

Cosmology in theories with spontaneous scalarization of neutron stars

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In a model of spontaneous scalarization of neutron stars proposed by Damour and Esposito-Farese, a general relativistic branch becomes unstable to trigger tachyonic growth of a scalar field ϕ toward a scalarized branch. Applying this scenario to cosmology, there is fatal tachyonic instability of ϕ during inflation and matter dominance being incompatible with solar-system constraints on today's field value ϕ_0 . In the presence of a four-point coupling $g^2\phi^2\chi^2/2$ between ϕ and an inflaton field χ , it was argued by Anson *et al.* that a positive mass squared heavier than the square of a Hubble expansion rate leads to the exponential suppression of ϕ during inflation and that ϕ_0 can remain small even with the growth of ϕ after the radiation-dominated epoch. For several inflaton potentials approximated as $V(\chi) = m^2\chi^2/2$ about the potential minimum, we study the dynamics of ϕ during reheating as well as other cosmological epochs in detail. For certain ranges of the coupling g , the homogeneous field ϕ can be amplified by parametric resonance during a coherent oscillation of the inflaton. Incorporating the backreaction of created particles under a Hartree approximation, the maximum values of ϕ reached during preheating are significantly smaller than those obtained without the backreaction. We also find that the minimum values of g consistent with solar system bounds on ϕ at the end of reheating are of order 10^{-5} and hence there is a wide range of acceptable values of g . Thus, the scenario proposed by Anson *et al.* naturally leads to the viable cosmological evolution of ϕ consistent with local gravity constraints, without modifying the property of scalarized neutron stars.

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I. INTRODUCTION

The physics in strong gravity regimes can be now probed by the observations of gravitational waves (GWs) emitted from a binary system containing black holes or neutron stars (NSs) [1–3]. With the future high-precision data of GWs, it will be possible to probe the accuracy of General Relativity (GR) and the possible deviation from it [4–6]. In theories beyond GR, there are usually additional degrees of freedom that can be coupled to gravity [7–13]. A simplest example is known as scalar-tensor theories, in which a scalar field ϕ has nonminimal or derivative couplings to gravity [14–20].

In the presence of a nonminimal coupling $F(\phi)R$, where F is a function of ϕ and R is the Ricci scalar, it is known that NSs can have scalar hairs through an indirect coupling between the scalar field and matter mediated by gravity [21–30]. For the exponential coupling $F(\phi) = e^{-2Q\phi/M_{\text{Pl}}}$, where Q is a constant and M_{Pl} is the reduced Planck mass, the fifth force propagates around weak gravitational objects like the Sun. From solar-system experiments, there is a tight bound $|Q| < 2 \times 10^{-3}$ on the coupling strength [31,32]. Then, the deviation from GR in the vicinity of NSs is also suppressed, so probing the modification of gravity from the GW observations is challenging for the nonminimal coupling $F(\phi) = e^{-2Q\phi/M_{\text{Pl}}}$.

If we consider nonminimal couplings containing even power-law functions of ϕ , it is possible to have a nontrivial branch $\phi'(r) \neq 0$ besides a GR branch $\phi'(r) = 0$ on a static and spherically symmetric background with the radial distance r . A typical example is the coupling $F(\phi) = e^{-\beta\phi^2/(2M_{\text{Pl}}^2)}$ proposed by Damour and Esposito-Farese (DEF), where β is a constant [21,22]. For strong gravitational objects like NSs, the GR branch can be unstable to trigger tachyonic instability of the scalar field to reach a nontrivial branch of ϕ . This phenomenon, which is called spontaneous scalarization, occurs for negative coupling constants in the range $\beta \leq -4.35$ [23,24,33,34]. Since the gravitational interaction for such scalarized NSs exhibits the appreciable deviation from that in GR, it is possible to probe the modification of gravity from binary pulsar measurements [35–37] as well as the observations of GWs emitted from a binary system containing at least one NS [38,39].

When spontaneous scalarization of NSs occurs, the asymptotic value of the scalar field ϕ_0 needs to be in the range $|\phi_0| \lesssim 10^{-3}M_{\text{Pl}}|\beta|^{-1}$ for the consistency with local gravity constraints [21]. This asymptotic value should be determined by the cosmological evolution of ϕ from the past to today. In the original DEF model, however, the

background cosmological scalar field largely deviates from 0 for the coupling range β allowing for spontaneous scalarization [40,41]. This is attributed to the fact that the negative coupling β leads to tachyonic growth of $|\phi|$ during the cosmological evolution. Without severely fine-tuned initial conditions, the amplitude of today's field value exceeds the upper limit constrained by solar system tests of gravity. We note that the similar type of instabilities is also present for spontaneously scalarized BHs arising from a scalar Gauss-Bonnet coupling [42–44].

On the other hand, Anson *et al.* [45] proposed a mechanism of reconciling spontaneous scalarization with cosmology by incorporating a coupling $g^2\phi^2\chi^2/2$ between the scalar field ϕ and an inflaton field χ . Since the field ϕ can have a positive effective mass squared $g^2\chi^2$ larger than the squared Hubble expansion rate during inflation, the amplitude of the homogeneous field ϕ exponentially decreases. Even though $|\phi|$ increases after the onset of the radiation era to today, it is expected that today's field value is still in the range $|\phi_0| \lesssim 10^{-3}M_{\text{Pl}}|\beta|^{-1}$. However, the analysis of Ref. [45] does not accommodate the evolution of ϕ during the post inflationary reheating period. Indeed, for certain ranges of the coupling g , the four-point coupling $g^2\phi^2\chi^2/2$ leads to parametric resonance of the homogeneous field ϕ and its perturbations during a preheating stage after inflation [46–53]. Even for small couplings g without the period of preheating, it can happen that the amplitude of ϕ grows during reheating by the dominance of the negative nonminimal coupling over the positive mass term $g^2\chi^2$. Hence, it is important to clarify the coupling range of g in which the model can be at work. We note that there are other mechanisms for reconciling spontaneous scalarization with cosmology [39,41,54,55], but it is typically nontrivial to realize the acceptable cosmological evolution of ϕ .

In the DEF model with the coupling $g^2\phi^2\chi^2/2$, we will study the cosmological evolution of the scalar field ϕ responsible for spontaneous scalarization. We pay particular attention to the dynamics during reheating in which the further growth of $|\phi|$ can be expected. Since the inflaton potentials are approximated as $V(\chi) \simeq m^2\chi^2/2$ around $\chi = 0$, the reheating period corresponds to an effective matter era driven by the oscillation of a massive inflaton field.

For coupling ranges of g in which the preheating stage is present, we need to take into account the backreaction of created ϕ particles that leads to the violation of coherent oscillations of χ . Without the backreaction, the maximum amplitude ϕ_{max} of the field ϕ reached during preheating can exceed a value constrained by solar-system tests of gravity at the end of reheating ($|\phi_{\text{R}}| \lesssim 10^{-11}M_{\text{Pl}}$). However, we will show that implementing the backreaction under the Hartree approximation [50,52] leads to ϕ_{max} significantly smaller than $10^{-11}M_{\text{Pl}}$. For two inflaton potentials considered in this paper, ϕ_{max} is less than the

order of $10^{-38}M_{\text{Pl}}$. After the system reaches an equilibrium state with the violation of coherent oscillations of the inflaton, the further significant amplification of ϕ is not expected by the end of reheating because the negative nonminimal coupling is suppressed compared to $g^2\chi^2$ in the background equation of ϕ . Thus, even with the preheating epoch, the presence of the coupling $g^2\phi^2\chi^2/2$ allows the cosmological evolution of ϕ consistent with local gravity constraints on ϕ_0 .

For small coupling ranges of g in which preheating does not occur, we do not need to implement the backreaction of created particles. In such cases, we will solve the background equations of motion by the end of reheating with a Born decay constant Γ taken into account. Depending on the form of inflaton potentials, the nonminimal coupling can overwhelm the term $g^2\chi^2$ during reheating. Since this leads to the growth of $|\phi|$ by the end of reheating, it can happen that $|\phi_{\text{R}}|$ exceeds the upper bound $10^{-11}M_{\text{Pl}}$. This is especially the case for a low-scale reheating scenario with the reheating temperature of order MeV. For two inflaton potentials, we will put lower bounds on the coupling g consistent with solar-system constraints. In both cases, the minimum values of g are of order 10^{-5} , so the mechanism proposed by Anson *et al.* [45] is at work for wide ranges of the coupling g (including the case where preheating occurs).

This paper is organized as follows. In Sec. II, we briefly review the DEF model and derive the background equations of motion on the spatially-flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime in the presence of the coupling $g^2\phi^2\chi^2/2$. In Sec. III, we discuss the cosmological evolution of ϕ from the radiation era to today and interpret a solar-system bound on ϕ_0 as the constraint on ϕ at the onset of radiation dominance. In Sec. IV, we study the dynamics of the scalar field during inflation for several inflaton potentials and find minimum values of g above which the amplitude of ϕ exponentially decreases. In Sec. V, we analyze the evolution of the homogeneous field ϕ and its perturbations during the reheating stage and derive minimum values of the coupling constant g consistent with solar-system constraints. Sec. VI is devoted to conclusions.

II. THEORIES WITH SPONTANEOUS SCALARIZATION

We consider theories given by the action

$$\mathcal{S} = \int d^4x \sqrt{-g_J} \left[\frac{M_{\text{Pl}}^2}{2} F(\phi)R + \omega(\phi)X + L_{\text{inf}} \right] + \mathcal{S}_m(g_{\mu\nu}, \Psi_m), \quad (2.1)$$

where g_J is a determinant of metric tensor $g_{\mu\nu}$ in the Jordan frame, and $X = -(1/2)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ is a scalar kinetic term with ∇_μ being a covariant derivative operator.

The ϕ -dependent function in front of X is chosen to be [39,56,57]

$$\omega(\phi) = \left(1 - \frac{3M_{\text{Pl}}^2 F_{,\phi}^2}{2F^2}\right) F(\phi), \quad (2.2)$$

where $F_{,\phi} = dF/d\phi$. Brans-Dicke (BD) theories [14] correspond to the particular nonminimal coupling $F(\phi) = e^{-2Q\phi/M_{\text{Pl}}}$, where a coupling constant Q is related to the BD parameter ω_{BD} as $3 + 2\omega_{\text{BD}} = 1/(2Q^2)$ [32,58]. In theories of spontaneous scalarization, $F(\phi)$ contains even power-law functions of ϕ .

We take into account the contribution of an inflaton field χ as the Lagrangian

$$L_{\text{inf}} = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - V(\chi) - \frac{1}{2}g^2\phi^2\chi^2, \quad (2.3)$$

where V is the potential of χ . The last term in Eq. (2.3) characterizes an interaction between ϕ and χ with a coupling constant g . During inflation, this can generate a large effective positive mass squared of ϕ relative to the square of the Hubble expansion rate. Then, it can compensate a negative mass squared induced by the non-minimal coupling $F(\phi)R$ responsible for spontaneous scalarization. This allows a possibility for avoiding the tachyonic growth of ϕ during inflation [45].

In the presence of the four-point coupling $g^2\phi^2\chi^2/2$, it is known that a phenomenon called preheating [46,52] can occur after inflation during the coherent oscillation of inflaton. In this stage, the scalar field ϕ and its perturbations $\delta\phi$ can be amplified by parametric resonance. Since the dynamics of the field ϕ during preheating were not addressed in Ref. [45], we will study whether the presence of this stage is harmless or not for the compatibility with local gravity constraints on today's field value ϕ_0 . For this purpose, we also scrutinize the scalar-field dynamics in other cosmological epochs from the onset of inflation to today.

In Eq. (2.1), the action \mathcal{S}_m incorporates the contributions of matter fields Ψ_m such as radiation, nonrelativistic matter, and dark energy. We assume that matter fields are minimally coupled to gravity.

Under the conformal transformation $\hat{g}_{\mu\nu} = F(\phi)g_{\mu\nu}$, the action (2.1) is transformed to [8,56]

$$\hat{\mathcal{S}} = \int d^4x \sqrt{-\hat{g}} \left[\frac{M_{\text{Pl}}^2}{2} \hat{R} - \frac{1}{2} \hat{g}^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \hat{L}_{\text{inf}} \right] + \mathcal{S}_m(F^{-1}(\phi)\hat{g}_{\mu\nu}, \Psi_m), \quad (2.4)$$

where a hat represents quantities in the Einstein frame. In the transformed frame, the scalar field ϕ is coupled to matter fields through the metric tensor $\hat{g}_{\mu\nu}$.

We deal with the Jordan frame as a physical one and perform all analyses in this frame by exploiting the action (2.1). We consider the nonminimal coupling chosen by DEF [21,22]

$$F(\phi) = e^{-\beta\phi^2/(2M_{\text{Pl}}^2)}, \quad (2.5)$$

where β is a constant. In this case, we have

$$\omega(\phi) = \left(1 - \frac{3\beta^2\phi^2}{2M_{\text{Pl}}^2}\right) e^{-\beta\phi^2/(2M_{\text{Pl}}^2)}. \quad (2.6)$$

Spontaneous scalarization of NSs can occur for the coupling $\beta \leq -4.35$ [23,24,33,34]. In such cases, the GR branch $\phi = 0$ can be unstable to trigger tachyonic instability toward the other nontrivial branch $\phi \neq 0$. From binary pulsar measurements of the energy loss through dipolar radiation, the coupling β was constrained to be $\beta \geq -4.5$ [35,36]. Thus, the coupling constant β is restricted in a limited range.

We consider a spatially-flat FLRW background given by the line element

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (2.7)$$

where $a(t)$ is a time-dependent scale factor. Incorporating the Born decay term $\Gamma\dot{\chi}$ to complete the reheating process (Γ is a decay constant and a dot represents a derivative with respect to t) into the inflaton equation of motion, it follows that

$$\ddot{\chi} + (3H + \Gamma)\dot{\chi} + V_{,\chi} + g^2\phi^2\chi = 0, \quad (2.8)$$

where $H = \dot{a}/a$ is the Hubble expansion rate. We will consider the case $H \gg \Gamma$ during inflation, so the decay term $\Gamma\dot{\chi}$ is important only at the late stage of reheating. Due to the energy transfer from the inflaton to radiation, the radiation energy density ρ_r satisfies the differential equation

$$\dot{\rho}_r + 4H\rho_r = \Gamma\dot{\chi}^2. \quad (2.9)$$

The energy density ρ_m of nonrelativistic matter (cold dark matter and baryons) obeys the continuity equation

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (2.10)$$

with a vanishing pressure. As a source for dark energy, we take the cosmological constant Λ into account.

The (00) and (11) components of gravitational field equations following from the action (2.1) are given, respectively, by

$$3M_{\text{Pl}}^2 H(FH + F_{,\phi}\dot{\phi}) = \frac{1}{2}\dot{\chi}^2 + V + \frac{1}{2}\omega\dot{\phi}^2 + \frac{1}{2}g^2\phi^2\chi^2 + \rho_r + \rho_m + \Lambda, \quad (2.11)$$

$$\begin{aligned}
-M_{\text{Pl}}^2 F(2\dot{H} + 3H^2) &= M_{\text{Pl}}^2 (F_{,\phi} \ddot{\phi} + F_{,\phi\phi} \dot{\phi}^2 + 2F_{,\phi} H \dot{\phi}) \\
&+ \frac{1}{2} \dot{\chi}^2 - V + \frac{1}{2} \omega \dot{\phi}^2 - \frac{1}{2} g^2 \phi^2 \chi^2 \\
&+ \frac{1}{3} \rho_r - \Lambda. \tag{2.12}
\end{aligned}$$

The scalar field ϕ obeys the differential equation

$$\ddot{\phi} + 3H\dot{\phi} + m_{\text{eff}}^2 \phi = 0, \tag{2.13}$$

where

$$m_{\text{eff}}^2 \equiv \frac{1}{\omega} \left[g^2 \chi^2 + 3\beta F(2H^2 + \dot{H}) - \frac{\beta \omega \dot{\phi}^2}{2M_{\text{Pl}}^2} - \frac{3\beta^2 F \dot{\phi}^2}{2M_{\text{Pl}}^2} \right]. \tag{2.14}$$

To discuss the dynamics of inflation and reheating, we consider the inflaton potentials of α -attractors [59] given by

$$V(\chi) = \frac{3}{4} \alpha m^2 M_{\text{Pl}}^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3\alpha}} \frac{\chi}{M_{\text{Pl}}}\right) \right]^2, \tag{2.15}$$

where α is a positive dimensionless constant, and m is a constant having a dimension of mass. In the limit $\alpha \rightarrow \infty$, the potential (2.15) reduces to that in chaotic inflation, i.e., $V(\chi) = m^2 \chi^2/2$ [60]. Starobinsky's model with the Lagrangian $L = R + R^2/(6m^2)$ [61] gives rise to the potential (2.15) with $\alpha = 1$ after a conformal transformation to the Einstein frame [8]. For the numerical simulation performed in Secs. IV and V, we will consider the two potentials: (i) $V(\chi) = m^2 \chi^2/2$, and (ii) the potential (2.15) with $\alpha = 1$.

III. COSMOLOGICAL DYNAMICS FROM RADIATION DOMINATION TO TODAY

In this section, we investigate a bound of the field value ϕ at the onset of radiation dominance constrained from solar system tests of gravity. In theories given by the action (2.1), the post-Newtonian parameter γ_{PPN} is [21,62]

$$\gamma_{\text{PPN}} - 1 = -\frac{2\alpha_{\text{PPN}}^2(\varphi_0)}{1 + \alpha_{\text{PPN}}^2(\varphi_0)} = -\frac{2\beta^2 \varphi_0^2}{1 + \beta^2 \varphi_0^2}, \tag{3.1}$$

where $\varphi = \phi/(\sqrt{2}M_{\text{Pl}})$ is a dimensionless field with today's value φ_0 , and $\alpha_{\text{PPN}} = -F_{,\varphi}/(2F) = \beta\varphi$ with $F = e^{-\beta\varphi^2}$. The Shapiro time delay measurements have given the bound $\gamma_{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [63]. Since $\gamma_{\text{PPN}} - 1$ is negative in the current theory, we adopt the limit $|\gamma_{\text{PPN}} - 1| \leq 0.2 \times 10^{-5}$. This corresponds to the bound $|\beta\varphi_0| \leq 1 \times 10^{-3}$, so today's field value ϕ_0 is constrained to be

$$|\phi_0| \leq 1.4 \times 10^{-3} M_{\text{Pl}} |\beta|^{-1}. \tag{3.2}$$

For $\beta = -4.4$, we have $|\phi_0| \leq 3.2 \times 10^{-4} M_{\text{Pl}}$. In the following, we will study how this constraint translates to the upper limit of $|\phi|$ at the onset of radiation era.

After the reheating period ends, we can neglect the contribution of the inflaton field χ in Eq. (2.14). Moreover, provided that the energy density of ϕ is negligible relative to that of the background and that $|\phi| \ll M_{\text{Pl}}$, the effective mass squared (2.14) is approximated as $m_{\text{eff}}^2 \simeq 3\beta(2H^2 + \dot{H})$. Then, the scalar-field equation (2.13) approximately reduces to

$$\phi'' + \frac{3}{2}(1 - w_{\text{eff}})\phi' + \frac{3}{2}(1 - 3w_{\text{eff}})\beta\phi \simeq 0, \tag{3.3}$$

where a prime represents the derivative with respect to $N = \ln a$, and $w_{\text{eff}} = -1 - 2H'/ (3H)$ is the effective equation of state. If $w_{\text{eff}} = \text{constant}$, there is the following growing-mode solution

$$\phi \propto a^\lambda, \tag{3.4}$$

where

$$\lambda = \frac{3}{4}(1 - w_{\text{eff}}) \left[\sqrt{1 - \frac{8\beta(1 - 3w_{\text{eff}})}{3(1 - w_{\text{eff}})^2}} - 1 \right]. \tag{3.5}$$

During the radiation era ($w_{\text{eff}} = 1/3$), we have $\lambda = 0$ and hence $\phi = \text{constant}$. This property is attributed to the fact that R vanishes on the exact radiation-dominated background. In the matter-dominated era ($w_{\text{eff}} = 0$), the negative mass squared of ϕ leads to the following tachyonic growth of the field

$$\phi \propto a^{(3/4)(\sqrt{1-8\beta/3}-1)}. \tag{3.6}$$

During the epoch of cosmological constant domination, the growth of ϕ is even stronger; $\phi \propto a^{(3/2)(\sqrt{1-8\beta/3}-1)}$. However, the dominance of dark energy over nonrelativistic matter occurs only at low redshifts $z \lesssim 0.3$, so we can approximately use Eq. (3.6) for the evolution of ϕ from radiation-matter equality to today. Then, the field value at radiation-matter equality can be estimated as

$$\phi_{\text{eq}} = \phi_0 (z_{\text{eq}} + 1)^{-(3/4)(\sqrt{1-8\beta/3}-1)}, \tag{3.7}$$

where z_{eq} is determined by $z_{\text{eq}} = \Omega_{m0}/\Omega_{r0} - 1$, with Ω_{m0} and Ω_{r0} being today's density parameters of nonrelativistic matter and radiation respectively.

In Fig. 1, we plot the evolution of ϕ/M_{Pl} and $\Omega_r = \rho_r/(3FH^2M_{\text{Pl}}^2)$, $\Omega_m = \rho_m/(3FH^2M_{\text{Pl}}^2)$, and $\Omega_{\text{DE}} = \Lambda/(3FH^2M_{\text{Pl}}^2)$ for $\beta = -4.4$ between the radiation era and today. We choose the initial conditions around redshift

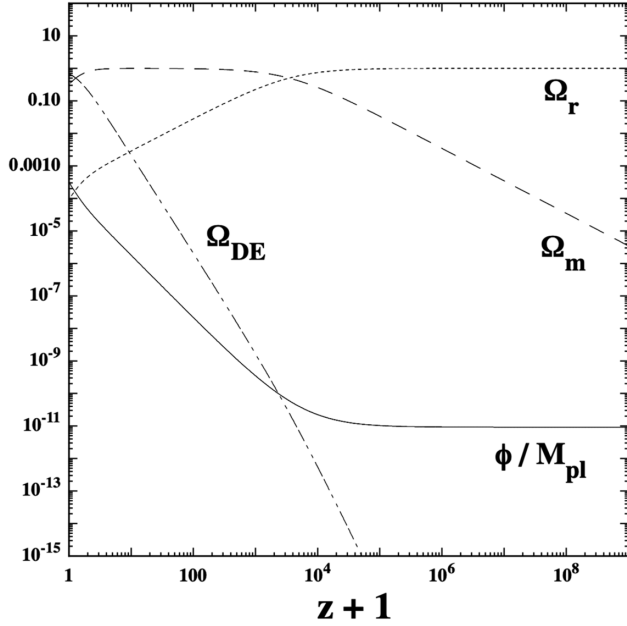


FIG. 1. Evolution of ϕ/M_{pl} and Ω_r , Ω_m , and Ω_{DE} versus $z+1 (=1/a)$ for $\beta = -4.4$. The initial conditions are chosen to realize today's values $\phi_0/M_{\text{pl}} = 3.2 \times 10^{-4}$, $\Omega_{r0} = 9.2 \times 10^{-5}$, and $\Omega_{\text{DE}0} = 0.68$.

$z = 10^9$ to realize today's value $\phi_0 = 3.2 \times 10^{-4} M_{\text{pl}}$, which corresponds to the upper limit consistent with local gravity constraints. In this case the redshift at radiation-matter equality is $z_{\text{eq}} \simeq 3470$, so the analytic estimation (3.7) gives $\phi_{\text{eq}} = 4.8 \times 10^{-11} M_{\text{pl}}$. This is fairly close to the numerical value $\phi_{\text{eq}} = 6.0 \times 10^{-11} M_{\text{pl}}$, even though we ignored the epoch of late-time cosmic acceleration for the analytic estimation of ϕ_0 .

In Fig. 1, we observe that ϕ slowly grows even at the late stage of radiation era. This reflects the fact that $R = 6(2H^2 + \dot{H})$ does not completely vanish due to the presence of nonrelativistic matter. Numerically, we find that ϕ_{eq} is larger than the initial value ϕ_{R} in the radiation era by one order of magnitude. This is consistent with the analytic estimation given in Ref. [45]. Multiplying the analytic value (3.7) by a factor 0.2, it is possible to reproduce the numerical value of ϕ_{R} approximately. Then, we have the following analytic formula

$$|\phi_{\text{R}}| \simeq 0.2 |\phi_0| \left(\frac{\Omega_{m0}}{\Omega_{r0}} \right)^{-(3/4)(\sqrt{1-8\beta/3}-1)}. \quad (3.8)$$

For the model parameters used in the numerical simulation of Fig. 1, i.e., $\phi_0 = 3.2 \times 10^{-4} M_{\text{pl}}$, $\Omega_{m0} = 0.32$, and $\Omega_{r0} = 9.2 \times 10^{-5}$, the analytic estimation (3.8) gives $|\phi_{\text{R}}| = 9.6 \times 10^{-12} M_{\text{pl}}$. This is close to the numerical value $|\phi_{\text{R}}| = 9.2 \times 10^{-12} M_{\text{pl}}$.

Applying the formula (3.8) to Eq. (3.2), we obtain the following upper bound

$$|\phi_{\text{R}}| \leq 2.8 \times 10^{-4} M_{\text{pl}} |\beta|^{-1} \left(\frac{\Omega_{m0}}{\Omega_{r0}} \right)^{-(3/4)(\sqrt{1-8\beta/3}-1)}. \quad (3.9)$$

For $\beta = -4.5$ and $\beta = -4.35$, the criterion (3.9) gives $|\phi_{\text{R}}| \leq 7.5 \times 10^{-12} M_{\text{pl}}$ and $|\phi_{\text{R}}| \leq 1.1 \times 10^{-11} M_{\text{pl}}$, respectively, where we used the same values of Ω_{m0} and Ω_{r0} mentioned above. These bounds are close to the numerically derived upper limits $|\phi_{\text{R}}| \leq 7.1 \times 10^{-12} M_{\text{pl}}$ and $|\phi_{\text{R}}| \leq 1.1 \times 10^{-11} M_{\text{pl}}$, respectively. Thus, in the coupling range $-4.5 \leq \beta \leq -4.35$, the initial value of $|\phi_{\text{R}}|$ at the onset of radiation era needs to be smaller than the order $10^{-11} M_{\text{pl}}$ for the consistency with local gravity constraints.

IV. INFLATIONARY EPOCH

In this section, we study the evolution of ϕ during inflation in the presence of the coupling $(1/2)g^2\phi^2\chi^2$ besides the nonminimal coupling (2.5) with $\beta < 0$. We consider two inflaton potentials: (i) $V(\chi) = m^2\chi^2/2$ and (ii) α -attractor potential (2.15) with $\alpha = 1$. The potential (i), which corresponds to the limit $\alpha \rightarrow \infty$ of Eq. (2.15), leads to the scalar spectral index $n_s \simeq 1-2/N$ and the tensor-to-scalar ratio $r \simeq 8/N$, where N is the number of e -foldings backward from the end of inflation to the epoch at which the perturbations relevant to observed cosmic microwave background (CMB) temperature anisotropies crossed the Hubble radius [53]. For $N = 60$ we have $n_s \simeq 0.967$ and $r = 0.133$, so the model is in tension with the Planck 2018 bound of the tensor-to-scalar ratio $r < 0.066$ (95% C.L.) [64].

Nevertheless, the potential (i) of chaotic inflation with the four-point interaction $g^2\phi^2\chi^2/2$ is a baseline model widely studied in the context of preheating after inflation [46–53]. We will accommodate this case in our analysis for the purpose of understanding the difference from the α -attractor with $\alpha = 1$. In the model (ii) with $\alpha \lesssim \mathcal{O}(1)$, we have $n_s \simeq 1-2/N$ and $r \simeq 12\alpha/N^2$ [59], and hence $r = 3.3 \times 10^{-3}$ for $\alpha = 1$ and $N = 60$. If $\alpha \lesssim 40$, the α -attractor model is compatible with the Planck CMB bound $r < 0.066$ and $n_s = 0.9661 \pm 0.0040$ (68% C.L.) [65].

In the current theory the field ϕ is present besides the inflaton χ , so the existence of the former field can modify the prediction of inflationary observables like n_s and r . To avoid this, we consider the case in which the contribution of ϕ to the background equations of motion is suppressed around the e -folding $N = 60$ backward from the end of inflation. This amounts to the conditions $\beta^2\phi^2/(2M_{\text{pl}}^2) \ll 1$, $\dot{\phi}^2 \ll H^2 M_{\text{pl}}^2$, and $g^2\phi^2\chi^2 \ll V$. Considering the coupling β in the range $-4.5 \leq \beta \leq -4.35$ with $|\dot{\phi}|$ at most of order $|H\phi|$, the first two conditions can be satisfied for $|\phi| \lesssim 0.1 M_{\text{pl}}$. Then, the field value ϕ_{inf} about the

60 e -folding backward from the end of inflation is in the range

$$|\phi_{\text{inf}}| \lesssim 0.1M_{\text{Pl}}, \quad \text{and} \quad |\phi_{\text{inf}}| \lesssim 0.1 \frac{\sqrt{V}}{g|\chi|}. \quad (4.1)$$

For the potential $V(\chi) = m^2\chi^2/2$, the latter condition translates to $|\phi_{\text{inf}}| \lesssim 0.1m/g$.

Under the conditions (4.1), the effective mass squared (2.14) during inflation ($|\dot{H}| \ll H^2$ and $3H^2M_{\text{Pl}}^2 \simeq V$) approximately reduces to

$$m_{\text{eff}}^2 \simeq g^2\chi^2 + 6\beta H^2 \simeq g^2\chi^2 + \frac{2\beta V}{M_{\text{Pl}}^2}. \quad (4.2)$$

When $g = 0$, we have $m_{\text{eff}}^2 < 0$ and hence there is the tachyonic growth of ϕ [45]. On the exact de Sitter background, the growing-mode solution to Eq. (2.13) for $g = 0$ is given by

$$\phi \propto \exp \left[\frac{3}{2} \left(\sqrt{1 - \frac{8}{3}\beta} - 1 \right) Ht \right]. \quad (4.3)$$

During the time interval $t = 10H^{-1}$, for instance, the field ϕ is amplified by a factor 5×10^{16} for $\beta = -4.4$. This enhancement of ϕ destroys the inflationary period driven by the potential energy of χ . Even if the initial field value ϕ_{inf} is fine-tuned to be extremely close to 0, there exists a field perturbation $\delta\phi$ whose amplitude does not vanish due to the uncertainty principle. After the Hubble radius crossing, the perturbation $\delta\phi$ is amplified in a manner analogous to the homogeneous field ϕ discussed above [45]. Hence, the existence of the nonminimal coupling (2.5) with $\beta = -\mathcal{O}(1)$ violates the successful inflationary prediction of primordial density perturbations sourced by the perturbations of χ .

The nonvanishing coupling g gives rise to a positive contribution to the mass squared (4.2). If g is in the range

$$g > \frac{\sqrt{2|\beta|V}}{M_{\text{Pl}}|\chi|}, \quad (4.4)$$

we have $m_{\text{eff}}^2 > 0$ and hence the exponential growth of $|\phi|$ can be avoided.

Let us first consider the effective mass in the range $0 < m_{\text{eff}}^2 \leq 9H^2/4$. Neglecting the variation of m_{eff}^2 during inflation, the growing-mode solution to Eq. (2.13) is given by

$$\phi \propto \exp \left[-\frac{3}{2} \left(1 - \sqrt{1 - \frac{4m_{\text{eff}}^2}{9H^2}} \right) Ht \right]. \quad (4.5)$$

When $m_{\text{eff}}^2 = 0$ we have $\phi = \text{constant}$, while, for increasing m_{eff}^2 , the decreasing rate of ϕ gets larger.

For the effective mass satisfying $m_{\text{eff}}^2 > 9H^2/4$, the coupling g is in the range

$$g > \frac{\sqrt{(3-8\beta)V}}{2M_{\text{Pl}}|\chi|}, \quad (4.6)$$

whose lower limit is larger than that in Eq. (4.4). In this mass range, the field ϕ evolves as

$$\phi \propto \exp \left(-\frac{3}{2} Ht \right) \cos \left(\sqrt{1 - \frac{9H^2}{4m_{\text{eff}}^2}} m_{\text{eff}} t \right). \quad (4.7)$$

Then, the amplitude of ϕ decreases as $|\phi| \propto e^{-3Ht/2} \propto a^{-3/2}$ with the oscillation induced by the effective mass m_{eff} [66–69], where we used the approximate solution $a \propto e^{Ht}$ during inflation. On using the number of e -foldings N_{inf} relevant to the observation of CMB temperature anisotropies, the amplitude of ϕ at the end of inflation (denoted as $|\phi_I|$) can be estimated as

$$|\phi_I| = |\phi_{\text{inf}}| e^{-3N_{\text{inf}}/2}. \quad (4.8)$$

For $N_{\text{inf}} = 60$ and $\phi_{\text{inf}} = 0.1M_{\text{Pl}}$, the estimation (4.8) gives $|\phi_I| = 8 \times 10^{-41}M_{\text{Pl}}$, so there is the strong suppression of $|\phi|$ during inflation. Notice that, for $m_{\text{eff}}^2 > 9H^2/4$, the decreasing rate of $|\phi|$ does not depend on g .

A. Quadratic potential

For the inflaton potential $V(\chi) = m^2\chi^2/2$, the inequalities (4.4) and (4.6) translate to $g > \sqrt{|\beta|m}/M_{\text{Pl}}$ and $g > \sqrt{3/8 - \beta}m/M_{\text{Pl}}$, respectively. We consider the mass $m = 6 \times 10^{-6}M_{\text{Pl}}$ constrained from the Planck normalization of CMB temperature anisotropies [64]. For $\beta = -4.4$, the conditions (4.4) and (4.6) correspond to $g > 1.26 \times 10^{-5}$ and $g > 1.31 \times 10^{-5}$, respectively.

In Fig. 2, we plot the evolution of $|\phi|/M_{\text{Pl}}$ for four different coupling constants g . We choose the initial field value ϕ_{inf} as a maximum satisfying the two conditions given in Eq. (4.1). In case (a) the coupling $g = 1.27 \times 10^{-5}$ is slightly larger than the value $g = 1.26 \times 10^{-5}$ corresponding to $m_{\text{eff}}^2 = 0$, so the field ϕ mildly decreases according to Eq. (4.5). As g increases in the range $1.26 \times 10^{-5} < g < 1.31 \times 10^{-5}$, the decreasing rate of ϕ tends to be larger as we observe in cases (b) and (c) of Fig. 2. For $g > 1.31 \times 10^{-5}$, the amplitude of ϕ decreases in proportion to $a^{-3/2}$ with oscillations. Thus, provided the condition (4.6), i.e.,

$$g > \sqrt{\frac{3}{8} - \beta} \frac{m}{M_{\text{Pl}}}, \quad (4.9)$$

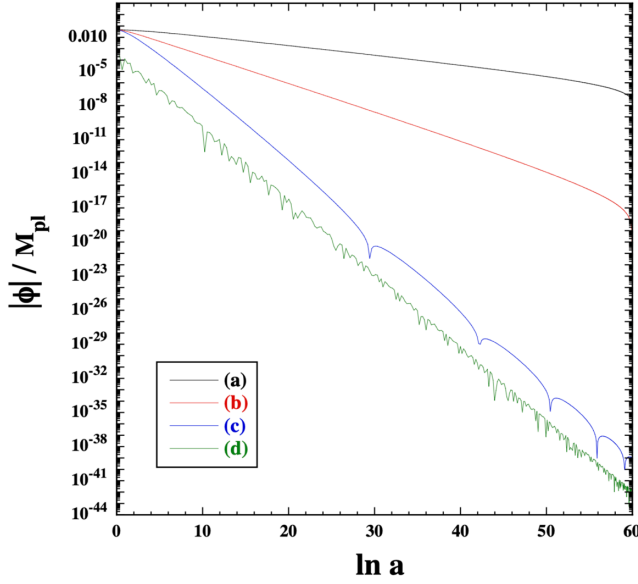


FIG. 2. Evolution of $|\phi|/M_{\text{Pl}}$ versus $\ln a$ during inflation for the potential $V(\chi) = m^2 \chi^2 / 2$ with $\beta = -4.4$ and $m = 6.0 \times 10^{-6} M_{\text{Pl}}$. The initial field values of scalar fields are chosen to be $\chi = 15.38 M_{\text{Pl}}$, with ϕ corresponding to the maximum of Eq. (4.1). In this case, the number of e -foldings during inflation is $N = 60$. Each line corresponds to (a) $g = 1.27 \times 10^{-5}$, (b) $g = 1.29 \times 10^{-5}$, (c) $g = 1.31 \times 10^{-5}$, and (d) $g = 1.0 \times 10^{-3}$.

is satisfied, the field value at the end of inflation is suppressed to be in the range $|\phi_I| \lesssim 10^{-40} M_{\text{Pl}}$. For such couplings, unless $|\phi|$ is amplified by a factor more than 10^{29} during reheating, the bound $|\phi_R| \lesssim 10^{-11} M_{\text{Pl}}$ can be satisfied at the onset of radiation era. For the coupling g in the range $\sqrt{2|\beta|V}/(M_{\text{Pl}}|\chi|) < g < \sqrt{(3-8\beta)V}/(2M_{\text{Pl}}|\chi|)$, the suppression of ϕ during inflation is not necessarily significant, so the growth of ϕ during reheating matters to satisfy the bound $|\phi_R| \lesssim 10^{-11} M_{\text{Pl}}$.

B. α -attractor with $\alpha = 1$

Let us consider the α -attractor potential (2.15) with $\alpha = 1$. In this case, the right hand-sides of Eqs. (4.4) and (4.6) depend on the value of χ . Under the slow-roll approximation, the number of e -foldings backward from the end of inflation (inflaton value χ_f) can be computed by $N = M_{\text{Pl}}^{-2} \int_{\chi_f}^{\chi} V/V_{,\chi} d\chi$. In the present case, we have $N = (3/4)[1/y - 1/y_f + \ln(y/y_f)]$, where $y = e^{-\sqrt{2/3}\chi/M_{\text{Pl}}}$ and $y_f = e^{-\sqrt{2/3}\chi_f/M_{\text{Pl}}}$. The field value at the end of inflation is determined by the condition $\epsilon_V = (M_{\text{Pl}}^2/2)(V_{,\chi}/V)^2 = 1$, and hence $\chi_f = 0.940 M_{\text{Pl}}$ and $y_f = 0.464$. For $N = 60$, we have $y = 1.165 \times 10^{-2}$ and $\chi = 5.453 M_{\text{Pl}}$, in which case the Planck normalization of primordial curvature perturbations gives the constraint $m \simeq 1.1 \times 10^{-5} M_{\text{Pl}}$.

Since $\chi \gtrsim \mathcal{O}(M_{\text{Pl}})$ during the inflationary period, the potential (2.15) is nearly constant in this regime and hence

the term $g^2 \chi^2$ in Eq. (4.2) decreases faster than $2|\beta|V/M_{\text{Pl}}^2$. Unlike the quadratic potential, there are some ranges of the coupling g for which $g^2 \chi^2$ is initially larger than $2|\beta|V/M_{\text{Pl}}^2$, but the latter dominates over the former during inflation. On using the value $\chi = 5.453 M_{\text{Pl}}$ at $N = 60$ with $\beta = -4.4$ and $m = 1.1 \times 10^{-5} M_{\text{Pl}}$, the condition (4.4) for the realization of positive m_{eff}^2 translates to $g > 5.12 \times 10^{-6}$.

In case (a) of Fig. 3, we plot the evolution of $|\phi|/M_{\text{Pl}}$ during inflation for $g = 5.5 \times 10^{-6}$. In this case the field initially decreases because $m_{\text{eff}}^2 > 0$, but it starts to grow at some point because $g^2 \chi^2$ drops below $2|\beta|V/M_{\text{Pl}}^2$. This enhancement of ϕ largely modifies the dynamics of inflation in such a way that the total number of e -foldings does not reach even 40. As we observe in case (b) of Fig. 3, the growth of ϕ also occurs for the coupling $g = 6.5 \times 10^{-6}$. In this case, the field value ϕ at the end of inflation is of order $10^{-11} M_{\text{Pl}}$, so the dynamics of inflation driven by the field χ is hardly modified. In Sec. V, we will study whether or not the existence of a subsequent reheating stage leads to additional growth of ϕ exceeding the bound $|\phi_R| < 10^{-11} M_{\text{Pl}}$.

If we demand that the condition (4.4), i.e., $m_{\text{eff}}^2 > 0$, holds by the end of inflation (at which $\chi_f = 0.940 M_{\text{Pl}}$), then the coupling g is constrained to be

$$g > 0.698 \sqrt{|\beta|} \frac{m}{M_{\text{Pl}}}. \quad (4.10)$$

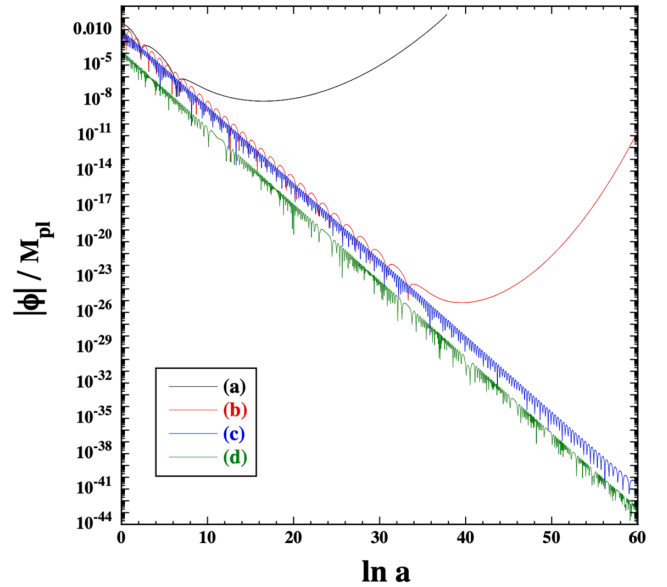


FIG. 3. Evolution of $|\phi|/M_{\text{Pl}}$ versus $\ln a$ during inflation for the α -attractor potential (2.15) with $\alpha = 1$, $\beta = -4.4$, and $m = 1.1 \times 10^{-5} M_{\text{Pl}}$. The initial field values of scalar fields are chosen to be $\chi = 5.418 M_{\text{Pl}}$, with ϕ corresponding to the maximum of Eq. (4.1). Each case corresponds to (a) $g = 5.5 \times 10^{-6}$, (b) $g = 6.5 \times 10^{-6}$, (c) $g = 1.68 \times 10^{-5}$, and (d) $g = 1.0 \times 10^{-3}$.

Substituting the values $\beta = -4.4$ and $m = 1.1 \times 10^{-5} M_{\text{Pl}}$ into Eq. (4.10), we have $g > 1.61 \times 10^{-5}$. Similarly, the condition (4.6) translates to $g > 1.68 \times 10^{-5}$. The evolution of $|\phi|/M_{\text{Pl}}$ for $g = 1.68 \times 10^{-5}$ is plotted as case (c) in Fig. 3. We observe that the field exhibits exponential decrease $|\phi| \propto a^{-3/2}$ by the end of inflation. Numerically, we find that, even for the marginal coupling $g = 1.61 \times 10^{-5}$, ϕ decreases in a similar manner to case (c). Hence the condition (4.10) is practically sufficient to ensure the exponential suppression of ϕ during the whole stage of inflation. As we see in case (d), the decreasing rate of ϕ for $g = 1.0 \times 10^{-3}$ is practically the same as in case (c). The only difference between cases (c) and (d) is the choice of initial conditions, where we have selected the maximum value of $|\phi_{\text{inf}}|$ satisfying the conditions (4.1). For larger g , $|\phi_{\text{inf}}|$ tends to be smaller.

V. REHEATING EPOCH

After the end of inflation, the Universe enters a reheating stage in which the inflaton field χ oscillates around the potential minimum. The field value χ_I at the onset of reheating is determined by the condition $\epsilon_V = (M_{\text{Pl}}^2/2)(V_{,\chi}/V)^2 = 1$, so that $\chi_I = 1.414 M_{\text{Pl}}$ for $V(\chi) = m^2 \chi^2/2$ and $\chi_I = 0.940 M_{\text{Pl}}$ for the α -attractor potential (2.15) with $\alpha = 1$. Around $\chi = 0$, the potential (2.15) approximately reduces to the quadratic one; $V(\chi) \simeq m^2 \chi^2/2$.

In the presence of the coupling $g^2 \phi^2 \chi^2/2$, the background field ϕ and its perturbations can be resonantly amplified by broad parametric resonance during reheating for the coupling g in the range $g^2 \chi^2 \gg m^2$ [46,52]. The resonant growth of the variance of ϕ affects the evolution of the homogeneous mode of χ through the backreaction of created particles [50,52,70]. To incorporate this effect, we expand the inhomogeneous field $\delta\phi(t, \mathbf{x})$ in terms of the Fourier series, as

$$\delta\phi(t, \mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3k \delta\phi_k(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (5.1)$$

where \mathbf{k} is a comoving wavenumber with $k = |\mathbf{k}|$. Under a Hartree approximation, the zero-momentum mode of χ is the only nonvanishing component of inflaton [50]. Using this approximation with $V(\chi) \simeq m^2 \chi^2/2$ in the reheating stage, Eq. (2.8) is modified to

$$\ddot{\chi} + (3H + \Gamma)\dot{\chi} + [m^2 + g^2(\phi^2 + \langle\delta\phi^2\rangle)]\chi = 0, \quad (5.2)$$

where ϕ is the homogeneous value of the field, and $\langle\delta\phi^2\rangle$ is the vacuum expectation value given by

$$\langle\delta\phi^2\rangle = \int \frac{d\mathbf{k}}{k} \mathcal{P}_{\delta\phi_k}, \quad (5.3)$$

with the power spectrum

$$\mathcal{P}_{\delta\phi_k} = \frac{k^3}{2\pi^2} |\delta\phi_k|^2. \quad (5.4)$$

The Fourier modes $\delta\phi_k(t)$ obey the differential equation

$$\delta\ddot{\phi}_k + \left(3H + \frac{\omega_{,\phi}\dot{\phi}}{\omega}\right)\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + M_{\text{eff}}^2\right)\delta\phi_k = 0, \quad (5.5)$$

where

$$M_{\text{eff}}^2 = \frac{1}{\omega} \left[g^2 \chi^2 - 3F_{,\phi\phi} M_{\text{Pl}}^2 (2H^2 + \dot{H}) + (\ddot{\phi} + 3H\dot{\phi})\omega_{,\phi} + \frac{\dot{\phi}^2}{2}\omega_{,\phi\phi} \right]. \quad (5.6)$$

To the right hand-sides of Eqs. (2.11) and (2.12), we take into account the perturbed density $\langle\rho_{\delta\phi}\rangle$ and the pressure $\langle P_{\delta\phi}\rangle$ of produced ϕ particles, respectively. Assuming that the conditions $m \gg H$ and $\langle\delta\phi^2\rangle \ll M_{\text{Pl}}^2$ hold during reheating, we have

$$\langle\rho_{\delta\phi}\rangle = \frac{\omega(\phi)}{2} \left[\langle\delta\dot{\phi}^2\rangle + \frac{1}{a^2} \langle\partial\delta\phi\rangle^2 \right] + \frac{1}{2} g^2 \chi^2 \langle\delta\phi^2\rangle, \quad (5.7)$$

$$\langle P_{\delta\phi}\rangle = \left[\frac{\omega(\phi)}{2} - \beta F(\phi) \right] \left[\langle\delta\dot{\phi}^2\rangle + \frac{1}{a^2} \langle\partial\delta\phi\rangle^2 \right] - \frac{1}{2} g^2 \chi^2 \langle\delta\phi^2\rangle, \quad (5.8)$$

where

$$\langle\delta\dot{\phi}^2\rangle = \int d\mathbf{k} \frac{k^2}{2\pi^2} |\delta\dot{\phi}_k|^2, \quad (5.9)$$

$$\langle\partial\delta\phi\rangle^2 = \int d\mathbf{k} \frac{k^4}{2\pi^2} |\delta\phi_k|^2. \quad (5.10)$$

Defining the rescaled perturbed field

$$\delta\varphi_k = a^{3/2} \omega^{1/2} \delta\phi_k, \quad (5.11)$$

Equation (5.5) can be expressed in the form

$$\delta\ddot{\varphi}_k + \Omega_k^2 \delta\varphi_k = 0, \quad (5.12)$$

where

$$\Omega_k^2 = \frac{k^2}{a^2} - \frac{9}{4} H^2 - \frac{3}{2} \dot{H} + \frac{1}{\omega} \left[g^2 \chi^2 - 3F_{,\phi\phi} M_{\text{Pl}}^2 (2H^2 + \dot{H}) + \frac{1}{2} (\ddot{\phi} + 3H\dot{\phi})\omega_{,\phi} + \frac{\dot{\phi}^2 \omega_{,\phi}^2}{4\omega} \right]. \quad (5.13)$$

The typical wave number relevant to the parametric excitation of ϕ particles is $k/a_I \sim m$, where a_I is the scale factor at the onset of reheating. The perturbations with $k/a_I \sim m$ are deep inside the Hubble radius during inflation ($k/a \gg H$), so the dominant contribution to Ω_k^2 is the term k^2/a^2 . For such modes, we choose a positive-frequency solution in the Bunch-Davies vacuum state as

$$\delta\varphi_k = \frac{1}{\sqrt{2\Omega_k}} e^{-i \int \Omega_k dt}, \quad (5.14)$$

which corresponds to an initial condition of $\delta\varphi_k$ at the onset of reheating. For the modes $k/a_I \sim m$, the frequency Ω_k can be approximated as $\Omega_k \simeq k/a$ during inflation except for the last short period in which k^2/a^2 drops below M_{eff}^2 . Then, the amplitude of $\delta\phi_k$ around the beginning of reheating is estimated as

$$|\delta\phi_k(t_I)| \simeq \frac{1}{a_I \sqrt{\omega} \sqrt{2k}}. \quad (5.15)$$

The square root of the power spectrum (5.4) for the mode $k/a_I \simeq m$ is given by

$$\sqrt{\mathcal{P}_{\delta\phi_k}(t_I)} \simeq \frac{1}{\sqrt{\omega}} \frac{k}{2\pi a_I} \simeq \frac{m}{2\pi}, \quad (5.16)$$

where we used the approximation $\omega \simeq 1$ in the second equality. The perturbation $\delta\phi_k$ excited by parametric resonance has a typical initial amplitude $m/(2\pi)$. For the quadratic potential $V = m^2\chi^2/2$ and the α -attractor potential (2.15) with $\alpha = 1$, we have $\sqrt{\mathcal{P}_{\delta\phi_k}(t_I)} \approx 10^{-6} M_{\text{Pl}}$.

Let us also estimate the power spectrum of larger-scale modes of $\delta\phi_k$ that exit the Hubble radius during inflation. We are interested in the range of coupling g where parametric resonance occurs during preheating, in which case $M_{\text{eff}}^2 > 9H^2/4$ during inflation. After the Hubble radius crossing during inflation ($k < aH$), Ω_k^2 is of order $M_{\text{eff}}^2 \simeq g^2\chi^2/\omega$. Around the onset of reheating, the amplitude of $|\delta\phi_k|$ can be estimated as

$$|\delta\phi_k|(t_I) \simeq \frac{1}{a_I^{3/2} \sqrt{\omega} \sqrt{2M_{\text{eff}}}}. \quad (5.17)$$

This corresponds to the power spectrum

$$\mathcal{P}_{\delta\phi_k}(t_I) \simeq \frac{1}{4\pi^2 \omega M_{\text{eff}}} \left(\frac{k}{a_I}\right)^3 \simeq \frac{1}{4\pi^2} \frac{H_{\text{inf}}^3}{M_{\text{eff}}} e^{-3N_{\text{inf}}}, \quad (5.18)$$

where H_{inf} is the value of H at the Hubble radius crossing. In the second equality of Eq. (5.18), we have substituted $k = a_{\text{inf}} H_{\text{inf}}$ and used the number of e -foldings $N_{\text{inf}} = \ln(a_I/a_{\text{inf}})$. For the perturbations relevant to the observed CMB temperature anisotropies, we have $N_{\text{inf}} = 55 \sim 60$.

Taking the value $N_{\text{inf}} = 60$, the amplitude of perturbations $\delta\phi_k$ at the beginning of reheating is of order

$$\sqrt{\mathcal{P}_{\delta\phi_k}(t_I)} \simeq 10^{-40} \frac{H_{\text{inf}}}{\sqrt{r_M}}, \quad (5.19)$$

where $r_M \equiv M_{\text{eff}}/H_{\text{inf}}$. For the inflationary scale $H_{\text{inf}} \simeq 10^{-4} M_{\text{Pl}}$, we have $\sqrt{\mathcal{P}_{\delta\phi_k}(t_I)} \simeq 10^{-44} M_{\text{Pl}}/\sqrt{r_M}$. This suppression of the large-scale modes of $\delta\phi_k$ is analogous to what happens for the homogeneous field ϕ , see Eq. (4.8). The positive mass squared M_{eff}^2 greater than the order H_{inf}^2 leads to the exponential decrease of $\delta\phi_k$ for the perturbations that exit the Hubble radius long before the end of inflation. From the above discussion, the large-scale perturbations $\delta\phi_k$ with wave numbers k in the range $k/a_I \ll m$ hardly contribute to the vacuum expectation value $\langle \delta\phi^2 \rangle$.

Provided the background field ϕ is suppressed during inflation, its contributions to Eqs. (2.8) and (2.11)–(2.12) can be ignored in the early stage of reheating. The initial reheating period in which χ oscillates coherently can be identified by a temporal matter era where the scale factor evolves as $a \propto t^{2/3}$ with $H = 2/(3t)$. For the approximate potential $V \simeq m^2\chi^2/2$ around $\chi = 0$, the background Eqs. (2.8) and (2.11) reduce to $\ddot{\chi} + 3H\dot{\chi} + m^2\chi \simeq 0$ and $4M_{\text{Pl}}^2/(3t^2) \simeq \dot{\chi}^2/2 + m^2\chi^2/2$, respectively. The solution consistent with the virial relation $\langle \dot{\chi}^2/2 \rangle = \langle m^2\chi^2/2 \rangle$ averaged over oscillations is given by

$$\chi = \chi_I \frac{t_I}{t} \sin(mt), \quad \text{with } \chi_I \equiv \sqrt{\frac{8}{3}} \frac{M_{\text{Pl}}}{mt_I}, \quad (5.20)$$

where χ_I is the amplitude at the onset of reheating ($t = t_I$). The inflaton field oscillates with the amplitude decreasing in proportion to $1/t$.

Ignoring the contributions of time derivatives of ϕ and terms $-9H^2/4 - 3\dot{H}/2$ in Eq. (5.13) during the coherent oscillation of χ , the perturbation $\delta\varphi_k$ obeys the Mathieu equation

$$\frac{d^2\delta\varphi_k}{dz^2} + [A_k - 2q \cos(2z)]\delta\varphi_k = 0, \quad (5.21)$$

where

$$A_k = \left(\frac{k}{ma}\right)^2 + 2q + \frac{2\beta}{3z^2}, \quad (5.22)$$

$$q = q_I \left(\frac{t_I}{t}\right)^2, \quad q_I = \frac{g^2\chi_I^2}{4m^2}, \quad (5.23)$$

$$z = mt. \quad (5.24)$$

As we see in Eq. (2.13), the homogeneous mode $\varphi = a^{3/2}\phi$ satisfies the same form of equation as (5.21) with the limit

$k/(ma) \rightarrow 0$ in Eq. (5.22). If the parameter q_I is in the range $q_I \gg 1$, it is known that there is an epoch of the preheating period in which $\delta\phi_k$ and ϕ are amplified by broad parametric resonance [46,52]. Since χ_I is of order M_{Pl} for the inflaton potentials discussed in Sec. IV, preheating can occur in the coupling range

$$g \gg \frac{m}{M_{\text{Pl}}}. \quad (5.25)$$

For the exact quadratic potential $V = m^2\chi^2/2$, the mass m is constrained to be $m = 6.0 \times 10^{-6}M_{\text{Pl}}$ from the Planck normalization, so the condition (5.25) translates to $g \gg 6.0 \times 10^{-6}$. Since the parameter q decreases as $q \propto 1/t^2$ due to cosmic expansion, the growth of the variance $\langle\delta\phi^2\rangle$ actually occurs for $g \gtrsim 10^{-4}$ [49,50,52]. In the α -attractor model with $\alpha = 1$, we have $m = 1.1 \times 10^{-5}M_{\text{Pl}}$ from the Planck normalization, so the bound (5.25) corresponds to $g \gg 1.1 \times 10^{-5}$. In this case, the field value χ_I at the onset of reheating is smaller than that for the exact quadratic potential, so the larger coupling g is required for the realization of broad parametric resonance.

For the coupling g in the range (5.25), the homogeneous field ϕ is exponentially suppressed ($|\phi| \propto a^{-3/2}$) during inflation. Since we are considering the coupling constant around $\beta \sim -4$, the term $2\beta/(3z^2)$ in Eq. (5.22) is less than the order 1 for $z \gtrsim 1$. Provided that $g \gg m/M_{\text{Pl}}$, the condition $2q \gg 2|\beta|/(3z^2)$ is satisfied during preheating, so the nonminimal coupling β hardly affects the dynamics of $\delta\phi_k$. If $g \sim m/M_{\text{Pl}}$, the parameter $2q$ is of the same order as $2|\beta|/(3z^2)$ and hence the coupling β cannot be ignored. In such cases, however, the absence of parametric resonance does not lead to the growth of the field variance $\langle\delta\phi^2\rangle$. For the range of g where the preheating does not occur, we do not take the backreaction into account.

For wave numbers in the range $k/a_I \gg m$, the term $(k/ma_I)^2$ is much larger than 1. Although $k/(ma)$ decreases during reheating, the parametric excitation of $\delta\phi_k$ is not significant for k/a_I very much larger than m . We recall that the perturbations $\delta\phi_k$ with $k/a_I \ll m$ as well as the homogeneous mode ϕ are subject to the exponential suppression during inflation. Then, the main contribution to $\langle\delta\phi^2\rangle$ at the beginning of preheating comes from the modes distributed around $k/a_I \sim m$. We compute the variances (5.3) and (5.9)–(5.10) for the wave numbers up to $k/a_I \lesssim 10^2m$ by using the initial condition (5.14) at the onset of reheating.

A. Quadratic potential

Let us first study the reheating dynamics for the inflaton potential $V(\chi) = m^2\chi^2/2$. As we showed in Sec. IVA, the amplitude of ϕ decreases as $|\phi| \propto a^{-3/2}$ during inflation for $g > 1.31 \times 10^{-5}$. Parametric resonance occurs for $g \gtrsim 10^{-4}$, in which regime the field value ϕ_I at the onset

of reheating is determined by Eq. (4.8), where ϕ_{inf} is limited as Eq. (4.1). When $g \gtrsim 10^{-4}$, the second condition of (4.1) gives the upper bounds of ϕ_{inf} and ϕ_I . For larger g , the maximum values of ϕ_{inf} and ϕ_I tend to be smaller. In the numerical simulation given below, we use the maximum allowed values of ϕ_I constrained by Eq. (4.1) as the initial condition of reheating.

In Fig. 4, we show the evolutions of $|\phi|$, χ^2 , and $\langle\delta\phi^2\rangle$ during the early stage of reheating for the coupling $g = 2.0 \times 10^{-3}$ with $\beta = -4.4$, $m = 6.0 \times 10^{-6}M_{\text{Pl}}$, and $\Gamma = 1.0 \times 10^{-13}M_{\text{Pl}}$, in which case $q_I = 5.6 \times 10^4$. In this case, the variance $\langle\delta\phi^2\rangle$ starts to increase by parametric resonance and eventually catches up with the background inflaton density χ^2 . After $\langle\delta\phi^2\rangle$ grows to the order m^2/g^2 , the backreaction of created particles starts to violate the coherent oscillation of χ . In Fig. 4, this property can be seen after χ^2 drops below $\langle\delta\phi^2\rangle$. The growth of $\langle\delta\phi^2\rangle$ terminates by the violation of coherent oscillations of χ . In the regime where q_I is sufficiently larger than 1, our numerical analysis shows that the maximum values of $\langle\delta\phi^2\rangle$ reached during preheating have the approximate dependence $\langle\delta\phi^2\rangle_{\text{max}} \propto 1/\sqrt{q_I}$. This property agrees with what was found for a minimally coupled theory ($\beta = 0$) [50]. This means that, for increasing g in the range $q_I \gg 1$, $\langle\delta\phi^2\rangle_{\text{max}}$ tends to be suppressed.

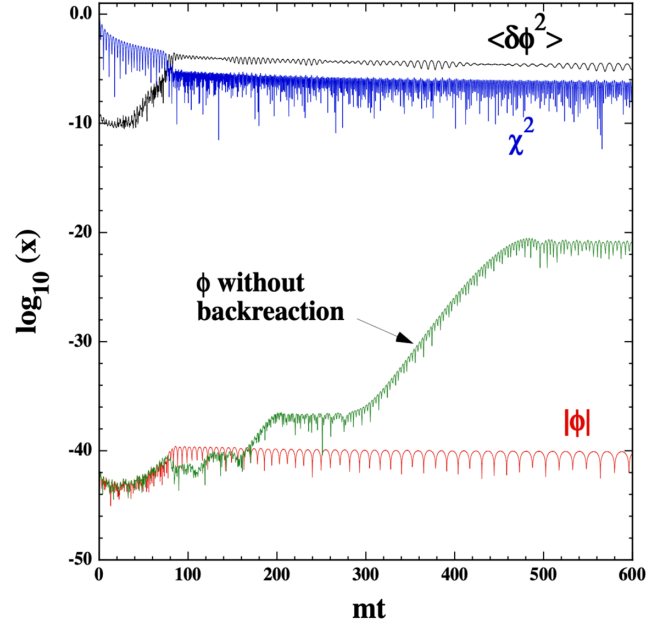


FIG. 4. Evolutions of $|\phi|$, χ^2 , $\langle\delta\phi^2\rangle$ during the early stage of reheating for the quadratic potential $V(\chi) = m^2\chi^2/2$ with $g = 2.0 \times 10^{-3}$, $\beta = -4.4$, $m = 6.0 \times 10^{-6}M_{\text{Pl}}$, and $\Gamma = 1.0 \times 10^{-13}M_{\text{Pl}}$ ($x = |\phi|, \chi^2, \langle\delta\phi^2\rangle$ in the figure). Here ϕ is normalized by M_{Pl} , while χ^2 and $\langle\delta\phi^2\rangle$ are normalized by M_{Pl}^2 . The initial field value ϕ_I at the onset of reheating is determined by the evolution of ϕ during inflation discussed in Sec. IV. The red and green lines correspond to the cases in which the backreaction of created particles is included and neglected, respectively.

In Fig. 4, we observe that the homogeneous field ϕ stops growing after $\langle \delta\phi^2 \rangle$ catches up with χ^2 . Hence the backreaction induced by the growth of $\langle \delta\phi^2 \rangle$ terminates the parametric excitation of ϕ as well. For $g = 2.0 \times 10^{-3}$, the maximum value of $|\phi|$ reached during preheating is of order $\phi_{\max} = 10^{-40} M_{\text{Pl}}$. If we do not take the backreaction of created particles into account, $|\phi|$ continues to grow by the time at which the resonance parameter q drops below the order 1. Without the backreaction effect, the maximum field value ϕ_{\max} for $g = 2.0 \times 10^{-3}$ is of order $10^{-21} M_{\text{Pl}}$, which is enormously larger than the value $10^{-40} M_{\text{Pl}}$ obtained by implementing the backreaction (see Fig. 4).

In Fig. 5, we plot the evolution of $|\phi|$ for four different values of g . Provided that $g \gtrsim 10^{-4}$, $|\phi|$ is initially amplified by parametric resonance. For $g = 1.0 \times 10^{-4}$, which corresponds to the parameter $q_I = 1.4 \times 10^2$, there is an initial short period in which $|\phi|$ is enhanced, but the growth is limited due to the early entry to the region $q \lesssim 1$. For $g \gtrsim 10^{-2}$, the growth of $|\phi|$ is terminated by the backreaction of created particles. When $g \simeq 10^{-2}$, we find that ϕ_{\max} is of order $10^{-38} M_{\text{Pl}}$. In this case, the maximum value of $|\phi|$ obtained without the backreaction is of order $10^{-3} M_{\text{Pl}}$, which is 10^{35} times as large as the value $10^{-38} M_{\text{Pl}}$. This shows the importance of properly implementing the backreaction effect to estimate the maximum value of ϕ_{\max} reached during preheating.

As g increases in the coupling range $g \gtrsim 10^{-2}$, ϕ_{\max} tends to be suppressed in comparison to the value around $g = 10^{-2}$. Indeed, we can confirm this property in Fig. 5

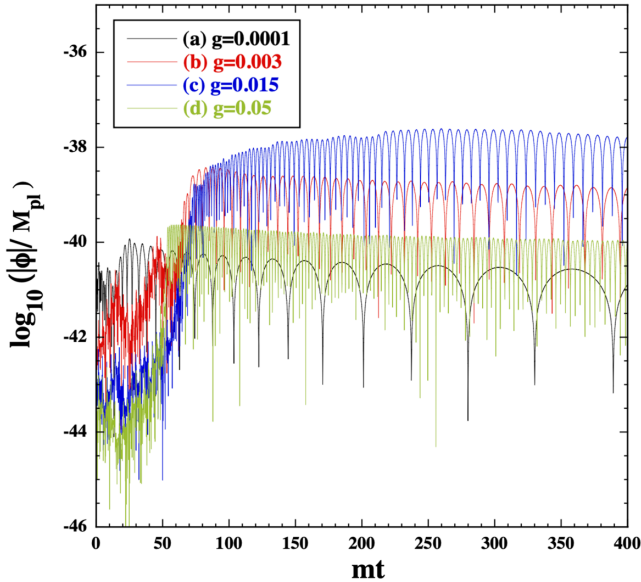


FIG. 5. Evolution of $|\phi|/M_{\text{Pl}}$ versus mt during the early stage of reheating for the quadratic potential $V(\chi) = m^2\chi^2/2$ with $\beta = -4.4$, $m = 6.0 \times 10^{-6} M_{\text{Pl}}$, and $\Gamma = 1.0 \times 10^{-13} M_{\text{Pl}}$. Each line corresponds to (a) $g = 1.0 \times 10^{-4}$ (black), (b) $g = 3.0 \times 10^{-3}$ (red), (c) $g = 1.5 \times 10^{-2}$ (blue), and (d) $g = 5.0 \times 10^{-2}$ (green).

for the coupling $g = 5.0 \times 10^{-2}$. Then, for $g \gtrsim 10^{-4}$, the maximum values of $|\phi|$ obtained under the Hartree approximation are in the range

$$\phi_{\max} \lesssim 10^{-38} M_{\text{Pl}}. \quad (5.26)$$

This is significantly smaller than the upper limit $|\phi_{\text{R}}| \simeq 10^{-11} M_{\text{Pl}}$ constrained from the post-Newtonian bound at the end of reheating.

We note that our approximation scheme does not incorporate nonlinear effects like rescattering. However, it is known that the rescattering of ϕ particles off the inflaton condensate tends to limit the growth of $\langle \delta\phi^2 \rangle$ further [49]. This means that the growth of the homogeneous field ϕ should be also limited, so it is expected that the maximum values of ϕ do not exceed those derived under the Hartree approximation. After the rescattering of produced particles, the variance $\langle \delta\phi^2 \rangle$ reaches an equilibrium state. At this stage, the significant amplification of the homogeneous field from $|\phi| \lesssim 10^{-38} M_{\text{Pl}}$ to the value exceeding the order $10^{-11} M_{\text{Pl}}$ by the end of reheating is unexpected.

In particular, the effective mass squared (2.14) is approximately given by $m_{\text{eff}}^2 \simeq g^2\chi^2 + 3\beta H^2/2$ in the temporal matter era during reheating. Until the onset of the radiation era at which the inflaton field χ completely decays to radiation, the negative coupling term $3\beta H^2/2$ does not completely dominate over the positive contribution $g^2\chi^2$ to m_{eff}^2 . Hence the strong tachyonic instability of ϕ induced by the negative nonminimal coupling constant β is not expected at the late stage of reheating. Eventually, the Born decay term $\Gamma\dot{\chi}$ in Eq. (2.8) starts to work to convert the inflaton density to the radiation density.

Thus, for $g \gtrsim 10^{-4}$, the growth of $|\phi|$ saturated by the backreaction during preheating allows the field value ϕ_{R} at the end of reheating consistent with the post-Newtonian constraint.

For $g \lesssim 10^{-4}$, the preheating stage is absent and hence we do not need to incorporate the backreaction effect. For $g \lesssim 10^{-5}$, the coupling g does not overwhelm the negative nonminimal coupling β in the effective mass of ϕ . Then, ϕ is not subject to the strong suppression during inflation. In Fig. 6, we plot the evolution of $|\phi|/M_{\text{Pl}}$ during inflation and reheating for three different values of g with the decay constant $\Gamma = 1.0 \times 10^{-9} M_{\text{Pl}}$. We integrate the background equations of motion by the end of reheating (time t_{R}) at which ρ_r catches up with the inflaton density $\rho_\chi = \dot{\chi}^2/2 + V(\chi)$. In case (a) the field value at $t = t_{\text{R}}$ is of order $10^{-10} M_{\text{Pl}}$, so it exceeds the solar-system bound $|\phi_{\text{R}}| \lesssim 10^{-11} M_{\text{Pl}}$. In cases (b) and (c), the field values at $t = t_{\text{R}}$ are of orders $10^{-15} M_{\text{Pl}}$ and $10^{-20} M_{\text{Pl}}$, respectively, which are well within the bound $|\phi_{\text{R}}| \lesssim 10^{-11} M_{\text{Pl}}$. This means that, for $\Gamma = 1.0 \times 10^{-9} M_{\text{Pl}}$, the coupling in the

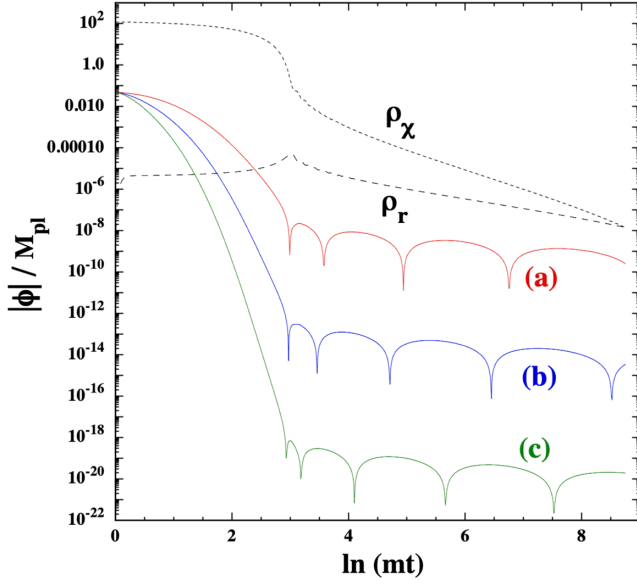


FIG. 6. Evolution of $|\phi|/M_{\text{Pl}}$ versus $\ln(mt)$ during inflation and reheating for the quadratic potential $V(\chi) = m^2\chi^2/2$ with $\beta = -4.4$, $m = 6.0 \times 10^{-6}M_{\text{Pl}}$, and $\Gamma = 1.0 \times 10^{-9}M_{\text{Pl}}$. Each line corresponds to (a) $g = 1.27 \times 10^{-5}$ (red), (b) $g = 1.28 \times 10^{-5}$ (blue), and (c) $g = 1.29 \times 10^{-5}$ (green). Two black dashed lines show the evolution of the inflaton density ρ_χ and radiation density ρ_r . We integrate the background equations by the time at which ρ_r catches up with ρ_χ .

range $g \geq 1.28 \times 10^{-5}$ can be consistent with the solar-system constraint.

In Fig. 6, we observe that, after the initial rapid decrease of $|\phi|$ during inflation, $|\phi|$ exhibits mild decrease in the reheating period. This reflects the fact that $g^2\chi^2$ is larger than the term $3|\beta|H^2/2$ during reheating. The field value ϕ_R at the end of reheating depends on the decay constant Γ . For decreasing Γ , $|\phi_R|$ tends to be smaller because of the longer period of reheating. We also note that the field value ϕ_I at the beginning of reheating depends on the duration of inflation. For the minimum number of e -foldings $N = 60$, the criterion consistent with the bound $|\phi_R| \lesssim 10^{-11}M_{\text{Pl}}$ is given by

$$g \geq 1.3 \times 10^{-5}, \quad (5.27)$$

irrespective of the values of Γ smaller than H at the end of inflation. Thus, the mechanism proposed by Anson *et al.* [42] works even for small couplings of order 10^{-5} .

B. α -attractor with $\alpha = 1$

We also study the dynamics of ϕ in the α -attractor model with $\alpha = 1$. In the α -attractor model, the field value $\chi_I = 0.940M_{\text{Pl}}$ at the onset of reheating is smaller than that for the exact quadratic potential. Hence we require larger couplings g to enhance both ϕ and $\langle \delta\phi^2 \rangle$ by parametric resonance in comparison to the case of quadratic

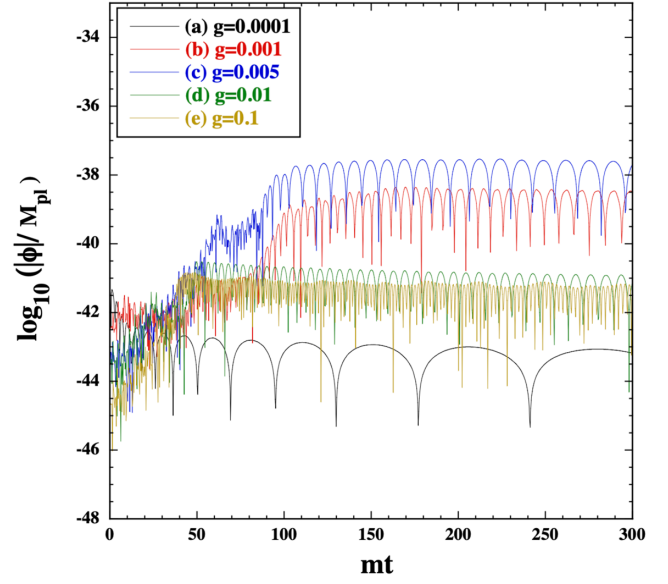


FIG. 7. Evolution of $|\phi|/M_{\text{Pl}}$ versus mt during the early stage of reheating for the α -attractor potential (2.15) with $\alpha = 1$, $\beta = -4.4$, $m = 1.1 \times 10^{-5}M_{\text{Pl}}$, and $\Gamma = 1.0 \times 10^{-13}M_{\text{Pl}}$. Each line corresponds to (a) $g = 1.0 \times 10^{-4}$ (black), (b) $g = 1.0 \times 10^{-3}$ (red), (c) $g = 5.0 \times 10^{-3}$ (blue), (d) $g = 1.0 \times 10^{-2}$ (green), and (e) $g = 1.0 \times 10^{-1}$ (brown).

potential. In the numerical simulation of Fig. 7, we observe that $|\phi|$ does not grow for $g = 1.0 \times 10^{-4}$, but parametric resonance occurs for $g = 1.0 \times 10^{-3}$.

Numerically, we find that the maximum value of $|\phi|$ reached for $g = 5.0 \times 10^{-3}$ is of order $\phi_{\text{max}} = 10^{-38}M_{\text{Pl}}$ under the Hartree approximation. As g increases in the range $g \gtrsim 5 \times 10^{-3}$, ϕ_{max} tends to be decreased. This property can be seen in Fig. 7 for the couplings $g = 1.0 \times 10^{-2}$ and $g = 1.0 \times 10^{-1}$. Thus, for any couplings with $g \gtrsim 10^{-4}$, the maximum values of $|\phi|$ reached during preheating are in the range

$$\phi_{\text{max}} \lesssim 10^{-38}M_{\text{Pl}}, \quad (5.28)$$

which are again much smaller than the post-Newtonian upper limit $\phi_R \simeq 10^{-11}M_{\text{Pl}}$ at the end of reheating.

If we ignore the backreaction of created ϕ particles, ϕ_{max} can be significantly larger than the upper limit (5.28). When $g = 1.0 \times 10^{-1}$, for example, the numerical value of ϕ_{max} derived by neglecting the backreaction effect is of order $10^{-2}M_{\text{Pl}}$, which is very much larger than $10^{-11}M_{\text{Pl}}$. Thus, inclusion of the backreaction is crucially important for the proper estimation of ϕ_{max} . The upper limit $\phi_{\text{max}} = \mathcal{O}(10^{-38}M_{\text{Pl}})$ derived in the presence of the preheating stage is similar to that obtained for the quadratic inflaton potential.

For the coupling g smaller than the order 10^{-5} , the field ϕ is not subject to strong suppression during inflation. Since parametric resonance does not occur for such small

couplings, we do not need to implement the backreaction of created ϕ particles. In Fig. 8, we show the evolution of $|\phi|$ during inflation and reheating for three different values of g with the decay constant $\Gamma = 1.0 \times 10^{-9} M_{\text{Pl}}$. In cases (a) and (b) the field values at the end of reheating (time $t = t_{\text{R}}$) are of order $10^{-11} M_{\text{Pl}}$ and $10^{-25} M_{\text{Pl}}$, respectively, so case (b) is consistent with the solar system limit $|\phi_{\text{R}}| \lesssim 10^{-11} M_{\text{Pl}}$.

We need to caution that, in both cases (a) and (b), $|\phi|$ grows during reheating due to the dominance of the negative nonminimal coupling $3\beta H^2/2$ relative to $g^2 \chi^2$ in m_{eff}^2 (whose dominance starts to occur during inflation). If we consider a very low energy-scale reheating where the reheating temperature is of order MeV [71,72], the decay constant is of order $\Gamma \simeq 1 \text{ sec}^{-1} \simeq 10^{-43} M_{\text{Pl}}$. In this case, the time at the end of reheating is estimated as $t_{\text{R}} \simeq 1/\Gamma \simeq 10^{43} M_{\text{Pl}}^{-1}$ and hence $\ln(mt_{\text{R}}) \simeq 88$. In case (b), for example, the order of $|\phi|$ increases by one order of magnitude during the time interval $\ln(m\Delta t) = 2$. Then, for the MeV scale reheating, the field value ϕ_{R} exceeds the limit even in case (b).

To avoid the increase of $|\phi|$ during reheating, we require that m_{eff}^2 is positive by the end of inflation. As we discussed in Sec. IV B, this translates to the condition $g > 1.61 \times 10^{-5}$ for the α -attractor with $\alpha = 1$. In case (c) of Fig. 8, we plot the evolution of $|\phi|$ for $g = 1.6 \times 10^{-5}$. This corresponds to

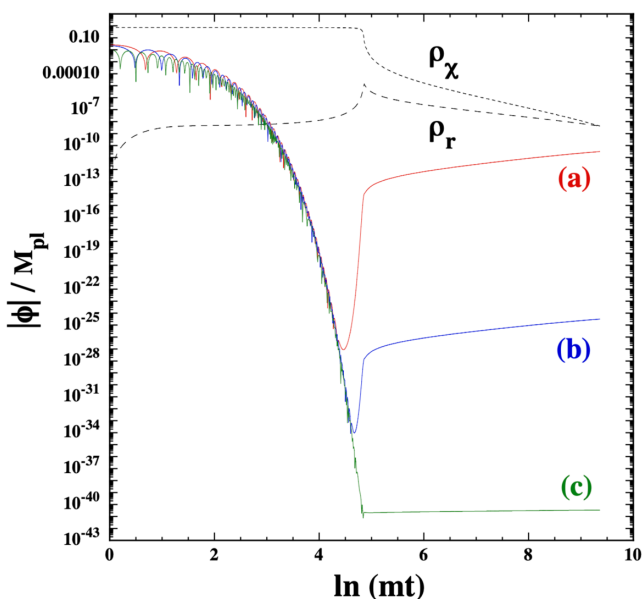


FIG. 8. Evolution of $|\phi|/M_{\text{Pl}}$ versus $\ln(mt)$ during inflation and reheating for the α -attractor potential (2.15) with $\alpha = 1$, $\beta = -4.4$, $m = 1.1 \times 10^{-5} M_{\text{Pl}}$, and $\Gamma = 1.0 \times 10^{-9} M_{\text{Pl}}$. Each line corresponds to (a) $g = 6.7 \times 10^{-6}$ (red), (b) $g = 8.0 \times 10^{-6}$ (blue), and (c) $g = 1.6 \times 10^{-5}$ (green). Two black-dashed lines show the evolution of the inflaton density ρ_χ and radiation density ρ_r . The background equations are integrated by the time at which ρ_r catches up with ρ_χ .

the marginal case in which the monotonic growth of $|\phi|$ during reheating can be avoided. Then, provided that

$$g \geq 1.6 \times 10^{-5}, \quad (5.29)$$

the field value at the end of reheating does not exceed the solar-system limit irrespective of the decay constant Γ . For the coupling range $g \gtrsim 10^{-3}$ in which the preheating epoch is present, the maximum field value is limited as Eq. (5.28). In this coupling regime, $g^2 \chi^2$ dominates over $3|\beta|H^2/2$ after inflation and hence the growth of $|\phi|$ from the end of preheating to the onset of radiation era is not expected to occur.

VI. CONCLUSIONS

In this paper, we studied the cosmological evolution of a scalar field ϕ in the presence of a nonminimal coupling $e^{-\beta\phi^2/(2M_{\text{Pl}}^2)}R$ and a four-point coupling $g^2\phi^2\chi^2/2$ between ϕ and the inflaton field χ . This nonminimal coupling gives rise to the phenomenon of spontaneous scalarization of NSs for $\beta \leq -4.35$. If we apply the original DEF scenario to cosmology, the scalar field ϕ is subject to tachyonic instability during the periods of cosmic acceleration and matter dominance. In Ref. [45], it was argued that the coupling $g^2\phi^2\chi^2/2$ allows a possibility for curing this problem by realizing a positive mass squared during inflation.

Since the dynamics of the field ϕ during the post inflationary reheating period was not addressed in the literature, we have studied the cosmological evolution from the onset of inflation to today including the reheating stage. To satisfy solar-system constraints, the field value at the beginning of radiation era is constrained as Eq. (3.9). For the coupling β relevant to the occurrence of spontaneous scalarization, this bound corresponds to $|\phi_{\text{R}}| \lesssim 10^{-11} M_{\text{Pl}}$.

Provided that the effective mass squared (2.14) is larger than the order of H^2 during inflation, the field ϕ is subject to the exponential suppression ($|\phi| \propto a^{-3/2}$) by the end of inflation. For the quadratic and α -attractor potentials with $\alpha = 1$, we showed that this suppression of ϕ occurs for the coupling g in the ranges (4.9) and (4.10), respectively, whose minimum values are both of order 10^{-5} . With these two potentials, the parametric excitation of ϕ and its perturbations during preheating can occur for $g \gtrsim 10^{-4}$ and $g \gtrsim 10^{-3}$, respectively. If we do not take the backreaction of created ϕ particles into account, the maximum values of $|\phi|$ reached during preheating can exceed the order of $10^{-11} M_{\text{Pl}}$. Incorporating the backreaction under the Hartree approximation, however, we found that ϕ_{max} is smaller than the order of $10^{-38} M_{\text{Pl}}$. After the termination of parametric resonance, the further growth of $|\phi|$ is unexpected by the end of reheating because the $g^2 \chi^2$ term dominates over the negative nonminimal coupling in the equation of motion of ϕ .

In the regime of small couplings g without the preheating stage, we also numerically solved the background equations of motion by the end of reheating with the Born decay term taken into account. For the quadratic inflaton potential, the amplitude of ϕ decreases during reheating in the range of couplings g that gives a positive mass squared during inflation. In this case, provided $g \geq 1.3 \times 10^{-5}$, the model is consistent with the solar-system bound $|\phi_R| \lesssim 10^{-11} M_{\text{Pl}}$. For the α -attractor with $\alpha = 1$, m_{eff}^2 can change its sign during inflation by the presence of a negative nonminimal coupling even if $m_{\text{eff}}^2 > 0$ at the onset of inflation. In such cases, the growth of $|\phi|$ also occurs in the reheating period, see Fig. 8. If we consider a low-scale reheating scenario with the reheating temperature of order MeV, the monotonic growth of $|\phi|$ during a long period of the reheating era can conflict with the limit $|\phi_R| \lesssim 10^{-11} M_{\text{Pl}}$. Provided $g \geq 1.6 \times 10^{-5}$, we found that the α -attractor with $\alpha = 1$ can be consistent with the solar-system bound irrespective of the decay constant Γ .

We thus showed that, for natural couplings in the range $g \gtrsim 10^{-5}$, the scenario proposed by Anson *et al.* leads to the viable cosmological evolution of ϕ consistent with today's local gravity constraints. Since the inflaton field decays to radiation by the onset of radiation era, it does not affect the

process of spontaneous scalarization of NSs which can occur in later cosmological epochs. Interestingly, the field ϕ responsible for spontaneous scalarization can also exhibit the phenomenon of parametric resonance in the early Universe. It is then possible to probe this scenario not only from the gravitational waveform emitted from compact binaries but also from the gravitational wave background.

While we focused on the cosmology in the DEF model with the coupling $g^2 \phi^2 \chi^2 / 2$, it may be of interest to extend the analysis to the cases in which higher-order ϕ -dependent terms like ϕ^4 in $F(\phi)$ [41] or k-essence terms like X^2 [39] are also present. This may widen parameter spaces of the coupling constant β consistent with binary pulsar constraints. These issues are left for future works.

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