Dihadron production in DIS at small x at next-to-leading order: Transverse photons

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We calculate the next-to-leading order corrections to dihadron production in deep inelastic scattering (DIS) at small x using the color glass condensate formalism for the case when the virtual photon is transverse polarized. Similar to the case of longitudinal photon exchange all UV and soft singularities cancel while the collinear divergences are absorbed into quark and antiquark-hadron fragmentation functions. Rapidity divergences lead to Jalilian-Marian, Iancu, McLerran, Wiegert, Leonidov, Kovner (JIMWLK) evolution of dipoles and quadrupoles which describe multiple scatterings of the quark antiquark dipole on the target proton/nucleus and contain all the QCD dynamics of the target leading to a finite final result for the dihadron production cross section.

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I. INTRODUCTION

A hadron or nucleus wave function at high energy (equivalently, small x) contains a large number of predominantly gluons leading to the phenomenon of gluon saturation [1–5]. Inclusive and diffractive two-particle production and angular correlations in high energy hadronic/nuclear collisions is a sensitive probe of gluon saturation in a proton or nucleus at small x [6–41]. The disappearance of the away side peak in proton (deuteron)-nucleus collisions in the forward rapidity region at RHIC [42,43] as predicted by gluon saturation models [8] provides the strongest hint for the presence of gluon saturation in the wave function of the target nucleus at small x. Nevertheless, because of complications arising from further interactions and radiation from both initial and final states, an unambiguous interpretation of the RHIC results remains illusive. Deep inelastic scattering (DIS) offers the cleanest environment in which the dynamics of gluon saturation can be investigated theoretically as the virtual photon probing the inner structure of the target does not interact strongly. The proposed electron-ion collider (EIC) will allow precision studies of the observables [44,45] in which gluon saturation is expected to play a dominant role and as such establish the presence of saturation and clarify the kinematics in which it is the main QCD effect. Because of this fact it is imperative that the existing predictions for saturation effects are made more precise by calculating higher orders in α_s corrections.

Next-to-leading order calculations for many processes using the color glass condensate effective theory of QCD at small x have recently become available [46–57]. In a recent paper [53] we calculated the one-loop corrections to inclusive dihadron production in DIS at small x for the case when the exchanged virtual photon is longitudinal. In this work we extend our studies of this process and calculate dihadron production in DIS with transverse photon exchange. As the calculational methods are identical to our earlier work we will skip a lot of the details of the calculation and refer the reader to [53]. As before there are several divergences that appear at the next-to-leading order. All divergences either cancel or can be absorbed into evolution of physical quantities. Our final results are then completely finite and can be used to calculate inclusive dihadron production and angular correlations in DIS at small x.

In the small x limit of DIS the virtual photon (transverse or longitudinal) splits into a quark antiquark pair (a dipole), which then multiply scatters from the target hadron or nucleus. To leading order (LO) accuracy the double inclusive production cross section can be written as

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$$\frac{\mathrm{d}\sigma^{\gamma^* A \to q\bar{q}X}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)\int \mathrm{d}^8\mathbf{x}[S_{122'1'} - S_{12} - S_{1'2'} + 1]$$
$$\times e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\bigg\{4z_1z_2K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2'}|Q_1)$$
$$+ (z_1^2+z_2^2)\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2'}|Q_1)\bigg\}, \tag{1}$$

where the first and second terms in the curly brackets above correspond to the contribution of the longitudinal and transverse polarizations of the virtual photon. The production cross section is a convolution of the probability for a photon to split into a quark at transverse position $\mathbf{x}_1$ and an antiquark at position $\mathbf{x}_2$ represented by the Bessel functions, with the probability for this quark antiquark pair to scatter from the target encoded in the dipoles S_{ij} and quadrupoles S_{ijkl}. The virtual photon has momentum l^μ with $l^2 = -Q^2$, and we have set the transverse momentum of the photon to zero without any loss of generality. Furthermore, p^μ (q^μ) is the momentum of the outgoing quark (antiquark) and z_1 (z_2) is its longitudinal momentum fraction relative to the photon. $\mathbf{x}_1$ ($\mathbf{x}_2$) is the transverse coordinate of the quark (antiquark), and primed coordinates are used in the conjugate amplitude. Quark and antiquark rapidities y_1 and y_2 are related to their momentum fractions z_1 and z_2 via $\mathrm{d}y_i = \mathrm{d}z_i/z_i$. For convenience we also define and use the following shorthand notations:

$$Q_i = Q\sqrt{z_i(1-z_i)}, \qquad \mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j,$$
$$\mathrm{d}^8\mathbf{x} = \mathrm{d}^2\mathbf{x}_1\mathrm{d}^2\mathbf{x}_2\mathrm{d}^2\mathbf{x}_{1'}\mathrm{d}^2\mathbf{x}_{2'}. \tag{2}$$

Dipoles S_{ij} and quadrupoles S_{ijkl} are normalized correlation functions of two and four Wilson lines defined as

$$S_{ij} = \frac{1}{N_c}\mathrm{tr}\Big\langle V_iV_j^\dagger\Big\rangle, \qquad S_{ijkl} = \frac{1}{N_c}\mathrm{tr}\Big\langle V_iV_j^\dagger V_kV_l^\dagger\Big\rangle, \tag{3}$$

which contain the full dynamics of gluon saturation. Here index i refers to the transverse coordinate $\mathbf{x}_i$, and we use the following notation for Wilson lines:

$$V_i = \hat{P}\exp\left(ig\int \mathrm{d}x^+A^-(x^+,\mathbf{x}_i)\right), \tag{4}$$

which resum the multiple scatterings of the quark and antiquark from the target hadron or nucleus.

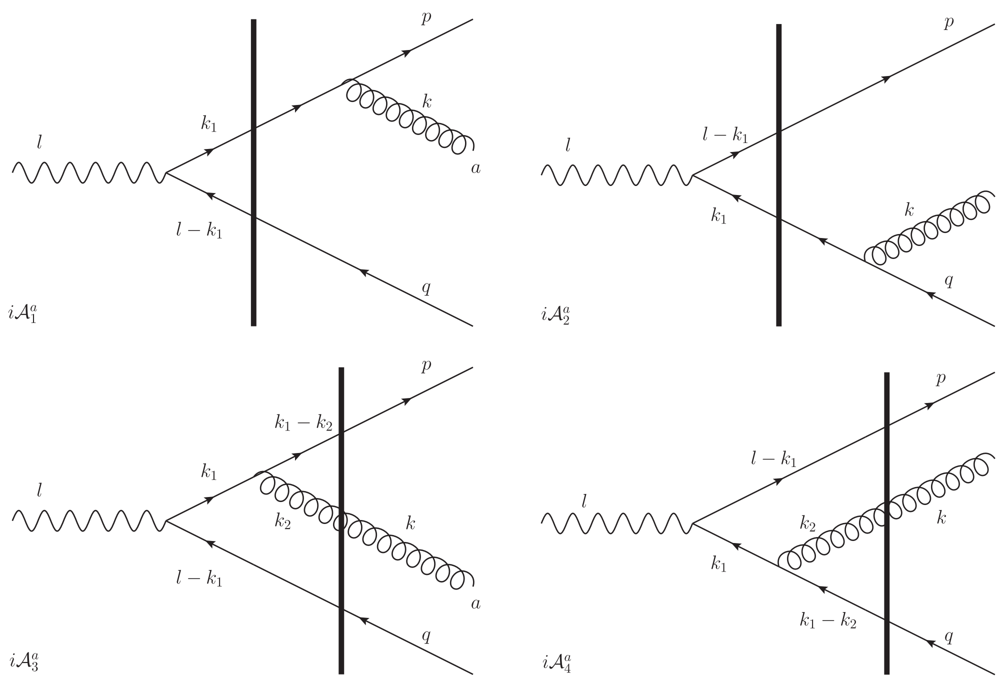

FIG. 1. The real corrections $i\mathcal{A}_1^a, \ldots, i\mathcal{A}_4^a$. The arrows on Fermion lines indicate Fermion number flow, where all momenta flow to the right. The thick solid line indicates interaction with the target.

II. ONE-LOOP CORRECTIONS

In [48,49,52,53,56] spinor helicity formalism was used to calculate the contribution of real diagrams to next-to-leading order (NLO) corrections to the leading order results. The real corrections are shown in Fig. 1 and involve radiation of a gluon either by the quark or antiquark before they scatter from the target, in which case the gluon also scatters from the target [58–60], or after they scatter from the target, in which case the radiated gluon does not scatter from the target.

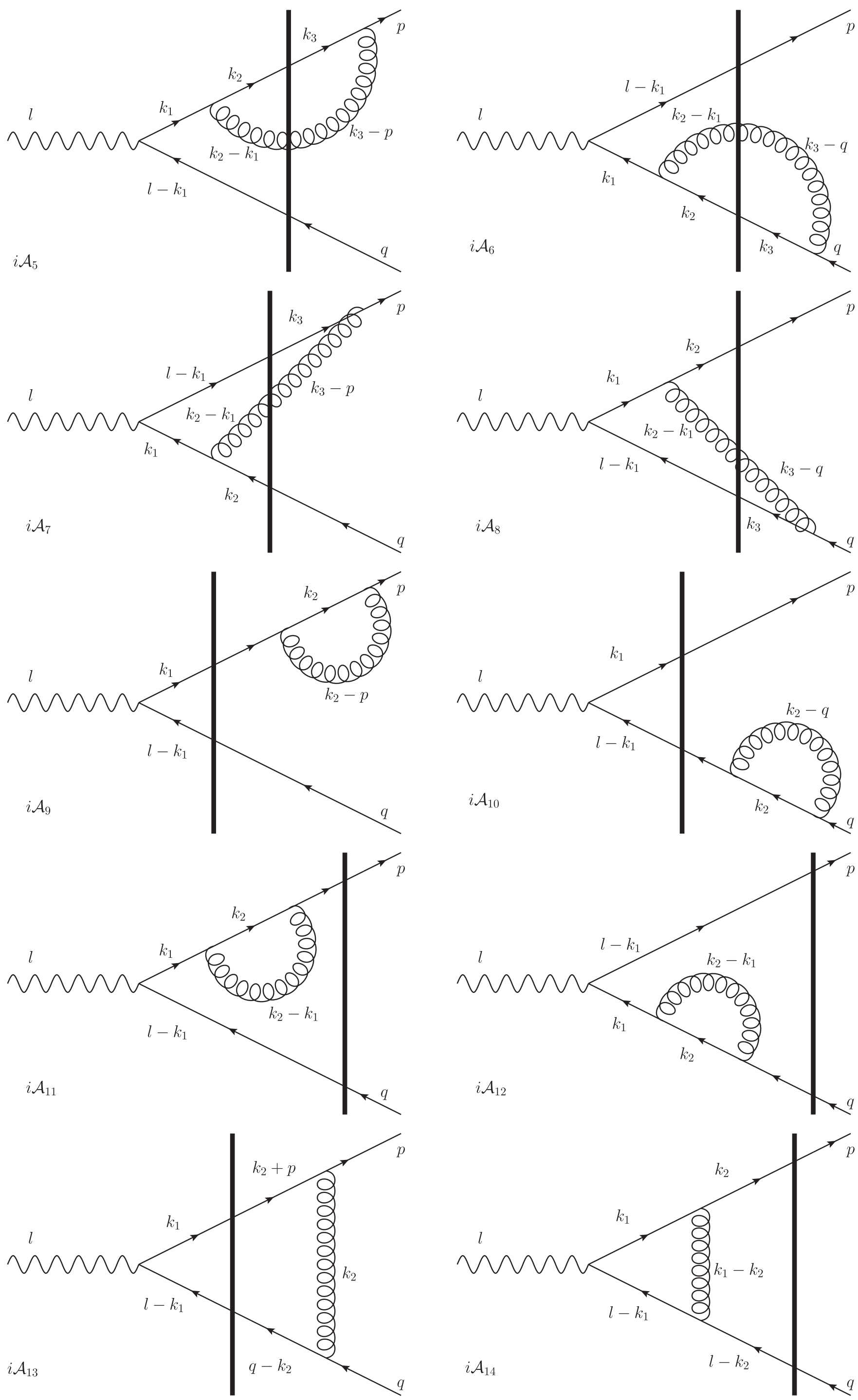

FIG. 2. The ten virtual NLO diagrams $iA_5, …, iA_{14}$. The arrows on fermion lines indicate fermion number flow, all momenta flow to the right, *except* for gluon momenta. The thick solid line indicates interaction with the target.

TABLE I. The minimal set of transverse photon numerators N_1 to N_4 in momentum fraction notation. Complex conjugation results in the numerator with all helicities flipped (while leaving longitudinal helicities unchanged), and any numerator where $\lambda_q = \lambda_{\bar{q}}$ is zero.

Numerator	$\lambda_\gamma;\lambda_q,\lambda_g$	$N_i^{\lambda_\gamma;\lambda_q,\lambda_g}$
N_1	$+;+,+$	$-(z_1)^{3/2}\sqrt{z_2}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\boldsymbol{\epsilon}]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}(\mathbf{k}_1\cdot\boldsymbol{\epsilon})$
	$+;+,-$	$-\sqrt{z_1z_2}(1-z_2)^2\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\boldsymbol{\epsilon}^*]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}(\mathbf{k}_1\cdot\boldsymbol{\epsilon})$
	$+;-,+$	$(z_2)^{3/2}\sqrt{z_1}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\boldsymbol{\epsilon}]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}(\mathbf{k}_1\cdot\boldsymbol{\epsilon})$
	$+,-,-$	$(z_1z_2)^{3/2}\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\boldsymbol{\epsilon}^*]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}(\mathbf{k}_1\cdot\boldsymbol{\epsilon})$
N_2	$+;+,+$	$-(z_1)^{3/2}\sqrt{z_2}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\boldsymbol{\epsilon}]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}(\mathbf{k}_1\cdot\boldsymbol{\epsilon})$
	$+;+,-$	$-(z_1z_2)^{3/2}\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\boldsymbol{\epsilon}^*]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}(\mathbf{k}_1\cdot\boldsymbol{\epsilon})$
	$+;-,+$	$(z_2)^{3/2}\sqrt{z_1}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\boldsymbol{\epsilon}]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}(\mathbf{k}_1\cdot\boldsymbol{\epsilon})$
	$+,-,-$	$\sqrt{z_1z_2}(1-z_1)^2\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\boldsymbol{\epsilon}^*]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}(\mathbf{k}_1\cdot\boldsymbol{\epsilon})$
N_3	$+;+,+$	$(z_1)^{3/2}\sqrt{z_2}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}}{z_3}-\frac{\mathbf{k}_1\cdot\boldsymbol{\epsilon}}{1-z_2}\right)\mathbf{k}_1\cdot\boldsymbol{\epsilon}$
	$+;+,-$	$\sqrt{z_1z_2}(1-z_2)^2\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}^*}{z_3}-\frac{\mathbf{k}_1\cdot\boldsymbol{\epsilon}^*}{1-z_2}\right)\mathbf{k}_1\cdot\boldsymbol{\epsilon}$
	$+;-,+$	$-(z_2)^{3/2}\sqrt{z_1}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}}{z_3}-\frac{\mathbf{k}_1\cdot\boldsymbol{\epsilon}}{1-z_2}\right)\mathbf{k}_1\cdot\boldsymbol{\epsilon}$
	$+,-,-$	$-(z_1z_2)^{3/2}\left[\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}^*}{z_3}-\frac{\mathbf{k}_1\cdot\boldsymbol{\epsilon}^*}{(1-z_2)}\right)\mathbf{k}_1\cdot\boldsymbol{\epsilon}+\frac{\mathbf{k}_1^2+z_2(1-z_2)Q^2}{2z_2(1-z_2)}\right]$
N_4	$+;+,+$	$(z_1)^{3/2}\sqrt{z_2}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}}{z_3}-\frac{\mathbf{k}_1\cdot\boldsymbol{\epsilon}}{1-z_1}\right)\mathbf{k}_1\cdot\boldsymbol{\epsilon}$
	$+;+,-$	$(z_1z_2)^{3/2}\left[\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}^*}{z_3}-\frac{\mathbf{k}_1\cdot\boldsymbol{\epsilon}^*}{(1-z_1)}\right)\mathbf{k}_1\cdot\boldsymbol{\epsilon}+\frac{\mathbf{k}_1^2+z_1(1-z_1)Q^2}{2z_1(1-z_1)}\right]$
	$+;-,+$	$-(z_2)^{3/2}\sqrt{z_1}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}}{z_3}-\frac{\mathbf{k}_1\cdot\boldsymbol{\epsilon}}{1-z_1}\right)\mathbf{k}_1\cdot\boldsymbol{\epsilon}$
	$+,-,-$	$-\sqrt{z_1z_2}(1-z_1)^2\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}^*}{z_3}-\frac{\mathbf{k}_1\cdot\boldsymbol{\epsilon}^*}{1-z_1}\right)\mathbf{k}_1\cdot\boldsymbol{\epsilon}$

The virtual corrections are shown in Fig. 2 and involve radiation of a gluon by either quark or antiquark, which is then absorbed by the quark or antiquark line still in the amplitude [53,56].

We refer the reader to [53] for the explicit expressions for the amplitudes for real and virtual corrections given by Eqs. (5), (8)–(14), respectively, and Eqs. (6), (14)–(19) for the Dirac numerators. In this study, we focus on the contribution from transversely polarized photons, and we compute the numerators using the spinor helicity formalism. The needed real numerators are in Table I and the virtual numerators are in Eqs. (5)–(15).

$$N_5^{+;+}=\frac{2^5(l^+)^2z_1^{3/2}\sqrt{z_2}}{(z_1-z_3)^2}\left\{z_1^2\left[\left(\mathbf{k}_3-\frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}\right]\left[\left(\mathbf{k}_2-\frac{z_3}{z_1}\mathbf{k}_1\right)\cdot\boldsymbol{\epsilon}^*\right]+z_3^2\left[\left(\mathbf{k}_3-\frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}^*\right]\left[\left(\mathbf{k}_2-\frac{z_3}{z_1}\mathbf{k}_1\right)\cdot\boldsymbol{\epsilon}\right]\right\}\mathbf{k}_1\cdot\boldsymbol{\epsilon},$$

$$N_5^{+;-}=\frac{-2^5(l^+)^2z_2^{3/2}\sqrt{z_1}}{(z_1-z_3)^2}\left\{z_1^2\left[\left(\mathbf{k}_3-\frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}^*\right]\left[\left(\mathbf{k}_2-\frac{z_3}{z_1}\mathbf{k}_1\right)\cdot\boldsymbol{\epsilon}\right]+z_3^2\left[\left(\mathbf{k}_3-\frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}\right]\left[\left(\mathbf{k}_2-\frac{z_3}{z_1}\mathbf{k}_1\right)\cdot\boldsymbol{\epsilon}^*\right]\right\}\mathbf{k}_1\cdot\boldsymbol{\epsilon}$$
$$+\frac{2^4(l^+)^2\sqrt{z_2}z_3^2}{\sqrt{z_1}(z_1-z_3)}[\mathbf{k}_1^2+z_1z_2Q^2]\left(\mathbf{k}_3-\frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}, \quad (5)$$

$$N_6^{+;+} = \frac{2^5(l^+)^2 z_1^{3/2}\sqrt{z_2}}{(z_2-z_3)^2}\left\{z_3^2\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}\right]\left[\left(\mathbf{k}_2 - \frac{z_3}{z_2}\mathbf{k}_1\right)\cdot\boldsymbol{\epsilon}^*\right] + z_2^2\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}^*\right]\left[\left(\mathbf{k}_2 - \frac{z_3}{z_2}\mathbf{k}_1\right)\cdot\boldsymbol{\epsilon}\right]\right\}\mathbf{k}_1\cdot\boldsymbol{\epsilon}$$
$$- \frac{2^4(l^+)^2\sqrt{z_1}z_3^2}{\sqrt{z_2}(z_2-z_3)}[\mathbf{k}_1^2 + z_1 z_2 Q^2]\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon},$$
$$N_6^{+;-} = \frac{-2^5(l^+)^2 z_2^{3/2}\sqrt{z_1}}{(z_2-z_3)^2}\left\{z_3^2\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}^*\right]\left[\left(\mathbf{k}_2 - \frac{z_3}{z_2}\mathbf{k}_1\right)\cdot\boldsymbol{\epsilon}\right] + z_2^2\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}\right]\left[\left(\mathbf{k}_2 - \frac{z_3}{z_2}\mathbf{k}_1\right)\cdot\boldsymbol{\epsilon}^*\right]\right\}\mathbf{k}_1\cdot\boldsymbol{\epsilon}, \tag{6}$$

$$N_7^{+;+} = \frac{-2^5(l^+)^2 z_3\sqrt{z_1 z_2}}{(1-z_3)(z_1-z_3)^2}\left\{z_1 z_2\left[\left(\mathbf{k}_3 - \frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}\right][(z_2\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}^*]\right.$$
$$\left. + z_3(1-z_3)\left[\left(\mathbf{k}_3 - \frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}^*\right][(z_2\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}]\right\}\mathbf{k}_1\cdot\boldsymbol{\epsilon}$$
$$- \frac{2^4(l^+)^2(z_1z_2)^{3/2}}{(z_1-z_3)(1-z_3)}[\mathbf{k}_1^2 + z_3(1-z_3)Q^2]\left[\left(\mathbf{k}_3 - \frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}\right],$$
$$N_7^{+;-} = \frac{2^5(l^+)^2\sqrt{z_1 z_2}}{(z_1-z_3)^2}\left\{z_1 z_2\left[\left(\mathbf{k}_3 - \frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}^*\right][(z_2\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}]\right.$$
$$\left. + z_3(1-z_3)\left[\left(\mathbf{k}_3 - \frac{z_3}{z_1}\mathbf{p}\right)\cdot\boldsymbol{\epsilon}\right][(z_2\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}^*]\right\}\mathbf{k}_1\cdot\boldsymbol{\epsilon}, \tag{7}$$

$$N_8^{+;+} = \frac{-2^5(l^+)^2\sqrt{z_1 z_2}}{(z_2-z_3)^2}\left\{z_3(1-z_3)\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}\right][(z_1\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}^*]\right.$$
$$\left. + z_1 z_2\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}^*\right][(z_1\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}]\right\}\mathbf{k}_1\cdot\boldsymbol{\epsilon},$$
$$N_8^{+;-} = \frac{2^5(l^+)^2 z_3\sqrt{z_1 z_2}}{(1-z_3)(z_2-z_3)^2}\left\{z_3(1-z_3)\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}^*\right][(z_1\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}]\right.$$
$$\left. + z_1 z_2\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}\right][(z_1\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}^*]\right\}\mathbf{k}_1\cdot\boldsymbol{\epsilon}$$
$$+ \frac{2^4(l^+)^2(z_1z_2)^{3/2}}{(z_2-z_3)(1-z_3)}[\mathbf{k}_1^2 + z_3(1-z_3)Q^2]\left[\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)\cdot\boldsymbol{\epsilon}\right], \tag{8}$$

$$N_9^{+;+} = 2^4(l^+)^2 z_1^{3/2}\sqrt{z_2}\left[k_2^2 + \frac{(2z_1 - z)}{z}(k_2-p)^2 - \frac{[z_1^2 + (z_1-z)^2]}{z_1 z}p^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon},$$
$$N_9^{+;+} = -2^4(l^+)^2 z_2^{3/2}\sqrt{z_1}\left[k_2^2 + \frac{(2z_1 - z)}{z}(k_2-p)^2 - \frac{[z_1^2 + (z_1-z)^2]}{z_1 z}p^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon}, \tag{9}$$

$$N_{10}^{+;+} = -2^4(l^+)^2 z_1^{3/2}\sqrt{z_2}\left[k_2^2 + \frac{(2z_2 - z)}{z}(k_2-q)^2 - \frac{[z_2^2 + (z_2-z)^2]}{z_2 z}q^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon},$$
$$N_{10}^{+;-} = 2^4(l^+)^2 z_2^{3/2}\sqrt{z_1}\left[k_2^2 + \frac{(2z_2 - z)}{z}(k_2-q)^2 - \frac{[z_2^2 + (z_2-z)^2]}{z_2 z}q^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon}, \tag{10}$$

$$N_{11}^{+;+} = 2^4(l^+)^2 z_1^{3/2}\sqrt{z_2}\left[\frac{[(k_2^+)^2+(p^+)^2]}{p^+(k_2^+-p^+)}k_1^2 + \frac{(p^++k_2^+)}{(p^+-k_2^+)}k_2^2 + (k_1-k_2)^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon},$$

$$N_{11}^{+;-} = -2^4(l^+)^2 z_2^{3/2}\sqrt{z_1}\left[\frac{[(k_2^+)^2+(p^+)^2]}{p^+(k_2^+-p^+)}k_1^2 + \frac{(p^++k_2^+)}{(p^+-k_2^+)}k_2^2 + (k_1-k_2)^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon} - \frac{2^4 z_2^{3/2} z_3 (l^+)^2}{\sqrt{z_1}(z_1-z_3)}k_1^2(z_3\mathbf{k}_1 - z_1\mathbf{k}_2)\cdot\boldsymbol{\epsilon}, \tag{11}$$

$$N_{12}^{+;+} = 2^4(l^+)^2 z_1^{3/2}\sqrt{z_2}\left[\frac{[(k_2^+)^2+(q^+)^2]}{q^+(k_2^+-q^+)}k_1^2 + \frac{(q^++k_2^+)}{(q^+-k_2^+)}k_2^2 + (k_1-k_2)^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon} + \frac{2^4(l^+)^2 z_1^{3/2} z_3}{\sqrt{z_2}(z_2-z_3)}k_1^2(z_3\mathbf{k}_1 - z_2\mathbf{k}_2)\cdot\boldsymbol{\epsilon},$$

$$N_{12}^{+;+} = -2^4(l^+)^2 z_2^{3/2}\sqrt{z_1}\left[\frac{[(k_2^+)^2+(q^+)^2]}{q^+(k_2^+-q^+)}k_1^2 + \frac{(q^++k_2^+)}{(q^+-k_2^+)}k_2^2 + (k_1-k_2)^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon}, \tag{12}$$

$$N_{13}^{+;+} = 2^5(l^+)^2(z_1+z)\sqrt{z_1z_2}\left[z_1 z\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1} - \frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1} - \frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}^*}{z}\right) + z^2\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}}{z} - \frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1} - \frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}^*}{z}\right)\right.$$
$$\left. - z_2 z\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}}{z} - \frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1} - \frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right) - p\cdot q - \frac{(z_1+z)}{2z}(k_2-q)^2 + \frac{(z_2-z)}{2z}(k_2+p)^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon},$$

$$N_{13}^{+;-} = -2^5(l^+)^2(z_2-z)\sqrt{z_1z_2}\left[z_1 z\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1} - \frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1} - \frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}}{z}\right) + z^2\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}^*}{z} - \frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1} - \frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}}{z}\right)\right.$$
$$\left. - z_2 z\left(\frac{\mathbf{k}_2\cdot\boldsymbol{\epsilon}^*}{z} - \frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1} - \frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right) - p\cdot q - \frac{(z_1+z)}{2z}(k_2-q)^2 + \frac{(z_2-z)}{2z}(k_2+p)^2\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon}, \tag{13}$$

$$N_{14(1)}^{+;+} = \frac{-2^5(l^+)^2\sqrt{z_1z_2}}{(1-z_3)(z_1-z_3)}\left[\frac{z_1z_2}{2}[\mathbf{k}_1^2 + z_3(1-z_3)Q^2] + \frac{z_3(1-z_3)}{2}[\mathbf{k}_2^2 + z_1z_2Q^2]\right.$$
$$\left. + (z_1-z_3)[(z_2\mathbf{k}_1 + z_3\mathbf{k}_2)\cdot\boldsymbol{\epsilon}][(z_1\mathbf{k}_1 + (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}^*]\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon}$$
$$+ \frac{2^4(l^+)^2(z_1z_2)^{3/2}}{z_3(z_1-z_3)(1-z_3)}[\mathbf{k}_1^2 + z_3(1-z_3)Q^2](z_1\mathbf{k}_1 - z_3\mathbf{k}_2)\cdot\boldsymbol{\epsilon},$$

$$N_{14(1)}^{+;-} = \frac{2^5(l^+)^2\sqrt{z_1z_2}}{z_3(z_1-z_3)}\left[\frac{z_1z_2}{2}[\mathbf{k}_1^2 + z_3(1-z_3)Q^2] + \frac{z_3(1-z_3)}{2}[\mathbf{k}_2^2 + z_1z_2Q^2]\right.$$
$$\left. + (z_1-z_3)[(z_2\mathbf{k}_1 + z_3\mathbf{k}_2)\cdot\boldsymbol{\epsilon}^*][(z_1\mathbf{k}_1 + (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}]\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon},$$

$$N_{14(2)}^{+;+} = \frac{-2^5(l^+)^2\sqrt{z_1z_2}}{(1-z_3)(z_3-z_1)}\left[\frac{z_1z_2}{2}[\mathbf{k}_1^2 + z_3(1-z_3)Q^2] + \frac{z_3(1-z_3)}{2}[\mathbf{k}_2^2 + z_1z_2Q^2]\right.$$
$$\left. + (z_3-z_1)[(z_2\mathbf{k}_1 + z_3\mathbf{k}_2)\cdot\boldsymbol{\epsilon}][(z_1\mathbf{k}_1 + (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}^*]\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon}, \tag{14}$$

$$N_{14(2)}^{+;-} = \frac{2^5(l^+)^2\sqrt{z_1z_2}}{z_3(z_3-z_1)}\left[\frac{z_1z_2}{2}[\mathbf{k}_1^2 + z_3(1-z_3)Q^2] + \frac{z_3(1-z_3)}{2}[\mathbf{k}_2^2 + z_1z_2Q^2]\right.$$
$$\left. + (z_3-z_1)[(z_2\mathbf{k}_1 + z_3\mathbf{k}_2)\cdot\boldsymbol{\epsilon}^*][(z_1\mathbf{k}_1 + (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}]\right]\mathbf{k}_1\cdot\boldsymbol{\epsilon}$$
$$+ \frac{2^4(l^+)^2(z_1z_2)^{3/2}}{z_3(z_1-z_3)(1-z_3)}[\mathbf{k}_1^2 + z_3(1-z_3)Q^2](z_2\mathbf{k}_1 - (1-z_3)\mathbf{k}_2)\cdot\boldsymbol{\epsilon}. \tag{15}$$

In all these expressions, the momentum fractions z and z_3 are all defined in the same way as in [53].

III. RESULTS

To calculate the $\mathcal{O}(\alpha_s)$ corrections for the production cross section we need to multiply the helicity amplitudes with the corresponding conjugate amplitudes. We will write the real corrections as $\sigma_{i\times j}$ for $i, j = 1, \ldots, 4$ and the virtual corrections as σ_i for $i = 5, \ldots, 14$. The details are shown in [53], and here we just show the final results. The T label signifies that we are including contributions only from transversely polarized photons, and we imply that we have summed over all outgoing polarizations. Furthermore and for the sake of brevity here, we omit a factor of $\delta(1 - z_1 - z_2 - z)$ in the real corrections and $\delta(1 - z_1 - z_2)$ in the virtual corrections and restore them at the end. In many cases, it is easiest to write the results in coordinate space with the radiation kernel $\Delta^{(3)}_{ij}$ defined as follows:

$$\Delta^{(3)}_{ij} = \frac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}^2_{3i}\mathbf{x}^2_{3j}}. \tag{16}$$

The next-to-leading order corrections are then

$$\frac{\mathrm{d}\sigma^T_{1\times1}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{e^2g^2Q^2N_c^2z_2^2(1-z_2)[z_1^2z_2^2+(z_1^2+z_2^2)(1-z_2)^2+(1-z_2)^4]}{2(2\pi)^{10}z_1}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{122'1'}-S_{12}-S_{1'2'}+1]$$
$$\times e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}K_1(|\mathbf{x}_{12}|Q_2)K_1(|\mathbf{x}_{1'2'}|Q_2)\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}e^{i\frac{z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}}\Delta^{(3)}_{1'1}; \tag{17}$$

$$\frac{\mathrm{d}\sigma^T_{2\times2}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{e^2g^2Q^2N_c^2z_1^2(1-z_1)[z_1^2z_2^2+(z_1^2+z_2^2)(1-z_1)^2+(1-z_1)^4]}{2(2\pi)^{10}z_2}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{122'1'}-S_{12}-S_{1'2'}+1]$$
$$\times e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2'}|Q_1)\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}e^{i\frac{z}{z_2}\mathbf{q}\cdot\mathbf{x}_{2'1}}\Delta^{(3)}_{2'2}; \tag{18}$$

$$\frac{\mathrm{d}\sigma^T_{1\times2}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{e^2g^2Q^2N_c^2\sqrt{z_1z_2(1-z_1)(1-z_2)}}{2(2\pi)^{10}}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{12}S_{1'2'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}$$
$$\times K_1(|\mathbf{x}_{12}|Q_2)K_1(|\mathbf{x}_{1'2'}|Q_1)4\mathrm{Re}\left[\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{1'2'}\cdot\boldsymbol{\epsilon}^*)}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}\left\{(z_1^2+z_2^2)(1-z_1)(1-z_2)\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{2'3}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}^2_{31}\mathbf{x}^2_{2'3}}\right.\right.$$
$$\left.\left.+z_1z_2((1-z_1)^2+(1-z_2)^2)\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)(\mathbf{x}_{2'3}\cdot\boldsymbol{\epsilon})}{\mathbf{x}^2_{31}\mathbf{x}^2_{2'3}}\right\}\right]e^{i\frac{z}{z_1}\mathbf{p}\cdot\mathbf{x}_{31}}e^{i\frac{z}{z_2}\mathbf{q}\cdot\mathbf{x}_{2'3}}; \tag{19}$$

$$\frac{\mathrm{d}\sigma^T_{3\times3}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{e^2g^2Q^2N_c^2z_1z_2^3}{2(2\pi)^{10}}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}$$
$$\times\frac{K_1(QX)K_1(QX')}{XX'}4\mathrm{Re}\left[(z_1^2+z_2^2)\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{31'}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}^2_{31}\mathbf{x}^2_{31'}}[(z_1\mathbf{x}_{12}+z\mathbf{x}_{32})\cdot\boldsymbol{\epsilon}][(z_1\mathbf{x}_{1'2'}+z\mathbf{x}_{32'})\cdot\boldsymbol{\epsilon}^*]\right.$$
$$+\left((1-z_2)^2+\frac{(z_1z_2)^2}{(1-z_2)^2}\right)\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)(\mathbf{x}_{31'}\cdot\boldsymbol{\epsilon})}{\mathbf{x}^2_{31}\mathbf{x}^2_{31'}}[(z_1\mathbf{x}_{12}+z\mathbf{x}_{32})\cdot\boldsymbol{\epsilon}][(z_1\mathbf{x}_{1'2'}+z\mathbf{x}_{32'})\cdot\boldsymbol{\epsilon}^*]$$
$$\left.-\frac{z_1^2z_2z}{2(1-z_2)^2}\left\{\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}^2_{31}}[(z_1\mathbf{x}_{12}+z\mathbf{x}_{32})\cdot\boldsymbol{\epsilon}]+\frac{(\mathbf{x}_{31'}\cdot\boldsymbol{\epsilon})}{\mathbf{x}^2_{31'}}[(z_1\mathbf{x}_{1'2'}+z\mathbf{x}_{32'})\cdot\boldsymbol{\epsilon}^*]\right\}+\frac{z_1^2z^2}{4(1-z_2)^2}\right]; \tag{20}$$

$$\frac{\mathrm{d}\sigma^T_{4\times4}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{e^2g^2Q^2N_c^2z_2z_1^3}{2(2\pi)^{10}}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}$$
$$\times\frac{K_1(QX)K_1(QX')}{XX'}4\mathrm{Re}\left[(z_1^2+z_2^2)\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}^2_{32}\mathbf{x}^2_{32'}}[(z_2\mathbf{x}_{21}+z\mathbf{x}_{31})\cdot\boldsymbol{\epsilon}][(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*]\right.$$
$$+\left((1-z_1)^2+\frac{(z_1z_2)^2}{(1-z_1)^2}\right)\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon}^*)(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon})}{\mathbf{x}^2_{32}\mathbf{x}^2_{32'}}[(z_2\mathbf{x}_{21}+z\mathbf{x}_{31})\cdot\boldsymbol{\epsilon}][(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*]$$
$$\left.-\frac{z_2^2z_1z}{2(1-z_1)^2}\left\{\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}^2_{32}}[(z_2\mathbf{x}_{21}+z\mathbf{x}_{31})\cdot\boldsymbol{\epsilon}]+\frac{(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon})}{\mathbf{x}^2_{32'}}[(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*]\right\}+\frac{z_2^2z^2}{4(1-z_1)^2}\right]; \tag{21}$$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^T_{3\times4}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = {} & \frac{e^2g^2Q^2N_c^2(z_1z_2)^2}{2(2\pi)^{10}}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \\
& \times\frac{K_1(QX)K_1(QX')}{XX'}4\mathrm{Re}\Bigg[(z_1^2+z_2^2)\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}_{31}^2\mathbf{x}_{32'}^2}[(z_1\mathbf{x}_{12}+z\mathbf{x}_{32})\cdot\boldsymbol{\epsilon}][(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*] \\
& +\frac{z_1z_2}{(1-z_1)(1-z_2)}[(1-z_1)^2+(1-z_2)^2]\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon})}{\mathbf{x}_{31}^2\mathbf{x}_{32'}^2}[(z_1\mathbf{x}_{12}+z\mathbf{x}_{32})\cdot\boldsymbol{\epsilon}][(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*] \\
& -\frac{z_2z(1-z_2)}{2(1-z_1)}\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}_{31}^2}[(z_1\mathbf{x}_{12}+z\mathbf{x}_{32})\cdot\boldsymbol{\epsilon}]-\frac{z_1z(1-z_1)}{2(1-z_2)}\frac{(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon})}{\mathbf{x}_{32'}^2}[(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*]\Bigg]; \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^T_{1\times3}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = {} & \frac{-e^2g^2Q^2N_c^2z_2^{5/2}\sqrt{1-z_2}}{2(2\pi)^{10}}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \\
& \times\frac{K_1(|\mathbf{x}_{12}|Q_2)K_1(QX')}{X'}4\mathrm{Re}\Bigg[(1-z_2)(z_1^2+z_2^2)\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{31'}\cdot\boldsymbol{\epsilon}^*)}{|\mathbf{x}_{12}|\mathbf{x}_{31'}^2}\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon})[(z_1\mathbf{x}_{1'2'}+z\mathbf{x}_{32'})\cdot\boldsymbol{\epsilon}^*]}{\mathbf{x}_{31}^2} \\
& +\left((1-z_2)^3+\frac{(z_1z_2)^2}{1-z_2}\right)\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{31'}\cdot\boldsymbol{\epsilon})}{|\mathbf{x}_{12}|\mathbf{x}_{31'}^2}\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)[(z_1\mathbf{x}_{1'2'}+z\mathbf{x}_{32'})\cdot\boldsymbol{\epsilon}^*]}{\mathbf{x}_{31}^2} \\
& -\frac{z_1^2z_2z}{2(1-z_2)}\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})}{|\mathbf{x}_{12}|}\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}_{31}^2}\Bigg]e^{i\frac{z}{z_1}\mathbf{p}\cdot\mathbf{x}_{31}}; \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^T_{1\times4}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = {} & \frac{-e^2g^2Q^2N_c^2z_1z_2^{3/2}\sqrt{1-z_2}}{2(2\pi)^{10}}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \\
& \times\frac{K_1(|\mathbf{x}_{12}|Q_2)K_1(QX')}{X'}4\mathrm{Re}\Bigg[(1-z_2)(z_1^2+z_2^2)\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon}^*)}{|\mathbf{x}_{12}|\mathbf{x}_{32'}^2}\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon})[(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*]}{\mathbf{x}_{31}^2} \\
& +\frac{z_1z_2}{1-z_1}((1-z_1)^2+(1-z_2)^2)\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon})}{|\mathbf{x}_{12}|\mathbf{x}_{32'}^2}\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)[(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*]}{\mathbf{x}_{31}^2} \\
& -\frac{z_2z(1-z_2)^2}{2(1-z_1)}\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})}{|\mathbf{x}_{12}|}\frac{(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}_{31}^2}\Bigg]e^{i\frac{z}{z_1}\mathbf{p}\cdot\mathbf{x}_{31}}; \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^T_{2\times3}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = {} & \frac{e^2g^2Q^2N_c^2z_2z_1^{3/2}\sqrt{1-z_1}}{2(2\pi)^{10}}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \\
& \times\frac{K_1(|\mathbf{x}_{12}|Q_1)K_1(QX')}{X'}4\mathrm{Re}\Bigg[(1-z_1)(z_1^2+z_2^2)\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{31'}\cdot\boldsymbol{\epsilon}^*)}{|\mathbf{x}_{12}|\mathbf{x}_{31'}^2}\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon})[(z_1\mathbf{x}_{1'2'}+z\mathbf{x}_{32'})\cdot\boldsymbol{\epsilon}^*]}{\mathbf{x}_{32}^2} \\
& +\frac{z_1z_2}{1-z_2}((1-z_1)^2+(1-z_2)^2)\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{31'}\cdot\boldsymbol{\epsilon})}{|\mathbf{x}_{12}|\mathbf{x}_{31'}^2}\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon}^*)[(z_1\mathbf{x}_{1'2'}+z\mathbf{x}_{32'})\cdot\boldsymbol{\epsilon}^*]}{\mathbf{x}_{32}^2} \\
& -\frac{z_1z(1-z_1)^2}{2(1-z_2)}\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})}{|\mathbf{x}_{12}|}\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}_{32}^2}\Bigg]e^{i\frac{z}{z_2}\mathbf{q}\cdot\mathbf{x}_{32}}; \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^T_{2\times4}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = {} & \frac{e^2g^2Q^2N_c^2z_1^{5/2}\sqrt{1-z_1}}{2(2\pi)^{10}}\int\frac{\mathrm{d}z}{z}\int\mathrm{d}^{10}\mathbf{x}[S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \\
& \times\frac{K_1(|\mathbf{x}_{12}|Q_1)K_1(QX')}{X'}4\mathrm{Re}\Bigg[(1-z_1)(z_1^2+z_2^2)\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon}^*)}{|\mathbf{x}_{12}|\mathbf{x}_{32'}^2}\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon})[(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*]}{\mathbf{x}_{32}^2} \\
& +\left((1-z_1)^3+\frac{(z_1z_2)^2}{1-z_1}\right)\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{32'}\cdot\boldsymbol{\epsilon})}{|\mathbf{x}_{12}|\mathbf{x}_{32'}^2}\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon}^*)[(z_2\mathbf{x}_{2'1'}+z\mathbf{x}_{31'})\cdot\boldsymbol{\epsilon}^*]}{\mathbf{x}_{32}^2} \\
& -\frac{z_2^2z_1z}{2(1-z_1)}\frac{(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})}{|\mathbf{x}_{12}|}\frac{(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon}^*)}{\mathbf{x}_{32}^2}\Bigg]e^{i\frac{z}{z_2}\mathbf{q}\cdot\mathbf{x}_{32}}; \qquad (26)
\end{aligned}
$$

$$\frac{\mathrm{d}\sigma_5^T}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{e^2g^2Q^2N_c^2z_2^{5/2}\sqrt{z_1}}{2(2\pi)^{10}}\int_0^{z_1}\frac{\mathrm{d}z}{z}\mathrm{d}^{10}\mathbf{x}[S_{322'1'}S_{13} - S_{13}S_{23} - S_{1'2'} + 1]$$
$$\times\frac{K_1(QX_5)K_1(|\mathbf{x}_{1'2'}|Q_1)}{X_5\mathbf{x}_{31}^2|\mathbf{x}_{1'2'}|}e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{-i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\mathbf{x}_{1'2'}\cdot[(z_1^2+z_2^2)(z_1^2+(z_1-z)^2)\mathbf{x}_{32}$$
$$+(z_1-z)(z_1(z_1^2+(z_1-z)^2)+z_2[z_1z_2+(z_1-z)(z_2+z)])\mathbf{x}_{13}]; \tag{27}$$

$$\frac{\mathrm{d}\sigma_6^T}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = -\frac{e^2g^2Q^2N_c^2z_1^{5/2}\sqrt{z_2}}{2(2\pi)^{10}}\int_0^{z_2}\frac{\mathrm{d}z}{z}\mathrm{d}^{10}\mathbf{x}[S_{132'1'}S_{23} - S_{13}S_{23} - S_{1'2'} + 1]$$
$$\times\frac{K_1(QX_6)K_1(|\mathbf{x}_{1'2'}|)}{X_6|\mathbf{x}_{1'2'}|\mathbf{x}_{32}^2}e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{-i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\mathbf{x}_{1'2'}\cdot[(z_1^2+z_2^2)(z_2^2+(z_2-z)^2)\mathbf{x}_{31}$$
$$+(z_2-z)[z_2(z_2^2+(z_2-z)^2)+z_1(z_1z_2+(z_2-z)(z_1+z))]\mathbf{x}_{23}]; \tag{28}$$

$$\frac{\mathrm{d}\sigma_7^T}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{e^2g^2Q^2N_c^2(z_1z_2)^{3/2}}{2(2\pi)^{10}}\int_0^{z_1}\frac{\mathrm{d}z(z_1-z)}{z}\mathrm{d}^{10}\mathbf{x}[S_{322'1'}S_{13} - S_{13}S_{23} - S_{1'2'} + 1]\frac{K_1(QX_5)K_1(|\mathbf{x}_{1'2'}|Q_1)}{X_5\mathbf{x}_{31}^2|\mathbf{x}_{1'2'}|}$$
$$\times\left[\frac{4\mathrm{Re}}{\mathbf{x}_{32}^2}\left\{(\mathbf{x}_{1'2'}\cdot\boldsymbol{\epsilon}^*)\left[\left(\mathbf{x}_{31}+\frac{z_2}{z_2+z}\mathbf{x}_{23}\right)\cdot\boldsymbol{\epsilon}\right]\left[\frac{z_2(z_1-z)}{z_1}(z_1^2+(z_2+z)^2)(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon}^*)\right.\right.\right.$$
$$\left.\left.\left.+(z_2+z)(z_2^2+(z_1-z)^2)(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)\right]\right\}-\frac{z_1z_2z}{z_2+z}\mathbf{x}_{31}\cdot\mathbf{x}_{1'2'}\right]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{-i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}; \tag{29}$$

$$\frac{\mathrm{d}\sigma_8^T}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{-e^2g^2Q^2N_c^2(z_1z_2)^{3/2}}{2(2\pi)^{10}}\int_0^{z_2}\frac{\mathrm{d}z(z_2-z)}{z}\mathrm{d}^{10}\mathbf{x}[S_{132'1'}S_{23} - S_{13}S_{23} - S_{1'2'} + 1]\frac{K_1(QX_6)K_1(|\mathbf{x}_{1'2'}|Q_1)}{X_6|\mathbf{x}_{1'2'}|\mathbf{x}_{32}^2}$$
$$\times\left[\frac{4\mathrm{Re}}{\mathbf{x}_{31}^2}\left\{(\mathbf{x}_{1'2'}\cdot\boldsymbol{\epsilon}^*)\left[\left(\mathbf{x}_{32}+\frac{z_1}{z_1+z}\mathbf{x}_{13}\right)\cdot\boldsymbol{\epsilon}\right]\left[(z_1+z)(z_1^2+(z_2-z)^2)(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon}^*)\right.\right.\right.$$
$$\left.\left.\left.+\frac{z_1(z_2-z)}{z_2}(z_2^2+(z_1+z)^2)(\mathbf{x}_{32}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{31}\cdot\boldsymbol{\epsilon}^*)\right]\right\}-\frac{z_1z_2z}{z_1+z}\mathbf{x}_{32}\cdot\mathbf{x}_{1'2'}\right]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{-i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}; \tag{30}$$

$$\frac{\mathrm{d}\sigma_9^T}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{-e^2g^2Q^2N_c^2(z_1z_2)^2(z_1^2+z_2^2)}{4(2\pi)^8}\int\mathrm{d}^8\mathbf{x}[S_{122'1'} - S_{12} - S_{1'2'} + 1]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2'}|Q_1)$$
$$\times e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\int_0^{z_1}\frac{\mathrm{d}z}{z}\left[\frac{z_1^2+(z_1-z)^2}{z_1^2}\right]\int\frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\frac{1}{(\mathbf{k}-\frac{z}{z_1}\mathbf{p})^2}; \tag{31}$$

$$\frac{\mathrm{d}\sigma_{10}^T}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{-e^2g^2Q^2N_c^2(z_1z_2)^2(z_1^2+z_2^2)}{4(2\pi)^8}\int\mathrm{d}^8\mathbf{x}[S_{122'1'} - S_{12} - S_{1'2'} + 1]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2'}|Q_1)$$
$$\times e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\int_0^{z_2}\frac{\mathrm{d}z}{z}\left[\frac{z_2^2+(z_2-z)^2}{z_2^2}\right]\int\frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\frac{1}{(\mathbf{k}-\frac{z}{z_2}\mathbf{q})^2}; \tag{32}$$

$$\frac{\mathrm{d}\sigma_{11}^T}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{ie^2g^2QN_c^2z_2^{3/2}\sqrt{z_1}(z_1^2+z_2^2)}{2(2\pi)^7}\int\mathrm{d}^8\mathbf{x}[S_{122'1'} - S_{12} - S_{1'2'} + 1]\frac{K_1(|\mathbf{x}_{1'2'}|Q_1)}{|\mathbf{x}_{1'2'}|}e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}$$
$$\times\int_0^{z_1}\frac{\mathrm{d}z}{z^2}\frac{[(z_1-z)^2+z_1^2]}{(z_1-z)}\int\frac{\mathrm{d}^2\mathbf{k}_2}{(2\pi)^2}\int\frac{\mathrm{d}^2\mathbf{k}_1}{(2\pi)^2}\frac{\mathbf{k}_1\cdot\mathbf{x}_{1'2'}e^{i\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{x}_2)}}{\left[\mathbf{k}_1^2+Q_1^2\right]\left[Q^2+\frac{\mathbf{k}_1^2}{z_1z_2}+\frac{z_1}{z(z_1-z)}\mathbf{k}_2^2\right]}; \tag{33}$$

$$\frac{\mathrm{d}\sigma^T_{12}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{-ie^2g^2QN_c^2z_1^{3/2}\sqrt{z_2}(z_1^2+z_2^2)}{2(2\pi)^7}\int \mathrm{d}^8\mathbf{x}[S_{122'1'}-S_{12}-S_{1'2'}+1]\frac{K_1(|\mathbf{x}_{1'2'}|Q_1)}{|\mathbf{x}_{1'2'}|}e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}$$
$$\times\int_0^{z_2}\frac{\mathrm{d}z}{z^2}\frac{[(z_2-z)^2+z_2^2]}{(z_2-z)}\int\frac{\mathrm{d}^2\mathbf{k}_2}{(2\pi)^2}\int\frac{\mathrm{d}^2\mathbf{k}_1}{(2\pi)^2}\frac{\mathbf{k}_1\cdot\mathbf{x}_{1'2'}e^{i\mathbf{k}_1\cdot(\mathbf{x}_2-\mathbf{x}_1)}}{\left[\mathbf{k}_1^2+Q_1^2\right]\left[Q^2+\frac{\mathbf{k}_1^2}{z_1z_2}+\frac{z_2}{z(z_2-z)}\mathbf{k}_2^2\right]}; \tag{34}$$

$$\frac{\mathrm{d}\sigma^T_{13(1)}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2}$$
$$=\frac{e^2g^2Q^2N_c^2(z_1z_2)^{3/2}}{2(2\pi)^8}\int_0^{z_2}\mathrm{d}z\sqrt{(z_1+z)(z_2-z)}\int\mathrm{d}^8\mathbf{x}[S_{12}S_{1'2'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}$$
$$\times\frac{K_1(|\mathbf{x}_{12}|Q\sqrt{(z_1+z)(z_2-z)})}{|\mathbf{x}_{12}|}\frac{K_1(|\mathbf{x}_{1'2'}|Q_1)}{|\mathbf{x}_{1'2'}|}\int\frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{x}_{21}}4\mathrm{Re}\left[(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{1'2'}\cdot\boldsymbol{\epsilon}^*)\left\{\frac{\frac{z_2(z_2-z)[z_1(z_1+z)+z_2(z_2-z)]}{2z}}{(z_2\mathbf{k}-z\mathbf{q})^2}\right.\right.$$
$$+z_1z_2z\left(\frac{z_1z\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1}-\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}^*}{z}\right)+z^2\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{z}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1}-\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}^*}{z}\right)-z_2z\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{z}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)-p\cdot q}{(z_2\mathbf{k}-z\mathbf{q})^2\left[\frac{(z_1\mathbf{k}-z\mathbf{p})^2}{z_1(z_1+z)}-\frac{(z_2\mathbf{k}-z\mathbf{q})^2}{z_2(z_2-z)}\right]}\right)$$
$$\left.\left.+z_2^2z(z_2-z)\left(\frac{z_1z\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1}-\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{z}\right)+z^2\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}^*}{z}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1}-\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{z}\right)-z_2z\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}^*}{z}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)-p\cdot q}{(z_1+z)(z_2\mathbf{k}-z\mathbf{q})^2\left[\frac{(z_1\mathbf{k}-z\mathbf{p})^2}{z_1(z_1+z)}-\frac{(z_2\mathbf{k}-z\mathbf{q})^2}{z_2(z_2-z)}\right]}\right)\right\}\right]; \tag{35}$$

$$\frac{\mathrm{d}\sigma^T_{13(2)}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2}$$
$$=\frac{e^2g^2Q^2N_c^2(z_1z_2)^{3/2}}{2(2\pi)^8}\int_0^{z_1}\mathrm{d}z\sqrt{(z_1-z)(z_2+z)}\int\mathrm{d}^8\mathbf{x}[S_{12}S_{1'2'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}$$
$$\times\frac{K_1(|\mathbf{x}_{12}|Q\sqrt{(z_1-z)(z_2+z)})}{|\mathbf{x}_{12}|}\frac{K_1(|\mathbf{x}_{1'2'}|Q_1)}{|\mathbf{x}_{1'2'}|}\int\frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{x}_{12}}4\mathrm{Re}\left[(\mathbf{x}_{12}\cdot\boldsymbol{\epsilon})(\mathbf{x}_{1'2'}\cdot\boldsymbol{\epsilon}^*)\left\{\frac{\frac{z_1(z_1-z)[z_1(z_1-z)+z_2(z_2+z)]}{2z}}{(z_1\mathbf{k}-z\mathbf{p})^2}\right.\right.$$
$$+z_1z_2z\left(\frac{z_1z\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1}-\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{z}\right)-z^2\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}^*}{z}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1}-\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{z}\right)-z_2z(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}^*}{z}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2})(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2})+p\cdot q}{(z_1\mathbf{k}-z\mathbf{p})^2\left[\frac{(z_1\mathbf{k}-z\mathbf{p})^2}{z_1(z_1-z)}-\frac{(z_2\mathbf{k}-z\mathbf{q})^2}{z_2(z_2+z)}\right]}\right)$$
$$\left.\left.+z_1^2z(z_1-z)\left(\frac{z_1z\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{z_1}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1}-\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}^*}{z}\right)-z^2\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{z}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1}-\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}^*}{z}\right)-z_2z\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{z}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}}{z_2}\right)\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}^*}{z_1}-\frac{\mathbf{q}\cdot\boldsymbol{\epsilon}^*}{z_2}\right)+p\cdot q}{(z_2+z)(z_1\mathbf{k}-z\mathbf{p})^2\left[\frac{(z_1\mathbf{k}-z\mathbf{p})^2}{z_1(z_1-z)}-\frac{(z_2\mathbf{k}-z\mathbf{q})^2}{z_2(z_2+z)}\right]}\right)\right\}\right]; \tag{36}$$

$$\frac{\mathrm{d}\sigma^T_{14(1)}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2}=\frac{-ie^2g^2QN_c^2(z_1z_2)^{3/2}}{2(2\pi)^7}\int_0^{z_1}\frac{\mathrm{d}z}{z}\mathrm{d}^8\mathbf{x}\frac{K_1(|\mathbf{x}_{1'2'}|Q_1)}{|\mathbf{x}_{1'2'}|}[S_{122'1'}-S_{1'2'}-S_{12}+1]e^{i(\mathbf{p}\cdot\mathbf{x}_{1'1}+\mathbf{q}\cdot\mathbf{x}_{2'2})}$$
$$\times\int\frac{\mathrm{d}^2\mathbf{k}_1}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{k}_2}{(2\pi)^2}e^{i\mathbf{k}_2\cdot\mathbf{x}_{12}}\left[\frac{[z_1(z_1-z)+z_2(z_2+z)-z(1-z)](\mathbf{k}_2\cdot\mathbf{x}_{1'2'})}{[\mathbf{k}_2^2+Q_1^2]\left[(\mathbf{k}_1-\frac{z_1-z}{z_1}\mathbf{k}_2)^2+\frac{z(z_1-z)}{z_2z_1^2}\mathbf{k}_2^2+\frac{z}{z_1}(z_1-z)Q^2\right]}\right.$$
$$+\frac{\frac{(z_1-z)}{z_1}(1+z^2-2z_2(z_1-z))(\mathbf{k}_1\cdot\mathbf{x}_{1'2'})}{[\mathbf{k}_1^2+(z_1-z)(z_2+z)Q^2]\left[(\mathbf{k}_1-\frac{z_1-z}{z_1}\mathbf{k}_2)^2+\frac{z(z_1-z)}{z_2z_1^2}\mathbf{k}_2^2+\frac{z}{z_1}(z_1-z)Q^2\right]}$$
$$\left.-\frac{Q^2\frac{(z_1-z)}{z_1}[2z_1z_2z(\mathbf{k}_1\cdot\mathbf{x}_{1'2'})+z(z+z_2-z_1)^2(\mathbf{k}_2\cdot\mathbf{x}_{1'2'})]}{[\mathbf{k}_1^2+(z_1-z)(z_2+z)Q^2][\mathbf{k}_2^2+Q_1^2]\left[(\mathbf{k}_1-\frac{z_1-z}{z_1}\mathbf{k}_2)^2+\frac{z(z_1-z)}{z_2z_1^2}\mathbf{k}_2^2+\frac{z}{z_1}(z_1-z)Q^2\right]}\right]; \tag{37}$$

$$\frac{\mathrm{d}\sigma^T_{14(2)}}{\mathrm{d}^2\mathbf{p}\mathrm{d}^2\mathbf{q}\mathrm{d}y_1\mathrm{d}y_2} = \frac{-ie^2g^2QN_c^2(z_1z_2)^{3/2}}{2(2\pi)^7}\int_0^{z_2}\frac{\mathrm{d}z}{z}\mathrm{d}^8\mathbf{x}\frac{K_1(|\mathbf{x}_{1'2'}|Q_1)}{|\mathbf{x}_{1'2'}|}[S_{122'1'} - S_{1'2'} - S_{12} + 1]e^{i(\mathbf{p}\cdot\mathbf{x}_{1'1}+\mathbf{q}\cdot\mathbf{x}_{2'2})}$$
$$\times\int\frac{\mathrm{d}^2\mathbf{k}_1}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{k}_2}{(2\pi)^2}e^{i\mathbf{k}_2\cdot\mathbf{x}_{12}}\left[\frac{[z_2(z_2-z)+z_1(z_1+z)-z(1-z)](\mathbf{k}_2\cdot\mathbf{x}_{1'2'})}{[\mathbf{k}_2^2+Q_1^2]\left[(\mathbf{k}_1-\frac{z_2-z}{z_2}\mathbf{k}_2)^2+\frac{z(z_2-z)}{z_1z_2^2}\mathbf{k}_2^2+\frac{z}{z_2}(z_2-z)Q^2\right]}\right.$$
$$+\frac{\frac{(z_2-z)}{z_2}(1+z^2-2z_1(z_2-z))(\mathbf{k}_1\cdot\mathbf{x}_{1'2'})}{[\mathbf{k}_1^2+(z_2-z)(z_1+z)Q^2]\left[(\mathbf{k}_1-\frac{z_2-z}{z_2}\mathbf{k}_2)^2+\frac{z(z_2-z)}{z_1z_2^2}\mathbf{k}_2^2+\frac{z}{z_2}(z_2-z)Q^2\right]}$$
$$\left.-\frac{Q^2\frac{(z_2-z)}{z_2}[2z_1z_2z(\mathbf{k}_1\cdot\mathbf{x}_{1'2'})+z(z+z_1-z_2)^2(\mathbf{k}_2\cdot\mathbf{x}_{1'2'})]}{[\mathbf{k}_1^2+(z_2-z)(z_1+z)Q^2][\mathbf{k}_2^2+Q_1^2]\left[(\mathbf{k}_1-\frac{z_2-z}{z_2}\mathbf{k}_2)^2+\frac{z(z_2-z)}{z_1z_2^2}\mathbf{k}_2^2+\frac{z}{z_2}(z_2-z)Q^2\right]}\right]. \tag{38}$$

These expressions constitute the full result for the one-loop corrections to the inclusive quark antiquark production cross section with transverse photon exchange. We have written these results all in terms of the dipole and quadrupole functions defined in Eq. (3) in the large N_c limit and ignored all subleading N_c terms.

We have also used the following notation for the coordinate dependence of some of the Bessel functions:

$$X = \sqrt{z_1z_2\mathbf{x}_{12}^2 + z_1z\mathbf{x}_{13}^2 + z_2z\mathbf{x}_{23}^2},$$
$$X_5 = \sqrt{z_2(z_1-z)\mathbf{x}_{12}^2 + z(z_1-z)\mathbf{x}_{13}^2 + z_2z\mathbf{x}_{23}^2},$$
$$X_6 = \sqrt{z_1(z_2-z)\mathbf{x}_{12}^2 + z_1z\mathbf{x}_{13}^2 + z(z_2-z)\mathbf{x}_{23}^2}. \tag{39}$$

Note that when $z\to 0$ these all become $|\mathbf{x}_{12}|\sqrt{z_1z_2}$. The primed version X' that appears in some real corrections is the same as X above but with $\mathbf{x}_1,\mathbf{x}_2\to\mathbf{x}_1',\mathbf{x}_2'$.

IV. DIVERGENCES

The above expressions are formal in the sense that they contain divergences that render them ill-defined unless regulated. As in the case of longitudinal exchange there are four types of divergences:

(i) Ultraviolet (UV) divergences when loop momentum $\mathbf{k}\to\infty$ or equivalently in coordinate space, when the transverse coordinate of the radiated gluon approaches the transverse coordinate $\mathbf{x}_i$ of either quark or antiquark when integrated, i.e., $\mathbf{x}_3\to\mathbf{x}_i$ such that $|\mathbf{x}_3-\mathbf{x}_i|\to 0$. The UV structure of the production cross section with transverse photon exchange is identical to that of longitudinal photon exchange so that cancellations are identical, i.e.,

$$[\mathrm{d}\sigma_5+\mathrm{d}\sigma_{11}]_{\mathrm{UV}} = 0,$$
$$[\mathrm{d}\sigma_6+\mathrm{d}\sigma_{12}]_{\mathrm{UV}} = 0,$$
$$[\mathrm{d}\sigma_9+\mathrm{d}\sigma_{10}+\mathrm{d}\sigma_{14(1)}+\mathrm{d}\sigma_{14(2)}]_{\mathrm{UV}} = 0, \tag{40}$$

with the rest of the contributions being UV finite.

(ii) Soft divergences when $k^\mu\to 0$, which in this context corresponds to *both* transverse momentum in the loop $\mathbf{k}$ *and* the radiated gluon momentum fraction z go to zero simultaneously, $\mathbf{k},z\to 0$. Both the real and virtual corrections contain soft divergences; however, all soft divergences cancel between real and virtual corrections as shown below,

$$[\mathrm{d}\sigma_{1\times1}+2\mathrm{d}\sigma_9]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_{2\times2}+2\mathrm{d}\sigma_{10}]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_{1\times2}+\mathrm{d}\sigma_{13(1)}+\mathrm{d}\sigma_{13(2)}]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_{3\times3}+\mathrm{d}\sigma_{4\times4}+2\mathrm{d}\sigma_{3\times4}]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_{1\times3}+\mathrm{d}\sigma_{1\times4}]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_{2\times3}+\mathrm{d}\sigma_{2\times4}]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_5+\mathrm{d}\sigma_7]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_6+\mathrm{d}\sigma_8]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_{11}+\mathrm{d}\sigma_{14(1)}]_{\mathrm{soft}} = 0,$$
$$[\mathrm{d}\sigma_{12}+\mathrm{d}\sigma_{14(2)}]_{\mathrm{soft}} = 0. \tag{41}$$

(iii) Collinear divergences when the radiated gluon momentum becomes parallel to either quark or antiquark momentum at *finite* $\mathbf{k}$ and z. They are present in diagrams $i\mathcal{A}_1$, $i\mathcal{A}_2$ (real corrections) and in $i\mathcal{A}_9$, $i\mathcal{A}_{10}$ (virtual corrections). These collinear divergences are absorbed into quark-hadron and antiquark-hadron fragmentation functions which make the fragmentation functions scale dependent, for example,

$$D_{h_1/q}(z_{h_1},\mu^2) = \int_{z_{h_1}}^1 \frac{\mathrm{d}\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h_1}}{\xi}\right)\left[\delta(1-\xi) + \frac{\alpha_s}{2\pi} P_{qq}(\xi)\log\left(\frac{\mu^2}{\Lambda^2}\right)\right], \tag{42}$$

defined using a cutoff scheme or

$$D_{h_1/q}(z_{h_1},\mu^2) = \int_{z_{h_1}}^1 \frac{\mathrm{d}\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h_1}}{\xi}\right)\left[\delta(1-\xi) + \frac{\alpha_s}{\pi} P_{qq}(\xi)\left(\frac{1}{\epsilon} - \log(\pi e^{\gamma_E}\mu|\mathbf{x}_1' - \mathbf{x}_1|)\right)\right], \tag{43}$$

when using a dimensional regularization scheme. We refer the reader to [53] for full details.

(iv) Rapidity divergences when the momentum fraction z of the gluon goes to zero while the transverse momentum $\mathbf{k}$ of the gluon remains finite. These are handled by introducing a longitudinal momentum fraction factorization scale z_f and dividing the z integration into two regions: $z > z_f$ and $z < z_f$,

$$\int_0^1 \frac{\mathrm{d}z}{z} f(z) = \left\{\int_0^{z_f} \frac{\mathrm{d}z}{z} + \int_{z_f}^1 \frac{\mathrm{d}z}{z}\right\} f(z). \tag{44}$$

The rapidity divergences are present only in the first term above and lead to evolution (renormalization) the dipoles and quadrupoles according to the Balitsky-Kovchegov (BK) and Jalilian-Marian, Iancu, McLerran, Wiegert, Leonidov, Kovner (JIMWLK) evolution equations [61–69]. The second term contains no rapidity divergences, it is completely finite, and it is part of the next-to-leading order corrections.

Our final result for the regulated dihadron production cross section can then be symbolically written as the sum of several terms [Eq. (45)] as shown below:

$$\mathrm{d}\sigma^{\gamma^* A\to h_1 h_2 X} = \mathrm{d}\sigma_{\mathrm{LO}} \otimes \mathrm{JIMWLK} + \mathrm{d}\sigma_{\mathrm{LO}} \otimes D_{h_1/q}(z_{h_1},\mu^2) \otimes D_{h_2/\bar{q}}(z_{h_2},\mu^2) + \mathrm{d}\sigma^{\mathrm{finite}}_{\mathrm{NLO}}. \tag{45}$$

The first term contains the z integration region below z_f where the leading order cross section is evolved with the BK/JIMWLK evolution equations. The second term includes the integration region $z > z_f$ where the leading order cross section is convoluted with the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolved fragmentation functions for both quark and antiquark. Finally the last term constitutes all the remaining contributions to the NLO cross section which is finite. The presence of the bare fragmentation functions in the first and last terms is implied.

It looks as if the limit $z \to 0$ will lead to a divergence. However, in the physical cross section the lowest possible value of z is set by the kinematics of the observed particles (in this case dihadrons). As a simpler example, in single inclusive hadron production in proton-proton collisions the lowest x that contributes to the physical cross section is cut off from below by a combination of the transverse momentum and rapidity of the produced particle and center of mass energy of the collision. An analogous relation for the z integral in Eq. (45) holds. The cross section is also sensitive to the chosen z_f cutoff. This is an artifact of perturbative calculation at finite order, and one expects that inclusion of higher order corrections will generally reduce sensitivity to the variation of z_f. The same is true for the renormalization scale μ^2 appearing in the evolved fragmentation functions: dependence on μ^2 decreases when higher order corrections are included. In principle, these two scales (z_f, μ^2) are independent, and so the choice for one cannot affect the other.

In summary, we have calculated the one-loop corrections to inclusive quark antiquark production in DIS at small x for transverse photons. We have shown the production cross section factorizes: all divergences that appear at the one-loop level are either canceled or absorbed into JIMWLK evolution of dipoles and quadrupoles, and into DGLAP evolution of parton-hadron fragmentation functions. These results are well suited for further phenomenological studies of angular correlations of the dihadrons produced in DIS at small x [52].

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