

Transition GPDs and exclusive electroproduction of $\pi^- \Delta(1232)$ final states

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We investigate exclusive electroproduction of $\pi^- \Delta^{++}$ within the handbag approach in which the helicity amplitudes factorize into generalized parton distributions (GPDs) and hard partonic subprocesses. We define the $p - \Delta$ transversity GPDs while the helicity nonflip GPDs are taken from the literature. For the numerical estimates of observables we utilize large- N_C results in order to relate a few of the $p - \Delta$ GPDs to the proton-proton ones and neglect all other GPDs. In the calculation of the twist-2 and twist-3 subprocess amplitudes we take into account quark transverse momenta in combination with Sudakov suppressions. The partial cross sections for $\gamma^* p \rightarrow \pi^- \Delta^{++}$ are predicted in the large- N_C limit.

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I. INTRODUCTION

In the last 25 years hard exclusive reactions have found much interest. The theoretical description of such processes, the handbag approach, is based on factorization of the process amplitudes into hard partonic subprocesses and soft hadronic matrix elements, parametrized as generalized parton distributions (GPDs). This factorization property has been shown to hold rigorously to leading-twist accuracy in the generalized Bjorken regime of large photon virtuality, Q^2 , and large invariant mass of the hadrons in the final state, W , but fixed Bjorken- x , x_B , and small Mandelstam- t [1–3]. However, there is no theoretical estimate of the strength of subasymptotic power corrections. In so far, the validity of the asymptotic leading-twist result in a given range of Q^2 and W is to be regarded as an assumption. The strength of power corrections have to be extracted from the analysis of relevant data. Still the handbag approach, with occasional power corrections, has been successfully applied to electroproduction of vector (e.g., ρ^0, ϕ, ω) and pseudo-scalar (e.g., π, η, η') mesons, see the reviews [4,5] and references therein. These processes require the diagonal proton-proton GPDs. Octet-octet transition GPDs occur for instance in kaon electroproduction. SU(3) flavor symmetry relates these GPDs to the $p - p$ ones provided that symmetry breaking effects are ignored [6]. This, for

instance, allows an analysis of the sparse kaon data (see [7] and references therein) within the handbag approach [8]. The situation for the octet-decuplet GPDs is more complicated. Only in the large- N_C limit these GPDs—or at least some of them—can be related to the $p - p$ ones [4,9].

Since in the near future data on the exclusive $\pi\Delta$ channels will come from the Jefferson Lab we think it timely to analyze such processes. First, still preliminary data, namely the beam spin asymmetry for $\pi^- \Delta^{++}(1232)$, have already been shown on conferences [10]. Here in this article we are going to analyze the process $\gamma^* p \rightarrow \pi^- \Delta^{++}(1232)$ in full analogy to $\gamma^* N \rightarrow \pi N'$ [11,12]. It however turned out for the latter process that the naive asymptotic leading-twist result is not readily applicable in the range of kinematics accessible to these experiments. In fact, large power corrections are required to the asymptotically dominant amplitudes for longitudinally polarized photons. There are also strong contributions from transversally polarized photons although they are asymptotically suppressed by $1/Q^2$ in the cross sections. In some cases, as for instance for π^0 electroproduction [13], the contributions from transversely polarized photons are even dominant. In [11,12], see also the review [14], a generalization of the handbag approach has been developed which allows us to model such power corrections. The decisive point is to retain the quark transverse momenta in the subprocess and to apply Sudakov suppressions. Implicitly, this way the transverse size of the meson is taken into account.¹ The contribution

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¹The role of quark transverse momenta and the transverse size of the meson has also been discussed in [15] in the case of light vector-meson production at HERA kinematics.

from transversely polarized photons are modeled by a combination of transversity GPDs and a twist-3 pion wave function. This twist-3 contribution is large in the case of pions since it is proportional to a mass parameter, μ_π , which is not the usual mass of the pion. It is rather enhanced by the chiral condensate

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} \quad (1)$$

by means of the divergence of the axial vector current.² The masses m_u and m_d are the current-quark masses of the pion's valence quarks. The mass parameter is large, about 2 GeV at the initial scale $\mu_0 = 2$ GeV. As in [11,12,17] the contribution from the pion pole is treated as a one-boson-exchange term which is much larger than its leading-twist approximation.

The plan of the paper is the following: After some kinematical preliminaries (Sec. II) we calculate the matrix elements, A^Δ , for the helicity nonflip $p - \Delta$ GPDs and the $\gamma^* p \rightarrow \pi^- \Delta^{++}$ helicity amplitudes (Sec. III). In the next section we define the transversity GPDs and calculate likewise the A^Δ and the helicity amplitudes for them. Section V is devoted to a study of the contribution from the pion pole and, in Sec. VI, we discuss the GPDs in the large- N_C limit. In Sec. VII the parametrization of the GPDs and the calculation of the subprocess amplitudes is presented. Our predictions for the $\gamma^* p \rightarrow \pi^- \Delta^{++}$ partial cross sections are shown and discussed in Sec. VIII. Finally, in Sec. IX, we give a summary.

II. KINEMATICAL PRELIMINARIES

We are interested in the hard exclusive process

$$\gamma^*(q, \mu) p(p, \nu) \rightarrow \pi^-(q', \mu') \Delta^{++}(p', \nu'), \quad (2)$$

where the labels in the parenthesis denote the momenta of the particles and their light-cone helicities. In light-cone components the momenta are defined as

$$\begin{aligned} p &= \left[(1 + \xi)P^+, \frac{m^2 + \Delta_\perp^2/4}{2(1 + \xi)P^+}, -\frac{\Delta_\perp}{2}, 0 \right], \\ p' &= \left[(1 - \xi)P^+, \frac{M^2 + \Delta_\perp^2/4}{2(1 - \xi)P^+}, \frac{\Delta_\perp}{2}, 0 \right], \\ q &= \left[\eta(1 + \xi)P^+, \frac{-Q^2 + \Delta_\perp^2/4}{2\eta(1 + \xi)P^+}, \frac{\Delta_\perp}{2}, 0 \right], \end{aligned} \quad (3)$$

²Twist-3 effects may also be generated by twist-3 GPDs in combination with a twist-2 meson wave function [16]. However, for these GPDs there is no similar enhancement known. Therefore, such contributions are expected to be small and neglected here.

where m and M denote the mass of the proton and the $\Delta(1232)$, respectively. The mass of the pion is neglected except in the pion pole term, see below. The negative of η equals the Bjorken variable, x_B , up to corrections of order m^2/Q^2 and Δ_\perp^2/Q^2 . It is convenient to introduce an average baryon momentum, P , and a momentum transfer, Δ :

$$P = \frac{1}{2}(p + p'), \quad \Delta = p' - p. \quad (4)$$

The skewness is defined by the ratio of light cone plus components of Δ and P

$$\xi = -\frac{\Delta^+}{2P^+} \quad (5)$$

and is related to Bjorken- x by

$$\xi = \frac{x_B}{2 - x_B}. \quad (6)$$

This relation strictly holds for $Q^2 \rightarrow \infty$ but we neglect here the corrections of order $1/Q^2$. Mandelstam t is given by

$$t = \Delta^2 = t_0 - \frac{\Delta_\perp^2}{1 - \xi^2}, \quad (7)$$

where t_0 is the minimal value of t implied by the positivity of Δ_\perp^2

$$t_0 = -\frac{2\xi}{1 - \xi^2} [(1 + \xi)(M^2 - m^2) + 2\xi m^2]. \quad (8)$$

For convenience we will frequently use in the following a variable t' defined by

$$t' = t - t_0 = -\frac{\Delta_\perp^2}{(1 - \xi^2)}. \quad (9)$$

For later use we also define the two quantities

$$\kappa_\pm = (1 + \xi)M \pm (1 - \xi)m. \quad (10)$$

III. THE HELICITY NONFLIP $p - \Delta$ TRANSITION GPDs

We will make use of the helicity nonflip $p - \Delta$ GPDs defined by Belitsky and Radyushkin [4]. There are four even-parity transition GPDs,³

³In [4] Δ is defined as $p - p'$ and P as $p + p'$.

$$\begin{aligned}
 & \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \Delta^{++}(p', \nu') | \bar{u}(-z/2) \gamma^+ d(z/2) | p(p, \nu) \rangle |_{z^+=0, z_\perp=0} \\
 &= \frac{1}{2P^+} \bar{u}_\delta(p', \nu') \left\{ \frac{\Delta^\delta n^\mu - \Delta^\mu n^\delta}{m} \left(\gamma_\mu G_1(x, \xi, t) \right. \right. \\
 & \quad \left. \left. + \frac{P^\mu}{m} G_2(x, \xi, t) + \frac{\Delta^\mu}{m} G_3(x, \xi, t) \right) + \frac{\Delta^+ \Delta^\delta}{m^2} G_4(x, \xi, t) \right\} \gamma_5 u(p, \nu), \tag{11}
 \end{aligned}$$

and four odd-parity ones,

$$\begin{aligned}
 & \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \Delta^{++}(p', \nu') | \bar{u}(-z/2) \gamma^+ \gamma^5 d(z/2) | p(p, \nu) \rangle |_{z^+=0, z_\perp=0} \\
 &= \frac{1}{2P^+} \bar{u}_\delta(p', \nu') \left\{ \frac{\Delta^\delta n^\mu - \Delta^\mu n^\delta}{m} \left(\gamma_\mu \tilde{G}_1(x, \xi, t) \right. \right. \\
 & \quad \left. \left. + \frac{P^\mu}{m} \tilde{G}_2(x, \xi, t) \right) + n^\delta \tilde{G}_3(x, \xi, t) + \frac{\Delta^+ \Delta^\delta}{m^2} \tilde{G}_4(x, \xi, t) \right\} u(p, \nu), \tag{12}
 \end{aligned}$$

where the vector n is

$$n = [0, 1, \mathbf{0}_\perp]. \tag{13}$$

In comparison with the diagonal proton-proton GPDs, see for example [18], an extra γ_5 is introduced in order to match the parity of the $\Delta(1232)$. The $\Delta(1232)$ is considered as a stable particle. Therefore, one can apply time-reversal invariance to show that the $p - \Delta$ transition GPDs are real-valued functions [5]. In the definitions (11) and (12) $u(-z/2)$ and $d(z/2)$ denote quark field operators of a specified flavor and we work in the light-cone gauge $A^+ = 0$. The GPDs also depend on the factorization scale which, for convenience, is suppressed in (11), (12), and in the following. The $u_\delta(p', \nu')$ in the above relations denotes a Rarita-Schwinger spinor for the $\Delta(1232)$. It satisfies the Dirac equation and is subject to the subsidiary conditions

$$p'_\mu u^\mu(p', \nu') = 0, \quad \gamma_\mu u^\mu(p', \nu') = 0. \tag{14}$$

The Rarita-Schwinger spinors can, for instance, be found in [19]. The lowest moments of the GPDs G_1 , G_2 , and G_3 are related to the proton- Δ transition form factors, see [4].

Isospin symmetry relates the $p - \Delta^{++}$ GPDs to those of other $\Delta(1232)$ states [4]

$$G_{p\Delta^{++}}^{ud} = -\frac{\sqrt{3}}{2} G_{p\Delta^+}^{uu-dd} = -\sqrt{3} G_{p\Delta^0}^{du}. \tag{15}$$

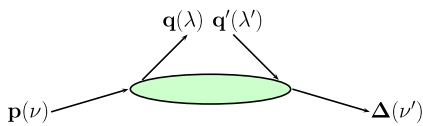


FIG. 1. The matrix element representing a proton- Δ GPD.

As a special exception we have indicated the flavor content of the GPDs in this relation. Thus, for instance, $G_{p\Delta^{++}}^{ud}$ is a GPD for which an d quark is emitted from the proton and u one reabsorbed forming the Δ^{++} . The relations (15) hold for all eight helicity nonflip GPDs as well as for the transversity GPDs that will be defined below. SU(3) flavor symmetry also relates the $p - \Delta$ GPDs to other octet-decuplet transition GPDs.

We are interested in matrix elements for emitted and reabsorbed partons with definite light-cone helicity. According to Diehl [18] this can be achieved by considering the following combinations of quark field operators:

$$\mathcal{O}_{\pm\pm} = \frac{1}{4} \bar{u}(-z/2) \gamma^+ (1 \pm \gamma_5) d(z/2), \tag{16}$$

where, in the region of $\xi < x < 1$, the corresponding matrix elements (see Fig. 1),

$$A_{\nu'\lambda\nu}^\Delta = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \Delta^{++}(p', \nu') | \mathcal{O}_{\lambda\lambda} | p(p, \nu) \rangle |_{z^+=0, z_\perp=0}. \tag{17}$$

describe the emission and reabsorption of on-shell quarks with helicity λ . In other regions of x one has, if necessary, to reinterpret an outgoing (incoming) quark with helicity λ as incoming (outgoing) quark with helicity $-\lambda$.

Using the definitions (11) and (12) one can readily work out the matrix elements for the kinematics described in Sec. II⁴:

⁴These results hold in any frame provided that \mathbf{p} and \mathbf{p}' lie in the 1-3 plane except for the fact that any $\sqrt{-t'}$ is to be multiplied by $\text{sign}(P^+ \Delta^1 - \Delta^+ P^1)$ [18].

$$\begin{aligned}
A_{3\pm\pm\pm}^\Delta &= +\frac{1}{2\sqrt{2}}\frac{\sqrt{-t'}}{m^2}\frac{1}{1-\xi}\left\{m(1-\xi^2)(G_1\pm\tilde{G}_1)+\kappa_-\left(\frac{1}{2}G_2-\xi G_3-\xi G_4\right)\pm\kappa_+\left(\frac{1}{2}\tilde{G}_2-\xi\tilde{G}_4\right)\right\}, \\
A_{3\pm\pm\pm}^\Delta &= +\frac{1}{2\sqrt{2}}\frac{t'}{m^2}\sqrt{\frac{1+\xi}{1-\xi}}\left\{-\frac{1}{2}G_2+\xi G_3+\xi G_4\mp\left(\frac{1}{2}\tilde{G}_2-\xi\tilde{G}_4\right)\right\}, \\
A_{1\pm\pm\pm}^\Delta &= \frac{1}{2\sqrt{6}}\frac{1}{m^2M}\frac{1}{(1-\xi^2)^{3/2}}\left\{(1-\xi^2)m[(1-\xi^2)t'(G_1\pm\tilde{G}_1)+2\xi M(\kappa_-G_1\pm\kappa_+\tilde{G}_1)]\right. \\
&\quad +\frac{1}{2}(1+\xi)(1-\xi^2)Mt'(G_2\pm\tilde{G}_2)+\frac{1}{2}[\xi(1+\xi)(3-\xi)M^2+\xi(1-\xi)^2m^2+(1-\xi^2)t'](\kappa_-G_2\pm\kappa_+\tilde{G}_2) \\
&\quad + (1-\xi)(1-\xi^2)m^2(\kappa_-G_3\pm\kappa_+\tilde{G}_3)+[\xi(1-3\xi)(1+\xi)\kappa_-M^2 \\
&\quad - (1-\xi^2)(\kappa_-+\xi(1+\xi)M)t'-(1+2\xi)(1-\xi)^2\kappa_-m^2]G_3 \\
&\quad \left.-\xi(1+\xi)(1-\xi^2)Mt'(G_4\pm\tilde{G}_4)-\xi((1-\xi^2)t'+\kappa_-\kappa_+)(\kappa_-G_4\pm\kappa_+\tilde{G}_4)\right\}, \\
A_{-1\pm\pm\pm}^\Delta &= \frac{1}{2\sqrt{6}}\frac{\sqrt{-t'}}{m^2M}\frac{1}{1-\xi^2}\left\{-2\xi(1-\xi^2)Mm(G_1\pm\tilde{G}_1)+(1-\xi)(1-\xi^2)m^2(G_1\mp\tilde{G}_1)\right. \\
&\quad -\frac{1}{2}(G_2\pm\tilde{G}_2)[(1-\xi^2)t'+m^2\xi(1-\xi)^2+M^2\xi(1+\xi)(3-\xi)]-\frac{1}{2}M(1+\xi)(\kappa_-G_2\pm\kappa_+\tilde{G}_2) \\
&\quad - (1-\xi)(1-\xi^2)m^2(G_3\pm\tilde{G}_3)+G_3[(1-\xi^2)t'+4\xi^2(1+\xi)M^2 \\
&\quad + (1+2\xi)(1-\xi)^2m^2-\xi(1-\xi^2)mM] \\
&\quad \left.+ (G_4\pm\tilde{G}_4)\xi((1-\xi^2)t'+\kappa_-\kappa_+)+\xi(1+\xi)M(\kappa_-G_4\pm\kappa_+\tilde{G}_4)\right\}. \tag{18}
\end{aligned}$$

Explicit helicities of photons, nucleons, and quarks are labeled by their signs, whereas the helicities of the $\Delta(1232)$ are denoted by ± 3 and ± 1 .

At leading-twist accuracy only longitudinally polarized photons contribute to the center-of-mass helicity amplitudes for the process $\gamma^*p \rightarrow \pi^-\Delta^{++}$. The amplitudes read [9]

$$\mathcal{M}_{0\nu'0\nu}^{tw2} = e_0 \int_{-1}^1 dx \sum_{\lambda} \mathcal{H}_{0\lambda 0\lambda}^{\pi^-} A_{\nu'\lambda\nu\lambda}^{\Delta(3)}. \tag{19}$$

The matrix elements $A_{\nu'\lambda\nu\lambda}^{\Delta(3)}$ appear in the isovector combination, i.e., any GPD contributes in the flavor combination

$$\tilde{G}_i^{(3)} = \tilde{G}_i^u - \tilde{G}_i^d. \tag{20}$$

Parity symmetry leads to the following relation for the $\gamma^*q \rightarrow \pi^-q$ subprocess amplitudes

$$\mathcal{H}_{0-\lambda'-\mu-\lambda}^{\pi} = -(-1)^{\mu-\lambda+\lambda'} \mathcal{H}_{0\lambda'\mu\lambda}^{\pi}. \tag{21}$$

With its help we can write $\mathcal{M}_{0\nu'0\nu}^{tw2}$ as

$$\mathcal{M}_{0\nu'0\nu}^{tw2} = e_0 \int_{-1}^1 dx \mathcal{H}_{0+0+}^{\pi} [A_{\nu'+\nu+}^{\Delta(3)} - A_{\nu'-\nu-}^{\Delta(3)}]. \tag{22}$$

Inspection of (18) reveals that only the odd-parity GPDs contribute to pion production. Explicitly, the helicity amplitudes read

$$\begin{aligned}
\mathcal{M}_{03.0+}^{tw2} &= \frac{e_0}{\sqrt{2}}\frac{\sqrt{-t'}}{m^2}\frac{1}{1-\xi}\left\{(1-\xi^2)m\langle\tilde{G}_1^{(3)}\rangle+\frac{\kappa_+}{2}\langle\tilde{G}_2^{(3)}\rangle-\xi\kappa_+\langle\tilde{G}_4^{(3)}\rangle\right\}, \\
\mathcal{M}_{0-3.0+}^{tw2} &= -\frac{e_0}{\sqrt{2}}\frac{t'}{m^2}\sqrt{\frac{1+\xi}{1-\xi}}\left\{\frac{1}{2}\langle\tilde{G}_2^{(3)}\rangle-\xi\langle\tilde{G}_4^{(3)}\rangle\right\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{01,0+}^{n\nu 2} &= \frac{e_0}{\sqrt{6}} \frac{1}{m^2 M} \frac{1}{(1-\xi^2)^{3/2}} \left\{ (1-\xi^2)m[(1-\xi^2)t' + 2\xi M\kappa_+] \langle \tilde{G}_1^{(3)} \rangle \right. \\
&\quad + \frac{1}{2} [(1+\xi)(1-\xi^2)Mt' + (\xi(1-\xi)^2m^2 + \xi(1+\xi)(3-\xi)M^2 + (1-\xi^2)t'\kappa_+) \langle \tilde{G}_2^{(3)} \rangle \\
&\quad \left. + (1-\xi^2)^2(1-\xi)m^2\kappa_+ \langle \tilde{G}_3^{(3)} \rangle - \xi[(1-\xi^2)t'(\kappa_+ + (1+\xi)M) + \kappa_-\kappa_+] \langle \tilde{G}_4^{(3)} \rangle \right\}, \\
\mathcal{M}_{0-1,0+}^{n\nu 2} &= \frac{e_0}{\sqrt{6}} \frac{\sqrt{-t'}}{m^2 M} \frac{1}{1-\xi^2} \left\{ -(1-\xi^2)m[2\xi M + (1-\xi)m] \langle \tilde{G}_1^{(3)} \rangle \right. \\
&\quad - \frac{1}{2} [(1-\xi^2)t' + \xi(1-\xi)^2m^2 + (1+\xi)M\kappa_+ + \xi(1+\xi)(3-\xi)M^2] \langle \tilde{G}_2^{(3)} \rangle \\
&\quad \left. - (1-\xi)(1-\xi^2)m^2 \langle \tilde{G}_3^{(3)} \rangle + \xi[(1-\xi^2)t' + \kappa_-\kappa_+ + (1+\xi)M\kappa_+] \langle \tilde{G}_4^{(3)} \rangle \right\}, \tag{23}
\end{aligned}$$

where

$$\langle \tilde{G}_i^{(3)} \rangle = \int_{-1}^1 dx \mathcal{H}_{0+0+}^{\pi^-} \tilde{G}_i^{(3)}. \tag{24}$$

Parity symmetry for the $\gamma^* p \rightarrow \pi^- \Delta^{++}$ helicity amplitudes leads to the relation

$$\mathcal{M}_{0-\nu'-\mu-\nu} = (-1)^{\mu-\nu+\nu'} \mathcal{M}_{0\nu'\mu\nu} \tag{25}$$

The generalization of these results to mesons in the final state other than the pion is straightforward.

IV. THE TRANSVERSITY $p - \Delta$ GPDs

For the definition of the eight transversity GPDs we have to consider the tensor matrix element

$$\begin{aligned}
&\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \Delta^{++}(p', \nu') | \bar{u}(-z/2) i\sigma^{+j} d(z/2) | p(p, \nu) \rangle |_{z^+=0, z_T=0} \\
&= \frac{1}{2P^+} \bar{u}_\delta(p', \nu') \left[G_{T1} \frac{p^\delta}{m} i\sigma^{+j} + G_{T2} p^\delta \frac{P^+ \Delta^j - \Delta^+ P^j}{m^3} \right. \\
&\quad + G_{T3} p^\delta \frac{\gamma^+ \Delta^j - \Delta^+ \gamma^j}{2m^2} + G_{T4} p^\delta \frac{\gamma^+ P^j - P^+ \gamma^j}{m^2} + G_{T5} (n^\delta \gamma^j - \gamma^\delta n^j) + G_{T6} \frac{n^\delta \Delta^j - \Delta^\delta n^j}{m} \left. \right] \gamma_5 u(p, \nu) \\
&\quad + \frac{1}{2P^+} \left[G_{T7} (\bar{u}^+(p', \nu') \gamma^j - \bar{u}^j(p', \nu') \gamma^+) + G_{T8} \frac{\bar{u}^+(p', \nu') \Delta^j - \bar{u}^j(p', \nu') \Delta^+}{m} \right] \gamma_5 u(p, \nu). \tag{27}
\end{aligned}$$

As the helicity nonflip GPDs the transversity ones are real-valued functions of x , ξ , and t . Any other antisymmetric tensor can be expressed as a linear combination of the eight tensors appearing in (27). This can be shown with the help of the Dirac equation and the generalized Gordon identities

$$\begin{aligned}
\bar{u}_\delta(p') i\sigma^{\alpha\beta} (p' \mp p)_\beta u(p) &= (M \pm m) \bar{u}_\delta(p') \gamma^\alpha u(p) - \bar{u}_\delta(p') (p' \pm p)^\alpha u(p), \\
\bar{u}_\delta(p') i\sigma^{\alpha\beta} (p' \pm p)_\beta \gamma_5 u(p) &= (M \pm m) \bar{u}_\delta(p') \gamma^\alpha \gamma_5 u(p) - \bar{u}_\delta(p') (p' \mp p)^\alpha \gamma_5 u(p). \tag{28}
\end{aligned}$$

Obviously, the set of GPDs is not unique.

$$\begin{aligned}
&\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \Delta^{++}(p', \nu') | \bar{u}(-z/2) i\sigma^{+j} d(z/2) | \\
&\quad \times p(p, \nu) \rangle |_{z^+=0, z_\perp=0} \\
&= \bar{u}_\delta(p', \nu') \Gamma^{\delta+j} \gamma_5 u(p, \nu). \tag{26}
\end{aligned}$$

Here, $\Gamma^{\delta+j}$ is a matrix in the Dirac space being antisymmetric in the Lorentz label $+$ and the transverse one $j(=1, 2)$. The matrix γ_5 occurs as a consequence of the spin and the parity of the $\Delta(1232)$. At disposal for the construction of Γ are the momenta P , Δ , and n as well as γ^α and $\sigma^{\alpha\beta}$ taking into account the conditions (14). The antisymmetric tensor can also be constructed directly with the Rarita-Schwinger spinor. A possible set of transversity GPDs is then

The matrix elements

$$A_{\nu'\lambda\nu\lambda}^{\Delta} = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \Delta^{++}(p'\nu') | \mathcal{O}_{-\lambda\lambda} | p(p, \nu) \rangle_{|z^+=0, z_{\perp}=0}, \quad (29)$$

where

$$\mathcal{O}_{\mp\pm} = \mp \frac{i}{4} \bar{u}(-z/2) (\sigma^{+1} \mp i\sigma^{+2}) d(z/2) \quad (30)$$

describe, in the region $\xi < x < 1$, the emission of an on-shell quark with helicity λ and the reabsorption of an on-shell quark with helicity $-\lambda$ [18]. Explicitly these matrix elements read

$$\begin{aligned} A_{3-,++}^{\Delta} &= -\frac{1}{4\sqrt{2}} \sqrt{\frac{1+\xi}{1-\xi}} \frac{t'}{m^2} \left\{ \frac{\kappa_-}{m} G_{T2} + (1+\xi)(G_{T3} - G_{T4}) \right\}, \\ A_{3+,+-}^{\Delta} &= \frac{1}{4\sqrt{2}} \sqrt{\frac{1+\xi}{1-\xi}} \left\{ \frac{t'}{m^2} \left[\frac{\kappa_-}{m} G_{T2} + (1-\xi)(G_{T3} + G_{T4}) \right] + 4(1-\xi)G_{T7} - 4\frac{\xi}{1+\xi} \frac{\kappa_-}{m} G_{T8} \right\}, \\ A_{-3-,++}^{\Delta} &= -\frac{1}{4\sqrt{2}} \frac{\sqrt{-t'}}{m} \left\{ -4(1+\xi)G_{T1} + (1+\xi) \frac{t'}{m^2} G_{T2} + 2\frac{\kappa_+}{m} \frac{\xi G_{T3} - G_{T4}}{1-\xi} - 4\xi G_{T8} \right\}, \\ A_{-3+,+-}^{\Delta} &= -\frac{1}{4\sqrt{2}} \frac{(-t')^{3/2}}{m^3} (1+\xi)G_{T2}, \\ A_{1-,++}^{\Delta} &= \frac{1}{4\sqrt{6}} \frac{\sqrt{-t'}}{m} \frac{1}{1-\xi} \left\{ -4(1-\xi^2)G_{T1} + \frac{\kappa_-^2 \kappa_+ + (1-\xi^2)t'(\kappa_- + (1+\xi)M)}{(1+\xi)m^2 M} G_{T2} \right. \\ &\quad \left. + \frac{\kappa_+ \kappa_- + (1-\xi^2)t'}{mM} (G_{T3} - G_{T4}) + 2\frac{\kappa_+}{m} (\xi G_{T3} - G_{T4}) \right. \\ &\quad \left. - 2(1-\xi) \left[(1-\xi^2) \frac{m}{M} G_{T5} + (1-\xi) \frac{\kappa_-}{M} G_{T6} + \frac{\kappa_-}{M} G_{T8} \right] \right\}, \\ A_{1+,+-}^{\Delta} &= -\frac{1}{4\sqrt{6}} \frac{\sqrt{-t'}}{m} \frac{1}{1+\xi} \left\{ \frac{\kappa_-^2 \kappa_+ + (1-\xi^2)t'(\kappa_- + M(1+\xi))}{(1-\xi)m^2 M} G_{T2} + \frac{\kappa_- \kappa_+ + (1-\xi^2)t'}{mM} [G_{T3} + G_{T4}] \right. \\ &\quad \left. + 2(1-\xi)(1-\xi^2) \frac{m}{M} G_{T5} - 2(1+\xi) \left[(1-\xi) \frac{\kappa_-}{M} G_{T6} - 2(1-\xi) \frac{m}{M} G_{T7} + \frac{\kappa_- + 2\xi M}{M} G_{T8} \right] \right\}, \\ A_{-1-,++}^{\Delta} &= \frac{1}{4\sqrt{6}} \frac{1}{\sqrt{1-\xi^2}} \left\{ -4 \frac{(1-\xi^2)t' + \kappa_- \kappa_+}{mM} G_{T1} + \frac{t'}{m^3 M} [\kappa_- (\kappa_+ + (1+\xi)M) + (1-\xi^2)t'] G_{T2} \right. \\ &\quad \left. + \frac{(1+\xi)^2 t'}{m^2} (G_{T3} - G_{T4}) + 2\frac{\kappa_+ (\kappa_- \kappa_+ + (1-\xi^2)t')}{(1-\xi^2)m^2 M} (\xi G_{T3} - G_{T4}) \right. \\ &\quad \left. - 4\frac{1-\xi}{M} \left(\kappa_+ G_{T5} + (1-\xi^2) \frac{t'}{2m} G_{T6} + (1-\xi)m G_{T7} \right) - 2\frac{(1-\xi^2)t' + 2\xi \kappa_- M}{mM} G_{T8} \right\}, \\ A_{-1+,+-}^{\Delta} &= -\frac{\sqrt{1-\xi^2}}{4\sqrt{6}} \frac{t'}{m^2} \left\{ \frac{\kappa_- (\kappa_+ + (1+\xi)M) + (1-\xi^2)t'}{(1-\xi^2)mM} G_{T2} + G_{T3} + G_{T4} - 2(1-\xi) \frac{m}{M} G_{T6} - 2\frac{m}{M} G_{T8} \right\}. \quad (31) \end{aligned}$$

Notice that the set of GPDs G_{Ti} , defined in (27), is linearly independent: the determinant of the 8×8 matrix that relates the GPDs to the matrix elements, A^{Δ} , is nonzero for $t' \neq 0$.

The matrix elements, A^{Δ} , are suitable for the calculation of amplitudes for any meson- $\Delta(1232)$ final state. In particular

the twist-3 $\gamma^* p \rightarrow \pi^- \Delta^{++}$ amplitudes are given by the convolutions of quark-helicity-flip subprocess amplitudes which are of twist-3 nature, with the matrix elements $A_{\nu'\lambda\nu\lambda}^{\Delta}$:

$$\mathcal{M}_{0\nu'\mu\nu}^{n\nu^3} = e_0 \int_{-1}^1 dx [\mathcal{H}_{0-\mu+}^{\pi^-} A_{\nu'-\nu+}^{\Delta} + \mathcal{H}_{0+\mu-}^{\pi^-} A_{\nu'+\nu-}^{\Delta}]. \quad (32)$$

Only transversally polarized photons contribute to the twist-3 amplitudes.

It can be checked that

$$A_{-\nu'-\lambda'-\nu-\lambda}^{\Delta} = (-1)^{\nu'-\lambda'-\nu+\lambda} A_{\nu'\lambda'\nu\lambda}^{\Delta}. \quad (33)$$

As we already mentioned the analogous property of the helicity amplitudes [see (25)] holds as a consequence of parity invariance. Inspection of (18) and (31) furthermore reveals that for $t' \rightarrow 0$ the matrix elements behave as

$$A_{\nu'\lambda'\nu\lambda}^{\Delta} \sim \sqrt{-t'}^{|\nu'-\lambda'-\nu+\lambda|}. \quad (34)$$

Also this property of the matrix elements is shared by the center-of-mass helicity amplitudes [20].

For $t \rightarrow 0$ the subprocess helicity amplitudes behave as [20]⁵

$$\mathcal{H}_{0\lambda'\mu\lambda} \sim \sqrt{-t'}^{-\lambda'-\mu+\lambda}. \quad (35)$$

This property of helicity amplitudes is a consequence of angular momentum conservation. Strictly speaking it reflects the conservation of the three-component of the

spin in the collinear situation at $t = 0$. In the generalized Bjorken regime, where $-t \ll Q^2$, the dominant contribution comes from subprocess amplitudes evaluated at $t = 0$. Hence, these subprocess amplitudes must be helicity non-flip ones implying

$$-\mu = \lambda' - \lambda. \quad (36)$$

Evidently, in this case (34) provides the correct t' dependence of the helicity amplitudes for $t' \rightarrow 0$ [20]

$$\mathcal{M}_{0\nu'\mu\nu} \sim \sqrt{-t'}^{|\nu'-\mu+\nu|}. \quad (37)$$

Since from (35) it follows that for $t \rightarrow 0$

$$\begin{aligned} \mathcal{H}_{0-++}^{\pi} &= -\mathcal{H}_{0+--}^{\pi} \propto \text{const}, \\ \mathcal{H}_{0--+}^{\pi} &= \mathcal{H}_{0+-+}^{\pi} \propto t \end{aligned} \quad (38)$$

the amplitudes (32) simplify—one of the two terms in this relation is zero. The $\gamma^* p \rightarrow \pi^- \Delta^{++}$ helicity amplitudes for transversally polarized photons read

$$\begin{aligned} \mathcal{M}_{03^{++}}^{tw3} &= -\frac{e_0}{4\sqrt{2}} \sqrt{\frac{1+\xi}{1-\xi}} \frac{t'}{m^2} \left[\frac{\kappa_-}{m} \langle G_{T2}^{(3)} \rangle + (1+\xi) (\langle G_{T3}^{(3)} \rangle - \langle G_{T4}^{(3)} \rangle) \right], \\ \mathcal{M}_{03^{-+}}^{tw3} &= -\frac{e_0}{4\sqrt{2}} \sqrt{\frac{1+\xi}{1-\xi}} \frac{t'}{m^2} \left(\frac{\kappa_-}{m} \langle G_{T2}^{(3)} \rangle + (1-\xi) (\langle G_{T3}^{(3)} \rangle + \langle G_{T4}^{(3)} \rangle) \right) + 4(1-\xi) \langle G_{T7}^{(3)} \rangle - 4 \frac{\xi}{1+\xi} \frac{\kappa_-}{m} \langle G_{T8}^{(3)} \rangle, \\ \mathcal{M}_{0-3^{++}}^{tw3} &= -\frac{e_0}{4\sqrt{2}} \frac{\sqrt{-t'}}{m} \left[-4(1+\xi) \langle G_{T1} \rangle + (1+\xi) \frac{t'}{m^2} \langle G_{T2}^{(3)} \rangle + 2 \frac{\kappa_+ \xi \langle G_{T3}^{(3)} \rangle - \langle G_{T4}^{(3)} \rangle}{1-\xi} - 4\xi \langle G_{T8}^{(3)} \rangle \right], \\ \mathcal{M}_{0-3^{-+}}^{tw3} &= \frac{e_0}{4\sqrt{2}} \frac{(-t')^{3/2}}{m^3} (1+\xi) \langle G_{T2}^{(3)} \rangle, \\ \mathcal{M}_{01^{++}}^{tw3} &= \frac{e_0}{2\sqrt{6}} \frac{\sqrt{-t'}}{m} \frac{1}{1-\xi} \left\{ -2(1+\xi) \langle G_{T1}^{(3)} \rangle + \frac{\kappa_-^2 \kappa_+ + (1-\xi^2) t' (\kappa_- + (1+\xi) M)}{2(1+\xi) m^2 M} \langle G_{T2}^{(3)} \rangle + \frac{\kappa_+ \kappa_- + (1-\xi^2) t'}{2mM} (\langle G_{T3}^{(3)} \rangle \right. \\ &\quad \left. - \langle G_{T4}^{(3)} \rangle) + \frac{\kappa_+}{m} (\xi \langle G_{T3}^{(3)} \rangle - \langle G_{T4}^{(3)} \rangle) - (1-\xi^2) (1-\xi) \frac{m}{M} \langle G_{T5}^{(3)} \rangle - (1-\xi)^2 \frac{\kappa_-}{M} \langle G_{T6}^{(3)} \rangle - (1-\xi) \frac{\kappa_-}{M} \langle G_{T8}^{(3)} \rangle \right\}, \\ \mathcal{M}_{01^{-+}}^{tw3} &= \frac{e_0}{4\sqrt{6}} \frac{\sqrt{-t'}}{m} \left\{ \frac{\kappa_-^2 \kappa_+ + (1-\xi^2) t' (\kappa_- + (1+\xi) M)}{(1-\xi^2) m^2 M} \langle G_{T2}^{(3)} \rangle + \frac{\kappa_- \kappa_+ + (1-\xi^2) t'}{(1+\xi) m M} \right. \\ &\quad \left. \times (\langle G_{T3}^{(3)} \rangle + \langle G_{T4}^{(3)} \rangle) + 2(1-\xi)^2 \frac{m}{M} \langle G_{T5}^{(3)} \rangle - 2(1-\xi) \left(\frac{\kappa_-}{M} \langle G_{T6}^{(3)} \rangle - 2 \frac{m}{M} \langle G_{T7}^{(3)} \rangle \right) - 2 \frac{\kappa_- + 2\xi M}{M} \langle G_{T8}^{(3)} \rangle \right\}, \\ \mathcal{M}_{0-1^{++}}^{tw3} &= \frac{e_0}{4\sqrt{6}} \frac{1}{\sqrt{1-\xi^2}} \left\{ -4 \frac{\kappa_- \kappa_+ + (1-\xi^2) t'}{mM} \langle G_{T1}^{(3)} \rangle + \frac{t'}{m^2} \frac{\kappa_- (\kappa_+ + (1+\xi) M) + (1-\xi^2) t'}{mM} \langle G_{T2}^{(3)} \rangle \right. \\ &\quad \left. + (1+\xi)^2 \frac{t'}{m^2} (\langle G_{T3}^{(3)} \rangle - \langle G_{T4}^{(3)} \rangle) \right\} \end{aligned}$$

⁵Since the quarks are considered as massless particles t_0 for the subprocess is zero.

$$\begin{aligned}
& + \frac{2\kappa_+(\kappa_-\kappa_+ + (1 - \xi^2)t')}{(1 - \xi^2)m^2M} (\xi \langle G_{T3}^{(3)} \rangle - \langle G_{T4}^{(3)} \rangle) \\
& - 4(1 - \xi) \frac{\kappa_+}{m} \langle G_{T5}^{(3)} \rangle - 2(1 - \xi)(1 - \xi^2) \frac{t'}{Mm} \langle G_{T6}^{(3)} \rangle \\
& - 4(1 - \xi)^2 \frac{m}{M} \langle G_{T7}^{(3)} \rangle - 2 \frac{2\xi\kappa_-m + (1 - \xi^2)t'}{mM} \langle G_{T8}^{(3)} \rangle \Big\}, \\
\mathcal{M}_{0-1\rightarrow+}^{tw3} & = \frac{e_0}{4\sqrt{6}} \frac{1}{\sqrt{1 - \xi^2}} \frac{t'}{m^2} \left\{ \frac{\kappa_-(\kappa_+ + (1 + \xi)M) + (1 - \xi^2)t'}{mM} \langle G_{T2}^{(3)} \rangle \right. \\
& \left. + (1 - \xi^2)(\langle G_{T3}^{(3)} \rangle + \langle G_{T4}^{(3)} \rangle) - 2(1 - \xi^2) \frac{m}{M} ((1 - \xi) \langle G_{T6}^{(3)} \rangle + \langle G_{T8}^{(3)} \rangle) \right\}, \tag{39}
\end{aligned}$$

where

$$\langle G_{Ti}^{(3)} \rangle = \int_{-1}^1 dx \mathcal{H}_{0\rightarrow++}^{\pi^-} G_{Ti}^{(3)}. \tag{40}$$

V. THE PION-POLE CONTRIBUTION

The pion-pole contribution, treated as a one-boson exchange term (see Fig. 2), reads

$$\mathcal{M}_{0\nu'\mu\nu}^{\text{pole}} = e_0 \frac{Q_{\pi\Delta}}{t - m_\pi^2} \bar{u}_\delta(p', \nu') \frac{\Delta^\delta}{m} u(p, \nu) (q - 2q')_\rho \epsilon^\rho(\mu), \tag{41}$$

where

$$Q_{\pi\Delta} = \sqrt{2} g_{\pi\Delta^{++}p} F_{\pi\Delta p}(t) F_\pi(Q^2). \tag{42}$$

As usual a possible t dependence of the pion form factor, F_π , is ignored (remember $-t \ll Q^2$ in the generalized Bjorken regime). The coupling of the pion to the proton and the Δ^{++} is described by a coupling constant, $g_{\pi\Delta^{++}p}$, and a t -dependent form factor, $F_{\pi\Delta p}$. Explicitly, the pole contribution to the $\gamma^* p \rightarrow \pi^- \Delta^{++}$ amplitudes reads

$$\begin{aligned}
\mathcal{M}_{030+}^{\text{pole}} & = -\frac{e_0}{\sqrt{2}} \frac{Q_{\pi\Delta}}{t - m_\pi^2} \frac{\sqrt{-t'}}{m} Q \frac{\kappa_+}{1 - \xi}, \\
\mathcal{M}_{0-30+}^{\text{pole}} & = \frac{e_0}{\sqrt{2}} \frac{Q_{\pi\Delta}}{t - m_\pi^2} \frac{t'}{m} Q \sqrt{\frac{1 + \xi}{1 - \xi}}, \\
\mathcal{M}_{03\pm+}^{\text{pole}} & = \mp \frac{e_0}{2} \frac{Q_{\pi\Delta}}{t - m_\pi^2} \sqrt{\frac{1 + \xi}{1 - \xi}} \frac{t'}{m} \kappa_+, \\
\mathcal{M}_{0-3\pm+}^{\text{pole}} & = \pm \frac{e_0}{2} \frac{Q_{\pi\Delta}}{t - m_\pi^2} \frac{(-t')^{3/2}}{m} (1 + \xi), \\
\mathcal{M}_{010+}^{\text{pole}} & = -\frac{e_0}{\sqrt{6}} \frac{Q_{\pi\Delta}}{t - m_\pi^2} \frac{Q}{mM} \frac{(1 - \xi^2)t'(\kappa_+ + (1 + \xi)M) + \kappa_-\kappa_+^2}{(1 - \xi^2)^{3/2}}, \\
\mathcal{M}_{0-10+}^{\text{pole}} & = \frac{e_0}{\sqrt{6}} \frac{Q_{\pi\Delta}}{t - m_\pi^2} \frac{\sqrt{-t'}}{mM} \frac{Q}{mM} \frac{(1 - \xi^2)t' + \kappa_+(\kappa_- + (1 + \xi)M)}{1 - \xi^2}, \\
\mathcal{M}_{01\pm+}^{\text{pole}} & = \pm \frac{e_0}{2\sqrt{3}} \frac{Q_{\pi\Delta}}{t - m_\pi^2} \frac{\sqrt{-t'}}{mM} \frac{Q}{mM} \frac{(1 - \xi^2)t'(\kappa_+ + (1 + \xi)M) + \kappa_-\kappa_+^2}{1 - \xi^2}, \\
\mathcal{M}_{0-1\pm+}^{\text{pole}} & = \pm \frac{e_0}{2\sqrt{3}} \frac{Q_{\pi\Delta}}{t - m_\pi^2} \frac{t'}{mM} \frac{(1 - \xi^2)t' + \kappa_+(\kappa_- + (1 + \xi)M)}{\sqrt{1 - \xi^2}} \Big]. \tag{43}
\end{aligned}$$

Taking into account the pion form factor which is hidden in $Q_{\pi\Delta}$ [see (42)], with its $1/Q^2$ behavior the pion-pole contribution to the longitudinal amplitudes behaves as $1/Q$ asymptotically whereas the transverse ones drop as $1/Q^2$. Thus, in this respect, the pion-pole contribution to the $\gamma^* p \rightarrow \pi^- \Delta^{++}$ amplitudes behaves as that one to the $\gamma^* p \rightarrow \pi^- p$ amplitudes. There are corrections to the amplitudes given in (43) suppressed by $1/Q^2$ that are extremely complicated. They can be neglected even for the longitudinal amplitudes since they are still suppressed by $1/Q$ compared to the transverse amplitudes.

The amplitudes satisfy the familiar symmetry relation for pion (unnatural parity) exchange

$$\mathcal{M}_{0\nu'\mp\nu}^{\text{pole}} = -\mathcal{M}_{0\nu'\pm\nu}^{\text{pole}}. \quad (44)$$

This is the same relation as for $\gamma^* p \rightarrow \pi N$ because both processes possess the same upper vertex. The relation (44) follows from the dynamics and forces the following amplitudes

$$\mathcal{M}_{0-1^{++}}^{\text{pole}}, \quad \mathcal{M}_{03^{++}}^{\text{pole}}, \quad \mathcal{M}_{0-3^{++}}^{\text{pole}} \quad (45)$$

to vanish by a factor of t' faster to zero for $t' \rightarrow 0$ than is forced by angular momentum conservation, see (37).

For longitudinally polarized photons the pion-pole contribution (41) can be cast into the form

$$\mathcal{M}_{0\nu'\nu}^{\text{pole}} = \frac{e_0}{2P^+} \bar{u}(p', \nu') \frac{\Delta^\delta \Delta^+}{m^2} u(p, \nu) \left[\frac{Q_{\pi\Delta}}{t - m_\pi^2} q' \cdot \epsilon(0) \frac{2m}{\xi} \right]. \quad (46)$$

If one is interested in the leading-twist contribution that dominates for $Q^2 \rightarrow \infty$, than the comparison with (12) and (23) reveals that the pion-pole contributes to the GPD \tilde{G}_4 asymptotically. The term in brackets is its convolution with the leading-twist subprocess amplitude

$$\mathcal{H}_{0^+0^+}^{\pi^-} = 2 \frac{Q F_\pi^{\text{pert.}}(Q^2)}{f_\pi \langle 1/\tau \rangle_\pi} \left[\frac{e_d}{x - \xi + i\epsilon} + \frac{e_u}{x + \xi - i\epsilon} \right], \quad (47)$$

where $f_\pi = 0.131$ GeV is the pion decay constant, $\langle 1/\tau \rangle_\pi$ denotes the $1/\tau$ moment of the leading-twist pion distribution amplitude, Φ_π , and e_a is the charge of the flavor- a quark in units of the positron charge, e_0 . Finally, $F_\pi^{\text{pert.}}$ is the leading-twist, leading-order result for the electromagnetic form factor of the pion [21–23]. This result for the convolution can be achieved if in analogy to the case of $\gamma^* p \rightarrow \pi^+ n$ [24,25] the GPD is

$$\begin{aligned} \tilde{G}_4^{\text{pole}} &= \tilde{G}_{4\text{pole}}^u - \tilde{G}_{4\text{pole}}^d = \Theta(|x| \leq \xi) \\ &\times \Phi_\pi \left(\frac{x + \xi}{2\xi} \right) \frac{m f_\pi g_{\pi\Delta N} F_{\pi\Delta N}(t)}{\sqrt{2}\xi t - m_\pi^2}. \end{aligned} \quad (48)$$

The difference between the leading-twist contribution from the pion pole and its full contribution (41) is the replacement of $F_\pi^{\text{pert.}}$ by the full form factor as extracted from the data on the longitudinal $\gamma^* p \rightarrow \pi^+ n$ cross section [26]. Since $F_\pi^{\text{pert.}}$ is substantially smaller than its experimental value a leading-twist analysis of pion electroproduction within the handbag approach evidently fails. In [11] the pion-pole term is, therefore, treated as an one-bose-exchange contribution in the analysis of exclusive π^+ data [27] and reasonable agreement with experiment is obtained, see also [28]. We are going to apply the same procedure to the process of interest here, $\gamma^* p \rightarrow \pi^- \Delta^{++}$.

In the evaluation of observables for $\gamma^* p \rightarrow \pi^- \Delta^{++}$ we will use the following parametrization of the form factors [11,12]

$$F_\pi = \frac{1}{1 + 2.0 \text{ GeV}^{-2} Q^2}, \quad F_{\pi\Delta p} = \frac{\Lambda_N^2 - m_\pi^2}{\Lambda_N^2 - t}, \quad (49)$$

with $\Lambda_N = 0.44$ GeV. For the $\pi\Delta^{++}p$ coupling constant we take the value [29]

$$g_{\pi\Delta^{++}p} = 14.8. \quad (50)$$

VI. GPDs IN THE LARGE- N_C LIMIT

As we have seen there are 12 GPDs contributing to the exclusive electroproduction of $\pi^- \Delta^{++}$. All of them are unknown functions and, at present, there are no relevant data to fix them. In order to be able to make predictions, or better estimates, we therefore take recourse to a theoretical approach, namely the large- N_C limit in which the $p - \Delta^+$ GPDs are related to the proton-proton ones [4,9]. In this limit the nucleon and the $\Delta(1232)$ are different rotational excitations of the same classical object, the chiral soliton, and they are degenerated in mass. This leads to relations between the matrix elements of the quark-field operators, (16) or (30), in the isovector combination [4]

$$\sqrt{2} \langle p_{\uparrow(\downarrow)} | \mathcal{O}^{(3)} | p_{\uparrow} \rangle = \langle \Delta_{\uparrow(\downarrow)}^+ | \mathcal{O}^{(3)} | p_{\uparrow} \rangle. \quad (51)$$

The arrows denote spin states. From (51) relations between the $p - \Delta^+$ GPDs and the proton-proton ones follow. Subsequently, isospin symmetry (15) links the $p - \Delta^+$ GPDs to the $p - \Delta^{++}$ ones. Since the baryon mass, M_C , is large, of order N_C , one has to make a nonrelativistic reduction of the Dirac bilinears appearing in (12) and (27). This can be done consistently in a Breit frame, see Fig. 3. In this frame there is no energy transfer but only a momentum transfer, Δ_3 . For the $p - p$ GPDs we use those defined in [18]. The matrix elements in (51) are then calculated in the Breit frame and expanded in powers of Δ_3/M_C . The coefficients of the various powers of Δ_3/M_C for the two matrix elements are equated which provides the

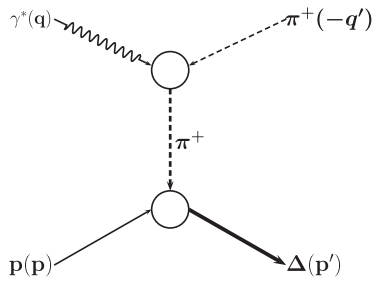


FIG. 2. The pion-pole contribution.

desired relations between the proton-proton GPDs and the $p - \Delta^+$ ones. In combination with the isospin relation (15) one obtains for the odd-parity $p - \Delta^{++}$ GPDs with this method [4,9]

$$\begin{aligned}\tilde{G}_3^{(3)} &= \frac{3}{2}(\tilde{H}^u - \tilde{H}^d), \\ \tilde{G}_4^{(3)} &= \frac{3}{8}(\tilde{E}^u - \tilde{E}^d),\end{aligned}\quad (52)$$

where, for each GPD, only the leading contribution in N_C is taken into account. It can be shown that [30]

$$\tilde{H}^u - \tilde{H}^d \sim N_C^2, \quad \tilde{E}^u - \tilde{E}^d \sim N_C^4. \quad (53)$$

The respective opposite flavor combinations are suppressed by $1/N_C$.

The corresponding application of this method to the matrix elements of the operator (30) leads to the relation

$$G_{T5}^{(3)} + \frac{1}{2}G_{T7}^{(3)} = -\frac{3}{2}(H_T^u - H_T^d) \quad (54)$$

between the $p - \Delta^{++}$ and the $p - p$ GPDs for the leading term of the Δ_3/M_C expansion. The higher-order terms relate even more complicated combinations of $p - \Delta$ GPDs to either the $p - p$ transversity GPD \tilde{E}_T or E_T which are unknown as yet and usually neglected in applications of the handbag approach [11,12,31]. The GPDs \tilde{H}_T and $\tilde{E}_T = 2\tilde{H}_T + E_T$ do not contribute to the matrix elements $\langle p_\downarrow | \mathcal{O}_{\mp\pm}^{(3)} | p_\uparrow \rangle$ evaluated in the Breit frame. This is in accordance with the large- N_C result, that the isovector

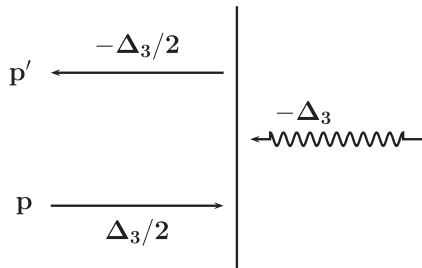


FIG. 3. The Breit frame.

combinations of these GPDs are suppressed compared to the sum of the corresponding u - and d -quark GPDs [30]. Hence, the large- N_C results for the transversity GPDs are of no help without additional assumptions. In the following we are going to probe two assumptions: all $p - \Delta$ transversity GPDs are zero except of either G_{T5} or G_{T7} and apply (54) to the nonzero one.

VII. THE SUBPROCESS AMPLITUDES AND THE $p - p$ GPDs

Due to the large- N_C results (52) and (54) and because the processes $\gamma^* p \rightarrow \pi^- \Delta^{++}$ and $\gamma^* n \rightarrow \pi^- p$ have the same hard subprocesses, namely $\gamma^* d \rightarrow \pi^- u$, the convolutions (24) and (40) are the same as those of the $p - p$ GPDs $\tilde{H}^{(3)}$, $\tilde{E}^{(3)}$, and $H_T^{(3)}$ up to numerical prefactors. The calculation of the latter has been performed in previous work [11,12,32] within the generalized handbag approach and is described in great detail therein.⁶ Therefore, only the basics facts will be sketched in this section.

The main idea of the generalized handbag approach is to keep the quark transverse momentum, k_\perp , in the subprocess while the emission and reabsorption of the partons from the baryons are still treated collinear to the baryon momenta. The subprocess amplitudes (at $t = 0$) read

$$\begin{aligned}\mathcal{H}_{0\lambda',\mu\lambda}^\pi &= \int d\tau d^2b \hat{\Psi}_{\pi,-\lambda'\lambda}(\tau, -\mathbf{b}, \mu_F) \hat{F}_{0\lambda',\mu\lambda}^\pi(x, \xi, \tau, Q^2, \mathbf{b}, \mu_R) \\ &\times \alpha_s(\mu_R) \exp[-S(\tau, \mathbf{b}, Q^2, \mu_F, \mu_R)]\end{aligned}\quad (55)$$

in the impact parameter space; \mathbf{b} is canonically conjugated to the quark transverse momenta. The Sudakov factor, S , has been calculated by Botts and Sterman [33] in next-to-leading-log approximation using resummation techniques and having recourse to the renormalization group. It takes into account the gluon radiation resulting from the separation of color charges, which is a consequence of the quark transverse momenta. The properties of the Sudakov factor force the following choice of the factorization scale: $\mu_F = 1/b$. The renormalization scale is taken to be the largest mass scale appearing in the subprocess, i.e., $\mu_R = \max(\tau Q, (1 - \tau)Q, 1/b)$ (τ is the momentum fraction of the quark entering the meson). The strong coupling constant, α_s , is evaluated from four flavors and $\Lambda_{\text{QCD}} = 0.181$ GeV. The inclusion of the quark transverse momenta and the Sudakov factor has two advantages—first the magnitude of the subprocess amplitudes are somewhat reduced as compared to a collinear calculation which leads to a better agreement with experiment (see, e.g., [12]) and, second, the

⁶For the process $\gamma^* n \rightarrow \pi^- p$ the $n - p$ transition GPDs occur that, by flavor symmetry [6,25], are related to the isovector combination of the proton-proton GPDs $K_{n \rightarrow p}^{ud} = K^u - K^d$ for any GPD $K = H, E, \tilde{H}, \dots$

infrared singularity occurring in a collinear calculation of the twist-3 subprocess amplitude is regularized.

For the twist-2 and twist-3 hard scattering amplitudes, F^π , evaluated to lowest order of perturbative QCD, we refer the reader to Refs. [11,32]. The last object to be explained is $\hat{\Psi}_{\pi,\lambda\lambda}$. It represents the Fourier transform of a soft pion light-cone wave function. For twist 2 one has $\lambda' = \lambda$ and the distribution amplitude associated to $\hat{\Psi}$, is the familiar twist-2 one. On the other hand, for twist-3 one has $\lambda' = -\lambda$ and a twist-3 wave function is required. The wave functions are parametrized as Gaussians in k_\perp [11]:

$$\begin{aligned}\Psi_{\pi,-+} &= 8\pi^2 \frac{f_\pi}{\sqrt{2N_C}} \frac{a_\pi^2}{\tau\bar{\tau}} \Phi_\pi(\tau) \exp[-a_\pi^2 k_\perp^2 / (\tau\bar{\tau})], \\ \Psi_{\pi,++} &= \frac{16\pi^{3/2}}{\sqrt{2N_C}} f_\pi a_p^3 k_\perp \exp[-a_p^2 k_\perp^2],\end{aligned}\quad (56)$$

where $\bar{\tau} = 1 - \tau$. For the twist-2 and twist-3 transverse size parameters of the pion we take the values (see [12])

$$a_\pi = 0.859 \text{ GeV}^{-1}, \quad a_p = 1.8 \text{ GeV}^{-1}. \quad (57)$$

For the familiar leading-twist pion distribution amplitude, Φ_π , we simply take the asymptotic form $6\tau\bar{\tau}$. The distribution amplitude associated with the twist-3 wave function, $\Psi_{\pi,++}$, is $\Phi_p \equiv 1$ as is fixed by the equation of motion if the three-body twist-3 distribution amplitude is assumed to be zero [34]. This ansatz for the twist-3 wave function makes it clear that the corresponding subprocess amplitude is calculated in the Wandzura-Wilczek approximation.⁷

For the numerical studies to be presented below, a value of 2.0 GeV is used for μ_π at the initial scale $\mu_0 = 2$ GeV. Since the current-quark masses decrease with increasing scale μ_π is scale dependent. The respective anomalous dimension is $4/\beta_0 = 12/25$ for four flavors.

We will make use of the $p-p$ GPDs determined in [11,12,32]. The idea is to parametrize the zero-skewness GPDs. Their products with suitable weight functions are considered as double distributions from which the full, skewness-dependent GPDs can be calculated [36]. The zero-skewness GPDs for flavor a are parametrized as

$$K_i^a(x, \xi = 0, t) = K_i^a(x, \xi = t = 0) \exp[(b_i^a - \alpha_i^a \ln(x))t]. \quad (58)$$

The forward limits, $\xi, t \rightarrow 0$, of the GPDs \tilde{H} and H_T are given by the polarized and transversity parton densities, respectively. In order to respect the Soffer bound the transversity density is parametrized as

⁷In [35] we have calculated the full twist-3 subprocess amplitude, i.e., its two-body as well as its three-body contribution. The application of this result to hard exclusive electroproduction of mesons is in progress.

TABLE I. Regge parameters and normalizations of the valence-quark $p-p$ GPDs, quoted at the initial scale $\mu_0 = 2$ GeV. Lacking parameters indicate that the corresponding parameters are part of the parton densities.

GPD	$\alpha(0)$	$\alpha'[\text{GeV}^{-2}]$	$b[\text{GeV}^{-2}]$	N^u	N^d
\tilde{H}	...	0.45	0.59
$\tilde{E}^{\text{n.p.}}$	0.48	0.45	0.9	14.0	4.0
H_T	...	0.45	0.3	1.1	-0.3

$$\delta^a = N_{H_T}^a \sqrt{x(1-x)} [q^a(x) + \Delta q^a(x)]. \quad (59)$$

The unpolarized and polarized parton densities are taken from [37,38], respectively. For the nonpole part of \tilde{E} the forward limit is not accessible in deep inelastic lepton-nucleon scattering and, hence, unknown. Therefore, it is parametrized like the PDFs

$$\tilde{E}^{\text{n.p.}a}(x, \xi = t = 0) = N_e^a x^{-\alpha_e^a(0)} (1-x)^{\beta_e^a} \quad (60)$$

with the additional parameters to be adjusted to the electroproduction data. The parameters of the zero-skewness GPDs, compiled in Table I, are taken from [12]. The powers β_e^a are set to the following values

$$\tilde{E}^{\text{n.p.}}: \quad \beta_e^u = \beta_e^d = 5. \quad (61)$$

A flavor-symmetric sea is assumed for all GPDs (i.e., $K^{\bar{u}} = K^{\bar{d}}$). Splitting any GPD into valance and sea quark GPDs

$$K^a = K^{a_{\text{val}}} + K^{\bar{a}} \quad (62)$$

one sees that only valance-quark GPDs contribute to the isovector combination (20):

$$K^u - K^d = K^{u_{\text{val}}} - K^{d_{\text{val}}}. \quad (63)$$

In this case the convolutions (29) and (40) are integrated from $-\xi$ to 1.

As is well known the GPDs evolve with the scale. Since the factorization of the amplitudes into GPDs and a subprocess is treated collinearly, the GPDs do not know about the impact-parameter dependence in the subprocess— b is integrated over. Hence, the subprocess factorization scale, μ_F , does not apply to the GPDs, it refers to the factorization of the soft meson wave function and the remaining hard part of the subprocess. The scale of the GPDs is therefore taken as the photon virtuality.

VIII. PREDICTIONS

The full $\gamma^* p \rightarrow \pi^- \Delta^{++}$ amplitudes for longitudinally polarized photons are

$$\mathcal{M}_{0\nu'0\nu} = \mathcal{M}_{0\nu'0\nu}^{tw2} + \mathcal{M}_{0\nu'0\nu}^{\text{pole}}, \quad (64)$$

and for transversally polarized photons ($\mu = \pm 1$)

$$\mathcal{M}_{0\nu'\mu\nu} = \mathcal{M}_{0\nu'\mu\nu}^{tw3} + \mathcal{M}_{0\nu'\mu\nu}^{\text{pole}}. \quad (65)$$

For the twist-3 contributions we consider the two scenarios

$$\begin{aligned} I: \quad G_{T5}^{(3)} &= -\frac{3}{2}H_T^{(3)}, \quad G_{Ti} = 0, \quad i \neq 5, \\ II: \quad G_{T7}^{(3)} &= -3H_T^{(3)}, \quad G_{Ti} = 0, \quad i \neq 7. \end{aligned} \quad (66)$$

For scenario I the twist-3 amplitudes (39) simplify to

$$\begin{aligned} \mathcal{M}_{01^{++}}^{tw3} &= \frac{e_0}{4} \sqrt{\frac{3}{2}} \frac{\sqrt{-t'}}{M} (1 - \xi^2) \langle H_T^{(3)} \rangle, \\ \mathcal{M}_{01^{-+}}^{tw3} &= -\frac{e_0}{4} \sqrt{\frac{3}{2}} \frac{\sqrt{-t'}}{M} (1 - \xi)^2 \langle H_T^{(3)} \rangle, \\ \mathcal{M}_{0^{-1}^{++}}^{tw3} &= \frac{e_0}{2} \sqrt{\frac{3}{2}} \sqrt{\frac{1 - \xi \kappa_+}{1 + \xi m}} \langle H_T^{(3)} \rangle, \end{aligned} \quad (67)$$

and all other twist-3 helicity amplitudes are zero. For scenario II the nonzero twist-3 amplitudes are

$$\begin{aligned} \mathcal{M}_{03^{-+}}^{tw3} &= 3 \frac{e_0}{\sqrt{2}} \sqrt{1 - \xi^2} \langle H_T^{(3)} \rangle, \\ \mathcal{M}_{01^{-+}}^{tw3} &= -e_0 \sqrt{\frac{3}{2}} \frac{\sqrt{-t'}}{M} (1 - \xi) \langle H_T^{(3)} \rangle, \\ \mathcal{M}_{0^{-1}^{++}}^{tw3} &= e_0 \sqrt{\frac{3}{2}} \sqrt{\frac{1 - \xi}{1 + \xi}} (1 - \xi) \frac{m}{M} \langle H_T^{(3)} \rangle. \end{aligned} \quad (68)$$

For both the scenarios the helicity nonflip amplitudes dominate at small $-t$, $\mathcal{M}_{0^{-1}^{++}}^{tw3}$ for scenario I and $\mathcal{M}_{03^{-+}}^{tw3}$ for scenario II.

The electroproduction cross section for the $\pi^- \Delta^{++}$ final state is defined by

$$\begin{aligned} \frac{d\sigma^4}{dW^2 dQ^2 dt d\phi} &= \frac{\alpha_{\text{em}}(W^2 - m^2)}{16\pi^2 E_L^2 m^2 Q^2 (1 - \varepsilon)} \\ &\times \left(\frac{d\sigma_T}{dt} + \varepsilon \frac{d\sigma_L}{dt} + \varepsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} \right. \\ &\left. + \sqrt{2\varepsilon(1 + \varepsilon)} \cos \phi \frac{d\sigma_{LT}}{dt} \right), \end{aligned} \quad (69)$$

where ϕ is the azimuthal angle between the lepton and the hadron plane, E_L is the beam energy and ε is the ratio of the longitudinal and transversal photon fluxes. If a value of the latter quantity is needed we evaluate it for a beam energy of 10.6 GeV and obtain $\varepsilon = 0.77$. The partial cross sections read

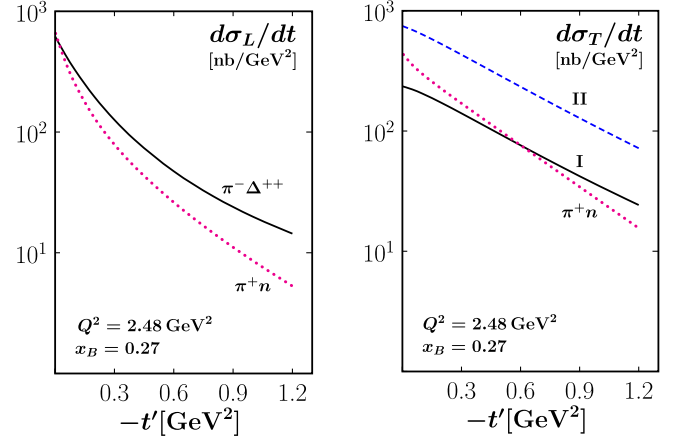


FIG. 4. The longitudinal (left) and the transverse (right) cross sections of $\gamma^* p \rightarrow \pi^- \Delta^{++}$ versus $-t'$. The solid (dashed) lines represent the predictions obtained for scenario I (II). For comparison the dotted lines are the results for $\gamma^* p \rightarrow \pi^+ n$ obtained with the same GPDs.

$$\begin{aligned} \frac{d\sigma_L}{dt} &= \frac{\sum_{\nu'} |\mathcal{M}_{0\nu'0+}|^2}{16\pi(W^2 - m^2) \sqrt{\Lambda(W^2, -Q^2, m^2)}}, \\ \frac{d\sigma_T}{dt} &= \frac{\sum_{\nu'} [|\mathcal{M}_{0\nu'++}|^2 + |\mathcal{M}_{0\nu'-+}|^2]}{32\pi(W^2 - m^2) \sqrt{\Lambda(W^2, -Q^2, m^2)}}, \\ \frac{d\sigma_{TT}}{dt} &= -\frac{\sum_{\nu'} \text{Re}[\mathcal{M}_{0\nu'++}^* \mathcal{M}_{0\nu'-+}]}{16\pi(W^2 - m^2) \sqrt{\Lambda(W^2, -Q^2, m^2)}}, \\ \frac{d\sigma_{LT}}{dt} &= -\sqrt{2} \frac{\sum_{\nu'} \text{Re}[\mathcal{M}_{0\nu'0+}^* (\mathcal{M}_{0\nu'++} - \mathcal{M}_{0\nu'-+})]}{32\pi(W^2 - m^2) \sqrt{\Lambda(W^2, -Q^2, m^2)}}. \end{aligned} \quad (70)$$

The Mandelstam function is defined by

$$\begin{aligned} \Lambda(W^2, -Q^2, m^2) &= W^4 + Q^4 + m^4 + 2W^2 Q^2 \\ &\quad - 2W^2 m^2 + 2Q^2 m^2. \end{aligned} \quad (71)$$

Equations (69) and (70) are in agreement with results quoted in [39] but, by definition, the interference cross sections have opposite signs here and in [39] and the longitudinal-transverse cross section is larger by a factor $\sqrt{2}$ in [39] than by us.

In Figs. 4 and 5 we display predictions for the partial cross sections for the two scenarios and compare with the corresponding $\gamma^* p \rightarrow \pi^+ n$ cross sections. We show the observables as functions of t' and the reader should be aware of the quite different values of t_0 (for the kinematics shown in the plots t_0 is -0.323 GeV² and -0.088 GeV² for the $\pi^- \Delta^{++}$ and the $\pi^+ n$ channel, respectively). Therefore, at a given value of t' , the convolutions of a GPD for $\pi^- \Delta^{++}$ and for $\pi^+ n$ are evaluated at different values of t . This partially compensates the prefactors in (52) and (54).

The longitudinal cross section for which the predictions from the two scenarios fall together, is dominated by the

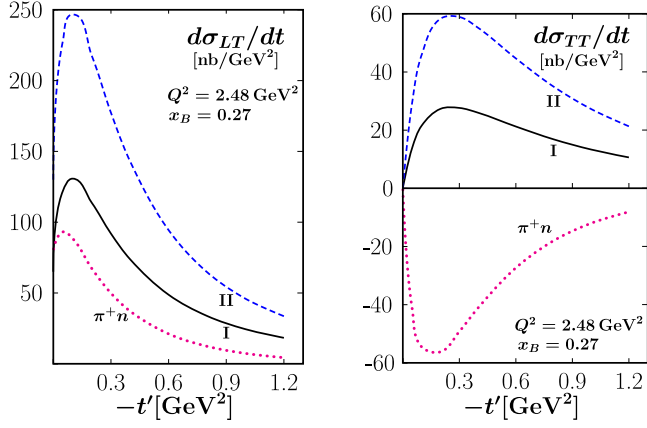


FIG. 5. The longitudinal-transverse (left) and the transverse-transverse (right) interference cross sections of $\gamma^* p \rightarrow \pi^- \Delta^{++}$. For other notations see Fig. 4.

pion-pole contribution. Since the $\pi \Delta^{++} p$ coupling constant (50) is somewhat larger than the familiar pion-nucleon one [19] and also the convolution of $\tilde{G}_{T3}^{(3)}$ is larger than that of $\tilde{H}^{(3)}$ [see (52)] it is clear that the $\pi^- \Delta^{++}$ cross section is larger than the $\pi^+ n$ one. For the other partial cross sections there are substantial differences between the scenarios I and II. One also sees that, at small $-t'$, the longitudinal cross section is larger than the transverse one as is the case for the $\pi^+ n$ channel [11]. The interference cross sections are larger for scenario II than for scenario I. Note that for the $\pi^+ n$ channel the transversity GPD \tilde{E}_T is taken into account but its contribution is very small [12] since, in agreement with large- N_C results [30], \tilde{E}_T for u and d quarks have the same sign and about the same size. This is to be contrasted with the $\pi^0 p$ channel to which \tilde{E}_T contributes in the combination

$$\frac{1}{\sqrt{2}} [e_u \tilde{E}_T^u - e_d \tilde{E}_T^d], \quad (72)$$

which is large and provides an important contribution to the $\pi^0 p$ cross sections [12]. Particularly interesting is the opposite sign of $d\sigma_{TT}$ for the $\pi^- \Delta^{++}$ and the $\pi^+ n$ channels. The dominant contribution to $d\sigma_{TT}$ comes from the interference of a twist-3 helicity nonflip amplitude, being proportional to the convolution $\langle H_T \rangle$, and a pion-pole contribution. The Q^2 and W dependencies of the $\pi^- \Delta^{++}$ partial cross sections is similar to those of the $\pi^- p$ ones.

The uncertainties of our predictions are very large. First, the large- N_C considerations do not really fix the transversity GPDs, only a sum of G_{T5} and G_{T7} is related to the proton-proton GPD H_T . This forces us to invent the two scenarios by an additional assumption. Furthermore, the quality of the large- N_C results (52) is unknown since there are as yet no $\pi \Delta(1232)$ cross section data available to check this relation. Only the qualitative results on the relative magnitudes of the $u + d$ and $u - d$ quark combinations of

$p - p$ GPDs [30] are known to be in fair agreement with phenomenology. We regard the differences between the results for the two scenarios as an indication of the uncertainties.

For most spin asymmetries the uncertainties are likely even larger than for the cross sections because they also depend on the imaginary part of products of helicity amplitudes, i.e., on the rather small differences between the phases of the amplitudes. Therefore, we refrain from showing predictions for asymmetries, like the beam spin asymmetry for which preliminary data are already available [10]. An exception is the asymmetry A_{LL} measured with longitudinally polarized beam and target which, like the transverse cross section, only depends on the absolute values of the amplitudes for transversally polarized photons:

$$A_{LL} = \sqrt{1 - \epsilon^2} \frac{1}{2\sigma_0} \sum_{\nu'} [|\mathcal{M}_{0\nu'+} |^2 - |\mathcal{M}_{0\nu'-} |^2], \quad (73)$$

where

$$\sigma_0 = \sum_{\nu'} [|\mathcal{M}_{0\nu'+} |^2 + |\mathcal{M}_{0\nu'-} |^2 + \epsilon |\mathcal{M}_{0\nu'0+} |^2]. \quad (74)$$

This asymmetry is obtained from an integral upon the electroproduction cross section. There is a correction due to the fact that the target polarization is defined with respect to the lepton beam direction in experiment and not with respect to the direction of the virtual photon, see [39]. For the kinematics of interest in this work this correction is small and neglected by us. Predictions of A_{LL} for the two scenarios are shown in Fig. 6 and compared to this asymmetry for the $\pi^+ n$ channel. The magnitudes of the predictions evaluated from the two scenarios are close to

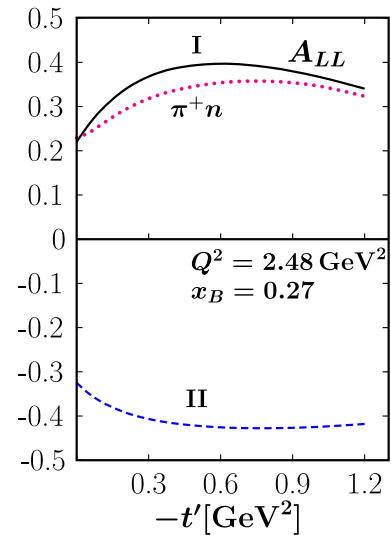


FIG. 6. The asymmetry A_{LL} for the $\pi^- \Delta^{++}$ and $\pi^+ n$ channels. For other notations see Fig. 4.

that one for the π^+n channel. However, the predictions for the two scenarios for $\pi^-\Delta^{++}$ have opposite signs. As an inspection of (67) and (68) reveals this is understandable: for scenario I the dominant helicity nonflip amplitude, $M_{0-1^{++}}^{tw3}$, provides a positive contribution to A_{LL} [see (73)] whereas for the second scenario the dominant amplitude, $M_{03^{-+}}^{tw3}$, gives a negative contribution to it. The pion pole is unimportant for this asymmetry, most of its contributions cancel (note that $\mathcal{M}_{0\nu'^{++}}^{\text{pole}} = -\mathcal{M}_{0\nu'^{-+}}^{\text{pole}}$).

IX. SUMMARY

We investigated exclusive electroproduction of $\pi^-\Delta^{++}$ in the generalized Bjorken regime within the handbag approach. In addition to the known eight helicity nonflip $p-\Delta$ GPDs [4] we defined a set of eight transversity GPDs. For both sets of GPDs we calculated the $p-\Delta^{++}$ matrix elements, $A_{\nu'\lambda'\lambda}^{\Delta}$ and the helicity amplitudes, $\mathcal{M}_{0\nu'\mu\nu}$, for $\gamma^*p \rightarrow \pi^-\Delta^{++}$. We also calculated the pion-pole contribution to this process and showed that, to leading-twist accuracy, the pion pole contributes to the GPD \tilde{G}_4 .

In order to generate predictions for the $\pi\Delta^{++}$ partial cross sections we had to take recourse to the large- N_C limit where a few of the $p-\Delta$ GPDs are related to the diagonal proton-proton ones. In the large N_C limit the odd-parity helicity nonflip GPDs \tilde{G}_3 and \tilde{G}_4 in the isovector combination are related to the corresponding combinations of \tilde{H} and \tilde{E} , respectively [4,9]. For the transversity GPDs we found rather complicated relations. Therefore, we only utilized the relation between the sum $G_{T5} + G_{T7}/2$ and the $p-p$ GPD H_T obtained from the leading-order term in the Δ_3/M_C expansion. We were forced to make the additional assumption that either G_{T7} is zero (scenario I) or G_{T5} (scenario II). All other transversity GPDs are neglected. Taking the parametrization of the $p-p$ GPDs from previous work [11,12] and evaluating the subprocess

amplitudes within the modified perturbative approach which effectively takes into account the transverse size of the meson, we are in the position to predict the partial cross section for $\gamma^*p \rightarrow \pi^-\Delta^{++}$.

A precise calculation of observables for exclusive electroproduction of $\pi^-\Delta^{++}$ or for other $\pi\Delta(1232)$ channels is beyond feasibility at present. There are many uncertainties: For instance due to the use of large- N_C results or due to the neglect of many of the $p-\Delta$ GPDs. Other uncertainties, although of weaker importance, are the parametrizations of the $p-p$ GPDs or the exact treatment of the twist-3 contribution, e.g., the neglect of possible three-particle configurations of the meson state (see [35]). With regard to all these uncertainties we consider our investigation of exclusive electroproduction of $\pi^-\Delta^{++}$ as a rough estimate. The trends and magnitudes are probably correct but not the details. This is also the reason why we did not discuss asymmetries—with A_{LL} as an exception. Most of the asymmetries depend on the badly known relative phases of the helicity amplitudes. With regard to the experimental program of the Jefferson lab our study seems to be timely. Its results are perhaps useful as a starting point of a GPD analysis of data to come.

The extension of our study to other octet-decuplet transitions is straightforward with the help of SU(3) flavor symmetry [4,9]. Of course, this way also various octet-octet transitions GPDs are related to the $p-p$ ones [6]. Some of these relations have already been used to evaluate the electroproduction cross sections for $K\Lambda$ and $K\Sigma$ [8,12].

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