Study of CP violation in $D^{\pm} \to K^{*}(892)^{0}\pi^{\pm} + \bar{K}^{*}(892)^{0}\pi^{\pm} \to K^{0}_{S,L}\pi^{0}\pi^{\pm}$ decays

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Within the Standard Model, we investigate the CP violations and the $K^0_S - K^0_L$ asymmetries in $D^{\pm} \to K^{*}(892)^{0}\pi^{\pm} + \bar{K}^{*}(892)^{0}\pi^{\pm} \to K^{0}_{S,L}\pi^{0}\pi^{\pm}$ decays basing on the factorization-assisted topological-amplitude (FAT) approach and the topological amplitude (TA) approach of Cheng and Chiang [Phys. Rev. D **104**, 073003 (2021).]. We find that the CP violations in these decays $A_{CP}^{K^0_{S,L}}$ can exceed the order of 10^{-3} in the two approaches and consist of three parts: the indirect CP violations in $K^0 - \bar{K}^0$ mixing $A^{\rm mix}_{CP,K^0_{S,L}}$, the direct CP violations in charm decays $A^{\rm dir}_{CP,K^0_{S,L}}$, and the new CP violation effects $A^{\rm int}_{CP,K^0_{S,L}}$, which are induced from the interference between two tree (Cabibbo-favored and doubly Cabibbo-suppressed) amplitudes with the neutral kaon mixing. The indirect CP violations in $K^0 - \bar{K}^0$ mixing play a dominant role in $A_{CP}^{K^0_{S,L}}$; the new CP violation effects have a non-negligible contribution to $A_{CP}^{K^0_{S,L}}$. We estimate the numerical results of the $K^0_S - K^0_L$ asymmetries $R^{D^{\pm}}_{K_S - K_L}$ and find that there exists a large difference between the numerical results in the FAT approach and that of the TA approach. We present the numbers of $D^{\pm}$ events-times-efficiency needed to observe the CP violations and the $K^0_S - K^0_L$ asymmetries at the level of 3 standard deviations (3σ). We also find that if one adopts the values of the decay time parameters $t_0 = 3.0\tau_S$ and $t_1 = 10.0\tau_S$, the new CP violation effect $A^{\rm int}_{CP,K^0_S}$ would dominate the CP violation in $D^{\pm} \to K^{*}(892)^{0}\pi^{\pm} + \bar{K}^{*}(892)^{0}\pi^{\pm} \to K^{0}_{S}\pi^{0}\pi^{\pm}$ decays and could be observed with 6.7×10^6 and 6.5×10^6 $D^{\pm}$ events-times-efficiency in the FAT approach and the TA approach, respectively. Our results could be tested by the LHCb (Large Hadron Collider beauty), Belle II, and BESIII experiments.

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I. INTRODUCTION

The exploration of CP violation is one of the main topics in particle physics and cosmology; heavy flavor meson decays provide an ideal place to study CP violation. In the Standard Model (SM), CP violation is due to a complex parameter in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. However, the strength of CP violation predicted by the Standard Model is insufficient to explain the baryon asymmetry of the unverse [1,2], so it is necessary to search for new sources of CP violation. It is important to investigate as many systems as possible, to see the correlation between different processes and understand the origin of CP violation.

CP violation in Kaon and B meson systems has been well established, but not yet in charmed meson decays. In 2019, the LHCb (Large Hadron Collider beauty) collaboration reported the first confirmed observation of the CP asymmetries in the charm sector via measuring the difference of time-integrated CP asymmetries of $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays with a significance of more than 5σ [3]. Combining the LHCb (Large Hadron Collider beauty) results in 2014 [4], 2016 [5], and 2019 [3] leads to a result of a nonzero value of ΔA_{CP}

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (1.54 \pm 0.29) \times 10^{-3}. \tag{1}$$

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In recent years, there have been a number of theoretical works, which concentrate on studying the CP violations in the charm sector [6–44]. Charmed meson decays have become one of the most important platforms for studying the CP violation and its origin.

The decays with final states including K_S^0 or K_L^0 can be used to study CP violation [45–73]. In these decays, the indirect CP violation induced by the $K^0-\bar{K}^0$ mixing has a non-negligible effect, and even plays a dominant role. There exists a 2.8σ discrepancy observed between the *BABAR* measurement and the SM prediction of the CP asymmetry in the $\tau^+\to\pi^+K_S\bar{\nu}_\tau$ decay [59–61]; this may imply the existence of the physics beyond the SM because of the absence of the direct CP violation in this decay. However, no unambiguous conclusion can be drawn due to the large uncertainty [63], so more precise data and more reactions with final states including K_S^0 or K_L^0 are needed in both experiment and theory.

In Ref. [51], the authors study the CP asymmetries in the $D^\pm\to K_S^0\pi^\pm$ decays; they show that besides the indirect CP violation due to the $K^0-\bar{K}^0$ mixing, a new CP violation effect induced by the interference between the Cabibbo-favored (CF) and doubly Cabibbo-suppressed (DCS) amplitudes with the $K^0-\bar{K}^0$ mixing may give a non-negligible contribution to the CP asymmetries in the $D^\pm\to K_S^0\pi^\pm$ decays. CP violations in the $D^\pm\to K^*(892)^0\pi^\pm$ and $D^\pm\to\bar{K}^*(892)^0\pi^\pm$ decays are rarely studied, especially CP violations in the $D^\pm\to K^*(892)^0\pi^\pm+\bar{K}^*(892)^0\pi^\pm\to K_{S,L}^0\pi^0\pi^\pm$ decays [74]. For example, the new CP violation effects induced by the interference between the CF and DCS amplitudes with the $K^0-\bar{K}^0$ mixing in the $D^\pm\to K^*(892)^0\pi^\pm+\bar{K}^*(892)^0\pi^\pm\to K_{S,L}^0\pi^0\pi^\pm$ decays have never been studied. For simplicity, we refer to $K^*(892)^0$ and $\bar{K}^*(892)^0$ as K^{*0} and $\bar{K}^{*0}$ hereafter, respectively.

In this paper, we will study the CP violations in the $D^\pm\to K^{*0}\pi^\pm+\bar{K}^{*0}\pi^\pm\to K_{S,L}^0\pi^0\pi^\pm$ decays, which consist of the indirect CP violations in $K^0-\bar{K}^0$ mixing, the direct CP asymmetries in charm decays, and the new CP violation effects induced by the interference between the CF and DCS amplitudes with the $K^0-\bar{K}^0$ mixing. We will present the formulas and the numerical results of the CP asymmetries; we will also investigate the possibility of observing the new CP violation effect in the $D^\pm\to K^{*0}\pi^\pm+\bar{K}^{*0}\pi^\pm\to K_S^0\pi^0\pi^\pm$ decays, which depends on the choice of the decay time of K_S^0. Additionally, we will study the $K_S^0-K_L^0$ asymmetries in the $D^\pm\to K^{*0}\pi^\pm+\bar{K}^{*0}\pi^\pm\to K_{S,L}^0\pi^0\pi^\pm$ decays and give the numerical results.

The paper is organized as follows. In Sec. II, we derive the branching ratio of the $D^\pm\to K^{*0}\pi^\pm+\bar{K}^{*0}\pi^\pm\to K_{S,L}^0\pi^0\pi^\pm$ decays. In Sec. III, we calculate the CP violations and the $K_S^0-K_L^0$ asymmetries for the $D^\pm\to K^{*0}\pi^\pm+\bar{K}^{*0}\pi^\pm\to K_{S,L}^0\pi^0\pi^\pm$ decays. The numerical results and discussions are present in Sec. IV. In Sec. V, we investigate the observation of the new CP violation effect in the $D^\pm\to K^{*0}\pi^\pm+\bar{K}^{*0}\pi^\pm\to K_S^0\pi^0\pi^\pm$ decays. And Sec. VI is the conclusion. In the Appendix, we collect the formulas for the evolutions of the Wilson coefficients in the scale $\mu<m_c$.

II. BRANCHING FRACTIONS

A. The amplitudes for the decays $D^\pm\to K^{*0}(\bar{K}^{*0})\pi^\pm\to K^0(\bar{K}^0)\pi^0\pi^\pm$

The $D^\pm\to K^{*0}\pi^\pm+\bar{K}^{*0}\pi^\pm\to K_{S,L}^0\pi^0\pi^\pm$ decays can proceed via the $D^\pm\to K^{*0}(\bar{K}^{*0})\pi^\pm$ processes, the $K^{*0}\to K^0\pi^0(\bar{K}^{*0}\to\bar{K}^0\pi^0)$ decays, and the $K^0-\bar{K}^0$ oscillation and decay. Within the SM, the CF decays $D^+\to\bar{K}^{*0}\pi^+$ and $D^-\to K^{*0}\pi^-$ can proceed via the color-allowed external W-emission tree diagram and the color-suppressed internal W-emission tree diagram, which are displayed in Fig. 1; the DCS channels $D^+\to K^{*0}\pi^+$ and $D^-\to\bar{K}^{*0}\pi^-$ can occur through the color-suppressed internal W-emission tree diagram and the W-annihilation diagram, which are displayed in Fig. 2. Here, all diagrams are meant to have all the strong interactions included, i.e., gluon lines are included implicitly in all possible ways [75]. The effective Hamiltonian relevant to the $D^\pm\to K^{*0}(\bar{K}^{*0})\pi^\pm$ decays is given by

$$\begin{aligned}\mathcal{H}_{\rm eff}=&\frac{G_F}{\sqrt{2}}C_1(\mu)[V_{cs}^*V_{ud}(\bar{s}_\alpha c_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A}+V_{cd}^*V_{us}(\bar{d}_\alpha c_\beta)_{V-A}(\bar{u}_\beta s_\alpha)_{V-A}]\\&+\frac{G_F}{\sqrt{2}}C_2(\mu)[V_{cs}^*V_{ud}(\bar{s}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A}+V_{cd}^*V_{us}(\bar{d}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta s_\beta)_{V-A}]+{\rm H.c.},\end{aligned}\qquad(2)$$

where G_F is the Fermi coupling constant, $V_{qq'}$ is the corresponding CKM matrix element, α and β are the color indices, and $(\bar{q}q')_{V-A}$ represents $\bar{q}\gamma_\mu(1-\gamma_5)q'$. $C_1(\mu)$ and $C_2(\mu)$ are the Wilson coefficients; the evolutions of these Wilson coefficients in the scale μ are given in Ref. [14]. For convenience, we duplicate these explicit expressions in the Appendix. Based on the topological amplitude approach [76,77], the decay amplitudes of the diagrams in Figs. 1 and 2 can be parametrized as

$$\langle\bar{K}^{*0}\pi^+|\mathcal{H}_{\rm eff}|D^+\rangle_{T_V}=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}T_V^0\,\varepsilon^*\cdot p_{D^+},\qquad(3)$$

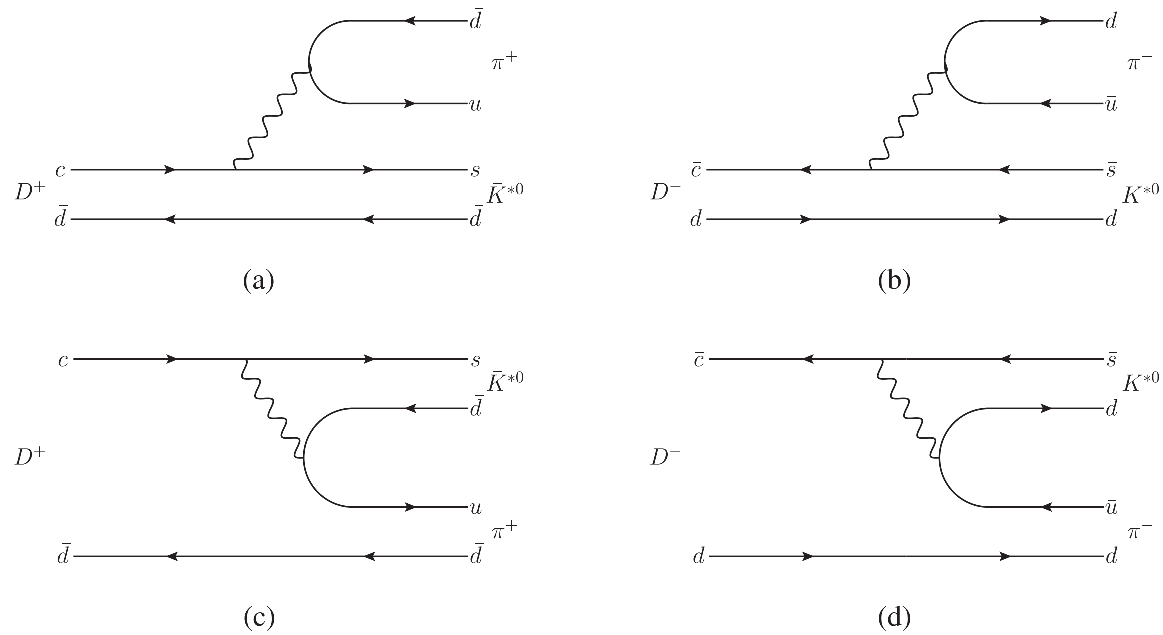

FIG. 1. Topological diagrams contributing to the Cabibbo-allowed $D^{+} \to \bar{K}^{*0}\pi^{+}$ and $D^{-} \to K^{*0}\pi^{-}$ decays: (a, b) the color-allowed external W-emission tree diagram and (c, d) the color-suppressed internal W-emission tree diagram.

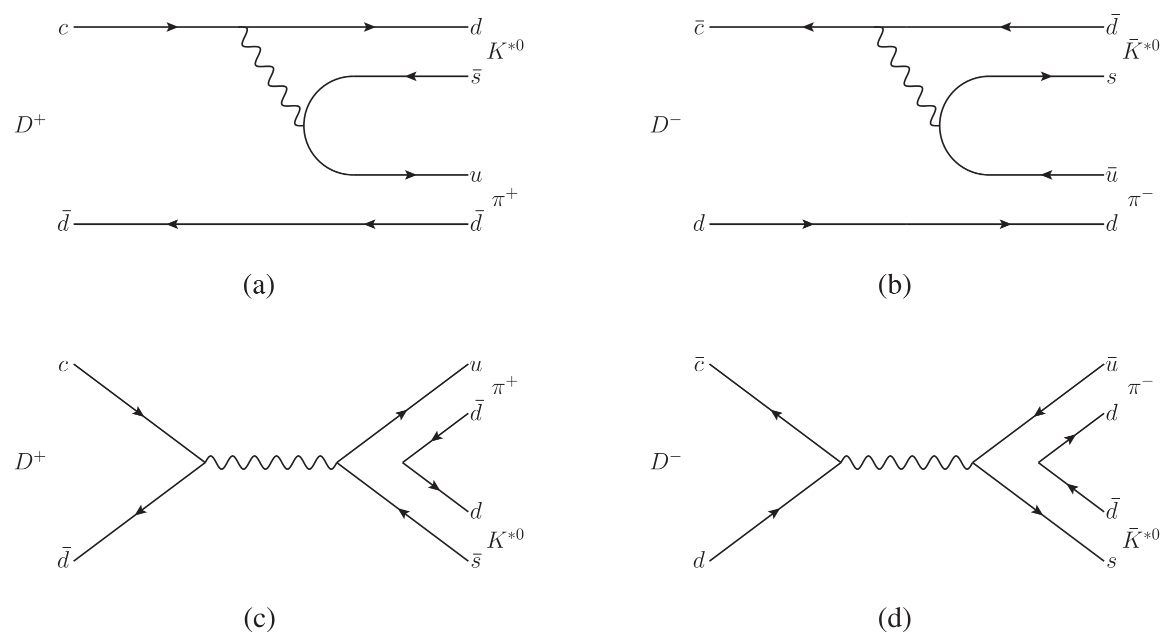

FIG. 2. Topological diagrams contributing to the DCS $D^{+} \to K^{*0}\pi^{+}$ and $D^{-} \to \bar{K}^{*0}\pi^{-}$ decays: (a, b) the color-suppressed internal W-emission tree diagram and (c, d) the W-annihilation diagram.

$$\langle K^{*0}\pi^{-}|\mathcal{H}_{\rm eff}|D^{-}\rangle_{T_V} = \frac{G_F}{\sqrt{2}} V_{cs}V_{ud}^{*}T_{V}^{0}\,\varepsilon^{*}\cdot p_{D^{-}}, \quad (4)$$

$$\langle \bar{K}^{*0}\pi^{+}|\mathcal{H}_{\rm eff}|D^{+}\rangle_{C_P} = \frac{G_F}{\sqrt{2}} V_{cs}^{*}V_{ud}C_{P}^{0}\,\varepsilon^{*}\cdot p_{D^{+}}, \quad (5)$$

$$\langle K^{*0}\pi^{-}|\mathcal{H}_{\rm eff}|D^{-}\rangle_{C_P} = \frac{G_F}{\sqrt{2}} V_{cs}V_{ud}^{*}C_{P}^{0}\,\varepsilon^{*}\cdot p_{D^{-}}, \quad (6)$$

$$\langle K^{*0}\pi^{+}|\mathcal{H}_{\rm eff}|D^{+}\rangle_{C_P} = \frac{G_F}{\sqrt{2}} V_{cd}^{*}V_{us}C_{P}^{0}\,\varepsilon^{*}\cdot p_{D^{+}}, \quad (7)$$

$$\langle \bar{K}^{*0}\pi^{-}|\mathcal{H}_{\rm eff}|D^{-}\rangle_{C_P} = \frac{G_F}{\sqrt{2}} V_{cd}V_{us}^{*}C_{P}^{0}\,\varepsilon^{*}\cdot p_{D^{-}}, \quad (8)$$

$$\langle K^{*0}\pi^{+}|\mathcal{H}_{\rm eff}|D^{+}\rangle_{A_V} = \frac{G_F}{\sqrt{2}} V_{cd}^{*}V_{us}A_{V}^{0}\,\varepsilon^{*}\cdot p_{D^{+}}, \quad (9)$$

$$\langle \bar{K}^{*0}\pi^-|\mathcal{H}_{\text{eff}}|D^-\rangle_{A_V} = \frac{G_F}{\sqrt{2}} V_{cd}V_{us}^* A_V^0\, \varepsilon^* \cdot p_{D^-}, \tag{10}$$

where the subscript T_V in Eqs. (3) and (4) denotes that the decay amplitude is the color-allowed external W-emission tree diagram amplitude with the $D^+ \to \bar{K}^{*0}$ and $D^- \to K^{*0}$ transitions, the subscript C_P in Eqs. (5)–(8) represents that the decay amplitude is the color-suppressed internal W-emission tree diagram amplitude with the $D^\pm \to \pi^\pm$ transitions, and the subscript A_V in Eqs. (9) and (10) denotes that the decay amplitude is the W-annihilation diagram amplitude with the s (or $\bar{s}$) quark from the weak decay entering in the $\bar{K}^{*0}$ (or K^{*0}) meson. We based our calculation on the results of two topological amplitude approaches: the factorization-assisted topological-amplitude (FAT) approach and the topological amplitude approach of Ref. [40] (hereinafter for brevity referred to as the TA approach). In the FAT approach, the topological amplitudes can be expressed as [14,23,78]

$$T_V^0 = \alpha_1^V 2m_{K^*} f_{\pi^+} A_0(m_{\pi^+}^2), \tag{11}$$

$$C_P^0 = \alpha_2^P 2m_{K^*} f_{K^*} f_+(p_{K^*}^2), \tag{12}$$

$$A_V^0 = C_1(\mu_A)\chi_q^A e^{i\phi_q^A} f_{D^+} m_{D^+} \frac{f_{K^*}}{f_\rho} e^{iS_\pi}, \tag{13}$$

where m_{K^*}, m_{π^+}, and m_{D^+} are the mass of the meson K^{*0}, π^+, and D^+, respectively. f_{π^+}, f_{K^*}, f_ρ, and f_{D^+} are the decay constants of the meson π^+, K^{*0}, ρ, and D^+, respectively. ε [we denote $\varepsilon \equiv \varepsilon(p_{K^*}, \lambda)$ for simplicity] is the polarization vector of the K^{*0} meson; it yields the following relations [40]:

$$\varepsilon_\mu(p_{K^*}, \lambda) p_{K^*}^\mu = 0, \tag{14}$$

$$\sum_\lambda \varepsilon^{*\mu}(p_{K^*}, \lambda)\varepsilon^\nu(p_{K^*}, \lambda) = -g^{\mu\nu} + \frac{p_{K^*}^\mu p_{K^*}^\nu}{m_{K^*}^2}. \tag{15}$$

The effective Wilson coefficients α_1^V and α_2^P in Eqs. (11) and (12) are

$$\alpha_1^V = C_2(\mu_T) + \frac{1}{3}C_1(\mu_T),$$
$$\alpha_2^P = C_1(\mu_C) + C_2(\mu_C)\left[\frac{1}{3} + \chi_P^C e^{i\phi_P^C}\right]; \tag{16}$$

χ_q^A, ϕ_q^A, χ_P^C, and ϕ_P^C in Eqs. (13) and (16) are the nonfactorizable parameters; and e^{iS_π} in Eq. (13) is a strong phase factor which is introduced for each pion involved in the nonfactorizable contributions of the W-annihilation diagram amplitude. We note that the parameters χ_q^A, ϕ_q^A, χ_P^C, ϕ_P^C, and S_π are free and universal; they can be determined by fitting the data. μ_A, μ_T, and μ_C in Eqs. (13) and (16) are, respectively, the scale for the W-annihilation diagram, the color-allowed external W-emission tree diagram, and the color-suppressed internal W-emission tree diagram [23,78]

$$\mu_A = \sqrt{\Lambda m_{D^+}(1-r_P^2)(1-r_V^2)}, \quad \mu_T = \sqrt{\Lambda m_{D^+}(1-r_P^2)},$$
$$\mu_C = \sqrt{\Lambda m_{D^+}(1-r_V^2)}, \tag{17}$$

with

$$r_P = \frac{m_{\pi^+}}{m_{D^+}}, \qquad r_V = \frac{m_{K^*}}{m_{D^+}}, \tag{18}$$

where Λ represents the momentum of the soft degree of freedom in the D decays, fixed to be $\Lambda = 0.5$ GeV in this work. $f_+(p_{K^*}^2)$ in Eq. (12) is the $D^\pm \to \pi^\pm$ transition form factor, which can be written as

$$\langle \pi^+|\bar{u}\gamma^\mu c|D^+\rangle = f_0(q^2)\left(\frac{m_{D^+}^2 - m_{\pi^+}^2}{q^2}q^\mu\right) + f_+(q^2)\left(p_{D^+}^\mu + p_{\pi^+}^\mu - \frac{m_{D^+}^2 - m_{\pi^+}^2}{q^2}q^\mu\right), \tag{19}$$

with $q = p_{D^+} - p_{\pi^+}$. $A_0(m_{\pi^+}^2)$ in Eq. (11) is the $D^\pm \to \bar{K}^{*0}$ transition form factor, which can be written as [79–81]

$$\langle \bar{K}^{*0}|\bar{s}\gamma^\mu\gamma_5 c|D^+\rangle = i\left[\varepsilon^{*\mu}(m_{D^+} + m_{K^*})A_1(q^2) - \frac{\varepsilon^* \cdot q}{m_{D^+} + m_{K^*}}(p_{D^+}^\mu + p_{K^*}^\mu)A_2(q^2) - \frac{\varepsilon^* \cdot q}{q^2}2m_{K^*}q^\mu A_3(q^2)\right] + i\frac{\varepsilon^* \cdot q}{q^2}2m_{K^*}q^\mu A_0(q^2), \tag{20}$$

with $q = p_{D^+} - p_{K^*}$ and

$$A_3(q^2) = \frac{m_{D^+} + m_{K^*}}{2m_{K^*}}A_1(q^2) - \frac{m_{D^+} - m_{K^*}}{2m_{K^*}}A_2(q^2). \tag{21}$$

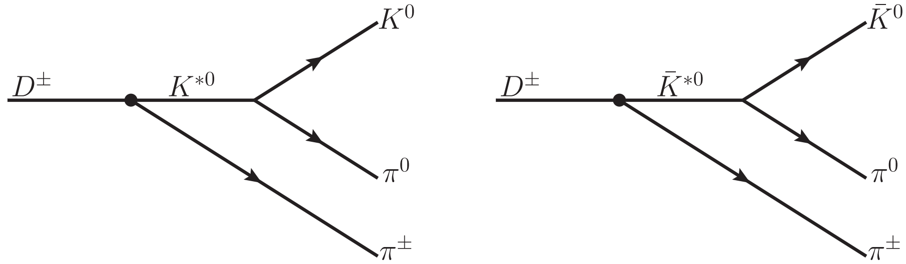

FIG. 3. Resonant contribution to the amplitudes of $D^{\pm} \to \pi^{\pm} K^0 \pi^0$ and $D^{\pm} \to \pi^{\pm} \bar{K}^0 \pi^0$ through the intermediate states K^{*0} and $\bar{K}^{*0}$, where the blob stands for a transition due to weak interactions.

There exist many model and lattice calculations for $D^{\pm}$ to $\pi^{\pm}$, K^{*0} transition form factors. In this paper, we shall use the following parametrization for form-factor q^2 dependence [33,75,82,83]:

$$F(q^2) = \frac{F(0)}{(1 - \frac{q^2}{m^2_{\text{pole}}})(1 - \alpha \frac{q^2}{m^2_{\text{pole}}})}, \tag{22}$$

where for the form factor $f_+(q^2)$, $m_{\text{pole}} = m_{D^*(2010)^+}$, $F(0) = 0.666$, and $\alpha = 0.24$, while for the form factor $A_0(q^2)$, $m_{\text{pole}} = m_{D_s^+}$, $F(0) = 0.78$ and $\alpha = 0.24$.

In the TA approach, based on the solution (S3') of the fitting result in Table II of Ref. [40], we can obtain the following numerical results of the topological amplitudes:

$$|T_V^0| = 0.266 \pm 0.004, \qquad \delta_{T_V^0} = 0° \tag{23}$$

$$|C_P^0| = 0.245 \pm 0.002, \qquad \delta_{C_P^0} = (201 \pm 1)°, \tag{24}$$

$$|A_V^0| = 0.028 \pm 0.002, \qquad \delta_{A_V^0} = (77 \pm 5)°. \tag{25}$$

Here, we note that the values of $|T_V^0|$, $|C_P^0|$, and $|A_V^0|$ are obtained by the products of the values of the corresponding topological amplitudes in Table II of Ref. [40] and $\sqrt{2}/G_F$; the values of $\delta_{T_V^0}$, $\delta_{C_P^0}$, and $\delta_{A_V^0}$ are obtained directly from Table II of Ref. [40].

In the overlapped region of the K^{*0} and $\bar{K}^{*0}$ resonances, the decay amplitudes of the cascade decays $D^{\pm} \to K^{*0}\pi^{\pm} \to K^0\pi^0\pi^{\pm}$ and $D^{\pm} \to \bar{K}^{*0}\pi^{\pm} \to \bar{K}^0\pi^0\pi^{\pm}$, which are depicted in Fig. 3, can be written as

$$A(D^+ \to K^{*0}\pi^+ \to K^0\pi^0\pi^+) = \langle K^0\pi^0|\mathcal{L}|K^{*0}\rangle T^{\text{BW}}_{K^*}(p^2_{K^*})(\langle K^{*0}\pi^+|\mathcal{H}_{\text{eff}}|D^+\rangle_{C_P} + \langle K^{*0}\pi^+|\mathcal{H}_{\text{eff}}|D^+\rangle_{A_V}), \tag{26}$$

$$A(D^+ \to \bar{K}^{*0}\pi^+ \to \bar{K}^0\pi^0\pi^+) = \langle \bar{K}^0\pi^0|\mathcal{L}|\bar{K}^{*0}\rangle T^{\text{BW}}_{K^*}(p^2_{K^*})(\langle \bar{K}^{*0}\pi^+|\mathcal{H}_{\text{eff}}|D^+\rangle_{C_P} + \langle \bar{K}^{*0}\pi^+|\mathcal{H}_{\text{eff}}|D^+\rangle_{T_V}), \tag{27}$$

$$A(D^- \to K^{*0}\pi^- \to K^0\pi^0\pi^-) = \langle K^0\pi^0|\mathcal{L}|K^{*0}\rangle T^{\text{BW}}_{K^*}(p^2_{K^*})(\langle K^{*0}\pi^-|\mathcal{H}_{\text{eff}}|D^-\rangle_{C_P} + \langle K^{*0}\pi^-|\mathcal{H}_{\text{eff}}|D^-\rangle_{T_V}), \tag{28}$$

$$A(D^- \to \bar{K}^{*0}\pi^- \to \bar{K}^0\pi^0\pi^-) = \langle \bar{K}^0\pi^0|\mathcal{L}|\bar{K}^{*0}\rangle T^{\text{BW}}_{K^*}(p^2_{K^*})(\langle \bar{K}^{*0}\pi^-|\mathcal{H}_{\text{eff}}|D^-\rangle_{C_P} + \langle \bar{K}^{*0}\pi^-|\mathcal{H}_{\text{eff}}|D^-\rangle_{A_V}), \tag{29}$$

with the Lagrangian [40]

$$\mathcal{L} = ig^{K^{*0}\to K^0\pi^0}(\bar{K}^{*0\mu}\pi^0 \overleftrightarrow{\partial}_\mu K^0 + K^{*0\mu}\bar{K}^0 \overleftrightarrow{\partial}_\mu \pi^0) \tag{30}$$

and the relativistic Breit-Wigner line shape for K^{*0},

$$T^{\text{BW}}_{K^*}(s) = \frac{1}{s - m^2_{K^*} + im_{K^*}\Gamma_{K^*}(s)}, \tag{31}$$

where $\Gamma_{K^*}(s)$ is the mass dependent width of K^{*0},

$$\Gamma_{K^*}(s) = \Gamma^0_{K^*}\left(\frac{q_{K^0}}{q^0_{K^0}}\right)^3 \frac{m_{K^*}}{\sqrt{s}} \frac{X_1^2(q_{K^0})}{X_1^2(q^0_{K^0})}, \tag{32}$$

and q_{K^0} denotes the c.m. momentum of K^0 in the rest frame of K^*; $q^0_{K^0}$ is the value of q_{K^0} when s is equal to $m^2_{K^*}$,

$$q_{K^0} = \frac{f(\sqrt{s}, m_{K^0}, m_{\pi^0})}{\sqrt{4s}}, \tag{33}$$

where the function f is

$$f(x,y,z) = \sqrt{x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 - 2y^2z^2}. \tag{34}$$

In Eq. (32), $\Gamma^0_{K^*}$ is the nominal total width of K^* with $\Gamma^0_{K^*} = \Gamma_{K^*}(m^2_{K^*})$ and X_1 is the Blatt-Weisskopf barrier factor

$$X_1(z) = \sqrt{\frac{1}{(zr_{\text{BW}})^2 + 1}}, \tag{35}$$

with $r_{\text{BW}} \approx 4.0\ \text{GeV}^{-1}$.

Using the Lagrangian in Eq. (30), one obtains

$$\langle K^0\pi^0|\mathcal{L}|K^{*0}\rangle = g^{K^{*0}\to K^0\pi^0}\varepsilon\cdot(p_{K^0}-p_{\pi^0}), \tag{36}$$

$$\langle \bar{K}^0\pi^0|\mathcal{L}|\bar{K}^{*0}\rangle = -g^{K^{*0}\to K^0\pi^0}\varepsilon\cdot(p_{\bar{K}^0}-p_{\pi^0}), \tag{37}$$

where $g^{K^{*0}\to K^0\pi^0}$ is the coupling of K^{*0} to $K^0\pi^0$, which can be extracted from

$$\mathcal{B}(K^{*0}\to K^0\pi^0) = \frac{1}{6\pi m_{K^*}^2\Gamma_{K^*}^0}(g^{K^{*0}\to K^0\pi^0})^2 \times\left(\frac{f(m_{K^*},m_{K^0},m_{\pi^0})}{\sqrt{4m_{K^*}^2}}\right)^3. \tag{38}$$

Substituting Eqs. (7), (9), (31), and (36) into Eq. (26), we can obtain the decay amplitude of the cascade decay $D^+\to K^{*0}\pi^+\to K^0\pi^0\pi^+$

$$\begin{aligned}&A(D^+\to K^{*0}\pi^+\to K^0\pi^0\pi^+)\\ &=\frac{G_F}{\sqrt{2}}V_{cd}^*V_{us}(C_P^0+A_V^0)\varepsilon^*\cdot p_{D^+}\\ &\times\frac{1}{p_{K^*}^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}(p_{K^*}^2)}\\ &\times g^{K^{*0}\to K^0\pi^0}\varepsilon\cdot(p_{K^0}-p_{\pi^0})F\left(\sqrt{p_{K^*}^2},m_{K^*}\right).\end{aligned} \tag{39}$$

In order to account for the off shell effect of K^*, we follow Ref. [40,84] to add a form factor $F(\sqrt{s},m_{K^*})$ into the above equation. The form factor $F(\sqrt{s},m_{K^*})$ can be parametrized as

$$F(\sqrt{s},m_{K^*}) = \frac{\Lambda^2+m_{K^*}^2}{\Lambda^2+s}, \tag{40}$$

with the cutoff Λ not far from the mass of the resonance K^*,

$$\Lambda = m_{K^*}+\beta\Lambda_{\rm QCD}, \tag{41}$$

where $\beta = 1.0\pm 0.2$ and $\Lambda_{\rm QCD} = 0.25$ GeV.

From Eqs. (14) and (15), we obtain

$$\sum_\lambda \varepsilon^*\cdot p_{D^+}\varepsilon\cdot(p_{K^0}-p_{\pi^0}) = 2\vec{p}_{\pi^+}\cdot\vec{p}_{K^0} \tag{42}$$

in the rest frame of K^0 and π^0. Substituting the above equation into Eq. (39), we obtain

$$A(D^+\to K^{*0}\pi^+\to K^0\pi^0\pi^+) = \sqrt{2}G_FV_{cd}^*V_{us}(C_P^0+A_V^0)\frac{1}{p_{K^*}^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}(p_{K^*}^2)}g^{K^{*0}\to K^0\pi^0}F\left(\sqrt{p_{K^*}^2},m_{K^*}\right)\vec{p}_{\pi^+}\cdot\vec{p}_{K^0}. \tag{43}$$

Similarly, we can obtain the following amplitudes:

$$A(D^+\to \bar{K}^{*0}\pi^+\to \bar{K}^0\pi^0\pi^+) = -\sqrt{2}G_FV_{cs}^*V_{ud}(C_P^0+T_V^0)\frac{1}{p_{K^*}^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}(p_{K^*}^2)}g^{K^{*0}\to K^0\pi^0}F\left(\sqrt{p_{K^*}^2},m_{K^*}\right)\vec{p}_{\pi^+}\cdot\vec{p}_{\bar{K}^0}, \tag{44}$$

$$A(D^-\to \bar{K}^{*0}\pi^-\to \bar{K}^0\pi^0\pi^-) = -\sqrt{2}G_FV_{cd}V_{us}^*(C_P^0+A_V^0)\frac{1}{p_{K^*}^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}(p_{K^*}^2)}g^{K^{*0}\to K^0\pi^0}F\left(\sqrt{p_{K^*}^2},m_{K^*}\right)\vec{p}_{\pi^-}\cdot\vec{p}_{\bar{K}^0}, \tag{45}$$

$$A(D^-\to K^{*0}\pi^-\to K^0\pi^0\pi^-) = \sqrt{2}G_FV_{cs}V_{ud}^*(C_P^0+T_V^0)\frac{1}{p_{K^*}^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}(p_{K^*}^2)}g^{K^{*0}\to K^0\pi^0}F\left(\sqrt{p_{K^*}^2},m_{K^*}\right)\vec{p}_{\pi^-}\cdot\vec{p}_{K^0}. \tag{46}$$

B. The effect of the $K^0-\bar{K}^0$ mixing

Now, we proceed to study the time evolution of the initially pure $K^0(\bar{K}^0)$ states. In the $K^0-\bar{K}^0$ system, the two mass eigenstates, K_S^0 of mass m_S and width Γ_S and K_L^0 of mass m_L and width Γ_L, are linear combinations of the flavor eigenstates K^0 and $\bar{K}^0$. Under the assumption of CPT invariance, these mass eigenstates can be expressed as [85]

$$|K_L^0\rangle = p|K^0\rangle - q|\bar{K}^0\rangle, \tag{47}$$

$$|K_S^0\rangle = p|K^0\rangle + q|\bar{K}^0\rangle, \tag{48}$$

where p and q are complex mixing parameters. CP conservation requires both $p = q = \sqrt{2}/2$. The mass and width eigenstates $K_{S,L}^0$ may also be described with the popular notations

$$|K_L^0\rangle = \frac{1+\epsilon}{\sqrt{2(1+|\epsilon|^2)}}|K^0\rangle - \frac{1-\epsilon}{\sqrt{2(1+|\epsilon|^2)}}|\bar{K}^0\rangle, \tag{49}$$

$$|K_S^0\rangle = \frac{1+\epsilon}{\sqrt{2(1+|\epsilon|^2)}}|K^0\rangle + \frac{1-\epsilon}{\sqrt{2(1+|\epsilon|^2)}}|\bar{K}^0\rangle, \tag{50}$$

where the complex parameter ϵ signifies deviation of the mass eigenstates from the CP eigenstates. The parameters p and q can be expressed in terms of ϵ:

$$p = \frac{1+\epsilon}{\sqrt{2(1+|\epsilon|^2)}}, \qquad q = \frac{1-\epsilon}{\sqrt{2(1+|\epsilon|^2)}}. \tag{51}$$

Combining Eqs. (47), (48), and (51) and neglecting the tiny direct CP asymmetry in the $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^+\pi^-\pi^0$ decays, we can derive

$$\frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} = \frac{p-q}{p+q} = \epsilon, \tag{52}$$

$$\frac{A(K_S^0 \to \pi^+\pi^-\pi^0)}{A(K_L^0 \to \pi^+\pi^-\pi^0)} = \frac{p-q}{p+q} = \epsilon. \tag{53}$$

The time-evolved states of the $K^0 - \bar{K}^0$ system can be expressed by the mass eigenstates

$$|K_{\rm phys}^0(t)\rangle = \frac{1}{2p}e^{-im_L t - \frac{1}{2}\Gamma_L t}|K_L^0\rangle + \frac{1}{2p}e^{-im_S t - \frac{1}{2}\Gamma_S t}|K_S^0\rangle, \tag{54}$$

$$|\bar{K}_{\rm phys}^0(t)\rangle = -\frac{1}{2q}e^{-im_L t - \frac{1}{2}\Gamma_L t}|K_L^0\rangle + \frac{1}{2q}e^{-im_S t - \frac{1}{2}\Gamma_S t}|K_S^0\rangle. \tag{55}$$

Using Eqs. (54) and (55), the time-dependent amplitudes of the cascade decays $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K^0(t)\pi^0\pi^{\pm} + \bar{K}^0(t)\pi^0\pi^{\pm} \to f_{K^0}\pi^0\pi^{\pm}$ (hereinafter for brevity referred to as $D^{\pm} \to K^*\pi^{\pm} \to K(t)\pi^0\pi^{\pm} \to f_{K^0}\pi^0\pi^{\pm}$) can be written as

$$\begin{aligned} &A(D^{\pm} \to K^*\pi^{\pm} \to K(t)\pi^0\pi^{\pm} \to f_{K^0}\pi^0\pi^{\pm}) \\ &= A(D^{\pm} \to K^{*0}\pi^{\pm} \to K^0\pi^0\pi^{\pm}) \cdot A(K_{\rm phys}^0(t) \to f_{K^0}) \\ &\quad + A(D^{\pm} \to \bar{K}^{*0}\pi^{\pm} \to \bar{K}^0\pi^0\pi^{\pm}) \cdot A(\bar{K}_{\rm phys}^0(t) \to f_{K^0}), \end{aligned} \tag{56}$$

where f_{K^0} denotes the final state from the decay of the K^0 or $\bar{K}^0$ meson. $A(K_{\rm phys}^0(t) \to f_{K^0})$ and $A(\bar{K}_{\rm phys}^0(t) \to f_{K^0})$ denote the amplitude of the $K_{\rm phys}^0(t) \to f_{K^0}$ and $\bar{K}_{\rm phys}^0(t) \to f_{K^0}$ decays, respectively. They have the following forms:

$$\begin{aligned} A(K_{\rm phys}^0(t) \to f_{K^0}) &= \frac{1}{2p}e^{-im_L t - \frac{1}{2}\Gamma_L t}A(K_L^0 \to f_{K^0}) \\ &\quad + \frac{1}{2p}e^{-im_S t - \frac{1}{2}\Gamma_S t}A(K_S^0 \to f_{K^0}), \end{aligned} \tag{57}$$

$$\begin{aligned} A(\bar{K}_{\rm phys}^0(t) \to f_{K^0}) &= -\frac{1}{2q}e^{-im_L t - \frac{1}{2}\Gamma_L t}A(K_L^0 \to f_{K^0}) \\ &\quad + \frac{1}{2q}e^{-im_S t - \frac{1}{2}\Gamma_S t}A(K_S^0 \to f_{K^0}). \end{aligned} \tag{58}$$

For convenience, we introduce the following substitutions:

$$r_{sf}e^{i\delta} = \frac{C_P^0 + A_V^0}{C_P^0 + T_V^0}, \qquad r_{wf}e^{i\phi} = -\frac{V_{cd}^* V_{us}}{V_{cs}^* V_{ud}}, \qquad r_f = r_{sf}r_{wf}, \tag{59}$$

where r_{sf}, r_{wf}, and r_f are positive numbers, r_f denotes the magnitude of the ratio of the DCS amplitude to the CF amplitude, and δ and ϕ are the strong phase difference and the weak phase difference, respectively. Making use of Eqs. (40), (43), (44), (56), and (59) and performing integration over phase space, we can obtain

$$\begin{aligned} \Gamma(D^+ \to K^*\pi^+ \to K(t)\pi^0\pi^+ \to f_{K^0}\pi^0\pi^+) = &\frac{G_F^2|g^{K^{*0}\to K^0\pi^0}|^2}{6144\pi^3 m_{D^+}^3}|V_{cs}|^2|V_{ud}|^2 \cdot \int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2 \\ &\times [r_f^2 g_{K_{\rm phys}^0} + r_f e^{i(\delta+\phi)} g_{K_{\rm phys}^0\bar{K}_{\rm phys}^0} + r_f e^{-i(\delta+\phi)} g_{K_{\rm phys}^0\bar{K}_{\rm phys}^0}^* + g_{\bar{K}_{\rm phys}^0}]dp_{K^*}^2, \end{aligned} \tag{60}$$

where we use the following substitutions:

$$g_{K_{\rm phys}^0} = |A(K_{\rm phys}^0(t) \to f_{K^0})|^2, \qquad g_{\bar{K}_{\rm phys}^0} = |A(\bar{K}_{\rm phys}^0(t) \to f_{K^0})|^2, \tag{61}$$

$$g_{K_{\rm phys}^0\bar{K}_{\rm phys}^0} = A(K_{\rm phys}^0(t) \to f_{K^0})A^*(\bar{K}_{\rm phys}^0(t) \to f_{K^0}), \tag{62}$$

$$g_{\rm in}(p_{K^*}^2) = \frac{(f(m_{D^+}, \sqrt{p_{K^*}^2}, m_{\pi^+}))^3 \cdot (f(\sqrt{p_{K^*}^2}, m_{K^0}, m_{\pi^0}))^3}{[(p_{K^*}^2 - m_{K^*}^2)^2 + m_{K^*}^2\Gamma_{K^*}^2(p_{K^*}^2)] \cdot p_{K^*}^6} \frac{(\Lambda^2 + m_{K^*}^2)^2}{(\Lambda^2 + p_{K^*}^2)^2}, \tag{63}$$

where p_0^2 and p_1^2 in Eq. (60) are the lower bound and the upper bound of $p_{K^*}^2$, respectively. In order to select the K^* event and suppress the background, we adopt $p_0^2 = (m_{K^*} - 3\Gamma_{K^*}^0)^2$ and $p_1^2 = (m_{K^*} + 3\Gamma_{K^*}^0)^2$ in our calculation, where m_{K^*} and $\Gamma_{K^*}^0$ are the mass and decay width of the K^* resonance, respectively.

Similarly, we can derive the decay width for the $D^- \to K^{*0}\pi^- + \bar{K}^{*0}\pi^- \to K^0(t)\pi^0\pi^- + \bar{K}^0(t))\pi^0\pi^- \to f_{K^0}\pi^0\pi^-$ decay

$$\Gamma(D^- \to K^*\pi^- \to K(t)\pi^0\pi^- \to f_{K^0}\pi^0\pi^-) = \frac{G_F^2|g^{K^{*0}\to K^0\pi^0}|^2}{6144\pi^3 m_{D^+}^3}|V_{cs}|^2|V_{ud}|^2 \cdot \int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2 \times [r_f^2 g_{\bar{K}^0_{\rm phys}} + r_f e^{i(\phi-\delta)} g_{K^0_{\rm phys}\bar{K}^0_{\rm phys}} + r_f e^{-i(\phi-\delta)} g^*_{K^0_{\rm phys}\bar{K}^0_{\rm phys}} + g_{K^0_{\rm phys}}] dp_{K^*}^2. \tag{64}$$

C. The decay widths for the $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays

In experiment, the K_S^0 state is defined via a final state $\pi^+\pi^-$ with $m_{\pi\pi} \approx m_S$ and a time difference between the $D^\pm$ decay and the K_S^0 decay [59,86,87]. By taking into account these experimental features, the partial decay width for the $D^+ \to K^{*0}\pi^+ + \bar{K}^{*0}\pi^+ \to K_S^0\pi^0\pi^+$ (hereinafter for brevity referred to as $D^+ \to K^*\pi^+ \to K_S^0\pi^0\pi^+$) decay can be defined as

$$\Gamma(D^+ \to K^*\pi^+ \to K_S^0\pi^0\pi^+) = \frac{\int_{t_0}^{t_1} \Gamma(D^+ \to K^*\pi^+ \to K(t)\pi^0\pi^+ \to \pi^+\pi^-\pi^0\pi^+)dt}{(e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}) \cdot \mathcal{B}(K_S^0 \to \pi^+\pi^-)}, \tag{65}$$

where $t_0 = 0.1\tau_S$ and $t_1 = 2\tau_S \sim 20\tau_S$ with τ_S being the K_S^0 lifetime; we adopt $t_1 = 10\tau_S$ in our calculation. Combining Eqs. (52), (57), (58), (60), and (65), we can obtain

$$\Gamma(D^+ \to K^*\pi^+ \to K_S^0\pi^0\pi^+) = \frac{G_F^2|g^{K^{*0}\to K^0\pi^0}|^2}{6144\pi^3 m_{D^+}^3}|V_{cs}|^2|V_{ud}|^2 \cdot \int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2 \times \left[r_f^2 g^{K_S^0}_{K^0_{\rm phys}} + r_f e^{i(\delta+\phi)} g^{K_S^0}_{K^0_{\rm phys}\bar{K}^0_{\rm phys}} + r_f e^{-i(\delta+\phi)} g^{K_S^0 *}_{K^0_{\rm phys}\bar{K}^0_{\rm phys}} + g^{K_S^0}_{\bar{K}^0_{\rm phys}}\right] dp_{K^*}^2, \tag{66}$$

with

$$g^{K_S^0}_{K^0_{\rm phys}} = \frac{\int_{t_0}^{t_1} |A(K^0_{\rm phys}(t) \to \pi^+\pi^-)|^2 dt}{(e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}) \cdot \mathcal{B}(K_S^0 \to \pi^+\pi^-)} = \frac{1}{4|p|^2}\left[1 + \frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\mathcal{B}(K_L^0 \to \pi^+\pi^-)}{\mathcal{B}(K_S^0 \to \pi^+\pi^-)} + 2{\rm Re}\left(\frac{p-q}{p+q} t_{K_S^0-K_L^0}\right)\right], \tag{67}$$

$$g^{K_S^0}_{K^0_{\rm phys}\bar{K}^0_{\rm phys}} = \frac{\int_{t_0}^{t_1} A(K^0_{\rm phys}(t) \to \pi^+\pi^-) A^*(\bar{K}^0_{\rm phys}(t) \to \pi^+\pi^-) dt}{(e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}) \cdot \mathcal{B}(K_S^0 \to \pi^+\pi^-)} = \frac{1}{4pq^*}\left[1 - \frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\mathcal{B}(K_L^0 \to \pi^+\pi^-)}{\mathcal{B}(K_S^0 \to \pi^+\pi^-)} + 2i{\rm Im}\left(\frac{p-q}{p+q} t_{K_S^0-K_L^0}\right)\right], \tag{68}$$

$$g^{K_S^0}_{\bar{K}^0_{\rm phys}} = \frac{\int_{t_0}^{t_1} |A(\bar{K}^0_{\rm phys}(t) \to \pi^+\pi^-)|^2 dt}{(e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}) \cdot \mathcal{B}(K_S^0 \to \pi^+\pi^-)} = \frac{1}{4|q|^2}\left[1 + \frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\mathcal{B}(K_L^0 \to \pi^+\pi^-)}{\mathcal{B}(K_S^0 \to \pi^+\pi^-)} - 2{\rm Re}\left(\frac{p-q}{p+q} t_{K_S^0-K_L^0}\right)\right], \tag{69}$$

where

$$t_{K_S^0-K_L^0} = \frac{e^{-i(m_L-m_S)t_0-\frac{\Gamma_S+\Gamma_L}{2}t_0} - e^{-i(m_L-m_S)t_1-\frac{\Gamma_S+\Gamma_L}{2}t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\Gamma_S}{\frac{\Gamma_S+\Gamma_L}{2} + i(m_L - m_S)}. \quad (70)$$

The terms in the square brackets of Eqs. (67)–(69) are related to the effect of the K_S^0 decay, the effect of the K_L^0 decay, and their interference, respectively. From the Particle Data Group [85], we can obtain

$$\frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} = 0.019, \quad (71)$$

$$\frac{\mathcal{B}(K_L^0 \to \pi^+\pi^-)}{\mathcal{B}(K_S^0 \to \pi^+\pi^-)} = (2.84 \pm 0.01) \times 10^{-3}, \quad (72)$$

with $t_0 = 0.1\tau_S = 0.1/\Gamma_S$ and $t_1 = 10\tau_S = 10/\Gamma_S$, so the second term in the square bracket of Eqs. (67)–(69), which corresponds to the effect of the K_L^0 decay, can be neglected. Combining Eqs. (51), (66)–(69) and neglecting the terms of $\mathcal{O}(\epsilon)$, we can derive

$$\Gamma(D^+ \to K^*\pi^+ \to K_S^0\pi^0\pi^+) = \frac{G_F^2|g^{K^{*0}\to K^0\pi^0}|^2}{6144\pi^3 m_{D^+}^3}|V_{cs}|^2|V_{ud}|^2 \cdot \int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2 \left\{\frac{r_f^2}{2}[1 - 2{\rm Re}(\epsilon) + 2{\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0})] + r_f\cos(\phi+\delta) + 2r_f\sin(\phi+\delta)[{\rm Im}(\epsilon) - {\rm Im}(\epsilon \cdot t_{K_S^0-K_L^0})] + \frac{1}{2}[1 + 2{\rm Re}(\epsilon) - 2{\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0})]\right\} dp_{K^*}^2. \quad (73)$$

Similarly, we can derive the decay width for the $D^- \to K^{*0}\pi^- + \bar{K}^{*0}\pi^- \to K_S^0\pi^0\pi^-$ (hereinafter for brevity referred to as $D^- \to K^*\pi^- \to K_S^0\pi^0\pi^-$) decay

$$\Gamma(D^- \to K^*\pi^- \to K_S^0\pi^0\pi^-) = \frac{G_F^2|g^{K^{*0}\to K^0\pi^0}|^2}{6144\pi^3 m_{D^+}^3}|V_{cs}|^2|V_{ud}|^2 \cdot \int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2 \left\{\frac{r_f^2}{2}[1 + 2{\rm Re}(\epsilon) - 2{\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0})] + r_f\cos(\phi-\delta) + 2r_f\sin(\phi-\delta)[{\rm Im}(\epsilon) - {\rm Im}(\epsilon \cdot t_{K_S^0-K_L^0})] + \frac{1}{2}[1 - 2{\rm Re}(\epsilon) + 2{\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0})]\right\} dp_{K^*}^2. \quad (74)$$

In experiment, the K_L^0 state is defined via a large time difference between the $D^\pm$ decay and the K_L^0 decay, so the K_L^0 states mostly decay outside the detector [88]. Based on these experimental features, the partial decay width for the $D^+ \to K^{*0}\pi^+ + \bar{K}^{*0}\pi^+ \to K_L^0\pi^0\pi^+$ (hereinafter for brevity referred to as $D^+ \to K^*\pi^+ \to K_L^0\pi^0\pi^+$) decay can be defined as

$$\Gamma(D^+ \to K^*\pi^+ \to K_L^0\pi^0\pi^+) = \frac{\int_{t_2}^{+\infty} \Gamma(D^+ \to K^*\pi^+ \to K(t)\pi^0\pi^+ \to \pi^+\pi^-\pi^0\pi^0\pi^+)dt}{e^{-\Gamma_L t_2} \cdot \mathcal{B}(K_L^0 \to \pi^+\pi^-\pi^0)}, \quad (75)$$

where $t_2 \geq 100\tau_S$. Using Eqs. (53), (57)–(58), (60), and (75), we can derive

$$\Gamma(D^+ \to K^*\pi^+ \to K_L^0\pi^0\pi^+) = \frac{G_F^2|g^{K^{*0}\to K^0\pi^0}|^2}{6144\pi^3 m_{D^+}^3}|V_{cs}|^2|V_{ud}|^2 \cdot \int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2 \times \left[r_f^2 g_{K^0_{\rm phys}}^{K_L^0} + r_f e^{i(\delta+\phi)} g_{K^0_{\rm phys}\bar{K}^0_{\rm phys}}^{K_L^0} + r_f e^{-i(\delta+\phi)} g_{K^0_{\rm phys}\bar{K}^0_{\rm phys}}^{K_L^0 *} + g_{\bar{K}^0_{\rm phys}}^{K_L^0}\right] dp_{K^*}^2, \quad (76)$$

with

$$g_{K^0_{\rm phys}}^{K_L^0} = \frac{\int_{t_2}^{+\infty} |A(K^0_{\rm phys}(t) \to \pi^+\pi^-\pi^0)|^2 dt}{e^{-\Gamma_L t_2} \cdot \mathcal{B}(K_L^0 \to \pi^+\pi^-\pi^0)} = \frac{1}{4|p|^2}\left[1 + e^{-(\Gamma_S-\Gamma_L)t_2} \cdot \frac{\mathcal{B}(K_S^0 \to \pi^+\pi^-\pi^0)}{\mathcal{B}(K_L^0 \to \pi^+\pi^-\pi^0)} + 2{\rm Re}\left(\frac{p-q}{p+q} t_{K_L^0-K_S^0}\right)\right], \quad (77)$$

$$g^{K_L^0}_{K^0_{\rm phys}\bar{K}^0_{\rm phys}} = \frac{\int_{t_2}^{+\infty} A(K^0_{\rm phys}(t)\to\pi^+\pi^-\pi^0)A^*(\bar{K}^0_{\rm phys}(t)\to\pi^+\pi^-\pi^0)dt}{e^{-\Gamma_L t_2}\cdot\mathcal{B}(K_L^0\to\pi^+\pi^-\pi^0)}$$
$$= \frac{1}{4pq^*}\left[-1+e^{-(\Gamma_S-\Gamma_L)t_2}\cdot\frac{\mathcal{B}(K_S^0\to\pi^+\pi^-\pi^0)}{\mathcal{B}(K_L^0\to\pi^+\pi^-\pi^0)} - 2i\mathrm{Im}\left(\frac{p-q}{p+q}t_{K_L^0-K_S^0}\right)\right], \tag{78}$$

$$g^{K_L^0}_{\bar{K}^0_{\rm phys}} = \frac{\int_{t_2}^{+\infty}|A(\bar{K}^0_{\rm phys}(t)\to\pi^+\pi^-\pi^0)|^2dt}{e^{-\Gamma_L t_2}\cdot\mathcal{B}(K_L^0\to\pi^+\pi^-\pi^0)}$$
$$= \frac{1}{4|q|^2}\left[1+e^{-(\Gamma_S-\Gamma_L)t_2}\cdot\frac{\mathcal{B}(K_S^0\to\pi^+\pi^-\pi^0)}{\mathcal{B}(K_L^0\to\pi^+\pi^-\pi^0)} - 2\mathrm{Re}\left(\frac{p-q}{p+q}t_{K_L^0-K_S^0}\right)\right], \tag{79}$$

where

$$t_{K_L^0-K_S^0} = \frac{\Gamma_L\cdot e^{i(m_L-m_S)t_2-\frac{\Gamma_S-\Gamma_L}{2}t_2}}{\frac{\Gamma_S+\Gamma_L}{2}-i(m_L-m_S)}. \tag{80}$$

Using the result from the Particle Data Group [85], $\Gamma_L/\Gamma_S = (1.75\pm 0.01)\times 10^{-3}$, we can obtain

$$e^{-(\Gamma_S-\Gamma_L)t_2} \leq 4.4\times 10^{-44}, \qquad e^{-\frac{\Gamma_S-\Gamma_L}{2}t_2} \leq 2.1\times 10^{-22}, \tag{81}$$

with $t_2 \geq 100/\Gamma_S$, so the last two terms in the square brackets of Eqs. (77)–(79) can be neglected safely. Substituting Eqs. (51), (77)–(79) into Eq. (76) and neglecting the terms of $\mathcal{O}(\epsilon)$, we can obtain

$$\Gamma(D^+\to K^*\pi^+\to K_L^0\pi^0\pi^+) = \frac{G_F^2|g^{K^{*0}\to K^0\pi^0}|^2}{6144\pi^3 m_{D^+}^3}|V_{cs}|^2|V_{ud}|^2\int_{p_0^2}^{p_1^2} g_{\rm in}(p^2_{K^*})|C_P^0+T_V^0|^2$$
$$\cdot\left[\frac{r_f^2}{2}(1-2\mathrm{Re}(\epsilon)) - r_f\cos(\phi+\delta) - 2r_f\sin(\phi+\delta)\mathrm{Im}(\epsilon)+\frac{(1+2\mathrm{Re}(\epsilon))}{2}\right]dp^2_{K^*}. \tag{82}$$

Similarly, we can derive the decay width for the $D^-\to K^{*0}\pi^- + \bar{K}^{*0}\pi^- \to K_L^0\pi^0\pi^-$ (hereinafter for brevity referred to as $D^-\to K^*\pi^-\to K_L^0\pi^0\pi^-$) decay

$$\Gamma(D^-\to K^*\pi^-\to K_L^0\pi^0\pi^-) = \frac{G_F^2|g^{K^{*0}\to K^0\pi^0}|^2}{6144\pi^3 m_{D^+}^3}|V_{cs}|^2|V_{ud}|^2\int_{p_0^2}^{p_1^2} g_{\rm in}(p^2_{K^*})|C_P^0+T_V^0|^2$$
$$\cdot\left[\frac{r_f^2}{2}(1+2\mathrm{Re}(\epsilon)) - r_f\cos(\phi-\delta) - 2r_f\sin(\phi-\delta)\mathrm{Im}(\epsilon)+\frac{(1-2\mathrm{Re}(\epsilon))}{2}\right]dp^2_{K^*}. \tag{83}$$

The branching ratios of the $D^\pm\to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays can be obtained by multiplying the partial decay widths for these decays, which are given in Eqs. (73)–(74) and (82)–(83), and the mean life of the $D^\pm$ meson.

III. *CP* VIOLATIONS AND $K_S^0 - K_L^0$ ASYMMETRIES

A. *CP* violations in the $D^\pm\to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays

Basing on the partial decay widths for the $D^\pm\to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays derived in Sec. II, we can proceed to study the *CP* violations and $K_S^0 - K_L^0$ asymmetries in these decays.

In the $D^\pm\to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays, the time-independent *CP* violation observables are defined as

$$A_{CP}^{K^0_{S,L}} = \frac{\Gamma(D^+\to K^*\pi^+\to K^0_{S,L}\pi^0\pi^+) - \Gamma(D^-\to K^*\pi^-\to K^0_{S,L}\pi^0\pi^-)}{\Gamma(D^+\to K^*\pi^+\to K^0_{S,L}\pi^0\pi^+) + \Gamma(D^-\to K^*\pi^-\to K^0_{S,L}\pi^0\pi^-)}. \tag{84}$$

Substituting Eqs. (73)–(74) into Eq. (84), we can derive

$$A_{CP}^{K_S^0} = A_{CP,K_S^0}^{\rm mix} + A_{CP,K_S^0}^{\rm dir} + A_{CP,K_S^0}^{\rm int}, \tag{85}$$

$$A_{CP,K_S^0}^{\rm mix} = \frac{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2\{2(1-r_f^2)[{\rm Re}(\epsilon) - {\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0})]\}dp_{K^*}^2}{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2(1 + r_f^2 + 2r_f\cos\delta\cos\phi)dp_{K^*}^2}, \tag{86}$$

$$A_{CP,K_S^0}^{\rm dir} = \frac{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2(-2r_f\sin\delta\sin\phi)dp_{K^*}^2}{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2(1 + r_f^2 + 2r_f\cos\delta\cos\phi)dp_{K^*}^2}, \tag{87}$$

$$A_{CP,K_S^0}^{\rm int} = \frac{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2\{4r_f\sin\delta\cos\phi[{\rm Im}(\epsilon) - {\rm Im}(\epsilon \cdot t_{K_S^0-K_L^0})]\}dp_{K^*}^2}{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2(1 + r_f^2 + 2r_f\cos\delta\cos\phi)dp_{K^*}^2}, \tag{88}$$

where $A_{CP,K_S^0}^{\rm mix}$ denotes the *CP* violation in kaon mixing [51,59] and the two terms in the square bracket of Eq. (86) correspond to the pure K_S^0 term, and the $K_L^0 - K_S^0$ interference term, respectively. The $K_L^0 - K_S^0$ interference term, which is a function of t_0 and t_1, is as important as the pure K_S^0 term [59]. $A_{CP,K_S^0}^{\rm dir}$ denotes the direct *CP* asymmetry induced by the interference between the tree level CF and DCS amplitudes. $A_{CP,K_S^0}^{\rm int}$ represents a new *CP* violating effect, which relates to the following expression:

$$4r_f\sin\delta\cos\phi[{\rm Im}(\epsilon) - {\rm Im}(\epsilon \cdot t_{K_S^0-K_L^0})] = \frac{(C_P^0 + A_V^0)(C_P^{0*} + T_V^{0*}) - (C_P^{0*} + A_V^{0*})(C_P^0 + T_V^0)}{|(C_P^0 + T_V^0)|^2} \cdot \frac{V_{cd}^* V_{us} V_{cs} V_{ud}^* + V_{cd} V_{us}^* V_{cs}^* V_{ud}}{2|V_{cs}|^2|V_{ud}|^2} \cdot \left(g_{K^0_{\rm phys}\bar{K}^0_{\rm phys}}^{K_S^0 *} - g_{K^0_{\rm phys}\bar{K}^0_{\rm phys}}^{K_S^0}\right), \tag{89}$$

i.e., this new *CP* violating effect arises from the interference between two tree (CF and DCS) amplitudes with the neutral kaon mixing [51,89,90]. Here, we also note that the $K_L^0 - K_S^0$ interference term $\epsilon \cdot t_{K_S^0-K_L^0}$ has a large contribution to the new *CP* violating effect, as shown in Eq. (88). In our calculation, we adopt $t_0 = 0.1\tau_S$ and $t_1 = 10\tau_S$. In addition, we will discuss the impact of the choice of t_0 on $A_{CP,K_S^0}^{\rm mix}$, $A_{CP,K_S^0}^{\rm int}$, and $A_{CP}^{K_S^0}$ in Sec. V.

Similarly, substituting Eqs. (82)–(83) into Eq. (84), we can derive the expression for *CP* asymmetry in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_L^0\pi^0\pi^{\pm}$ decays

$$A_{CP}^{K_L^0} = A_{CP,K_L^0}^{\rm mix} + A_{CP,K_L^0}^{\rm dir} + A_{CP,K_L^0}^{\rm int}, \tag{90}$$

$$A_{CP,K_L^0}^{\rm mix} = \frac{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2[2(1-r_f^2){\rm Re}(\epsilon)]dp_{K^*}^2}{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2(1 + r_f^2 - 2r_f\cos\delta\cos\phi)dp_{K^*}^2}, \tag{91}$$

$$A_{CP,K_L^0}^{\rm dir} = \frac{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2(2r_f\sin\delta\sin\phi)dp_{K^*}^2}{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2(1 + r_f^2 - 2r_f\cos\delta\cos\phi)dp_{K^*}^2}, \tag{92}$$

$$A_{CP,K_L^0}^{\rm int} = \frac{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2[-4r_f\sin\delta\cos\phi{\rm Im}(\epsilon)]dp_{K^*}^2}{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2(1 + r_f^2 - 2r_f\cos\delta\cos\phi)dp_{K^*}^2}, \tag{93}$$

where $A_{CP,K_L^0}^{\rm mix}$, $A_{CP,K_L^0}^{\rm dir}$, and $A_{CP,K_L^0}^{\rm int}$ denote the indirect *CP* violation in kaon mixing, the direct *CP* violation in charm decays, and the new *CP* violation effect, respectively. From Eqs. (90)–(93), one can find that all *CP* violation effects in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_L^0\pi^0\pi^{\pm}$ decays receive no contribution from the $K_L^0 - K_S^0$ interference and are independent of the decay time t_2.

B. $K_S^0 - K_L^0$ asymmetries in the $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K_{S,L}^0\pi^0\pi^\pm$ decays

The $K_S^0 - K_L^0$ asymmetries in the D meson decays are induced by the interference between the CF and DCS amplitudes, which was first pointed out by Bigi and Yamamoto [91]. The determination on the $K_S^0 - K_L^0$ asymmetries in the D meson decays can be useful to study the DCS processes and understand the dynamics of charm decay [78,92]. In the $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K_{S,L}^0\pi^0\pi^\pm$ decays, the $K_S^0 - K_L^0$ asymmetries are defined by

$$R_{K_S-K_L}^{D^+} = \frac{\Gamma(D^+ \to K^*\pi^+ \to K_S^0\pi^0\pi^+) - \Gamma(D^+ \to K^*\pi^+ \to K_L^0\pi^0\pi^+)}{\Gamma(D^+ \to K^*\pi^+ \to K_S^0\pi^0\pi^+) + \Gamma(D^+ \to K^*\pi^+ \to K_L^0\pi^0\pi^+)}, \tag{94}$$

$$R_{K_S-K_L}^{D^-} = \frac{\Gamma(D^- \to K^*\pi^- \to K_S^0\pi^0\pi^-) - \Gamma(D^- \to K^*\pi^- \to K_L^0\pi^0\pi^-)}{\Gamma(D^- \to K^*\pi^- \to K_S^0\pi^0\pi^-) + \Gamma(D^- \to K^*\pi^- \to K_L^0\pi^0\pi^-)}. \tag{95}$$

Using Eqs. (73), (82), and (94), we can obtain

$$R_{K_S-K_L}^{D^+} = \frac{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2 A_{K_S-K_L}^{D^+} dp_{K^*}^2}{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2[1 + r_f^2 + 2{\rm Re}(\epsilon) - {\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0})]dp_{K^*}^2}, \tag{96}$$

with

$$A_{K_S-K_L}^{D^+} = 2r_f\cos(\phi+\delta) + 2r_f\sin(\phi+\delta)(2{\rm Im}(\epsilon) - {\rm Im}(\epsilon \cdot t_{K_S^0-K_L^0})) - {\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0}). \tag{97}$$

From the above equation, we can see that the main contribution to $R_{K_S-K_L}^{D^+}$ comes from the pure K_S^0 and K_L^0 decay; the contribution from the $K_L^0 - K_S^0$ interference terms $\epsilon \cdot t_{K_S^0-K_L^0}$ is small because of the suppression of the parameter ϵ. Similarly, combining Eqs. (74), (83), and (95), we can derive the expression for $K_S^0 - K_L^0$ asymmetry in $D^- \to K^{*0}\pi^- + \bar{K}^{*0}\pi^- \to K_{S,L}^0\pi^0\pi^-$ decays

$$R_{K_S-K_L}^{D^-} = \frac{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2 A_{K_S-K_L}^{D^-} dp_{K^*}^2}{\int_{p_0^2}^{p_1^2} g_{\rm in}(p_{K^*}^2)|C_P^0 + T_V^0|^2[1 + r_f^2 - 2{\rm Re}(\epsilon) + {\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0})]dp_{K^*}^2}, \tag{98}$$

with

$$A_{K_S-K_L}^{D^-} = 2r_f\cos(\phi-\delta) + 2r_f\sin(\phi-\delta)(2{\rm Im}(\epsilon) - {\rm Im}(\epsilon \cdot t_{K_S^0-K_L^0})) + {\rm Re}(\epsilon \cdot t_{K_S^0-K_L^0}). \tag{99}$$

According to the definition of the weak phase difference in Eq. (59), we have $\sin\phi = \mathcal{O}(10^{-3})$ and $\cos\phi \approx 1$, hence as a good approximation, $\cos(\phi \pm \delta) \approx \cos\delta$ and $\sin(\phi \pm \delta) \approx \pm\sin\delta$. Therefore, the determinations of $R_{K_S-K_L}^{D^+}$ and $R_{K_S-K_L}^{D^-}$ are useful for understanding the strong phase difference between the DCS and CF amplitudes [78].

IV. NUMERICAL RESULTS

A. Input parameters

Using the theoretical expressions for the branching ratios, the CP asymmetries, and the $K_S^0 - K_L^0$ asymmetries derived in Secs. II and III, we are able to calculate these observables numerically. Firstly, we collect the input parameters used in this work as below [85,93–98]:

$$\begin{aligned} m_{D^+} &= 1.870\ {\rm GeV}, \qquad \tau_{D^+} = (1033 \pm 5) \times 10^{-15}\ {\rm s}, \\ m_S &= 0.498\ {\rm GeV}, \qquad m_L = 0.498\ {\rm GeV}, \\ m_L - m_S &= 3.484 \times 10^{-15}\ {\rm GeV}, \qquad m_{K^0} = 0.498\ {\rm GeV}, \\ \Gamma_S &= (7.351 \pm 0.003) \times 10^{-15}\ {\rm GeV}, \qquad \Gamma_L = (1.287 \pm 0.005) \times 10^{-17}\ {\rm GeV}, \end{aligned}$$

$$\begin{aligned}
m_{D^*(2010)^{\pm}} &= 2.010\ \text{GeV}, \qquad m_{D_S^{\pm}} = 1.968\ \text{GeV}, \\
m_{K^*} &= 0.892\ \text{GeV}, \qquad \Gamma^0_{K^*} = (5.14 \pm 0.08) \times 10^{-2}\ \text{GeV}, \\
m_{\pi^+} &= 0.140\ \text{GeV}, \qquad m_{\pi^0} = 0.135\ \text{GeV}, \\
f_{D^+} &= (0.205 \pm 0.004)\ \text{GeV}, \qquad f_{K^*} = (0.220 \pm 0.005)\ \text{GeV}, \\
f_{\pi^+} &= (0.130 \pm 0.001)\ \text{GeV}, \qquad f_{\rho^0} = (0.216 \pm 0.003)\ \text{GeV}, \\
\text{Re}(\epsilon) &= (1.66 \pm 0.02) \times 10^{-3}, \qquad \text{Im}(\epsilon) = (1.57 \pm 0.02) \times 10^{-3}.
\end{aligned} \tag{100}$$

The branching ratios used in this paper have been taken from the Particle Data Group [85]:

$$\begin{aligned}
&\mathcal{B}(K^{*0} \to K^0\pi^0) = (33.251 \pm 0.007) \times 10^{-2}, \\
&\mathcal{B}(K_S^0 \to \pi^+\pi^-) = (69.20 \pm 0.05) \times 10^{-2}, \qquad \mathcal{B}(K_L^0 \to \pi^+\pi^-) = (1.967 \pm 0.010) \times 10^{-3}, \\
&\mathcal{B}(K_S^0 \to \pi^+\pi^-\pi^0) = (3.5^{-0.9}_{+1.1}) \times 10^{-7}, \qquad \mathcal{B}(K_L^0 \to \pi^+\pi^-\pi^0) = (12.54 \pm 0.05) \times 10^{-2}.
\end{aligned} \tag{101}$$

As for the universal nonfactorizable parameters, we use the results fitted in Ref. [78], which are based on the factorization-assisted topological-amplitudes approach:

$$\begin{aligned}
&\chi_P^C = -0.443 \pm 0.007, \qquad \phi_P^C = 0.497 \pm 0.027, \\
&\chi_q^A = 0.147 \pm 0.021, \qquad \phi_q^A = -0.584 \pm 0.211, \qquad S_\pi = 1.28 \pm 0.14.
\end{aligned} \tag{102}$$

In order to see physics more transparently, we use the Wolfenstein parametrization of the CKM matrix elements, whose imaginary part satisfies the unitarity relation to order λ^5 [85,99–101]:

$$V_{ud} = 1 - \frac{\lambda^2}{2}, \qquad V_{us} = \lambda, \qquad V_{cd} = -\lambda, \qquad V_{cs} = 1 - \frac{\lambda^2}{2} - A^2\lambda^4(\rho + i\eta), \tag{103}$$

where λ, A, ρ, and η are the real parameters. The latest results fitted by the UTfit collaboration are presented as follows [102]:

$$\lambda = 0.225 \pm 0.001, \qquad A = 0.826 \pm 0.012, \qquad \rho = 0.152 \pm 0.014, \qquad \eta = 0.357 \pm 0.010. \tag{104}$$

By substituting the values of the parameters listed above into Eqs. (73)–(74) and (82)–(83), we can obtain the numerical values of the branching ratios, which are shown in Table I.

Here, the results in the last two lines of Table I are the averaged branching ratios of the decay and its charge conjugate. The results given in Table I are consistent with the experimental measurement of $\mathcal{B}(D^+ \to K^*\pi^+ \to K_S^0\pi^0\pi^+) = (2.64 \pm 0.32) \times 10^{-3}$ from BESIII [85,103]. We also note that the reasons for the differences between the results of the FAT approach and that of the TA approach are the small values of $\cos\delta$ and $|(C_P^0 + T_V^0)|^2$ in the TA approach.

TABLE I. The values of the branching ratios for the $D^{\pm} \to K^*\pi^{\pm} \to K_{S,L}^0\pi^0\pi^{\pm}$ decays in the FAT approach and the TA approach.

Observables	The FAT approach	The TA approach
$\mathcal{B}(D^+ \to K^*\pi^+ \to K_S^0\pi^0\pi^+)$	$(3.12^{-0.34}_{+0.36}) \times 10^{-3}$	$(2.25^{-0.21}_{+0.23}) \times 10^{-3}$
$\mathcal{B}(D^- \to K^*\pi^- \to K_S^0\pi^0\pi^-)$	$(3.14^{-0.34}_{+0.36}) \times 10^{-3}$	$(2.28^{-0.22}_{+0.23}) \times 10^{-3}$
$\mathcal{B}(D^+ \to K^*\pi^+ \to K_L^0\pi^0\pi^+)$	$(2.14^{-0.24}_{+0.25}) \times 10^{-3}$	$(2.43^{-0.21}_{+0.23}) \times 10^{-3}$
$\mathcal{B}(D^- \to K^*\pi^- \to K_L^0\pi^0\pi^-)$	$(2.12^{-0.24}_{+0.25}) \times 10^{-3}$	$(2.41^{-0.21}_{+0.22}) \times 10^{-3}$
$\mathcal{B}(D^{\pm} \to K^*\pi^{\pm} \to K_S^0\pi^0\pi^{\pm})$	$(3.13^{-0.34}_{+0.36}) \times 10^{-3}$	$(2.27^{-0.22}_{+0.23}) \times 10^{-3}$
$\mathcal{B}(D^{\pm} \to K^*\pi^{\pm} \to K_L^0\pi^0\pi^{\pm})$	$(2.13^{-0.24}_{+0.25}) \times 10^{-3}$	$(2.42^{-0.21}_{+0.22}) \times 10^{-3}$

TABLE II. The values of the CP asymmetries in the $D^\pm \to K^*\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays in the FAT approach and the TA approach.

Observables	The FAT approach	The TA approach
$A^{\rm mix}_{CP,K^0_S}$	$(-2.92 \pm 0.06) \times 10^{-3}$	$(-3.64^{-0.07}_{+0.06}) \times 10^{-3}$
$A^{\rm dir}_{CP,K^0_S}$	$(-1.18 \pm 0.11) \times 10^{-4}$	$(-1.67 \pm 0.12) \times 10^{-4}$
$A^{\rm int}_{CP,K^0_S}$	$(-6.50^{-0.51}_{+0.52}) \times 10^{-4}$	$(-9.17^{-0.52}_{+0.48}) \times 10^{-4}$
$A^{K^0_S}_{CP} = A^{\rm mix}_{CP,K^0_S} + A^{\rm dir}_{CP,K^0_S} + A^{\rm int}_{CP,K^0_S}$	$(-3.69 \pm 0.09) \times 10^{-3}$	$(-4.72 \pm 0.09) \times 10^{-3}$
$A^{\rm mix}_{CP,K^0_L}$	$(3.92^{-0.09}_{+0.08}) \times 10^{-3}$	$(3.11^{-0.06}_{+0.05}) \times 10^{-3}$
$A^{\rm dir}_{CP,K^0_L}$	$(1.74^{-0.14}_{+0.15}) \times 10^{-4}$	$(1.56 \pm 0.11) \times 10^{-4}$
$A^{\rm int}_{CP,K^0_L}$	$(8.52^{-0.59}_{+0.61}) \times 10^{-4}$	$(7.65^{-0.41}_{+0.40}) \times 10^{-4}$
$A^{K^0_L}_{CP} = A^{\rm mix}_{CP,K^0_L} + A^{\rm dir}_{CP,K^0_L} + A^{\rm int}_{CP,K^0_L}$	$(4.95 \pm 0.10) \times 10^{-3}$	$(4.03 \pm 0.07) \times 10^{-3}$

B. The numerical results of the *CP* asymmetries

Now, we move on to calculate the numerical results of the CP asymmetries in $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays. By substituting the values of the parameters in Eqs. (100), (102), and (104) into Eqs. (85)–(88) and (90)–(93), we can obtain the numerical results of the CP asymmetries in $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays, which are shown in Table II. From these numerical values, we can obtain the following points:

(1) The indirect CP violation in $K^0 - \bar{K}^0$ mixing $A^{\rm mix}_{CP,K^0_S}$ is dominant in the CP asymmetry in $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_S\pi^0\pi^\pm$ decays $A^{K^0_S}_{CP}$. The contributions from the $K^0_L - K^0_S$ interference term ${\rm Re}(\epsilon \cdot t_{K^0_S-K^0_L})$ are more than twice of that from the pure K^0_S decay term ${\rm Re}(\epsilon)$ in $A^{\rm mix}_{CP,K^0_S}$, and they interfere destructively.

(2) The direct CP asymmetry $A^{\rm dir}_{CP,K^0_S}$ suffers from both the r_{wf} and $\sin\phi$ suppression; thus its numerical value is small.

(3) The value of r_{sf} and $\sin\delta$ vary from 2.49 to 2.97 and from -0.91 to -0.57 in the integral interval of $p^2_{K^*}$ in the FAT approach, respectively. In the TA approach, the value of r_{sf} and $\sin\delta$ is 2.42 and -0.99, respectively, so the new CP violation effect $A^{\rm int}_{CP,K^0_S}$ only suffers from the r_{wf} suppression relative to the indirect CP violation in $K^0 - \bar{K}^0$ mixing, as shown in Eqs. (86) and (88). Moreover, the pure K^0_S decay term ${\rm Im}(\epsilon)$ and the $K^0_L - K^0_S$ interference term ${\rm Im}(\epsilon \cdot t_{K^0_S-K^0_L})$ interfere constructively in $A^{\rm int}_{CP,K^0_S}$; all these reasons result in a non-negligible contribution of the new CP violation effect to the CP asymmetry in $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_S\pi^0\pi^\pm$ decays.

(4) The value of r_f and $\cos\delta$ vary from 0.13 to 0.16 and from 0.42 to 0.82 in the integral interval of $p^2_{K^*}$ in the FAT approach, respectively; however, the value of r_f and $\cos\delta$ is 0.13 and -0.13 in the TA approach, respectively.

(5) Based on the numerical values of $\sin\delta$ and $\cos\delta$ in the FAT approach and the TA approach and according to the expressions for CP asymmetries in Eqs. (85)–(88) and (90)–(93), we can derive that the large value of $|\sin\delta|$ and the negative value of $\cos\delta$ in the TA approach result in the differences between the numerical values of the CP asymmetries in the FAT approach and that in the TA approach.

According to the numerical results of the CP asymmetries in $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm$ decays, we can estimate how many $D^\pm$ events-times-efficiency are needed to establish the CP asymmetries to 3 standard deviations (3σ). When the CP violations are observed at the 3 standard deviation (3σ) level, the number of $D^\pm$ events-times-efficiency needed reads as [104–106]

$$(\epsilon_f N)^{K^0_{S,L}}_{CP} = \frac{9}{2 \cdot \mathcal{B}(D^\pm \to K^*\pi^\pm \to K^0_{S,L}\pi^0\pi^\pm) \cdot \mathcal{B}(K^0_{S,L} \to f_{K^0_{S,L}}) \cdot |A^{K^0_{S,L}}_{CP}|}, \tag{105}$$

where $f_{K^0_S}$ and $f_{K^0_L}$ denote $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$, respectively. Combining Eqs. (101), (105), and the numerical results of the branching ratios and the CP asymmetries in Tables I and II, we can obtain

$$(\epsilon_f N)^{K^0_S}_{CP} = \begin{cases} (5.0 \sim 6.3) \times 10^5, & \text{the FAT approach,} \\ (5.5 \sim 6.7) \times 10^5, & \text{the TA approach.} \end{cases} \tag{106}$$

Similarly, substituting Eq. (101) and the numerical results of the branching ratios and the CP asymmetries in Tables I and II into Eq. (105), we have

$$(\epsilon_f N)^{K_L^0}_{CP} = \begin{cases} (3.0\sim3.8)\times10^6, & \text{the FAT approach,} \\ (3.4\sim4.0)\times10^6, & \text{the TA approach.} \end{cases} \quad (107)$$

C. The numerical results of the $K_S^0 - K_L^0$ asymmetries

Now, we turn to calculate the numerical results of the $K_S^0 - K_L^0$ asymmetries $R^{D^{\pm}}_{K_S-K_L}$. The explicit expressions for $R^{D^{\pm}}_{K_S-K_L}$ have been given in Eqs. (96)–(99). With the values of the parameters in Eqs. (100), (102), and (104), we can obtain the numerical results of $R^{D^{\pm}}_{K_S-K_L}$:

$$R^{D^{+}}_{K_S-K_L} = \begin{cases} 0.186^{-0.017}_{+0.015}, & \text{the FAT approach,} \\ -0.038^{-0.013}_{+0.012}, & \text{the TA approach} \end{cases} \quad (108)$$

and

$$R^{D^{-}}_{K_S-K_L} = \begin{cases} 0.194^{-0.016}_{+0.015}, & \text{the FAT approach,} \\ -0.029^{-0.013}_{+0.012}, & \text{the TA approach.} \end{cases} \quad (109)$$

Based on these numerical values, we can obtain the following points:

(1) From Eqs. (96)–(99), we can see that the $K_S^0 - K_L^0$ asymmetries $R^{D^{\pm}}_{K_S-K_L}$ only suffer from the r_{wf} suppression, so they have a large value, which indicates that there exists a large difference between the branching ratios of $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ and the branching ratios of $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_L^0\pi^0\pi^{\pm}$.

(2) The numerical results of $R^{D^{\pm}}_{K_S-K_L}$ of the FAT approach are many times (about 5 times for $R^{D^{+}}_{K_S-K_L}$ and about 6 times for $R^{D^{-}}_{K_S-K_L}$) larger than that of the TA approach. Moreover, the signs of $R^{D^{\pm}}_{K_S-K_L}$ in these two approaches are opposite to each other; the reason is that the values of $\cos\delta$ are different in these two approaches. In addition, the $K_L^0 - K_S^0$ interference term $\mathrm{Re}(\epsilon \cdot t_{K_S^0-K_L^0})$ has a non-negligible contribution to $R^{D^{\pm}}_{K_S-K_L}$ in the TA approach.

(3) The measurement of $R^{D^{\pm}}_{K_S-K_L}$ can help to discriminate the FAT approach and the TA approach.

In the same way as the CP asymmetries in $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_{S,L}^0\pi^0\pi^{\pm}$ decays, the number of $D^{\pm}$ events-times-efficiency needed for observing the $K_S^0 - K_L^0$ asymmetries at the 3 standard deviation (3σ) level is

$$(\epsilon_f N)^{D^{\pm}}_{K_S-K_L} = \frac{9}{[\mathcal{B}(D^{\pm} \to K^{*}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}) + \mathcal{B}(D^{\pm} \to K^{*}\pi^{\pm} \to K_L^0\pi^0\pi^{\pm})] \cdot |R^{D^{\pm}}_{K_S-K_L}|}. \quad (110)$$

Using the numerical results of the branching ratios in Table I, Eq. (108), and Eq. (110), we can obtain

$$(\epsilon_f N)^{D^{+}}_{K_S-K_L} = \begin{cases} (0.8\sim1.0)\times10^4, & \text{the FAT approach,} \\ (3.8\sim7.8)\times10^4, & \text{the TA approach.} \end{cases} \quad (111)$$

Similarly, using the numerical results of the branching ratios in Table I, Eq. (109), and Eq. (110), we have

$$(\epsilon_f N)^{D^{-}}_{K_S-K_L} = \begin{cases} (0.8\sim1.0)\times10^4, & \text{the FAT approach,} \\ (0.5\sim1.2)\times10^5. & \text{the TA approach.} \end{cases} \quad (112)$$

V. THE OBSERVATION OF THE NEW CP VIOLATION EFFECT

In this section, we will study the new CP violation effect in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ decays. As discussed in Sec. III A, the CP violation in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ decays $A^{K_S^0}_{CP}$ consists of three parts: the indirect CP violation in $K^0 - \bar{K}^0$ mixing $A^{\mathrm{mix}}_{CP,K_S^0}$, the direct CP violation in charm decays $A^{\mathrm{dir}}_{CP,K_S^0}$, and the new CP violation effect from the interference between two tree (CF and DCS) amplitudes with the neutral kaon mixing $A^{\mathrm{int}}_{CP,K_S^0}$. Moreover, the CP violation in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ decays is dominated by the indirect CP violation in $K^0 - \bar{K}^0$ mixing, which is shown in Table II; all of these make the observation of the new CP violation effect more difficult.

Now, it is important to note the following features of the three parts of the CP violation in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ decays:

(1) The $K_L^0 - K_S^0$ interference term $\epsilon \cdot t_{K_S^0-K_L^0}$ makes a large contribution to both the indirect CP violation in $K^0 - \bar{K}^0$ mixing $A_{CP,K_S^0}^{\rm mix}$ and the new CP violation effect from the interference between two tree (CF and DCS) amplitudes with the neutral kaon mixing $A_{CP,K_S^0}^{\rm int}$, which can be seen from Eqs. (86) and (88).

(2) The $K_L^0 - K_S^0$ interference term is the function of the decay time parameters t_0 and t_1; we adopt $t_0 = 0.1\tau_S$ and $t_1 = 10\tau_S$ in our above calculation.

(3) As discussed in Sec. IV B, the contributions from the $K_L^0 - K_S^0$ interference term $\mathrm{Re}(\epsilon \cdot t_{K_S^0-K_L^0})$ and that from the pure K_S^0 decay term $\mathrm{Re}(\epsilon)$ interfere destructively in $A_{CP,K_S^0}^{\rm mix}$; however, the contributions from the $K_L^0 - K_S^0$ interference term $\mathrm{Im}(\epsilon \cdot t_{K_S^0-K_L^0})$ and that from the pure K_S^0 decay term $\mathrm{Im}(\epsilon)$ interfere constructively in $A_{CP,K_S^0}^{\rm int}$.

So there is a possibility that the numerical value of the indirect CP violation in $K^0 - \bar{K}^0$ mixing $A_{CP,K_S^0}^{\rm mix}$ becomes smaller and the numerical value of the new CP violation effect $A_{CP,K_S^0}^{\rm int}$ becomes larger if we adopt some specific values of t_0; as a result, the new CP violation effect $A_{CP,K_S^0}^{\rm int}$ would dominate the CP violation in the $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K_S^0\pi^0\pi^\pm$ decays, and the observation of the new CP violation effect becomes possible.

According the Eqs. (85)–(88), we calculate the dependence of $A_{CP,K_S^0}^{\rm mix}$, $A_{CP,K_S^0}^{\rm int}$, and $A_{CP}^{K_S^0}$ on the selection of t_0 in the FAT approach and the TA approach, which is shown in Fig. 4. Here, we note that we still adopt $t_1 = 10\tau_S$ in the calculations. It can be seen from Fig. 4 that the maximum value of $|A_{CP}^{K_S^0}|$ can reach up to 9.31×10^{-3} and 1.23×10^{-2} in the FAT approach and the TA approach, respectively. When $|A_{CP}^{K_S^0}|$ adopt these values, the new CP violation effect $A_{CP,K_S^0}^{\rm int}$ is comparable with the indirect CP violation in $K^0 - \bar{K}^0$ mixing $A_{CP,K_S^0}^{\rm mix}$. In addition, it can be seen from Fig. 4 that the numerical value of the indirect CP violation in $K^0 - \bar{K}^0$ mixing $A_{CP,K_S^0}^{\rm mix}$ becomes smaller and the new CP violation effect $A_{CP,K_S^0}^{\rm int}$ plays a dominant pole in the CP violation in the $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K_S^0\pi^0\pi^\pm$ decays $A_{CP}^{K_S^0}$ at some values of t_0. For example, when $t_0 = 3.0\tau_S$, we have

$$A_{CP,K_S^0}^{\rm mix} = (-0.84 \pm 0.25) \times 10^{-3}, \tag{113}$$

$$A_{CP,K_S^0}^{\rm dir} = (-1.18 \pm 0.11) \times 10^{-4}, \tag{114}$$

$$A_{CP,K_S^0}^{\rm int} = (-6.15 \pm 0.48) \times 10^{-3}, \tag{115}$$

$$A_{CP}^{K_S^0} = A_{CP,K_S^0}^{\rm mix} + A_{CP,K_S^0}^{\rm dir} + A_{CP,K_S^0}^{\rm int} = (-7.11 \pm 0.56) \times 10^{-3} \tag{116}$$

in the FAT approach and

$$A_{CP,K_S^0}^{\rm mix} = (-1.05 \pm 0.31) \times 10^{-3}, \tag{117}$$

$$A_{CP,K_S^0}^{\rm dir} = (-1.67 \pm 0.12) \times 10^{-4}, \tag{118}$$

$$A_{CP,K_S^0}^{\rm int} = (-8.68^{-0.48}_{+0.45}) \times 10^{-3}, \tag{119}$$

$$A_{CP}^{K_S^0} = A_{CP,K_S^0}^{\rm mix} + A_{CP,K_S^0}^{\rm dir} + A_{CP,K_S^0}^{\rm int} = (-9.90^{-0.59}_{+0.56}) \times 10^{-3} \tag{120}$$

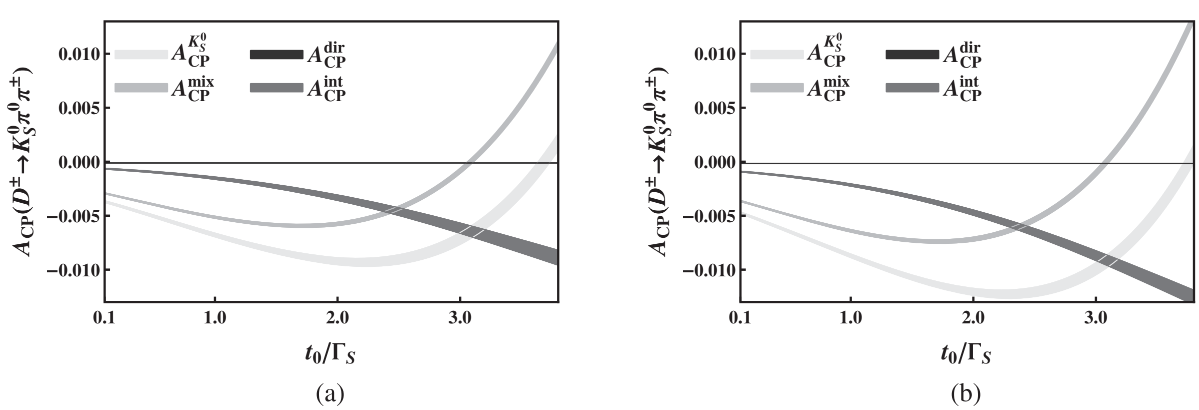

FIG. 4. The dependence of the indirect CP violation in $K^0 - \bar{K}^0$ mixing $A_{CP,K_S^0}^{\rm mix}$, the direct CP violation in the charm decay $A_{CP,K_S^0}^{\rm dir}$, the new CP violation effect $A_{CP,K_S^0}^{\rm int}$, and the CP violation in the $D^\pm \to K^{*0}\pi^\pm + \bar{K}^{*0}\pi^\pm \to K_S^0\pi^0\pi^\pm$ decays $A_{CP}^{K_S^0}$ on the selection of t_0 with $t_1 = 10/\Gamma_S$: (a) in the FAT approach and (b) in the TA approach.

in the TA approach. Obviously, if we adopt $t_0 = 3.0\tau_S$ and $t_1 = 10.0\tau_S$, the new CP violation effect $A^{\rm int}_{CP,K^0_S}$ is possible to be observed.

However, the method mentioned above has a drawback: if we adopt $t_0 = 3.0\tau_S$ and $t_1 = 10.0\tau_S$, we would lose a lot of the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K^0_S\pi^0\pi^{\pm}$ event. The reason is that the decay of a K^0_S meson to final state $\pi^+\pi^-$ occurs mainly at time less than $5\tau_S$, and the decay rate of K^0_S meson decreases rapidly with time. The event selection efficiency of $t_0 = 3.0\tau_S$ and $t_1 = 10.0\tau_S$ can be written as

$$\epsilon_{t_0} = \frac{\int_{3\tau_S}^{10\tau_S} \Gamma(D^{\pm} \to K^{*}\pi^{\pm} \to K(t)\pi^0\pi^{\pm} \to \pi^+\pi^-\pi^0\pi^{\pm})dt}{\int_0^{+\infty} \Gamma(D^{\pm} \to K^{*}\pi^{\pm} \to K(t)\pi^0\pi^{\pm} \to \pi^+\pi^-\pi^0\pi^{\pm})dt}. \tag{121}$$

Substituting Eqs. (60) and (64) into Eq. (121) and using the values of the parameters in Eqs. (100), (102), and (104), we can obtain the numerical result of ϵ_{t_0}:

$$\epsilon_{t_0} = 5.0 \times 10^{-2}, \tag{122}$$

where the above result is the averaged efficiency of the decay and its charge conjugate. So, if the CP violations in Eqs. (116) and (120) are observed at the 3 standard deviation (3σ) level, the number of $D^{\pm}$ events-times-efficiency needed reads as

$$(\epsilon_f N)^{K^0_S}_{CP,t_0=3\tau_S} = \frac{9}{2 \cdot \mathcal{B}(D^{\pm} \to K^{*}\pi^{\pm} \to K^0_S\pi^0\pi^{\pm}) \cdot \mathcal{B}(K^0_S \to \pi^+\pi^-) \cdot |A^{K^0_S}_{CP}| \cdot \epsilon_{t_0}}. \tag{123}$$

Substituting Eqs. (101), (116), (120), (122), and the numerical results of the branching ratios in Table I into Eq. (123), we can obtain

$$(\epsilon_f N)^{K^0_S}_{CP,t_0=3\tau_S} = \begin{cases} (5.1 \sim 6.7) \times 10^6, & \text{the FAT approach,} \\ (5.2 \sim 6.5) \times 10^6, & \text{the TA approach ,} \end{cases} \tag{124}$$

where ϵ_f is the selection efficiency in experiment; it does not contain ϵ_{t_0}. In a word, if one adopts the scenario $t_0 = 3.0\tau_S$ and $t_1 = 10.0\tau_S$ and wants to observe the new CP violation effect $A^{\rm int}_{CP,K^0_S}$ in $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K^0_S\pi^0\pi^{\pm}$ decays, the number of $D^{\pm}$ events-times-efficiency needed is $(5.1 \sim 6.7) \times 10^6$ and $(5.2 \sim 6.5) \times 10^6$ in the FAT approach and the TA approach, respectively.

VI. CONCLUSIONS

In this work, we derive the expressions for the CP violations in $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K^0_{S,L}\pi^0\pi^{\pm}$ decays $A^{K^0_{S,L}}_{CP}$, which consists of three parts: the indirect CP violations in $K^0 - \bar{K}^0$ mixing $A^{\rm mix}_{CP,K^0_{S,L}}$, the direct CP violations in charm decay $A^{\rm dir}_{CP,K^0_{S,L}}$, and the new CP violation effects $A^{\rm int}_{CP,K^0_{S,L}}$, which are induced from the interference between two tree (CF and DCS) amplitudes with the neutral kaon mixing. We calculate the numerical results of the CP violations in $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K^0_{S,L}\pi^0\pi^{\pm}$ decays based on the FAT approach and the TA approach:

$$A^{K^0_S}_{CP} = \begin{cases} (-3.69 \pm 0.09) \times 10^{-3}, & \text{the FAT approach,} \\ (-4.72 \pm 0.09) \times 10^{-3}, & \text{the TA approach,} \end{cases} \tag{125}$$

and

$$A^{K^0_L}_{CP} = \begin{cases} (4.95 \pm 0.10) \times 10^{-3}, & \text{the FAT approach,} \\ (4.03 \pm 0.07) \times 10^{-3}, & \text{the TA approach.} \end{cases} \tag{126}$$

We find that the indirect CP violations in $K^0 - \bar{K}^0$ mixing play a dominant role in the CP violations in $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K^0_{S,L}\pi^0\pi^{\pm}$ decays; the new CP violation effect has a non-negligible contribution to the CP violations in $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K^0_{S,L}\pi^0\pi^{\pm}$ decays. In order to observe the CP violations at the 3 standard deviation (3σ) level, 6.3×10^5 and 3.8×10^6 $D^{\pm}$ events-times-efficiency are needed for the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K^0_S\pi^0\pi^{\pm}$ decays and

$D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_L^0\pi^0\pi^{\pm}$ decays in the FAT approach, respectively. In the TA approach, 6.7×10^5 and 4.0×10^6 $D^{\pm}$ events-times-efficiency are needed to observe the CP violations at the 3 standard deviation (3σ) level for the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ decays and $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_L^0\pi^0\pi^{\pm}$ decays, respectively.

We present the formulas of the $K_S^0 - K_L^0$ asymmetries $R_{K_S-K_L}^{D^{\pm}}$ in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_{S,L}^0\pi^0\pi^{\pm}$ decays and predict the numerical values of them in the FAT approach and the TA approach:

$$R_{K_S-K_L}^{D^+} = \begin{cases} 0.186^{-0.017}_{+0.015}, & \text{the FAT approach,} \\ -0.038^{-0.013}_{+0.012}, & \text{the TA approach,} \end{cases} \tag{127}$$

and

$$R_{K_S-K_L}^{D^-} = \begin{cases} 0.194^{-0.016}_{+0.015}, & \text{the FAT approach,} \\ -0.029^{-0.013}_{+0.012}, & \text{the TA approach.} \end{cases} \tag{128}$$

Because the $K_S^0 - K_L^0$ asymmetries $R_{K_S-K_L}^{D^{\pm}}$ only suffer from the r_{wf} suppression, they have a large value, which indicates that there exists a large difference between the branching ratios of $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ and the branching ratios of $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_L^0\pi^0\pi^{\pm}$. In addition, because the values of $\cos\delta$ are different in the FAT approach and the TA approach, the numerical results of $R_{K_S-K_L}^{D^{\pm}}$ in the FAT approach are many times (about 5 times for $R_{K_S-K_L}^{D^+}$ and about 6 times for $R_{K_S-K_L}^{D^-}$) larger than that in the TA approach. Moreover, the signs of $R_{K_S-K_L}^{D^{\pm}}$ in these two approaches are opposite to each other. Based on the FAT approach, we estimate that the range of the numbers of $D^{\pm}$ events-times-efficiency needed for observing the $K_S^0 - K_L^0$ asymmetries at the 3 standard deviation (3σ) level is from 0.8×10^4 to 1.0×10^4 both for the $D^+ \to K^{*0}\pi^+ + \bar{K}^{*0}\pi^+ \to K_{S,L}^0\pi^0\pi^+$ decays and for the $D^- \to K^{*0}\pi^- + \bar{K}^{*0}\pi^- \to K_{S,L}^0\pi^0\pi^-$ decays. In the TA approach, we derive that the range of the numbers of $D^{\pm}$ events-times-efficiency needed for observing the $K_S^0 - K_L^0$ asymmetries at the 3 standard deviation (3σ) level is $3.8 \times 10^4 \sim 7.8 \times 10^4$ for the $D^+ \to K^{*0}\pi^+ + \bar{K}^{*0}\pi^+ \to K_{S,L}^0\pi^0\pi^+$ decays and $0.5 \times 10^5 \sim 1.2 \times 10^5$ for the $D^- \to K^{*0}\pi^- + \bar{K}^{*0}\pi^- \to K_{S,L}^0\pi^0\pi^-$ decays.

We also investigate the possibility of observing the new CP violation effect $A_{CP,K_S^0}^{\text{int}}$ in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ decays in the FAT approach and the TA approach. We find that the new CP violation effect can dominate the CP violation in the $D^{\pm} \to K^{*0}\pi^{\pm} + \bar{K}^{*0}\pi^{\pm} \to K_S^0\pi^0\pi^{\pm}$ decays when the scenario with $t_0 = 3.0\tau_S$ and $t_1 = 10.0\tau_S$ is adopted. However, the observation of the new CP violation effect $A_{CP,K_S^0}^{\text{int}}$ in the above mentioned scenario is at the expense of the event selection's efficiency. If the clean signal of the new CP violation effect $A_{CP,K_S^0}^{\text{int}}$ is established, the number of $D^{\pm}$ events-times-efficiency needed is 6.7×10^6 and 6.5×10^6 in the FAT approach and the TA approach, respectively.

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APPENDIX: WILSON COEFFICIENTS

Below we present the evolution of the Wilson coefficients in the scale $\mu < m_c$ [14,107],

$$C_1(\mu) = 0.2334(\alpha_s)^{1.444} + 0.0459(\alpha_s)^{0.7778} - 1.313(\alpha_s)^{0.4444} + 0.3041(\alpha_s)^{-0.2222}, \tag{A1}$$

$$C_2(\mu) = -0.2334(\alpha_s)^{1.444} + 0.0459(\alpha_s)^{0.7778} + 1.313(\alpha_s)^{0.4444} + 0.3041(\alpha_s)^{-0.2222}, \tag{A2}$$

where α_s is the strong running coupling constant

$$\alpha_s = \alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\overline{\text{MS}}}^2)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln\ln(\mu^2/\Lambda_{\overline{\text{MS}}}^2)}{\ln(\mu^2/\Lambda_{\overline{\text{MS}}}^2)}\right], \tag{A3}$$

with

$$\beta_0 = \frac{33 - 2f}{3}, \qquad \beta_1 = 102 - \frac{38}{3}f, \tag{A4}$$

where $\Lambda_{\overline{\text{MS}}}$ is the QCD scale characteristic for the $\overline{\text{MS}}$ scheme, f is the number of "effective" flavors, and their values are

$$\Lambda_{\overline{\text{MS}}} = \Lambda_{\overline{\text{MS}}}^{(3)} = 375\ \text{MeV}, \quad f = 3, \tag{A5}$$

for $\mu < m_c$.

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