Relativistic corrections to $J/\psi \to p\bar{p}$ decay

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We study relativistic corrections to the exclusive decay of J/ψ into proton-antiproton final state. We calculate the relativistic QCD corrections to the dominant decay amplitude, which depend on the nucleon twist-3 light-cone distribution amplitudes only. It is shown that in this case the collinear factorization is also valid beyond the leading-order approximation. Our numerical estimates show that relativistic correction of relative order v^2 provides a large numerical impact.

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I. INTRODUCTION

In recent years, the BESII and BESIII Collaborations have obtained many accurate data on baryon-antibaryon decays of S-wave charmonia; see, e.g., Refs. [1–8]. These data allow one to get information about various important hadronic parameters and provide an interesting possibility to study QCD dynamics.

There are different approaches for a description of the baryon-antibaryon decays. In Refs. [9,10], it was proposed to use a phenomenological model based on an empirical effective Lagrangian with unknown parameters, which are subject to restrictions from $SU(3)$ flavor symmetry and experimental data.

Another way is to use effective field theory framework, combining nonrelativistic expansion and collinear factorization [11–14]. The appropriate effective Lagrangian is described by nonrelativistic QCD (NRQCD) [15,16] and collinear copies of QCD [11,12,14].

Such an approach allows one to build a systematic expansion in powers of $1/m_c$ describing decay amplitudes as a superposition of perturbative and universal nonperturbative functions. The perturbative part can be computed in perturbative QCD at the scale of order m_c. The nonperturbative matrix elements are defined at soft hadronic scale $\Lambda \ll m_c$. They can be limited to other processes or evaluated using nonperturbative methods such as QCD sum rules or lattice QCD.

Since the real mass of the charm quark is not large enough, a systematic description requires careful analysis of the various corrections. Existing calculations of exclusive and inclusive reactions show that in many cases radiative corrections in $\alpha_s(m_c)$ and relativistic corrections can have a significant numerical effect; see, for instance, reviews [17,18].

The first estimates for exclusive proton-antiproton decay based on the leading-order (LO) approximation were obtained already a long time ago in Refs. [11,14]. Recently, this analysis has been extended in Refs. [19–21], where the power-law corrections $\sim\Lambda/m_c$ associated with the collinear expansion were taken into account for different amplitudes. The collinear power corrections are associated with the higher twist baryon light-cone distribution amplitudes (LCDAs). It has been established that the effect of such corrections is not very strong, and one can describe various channels of baryon-antibaryon decay with good accuracy. However, relativistic and QCD radiative corrections for baryon-antibaryon decays have not yet been studied.

The available data for various baryons indicate the possible presence of effects from relativistic corrections. For example, the so-called "12% rule"

$$Q_B = \frac{Br[\psi(2S) \to B\bar{B}]}{Br[J/\psi \to B\bar{B}]} \simeq \frac{Br[\psi(2S) \to e^+e^-]}{Br[J/\psi \to e^+e^-]} \simeq 0.13 \qquad (1)$$

must hold if the width is dominated by the LO NRQCD approximation. The available experimental data [22] show the following results: $Q_p = 0.1386(3)$, $Q_n = 0.146(1)$, $Q_\Lambda = 0.204(1)$, $Q_{\Sigma^0} = 0.21(3)$, $Q_{\Sigma^+} = 0.072(5)$, and $Q_\Xi^+ = 0.276(5)$. Consequently, the data for baryons other than the nucleon indicate a violation of this expectation.

For nucleon-antinucleon decays, the rule (1) works quite well, but, on the other hand, the values of the polar angular distribution coefficient α_p for ground and excited S-wave charmonium $[\psi(2S) \equiv \psi']$ are very different [3,7]:

$$\alpha_p|_{J/\psi} = 0.595 \pm 0.012, \qquad \alpha_p|_{\psi'} = 1.03 \pm 0.06. \qquad (2)$$

This observation may also indicate a significant contribution of relativistic corrections.

The purpose of this work is to study the effects of relativistic corrections in proton-antiproton decays. The technique for calculating relativistic corrections is well known. Such corrections were already studied for different exclusive decays as $J/\psi \to e^+e^-$ [23,24], $H \to \gamma + J/\psi$ [25,26], and $\chi_{cJ} \to \gamma\gamma$ [27,28] and for exclusive production $e^+e^- \to J/\psi + \eta_c$ [29,30]. However, relativistic corrections to exclusive hadronic decays have not yet been considered.

The feature of hadronic decays is that the hard kernels are not simple numbers but depend on light quark momentum fractions. The corresponding decay amplitude is described by the convolution integral over the momentum fractions of the hard kernel with LCDAs. In this case, it is more convenient to perform the hard matching at the amplitude level. In some cases, the convolution integrals have infrared divergencies that indicate violation of collinear factorization. This is often the case for next-to-leading power contributions in exclusive processes. However, for the dominant amplitude describing $n^3S_1 \to B\bar{B}$ decay, collinear factorization is still applicable, and this makes it possible to systematically study the relativistic effects.

Our paper is organized as follows. In Sec. II, we give important definitions and kinematical notation. In Sec. III, we calculate the hard kernels and perform the resummation of some class of relativistic effects. After that, we provide a qualitative numerical analysis for different choices of the nucleon LCDAs. In Sec. IV, we discuss the obtained results and make conclusions. In the appendixes, we give important details about the nucleon twist-3 LCDAs and provide analytical expressions for the hard kernels.

II. KINEMATICS AND DECAY AMPLITUDES

In this work, we use notation from Ref. [21]. We define the kinematics of $J/\psi(P) \to p(k)\bar{p}(k')$ decay in the charmonium rest frame

$$P = M_\psi \omega, \qquad \omega = (1, \vec{0}). \tag{3}$$

The outgoing momenta k and k' are directed along the z axis and read, respectively,

$$k = (M_\psi/2, 0, 0, M_\psi\beta/2), \qquad k' = (M_\psi/2, 0, 0, -M_\psi\beta/2), \tag{4}$$

where m_N is the nucleon mass and $\beta = \sqrt{1 - 4m_N^2/M_\psi^2}$. We also define auxiliary light-cone vectors

$$n = (1, 0, 0, -1), \qquad \bar{n} = (1, 0, 0, 1), \tag{5}$$

so that any four-vector V can be represented as

$$V = V_+ \frac{\bar{n}}{2} + V_- \frac{n}{2} + V_\perp, \tag{6}$$

where $V_+ \equiv (V \cdot n) = V_0 + V_3$ and $V_- \equiv (V \cdot \bar{n}) = V_0 - V_3$.

The decay amplitude $J/\psi \to p\bar{p}$ is defined as

$$\langle k, k'|i\hat{T}|P\rangle = (2\pi)^4 \delta(P - k - k') iM, \tag{7}$$

with

$$M = \bar{N}(k) \left\{ A_1 \not{\epsilon}_\psi + A_2 (\epsilon_\psi)_\mu (k' + k)_\nu \frac{i\sigma^{\mu\nu}}{2m_N} \right\} V(k'), \tag{8}$$

where $\not{p} = p_\mu \gamma^\mu$. The nucleon $\bar{N}(k)$ and antinucleon $V(k')$ spinors have standard normalization $\bar{N}N = 2m_N$ and $\bar{V}V = -2m_N$. The charmonium polarization vector $\epsilon_\psi^\mu \equiv \epsilon_\psi^\mu(P, \lambda)$ satisfies

$$\sum_\lambda \epsilon_\psi^\mu(P, \lambda) \epsilon_\psi^\nu(P, \lambda) = -g^{\mu\nu} + \frac{P^\mu P^\nu}{M_\psi^2}. \tag{9}$$

The scalar amplitudes A_1 and A_2 describe the decay process. These amplitudes are computed in the effective field framework by expansion with respect to small relative heavy quark velocity v and small ratio $\lambda^2 \sim \Lambda/m_Q$. The amplitude A_2 is suppressed as

$$A_2/A_1 \sim \lambda^2. \tag{10}$$

The decay width is conveniently described in terms of two combinations

$$\mathcal{G}_M = A_1 + A_2, \qquad \mathcal{G}_E = A_1 + \frac{M_\psi^2}{4m_N^2} A_2 \tag{11}$$

and reads

$$\Gamma[J/\psi \to p\bar{p}] = \frac{M_\psi \beta}{12\pi} \left(|\mathcal{G}_M|^2 + \frac{2m_N^2}{M_\psi^2} |\mathcal{G}_E|^2 \right). \tag{12}$$

The ratio $|\mathcal{G}_E|/|\mathcal{G}_M|$ can be measured through the angular behavior of the cross section $e^+e^- \to J/\psi \to p\bar{p}$; see, e.g., Ref. [31]. Available data indicate that the contribution with the amplitude $\mathcal{G}_E$ is quite small (about 10%) and the value of width is dominated by the amplitude $|\mathcal{G}_M|^2$.

To the leading-order approximation

$$\mathcal{G}_M \simeq A_1^{\mathrm{lo}} + \mathcal{O}(\alpha_s) + \mathcal{O}(v^2) + \mathcal{O}(\lambda^2), \tag{13}$$

where in the rhs we indicate various possible corrections. These are next-to-leading-order (NLO) contributions with respect to α_s, relativistic corrections of relative order v^2, and power corrections associated with the higher twist light-cone distribution amplitudes, respectively. The NLO

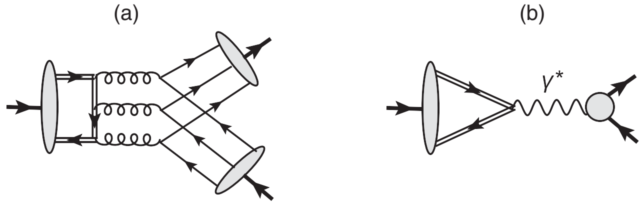

FIG. 1. Hadronic (a) and electromagnetic (b) contributions to the decay amplitudes.

radiative QCD corrections and relativistic corrections are not known yet. The power-suppressed contribution have been estimated in Ref. [21], and the corresponding numerical effect is moderate, about 25%. In this paper, we want to study the relativistic corrections to the amplitude A_1^{lo}.

The amplitude A_1^{lo} is described by the sum of two different contributions: the hard and the electromagnetic one, which correspond to the annihilation into three gluons or one hard photon as shown in Fig. 1. The electromagnetic amplitude is relatively small; however, it can provide numerical impact through interference with the QCD amplitude [21]. In this paper, for simplicity, the electromagnetic contribution is not considered.

The leading-order factorization formula for the amplitude A_1^{lo} reads [11,12]

$$A_1^{(0)} = \frac{\sqrt{2M_\psi}\langle 0|O|J/\psi\rangle}{m_c^2}\frac{f_N^2}{m_c^4}(\pi\alpha_s)^3\frac{10}{81}J_0, \qquad (14)$$

where the NRQCD matrix element is defined in the standard way:

$$\langle 0|O|J/\psi\rangle = \langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\psi|J/\psi(\lambda)\rangle = \sqrt{\frac{N_c}{2\pi}}R_{10}(0), \quad (15)$$

where $R_{10}(0)$ is the radial charmonium wave function at the origin. The constant f_N has the dimension of the square of the mass and describes the normalization of the matrix element of the nucleon. The decay amplitude is sensitive to nucleon structure that is described by the dimensionless collinear convolution integral J_0, which reads

$$J_0 = \frac{1}{4}\int Dx_i \int Dy_i \frac{1}{x_1x_2x_3}\frac{1}{y_1y_2y_3}\frac{y_1x_3}{D_1D_3} \times \left\{\varphi_3(y_{123})\varphi_3(x_{123}) + \frac{1}{2}(\varphi_3(y_{123}) + \varphi_3(y_{321}))(\varphi_3(x_{123}) + \varphi_3(x_{321}))\right\}, \qquad (16)$$

where $D_i = x_i + y_i - 2x_iy_i$ and we used short notation for the twist-3 nucleon LCDA $\varphi_3(x_{123}) \equiv \varphi_3(x_1, x_2, x_3)$. This nonperturbative function depends on the quark momentum fractions $0 < x_i < 1$ and the factorization scale, which is not shown explicitly. The light-cone fractions x_i satisfy the momentum conservation condition $x_1 + x_2 + x_3 = 1$. Therefore, the measure of the convolution integrals in Eq. (16) includes the δ function

$$Dx_i = dx_1dx_2dx_3\delta(1 - x_1 - x_2 - x_3). \qquad (17)$$

The nucleon LCDA φ_3 is defined as the matrix element of a three-quark operator and is well known in the literature; see, e.g., Refs. [12,42]. For a convenience, the definition and some important details are given in Appendix A.

Our task is to calculate the relativistic corrections associated with the higher-order matrix elements in NRQCD:

$$\langle 0|O_n|J/\psi\rangle = \langle 0|\chi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^{2n}\psi|J/\psi(\lambda)\rangle, \qquad (18)$$

where $\langle 0|O_0|J/\psi\rangle \equiv \langle 0|O|J/\psi\rangle$. Usually, it is convenient to describe the contribution of these matrix elements by introducing the ratio

$$\langle \boldsymbol{q}^{2n}\rangle = m_c^2\langle v^{2n}\rangle = \frac{\langle 0|O_n|J/\psi\rangle}{\langle 0|O_0|J/\psi\rangle}. \qquad (19)$$

The set of the operators in Eq. (18) does not represent the complete set of all possible NRQCD operators. The operator O_2 provides the correct description of the relativistic correction of relative order v^2. But, to describe relativistic corrections of a higher order, many more different operators are used, for example, color-octet ones. However, resumming the contributions of all orders associated with the subset (19) is useful, because it allows one to study the convergence of the nonrelativistic expansion.

The value of the NLO matrix element (19) can be exactly found with the help of Kapustin-Gremm relation [33]

$$\langle 0|O_1|J/\psi\rangle = m_cE_{J/\psi}\langle 0|O_0|J/\psi\rangle, \qquad (20)$$

where $E_{J/\psi}$ denotes the binding energy. This gives

$$\langle \boldsymbol{q}^2\rangle = m_cE_{J/\psi}. \qquad (21)$$

In Ref. [34], it was found that the higher-order operators satisfy the approximate relation

$$\langle \boldsymbol{q}^{2n}\rangle = \langle \boldsymbol{q}^2\rangle^n + \mathcal{O}(v^2). \qquad (22)$$

Neglecting the higher-order corrections in the rhs in Eq. (22) allows one to resum the power series of the higher-order terms with the matrix elements (18) to all orders. Such resummation includes those relativistic corrections that are contained in the quark-antiquark quarkonium wave function in the leading potential model for the wave function [30].

The resummation of the relativistic corrections associated with the matrix elements (19) is already considered for various processes in Refs. [23,24,30], and this technique can be easily adapted for calculations of exclusive hadronic decays.

III. MATCHING AND ALL-ORDER RESUMMATION

In order to calculate the decay amplitude, we use the covariant spin-projector technique [35]. The generalization of this technique to higher orders in v^2 is considered in Refs. [23,24]. Here, we briefly describe the NRQCD matching and clarify some features of the present calculation.

Our task is to compute the hard coefficient functions in front of higher-order operators (18). The matching can be conveniently done using the $Q\bar{Q}$ state as the initial state instead of the charmonium state. The corresponding heavy quark and antiquark have the momenta p and p', respectively:

$$p = (E, \boldsymbol{q}), \qquad p' = (E, -\boldsymbol{q}), \qquad E = \sqrt{m_Q^2 + \boldsymbol{q}^2}. \quad (23)$$

Then the total and relative momenta read, respectively,

$$P = p + p', \qquad q = \frac{1}{2}(p - p'). \quad (24)$$

In the rest frame,

$$P = (2E, \boldsymbol{0}), \qquad q = (0, \boldsymbol{q}). \quad (25)$$

Let the perturbative amplitude be described as

$$A_{QQ} = \bar{v}_Q(p', s'), \qquad \hat{A}_{QQ} u_Q(p, s), \quad (26)$$

where $\bar{v}_Q$ and u_Q denote heavy quark spinors. The amplitude projected onto the S-wave state of heavy quark-antiquark $Q\bar{Q}(^3S_1)$ reads

$$A_{QQ}(^3S_1) = \mathrm{Tr}[\Pi_1 \hat{A}_{QQ}(x_i, y_i)], \quad (27)$$

where the spin-triplet projector Π_1 is given by [23]

$$\Pi_1 = \frac{-1}{2\sqrt{2}E(E + m_Q)}\left(\frac{1}{2}\not{P} + \not{q} + m_Q\right) \times \frac{\not{P} + 2E}{4E}\not{\epsilon}\left(\frac{1}{2}\not{P} - \not{q} - m_Q\right) \otimes \frac{\mathbf{1}}{\sqrt{N_c}}. \quad (28)$$

This projector is relativistically normalized:

$$\mathrm{Tr}[\Pi_1 \Pi_1^\dagger] = 4E^2. \quad (29)$$

In order to project the amplitude onto a state with $L = 0$, the expression in Eq. (27) must be averaged over the angles of the momentum $\boldsymbol{q}$:

$$A_{QQ}(\boldsymbol{q}^2)|_{^3S_1, L=0} = \bar{A}_{QQ}(\boldsymbol{q}^2) = \frac{1}{4\pi}\int d\Omega \mathrm{Tr}[\Pi_1 \hat{A}_{QQ}(x_i, y_i)]. \quad (30)$$

In NRQCD, the amplitude reads

$$A_{\mathrm{NRQCD}} = \sqrt{2M_\psi}\sum_{n\geq 0} c_n \langle 0|O_n|J/\psi\rangle, \quad (31)$$

where $\sqrt{2M_\psi}$ arises from relativistic normalization. The coefficients c_n can be obtained from the amplitude (30) using that

$$\langle 0|O_n|Q\bar{Q}\rangle = \sqrt{2N_c}2E\boldsymbol{q}^{2n} \quad (32)$$

and

$$\bar{A}_{QQ}(\boldsymbol{q}^2) = \sum_n c_n \langle 0|O_n|Q\bar{Q}\rangle = \sum_n c_n \sqrt{2N_c}2E\boldsymbol{q}^{2n}. \quad (33)$$

Using this, one finds

$$c_n = \frac{1}{\sqrt{2N_c}}\frac{1}{n!}\frac{\partial}{\partial \boldsymbol{q}^{2n}}\frac{\bar{A}_{QQ}(\boldsymbol{q}^2)}{2E}. \quad (34)$$

Substituting this into Eq. (31) and using Eq. (22), one finds

$$A_{\mathrm{NRQCD}} \simeq \sqrt{2M_\psi}\langle 0|O|J/\psi\rangle \sum_n \frac{1}{n!}\langle \boldsymbol{q}^2\rangle^n \frac{\partial^n}{\partial \boldsymbol{q}^{2n}}\frac{\bar{A}_{QQ}(^3S_1)}{\sqrt{2N_c}2E} \quad (35)$$

$$= \sqrt{2M_\psi}\langle 0|O|J/\psi\rangle \frac{\bar{A}_{QQ}(\langle \boldsymbol{q}^2\rangle)}{\sqrt{2N_c}2E}. \quad (36)$$

The specific of our calculation is that the amplitude $\bar{A}_{QQ}(^3S_1)$ also includes the nonperturbative matrix elements describing the couplings with the nucleons. The amplitude $Q\bar{Q} \to p\bar{p}$ is described by the diagrams in Fig. 2. Each diagram can be described as consisting of a heavy quark subdiagram Γ_Q describing the $Q\bar{Q}$ annihilation into three gluons and a residual part describing the transition of three virtual gluons to the $p\bar{p}$ state. The expression for each diagram can be written in the following form:

$$A_{QQ}^{icj} = \int Dx_i \int Dy_i \Delta_1^{\mu_1\mu_1'}\Delta_2^{\mu_2\mu_2'}\Delta_3^{\mu_3\mu_3'} B_{\mu_1\mu_2\mu_3}(x_i, y_i) \times \bar{v}_Q(p', s')\Gamma_Q^{\mu_i'\mu_c'\mu_j'} u_Q(p, s), \quad (37)$$

where $\bar{v}_Q$ and u_Q denote heavy quark spinors and $\Delta_i^{\mu_i\mu_i'}$ denotes the gluon propagator (we use Feynman gauge and do not write the color indices for simplicity)

$$\Delta_i^{\mu_i\mu_i'} = \frac{(-i)g^{\mu_i\mu_i'}}{(k_i + k_i')^2}, \qquad k_i = x_i k, \qquad k_i' = y_i k', \tag{38}$$

where the proton and antiproton momenta read

$$k \simeq En, \qquad k' \simeq E\bar{n}, \qquad k^2 = k'^2 = 0. \tag{39}$$

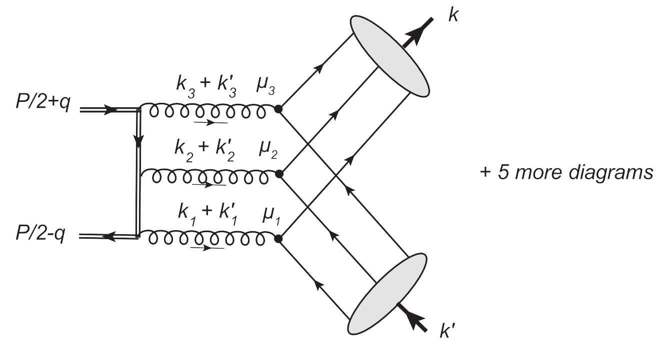

FIG. 2. The diagrams, which describe the $Q\bar{Q} \to p\bar{p}$ subprocess.

The heavy quark subdiagram reads

$$\Gamma_Q^{\mu_i'\mu_c'\mu_j'}(x_i, y_i) = i^2 (ig)^3 \frac{\gamma^{\mu_i}(-\not{P}/2 + \not{q} + \not{k}_i + \not{k}_i' + m_Q)\gamma^{\mu_c}(\not{P}/2 + \not{q} - \not{k}_j - \not{k}_j' + m_Q)\gamma^{\mu_j}}{[(k_i + k_i' - P/2 + q)^2 - m_Q^2][(P/2 + q - k_j - k_j')^2 - m_Q^2]}. \tag{40}$$

The set of indices $\{\mu_i, \mu_c, \mu_j\}$ describes six possible transpositions of the indices $\{\mu_1, \mu_2, \mu_3\}$, which correspond to the different diagrams: $\{\mu_1, \mu_2, \mu_3\}$, $\{\mu_3, \mu_2, \mu_1\}$, $\{\mu_2, \mu_1, \mu_3\}$, $\{\mu_3, \mu_1, \mu_2\}$, $\{\mu_1, \mu_3, \mu_2\}$, and $\{\mu_2, \mu_3, \mu_1\}$. Therefore, each diagram can be coded by the set icj, and the sum of these diagrams gives the total result.

The term $B_{\mu_1\mu_2\mu_3}(x_i, y_i)$ in Eq. (37) describes the trace, which occurs after contractions of the Dirac indices α_i and α_i' of the proton and antiproton light-cone matrix elements with the γ matrices of the quark-gluon vertices

$$\langle 0|[O_{\mathrm{tw3}}]_{\alpha_1'\alpha_2'\alpha_3'}|\bar{p}(k')\rangle [(ig)\gamma_{\mu_1}]_{\alpha_1'\alpha_1}[(ig)\gamma_{\mu_2}]_{\alpha_2'\alpha_2}[(ig)\gamma_{\mu_3}]_{\alpha_3'\alpha_3} \times \langle 0|[O_{\mathrm{tw3}}]_{\alpha_1\alpha_2\alpha_3}|p(k)\rangle \tag{41}$$

$$= \mathrm{FT}[B_{\mu_1\mu_2\mu_3\rho}(x_i, y_i)]. \tag{42}$$

The symbol "FT" denotes Fourier transformations, which occur from the light-cone matrix elements (the definition of the light-cone operator O_{tw3} is given in Appendix A). The nucleon matrix elements are described by the convenient combinations of LCDAs V_1,A_1, and T_1, which are defined as

$$V_1(x_{123}) = \frac{1}{2}(\varphi_3(x_{123}) + \varphi_3(x_{213})), \tag{43}$$

$$A_1(x_{123}) = \frac{1}{2}(\varphi_3(x_{123}) - \varphi_3(x_{213})), \tag{44}$$

$$T_1(x_{123}) = \frac{1}{2}(\varphi_3(x_{132}) + \varphi_3(x_{231})). \tag{45}$$

The B term (42) is universal for all diagrams. The calculation of the Dirac contractions yields

$$B_{\mu_1\mu_2\mu_3}(x_i, y_i) = \bar{N}_{\bar{n}}\gamma_\perp^\rho V_n (ig)^3 \frac{1}{4}(k\cdot k') T_{\mu_1\mu_2\mu_3\rho}(x_i, y_i), \tag{46}$$

with

$$\begin{aligned} T_{\mu_1\mu_2\mu_3\rho}(x_i, y_i) &= g^\perp_{\mu_1\mu_2} g^\perp_{\rho\mu_3}\{V_1(x_{123})V_1(y_{123}) + A_1(x_{123})A_1(y_{123})\} \\ &\quad - i\varepsilon^\perp_{\mu_1\mu_2} i\varepsilon^\perp_{\mu_3\rho}\{V_1(y_{123})A_1(x_{123}) + A_1(y_{123})V_1(x_{123})\} \\ &\quad + G^\perp_{\mu_1\mu_2\alpha\sigma} G^{\perp\alpha\sigma}_{\rho\mu_3} T_1(y_{123})T_1(x_{123}), \end{aligned} \tag{47}$$

where we used the following short notations:

$$g^\perp_{\alpha\beta} = g_{\alpha\beta} - \frac{1}{2}(n_\alpha\bar{n}_\beta + n_\beta\bar{n}_\alpha), \qquad i\varepsilon^\perp_{\alpha\beta} = \frac{1}{2} i\varepsilon_{\alpha\beta\sigma\lambda} n^\sigma \bar{n}^\lambda, \tag{48}$$

$$G^\perp_{\alpha\beta\sigma\rho} = g^\perp_{\alpha\sigma} g^\perp_{\beta\rho} + g^\perp_{\alpha\rho} g^\perp_{\beta\sigma} - g^\perp_{\sigma\rho} g^\perp_{\alpha\beta}. \tag{49}$$

For the projected nucleon spinors $\bar{N}(k)$ and $V(k')$, we used

$$\bar{N}(k)\frac{\not{n}\not{\bar{n}}}{4} = \bar{N}_{\bar{n}}, \qquad \frac{\not{n}\not{\bar{n}}}{4} V(k') = V_n. \tag{50}$$

Therefore, the expression for the amplitude (30) reads

$$i\bar{A}_{QQ}(\boldsymbol{q}^2) = \frac{1}{2}\frac{1}{4\pi}\int d\Omega \sum_{\{icj\}} \int Dx_i \times \int Dy_i T_{\mu_1\mu_2\mu_3} \Delta_1^{\mu_1\mu_1'} \Delta_2^{\mu_2\mu_2'} \Delta_3^{\mu_3\mu_3'} \mathrm{Tr}\left[\Pi_1 \hat{A}_{QQ}^{\mu_i'\mu_c'\mu_j'}\right], \tag{51}$$

where we also show explicitly the symmetry factor 1/2. Calculating the trace and contracting the indices, we obtain the following result[1]:

$$\bar{A}_{QQ}(\boldsymbol{q}^2) = \sqrt{2N_c}\frac{2}{m_Q}\frac{f_N^2}{(m_Q^2+\boldsymbol{q}^2)^2}(\pi\alpha_s)^3 \times \frac{10}{81}\frac{m_Q+E}{2m_Q}J(\boldsymbol{q}^2/m_Q^2), \tag{52}$$

where the dimensionless collinear integral reads

$$J(\boldsymbol{v}^2) = \frac{1}{32}\frac{1}{f_N^2}\int \frac{Dx_i}{x_1x_2x_3}\int Dy_i\frac{1}{y_1y_2y_3} \times \left\{\frac{A(x_i,y_i)}{D_1D_3}+\frac{B(x_i,y_i)}{D_1D_2}+\frac{C(x_i,y_i)}{D_2D_3}\right\}. \tag{53}$$

The integrand in Eq. (53) has the following structure:

$$D_i = x_i + y_i - 2x_iy_i = x_i(1-y_i) + y_i(1-x_i) \geq 0. \tag{54}$$

The functions A, B, and C also depend on $\boldsymbol{v}^2 = \boldsymbol{q}^2/m_Q^2$, which is not explicitly shown for simplicity. They can be presented in the following way:

$$A(x_{123},y_{123}) = \sum_{i=0}^{4} A_k(x_{123},y_{123})I_k(1,3), \tag{55}$$

$$B(x_{123},y_{123}) = \sum_{i=0}^{4} B_k(x_{123},y_{123})I_k(1,2), \tag{56}$$

$$C(x_{123},y_{123}) = \sum_{i=0}^{4} C_k(x_{123},y_{123})I_k(2,3). \tag{57}$$

These expressions include the angular integrals I_k which appear due to the integration over $d\Omega = d\varphi d\cos\theta$. The polar integration can be easily computed, and the remaining azimuth integrals read

$$I_k[ij] = \frac{1}{2}\int_{-1}^{1} d\eta \frac{|\boldsymbol{v}|^k\eta^k}{[1+|\boldsymbol{v}|a_i\eta][1-|\boldsymbol{v}|a_j\eta]}, \tag{58}$$

where we used $\eta \equiv \cos\theta$, $|\boldsymbol{v}| = |\boldsymbol{q}|/m_Q$, and convenient short notation

$$a_i = \frac{m_Q}{E}\frac{x_i - y_i}{D_i} < 1. \tag{59}$$

The numerator in Eq. (58) arises from the trace and contractions in Eq. (51). The denominators $[1+|\boldsymbol{v}|a_i\eta]$ $[1-|\boldsymbol{v}|a_j\eta]$ occur from the heavy quark propagators. The angular dependence occurs from the scalar products

$$(qk) = -Eq_z = -E|\boldsymbol{q}|\cos\theta, \qquad (qk') = Eq_z = E|\boldsymbol{q}|\cos\theta. \tag{60}$$

From the definition (58), it follows that

$$I_0 \sim \mathcal{O}(1), \qquad I_{1,2} \sim \mathcal{O}(v^2), \qquad I_{3,4} \sim \mathcal{O}(v^4). \tag{61}$$

If one neglects by the contribution form the denominators setting $a_i = a_j = 0$, then

$$I_0[ij] = 1, \qquad I_1[ij] = 0, \qquad I_2[ij] = \frac{\boldsymbol{v}^2}{3}, \qquad I_3[ij] = 0, \qquad I_4 = \frac{\boldsymbol{v}^2}{5}. \tag{62}$$

One can also easily find that

$$I_k[ji] = (-1)^k I_k[ij]. \tag{63}$$

These integrals can be computed analytically that gives

$$I_0[ij] = \frac{1}{2}\frac{1}{a_i+a_j}\left(\ln\frac{1+|\boldsymbol{v}|a_i}{1-|\boldsymbol{v}|a_i} + \ln\frac{1+|\boldsymbol{v}|a_j}{1-|\boldsymbol{v}|a_j}\right) = \frac{1}{a_i+a_j}\sum_{n=0}^{\infty}\frac{|\boldsymbol{v}|^n}{n+1}(a_j^{n+1}+(-1)^n a_i^{n+1}) \times \frac{1}{2}[1+(-1)^n], \tag{64}$$

and for $k > 0$

$$I_k[ij] = \frac{1}{2}\frac{1}{a_i+a_j}\frac{(-1)^k}{a_i^{k-1}}\left(\frac{1}{|\boldsymbol{v}|a_i}\ln\frac{1+|\boldsymbol{v}|a_i}{1-|\boldsymbol{v}|a_i} - \sum_{l=0}^{k-1}|\boldsymbol{v}|^l a_i^l(-1)^l\frac{1+(-1)^l}{l+1}\right) \tag{65}$$

$$+\frac{1}{2}\frac{1}{a_i+a_j}\frac{1}{a_j^{k-1}}\left(\frac{1}{|\boldsymbol{v}|a_i}\ln\frac{1+|\boldsymbol{v}|a_i}{1-|\boldsymbol{v}|a_i} - \sum_{l=0}^{k-1}|\boldsymbol{v}|^l a_j^l\frac{1+(-1)^l}{l+1}\right) \tag{66}$$

$$=\frac{|\boldsymbol{v}|^k}{a_i+a_j}\sum_{n=0}^{\infty}\frac{|\boldsymbol{v}|^n}{n+1+k}(a_j^{n+1}+(-1)^n a_i^{n+1})\frac{1}{2}\left[1+(-1)^{n+k}\right]. \tag{67}$$

The coefficients A_k, B_k, and C_k in Eqs. (55)–(57) depend on the nucleon LCDAs, and their general structure reads ($X = \{A, B, C\}$)

[1]We used the package FeynCalc [36].

$$\begin{aligned}X_k(x_{123}, y_{123}) = [X_k]_{VV}\{&V_1(x_{123})V_1(y_{123})\\ &+ A_1(x_{123})A_1(y_{123})\}\\ &+ [X_k]_{AV}\{A_1(x_{123})V_1(y_{123})\\ &+ V_1(x_{123})A_1(y_{123})\}\\ &+ [X_k]_{TT}T_1(x_{123})T_1(y_{123}),\end{aligned} \tag{68}$$

where coefficients $[X_k]_{VV,AV,TT}$ are polynomial functions of the momentum fractions x_i and y_i. Their analytical expressions are presented in Appendix B.

The convolution integral J in Eq. (53) is well defined that can be easily understood taking into account the general behavior of the LCDAs:

$$\begin{aligned}Z(x_{123}) = x_1x_2x_3 \times (\text{polynomial function in } x_i),\\ Z = \{V_1, A_1, T_1\}.\end{aligned} \tag{69}$$

Using the expression in Eq. (52), one finds the NRQCD amplitude A_1 defined in Eq. (8):

$$\begin{aligned}(A_1)_{\text{NRQCD}} = &\frac{\sqrt{2M_\psi}\langle 0|O|J/\psi\rangle}{m_c^2\sqrt{1+\langle v^2\rangle}}\frac{f_N^2}{m_c^4(1+\langle v^2\rangle)^2}(\pi\alpha_s)^3\\ &\times\frac{10}{81}(1+\sqrt{1+\langle v^2\rangle})\frac{1}{2}J(\langle v^2\rangle).\end{aligned} \tag{70}$$

In the limit $\langle v^2\rangle \to 0$, this result reproduces the well-known leading-order approximation in Eq. (14). The obtained expression depends on the power of heavy quark mass m_c^{-6} as required by the scale properties of the amplitude. The charm mass in the factor $\sqrt{2M_\psi}$ is usually calculated as $M_\psi \simeq 2m_c\sqrt{1+\langle v^2\rangle}$. The factor $(1+\langle v^2\rangle)^{-2}$ is naturally provided by the hard propagators in the diagrams. The essential part of the relativistic corrections is included in the convolution integral, which depends on the twist-3 LCDAs, which describe the nonperturbative overlaps with outgoing nucleons. Therefore, the total effect of the relativistic corrections also depends on the nonperturbative structure of nucleon.

To see the numerical impact of the obtained relativistic corrections, we need to compute the convolution integral J. This integral is not simple and, in the general case, can be calculated only numerically. In order to understand better the dependence of the integral on the model of the LCDA, we consider two different models. As an example of a relatively simple case, consider the asymptotic LCDA, which is defined as

$$V_1^{as} = T_1^{as} = 120 f_N x_1x_2x_3, \qquad A_1^{as} = 0. \tag{71}$$

Appropriate analysis allows better understanding numerical values of various convolution integrals. For a more realistic description, we use the Anikin-Braun-Offen (ABO) model [37], which provides a reliable description of various data.

For our consideration, we use $m_c = 1.4$ GeV for the pole heavy quark mass and use the estimate for the binding energy J/ψ from Ref. [24]:

$$E_{J/\psi} = 0.306 \text{ GeV}, \tag{72}$$

that gives

$$\langle v^2\rangle_{J/\psi} \simeq 0.225. \tag{73}$$

The value $\langle v^2\rangle_{\psi(2S)} \equiv \langle v^2\rangle_{\psi'}$ can be estimated as

$$\langle v^2\rangle_{\psi'} = \frac{E_{\psi'}}{m_c} \simeq \frac{M_{\psi'} - M_{J/\psi} + E_{J/\psi}}{(M_{J/\psi} - E_{J/\psi})/2} = 0.64, \tag{74}$$

which is quite large due to the large difference $M_{\psi'} - M_{J/\psi} \simeq 589$ MeV.

A. Relativistic corrections with asymptotic LCDAs

The asymptotic DA (71) is not a realistic model, but the use of this approximation makes it possible to simplify analytical expressions and study the properties of convolution integrals. As the first step, we also calculate the correction of relative order v^2 only.

The results in Eqs. (64) and (67) yield

$$\begin{aligned}I_0[ij] &= 1 + \langle v^2\rangle\frac{1}{3}(a_i^2 - a_ia_j + a_j^2) + \mathcal{O}(v^4)\\ &\simeq 1 + \langle v^2\rangle I_0^{(1)}[ij],\end{aligned} \tag{75}$$

$$I_1[ij] = \frac{\langle v^2\rangle}{3}(a_j - a_i) + \mathcal{O}(v^4) \simeq I_1^{(1)}[ij], \tag{76}$$

$$I_2[ij] = \frac{\langle v^2\rangle}{3} + \mathcal{O}(v^4), \qquad I_{3,4} \sim \mathcal{O}(v^4), \tag{77}$$

where a_i is defined in Eq. (59). Therefore, the contributions with $I_{3,4}$ can be neglected. It is convenient to define

$$J_{as}(\langle v^2\rangle) = \frac{120^2}{32}\sum_{k=0}^{4} J_{as}^{(k)}(\langle v^2\rangle), \tag{78}$$

where the coefficient in front of the sum is chosen in order to get a convenient normalization. In what follows, we are going to discuss the values of the integrals

$$J_{as}^{(k)}(\langle v^2\rangle) = \frac{1}{120^2 f_N^2}\int\frac{Dx_i}{x_1x_2x_3}\int\frac{Dy_i}{y_1y_2y_3} \times\left\{\frac{A_k^{as}I_k[13]}{D_1D_3}+\frac{B_k^{as}I_k[12]}{D_1D_2}+\frac{C_k^{as}I_k[23]}{D_2D_3}\right\} \quad (79)$$

and their dependence on the parameter $\langle v^2\rangle$.

For the coefficients A_k^{as}, we get

$$A_k^{as} = [A_k]_{VV}\{V_1^{as}(x_{123})V_1^{as}(y_{123})\} + [A_k]_{TT}T_1^{as}(x_{123})T_1^{as}(y_{123}) \quad (80)$$

$$= f_N^2 120^2 (x_1x_2x_3)(y_1y_2y_3)([A_k]_{VV}+[A_k]_{TT}) = f_N^2 120^2 (x_1x_2x_3)(y_1y_2y_3)\bar{A}_k^{as}, \quad (81)$$

with

$$\bar{A}_k^{as} = [A_k]_{VV}+[A_k]_{TT}, \quad (82)$$

and similarly for B_k and C_k. Substituting this in Eq. (79), we obtain

$$J_{as}^{(k)} \simeq \int Dx_i\int Dy_i\left\{\frac{\bar{A}_k^{as}I_k[13]}{D_1D_3}+\frac{\bar{B}_k^{as}I_k[12]}{D_1D_2}+\frac{\bar{C}_k^{as}I_k[23]}{D_2D_3}\right\} \quad (83)$$

$$= \int Dx_i\int Dy_i\frac{I_k[12]}{D_1D_2}\{\bar{B}_k^{as}+\hat{P}_{23}\bar{A}_k^{as}+(-1)^k\hat{P}_{13}\bar{C}_k^{as}\}, \quad (84)$$

where the transposition operator $\hat{P}_{ij}$ interchanges the arguments $\{x_i,y_i\}\leftrightarrow\{x_j,y_j\}$; for instance,

$$\hat{P}_{23}f(x_{123},y_{123}) = f(x_{132},y_{132}). \quad (85)$$

Substituting the explicit expressions for the coefficients $\bar{A}_k^{as}$, $\bar{B}_k^{as}$, and $\bar{C}_k^{as}$ and neglecting the higher-order terms, we get

$$\sum_{k=0}^{2}J_{as}^{(k)}(\langle v^2\rangle) = \int Dx_i\int Dy_i\frac{1}{D_1D_2}\{24x_1y_2 - 8\langle v^2\rangle + 24\langle v^2\rangle x_1y_2(I_0^{(1)}[12]+I_1^{(1)}[12]) + \langle v^2\rangle\left(2x_1y_2-\frac{1}{6}(x_1+x_2+y_1+y_2)\right)\right\}. \quad (86)$$

The integrals which enter in these formulas can be easily calculated numerically:

$$\text{LO:}\quad \int Dx_i\int Dy_i\frac{x_1y_2}{D_1D_2} = 0.140. \quad (87)$$

$$\text{NLO:}\quad 1^{\text{st}} = \int Dx_i\int Dy_i\frac{1}{D_1D_2} = 2.029, \quad (88)$$

$$\text{NLO:}\quad 2^{\text{nd}} = \int Dx_i\int Dy_i\frac{x_1y_2}{D_1D_2}(I_0^{(1)}[12]+I_1^{(1)}[12]) = 0.012, \quad (89)$$

$$\text{NLO:}\quad 3^{d} = \int Dx_i\int Dy_i\frac{1}{D_1D_2}\left(2x_1y_2 - \frac{1}{6}(x_1+x_2+y_1+y_2)\right) = -0.121. \quad (90)$$

This gives

$$\sum J_{as}^{(k)}(\langle v^2\rangle) = 24*0.140 - 8\langle v^2\rangle*(2.029)_{1\text{st}} + 24\langle v^2\rangle(0.012)_{2\text{nd}} + \langle v^2\rangle(-0.121)_{3d} \quad (91)$$

$$= 3.36_{\text{lo}} - 16.232\langle v^2\rangle_{1\text{st}} + 0.167\langle v^2\rangle_{2\text{nd}+3d} = 3.36(1-4.90\langle v^2\rangle). \quad (92)$$

Substituting the numerical values for $\langle v^2\rangle_{J/\psi}$ (73) and $\langle v^2\rangle_{\psi'}$ (74), we obtain

$$\sum J_{as}^{(k)}(\langle v^2\rangle_{J/\psi}) = -0.33, \quad (93)$$

$$\sum J_{as}^{(k)}(\langle v^2\rangle_{\psi'}) = -7.14. \quad (94)$$

These results show that the relativistic correction is negative and provides about 100% and 300% numerical effect for $1S$ and $2S$ states, respectively. To better understand the origin of such a large numerical impact, consider the structure of the expression in Eq. (91).

From this result, it follows that relativistic correction is dominated by the terms which appear from the integral (88). This integral is about an order of magnitude larger than the leading-order integral (87). This has a natural mathematical explanation: Both integrands in Eqs. (87) and (88) are positive, but the leading-order integrand in Eq. (87) has in the numerator a small factor x_1y_2 that reduces the value of the integral (87) comparing to the one in Eq. (88). The actual values of the NRQCD parameter $\langle v^2\rangle$ for charmonium states cannot compensate for the numerical enhancement of the subleading integral. This indicates the relative smallness of the charm quark mass.

The mechanism of enhancement of the subleading integral does not depend on the model of LCDA of the nucleon. The structure of leading and subleading integrals is closely related to the structure of the numerators of the Feynman diagrams and can be interpreted as a specific feature of the hard perturbative subprocess. In order to see this, consider the leading-order approximation with $q=0$; then the trace in Eq. (51) reads

$$\mathrm{Tr}\left[\Pi_1^{(\mathrm{lo})}\Gamma_Q^{\mu_i'\mu_c'\mu_j'}(x_i,y_i)\right] \sim \mathrm{Tr}[(\not\omega+1)\not\epsilon_\psi \gamma_\perp^{\mu_i} \times(-\not P/2+m_Q+\not k_i+\not k_i')\gamma_\perp^{\mu_c} \times(\not P/2+\not k_j+\not k_j'+m_Q)\gamma_\perp^{\mu_j}] \quad (95)$$

where we used the expression from Eq. (40) and

$$\Pi_1^{(\mathrm{lo})} = \frac{-1}{2\sqrt{2}}(\not\omega+1)\not\epsilon_\psi. \quad (96)$$

Notice that all Dirac matrices with open Lorentz indices in Eq. (95) are transverse because of contraction with transverse tensor $T_{\mu_1\mu_2\mu_3\rho}$ in Eq. (47). Recall that the heavy quark velocity

$$\omega^\mu = \delta_{\mu 0} = \frac{1}{2}(n^\mu + \bar{n}^\mu). \quad (97)$$

If one neglects collinear momenta k_i, k_i' or k_j, k_j' in the trace Eq. (95), then the trace vanishes; for instance,

$$\mathrm{Tr}[(\not\omega+1)\not\epsilon_\psi\gamma_\perp^{\mu_i}(-\not P/2+m_Q)\gamma_\perp^{\mu_c}(\not P/2+\not k_j+\not k_j'+m_Q)\gamma_\perp^{\mu_j}] \quad (98)$$

$$= m_Q \mathrm{Tr}[(\not\omega+1)\not\epsilon_\psi\gamma_\perp^{\mu_i}(-\not\omega+1)\gamma_\perp^{\mu_c}(\ldots)\gamma_\perp^{\mu_j}] \quad (99)$$

$$= m_Q \mathrm{Tr}[\underline{(\not\omega+1)(-\not\omega+1)}\not\epsilon_\psi\gamma_\perp^{\mu_i}\gamma_\perp^{\mu_c}(\ldots)\gamma_\perp^{\mu_j}] = 0, \quad (100)$$

where we used $[\not\omega,\gamma_\perp]=[\not\omega,\not\epsilon_\psi]=0$. Therefore, the nontrivial result is obtained only from the term with collinear momenta in both quark propagators:

$$\mathrm{Tr}\left[\Pi_1^{(\mathrm{lo})}\hat{A}_{QQ}^{\mu_i'\mu_c'\mu_j'}(x_i,y_i)\right] \sim \mathrm{Tr}[(\not\omega+1)\not\epsilon_\psi\gamma_\perp^{\mu_i}(\not k_i+\not k_i') \times\gamma_\perp^{\mu_c}(\not k_j+\not k_j')\gamma_\perp^{\mu_j}] \sim x_i y_j + x_j y_i. \quad (101)$$

This explains the structure of the leading-order integral in Eq. (87).

The calculation of the subleading correction involves the relative momentum q that introduces the traces with two insertions of q instead of collinear momenta; for instance,

$$\mathrm{Tr}\left[\Pi_1\Gamma_Q^{\mu_i'\mu_c'\mu_j'}(x_i,y_i)\right] \sim \mathrm{Tr}[(\not\omega+1)\not\epsilon_\psi\gamma_\perp^{\mu_i}\not q\gamma_\perp^{\mu_c}\not q\gamma_\perp^{\mu_j}] + \cdots. \quad (102)$$

Such subleading contributions give terms of relative order v^2, but the corresponding integral has no momentum fractions in the numerator; see Eq. (88). As a result, the large numerical contributions are generated by the integrals $J_{as}^{(0)}$ and $J_{as}^{(2)}$ only, which can get appropriate contributions only from the trace and contractions in Eq. (51). Parametrically, such integrals are suppressed by small velocity; however, in reality, such suppression is not sufficiently strong in order to provide a small numerical correction with respect to the leading-order contribution.

The enhanced integral also takes place in the relative order v^4, but the corresponding numerical effect is already suppressed by additional factor $\langle v^2\rangle$, and for J/ψ such a contribution can be estimated at about 20%–30%. But for the excited state such a contribution still remains quite large because of the large value of $\langle v^2\rangle_{\psi'}$.

In order to study the more general situation and the possible numerical effects of higher-order terms in v^2, we present in Table I the results for the integrals $J_{as}^{(k)}$, which are calculated with the resummed relativistic corrections. Recall that the leading-order approximation is given by the integral $J_{as}^{(0)}(\langle v^2\rangle = 0) = 3.36$.

The largest numerical effects are provided by the NLO corrections in integrals $J_{as}^{(0,2)}$ for the same reason as discussed above. The numerical effect from the resummation is most visible for $J_{as}^{(0)}$, but the corresponding effects are not sufficiently large and do not change the qualitative picture discussed for the NLO approximation. Therefore, we can conclude that relativistic expansion for J/ψ is well convergent. The large numerical impact is generated only by the correction of relative order v^2 and associated with the numerical enhancement of the NLO convolution integral.

In Table I (bottom), we also show the results for the $2S$ state, which has much larger relativistic corrections. In this case, the general structure remains quite similar: The largest numerical effect is provided by the NLO contribution in $J_{as}^{(0,2)}$, but the higher-order power corrections also

TABLE I. Numerical results for integrals $J_{as}^{(k)}$ for J/ψ and ψ', top and bottom, respectively.

$\langle v^2\rangle_{J/\psi} = 0.225$	$J_{as}^{(0)}$	$J_{as}^{(1)}$	$J_{as}^{(2)}$	$J_{as}^{(3)}$	$J_{as}^{(4)}$	$\sum J_{as}^{(k)}$
NLO	−1.84	−0.18	1.89	⋯	⋯	−0.13
All orders	−1.00	−0.19	1.66	−0.01	0.13	0.59

$\langle v^2\rangle_{\psi'} = 0.64$	$J_{as}^{(0)}$	$J_{as}^{(1)}$	$J_{as}^{(2)}$	$J_{as}^{(3)}$	$J_{as}^{(4)}$	$\sum J_{as}^{(k)}$
NLO	−11.68	−0.50	5.37	⋯	⋯	−6.81
All orders	−6.78	−0.69	4.53	−0.14	0.98	−2.10

make a significant numerical contribution, reducing the total sum by a factor of 3. This clearly illustrates the expected conclusion: The relativistic expansion for excited charmonium has very large relativistic corrections and converges rather slowly.

B. Relativistic corrections with realistic nucleon LCDA

The value of the collinear integral also depends on the model of the LCDA φ_3, which describes a distribution of the quark longitudinal momenta at zero transverse separation. To illustrate the possible effect of the nucleon structure, we calculate in this section the convolution integrals with a realistic LCDA model. For that purpose, we use the ABO-I model from Ref. [37]. This model gives a reliable description of the electromagnetic form factor data within the light-cone sum rules [38] and also provides a good description of $J/\psi \to p\bar{p}$ decay data in the leading-order approximation [21]. The expression for the model reads

$$\varphi_3^{\mathrm{ABO}}(x_i) = 120x_1x_2x_3\{1 + \varphi_{10}\mathcal{P}_{10}(x_i) + \varphi_{11}\mathcal{P}_{11}(x_i) + \varphi_{20}\mathcal{P}_{20}(x_i) + \varphi_{21}\mathcal{P}_{21}(x_i) + \varphi_{22}\mathcal{P}_{22}(x_i)\}, \tag{103}$$

where the orthogonal polynomials $\mathcal{P}_{ij}(x_i)$ read

$$\mathcal{P}_{10}(x_i) = 21(x_1 - x_3), \qquad \mathcal{P}_{11}(x_i) = 7(x_1 - 2x_2 + x_3), \tag{104}$$

$$\mathcal{P}_{20}(x_i) = \frac{63}{10}[3(x_1 - x_3)^2 - 3x_2(x_1 + x_3) + 2x_2^2], \tag{105}$$

$$\mathcal{P}_{21}(x_i) = \frac{63}{2}(x_1 - 3x_2 + x_3)(x_1 - x_3), \tag{106}$$

$$\mathcal{P}_{22}(x_i) = \frac{9}{5}[x_1^2 + 9x_2(x_1 + x_3) - 12x_1x_3 - 6x_2^2 + x_3^2]. \tag{107}$$

The moments $\varphi_{ij} \equiv \varphi_{ij}(\mu)$ are multiplicatively renormalizable; more details about properties of the polynomials $\mathcal{P}_{ij}$ and about the evolution of the moments can be found in Ref. [39]. In Appendix A, we also provide some useful details. In our calculation, we fix the relatively low normalization scale $\mu^2 = 1.5$ GeV following Ref. [21]. Then the values of the twist-3 moments read

$$\varphi_{10} = 0.051, \qquad \varphi_{11} = 0.052, \qquad \varphi_{20} = 0.078, \qquad \varphi_{21} = -0.028, \qquad \varphi_{22} = 0.179. \tag{108}$$

Again, we rewrite the corresponding collinear integral $J \equiv J_{\mathrm{ABO}}$ defined in Eq. (53) as the sum

$$J_{\mathrm{ABO}}(\langle v^2\rangle) = \frac{120^2}{32}\sum_{k=0}^{4} J_{\mathrm{ABO}}^{(k)}(\langle v^2\rangle), \tag{109}$$

where each integral $J_{\mathrm{ABO}}^{(k)}$ is defined as

$$J_{\mathrm{ABO}}^{(k)}(\langle v^2\rangle) = \frac{1}{120^2 f_N^2}\int \frac{Dx_i}{x_1x_2x_3}\int \frac{Dy_i}{y_1y_2y_3} \times \left\{\frac{A_k^{\mathrm{ABO}} I_k[13]}{D_1D_3} + \frac{B_k^{\mathrm{ABO}} I_k[12]}{D_1D_2} + \frac{C_k^{\mathrm{ABO}} I_k[23]}{D_2D_3}\right\}. \tag{110}$$

The value of the leading-order integral reads

$$J_{\mathrm{ABO}}^{(0)}(\langle v^2\rangle = 0) = 5.08. \tag{111}$$

The numerical results for different integrals $J_{\mathrm{ABO}}^{(k)}$ for J/ψ and ψ' are presented in Table II.

The qualitative picture remains the same as described above; the sum of integrals $J_{\mathrm{ABO}}^{(0)} + J_{\mathrm{ABO}}^{(2)}$ provides the largest numerical impact, but the values of all integrals $J_{\mathrm{ABO}}^{(k)}$ are larger. We can conclude that the described mechanism of the large numerical effect also works for the realistic model of LCDAs. In the case of J/ψ, the total sum is negative, which indicates that the negative relativistic correction is somewhat larger than the LO contribution.

To better show the effect of a large contribution of the order of v^2, we also present the results in Table II in the following form:

TABLE II. Numerical results for the integrals $J_{\mathrm{ABO}}^{(k)}$ for J/ψ and ψ', top and bottom, respectively.

$\langle v^2\rangle_{J/\psi} = 0.225$	$J_{\mathrm{ABO}}^{(0)}$	$J_{\mathrm{ABO}}^{(1)}$	$J_{\mathrm{ABO}}^{(2)}$	$J_{\mathrm{ABO}}^{(3)}$	$J_{\mathrm{ABO}}^{(4)}$	$\sum J_{\mathrm{ABO}}^{(k)}$
NLO	−5.18	−0.27	3.66	⋯	⋯	−1.79
All orders	−3.65	−0.35	3.30	−0.03	0.22	−0.51

$\langle v^2\rangle_{\psi'} = 0.64$	$J_{\mathrm{ABO}}^{(0)}$	$J_{\mathrm{ABO}}^{(1)}$	$J_{\mathrm{ABO}}^{(2)}$	$J_{\mathrm{ABO}}^{(3)}$	$J_{\mathrm{ABO}}^{(4)}$	$\sum J_{\mathrm{ABO}}^{(k)}$
NLO	−24.09	−0.75	10.42	⋯	⋯	−14.42
All orders	−15.69	−1.39	9.31	−0.30	1.78	−6.29

$$J_{\mathrm{ABO}}(\langle v^2\rangle) \approx \frac{120^2}{32} \times 5.08(1 - 6.0\langle v^2\rangle_\psi + \delta_\psi), \quad (112)$$

where δ_ψ denotes the sum of the higher-order contributions starting from the terms $\langle v^2\rangle^2 \sim \mathcal{O}(v^4)$. Their values are as follows: $\delta_{J/\psi} = 0.25$ and $\delta_{\psi'} = 1.60$. Taking into account the values of $\langle v^2\rangle_\psi$ from Eqs. (73) and (74), it is easy to find that the contribution of the order of v^2 is about 5 and 2 times larger than $\delta_{J/\psi}$ and $\delta_{\psi'}$, respectively.

Let us notice that the ABO model (103) gives a reasonable description of the branching ratio $J/\psi \to p\bar{p}$ in the leading-order approximation [19,21], as well as electromagnetic nucleon form factors at large momentum transfer [37]. But now the sum of different contributions in the parentheses in Eq. (112) yields ≈ -0.1 instead of one. As a result, the value of the branching ratio turns out to be 2 orders of magnitude smaller than the data. Qualitatively, this is a consequence of the strong cancellation between the leading-order and next-to-leading-order contributions in Eq. (112). It seems extremely unlikely that such a large effect can somehow be compensated for by a different choice of parameters of the nucleon LCDA. This is also confirmed by comparing the collinear integrals J for the asymptotic LCDA in Eq. (92) and for the ABO model in Eq. (112), which shows that the numerical effect from the higher moments of LCDA is not large.

Additional numerical impact is also provided by $\langle v^2\rangle$ dependence of the coefficient in front of the convolution integral in Eq. (70). The dominant numerical effect is provided by the factor $1/(1 + \langle v^2\rangle)^2$, which occurs from the hard propagators. It seems that this factor can be understood as an indication that the charmonium mass $M_\psi^2 \simeq 4m_c^2(1 + \langle v^2\rangle)$ is a more natural scale for the hard gluon propagators in the Feynman diagrams. The corresponding numerical effect is smaller in comparison with one in the convolution integral discussed above. For J/ψ, the modification of the coefficient in Eq. (70) gives a reduction about 37%. For ψ', a similar effect is already 67%.

IV. DISCUSSION

We presented the first study of relativistic corrections in exclusive S-wave charmonium decays into a proton-antiproton final state. The relativistic corrections are calculated within the NRQCD and collinear factorization frameworks. We consider only the helicity-conserving amplitude A_1, which provides the dominant numerical contribution to the decay width. In this case, the baryon nonperturbative light-cone matrix elements depend on the twist-3 LCDAs only, and the formula for the amplitude with relativistic corrections includes the collinear convolution integral, which is free from any infrared singularities.

The result obtained provides a full correction of the relative order v^2 and also includes the summation of all orders of relativistic corrections associated with the quark-antiquark quarkonia wave function in the potential model [30].

The largest numerical impact is provided by the NLO relativistic correction with the matrix element, which can be computed using the equation of motion and is proportional to the binding energy. Because of a specific structure of the LO and NLO hard scattering kernels, the value of the NLO collinear integral is about an order of magnitude larger than the value of the LO one. The charmonium parameter $\langle v^2\rangle$ is not small enough to compensate for this effect, and the magnitude of the resulting relativistic correction is numerically of the same order as the LO contribution.

The order v^2 correction is dominant and has the opposite sign with respect to the leading-order term; in the case of J/ψ, the sum of these contributions almost cancels out. The calculated higher-order corrections are relatively small, and their partial resummation shows that relativistic expansion converges sufficiently well. The numerical effect also depends on the model of the nucleon LCDA; however, this dependence does not change the main qualitative conclusions about the large relativistic corrections. For instance, in order to compensate for the effect of large relativistic corrections, it is necessary to increase the nucleon normalization coupling f_N by a factor of 3, which strongly contradicts available sum rule estimates [12,42] and lattice calculations [43].

A description of excited state ψ' is more challenging because of the relatively large value of $\langle v^2\rangle_{\psi'}$. In this case, the numerical effect from relativistic corrections is much larger than the LO contribution and is more sensitive to the higher-order contributions. Therefore, a description of ψ' baryonic decays suffers from large uncertainties, which are associated with the higher-order contributions of the relativistic expansions.

Taking into account these large relativistic corrections, one cannot expect the 13% rule in Eq. (1) to hold in the general case. The rather good agreement for the nucleon channel is probably an accidental consequence of different numerical cancellations.

The large effect from relativistic corrections raises the question about a phenomenological description of baryon decays in the effective field theory framework. Various existing phenomenological estimates for J/ψ decay width and angular behavior are based on the LO approximation and provide qualitative reliable estimates [21]. However, such a qualitative picture is violated by the large value of relativistic corrections. The large negative correction almost cancels the LO contribution, which greatly reduces the amplitude value and, therefore, makes the description problematic. A possible solution of this problem might be associated with the large and positive NLO radiative correction, which is not yet known. Cancellation of radiative and relativistic corrections could resolve the situation. Large NLO radiative corrections are already observed in the exclusive production $e^+e^- \to J/\psi + \eta_c$; see, e.g., Ref. [40]. Probably a similar situation also arises in the baryonic decays. Therefore, the calculation of the NLO radiative correction is necessary in order to better understand the hadronic decay dynamics.

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APPENDIX A: DEFINITION AND PROPERTIES OF NUCLEON LCDA

In this appendix, we provide a definition of the light-cone matrix element for a proton state. The formulas for the antiproton state can be obtained by charge conjugation. In order to simplify formulas, we also use the light-cone gauge

$$n \cdot A(x) = 0. \tag{A1}$$

In this case, the light-cone three-quark operator which we need for the proton state can be defined as (i, j, k are the color indices)

$$O_{\mathrm{tw3}} = \varepsilon^{ijk} \xi^i_\alpha(z_{1-}) \xi^j_\beta(z_{2-}) \xi^k_\sigma(z_{3-}), \tag{A2}$$

where the projected quark field $\xi(z) \equiv \frac{\bar{n\!\!\!/} n\!\!\!/}{4}\psi(z)$ and the arguments of the fields read

$$z_{i-} = (\bar{n} \cdot z_i)\frac{n}{2}. \tag{A3}$$

The proton flavor structure implies that $O_{\mathrm{tw3}} = uud$.

The twist-3 light-cone matrix element is defined as

$$\begin{aligned}
&\langle 0|O_{\mathrm{tw3}}(z_1, z_2, z_3)|p(k)\rangle \\
&= \frac{1}{4}[k\!\!\!/ C]_{\alpha\beta}[\gamma_5 N_{\bar{n}}]_\sigma \mathrm{FT}[V_1(y_i)] \\
&\quad + \frac{1}{4}[k\!\!\!/ \gamma_5 C]_{\alpha\beta}[N_{\bar{n}}]_\sigma \mathrm{FT}[A_1(y_i)] \\
&\quad + \frac{1}{4}k^\nu [i\sigma_{\mu\nu} C]_{\alpha\beta}[\gamma^\mu_\perp \gamma_5 N_{\bar{n}}]_\sigma \mathrm{FT}[T_1(y_i)],
\end{aligned} \tag{A4}$$

where the projected nucleon spinor reads

$$N_{\bar{n}} = \frac{\bar{n\!\!\!/} n\!\!\!/}{4} N(k) \tag{A5}$$

and C is the charge conjugation matrix. The Fourier transformation "FT" is defined as

$$\begin{aligned}
\mathrm{FT}[F(y_i)] = \int Dy_i e^{-iy_1 k_- z_{1+}/2 - iy_2 k_- z_{2+}/2 - iy_3 k_- z_{3+}/2} \\
\times F(y_1, y_2, y_3),
\end{aligned} \tag{A6}$$

with the integration measure

$$Dy_i = dy_1 dy_2 dy_3 \delta(1 - y_1 - y_2 - y_3). \tag{A7}$$

Three LCDAs V_1, A_1, and T_1 satisfy the following properties [42]:

$$V_1(y_2, y_1, y_3) = V_1(y_1, y_2, y_3), \tag{A8}$$

$$A_1(y_2, y_1, y_3) = -A_1(y_1, y_2, y_3), \tag{A9}$$

$$T_1(y_2, y_1, y_3) = T_1(y_1, y_2, y_3). \tag{A10}$$

The isospin symmetry allows one to get the following relation (see details in Refs. [41,42]):

$$\begin{aligned}
T_1(y_1, y_2, y_3) = \frac{1}{2}(V_1 - A_1)(y_1, y_3, y_2) \\
+ \frac{1}{2}(V_1 - A_1)(y_2, y_3, y_1).
\end{aligned} \tag{A11}$$

It is convenient to define the following combination:

$$f_N \varphi_3(y_1, y_2, y_3) = V_1(y_1, y_2, y_3) - A_1(y_1, y_2, y_3), \tag{A12}$$

which allows one to describe the set of three LCDAs V_1, A_1, and T_1 in terms of the one function. The coupling f_N describes the normalization so that φ_3 is dimensionless and normalized as

$$\int Dy_i \varphi_3(y_1, y_2, y_3) = 1. \tag{A13}$$

The LCDA φ_3 also depends on the factorization scale, which is not shown for simplicity:

$$\varphi_3(y_1, y_2, y_3) \equiv \varphi_3(y_1, y_2, y_3; \mu). \tag{A14}$$

The evolution properties of this LCDA were studied in Ref. [39]. The moments in Eq. (103) are multiplicatively renormalizable and can be calculated as

$$\phi_{ij}(\mu) = \phi_{ij}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_{ij}/\beta_0}, \tag{A15}$$

where $\beta_0 = 11 - 2n_f/3$ and γ_{ij} are the corresponding anomalous dimensions:

$$\gamma_{10} = \frac{20}{9}, \quad \gamma_{11} = \frac{8}{3}, \quad \gamma_{20} = \frac{32}{9}, \quad \gamma_{21} = \frac{40}{9}, \quad \gamma_{22} = \frac{14}{3}. \tag{A16}$$

APPENDIX B: ANALYTICAL EXPRESSIONS FOR THE COEFFICIENTS A_k, B_k, AND C_k DEFINED IN EQS. (55)–(57)

Here, we provide the explicit expressions for the coefficients $[X_k]_{VV,AV,TT}$ defined in Eq. (68). These coefficients are functions of the momentum fractions

$$[X_k]_{VV,AV,TT} \equiv [X_k]_{VV,AV,TT}(x_{123}, y_{123}). \tag{B1}$$

Below, we also use the following short notation:

$$\bar{m} = \frac{m_c}{E} = \frac{1}{\sqrt{1+\mathbf{v}^2}}, \qquad C_{ij} = x_i y_j + x_j y_i. \tag{B2}$$

The coefficients C_k can be obtained using the simple relations

$$[C_k]_{VV,TT}(x_{123}, y_{123}) = [A_k]_{VV,TT}(x_{213}, y_{213}), \tag{B3}$$

$$[C_k]_{AV}(x_{123}, y_{123}) = -[A_k]_{AV}(x_{213}, y_{213}). \tag{B4}$$

The other coefficients read

$$[A_0]_{VV} = 2C_{13} - (1-\bar{m})(x_3+y_3) - 2\bar{m}(1-\bar{m}), \tag{B5}$$

$$[A_0]_{AV} = \frac{2}{1+\bar{m}}(-2C_{13} + (1-\bar{m}^2)(x_1 - x_2 + y_1 - y_2)), \tag{B6}$$

$$[A_0]_{TT} = -4(1-\bar{m})(\bar{m}+1-x_3-y_3), \tag{B7}$$

$$[B_0]_{VV} = -(1-\bar{m})(2\bar{m}+x_3+y_3), \tag{B8}$$

$$[B_0]_{AV} = 2(1-\bar{m})(x_1-x_2+y_1-y_2), \tag{B9}$$

$$[B_0]_{TT} = 4(2C_{12} + (1-\bar{m})(x_3+y_3) + \bar{m}^2 - 1); \tag{B10}$$

$$[A_1]_{VV} = \frac{\bar{m}}{1+\bar{m}}(4(x_1 y_3 - x_3 y_1) + (1-\bar{m})(x_3 - x_1 + y_1 - y_3)), \tag{B11}$$

$$[A_1]_{AV} = -[A_1]_{VV}, \tag{B12}$$

$$[A_1]_{TT} = 0, \tag{B13}$$

$$[B_1]_{VV} = 0, \tag{B14}$$

$$[B_1]_{AV} = 0, \tag{B15}$$

$$[B_1]_{TT} = \frac{4\bar{m}}{1+\bar{m}}(4(x_1 y_2 - x_2 y_1) + (1-\bar{m})(x_2 - x_1 + y_1 - y_2)); \tag{B16}$$

$$[A_2]_{VV} = \frac{\bar{m}^2}{(1+\bar{m})^2}(2C_{13} + (1+\bar{m})(2+x_3+y_3) - 3(1-\bar{m}^2)), \tag{B17}$$

$$[A_2]_{AV} = -\frac{\bar{m}^2}{1+\bar{m}}(4(x_1+y_1-1) + 2(x_3+y_3) + \bar{m} - 1), \tag{B18}$$

$$[A_2]_{TT} = \frac{4\bar{m}^2}{1+\bar{m}}(1 - x_3 - y_3 + \bar{m}), \tag{B19}$$

$$[B_2]_{VV} = \frac{\bar{m}^2}{1+\bar{m}}(x_3 + y_3 + 2\bar{m}), \tag{B20}$$

$$[B_2]_{AV} = \frac{2\bar{m}^2}{1+\bar{m}}(x_2 - x_1 + y_2 - y_1), \tag{B21}$$

$$[B_2]_{TT} = \frac{4\bar{m}^2}{(1+\bar{m})^2}(2C_{12} + (1-\bar{m})(2 - x_3 - y_3) - 2(1-\bar{m}^2)); \tag{B22}$$

$$[A_3]_{VV} = \frac{\bar{m}^3}{(1+\bar{m})^2}(2(x_1 - y_1) + y_3 - x_3), \tag{B23}$$

$$[A_3]_{AV} = 0, \tag{B24}$$

$$[A_3]_{TT} = -\frac{4\bar{m}^3}{(1+\bar{m})^2}(x_3 - y_3), \tag{B25}$$

$$[B_3]_{VV} = \frac{\bar{m}^3}{(1+\bar{m})^2}(x_1 - x_2 - y_1 + y_2), \tag{B26}$$

$$[B_3]_{AV} = 0, \tag{B27}$$

$$[B_3]_{TT} = \frac{4\bar{m}^3}{(1+\bar{m})^2}(x_1 - x_2 - y_1 + y_2); \tag{B28}$$

$$[A_4]_{VV} = [B_4]_{VV} = \frac{2\bar{m}^4}{(1+\bar{m})^2}, \tag{B29}$$

$$[A_4]_{AV} = [B_4]_{AV} = 0, \tag{B30}$$

$$[A_4]_{TT} = [B_4]_{TT} = \frac{4\bar{m}^4}{(1+\bar{m})^2}. \tag{B31}$$

[1] M. Ablikim *et al.* (BES Collaboration), Phys. Lett. B **648**, 149 (2007).
[2] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. D **78**, 092005 (2008).
[3] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **86**, 032014 (2012).
[4] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **93**, 072003 (2016).
[5] M. Ablikim *et al.* (BESIII Collaboration), Phys. Lett. B **770**, 217 (2017).
[6] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **95**, 052003 (2017).
[7] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **98**, 032006 (2018).
[8] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **125**, 052004 (2020).
[9] M. Alekseev, A. Amoroso, R. B. Ferroli, I. Balossino, M. Bertani, D. Bettoni, F. Bianchi, J. Chai, G. Cibinetto, F. Cossio *et al.*, Chin. Phys. C **43**, 023103 (2019).
[10] R. Baldini Ferroli, A. Mangoni, S. Pacetti, and K. Zhu, Phys. Lett. B **799**, 135041 (2019); Phys. Rev. D **103**, 016005 (2021); A. Mangoni, arXiv:2202.08542.
[11] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981).
[12] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
[13] N. Brambilla *et al.* (Quarkonium Working Group), arXiv: hep-ph/0412158.
[14] V. L. Chernyak, A. A. Ogloblin, and I. R. Zhitnitsky, Yad. Fiz. **48**, 1398 (1988) [Z. Phys. C **42**, 583 (1989)] [Sov. J. Nucl. Phys. **48**, 889 (1988)].
[15] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853 (1997).
[16] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Rev. Mod. Phys. **77**, 1423 (2005).
[17] N. Brambilla *et al.* (Quarkonium Working Group), *Heavy Quarkonium Physics* (CERN, Geneva, 2005).
[18] M. B. Voloshin, Prog. Part. Nucl. Phys. **61**, 455 (2008).
[19] N. Kivel, Eur. Phys. J. A **56**, 64 (2020); **57**, 271(E) (2021).
[20] N. Kivel, Eur. Phys. J. A **58**, 26 (2022).
[21] N. Kivel, Eur. Phys. J. A **58**, 138 (2022).
[22] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[23] G. T. Bodwin and A. Petrelli, Phys. Rev. D **66**, 094011 (2002); **87**, 039902(E) (2013).
[24] G. T. Bodwin, H. S. Chung, D. Kang, J. Lee, and C. Yu, Phys. Rev. D **77**, 094017 (2008).
[25] G. T. Bodwin, H. S. Chung, J. H. Ee, J. Lee, and F. Petriello, Phys. Rev. D **90**, 113010 (2014).
[26] N. Brambilla, H. S. Chung, W. K. Lai, V. Shtabovenko, and A. Vairo, Phys. Rev. D **100**, 054038 (2019).
[27] J. P. Ma and Q. Wang, Phys. Lett. B **537**, 233 (2002).
[28] N. Brambilla, E. Mereghetti, and A. Vairo, J. High Energy Phys. 08 (2006) 039; 04 (2011) 058(E).
[29] E. Braaten and J. Lee, Phys. Rev. D **67**, 054007 (2003); **72**, 099901(E) (2005).
[30] G. T. Bodwin, J. Lee, and C. Yu, Phys. Rev. D **77**, 094018 (2008).
[31] T. Barnes, X. Li, and W. Roberts, Phys. Rev. D **77**, 056001 (2008).
[32] V. Braun, R. J. Fries, N. Mahnke, and E. Stein, Nucl. Phys. **B589**, 381 (2000); **607**, 433 (2001).
[33] M. Gremm and A. Kapustin, Phys. Lett. B **407**, 323 (1997).
[34] G. T. Bodwin, D. Kang, and J. Lee, Phys. Rev. D **74**, 014014 (2006).
[35] J. H. Kuhn, J. Kaplan, and E. G. O. Safiani, Nucl. Phys. **B157**, 125 (1979).
[36] J. Kublbeck, H. Eck, and R. Mertig, Nucl. Phys. B, Proc. Suppl. **29**, 204 (1992); V. Shtabovenko, R. Mertig, and F. Orellana, Comput. Phys. Commun. **256**, 107478 (2020).
[37] I. V. Anikin, V. M. Braun, and N. Offen, Phys. Rev. D **88**, 114021 (2013).
[38] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Sov. J. Nucl. Phys. **44**, 1028 (1986); I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Nucl. Phys. **B312**, 509 (1989); V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B345**, 137 (1990).
[39] V. M. Braun, S. E. Derkachov, G. P. Korchemsky, and A. N. Manashov, Nucl. Phys. **B553**, 355 (1999); V. M. Braun, A. N. Manashov, and J. Rohrwild, Nucl. Phys. **B807**, 89 (2009).
[40] Y. J. Zhang, Y. j. Gao, and K. T. Chao, Phys. Rev. Lett. **96**, 092001 (2006); B. Gong and J. X. Wang, Phys. Rev. D **77**, 054028 (2008).
[41] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B246**, 52 (1984).
[42] V. Braun, R. J. Fries, N. Mahnke, and E. Stein, Nucl. Phys. **B589**, 381 (2000); **B607**, 433(E) (2001).
[43] G. S. Bali *et al.* (RQCD Collaboration), Eur. Phys. J. A **55**, 116 (2019).