Independence of current components, polarization vectors, and reference frames in the light-front quark model analysis of meson decay constants

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The issue of resulting in the same physical observables with different current components, in particular from the minus current, has been challenging in the light-front quark model (LFQM) even for the computation of the two-point functions such as meson decay constants. At the level of one-body current matrix element computation, we show the uniqueness of pseudoscalar and vector meson decay constants using all available components including the minus component of the current in the LFQM consistent with the Bakamjian-Thomas construction. Regardless of the current components, the polarization vectors, and the reference frames, the meson decay constants are uniquely determined in the noninteracting constituent quark and antiquark basis while the interactions of the constituents are added to the meson mass operator in the LFQM.

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I. INTRODUCTION

Light-front dynamics (LFD) [1–3] is a useful framework for studying hadron structures with its direct applications in Minkowski space. The distinct features of LFD compared with other forms of Hamiltonian dynamics include that the rational energy-momentum dispersion relation in the LFD induces the suppression of vacuum fluctuations and that the LFD carries the maximal number (seven) of the kinematic generators of transformations for the Poincaré group.

The light-front quark model (LFQM) based on the LFD has been quite successful in describing the mass spectra and electroweak properties of mesons by treating mesons as quark-antiquark bound states [4–17]. Typically in the LFQM [4–12], the constituent quark (Q) and antiquark ($\bar{Q}$) are constrained to be on their respective mass shells, and the spin-orbit (SO) wave function is thus obtained by the interaction-independent Melosh transformation [18] from the ordinary equal-time static representation. While the hadronic form factors and decay constants are obtained from the matrix elements of a one-body current directly in the three-dimensional light front (LF) momentum space effectively with the plus current, $J^+ = J^0 + J^3$, the calculation with different current components such as the transverse current $\mathbf{J}_\perp$ and the minus current, $J^- = J^0 - J^3$, should in principle yield the same results of the hadronic form factors and decay constants as the physical observables must be Lorentz invariant. However, in practice, the issue of resulting in the same physical observables with different current components has been challenging in LFQM and led discussions on the Fock space truncation [19], the zero-mode contribution [20,21], etc., in a variety of contexts [22–27]. Thus, clarifying this long-standing issue even in the two-point function level, such as the computation of decay constants, is of great importance to construct a reliable light-front model to study hadron structure.

Focusing on the vector meson decay amplitudes with the matrix element of one-body current [28], two of us showed that the decay constants obtained from J^+ with longitudinal polarization and $\mathbf{J}_\perp$ with transverse polarization are numerically the same by imposing the on-mass shell condition of the constituents consistently throughout the LFQM analysis. In fact, it was demonstrated that those two decay constants obtained from using the so-called "Type II" [28] link between the manifestly covariant Bethe-Salpeter (BS) model and the standard LFQM are exactly equal to those obtained directly in the standard LFQM imposing the on-mass shell condition of the constituents.

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This on-mass shell condition is equivalent to imposing the four-momentum conservation $P = p_1 + p_2$ at the meson-quark vertex, where P and $p_{1(2)}$ are the meson and quark (antiquark) momenta, respectively, which implies the self-consistent replacement of the physical meson mass M with the invariant mass M_0 of the quark-antiquark system. The generalization of the results in Ref. [28] to any possible combination of current components and of polarization is the main object of the present paper.

We notice in retrospect that this condition for the one-body current matrix element computation is consistent with the Bakamjian-Thomas (BT) construction [29,30] up to that level of computation, where the meson state is constructed by the noninteracting $Q\bar{Q}$ representations while the interaction is included into the mass operator $M \coloneqq M_0 + V_{Q\bar{Q}}$ to satisfy the group structure or commutation relations. The main purpose of the present work is to demonstrate that the long-standing issue of resulting in the same physical observables with different current components can be resolved for the two-point physical observables, explicitly in the analysis of the decay constants for the one-body current matrix element computation with the aforementioned self-consistent condition stemmed from the BT construction. We note that the meson system of the constituent quark and antiquark presented in this work is immune to the limitation of the BT construction regarding the cluster separability for the systems of more than two particles [31].

Within the scope described above, we show for the first time the uniqueness of pseudoscalar and vector meson decay constants using all available components of the current in our LFQM being consistent with the BT construction for the one-body current matrix element computation. We explicitly demonstrate that the same decay constants are resulted not only for all possible current components but also for the polarization vectors independent of the reference frame. Our explicit demonstration is in fact related to the Lorentz invariant property, which could not be obtained in the relativistic quark models based on LFD without implementing the aforementioned self-consistency condition.

II. THEORETICAL FRAMEWORK

While our demonstration can be applied to the mesons composed of unequal-mass constituents in general, here we focus on the equal-mass case of the constituents for simplicity. The essential aspect of the standard LFQM for the meson state [4–10] is to saturate the Fock state expansion by the constituent quark and antiquark and treat the Fock state in a noninteracting representation. The interactions are then encoded in the LF wave function $\Psi^{JJ_z}_{\lambda_1\lambda_2}(\mathbf{p}_1, \mathbf{p}_2)$, which is the mass eigenfunction. The meson state $|M(P, J, J_z)\rangle \equiv |\mathcal{M}\rangle$ of momentum P and spin state (J, J_z) can be constructed as

$$|\mathcal{M}\rangle = \int [\mathrm{d}^3\mathbf{p}_1][\mathrm{d}^3\mathbf{p}_2] 2(2\pi)^3 \delta^3(\mathbf{P} - \mathbf{p}_1 - \mathbf{p}_2) \times \sum_{\lambda_1,\lambda_2} \Psi^{JJ_z}_{\lambda_1\lambda_2}(\mathbf{p}_1, \mathbf{p}_2) |Q(p_1, \lambda_1)\bar{Q}(p_2, \lambda_2)\rangle, \quad (1)$$

where p_i^μ and λ_i are the momenta and the helicities of the on-mass shell ($p_i^2 = m_i^2$) constituent quarks, respectively. For the equal-mass case, we set $m_i = m$. Here, $\mathbf{p} = (p^+, \mathbf{p}_\perp)$ and $[\mathrm{d}^3\mathbf{p}_i] \equiv \mathrm{d}p_i^+ \mathrm{d}^2\mathbf{p}_{i\perp}/(16\pi^3)$. The LF relative momentum variables $(x, \mathbf{k}_\perp)$ are defined as $x_i = p_i^+/P^+$ and $\mathbf{k}_{i\perp} = \mathbf{p}_{i\perp} - x_i \mathbf{P}_\perp$, which satisfy $\sum_i x_i = 1$ and $\sum_i \mathbf{k}_{i\perp} = 0$. By setting $x \equiv x_1$ and $\mathbf{k}_\perp \equiv \mathbf{k}_{1\perp}$, we decompose the LF wave function as $\Psi^{JJ_z}_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) = \phi(x, \mathbf{k}_\perp) \mathcal{R}^{JJ_z}_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp)$, where $\phi(x, \mathbf{k}_\perp)$ is the radial wave function and $\mathcal{R}^{JJ_z}_{\lambda_1\lambda_2}$ is the SO wave function obtained by the interaction-independent Melosh transformation.

The covariant forms of the SO wave functions are $\mathcal{R}^{JJ_z}_{\lambda_1\lambda_2} = \bar{u}_{\lambda_1}(p_1)\Gamma v_{\lambda_2}(p_2)/(\sqrt{2}M_0)$, where $\Gamma = \gamma_5$ and $-\hat{\not{\epsilon}}(J_z) + \hat{\epsilon}(J_z)\cdot(p_1 - p_2)/(M_0 + 2m)$ for pseudoscalar and vector mesons, respectively,[1] and $M_0^2 = \sum_i (\mathbf{k}_{i\perp}^2 + m_i^2)/x_i$. The polarization vectors $\hat{\epsilon}^\mu(J_z)$ of the vector meson are given by $\hat{\epsilon}^\mu(\pm 1) = (0, 2\boldsymbol{\epsilon}_\perp(\pm 1)\cdot\mathbf{P}_\perp/P^+, \boldsymbol{\epsilon}_\perp(\pm 1))$ with $\boldsymbol{\epsilon}_\perp(\pm 1) = \mp(1, \pm i)/\sqrt{2}$ for transverse polarizations and $\hat{\epsilon}^\mu(0) = (P^+, (\mathbf{P}_\perp^2 - M_0^2)/P^+, \mathbf{P}_\perp)/M_0$ for longitudinal polarization [4,5]. One of the important characteristics of our LFQM reflecting the feature of the BT construction is to use the invariant M_0 rather than the physical mass M in defining both $\mathcal{R}^{JJ_z}_{\lambda_1\lambda_2}$ and $\hat{\epsilon}^\mu(0)$, which should be contrasted with other covariant field theoretic computations in LFD [13–15] where the physical mass M is used in defining both the SO wave function and the longitudinal polarization vector. Because of this property imposed by the on-mass shell condition of the constituents, which is consistent with the BT construction, the SO wave functions satisfy the unitary condition, $\sum_{\lambda_1,\lambda_2} \mathcal{R}^{JJ_z\dagger}_{\lambda_1\lambda_2}\mathcal{R}^{JJ_z}_{\lambda_1\lambda_2} = 1$, independent of model parameters. Furthermore, the longitudinal polarization vector satisfies $P\cdot\hat{\epsilon}(0) = 0$ only when $P = p_1 + p_2$ or equivalently $P^2 = M_0^2$, which we call the self-consistency condition. We should note that the LF energy conservation ($P^- = p_1^- + p_2^-$) in addition to the LF three-momentum conservation at the meson-quark vertex is required for the calculations of the physical observables using the matrix element with the one-body current to be consistent with the BT construction [29,30] up to the level of computation presented in this work as the meson state is constructed by the noninteracting $Q\bar{Q}$ representations. The interaction between quark and antiquark is implemented in the radial

[1] The Lorentz invariant properties with the BT construction discussed here would in general apply to other types of the wave function vertices as well, e.g., the axial vector coupling for the pseudoscalar meson vertex in the analysis of axial anomaly.

wave function through the mass spectroscopic analysis as discussed below. This condition will be shown to be important in the complete covariant analysis of the meson decay constants in the LFQM.

The interactions between quark and antiquark are included in the mass operator [29,30] to compute the mass eigenvalue of the meson state. In our LFQM, we treat the radial wave function as a trial function for the variational principle to the QCD-motivated effective Hamiltonian $H_{Q\bar{Q}}$, i.e., $H_{Q\bar{Q}}|\Psi\rangle = (M_0 + V_{Q\bar{Q}})|\Psi\rangle = M|\Psi\rangle$, so that the mass eigenvalue is obtained from the interaction potential $V_{Q\bar{Q}}$ in addition to the relativistic free energies of quark and antiquark. The detailed mass spectroscopic analysis can be found in Refs. [11,12]. For the radial wave function of the $1S$ state meson, we use the Gaussian wave function $\phi(x, \mathbf{k}_\perp) = \sqrt{\partial k_z/\partial x}\hat{\phi}(\mathbf{k})$ as a trial wave function, where $\hat{\phi}(\mathbf{k}) = (4\pi^{3/4}/\beta^{3/2})\exp(-\mathbf{k}^2/2\beta^2)$ and β is the variational parameter fixed by mass spectroscopic analysis. It should be mentioned, however, that our observation and discussion about the independence of the model predictions with respect to the components of the current, the polarization vectors, and the reference frames is completely irrelevant to any specific form of the radial wave function as long as the Jacobian factor $\sqrt{\partial k_z/\partial x}$, which is crucial for the Lorentz invariance of the LFQM, is properly included.

III. DECAY CONSTANTS

The decay constants, $f_{\rm P}$ for the pseudoscalar (P) meson, and $f_{\rm V}$ and $f_{\rm V}^T$ for the longitudinally and transversely polarized vector (V) mesons, with their corresponding one-body currents are defined as

$$\begin{aligned}
\langle 0|\bar{q}\gamma^\mu\gamma_5 q|{\rm P}(P)\rangle &= i f_{\rm P} P^\mu, \\
\langle 0|\bar{q}\gamma^\mu q|{\rm V}(P, J_z)\rangle &= f_{\rm V} M \epsilon^\mu(J_z), \\
\langle 0|\bar{q}\sigma^{\mu\nu} q|{\rm V}(P, J_z)\rangle &= i f_{\rm V}^T[\epsilon^\mu(J_z)P^\nu - \epsilon^\nu(J_z)P^\mu],
\end{aligned} \quad (2)$$

where P^μ and M are the meson momentum and mass, respectively, and $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$.

In principle, the Lorentz structures on the right-hand side of Eq. (2) should be independent of the internal momentum of the quark-antiquark system. For instance, the longitudinal polarization vector of the vector meson defined on the right-hand side of Eq. (2) should be used with the physical mass M, i.e., $\epsilon^\mu(0) = (P^+, (\mathbf{P}_\perp^2 - M^2)/P^+, \mathbf{P}_\perp)/M$. Typically, one can obtain the decay constants using some particular choice of the currents and polarizations to preserve the Lorentz structures as given on the right-hand side of Eq. (2) [5,10], (i) $f_{\rm P}$ from $\gamma^{(+,\perp)}\gamma_5$, (ii) $f_{\rm V}$ from γ^+ and $\epsilon(0)$, and (iii) $f_{\rm V}^T$ from $\sigma^{\perp+}$ and $\epsilon(+1)$ as one can see from Eq. (2). Those results of $(f_{\rm P}, f_{\rm V}, f_{\rm V}^T)$ obtained from (i)–(iii) have already been provided as the standard LFQM results [10] [see Eqs. (18)–(20) in Ref. [10]], rewriting the decay constants $\mathcal{F} = \{f_{\rm P}, f_{\rm V}, f_{\rm V}^T\}$ as

$$\mathcal{F} = \sqrt{N_c}\int_0^1 {\rm d}x \int \frac{{\rm d}^2\mathbf{k}_\perp}{16\pi^3}\phi(x, \mathbf{k}_\perp) \times \frac{1}{\mathcal{P}}\sum_{\lambda_1,\lambda_2}\mathcal{R}^{JJ_z}_{\lambda_1\bar{\lambda}_2}\left[\frac{\bar{v}_{\lambda_2}(p_2)}{\sqrt{x_2}}\mathcal{G}\frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right], \quad (3)$$

where $N_c = 3$ is the number of color, and the current operators $\mathcal{G} = \{\gamma^\mu\gamma_5, \gamma^\mu, \sigma^{\mu\nu}\}$ pair with the corresponding Lorentz structures $\mathcal{P} = \{P^\mu, M\epsilon^\mu(J_z), i[\epsilon^\mu(J_z)P^\nu - \epsilon^\nu(J_z)P^\mu]\}$ defined on the right hand side of Eq. (2).

However, we note here that the Lorentz structures $\mathcal{P} = \{P^\mu, M\epsilon^\mu(J_z), i[\epsilon^\mu(J_z)P^\nu - \epsilon^\nu(J_z)P^\mu]\}$ in Eq. (3) for the particular choices of the currents and polarizations taken in (i)–(iii) apparently satisfy the self-consistency condition, $P = p_1 + p_2$ in $\mathcal{P}$, due to the momentum conservation for the $+$ and $\perp$ components. Such manifest realization of the self-consistency condition cannot be attained for the choices beyond (i)–(iii) taken in the computation. Nevertheless, we realize that the identical self-consistency condition can still be verified by linking the computation of the same physical observables between the manifestly covariant BS model and the standard LFQM as shown in Refs. [28,32]. Using different components of the currents and polarization vectors such as $f_{\rm P}$ from $\gamma^-\gamma_5$ [32] and $f_{\rm V}$ from $\gamma^\perp$ and $\epsilon(+1)$ [28], we find in this work that the same self-consistency condition, $P = p_1 + p_2$ in $\mathcal{P}$, is applicable to all the Lorentz structures $\mathcal{P}$ in Eq. (3) to attain the complete covariance of the decay constants for all possible combinations of currents and polarization vectors including the ones not discussed in Refs. [5,10,28,32]. As mentioned in the Introduction, this self-consistency condition for the one-body current matrix element computation is consistent with the BT construction up to that level of computation in which the meson state is constructed by the noninteracting $Q\bar{Q}$ representations while the interaction is included in the mass operator $M \coloneqq M_0 + V_{Q\bar{Q}}$.

IV. LINK BETWEEN THE BS MODEL AND THE LFQM

For a full demonstration of the validity of the identical self-consistency condition, $P = p_1 + p_2$ or $M \to M_0$ in $\mathcal{P}$, engaging any combination of current component and of polarization vector in Eq. (3), we briefly discuss the link between the manifestly covariant BS model and the standard LFQM. In the manifestly covariant BS model [28,32], the generic form of the matrix element for the decay amplitude $A_{\rm BS} \equiv \langle 0|\bar{q}\mathcal{G}q|V(P, J_z)\rangle$ in the one-loop approximation is given by

$$A_{\mathrm{BS}} = N_c \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4} \frac{H_V S_{\mathrm{BS}}}{(p_1^2 - m^2 + i\epsilon)(p_2^2 - m^2 + i\epsilon)},$$
$$= N_c \int_0^1 \frac{\mathrm{d}x}{(1-x)} \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \chi(x, \mathbf{k}_\perp)[S_{\mathrm{BS}}]_{\mathrm{on}}, \qquad (4)$$

where the trace term $S_{\mathrm{BS}} = \mathrm{Tr}[\mathcal{G}(\not{p}_1 + m)\Gamma(-\not{p}_2 + m)]$ in the first line becomes $[S_{\mathrm{BS}}]_{\mathrm{on}}$ in the second line after the light-front energy integration p_2^- picking up the on-mass shell pole $p_2^2 = m^2$ and the resulted light-front BS vertex function $\chi(x, \mathbf{k}_\perp)$ after the pole integration is given by $\chi(x, \mathbf{k}_\perp) = g/[x(M^2 - M_0^2)]$. We note that the manifestly covariant meson vertex $\Gamma_{\mathrm{V}} = \not{\epsilon}(J_z) - (p_1 - p_2) \cdot \epsilon(J_z)/(M + 2m)$ carries the longitudinal polarization $\epsilon^\mu(0)$ including the physical meson mass M in contrast to the standard LFQM where $\hat{\epsilon}^\mu(0)$ is used for the spin-orbit wave function. While we take here a constant $Q\bar{Q}$ bound-state vertex function, i.e., $H_V = g$, for simplicity, we should note that the usual multipole ansatz [28] for the $Q\bar{Q}$ bound-state vertex function such as $H_V = g/(p^2 - \Lambda^2 + i\epsilon)^n$ with the parameter Λ only alters the form of $\chi(x, \mathbf{k}_\perp)$ but not the generic form of Eq. (4). Comparing the computation between the covariant BS model and the standard LFQM, we find that the link, i.e., $\sqrt{2N_c}\chi(x, \mathbf{k}_\perp)/(1-x) \to \phi(x, \mathbf{k}_\perp)/\sqrt{m^2 + \mathbf{k}_\perp^2}$ and $M \to M_0$, applies to all possible components of the currents and polarization vectors as has already been found for the case of f_{P} obtained from $\mathcal{G} = (\gamma^+, \gamma^-)\gamma_5$ [32] and f_{V} obtained from $\mathcal{G} = (\gamma^+, \gamma^\perp)$ with $(\epsilon(0), \epsilon(+))$ [28], respectively. One should note that the possible instantaneous and zero-mode contributions vanish with the above link as shown in Refs. [28,32]. The instantaneous contribution with the γ^+ operator appears always proportional to $(M^2 - M_0^2)$, and the zero-mode operator found in the two-point function [28] is proportional to $Z_2 = x(M^2 - M_0^2) + (1 - 2x)M^2$ for the equal quark mass case. These contributions vanish under the link $M \to M_0$ discussed in Refs. [28,32]. Note that the term $(1 - 2x)M^2$ in Z_2 vanishes as well after the replacement of $M \to M_0$ because it is an odd function of x while other terms in the integrand are even in x as shown in Ref. [28] and as can be seen later also in this work. For the complete analysis of $(f_{\mathrm{P}}, f_{\mathrm{V}}, f_{\mathrm{V}}^T)$ on the validity of the link between the BS model and the standard LFQM extending the previous works [28,32], we show the generic form of the decay constants in Eq. (4) obtained from the on-mass shell quark propagating part as

$$\mathcal{F}_{\mathrm{BS}} = N_c \int_0^1 \frac{\mathrm{d}x}{(1-x)} \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{8\pi^3} \chi(x, \mathbf{k}_\perp) \mathcal{O}_{\mathrm{BS}}(x, \mathbf{k}_\perp), \qquad (5)$$

where the operators $\mathcal{O}_{\mathrm{BS}}$ are defined by $\mathcal{O}_{\mathrm{BS}} = [S_{\mathrm{BS}}]_{\mathrm{on}}/2\mathcal{P}$, and $\mathcal{O}_{\mathrm{BS}} = \{\mathcal{O}_{\mathrm{P}}, \mathcal{O}_{\mathrm{V}}(J_z), \mathcal{O}_{\mathrm{V}}^T(J_z)\}$ corresponding to $\mathcal{F}_{\mathrm{BS}} = \{f_{\mathrm{P}}, f_{\mathrm{V}}, f_{\mathrm{V}}^T\}$ for the equal quark and antiquark mass case are summarized in Table I.

TABLE I. The operators $\mathcal{O}_{\mathrm{BS}}$ defined in Eq. (5). Note that $\mathcal{O}_{\mathrm{BS}}$ turns into $\mathcal{O}_{\mathrm{LFQM}}$ if $M \to M_0$ is made, which are exactly the same as those defined in Eq. (6) for the standard LFQM.

$\mathcal{F}$	$\mathcal{G}$	$\epsilon(J_z)$	$\mathcal{O}_{\mathrm{BS}}$	$\mathcal{O}_{\mathrm{LFQM}}$
f_{P}	$\gamma^{(+,\perp)}\gamma_5$		$2m$	$2m$
	$\gamma^-\gamma_5$		$2m\frac{M_0^2+\mathbf{P}_\perp^2}{M^2+\mathbf{P}_\perp^2}$	$2m$
f_{V}	$\gamma^{(+,\perp)}$	$\epsilon(0)$	$\frac{(M_0^2+M^2)[m+2x(1-x)M]}{M(M+2m)}$	$2m + \frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	γ^-	$\epsilon(0)$	$\frac{\hat{\epsilon}^-(0)M_0(M_0^2+M^2)[m+2x(1-x)M]}{\epsilon^-(0)M^2(M+2m)}$	$2m + \frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\gamma^{(\perp,-)}$	$\epsilon(+1)$	$\frac{1}{M}(M_0^2 - \frac{2M\mathbf{k}_\perp^2}{M+2m})$	$M_0 - \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
f_{V}^T	$\sigma^{\perp+}$	$\epsilon(+1)$	$2m + \frac{2\mathbf{k}_\perp^2}{M+2m}$	$2m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\sigma^{\perp-}$	$\epsilon(+1)$	$2\frac{M_0^2}{M^2}(m + \frac{\mathbf{k}_\perp^2}{M+2m})$	$2m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	σ^{+-}	$\epsilon(0)$	$\frac{M_0^2+M^2}{2M^2}(\frac{2mM+M_0^2}{M+2m} - \frac{4\mathbf{k}_\perp^2}{M+2m})$	$M_0 - \frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$

As we shall show later in Eq. (6), the standard LFQM results $\mathcal{F}$ obtained directly from Eq. (3) are indeed exactly the same as the ones obtained from $\mathcal{F}_{\mathrm{BS}}$ applying the "Type II" [28] link, i.e., $\sqrt{2N_c}\chi(x, \mathbf{k}_\perp)/(1-x) \to \phi(x, \mathbf{k}_\perp)/\sqrt{m^2 + \mathbf{k}_\perp^2}$ and $M \to M_0$, in Eq. (5). The corresponding operators $\mathcal{O}_{\mathrm{LFQM}}$ obtained from replacement of $M \to M_0$ in $\mathcal{O}_{\mathrm{BS}}$ are also summarized in Table I. In other words, the same self-consistency condition, $P = p_1 + p_2$ or $M \to M_0$ in $\mathcal{P}$, should be applied to all the Lorentz structures $\mathcal{P}$ in Eq. (3) to attain the complete covariance of the decay constants in the standard LFQM for all possible combinations of currents and polarization vectors including the ones not discussed in Refs. [5,10,28,32].

In the covariant BS model, we also note that some combinations of the current components and polarization vectors [28,32] encounter the LF zero modes and give correct results only if the zero-mode contributions are not missed but taken into account properly. One may note from Table I that only the operator $\mathcal{O}_{\mathrm{BS}} = 2m$ for f_{P} obtained from $\gamma^{(+,\perp)}\gamma_5$ exactly matches with $\mathcal{O}_{\mathrm{LFQM}}$ in the standard LFQM, indicating that all other BS results for the decay constants except that case would require zero-mode contributions to give correct covariant results.

As the zero-mode contribution is locked into a single point of the LF longitudinal momentum in the meson decay process, one of the constituents of the meson carries the entire momentum of the meson, and it is important to capture the effect from a pair creation of particles with zero LF longitudinal momenta indicating an intensive interaction with the vacuum. The zero modes that appeared for some particular combinations of the current and polarization in the BS model are found to match with the substitution of $M \to M_0$ for those combinations in the standard LFQM. The present analysis of the meson decay constant with all possible combinations of the current and polarization confirmed the previous interpretation [28] for the substitution $M \to M_0$ in the standard LFQM with effective degrees of freedom represented by the constituent quark and antiquark as providing the view of an effective zero-mode cloud around the quark and antiquark inside the meson.

In a nutshell, we show the explicit final formula of the decay constants directly obtained from Eq. (3) for the equal quark and antiquark mass case:

$$\mathcal{F} = \sqrt{6}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{m^2+\mathbf{k}_\perp^2}} \mathcal{O}_{\mathrm{LFQM}}(x,\mathbf{k}_\perp), \qquad (6)$$

where the operators $\mathcal{O}_{\mathrm{LFQM}} = \{\mathcal{O}_{\mathrm{P}}, \mathcal{O}_{\mathrm{V}}(J_z), \mathcal{O}_{\mathrm{V}}^T(J_z)\}$ corresponding to $\mathcal{F} = \{f_{\mathrm{P}}, f_{\mathrm{V}}, f_{\mathrm{V}}^T\}$, respectively, are obtained from the sum of each helicity contribution, $\mathcal{O}_{\mathrm{LFQM}} = \sum_{\lambda_1,\lambda_2} H_{\lambda_1\lambda_2}$. It should be noted that Eq. (6) is a generalized formula for the previous standard LFQM results [10] for $(f_{\mathrm{P}}, f_{\mathrm{V}}^{(T)})$ where the substitution $M \to M_0$ is manifest due to the $+$ and $\perp$ momentum conservation. Equation (6) is indeed exactly the same as the one obtained from applying the link between the BS model and the standard LFQM to Eq. (5).

We summarize our results of $\mathcal{O}_{\mathrm{LFQM}}$ and the helicity contributions $H_{\lambda_1\lambda_2}$ to $\mathcal{O}_{\mathrm{LFQM}}$ for all possible components of the current $\mathcal{G}$ and the polarization vectors $\epsilon(J_z)$ in Table II. The results of $J_z = -1$ are not explicitly given for f_{V} and f_{V}^T as they correspond to those of $J_z = +1$ with $H_{\lambda_1\lambda_2}(J_z = -1) = H_{-\lambda_1-\lambda_2}(J_z = +1)$ absorbing the usual parity-related phase factor [33,34] within the definition of $H_{\lambda_1\lambda_2}$ as the contribution leading to the identical $\mathcal{O}_{\mathrm{LFQM}}$ after summing over the helicities. To obtain the results, we used the Dirac spinor basis with the chiral representation defined in Refs. [3,4]. The combinations of the current components and polarizations shown in Table II are the complete set, and other combinations are not possible to extract the decay constants. Equation (6) shows that the decay constants are not dependent on the energy of the bound states but on the mass of the constituents. This feature reflects the BT construction with the noninteracting $Q\bar{Q}$ representations including the interaction only in the mass operator $M \coloneqq M_0 + V_{Q\bar{Q}}$ and appears essential for the Lorentz-invariant quark phenomenology of decay constants in the LFQM.

V. OBSERVATION AND DISCUSSION

The results shown in Eq. (6) and Table II exhibit the Lorentz invariance of the physical observables represented by the decay constant $\mathcal{F}$, although each helicity contribution $H_{\lambda_1\lambda_2}$ obtained in our LFQM apparently depends on (a) the current components ($\mu = \pm, \perp$), (b) the polarization vectors $\epsilon^\mu(J_z)$, and (c) the transverse momentum $\mathbf{P}_\perp$ of the meson. We find that the decay constants $\mathcal{F}$ obtained from Eq. (6) turn out to be completely independent of (a), (b), and (c) and yield unique predictions of our LFQM.

For the quantitative estimation of decay constants, we exemplify the (π, ρ) mesons since they are good examples of the relativistic $Q\bar{Q}$ bound states. The model parameters are chosen as $(m, \beta) = (0.25, 0.3194)$ GeV following Refs. [8–10]. This parameter set gives $f_\pi = 131$ MeV, $f_\rho = 215$ MeV, and $f_\rho^T = 173$ MeV [10], which are in a good agreement with the experimental data, $f_\pi^{\mathrm{expt.}} = 130.3 \pm 0.3$ MeV and $f_\rho^{\mathrm{expt.}} = 210 \pm 4$ MeV [35]. However, what we would like to stress here is the uniqueness of the model predictions on the physical observables beyond just a good agreement with the data. Namely, the decay constant predicted by our LFQM is identical regardless of the aforementioned (a), (b), and (c). In particular, it is

TABLE II. The operators $\mathcal{O}_{\mathrm{LFQM}}$ and the helicity contributions $H_{\lambda_1\lambda_2}$ to $\mathcal{O}_{\mathrm{LFQM}}$ defined in Eq. (6) for all possible components of the current $\mathcal{G}$ and the polarization vectors $\epsilon(J_z)$, where $x_1 = x, x_2 = 1 - x$, and $\mathcal{D}_0 = M_0 + 2m$.

$\mathcal{F}$	$\mathcal{G}$	$\epsilon(J_z)$	$H_{\uparrow\uparrow}$	$H_{\uparrow\downarrow}$	$H_{\downarrow\uparrow}$	$H_{\downarrow\downarrow}$	$\mathcal{O}_{\mathrm{LFQM}}$
f_{P}	$\gamma^{(+,\perp)}\gamma_5$		0	m	m	0	$2m$
	$\gamma^-\gamma_5$		$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m - \frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m - \frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$2m$
f_{V}	$\gamma^{(+,\perp)}$	$\epsilon(0)$	0	$m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	$2m + \frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	γ^-	$\epsilon(0)$	0	$m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	$2m + \frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\gamma^{(\perp,-)}$	$\epsilon(+1)$	$M_0 - \frac{(M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	$\frac{x_1(x_1M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	$\frac{x_2(x_2M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	0	$M_0 - \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
f_{V}^T	$\sigma^{\perp+}$	$\epsilon(+1)$	$2m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	0	0	$2m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\sigma^{\perp-}$	$\epsilon(+1)$	$2m - \frac{2m(m+M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2m(m+x_1M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2m(m+x_2M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2\mathbf{k}_\perp^4}{x_1x_2M_0^2\mathcal{D}_0}$	$2m + \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	σ^{+-}	$\epsilon(0)$	$\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0} - \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$\frac{M_0}{2} - \frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}$	$\frac{M_0}{2} - \frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}$	$\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0} - \frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$M_0 - \frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$

remarkable to see from Table II that our analytic forms of the decay constants completely satisfy the SU(6) symmetry relation [36], $f_{\rm P} + f_{\rm V}(J_z) = 2f_{\rm V}^{T}(J_z)$, for each polarization vector $\epsilon(J_z)$ of the vector meson regardless of the components of the currents used in the calculation. Although the analytic forms of $f_{\rm V}^{(T)}(J_z)$ do not look the same for different J_z, they are in fact the same. This can be shown explicitly by converting Eq. (6) into the integral form of the ordinary three vector $\mathbf{k} = (k_z, \mathbf{k}_\perp)$ by taking into account the Jacobian of the variable transformation, $\{x, \mathbf{k}_\perp\} \to \{k_z, \mathbf{k}_\perp\}$, i.e.,

$$\mathcal{F} = \sqrt{6}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\hat{\phi}(\mathbf{k})}{M_0^{3/2}}\mathcal{O}_{\rm LFQM}(\mathbf{k}), \tag{7}$$

where $M_0 = 2\sqrt{m^2+\mathbf{k}^2}$ and $\hat{\phi}(\mathbf{k})$ corresponds to $\phi(x,\mathbf{k}_\perp)$ under the variable change $\{x,\mathbf{k}_\perp\} \to \{k_z,\mathbf{k}_\perp\}$. The difference of the two operators $\tilde{\mathcal{O}}_{\rm V}^{(T)} = \mathcal{O}_{\rm V}^{(T)}(J_z=1) - \mathcal{O}_{\rm V}^{(T)}(J_z=0)$ is then obtained as

$$\tilde{\mathcal{O}}_{\rm V}^{(T)} = \frac{2}{\mathcal{D}_0}(\mathbf{k}_\perp^2 - 2k_z^2), \tag{8}$$

and the integration of $\tilde{\mathcal{O}}_{\rm V}^{(T)}$ in Eq. (7) vanishes since the integrand except the term $(\mathbf{k}_\perp^2 - 2k_z^2)$ is rotationally invariant. This proves that $f_V^{(T)}(J_z=1) = f_V^{(T)}(J_z=0)$. Defining the integrand $\psi_V^{(J_z)}(\mathbf{k})$ for the computation of $f_V(J_z)$ as $f_V(J_z) = \int d^3\mathbf{k}\psi_V^{(J_z)}(\mathbf{k})$, we display 3D plots of $\psi_\rho^{(J_z)}$ for the longitudinally polarized ρ meson with $J_z = (0,+1)$ and their difference $\tilde{\psi}_\rho(\mathbf{k}) = \psi_\rho^{(0)} - \psi_\rho^{(+1)}$ in Fig. 1. As one can see, $\psi_\rho^{(0)}$ and $\psi_\rho^{(+1)}$ show the oblate and prolate ellipsoids, respectively, and their difference $\tilde{\psi}_\rho(\mathbf{k}) \propto (2k_z^2 - \mathbf{k}_\perp^2)$ reveals the d-wave orbital corresponding to the spherical harmonic function $Y_{20} \propto (3z^2 - r^2)$. For the transversely polarized ρ meson, the shapes of $\psi_\rho^{T(J_z)}(\mathbf{k})$ are very similar to those of

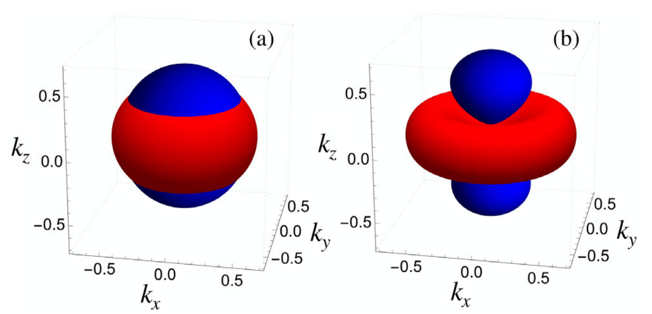

FIG. 1. The 3D plots of the wave functions (a) $\psi_\rho^{(J_z)}(\mathbf{k})$ for the ρ meson and (b) $\tilde{\psi}_\rho(\mathbf{k}) = \psi_\rho^{(0)} - \psi_\rho^{(+1)}$ defined by $f_\rho(J_z) = \int d^3\mathbf{k}\psi_\rho^{(J_z)}(\mathbf{k})$, where $\phi_\rho^{(0)}$ and $\phi_\rho^{(+1)}$ are depicted by red and blue, respectively. Each region represents the momentum distribution of the corresponding wave function.

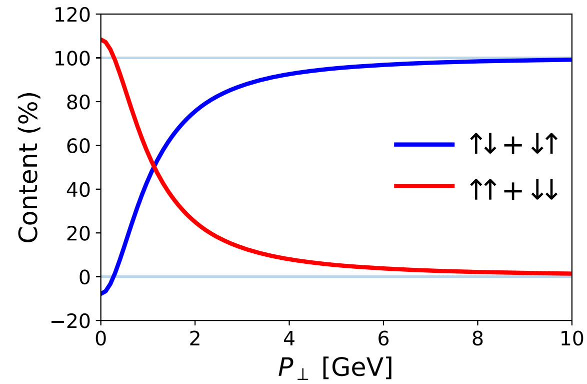

FIG. 2. The relative helicity contributions to f_π as a function of $\mathbf{P}_\perp$ calculated with the minus current. The blue and red lines represent the ordinary helicity $(\uparrow\downarrow, \downarrow\uparrow)$ and the higher helicity $(\uparrow\uparrow, \downarrow\downarrow)$ contributions, respectively. The sum is always the same regardless of $\mathbf{P}_\perp$.

$\psi_\rho^{(J_z)}(\mathbf{k})$. The shape of $\psi_\pi(\mathbf{k})$, on the other hand, shows the complete spherical symmetry.

The $\mathbf{P}_\perp$-independence of our results deserves the remarks below. As one can see from Table II, not only the final operators $\mathcal{O}_{\rm LFQM}$ but also each helicity contribution to $\mathcal{O}_{\rm LFQM}$ for the cases of $f_{\rm V}^{(T)}$ and $f_{\rm P}$ with $J^\mu = (J^+, \mathbf{J}_\perp)$ are obtained to be independent of $\mathbf{P}_\perp$ for the equal quark mass case. For the case of $f_{\rm P}$ with the minus current, however, each helicity contribution depends on $\mathbf{P}_\perp$ while the final operator $\mathcal{O}_{\rm P}$ is independent of $\mathbf{P}_\perp$. For the illustration of $\mathbf{P}_\perp$-independence of the final result in the case of the minus current, we show in Fig. 2 the relative helicity contributions to f_π ($\approx$131 MeV) as a function of $\mathbf{P}_\perp$. The blue and red lines represent the ordinary helicity $(\uparrow\downarrow, \downarrow\uparrow)$ and the higher helicity $(\uparrow\uparrow, \downarrow\downarrow)$ contributions, respectively. The higher helicity contributions are apparently important for the low and intermediate $\mathbf{P}_\perp$ regions although only the ordinary helicity contribution survives for the $\mathbf{P}_\perp \to \infty$ limit as in the case of plus and transverse components.

Although $\mathcal{O}_{\rm P}^- = \mathcal{O}_{\rm P}^{(+,\perp)}$ attained for the equal-mass case looks rather trivial, we note that $\mathcal{O}_{\rm P}^-$ has in fact more complicated structure in the unequal-mass case [12]. For $\tilde{\mathcal{O}}_{\rm P} \equiv \mathcal{O}_{\rm P}^- - \mathcal{O}_{\rm P}^+$, we find

$$\tilde{\mathcal{O}}_{\rm P} = \frac{4(m_1 - m_2)M_0}{(\mathbf{P}_\perp^2 + M_0^2)}k_z \tag{9}$$

for the unequal-mass case. While the result of $f_{\rm P}^- = f_{\rm P}^+$ is trivial in the equal-mass case due to the factor of $m_1 - m_2$ in Eq. (9), it is highly nontrivial that this equality $f_{\rm P}^- = f_{\rm P}^+$ prevails even in the unequal-mass case. The quantity $\tilde{\mathcal{O}}_{\rm P}$ contains the odd power of k_z as one may intuitively anticipate its appearance from $p^- - p^+ = -2p^3$. For the case that $m_1 \neq m_2$, we have $k_z = (x - 1/2)M_0 + (m_2^2 - m_1^2)/(2M_0)$ with $M_0 = \sqrt{m_1^2 + \mathbf{k}^2} + \sqrt{m_2^2 + \mathbf{k}^2}$, and the corresponding Jacobian $\sqrt{\partial k_z/\partial x}$ included in the radial wave function $\phi(x, \mathbf{k}_\perp)$ recovers the same spherically symmetric factor $\hat{\phi}(\mathbf{k})M_0^{-3/2}$ in the integrand of Eq. (7). The result of $f_{\rm P}^- = f_{\rm P}^+$ in the unequal-mass case is thus due to the symmetry under $k_z \leftrightarrow -k_z$ for all other terms besides $\tilde{\mathcal{O}}_{\rm P}$ in the integration. Similar behavior is also observed for the case of $f_{\rm V}^-(0) = f_{\rm V}^+(0)$. These results indicate that one should make sure that the rotational symmetry is not explicitly broken in the wave function level, if one constructs the radial wave function by assuming the separation of the longitudinal and transverse components [27], e.g., $\phi(x, \mathbf{k}_\perp) = \chi(x)\psi(\mathbf{k}_\perp)$.

VI. CONCLUSION

To assert the complete covariance of the decay constants defined by the matrix elements of one-body currents, it should be shown that they are completely independent of the current components ($\mu = \pm, \perp$) and the polarization vectors ($J_z = \pm 1, 0$). In this work, for the first time in the standard LFQM, we show this complete covariance by analyzing all the possible components of the currents and polarization vectors in the general LF frame with $\mathbf{P}_\perp \neq 0$.

From the analysis of the respective one-body current matrix elements in LFQM consistent with the BT construction at the level of one-body current computation, we obtained the complete Lorentz-invariant results of the decay constants, $(f_{\rm P}, f_{\rm V}, f_{\rm V}^T)$. We analyzed all possible combinations of the current components and the polarizations in the $\mathbf{P}_\perp \neq 0$ frame applying the self-consistency condition, $P = p_1 + p_2$ or equivalently $M \to M_0$. This condition reflects effectively the BT construction in the computation of the one-body current matrix elements where the meson state is described in the noninteracting $Q\bar{Q}$ basis while the interaction is added to the mass operator via $M \coloneqq M_0 + V_{Q\bar{Q}}$.

It is important to realize that the decay constants give identical results for the Fock space saturated to the $Q\bar{Q}$ state. While the equivalence should not be limited in principle by the Fock space truncation, it would deserve further analyses to explore the higher Fock states in practice regarding the issue of the cluster separability for the systems of more than two constituents [31]. In addition to the frame independence of the results, the verification of the identical results for the physical observables regardless of the current components and the polarizations taken in the computation can be used as an important guideline for the inclusion of the higher Fock space. It is also worthy to mention that the self-consistency condition for the calculation of the matrix elements with one-body current has been successfully applied to other higher-twist distribution amplitudes of pseudoscalar mesons and semileptonic and rare decays between two pseudoscalar mesons [32,37–39]. Further applications of our method to other exclusive processes of mesons are under investigation.

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