

**Erratum: Nonperturbative renormalization
in lattice QCD with three flavors of clover fermions:
Using periodic and open boundary conditions
[Phys. Rev. D **103**, 094511 (2021)]**

G. S. Bali, S. Bürger, S. Collins, M. Göckeler , M. Gruber, S. Piemonte, A. Schäfer, A. Sternbeck, and P. Wein

(RQCD Collaboration)



(Received 24 January 2023; published 21 February 2023)

DOI: [10.1103/PhysRevD.107.039901](https://doi.org/10.1103/PhysRevD.107.039901)

After the publication of our article we noticed that some results from the literature used were incorrect. This affects, in particular, the three-loop conversion coefficient for the tensor density between the RI'-MOM and the $\overline{\text{MS}}$ schemes and the subtraction of lattice artifacts at order g^2 in the coupling for our operators of dimensions three and four. The resulting changes are below 1‰ and smaller than the systematic uncertainties given, in most cases even considerably smaller. This will not lead to relevant differences in phenomenological results. Nevertheless, we prefer to correct our article accordingly. We also take this erratum as an opportunity to include the four- and five-loop anomalous dimensions of the tensor density and the quark field, respectively, in the determination. The details are as follows.

In the perturbative subtraction of lattice artifacts we need the gluon propagator for the Lüscher-Weisz action with tree-level coefficients. This has been taken from Eqs. (11.26)–(11.28) in Ref. [1]. Unfortunately, there is an error in Eq. (11.28). The last line should read

$$-4c_1^3 \sum_{\rho} \hat{k}_{\rho}^4 \prod_{\tau \neq \rho} \hat{k}_{\tau}^2, \quad (1)$$

as can be seen from the original literature [2,3].

Moreover, the conversion factor (up to three loops) for the tensor density $T_{\mu\nu}$ in the RI'-MOM scheme was extracted from results given in Ref. [4]. In Ref. [5] it is pointed out that the three-loop contribution is incorrect. The coefficients for the conversion factor (in the conventions of our paper) should read

$$\begin{aligned} c_1 &= 0, \\ c_2 &= -\frac{3847}{54} + \frac{184}{9} \zeta_3 + \frac{313}{81} n_f, \\ c_3 &= -\frac{9888899}{2916} + \frac{694633}{486} \zeta_3 + \frac{464}{27} \zeta_4 - \frac{31720}{81} \zeta_5 + \left(\frac{286262}{729} - \frac{2096}{27} \zeta_3 + \frac{80}{9} \zeta_4 \right) n_f - \left(\frac{13754}{2187} + \frac{32}{81} \zeta_3 \right) n_f^2. \end{aligned} \quad (2)$$

For $n_f = 3$ one finds $c_3 = -1194.56$ to be compared with the value $c_3 = -1207.96$ given in Table V of our paper.

We also realized that the $\overline{\text{MS}}$ anomalous dimension of $T_{\mu\nu}$ is actually known to four-loop accuracy [6], while in our paper we used only the three-loop anomalous dimension. The four-loop coefficient reads

$$\begin{aligned} \gamma_3 &= \frac{2208517}{81} - \frac{247456}{243} \zeta_3 + \frac{10208}{9} \zeta_4 - \frac{1339520}{243} \zeta_5 + \left(-\frac{3074758}{729} - \frac{607328}{243} \zeta_3 + \frac{13984}{27} \zeta_4 + \frac{36800}{27} \zeta_5 \right) n_f \\ &+ \left(\frac{39844}{729} + \frac{7360}{81} \zeta_3 - \frac{320}{9} \zeta_4 \right) n_f^2 + \left(\frac{56}{243} + \frac{128}{81} \zeta_3 \right) n_f^3. \end{aligned} \quad (3)$$

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TABLE I. Results from fits for operators with less than two derivatives based on the RI'-MOM scheme without the perturbative subtraction of lattice artifacts. The chiral extrapolation has been performed globally. The first number in parentheses gives the statistical error, while the second number is an estimate of the systematic uncertainty. All values refer to the $\overline{\text{MS}}$ scheme at the scale $\mu_0^2 = 4 \text{ GeV}^2$.

	$\beta = 3.34$	$\beta = 3.40$	$\beta = 3.46$	$\beta = 3.55$	$\beta = 3.70$	$\beta = 3.85$
Z_q	0.7822(11)(67)	0.7901(5)(61)	0.7971(2)(54)	0.8071(4)(48)	0.8223(10)(34)	0.8360(13)(30)
Z_S	0.6322(154)(300)	0.6202(71)(255)	0.6089(52)(261)	0.5964(78)(267)	0.5831(119)(233)	0.5729(138)(183)
Z_V	0.6915(22)(78)	0.7011(13)(61)	0.7102(7)(53)	0.7229(11)(52)	0.7419(24)(48)	0.7589(33)(54)
Z'_V	0.6968(21)(63)	0.7065(11)(47)	0.7156(7)(39)	0.7279(11)(38)	0.7465(24)(37)	0.7628(32)(45)
Z_A	0.7372(24)(74)	0.7449(12)(56)	0.7525(7)(55)	0.7628(13)(59)	0.7782(22)(58)	0.7919(29)(57)
Z'_A	0.7440(22)(42)	0.7514(12)(30)	0.7582(6)(27)	0.7678(13)(29)	0.7823(22)(30)	0.7952(30)(36)
Z_T	0.8064(21)(125)	0.8204(10)(98)	0.8338(6)(97)	0.8522(11)(102)	0.8796(23)(98)	0.9051(31)(98)
$Z_{v_{2a}}$	1.0527(18)(179)	1.0786(9)(167)	1.1020(6)(164)	1.1335(10)(162)	1.1798(21)(161)	1.2228(27)(187)
$Z_{v_{2b}}$	1.0672(17)(136)	1.0916(9)(128)	1.1136(5)(122)	1.1431(10)(118)	1.1868(19)(116)	1.2269(25)(134)
$Z_{r_{2a}}$	1.0514(20)(185)	1.0776(10)(172)	1.1012(6)(171)	1.1329(11)(171)	1.1791(23)(167)	1.2217(30)(183)
$Z_{r_{2b}}$	1.0907(19)(142)	1.1146(10)(135)	1.1358(5)(129)	1.1645(10)(120)	1.2067(20)(107)	1.2452(26)(113)
$Z_{h_{1a}}$	1.0794(19)(183)	1.1083(8)(172)	1.1344(5)(169)	1.1696(11)(167)	1.2218(22)(166)	1.2702(30)(194)
$Z_{h_{1b}}$	1.0896(20)(186)	1.1182(11)(175)	1.1440(6)(170)	1.1788(11)(163)	1.2305(22)(158)	1.2787(30)(186)
Z_A/Z_V	1.0643(32)(47)	1.0613(15)(42)	1.0585(11)(36)	1.0546(16)(28)	1.0486(30)(10)	1.0433(41)(9)
Z'_A/Z'_V	1.0660(32)(40)	1.0624(17)(29)	1.0589(10)(24)	1.0543(16)(21)	1.0477(30)(19)	1.0424(40)(24)

TABLE II. Results from fits for operators with less than two derivatives based on the RI'-MOM scheme with the perturbative subtraction of lattice artifacts. The chiral extrapolation has been performed globally. The first number in parentheses gives the statistical error, while the second number is an estimate of the systematic uncertainty. All values refer to the $\overline{\text{MS}}$ scheme at the scale $\mu_0^2 = 4 \text{ GeV}^2$.

	$\beta = 3.34$	$\beta = 3.40$	$\beta = 3.46$	$\beta = 3.55$	$\beta = 3.70$	$\beta = 3.85$
Z_q	0.7809(11)(48)	0.7894(5)(45)	0.7971(2)(41)	0.8077(4)(37)	0.8234(10)(30)	0.8373(13)(25)
Z_S	0.6044(154)(240)	0.5942(71)(209)	0.5850(49)(212)	0.5748(78)(214)	0.5643(119)(186)	0.5564(136)(146)
Z_V	0.6996(22)(35)	0.7099(13)(28)	0.7191(7)(22)	0.7318(11)(17)	0.7507(24)(13)	0.7670(32)(20)
Z'_V	0.7028(21)(25)	0.7131(11)(17)	0.7224(7)(12)	0.7349(11)(8)	0.7534(24)(11)	0.7691(32)(17)
Z_A	0.7433(24)(28)	0.7517(12)(23)	0.7594(7)(24)	0.7699(13)(25)	0.7853(23)(26)	0.7984(29)(24)
Z'_A	0.7476(22)(6)	0.7556(12)(4)	0.7627(6)(5)	0.7725(13)(5)	0.7871(22)(9)	0.7996(30)(11)
Z_T	0.8185(22)(67)	0.8333(10)(61)	0.8468(6)(58)	0.8652(11)(56)	0.8925(23)(51)	0.9170(31)(49)
$Z_{v_{2a}}$	1.0632(18)(167)	1.0894(9)(159)	1.1128(6)(153)	1.1444(10)(145)	1.1909(21)(137)	1.2333(27)(154)
$Z_{v_{2b}}$	1.0726(17)(133)	1.0973(9)(128)	1.1193(5)(120)	1.1489(10)(111)	1.1927(19)(102)	1.2323(25)(111)
$Z_{r_{2a}}$	1.0616(19)(168)	1.0880(10)(161)	1.1116(6)(156)	1.1433(11)(148)	1.1897(23)(139)	1.2316(30)(148)
$Z_{r_{2b}}$	1.0969(19)(142)	1.1210(10)(136)	1.1423(5)(130)	1.1711(10)(119)	1.2135(20)(102)	1.2516(26)(99)
$Z_{h_{1a}}$	1.0889(19)(174)	1.1182(8)(166)	1.1443(5)(160)	1.1797(11)(151)	1.2322(22)(141)	1.2800(30)(157)
$Z_{h_{1b}}$	1.1002(21)(182)	1.1291(11)(175)	1.1549(6)(167)	1.1900(11)(154)	1.2419(22)(141)	1.2896(30)(155)
Z_A/Z_V	1.0622(32)(52)	1.0592(15)(46)	1.0565(11)(41)	1.0527(16)(33)	1.0468(31)(17)	1.0418(41)(10)
Z'_A/Z'_V	1.0636(31)(37)	1.0600(17)(28)	1.0566(10)(21)	1.0521(16)(14)	1.0456(30)(8)	1.0406(39)(15)

Similarly, the five-loop $\overline{\text{MS}}$ anomalous dimension of the quark field in the Landau gauge can be found from Ref. [7]. In our paper we used only the four-loop result. The five-loop coefficient reads

$$\begin{aligned}
\gamma_4 = & \frac{4678028371}{7776} - \frac{161381953}{1728} \zeta_3 + \frac{7530725}{108} \zeta_4 - \frac{447446437}{1944} \zeta_5 + \frac{41883875}{1296} \zeta_6 + \frac{1914069143}{13824} \zeta_7 + \frac{64558441}{5184} \zeta_3^2 \\
& + \left(-\frac{732321613}{5832} - \frac{157885211}{5832} \zeta_3 - \frac{355483}{162} \zeta_4 - \frac{27229511}{2916} \zeta_5 + \frac{1894775}{1944} \zeta_6 + \frac{825587}{27} \zeta_7 + \frac{4989469}{972} \zeta_3^2 \right) n_f \\
& + \left(\frac{4296115}{729} + \frac{1103240}{243} \zeta_3 - \frac{20422}{27} \zeta_4 - \frac{11936}{27} \zeta_5 - \frac{1600}{9} \zeta_6 - 1323 \zeta_7 - \frac{256}{9} \zeta_3^2 \right) n_f^2 \\
& + \left(-\frac{42638}{2187} - \frac{2624}{81} \zeta_3 + \frac{320}{9} \zeta_4 \right) n_f^3 + \left(\frac{664}{729} - \frac{128}{81} \zeta_3 \right) n_f^4. \tag{4}
\end{aligned}$$

TABLE III. Results for operators with less than two derivatives based on the RI'-MOM scheme, obtained by means of the fixed-scale method without the perturbative subtraction of lattice artifacts. The chiral extrapolation has been performed globally. The first number in parentheses gives the statistical error, while the second number is an estimate of the systematic uncertainty. All values refer to the $\overline{\text{MS}}$ scheme at the scale $\mu_0^2 = 4 \text{ GeV}^2$.

	$\beta = 3.34$	$\beta = 3.40$	$\beta = 3.46$	$\beta = 3.55$	$\beta = 3.70$	$\beta = 3.85$
Z_q	0.8092(4)(24)	0.8114(2)(24)	0.8147(1)(24)	0.8207(1)(24)	0.8317(4)(28)	0.8430(5)(27)
Z_S	0.6978(54)(134)	0.6713(31)(113)	0.6473(23)(110)	0.6220(30)(100)	0.5880(62)(102)	0.5648(85)(118)
Z_V	0.7005(8)(12)	0.7074(5)(16)	0.7156(2)(13)	0.7275(3)(10)	0.7472(9)(10)	0.7639(13)(18)
Z'_V	0.7070(8)(15)	0.7137(4)(14)	0.7213(3)(12)	0.7322(3)(6)	0.7501(9)(3)	0.7648(13)(18)
Z_A	0.7456(8)(18)	0.7509(4)(15)	0.7565(3)(14)	0.7660(4)(10)	0.7812(9)(12)	0.7947(12)(23)
Z'_A	0.7561(9)(17)	0.7604(5)(12)	0.7648(3)(15)	0.7725(4)(9)	0.7851(10)(3)	0.7965(14)(13)
Z_T	0.8085(7)(37)	0.8216(4)(38)	0.8347(2)(39)	0.8543(3)(40)	0.8855(8)(43)	0.9135(11)(45)
$Z_{v_{2a}}$	1.0482(4)(111)	1.0760(3)(114)	1.1023(2)(118)	1.1389(3)(124)	1.1923(7)(140)	1.2399(10)(137)
$Z_{v_{2b}}$	1.0666(4)(86)	1.0920(3)(88)	1.1155(2)(91)	1.1478(3)(95)	1.1944(7)(108)	1.2364(9)(106)
$Z_{r_{2a}}$	1.0438(5)(111)	1.0726(3)(115)	1.0993(2)(119)	1.1363(3)(124)	1.1893(8)(142)	1.2363(11)(138)
$Z_{r_{2b}}$	1.0898(5)(88)	1.1147(3)(91)	1.1374(2)(93)	1.1685(3)(97)	1.2135(7)(108)	1.2537(10)(105)
$Z_{h_{1a}}$	1.0761(5)(116)	1.1069(3)(121)	1.1357(2)(125)	1.1756(3)(131)	1.2341(8)(149)	1.2868(11)(147)
$Z_{h_{1b}}$	1.0887(5)(117)	1.1185(3)(122)	1.1465(2)(126)	1.1857(3)(132)	1.2433(8)(150)	1.2956(11)(147)
Z_A/Z_V	1.0636(10)(5)	1.0605(6)(7)	1.0566(4)(7)	1.0524(5)(10)	1.0451(11)(7)	1.0397(17)(13)
Z'_A/Z'_V	1.0688(11)(7)	1.0648(6)(6)	1.0597(4)(5)	1.0541(6)(8)	1.0456(13)(3)	1.0399(19)(7)

TABLE IV. Results for operators with less than two derivatives based on the RI'-MOM scheme, obtained by means of the fixed-scale method with the perturbative subtraction of lattice artifacts. The chiral extrapolation has been performed globally. The first number in parentheses gives the statistical error, while the second number is an estimate of the systematic uncertainty. All values refer to the $\overline{\text{MS}}$ scheme at the scale $\mu_0^2 = 4 \text{ GeV}^2$.

	$\beta = 3.34$	$\beta = 3.40$	$\beta = 3.46$	$\beta = 3.55$	$\beta = 3.70$	$\beta = 3.85$
Z_q	0.7897(4)(23)	0.7965(2)(23)	0.8031(1)(23)	0.8126(1)(24)	0.8270(4)(28)	0.8403(5)(28)
Z_S	0.6509(54)(127)	0.6300(31)(105)	0.6107(23)(104)	0.5916(30)(94)	0.5657(64)(98)	0.5485(85)(117)
Z_V	0.7039(8)(11)	0.7129(5)(15)	0.7221(2)(13)	0.7342(3)(10)	0.7530(9)(10)	0.7684(13)(19)
Z'_V	0.7081(8)(14)	0.7169(4)(14)	0.7256(3)(12)	0.7369(3)(6)	0.7542(9)(3)	0.7681(13)(19)
Z_A	0.7447(8)(18)	0.7525(4)(15)	0.7595(3)(14)	0.7698(4)(10)	0.7849(9)(12)	0.7978(12)(24)
Z'_A	0.7523(9)(17)	0.7593(5)(12)	0.7651(3)(15)	0.7738(4)(9)	0.7869(10)(3)	0.7982(14)(13)
Z_T	0.8189(7)(38)	0.8338(4)(39)	0.8474(2)(40)	0.8664(3)(40)	0.8955(8)(43)	0.9211(11)(45)
$Z_{v_{2a}}$	1.0615(4)(112)	1.0892(3)(115)	1.1148(2)(119)	1.1500(3)(124)	1.2010(7)(140)	1.2465(10)(137)
$Z_{v_{2b}}$	1.0730(4)(87)	1.0986(3)(89)	1.1219(2)(92)	1.1535(3)(95)	1.1989(7)(108)	1.2397(9)(106)
$Z_{r_{2a}}$	1.0571(5)(112)	1.0856(3)(116)	1.1117(2)(119)	1.1473(3)(124)	1.1979(8)(142)	1.2428(11)(138)
$Z_{r_{2b}}$	1.0961(5)(89)	1.1214(3)(91)	1.1441(2)(94)	1.1748(3)(97)	1.2187(7)(108)	1.2578(10)(105)
$Z_{h_{1a}}$	1.0873(5)(118)	1.1182(3)(122)	1.1466(2)(126)	1.1855(3)(131)	1.2418(8)(149)	1.2926(11)(147)
$Z_{h_{1b}}$	1.1003(5)(119)	1.1305(3)(123)	1.1582(2)(127)	1.1964(3)(132)	1.2520(8)(150)	1.3023(11)(147)
Z_A/Z_V	1.0593(10)(6)	1.0566(6)(7)	1.0531(4)(8)	1.0495(5)(10)	1.0430(12)(7)	1.0383(17)(13)
Z'_A/Z'_V	1.0639(11)(8)	1.0604(6)(7)	1.0557(5)(5)	1.0508(6)(8)	1.0433(14)(3)	1.0383(21)(7)

We have recalculated the lattice artifacts in one-loop lattice perturbation theory for the quark-antiquark operators with less than two derivatives using the correct gluon propagator and implemented the above conversion factor for $\mathcal{T}_{\mu\nu}$ in the RI'-MOM scheme as well as the four-loop anomalous dimension of $\mathcal{T}_{\mu\nu}$ and the five-loop anomalous dimension of the quark field.

With all these corrections in place, our analysis yields the following tables of results, where we include also the numbers that have not changed.

Tables I–IV are the corrected versions of Tables VI, VII, IX, and X, respectively, in our paper. They differ from the latter tables due to the corrected subtraction, the updated anomalous dimensions of $\mathcal{T}_{\mu\nu}$ and the quark field, as well as due to the corrected value of the three-loop coefficient in the conversion factor for $\mathcal{T}_{\mu\nu}$.

TABLE V. Results from fits for operators with less than two derivatives based on the RI'-SMOM scheme without the perturbative subtraction of lattice artifacts. The chiral extrapolation has been performed globally. The first number in parentheses gives the statistical error, while the second number is an estimate of the systematic uncertainty. All values refer to the $\overline{\text{MS}}$ scheme at the scale $\mu_0^2 = 4 \text{ GeV}^2$.

	$\beta = 3.34$	$\beta = 3.40$	$\beta = 3.46$	$\beta = 3.55$	$\beta = 3.70$	$\beta = 3.85$
Z_q	0.7858(9)(93)	0.7938(4)(85)	0.8006(2)(81)	0.8106(3)(72)	0.8255(9)(55)	0.8391(12)(44)
Z_S	0.6233(14)(79)	0.6179(7)(73)	0.6128(4)(66)	0.6062(5)(55)	0.5962(11)(37)	0.5864(16)(28)
Z_P	0.4958(16)(255)	0.4968(6)(229)	0.4978(3)(205)	0.5006(5)(176)	0.5056(13)(116)	0.5092(19)(66)
Z_V	0.7151(11)(96)	0.7236(5)(87)	0.7310(3)(79)	0.7417(4)(67)	0.7581(10)(45)	0.7730(15)(40)
Z'_V	0.7104(13)(86)	0.7192(6)(72)	0.7268(3)(63)	0.7378(5)(53)	0.7550(13)(38)	0.7702(18)(49)
Z_A	0.7464(10)(66)	0.7543(4)(62)	0.7612(3)(58)	0.7711(4)(49)	0.7857(10)(37)	0.7988(14)(34)
Z'_A	0.7625(9)(74)	0.7679(4)(63)	0.7724(2)(62)	0.7795(4)(60)	0.7909(10)(46)	0.8017(14)(35)
Z_T	0.8312(12)(101)	0.8446(5)(95)	0.8565(3)(88)	0.8731(4)(77)	0.8982(12)(61)	0.9215(16)(62)
$Z_{v_{2a}}$	1.0707(17)(304)	1.0985(9)(290)	1.1226(6)(278)	1.1552(8)(251)	1.2026(21)(203)	1.2451(28)(177)
$Z_{v_{2b}}$	1.0655(16)(315)	1.0920(8)(300)	1.1150(6)(288)	1.1465(8)(260)	1.1927(20)(209)	1.2347(28)(180)
$Z_{r_{2a}}$	1.0640(18)(298)	1.0918(9)(286)	1.1160(6)(273)	1.1486(8)(248)	1.1959(21)(202)	1.2384(30)(172)
$Z_{r_{2b}}$	1.0842(18)(314)	1.1106(9)(301)	1.1334(6)(288)	1.1644(9)(261)	1.2096(22)(210)	1.2505(30)(177)
$Z_{h_{1a}}$	1.1388(18)(374)	1.1677(9)(361)	1.1934(6)(351)	1.2274(9)(334)	1.2756(23)(299)	1.3194(32)(270)
$Z_{h_{1b}}$	1.1529(18)(379)	1.1817(10)(365)	1.2071(6)(356)	1.2408(10)(338)	1.2884(24)(302)	1.3315(32)(273)
Z_S/Z_P	1.2119(39)(635)	1.2065(17)(586)	1.2001(10)(546)	1.1880(14)(477)	1.1657(33)(318)	1.1437(44)(160)
Z_A/Z_V	1.0412(7)(59)	1.0407(4)(55)	1.0400(2)(51)	1.0387(2)(45)	1.0360(5)(29)	1.0333(8)(18)
Z'_A/Z'_V	1.0705(12)(60)	1.0655(6)(46)	1.0609(3)(35)	1.0550(4)(25)	1.0466(9)(22)	1.0400(13)(29)
Z_A/Z_P	1.4529(39)(462)	1.4732(16)(421)	1.4908(8)(412)	1.5102(12)(384)	1.5335(35)(281)	1.5543(51)(212)
Z'_A/Z_P	1.4582(47)(335)	1.4753(15)(316)	1.4891(9)(297)	1.5045(15)(258)	1.5239(41)(169)	1.5422(59)(149)
$Z_P/(Z_S Z_A)$	1.0893(34)(439)	1.0847(16)(418)	1.0823(10)(399)	1.0816(12)(339)	1.0860(31)(204)	1.0916(43)(126)
$Z_P/(Z_S Z'_A)$	1.0661(32)(377)	1.0655(14)(351)	1.0666(10)(330)	1.0702(13)(265)	1.0790(34)(137)	1.0876(46)(94)

TABLE VI. Results from fits for operators with less than two derivatives based on the RI'-SMOM scheme with the perturbative subtraction of lattice artifacts. The chiral extrapolation has been performed globally. The first number in parentheses gives the statistical error, while the second number is an estimate of the systematic uncertainty. All values refer to the $\overline{\text{MS}}$ scheme at the scale $\mu_0^2 = 4 \text{ GeV}^2$.

	$\beta = 3.34$	$\beta = 3.40$	$\beta = 3.46$	$\beta = 3.55$	$\beta = 3.70$	$\beta = 3.85$
Z_q	0.7822(9)(72)	0.7908(4)(67)	0.7983(2)(64)	0.8088(3)(58)	0.8243(9)(46)	0.8383(12)(36)
Z_S	0.6115(14)(92)	0.6068(7)(86)	0.6025(4)(79)	0.5968(5)(66)	0.5880(11)(44)	0.5793(16)(31)
Z_P	0.4881(15)(215)	0.4901(6)(194)	0.4921(3)(172)	0.4959(5)(145)	0.5020(13)(92)	0.5064(19)(49)
Z_V	0.7137(11)(76)	0.7228(5)(69)	0.7308(3)(64)	0.7420(4)(56)	0.7588(10)(41)	0.7738(15)(34)
Z'_V	0.7072(13)(59)	0.7168(6)(49)	0.7250(3)(43)	0.7367(5)(37)	0.7544(13)(27)	0.7699(19)(33)
Z_A	0.7457(10)(57)	0.7541(4)(54)	0.7614(3)(51)	0.7716(4)(45)	0.7864(10)(37)	0.7996(14)(30)
Z'_A	0.7579(9)(41)	0.7641(4)(35)	0.7695(2)(36)	0.7774(4)(36)	0.7895(10)(30)	0.8006(14)(21)
Z_T	0.8321(12)(94)	0.8462(5)(88)	0.8585(3)(84)	0.8756(4)(76)	0.9010(12)(62)	0.9243(16)(53)
$Z_{v_{2a}}$	1.0731(17)(306)	1.1010(9)(293)	1.1251(6)(282)	1.1578(8)(256)	1.2053(21)(207)	1.2476(28)(176)
$Z_{v_{2b}}$	1.0672(16)(317)	1.0938(8)(302)	1.1169(6)(291)	1.1484(8)(264)	1.1948(20)(212)	1.2367(28)(179)
$Z_{r_{2a}}$	1.0666(18)(303)	1.0946(9)(290)	1.1187(6)(280)	1.1515(8)(254)	1.1989(22)(206)	1.2412(30)(173)
$Z_{r_{2b}}$	1.0861(18)(318)	1.1127(9)(304)	1.1355(6)(293)	1.1666(9)(266)	1.2119(22)(215)	1.2527(30)(177)
$Z_{h_{1a}}$	1.1412(18)(372)	1.1704(9)(360)	1.1961(6)(351)	1.2302(9)(333)	1.2786(23)(297)	1.3223(32)(268)
$Z_{h_{1b}}$	1.1553(18)(378)	1.1843(10)(365)	1.2098(6)(356)	1.2436(10)(337)	1.2913(24)(302)	1.3343(32)(270)
Z_S/Z_P	1.2084(39)(615)	1.2025(17)(566)	1.1958(10)(528)	1.1835(13)(461)	1.1609(33)(308)	1.1388(45)(155)
Z_A/Z_V	1.0423(7)(52)	1.0415(4)(47)	1.0406(2)(44)	1.0390(2)(36)	1.0362(6)(24)	1.0333(9)(13)
Z'_A/Z'_V	1.0692(12)(64)	1.0644(6)(50)	1.0599(3)(39)	1.0542(4)(26)	1.0459(9)(16)	1.0394(13)(27)
Z_A/Z_P	1.4583(39)(426)	1.4787(16)(394)	1.4960(8)(382)	1.5150(12)(350)	1.5380(35)(248)	1.5583(50)(190)
Z'_A/Z_P	1.4600(46)(326)	1.4772(15)(308)	1.4910(9)(287)	1.5064(14)(246)	1.5260(41)(157)	1.5442(59)(148)
$Z_P/(Z_S Z_A)$	1.0945(34)(445)	1.0897(16)(423)	1.0871(10)(404)	1.0862(12)(342)	1.0903(31)(205)	1.0953(43)(128)
$Z_P/(Z_S Z'_A)$	1.0748(32)(404)	1.0737(14)(373)	1.0744(10)(351)	1.0773(13)(285)	1.0854(34)(153)	1.0935(46)(102)

TABLE VII. Results for operators with less than two derivatives based on the RI'-SMOM scheme, obtained by means of the fixed-scale method without the perturbative subtraction of lattice artifacts. The chiral extrapolation has been performed globally. The first number in parentheses gives the statistical error, while the second number is an estimate of the systematic uncertainty. All values refer to the $\overline{\text{MS}}$ scheme at the scale $\mu_0^2 = 4 \text{ GeV}^2$.

	$\beta = 3.34$	$\beta = 3.40$	$\beta = 3.46$	$\beta = 3.55$	$\beta = 3.70$	$\beta = 3.85$
Z_q	0.8311(3)(26)	0.8305(2)(25)	0.8310(1)(24)	0.8338(1)(24)	0.8403(3)(31)	0.8487(4)(28)
Z_S	0.6343(4)(28)	0.6268(2)(28)	0.6199(1)(28)	0.6111(1)(27)	0.5981(4)(26)	0.5863(5)(25)
Z_P	0.5769(5)(40)	0.5635(2)(36)	0.5537(1)(34)	0.5439(1)(32)	0.5344(4)(30)	0.5283(5)(28)
Z_V	0.7629(4)(24)	0.7617(2)(22)	0.7621(1)(22)	0.7653(1)(21)	0.7734(4)(23)	0.7831(5)(22)
Z'_V	0.7611(5)(13)	0.7601(3)(11)	0.7606(1)(10)	0.7636(2)(10)	0.7716(5)(9)	0.7807(7)(4)
Z_A	0.7776(4)(23)	0.7787(2)(22)	0.7807(1)(22)	0.7856(1)(22)	0.7949(4)(25)	0.8048(5)(25)
Z'_A	0.8171(3)(13)	0.8123(2)(10)	0.8092(1)(9)	0.8074(1)(7)	0.8082(4)(10)	0.8120(5)(12)
Z_T	0.8700(4)(35)	0.8748(2)(35)	0.8805(1)(35)	0.8911(1)(36)	0.9100(4)(41)	0.9298(6)(41)
$Z_{v_{2a}}$	1.0890(6)(107)	1.1133(3)(111)	1.1356(2)(114)	1.1669(3)(118)	1.2127(8)(134)	1.2539(11)(130)
$Z_{v_{2b}}$	1.0918(6)(107)	1.1135(3)(110)	1.1340(2)(113)	1.1629(3)(117)	1.2055(8)(133)	1.2450(11)(130)
$Z_{r_{2a}}$	1.0800(6)(107)	1.1046(3)(110)	1.1273(2)(113)	1.1590(3)(117)	1.2049(8)(135)	1.2463(11)(131)
$Z_{r_{2b}}$	1.1052(6)(109)	1.1278(3)(112)	1.1486(2)(115)	1.1780(3)(119)	1.2206(9)(136)	1.2595(12)(133)
$Z_{h_{1a}}$	1.1386(6)(197)	1.1650(3)(202)	1.1897(2)(207)	1.2246(3)(214)	1.2761(8)(231)	1.3236(12)(234)
$Z_{h_{1b}}$	1.1510(6)(199)	1.1777(3)(204)	1.2024(2)(209)	1.2372(3)(216)	1.2884(9)(233)	1.3355(13)(236)
Z_S/Z_P	1.0988(6)(32)	1.1118(3)(26)	1.1194(2)(23)	1.1235(2)(20)	1.1196(5)(25)	1.1105(8)(24)
Z_A/Z_V	1.0190(2)(5)	1.0222(1)(4)	1.0243(1)(4)	1.0265(1)(4)	1.0279(2)(4)	1.0279(2)(5)
Z'_A/Z'_V	1.0736(4)(4)	1.0688(2)(4)	1.0639(1)(7)	1.0574(1)(10)	1.0476(3)(21)	1.0401(5)(20)
Z_A/Z_P	1.3472(8)(94)	1.3815(4)(94)	1.4098(2)(95)	1.4444(3)(96)	1.4884(9)(103)	1.5245(12)(102)
Z'_A/Z_P	1.3977(10)(94)	1.4231(5)(93)	1.4430(3)(94)	1.4660(4)(94)	1.4946(10)(97)	1.5194(14)(99)
$Z_P/(Z_S Z_A)$	1.1706(7)(43)	1.1552(3)(38)	1.1443(2)(37)	1.1329(3)(36)	1.1231(7)(47)	1.1183(11)(46)
$Z_P/(Z_S Z'_A)$	1.1140(6)(20)	1.1073(3)(11)	1.1039(2)(10)	1.1021(3)(10)	1.1043(7)(26)	1.1081(10)(31)

TABLE VIII. Results for operators with less than two derivatives based on the RI'-SMOM scheme, obtained by means of the fixed-scale method with the perturbative subtraction of lattice artifacts. The chiral extrapolation has been performed globally. The first number in parentheses gives the statistical error, while the second number is an estimate of the systematic uncertainty. All values refer to the $\overline{\text{MS}}$ scheme at the scale $\mu_0^2 = 4 \text{ GeV}^2$.

	$\beta = 3.34$	$\beta = 3.40$	$\beta = 3.46$	$\beta = 3.55$	$\beta = 3.70$	$\beta = 3.85$
Z_q	0.7974(3)(24)	0.8032(2)(23)	0.8086(1)(24)	0.8170(1)(24)	0.8297(3)(31)	0.8419(4)(29)
Z_S	0.6072(4)(26)	0.6027(2)(25)	0.5985(1)(25)	0.5930(1)(25)	0.5846(4)(24)	0.5764(5)(24)
Z_P	0.5369(5)(34)	0.5302(2)(30)	0.5257(1)(29)	0.5222(1)(28)	0.5202(4)(27)	0.5189(5)(26)
Z_V	0.7315(4)(21)	0.7372(2)(20)	0.7428(1)(21)	0.7514(1)(21)	0.7652(4)(22)	0.7782(5)(22)
Z'_V	0.7269(5)(5)	0.7329(3)(6)	0.7387(1)(7)	0.7474(2)(8)	0.7615(5)(8)	0.7743(7)(3)
Z_A	0.7552(4)(22)	0.7615(2)(21)	0.7673(1)(21)	0.7762(1)(22)	0.7894(4)(25)	0.8015(6)(25)
Z'_A	0.7761(3)(5)	0.7794(2)(3)	0.7824(1)(3)	0.7874(1)(2)	0.7956(4)(9)	0.8040(5)(12)
Z_T	0.8443(4)(35)	0.8560(2)(35)	0.8665(1)(36)	0.8820(1)(37)	0.9055(4)(42)	0.9276(6)(42)
$Z_{v_{2a}}$	1.0881(6)(108)	1.1140(3)(111)	1.1372(2)(114)	1.1691(3)(118)	1.2150(8)(134)	1.2560(11)(130)
$Z_{v_{2b}}$	1.0869(6)(108)	1.1108(3)(111)	1.1326(2)(114)	1.1627(3)(117)	1.2062(8)(133)	1.2458(11)(130)
$Z_{r_{2a}}$	1.0802(6)(107)	1.1063(3)(111)	1.1298(2)(114)	1.1620(3)(118)	1.2078(8)(135)	1.2488(11)(131)
$Z_{r_{2b}}$	1.1025(6)(110)	1.1269(3)(113)	1.1488(2)(116)	1.1790(3)(119)	1.2221(9)(136)	1.2611(13)(132)
$Z_{h_{1a}}$	1.1346(6)(197)	1.1634(3)(202)	1.1895(2)(207)	1.2256(3)(214)	1.2779(8)(232)	1.3254(12)(235)
$Z_{h_{1b}}$	1.1476(6)(199)	1.1765(3)(205)	1.2025(2)(210)	1.2384(3)(216)	1.2903(9)(234)	1.3373(13)(236)
Z_S/Z_P	1.1128(5)(28)	1.1220(3)(22)	1.1266(2)(20)	1.1275(2)(18)	1.1205(5)(24)	1.1100(8)(24)
Z_A/Z_V	1.0284(2)(3)	1.0298(1)(3)	1.0306(0)(3)	1.0312(1)(3)	1.0309(1)(4)	1.0297(2)(5)
Z'_A/Z'_V	1.0667(4)(5)	1.0630(2)(4)	1.0590(1)(7)	1.0536(1)(10)	1.0450(3)(21)	1.0384(5)(20)
Z_A/Z_P	1.3687(7)(94)	1.4012(4)(93)	1.4276(2)(94)	1.4592(3)(95)	1.4990(9)(102)	1.5319(12)(101)
Z'_A/Z_P	1.4007(10)(96)	1.4271(5)(94)	1.4473(3)(94)	1.4702(4)(94)	1.4980(10)(97)	1.5220(14)(98)
$Z_P/(Z_S Z_A)$	1.1804(6)(44)	1.1632(3)(39)	1.1513(2)(38)	1.1389(3)(36)	1.1280(8)(47)	1.1222(11)(46)
$Z_P/(Z_S Z'_A)$	1.1410(6)(24)	1.1301(3)(15)	1.1235(2)(13)	1.1181(3)(12)	1.1160(7)(26)	1.1165(11)(31)

Tables [V–VIII](#) are the corrected versions of Tables XII, XIII, XV, and XVI, respectively, in our paper. They differ from the latter tables due to the corrected subtraction as well as due to the updated anomalous dimensions of $\mathcal{T}_{\mu\nu}$ and the quark field.

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