle with momentum p and mass m, p_i (p_i') is the initial (final) momentum of the particle b_i in the cluster, k (k') represents the initial (final) momentum of the projectile A, and k_c (k_c') represents the total momentum of the initial (final) cluster. On the other hand, the general form of the S matrix of the three-body interaction is

$$S = -iT(2\pi)^4\delta(k + k_c - k' - k_c')\frac{1}{\mathcal{V}^2} \times \frac{1}{\sqrt{2\omega_k}}\frac{1}{\sqrt{2\omega_{k'}}}\frac{1}{\sqrt{2\omega_{k_c}}}\frac{1}{\sqrt{2\omega_{k_c'}}}, \tag{9}$$

and comparing this equation with Eq. (8), we get that the FCA equations (1) take the form

$$\bar{T}_1 = \bar{t}_1 + \bar{t}_1 G_0 \bar{T}_2$$
$$\bar{T}_2 = \bar{t}_2 + \bar{t}_2 G_0 \bar{T}_1$$
$$\bar{T} = \bar{T}_1 + \bar{T}_2 \tag{10}$$

with

$$\bar{t}_i = \sqrt{\frac{\omega_{k_c}\omega_{k'_c}}{\omega_{p_i}\omega_{p'_i}}} t_{Ab_i}(s_i), \tag{11}$$

which in our case can be approximated by

$$\bar{t}_1 = \bar{t}_2 = \frac{M_c}{m_{\bar{B}^*}} t_1. \tag{12}$$

With all these ingredients, Eq. (10) can be algebraically solved and gives, for the total three-body amplitude,

$$\bar{T} = 2\bar{T}_1 = \frac{2}{\bar{t}_1^{-1} - G_0}. \tag{13}$$

B. Two-body interaction

For the evaluation of the two-body t_i amplitudes, represented by the full squares in Fig. 1, we need the $\bar{K}^*\bar{B}^*$ interaction, whose amplitudes are obtained from Ref. [32]. Note that the amplitudes in Ref. [34] are provided for a given isospin and spin; therefore, we have to write the total isospin state of the global system in terms of the coupled isospin state of the $\bar{K}^*$ and a $\bar{B}^*$ of the cluster. For a general situation where the incident particle is called A and the cluster B is made of two particles, b_1 and b_2, the amplitudes t_i in Eq. (1) actually stand for matrix elements between the eigenstates

$$|I_A, I_B, I, M\rangle \tag{14}$$

where I_A is the isospin of the particle A, I_B the isospin of the cluster B, I is the total AB isospin, and M is the third component of the total isospin I. This state must then be first written in terms of the ket

$$|I_A, I_i, I_{Ai}, M_{Ai}\rangle \tag{15}$$

where I_i is the isospin of particle b_i and I_{Ai} the global isospin of the $A - b_i$ system. In order to do this, one can first write $|I_A, I_B, I, M\rangle$ in terms of $|I_A, M_A; I_B, M_B\rangle$, then write $|I_B, M_B\rangle$ in terms of $|I_1, M_1, I_2, M_2\rangle$, and finally $|I_A, M_A; I_i, M_i\rangle$ in terms of $|I_A, I_i, I_{Ai}, M_{Ai}\rangle$. Thus the expression of $|I_A, I_B, I, M\rangle$ in terms of $|I_A, I_i, I_{Ai}, M_{Ai}\rangle \otimes |I_j, M - M_{Ai}\rangle$ is [67]

$$\begin{aligned} |I_A, I_B, I, M\rangle^{(i)} &= \sum_{I_{Ai}}\sum_{M_{Ai}}\sum_{M_A} \mathcal{C}(I_A, I_B, I|M_A, M - M_A, M) \\ &\times \mathcal{C}(I_i, I_j, I_B|M_{Ai} - M_A, M - M_{Ai}, M - M_A) \\ &\times \mathcal{C}(I_A, I_i, I_{Ai}|M_A, M_{Ai} - M_A, M_{Ai}) \\ &\times |I_A, I_i, I_{Ai}, M_{Ai}\rangle \otimes |I_j, M - M_{Ai}\rangle \end{aligned} \tag{16}$$

where the superscript (i) indicates that we are correlating the particle A with b_i, and $\mathcal{C}(j_1, j_2, j_3|m_1, m_2, m_3)$ represent Clebsch-Gordan coefficients.

In the present case one has to consider that the $\bar{B}^*\bar{B}^*$ cluster is in isospin 0 ($I_B = 0$), that the total three-body isospin is $I = 1/2$, and that the isospin doublets are $(\bar{K}^{*0}, -\bar{K}^{*-})$ and $(\bar{B}^{*0}, -\bar{B}^{*-})$. Then using, Eq. (16), we have

$$\begin{aligned} |\bar{K}^*(\bar{B}^*\bar{B}^*)\rangle^{(1)}_{I=1/2,M=1/2} &= -\frac{1}{2}|\bar{K}^*\bar{B}^*\rangle_{I=0,M=0}|\bar{B}^{*0}\rangle \\ &-\frac{1}{2}|\bar{K}^*\bar{B}^*\rangle_{I=1,M=0}|\bar{B}^{*0}\rangle - \frac{1}{\sqrt{2}}|\bar{K}^*\bar{B}^*\rangle_{I=1,M=1}|\bar{B}^{*-}\rangle. \end{aligned} \tag{17}$$

The amplitude for the single-scattering contribution can be written in terms of the two-body amplitudes, $t^{(I_{Ai})}_{Ab_i}$, for the transition $Ab_i \to Ab_i$ with isospin I_{Ai}:

$$\begin{aligned} \langle I_A, I_B, I, M|t_i|I_A, I_B, I, M\rangle &= \sum_{I_{Ai}}\Bigg[\sum_{M_{Ai}}\sum_{M_A}\sum_{M'_A} \mathcal{C}(I_A, I_B, I|M_A, M - M_A, M)\mathcal{C}(I_A, I_B, I|M'_A, M - M'_A, M) \\ &\times \mathcal{C}(I_i, I_j, I_B|M_{Ai} - M_A, M - M_{Ai}, M - M_A)\mathcal{C}(I_i, I_j, I_B|M_{Ai} - M_A, M - M_{Ai}, M - M_{A'}) \\ &\times \mathcal{C}(I_A, I_i, I_{Ai}|M_A, M_{Ai} - M_A, M_{Ai})\mathcal{C}(I_A, I_i, I_{Ai}|M_{A'}, M_{Ai} - M_{A'}, M_{Ai})\Bigg] \times t^{(I_{Ai})}_{Ab_i} \\ &\equiv \sum_{I_{Ai}} \alpha_i t^{(I_{Ai})}_{Ab_i}, \end{aligned} \tag{18}$$

which is easily implementable for computer evaluation for a general case, and thus it is why we quote it here, since it may be useful in other works. In our case it is more direct to obtain it from Eq. (17),

$$t_1 = \frac{1}{4} t^{I=0}_{\bar{K}^*\bar{B}^*} + \frac{3}{4} t^{I=1}_{\bar{K}^*\bar{B}^*}. \tag{19}$$

On the other hand, we also have to consider the different spin combinations and write the total three-body spin amplitudes in terms of the spin of the two-body $\bar{B}^*\bar{B}^*$. The reasoning is then totally analogous to the previous discussion about the isospin, and hence we can use the master formula [Eq. (18)], but changing isospin by spin. Then, using the actual spins of the particles involved and taking into account that the bound $\bar{B}^*\bar{B}^*$ state has $J=1$, we get, for the different possible values of the total spin, $J=0,1,2$,

$$\begin{aligned} t_1^{J=0} &= t_{\bar{K}^*\bar{B}^*}^{J=1} \\ t_1^{J=1} &= \frac{1}{3}t_{\bar{K}^*\bar{B}^*}^{J=0} + \frac{1}{4}t_{\bar{K}^*\bar{B}^*}^{J=1} + \frac{5}{12}t_{\bar{K}^*\bar{B}^*}^{J=2} \\ t_1^{J=2} &= \frac{1}{4}t_{\bar{K}^*\bar{B}^*}^{J=1} + \frac{3}{4}t_{\bar{K}^*\bar{B}^*}^{J=2}. \end{aligned} \tag{20}$$

Combining Eqs. (19) and (20) we finally get

$$\begin{aligned} t_1^{J=0} &= \frac{1}{4}t_{\bar{K}^*\bar{B}^*}^{(I=0,J=1)} + \frac{3}{4}t_{\bar{K}^*\bar{B}^*}^{(I=1,J=1)} \\ t_1^{J=1} &= \frac{1}{12}t_{\bar{K}^*\bar{B}^*}^{(I=0,J=0)} + \frac{1}{16}t_{\bar{K}^*\bar{B}^*}^{(I=0,J=1)} + \frac{5}{48}t_{\bar{K}^*\bar{B}^*}^{(I=0,J=2)} \\ &\quad + \frac{1}{4}t_{\bar{K}^*\bar{B}^*}^{(I=1,J=0)} + \frac{3}{16}t_{\bar{K}^*\bar{B}^*}^{(I=1,J=1)} + \frac{5}{16}t_{\bar{K}^*\bar{B}^*}^{(I=1,J=2)} \\ t_1^{J=2} &= \frac{1}{16}t_{\bar{K}^*\bar{B}^*}^{(I=0,J=1)} + \frac{3}{16}t_{\bar{K}^*\bar{B}^*}^{(I=0,J=2)} \\ &\quad + \frac{3}{16}t_{\bar{K}^*\bar{B}^*}^{(I=1,J=1)} + \frac{9}{16}t_{\bar{K}^*\bar{B}^*}^{(I=1,J=2)}. \end{aligned} \tag{21}$$

Note that the argument of the function T_i in Eq. (1) is the total invariant mass energy, s, of the three-body system. However the argument of the two-body $\bar{K}^*\bar{B}^*$ amplitudes in Eq. (21), and hence t_1 and t_2 in Eq. (1), are s_1 and s_2, where s_i ($i=1,2$) is the invariant mass of the interacting particle A and the particle b_i of the B molecule and is given by [66]

$$s_i = m_A^2 + m_{b_i}^2 + \frac{1}{2m_B^2}(s - m_A^2 - m_B^2)(m_B^2 + m_{b_i}^2 - m_{b_{j\neq i}}^2), \tag{22}$$

which in our case gives

$$s_1 = s_2 = m_{K^*}^2 + m_{B^*}^2 + \frac{1}{2}(s - m_{K^*}^2 - M_c^2), \tag{23}$$

where M_c is the mass of the $\bar{B}^*\bar{B}^*$ bound state.

The $t_{\bar{K}^*\bar{B}^*}$ amplitudes for $I=0$ are obtained from Ref. [34] by implementing unitarity by means of the Bethe-Salpeter equation, starting with potential kernels, V, obtained from the dominant vector meson exchange interaction plus four vector contact interaction:

$$t_{\bar{K}^*\bar{B}^*} = [1 - VG_{\bar{K}^*\bar{B}^*}]^{-1}V. \tag{24}$$

The elementary vertices in the evaluation of V are supplied by local hidden gauge symmetry Lagrangians properly extended to the bottom sector. In this model, the $\bar{K}^*\bar{B}^*$ scattering amplitudes present poles for $I(J^P) = 0(0^+)$, $0(1^+)$, and $0(2^+)$ with binding energies of the order of 100 MeV. In Ref. [34] the widths of the generated states were also evaluated by identifying the main sources of the imaginary part, which turned out to be the width of the $\bar{K}^*$ and the box diagrams with intermediate $\bar{K}\,\bar{B}$ and $\bar{K}\bar{B}^*$ states. (See details of the formalism and calculations in Ref. [34]).

The $t_{\bar{K}^*\bar{B}^*}$ amplitudes in $I=1$ are not calculated in Ref. [34], and thus we evaluate them in the present work considering the same contact term and exchange of ρ, ω, and B_s^* mesons. The contact term contribution is

$$V_{\text{contact}}^{I=1} = \begin{cases} -4g^2 & \text{for } J=0, \\ 0 & \text{for } J=1, \\ 2g^2 & \text{for } J=2. \end{cases} \tag{25}$$

where $g = 800\ \text{MeV}/(2f_\pi)$, with $f_\pi = 93$ MeV, and

$$\begin{aligned} V_{\text{exch.}}^{I=1,J=0,2} &= \frac{g^2}{m_{B_s^*}^2}(p_1+p_4)(p_2+p_3) \\ &\quad + \frac{1}{2}g^2\left(\frac{1}{m_\omega^2} + \frac{1}{m_\rho^2}\right)(p_1+p_3)(p_2+p_4), \end{aligned} \tag{26}$$

$$\begin{aligned} V_{\text{exch.}}^{I=1,J=1} &= -\frac{g^2}{m_{B_s^*}^2}(p_1+p_4)(p_2+p_3) \\ &\quad + \frac{1}{2}g^2\left(\frac{1}{m_\omega^2} + \frac{1}{m_\rho^2}\right)(p_1+p_3)(p_2+p_4), \end{aligned} \tag{27}$$

where we have to carry out an s-wave projection of the momentum structures which gives [34,70]

$$\begin{aligned} (p_1+p_3)(p_2+p_4) \to \frac{1}{2}\Big[&3s - (m_1^2 + m_2^2 + m_3{}^2 + m_4{}^2). \\ &- \frac{1}{s}(m_1^2 - m_2^2)(m_3{}^2 - m_4{}^2)\Big], \end{aligned} \tag{28}$$

$$\begin{aligned} (p_1+p_4)(p_2+p_3) \to \frac{1}{2}\Big[&3s - (m_1^2 + m_2^2 + m_3{}^2 + m_4{}^2) \\ &+ \frac{1}{s}(m_1^2 - m_2^2)(m_3{}^2 - m_4{}^2)\Big], \end{aligned} \tag{29}$$

with m_i the mass of the particle with momentum p_i, of the process $\bar{K}^*(p_1)\bar{B}^*(p_2) \to \bar{K}^*(p_3)\bar{B}^*(p_4)$. This potential at threshold takes the value $29g^2$ for $J=0$, $30g^2$ for $J=1$, and $35g^2$ for $J=2$, which are strongly repulsive, and then the $I=1$ $\bar{K}^*\bar{B}^*$ amplitudes do not develop bound states unlike the $I=0$ ones when implementing the unitarization procedure through the Bethe-Salpeter equation.

For the evaluation of the $\bar{K}^*\bar{B}\bar{B}^*$ interaction the formalism is similar but now the particles in the cluster are different. This implies that now in Eq. (10) the partition amplitudes $\bar{t}_1$ and $\bar{t}_2$ are different since $\bar{t}_1$ represents the process where the projectile starts interacting with the $\bar{B}$ in the cluster and $\bar{t}_2$ with the $\bar{B}^*$, and they sum up to the total three-body amplitude

$$\bar{T} = \frac{\bar{t}_1 + \bar{t}_2 + 2\bar{t}_1\bar{t}_2 G_0}{1 - \bar{t}_1\bar{t}_2 G_0^2}, \tag{30}$$

with

$$\bar{t}_1 = \frac{M_c}{m_{\bar{B}}} t_1, \qquad \bar{t}_2 = \frac{M_c}{m_{\bar{B}^*}} t_2 \tag{31}$$

where now $M_c = 10583$ MeV stands for the mass of the $\bar{B}^*\bar{B}$ cluster [32]. The amplitudes for the different total three-body spins are

$$\begin{aligned}
t_1^{J=0} &= \frac{1}{4} t_{\bar{K}^*\bar{B}}^{(I=0,J=1)} + \frac{3}{4} t_{\bar{K}^*\bar{B}}^{(I=1,J=1)} \\
t_2^{J=0} &= \frac{1}{4} t_{\bar{K}^*\bar{B}^*}^{(I=0,J=0)} + \frac{3}{4} t_{\bar{K}^*\bar{B}^*}^{(I=1,J=0)} \\
t_1^{J=1} &= \frac{1}{4} t_{\bar{K}^*\bar{B}}^{(I=0,J=1)} + \frac{3}{4} t_{\bar{K}^*\bar{B}}^{(I=1,J=1)} \\
t_2^{J=1} &= \frac{1}{4} t_{\bar{K}^*\bar{B}^*}^{(I=0,J=1)} + \frac{3}{4} t_{\bar{K}^*\bar{B}^*}^{(I=1,J=1)} \\
t_1^{J=2} &= \frac{1}{4} t_{\bar{K}^*\bar{B}}^{(I=0,J=1)} + \frac{3}{4} t_{\bar{K}^*\bar{B}}^{(I=1,J=1)} \\
t_2^{J=2} &= \frac{1}{4} t_{\bar{K}^*\bar{B}^*}^{(I=0,J=2)} + \frac{3}{4} t_{\bar{K}^*\bar{B}^*}^{(I=1,J=2)}.
\end{aligned} \tag{32}$$

The amplitude $t_{\bar{K}^*\bar{B}}^{(I=0,J=1)}$ is taken from [34]. The isospin 1 amplitude $t_{\bar{K}^*\bar{B}}^{(I=1,J=1)}$ has not been previously calculated within our formalism but can be easily evaluated from the analogous Bethe-Salpeter equation (24) with the potential given by

$$V^{I=1,J=1} = \frac{1}{2} g^2 \left(\frac{1}{m_\omega^2} + \frac{1}{m_\rho^2} \right) (p_1 + p_3)(p_2 + p_4). \tag{33}$$

III. RESULTS

For the numerical evaluation, the three-momentum cutoff $q_{\rm max}$ of Eq. (5) is, in principle, a free parameter of the model. However it is conceptually analogous to the regularization cutoff used in the calculation of the $\bar{B}^*\bar{B}^*$ loop function needed in the Bethe-Salpeter equation to obtain the $\bar{B}^*\bar{B}^*$ bound state in Ref. [32]. Indeed it was shown in Ref. [71] that the use of a separable two-body potential in momentum space with a maximum momentum $q_{\rm max}$, $V = v\theta(q_{\rm max} - q)\theta(q_{\rm max} - q')$, with q (q') the modulus of the initial (final) scattering momenta, converts the coupled integral Bethe-Salpeter equation into an algebraic one with on shell prescriptions, and the $q_{\rm max}$ translate into the cutoff of the loop function as in Eq. (5). Therefore we use the same value $q_{\rm max} \in [400, 450]$ as was used for the evaluation of the $\bar{B}^*\bar{B}^*$ and $\bar{B}\bar{B}^*$ loops in Ref. [32]. The values obtained in Ref. [32] for the mass of the $\bar{B}^*\bar{B}^*$ bound state were 10612 MeV and 10607 MeV for $q_{\rm max} = 400$ MeV and $q_{\rm max} = 450$ MeV respectively. On the other hand, in the model for the two-body $\bar{K}^*\bar{B}^*$ amplitudes [34] there was also an uncertainty from the value of the cutoff used in the $\bar{K}^*\bar{B}^*$ loop function. This cutoff was obtained in that work by fitting the experimental mass of the $X_0(2866)$ state, obtained in the $\bar{K}^*D^*$ interaction. The use of the same value for the cutoff for $\bar{K}^*\bar{B}^*$ as for $\bar{K}^*D^*$ is justified within the heavy quark spin symmetry, since the value of the cutoff is independent of the heavy quark flavor, up to corrections of order $\mathcal{O}(1/m_Q)$ with m_Q the mass of the heavy quark [72]. This value of the $\bar{K}^*\bar{B}^*$ cutoff found in [34] was 1050 MeV, which we will call $q_{\rm max}^{\bar{K}^*\bar{B}^*}$ in the following to distinguish it from the $q_{\rm max}$ of the $\bar{B}^*\bar{B}^*$ regularization and form factor described above. For the estimation of uncertainties, a range between 900 MeV and 1050 MeV was used in [34], and then we will also use that range for $q_{\rm max}^{\bar{K}^*\bar{B}^*}$ in the present work. The reason for the consideration also of the lower value of $q_{\rm max}^{\bar{K}^*\bar{B}^*}$ is that one expects the value of the cutoff to be of the order of the inverse of the range of the interaction, and since the potential is dominated by vector meson exchange one expects to be closer to the mass of the vector mesons considered. The value of $q_{\rm max}^{\bar{K}^*\bar{B}^*}$ is the largest source of uncertainty of the present calculation.

In Fig. 4 we show the results for the three-body scattering amplitudes $|\bar{T}|^2$ for the $\bar{K}^*\bar{B}^*\bar{B}^*$ system as a function of the total invariant mass energy, $\sqrt{s}$, for the three possible values of the global spin, $J = 0$, 1, and 2. We show the calculations for the extreme values, 900 MeV and 1050 MeV, of the range considered for $q_{\rm max}^{\bar{K}^*\bar{B}^*}$. The difference between these results can be considered as an estimation of the largest uncertainty of our calculation. The consideration of the value of $q_{\rm max}$ for $\bar{B}^*\bar{B}^*$ in the range 400–450 MeV has an effect of a shift in the peaks of less than 10 MeV which is smaller than the uncertainty from $q_{\rm max}^{\bar{K}^*\bar{B}^*}$, and hence we show the results only for an intermediate value of $q_{\rm max} = 420$ MeV. Note, however, that the uncertainty from these cutoffs comes inherited from the two-body amplitudes, and thus it is not genuine of the three-body model.

For $J = 0$ we can see a sharp peak at $\sqrt{s} = 11393$ MeV, for $q_{\rm max}^{\bar{K}^*\bar{B}^*} = 1050$ MeV, which is below the $\bar{K}^*[\bar{B}^*\bar{B}^*]$ correlated threshold, 11503 MeV. This peak can thus be considered as a three-body $\bar{K}^*\bar{B}^*\bar{B}^*$ bound state, with a binding energy of about 150 MeV defined from the

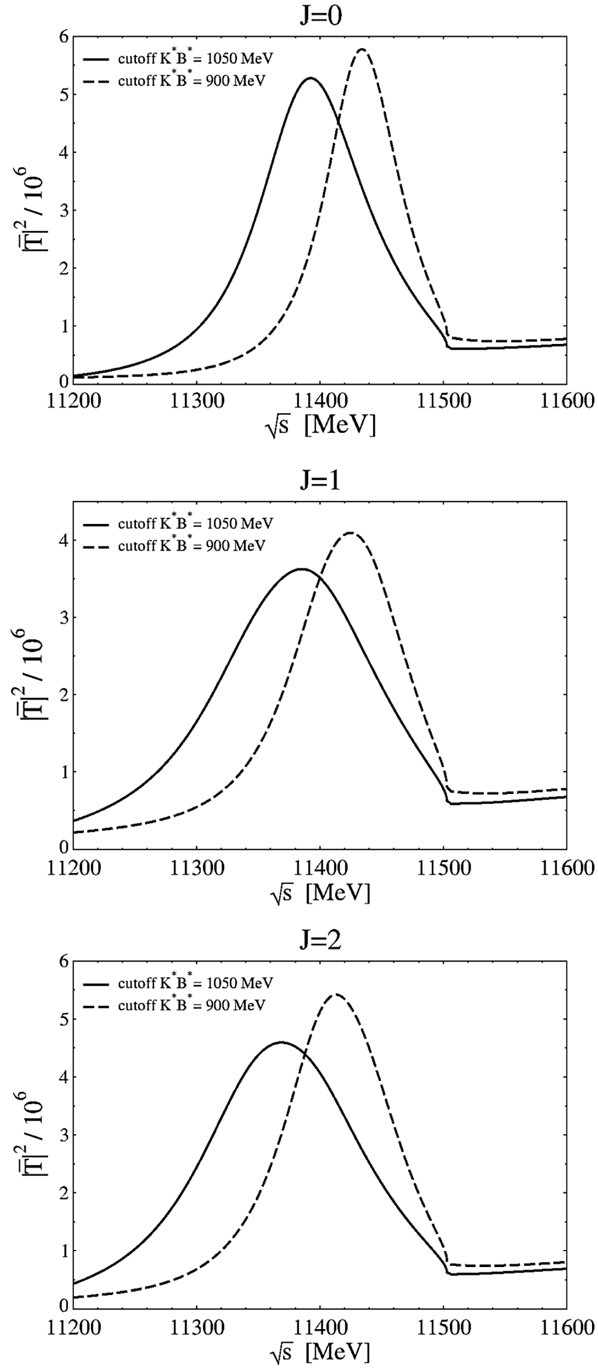

FIG. 4. The three-body amplitude $|\bar{T}|^2$ for the $\bar{K}^*\bar{B}^*\bar{B}^*$ system as a function of the three-body invariant mass energy, $\sqrt{s}$, for the three different values of the total spin, J, and for $q_{\rm max}^{\bar{K}^*\bar{B}^*} = 900$ MeV and $q_{\rm max}^{\bar{K}^*\bar{B}^*} = 1050$ MeV.

uncorrelated threshold $m_{\bar{K}^*} + 2m_{\bar{B}^*} = 11543$ MeV. Out of this energy, 40 MeV comes from the binding of the $\bar{B}^*\bar{B}^*$ cluster [32], and the rest comes from the three-body dynamics.

The results, also for $J = 1$ and $J = 2$, are summarized in Table I, where we show the binding energies, E_B, and the

TABLE I. Binding energy, E_B, and width, Γ, of the three-body systems for the three different possible total spins J. The first number in the numerical cells represents the value obtained with the cutoff $q_{\rm max}^{\bar{K}^*\bar{B}^*} = 900$ MeV, and the second one using 1050 MeV. All units are MeV. The binding energies refer to the uncorrelated thresholds, $m_{\bar{K}^*} + 2m_{\bar{B}^*} = 11543$ MeV and $m_{\bar{K}^*} + m_{\bar{B}} + m_{\bar{B}^*} = 11498$ MeV for $\bar{K}^*\bar{B}^*\bar{B}^*$ and $\bar{K}^*\bar{B}\bar{B}^*$ respectively.

	J	E_B	Γ
$\bar{K}^*\bar{B}^*\bar{B}^*$	0	109–150	72–104
	1	118–158	106–153
	2	130–174	103–149
$\bar{K}^*\bar{B}\bar{B}^*$	0	94–144	11–4
	1	95–144	16–6
	2	92–143	13–5

widths obtained for the two values of $q_{\rm max}^{\bar{K}^*\bar{B}^*}$ considered. The binding energies shown in the table are defined as the difference between the position of the maximum of the peak and the uncorrelated threshold $m_{\bar{K}^*} + 2m_{\bar{B}^*} = 11543$ MeV.

According to Eq. (21) the single-scattering three-body $J = 0$ amplitude is proportional to the two-body $J = 1$ one. Therefore, the origin of the three-body peak found for total $J = 0$ can be traced to the bound $\bar{K}^*\bar{B}^*$ $I(J^P) = 0(1^+)$ state which has a mass of 6113 MeV, (see Table I in Ref. [34]). This value for the two-body energy, $\sqrt{s_1}$, corresponds, using Eq. (23), to a three-body energy $\sqrt{s} = 11392$ MeV, which is almost where the three-body state is located. This is an indication that the multiple scattering [Figs. 1(b) and (c)] is small since, if this were the case, we could neglect G_0 in Eq. (13) and then $\bar{T} \simeq 2\bar{t}_1$.

The state found for $J = 1$ and $J = 2$ can be traced to the $\bar{K}^*\bar{B}^*$ two-body states following a similar argument as done above for $J = 0$. For $J = 1$ the three-body amplitude depends on a combination of the three possible two-body spin amplitudes, and for $J = 2$ on the three two-body spins, Eq. (21). According to the results of Ref. [34] (see Table I in Ref. [34]) three $\bar{K}^*\bar{B}^*$ states were found with energies 6125 MeV ($J = 0$), 6113 MeV ($J = 1$), and 6074 MeV ($J = 2$) and widths 160 MeV, 98 MeV, and 138 MeV respectively. These energies correspond to $\sqrt{s} = 11405$ MeV, 11392 MeV, and 11351 MeV respectively, from Eq. (23). Therefore, up to effects of the nonresonant isospin $I = 1$ amplitudes, and the multiple scattering mechanisms, the three-body pole found is essentially the effect of an overlap of these three states due to their large width. Indeed if, for illustrative purposes, we artificially reduced the main source of imaginary part of the $\bar{K}^*\bar{B}^*$ amplitudes, which are the box diagrams with intermediate $\bar{K}\,\bar{B}$ and $\bar{K}\bar{B}^*$ [34], to 5% of its true value, then we would see three clear narrow peaks in the three-body amplitudes, as is shown in Fig. 5 for the $J = 1$ case. The $J = 2$ case is qualitatively analogous.

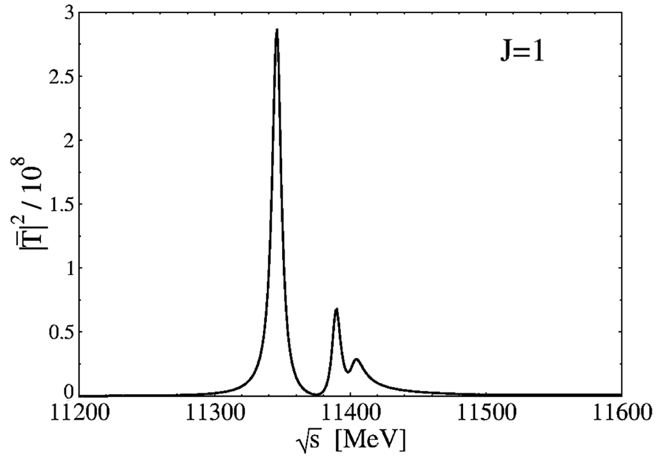

FIG. 5. Three-body amplitude $|\bar{T}|^2$ for $J = 1$ and $q_{\max}^{\bar{K}^*\bar{B}^*} = 1050$ MeV reducing artificially the main source of the imaginary part of the $\bar{K}^*\bar{B}^*$ amplitudes to 5% its true value.

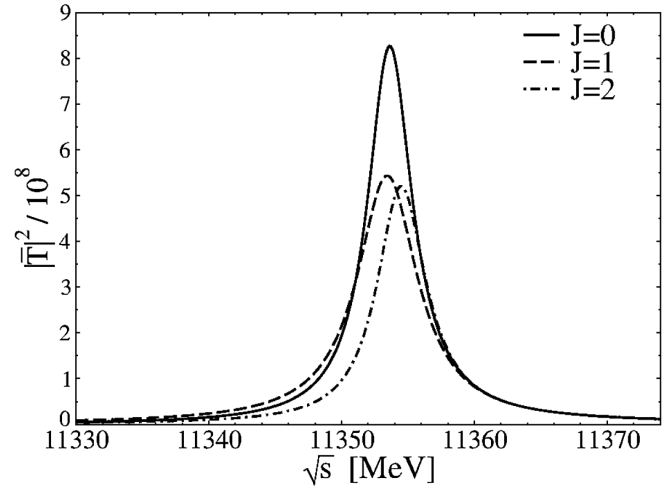

FIG. 6. Three-body amplitude $|\bar{T}|^2$ for the $\bar{K}^*\bar{B}\bar{B}^*$ system for the three different values of the total spin and $q_{\max}^{\bar{K}^*\bar{B}} = 1050$ MeV.

Another point worth commenting on is that in [41] the bound states for the $D^*D^*\bar{K}^*$ follow a behavior similar to ours, as far as the number of poles found and its nature is concerned, but their binding energies are smaller than the values that we obtain for the $\bar{K}^*\bar{B}^*\bar{B}^*$ system. This trend of the binding energy with the heavy meson mass has also been commonly observed in other studies when passing from the charm to the bottom sector [73–76].

In Fig. 6 we show the results for the $\bar{K}^*\bar{B}\bar{B}^*$ amplitude for the different channels calculated with a cutoff for the $\bar{K}^*\bar{B}^*$ and $\bar{K}^*\bar{B}$ loops of 1050 MeV (the use of the other cutoff, 900 MeV produces a similar shift as in the previous figures). We can appreciate that we find states peaking around 11350 MeV for the three different spins. This corresponds to a binding energy about 145 MeV below the uncorrelated $\bar{K}^*\bar{B}\bar{B}^*$ threshold (see Table I). Note that the binding energies are similar for the $\bar{K}^*\bar{B}^*\bar{B}^*$ and $\bar{K}^*\bar{B}\bar{B}^*$ cases, and the difference between both thresholds is of about 45 MeV. In principle this could imply that it could be difficult to differentiate experimentally between the states associated to the $\bar{K}^*\bar{B}^*\bar{B}^*$ and those corresponding to $\bar{K}^*\bar{B}\bar{B}^*$. However note that the widths of the $\bar{K}^*\bar{B}\bar{B}^*$ states are about 1 order of magnitude smaller than those of the $\bar{K}^*\bar{B}^*\bar{B}^*$ case, and then this could be a criterion to distinguish them. Anyway, the possible experimental difficulties of differentiating the states do not weaken our theoretical prediction that there must be exotic states with open strange and double-bottom flavors, and for the three different spins, around that energy (within the given uncertainties).

IV. SUMMARY

We have studied theoretically the three-body system $\bar{K}^*\bar{B}^*\bar{B}^*$ and the similar $\bar{K}^*\bar{B}\bar{B}^*$ to look for possible mesonic states with open s and two b flavors. The work is motivated by the results of a previous work where the $\bar{B}^*\bar{B}^*$ and $\bar{B}\bar{B}^*$ interactions with $I(J^P) = 0(1^+)$ were found to bind, and in other work, the $\bar{K}^*\bar{B}^*$ and $\bar{K}^*\bar{B}$ interactions were also found to be attractive in $I = 0$. This allows us to apply the fixed center approximation (FCA) to the Faddeev equations where the $\bar{K}^*$ interacts with each of the particles in the $\bar{B}^*\bar{B}^*$ or $\bar{B}\bar{B}^*$ cluster and undergoes multiple rescattering. The total three-body amplitude can then be written algebraically in terms of the two-body $\bar{K}^*\bar{B}^*$ and $\bar{K}^*\bar{B}$ obtained from the unitarization of interacting potentials obtained from suitable extensions to the heavy bottom of local hidden gauge symmetry Lagrangians. The method contains no further degrees of freedom besides the uncertainties already implied in the two-body scattering amplitudes.

We find resonant three-body structures with quantum numbers $I(J^P) = 1/2(0^-)$, $1/2(1^-)$, and $1/2(2^-)$ with binding energies and widths of the order of 100 MeV (see Table I and Fig. 4). We hope that these superexotic mesons, with open strange and double-bottom flavor, can be experimentally found in the not very distant future.

ACKNOWLEDGMENTS

We thank Eulogio Oset for reading a draft version of the manuscript. The work of N. I. was partly supported by JSPS KAKENHI Grants No. JP19K14709 and No. JP21KK0244. L. R. acknowledges support by the Spanish Ministerio de Ciencia e Innovación (MICINN) and the European Regional Development Fund (ERDF) under Contract No. PID2020–112777 GB-I00.

[1] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, Rep. Prog. Phys. **80**, 076201 (2017).
[2] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo, and C. Z. Yuan, Phys. Rep. **873**, 1 (2020).
[3] S. Godfrey and S. L. Olsen, Annu. Rev. Nucl. Part. Sci. **58**, (2008).
[4] X. Liu, Chin. Sci. Bull. **59**, 3815 (2014).
[5] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, Prog. Theor. Exp. Phys. **2016**, 062C01 (2016).
[6] H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rep. **639**, 1 (2016).
[7] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **125**, 242001 (2020).
[8] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **102**, 112003 (2020).
[9] Z. G. Wang, Int. J. Mod. Phys. A **35**, 2050187 (2020).
[10] X. G. He, W. Wang, and R. Zhu, Eur. Phys. J. C **80**, 1026 (2020).
[11] J. R. Zhang, Phys. Rev. D **103**, 054019 (2021).
[12] G. J. Wang, L. Meng, L. Y. Xiao, M. Oka, and S. L. Zhu, Eur. Phys. J. C **81**, 188 (2021).
[13] M. Z. Liu, J. J. Xie, and L. S. Geng, Phys. Rev. D **102**, 091502 (2020).
[14] H. X. Chen, W. Chen, R. R. Dong, and N. Su, Chin. Phys. Lett. **37**, 101201 (2020).
[15] Y. Huang, J. X. Lu, J. J. Xie, and L. S. Geng, Eur. Phys. J. C **80**, 973 (2020).
[16] Y. Xue, X. Jin, H. Huang, and J. Ping, Phys. Rev. D **103**, 054010 (2021).
[17] L. R. Dai, R. Molina, and E. Oset, Phys. Rev. D **105**, 016029 (2022).
[18] S. S. Agaev, K. Azizi, and H. Sundu, J. Phys. G **48**, 085012 (2021).
[19] H. Mutuk, J. Phys. G **48**, 055007 (2021).
[20] C. J. Xiao, D. Y. Chen, Y. B. Dong, and G. W. Meng, Phys. Rev. D **103**, 034004 (2021).
[21] J. He and D. Y. Chen, Chin. Phys. C **45**, 063102 (2021).
[22] R. Molina and E. Oset, Phys. Lett. B **811**, 135870 (2020).
[23] X. H. Liu, M. J. Yan, H. W. Ke, G. Li, and J. J. Xie, Eur. Phys. J. C **80**, 1178 (2020).
[24] T. J. Burns and E. S. Swanson, Phys. Lett. B **813**, 136057 (2021).
[25] R. Aaij *et al.* (LHCb Collaboration), Nat. Phys. **18**, 751 (2022).
[26] R. Aaij *et al.* (LHCb Collaboration), Nat. Commun. **13**, 3351 (2022).
[27] A. Feijoo, W. H. Liang, and E. Oset, Phys. Rev. D **104**, 114015 (2021).
[28] L. Meng, G. J. Wang, B. Wang, and S. L. Zhu, Phys. Rev. D **104**, 051502 (2021).
[29] X. Z. Ling, M. Z. Liu, L. S. Geng, E. Wang, and J. J. Xie, Phys. Lett. B **826**, 136897 (2022).
[30] M. Albaladejo, Phys. Lett. B **829**, 137052 (2022).
[31] M. L. Du, V. Baru, X. K. Dong, A. Filin, F. K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, Phys. Rev. D **105**, 014024 (2022).
[32] L. R. Dai, E. Oset, A. Feijoo, R. Molina, L. Roca, A. M. Torres, and K. P. Khemchandani, Phys. Rev. D **105**, 074017 (2022); **106**, 099904(E) (2022).
[33] S. Sakai, L. Roca, and E. Oset, Phys. Rev. D **96**, 054023 (2017).
[34] E. Oset and L. Roca, Eur. Phys. J. C **82**, 882 (2022); **82**, 1014(E) (2022).
[35] J. A. Oller, E. Oset, and A. Ramos, Prog. Part. Nucl. Phys. **45**, 157 (2000).
[36] X. K. Dong, F. K. Guo, and B. S. Zou, Commun. Theor. Phys. **73**, 125201 (2021).
[37] X. K. Dong, F. K. Guo, and B. S. Zou, Prog. Phys. **41**, 65 (2021).
[38] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao, and B. S. Zou, Rev. Mod. Phys. **90**, 015004 (2018); **94**, 029901(E) (2022).
[39] A. Martinez Torres, K. P. Khemchandani, L. Roca, and E. Oset, Few Body Syst. **61**, 35 (2020).
[40] H. Garcilazo and A. Valcarce, Phys. Lett. B **784**, 169 (2018).
[41] N. Ikeno, M. Bayar, and E. Oset, Phys. Rev. D **107**, 034006 (2023).
[42] M. P. Valderrama, Phys. Rev. D **98**, 014022 (2018).
[43] X. L. Ren and Z. F. Sun, Phys. Rev. D **99**, 094041 (2019).
[44] L. D. Faddeev, Zh. Eksp. Teor. Fiz. **39**, 1459 (1960) [Sov. Phys. JETP **12**, 1014 (1961)].
[45] E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. **B2**, 167 (1967).
[46] E. Epelbaum, A. Nogga, W. Gloeckle, H. Kamada, U. G. Meißner, and H. Witala, Phys. Rev. C **66**, 064001 (2002).
[47] Y. Nogami, Phys. Lett. **7**, 288 (1963).
[48] Y. Ikeda and T. Sato, Phys. Rev. C **76**, 035203 (2007).
[49] A. Martinez Torres, K. P. Khemchandani, and E. Oset, Phys. Rev. C **77**, 042203 (2008).
[50] D. Jido and Y. Kanada-En'yo, Phys. Rev. C **78**, 035203 (2008).
[51] G. Mennessier, J. Y. Pasquier, and R. Pasquier, Phys. Rev. D **6**, 1351 (1972).
[52] A. Martinez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale, and E. Oset, Phys. Rev. D **78**, 074031 (2008).
[53] A. Martinez Torres, K. P. Khemchandani, D. Gamermann, and E. Oset, Phys. Rev. D **80**, 094012 (2009).
[54] A. Martinez Torres, E. J. Garzon, E. Oset, and L. R. Dai, Phys. Rev. D **83**, 116002 (2011).
[55] R. Chand and R. H. Dalitz, Ann. Phys. (N.Y.) **20**, 1 (1962).
[56] R. C. Barrett and A. Deloff, Phys. Rev. C **60**, 02520 (1999).
[57] A. Deloff, Phys. Rev. C **61**, 024004 (2000).
[58] S. S. Kamalov, E. Oset, and A. Ramos, Nucl. Phys. **A690**, 494 (2001).
[59] A. Gal, Int. J. Mod. Phys. A **22**, 226 (2007).
[60] T. W. Wu, M. Z. Liu, and L. S. Geng, Phys. Rev. D **103**, L031501 (2021).
[61] X. Wei, Q. H. Shen, and J. J. Xie, Eur. Phys. J. C **82**, 718 (2022).
[62] S. Q. Luo, T. W. Wu, M. Z. Liu, L. S. Geng, and X. Liu, Phys. Rev. D **105**, 074033 (2022).
[63] M. Bayar, A. Martinez Torres, K. P. Khemchandani, R. Molina, and E. Oset, Eur. Phys. J. C **83**, 46 (2023).
[64] In Ref. [32] no bound states were found for other possible isospin and spin $\bar{B}^*\bar{B}^*$ channels.
[65] L. Roca and E. Oset, Phys. Rev. D **82**, 054013 (2010).

[66] J. Yamagata-Sekihara, L. Roca, and E. Oset, Phys. Rev. D **82**, 094017 (2010); **85**, 119905(E) (2012).
[67] L. Roca, Phys. Rev. D **84**, 094006 (2011).
[68] J. Yamagata-Sekihara, J. Nieves, and E. Oset, Phys. Rev. D **83**, 014003 (2011).
[69] F. Mandl and G. Shaw, *Quantum Field Theory*, 2nd ed. (John Wiley and Sons, New York, 2010).
[70] L. Roca, E. Oset, and J. Singh, Phys. Rev. D **72**, 014002 (2005).
[71] D. Gamermann, J. Nieves, E. Oset, and E. Ruiz Arriola, Phys. Rev. D **81**, 014029 (2010).
[72] J. Nieves and M. P. Valderrama, Phys. Rev. D **84**, 056015 (2011).
[73] J. P. Ader, J. M. Richard, and P. Taxil, Phys. Rev. D **25**, 2370 (1982).
[74] H. W. Ke, X. H. Liu, and X. Q. Li, Eur. Phys. J. C **82**, 144 (2022).
[75] J. Carlson, L. Heller, and J. A. Tjon, Phys. Rev. D **37**, 744 (1988).
[76] S. Zouzou, B. Silvestre-Brac, C. Gignoux, and J. M. Richard, Z. Phys. C **30**, 457 (1986).