Theoretical study of $D_s^+ \to \pi^+\pi^0\pi^0$

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Inspired by the BESIII Collaboration's recent amplitude analysis on $D_s^+ \to \pi^+\pi^0\pi^0$, we develop a model based on the chiral unitary approach to describe the $D_s^+ \to \pi^+\pi^0\pi^0$ decay, where D_s^+ decays into π^+ and a quark-antiquark pair through weak decay of a charm quark. The interaction of pseudoscalar mesons produced by the hadronization of a quark-antiquark pair will lead to dynamically generated $f_0(980)$. The dominant amplitudes from resonances $f_0(1370)$ and $f_2(1270)$ in the BESIII fit are also included in our model. In addition to the above contributions, to obtain a good fit, we also add an amplitude from the resonance $f_2(1430)$ to our model. Our model provides a good fit for the $M_{\pi^0\pi^0}$ and $M_{\pi^+\pi^0}$ invariant mass distributions simultaneously.

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I. INTRODUCTION

Heavy mesons are mesons containing b and c quarks. Their weak decay involves the final state interaction of meson pairs and the production of scalar resonances. The study of scalar resonances includes the mechanism of weak decay at the quark level. The Dalitz plot and the projection of the invariant mass distribution for weak decay of the heavy meson provide important information about the scalar resonances, which are thought to be dynamically generated by the final state interaction of meson pairs. The $f_0(980)$ meson is a one such scalar resonance, which can be obtained by weak decay of D_s^+.

The $f_0(980)$ meson is widely studied as a candidate for the tetraquark [1] or a quark-antiquark pair [2], mainly through the processes $D_s^+ \to \pi^+\pi^+\pi^-$, $D_s^+ \to \pi^+K^+K^-$, and $D_s^+ \to \pi^+\pi^0\pi^0$. Reference [3] adopted the chiral unitary approach in coupled channels to study the final state interaction of two pseudoscalar mesons in $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$ decays, and the $f_0(980)$ signal was found in both $\pi^+\pi^-$ and K^+K^- invariant mass distributions. Then, Ref. [4] found that the $f_0(980)$ resonance was the dominant contribution close to the K^+K^- threshold in the decay of $D_s^+ \to \pi^+K^+K^-$. Later, Ref. [5] not only provided a full analysis of $K\bar{K}$ and $K\pi$ mass spectra but also calculated the branching ratios of the dominant decay channels in both the s-wave and p-wave, which had good agreement with the experimental data from Ref. [6]. In addition, the $f_0(980)$ meson is studied in B decay (more details can be found about B decay in Refs. [7–16]).

Other studies on D_s^+ decays, for the process of $D_s^+ \to \pi^+\pi^0\eta$, instead of the W-annihilation mechanism, Ref. [17] proposed that the decay of D_s^+ occurred through internal W-emission, where the $a_0(980)$ resonance was proved to be a dynamically generated resonance by the final state interaction in coupled channels. Furthermore, in Ref. [18], the branching ratios of $D_s^+ \to \pi^+(a_0(980)\to)\pi^0\eta$ and $D_s^+ \to \pi^0(a_0(980)\to)\pi^+\eta$ decays were calculated by using the triangular rescattering mechanism, and they were also consistent with the experimental data. The research on K_s^0 also plays an important role in the discovery of resonance natures. The contribution of $a_0(1710)$ was analyzed in detail in Ref. [19] within the process of $D_s^+ \to \pi^+K_s^0K_s^0$, which was dynamically generated by the $K^*\bar{K}^*$ final state interaction and finally decayed to $K_s^0K_s^0$. The contribution of $a_0(1710)$ was also found in the process of $D_s^+ \to \pi^+K^+K_s^0$ [20] and finally decayed to $K^+\bar{K}^0$.

As an isospin partner of $f_0(980)$, $a_0(980)$ has similar properties to $f_0(980)$, and there are many works that address this. For the singly Cabibbo-suppressed process of $D^+ \to \pi^+\pi^0\eta$, the Dalitz plot was predicted in Ref. [21]. Then, Ref. [22] tested the mechanism of the W boson and found that both internal and external W-emission mechanisms were possible, in which the final state interaction of the meson pairs led to the $a_0(980)$ resonance. Similarly, Ref. [23] studied the resonance signal of $a_0(980)$ within the $\pi^0\eta$ invariant mass distribution for $D^0 \to \pi^0\eta\eta$ decay and predicted the possible $f_0(500)$, $f_0(980)$, $a_0(980)$

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resonances in $D^0 \to \pi^0\pi^0\pi^0$ and $D^0 \to \pi^0\pi^0\eta$ decays. The Cabibbo-favored decay $D^0 \to K^-\pi^+\eta$ was studied in Ref. [24] by using the external W-emission, which obtained the $M_{K\pi}$ distribution containing the contributions of $a_0(980)$ and $K_0^*(700)$ in good agreement with experimental data. For the doubly Cabibbo-suppressed process of $D^+ \to K^-K^+K^+$, in Ref. [25], it was found that the resonant contribution mainly came from the s-wave component of the system. Later, the resonant contribution from the s-wave was theoretically demonstrated in Ref. [26], where it was explained by the mechanism of the final state interaction of pseudoscalar pairs.

Recently, the BESIII Collaboration reported a more accurate measurement of the absolute branching fraction of $D_s^+ \to \pi^+\pi^0\pi^0$ decay [27] than what the CLEO Collaboration found [28], and it performed an amplitude analysis of this process. Compared to the decays of $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$, there is no contribution from $a_0(980) \to K^+K^-$ or $\rho \to \pi^+\pi^-$ in the $D_s^+ \to \pi^+\pi^0\pi^0$ decay. Therefore, it can be used as a relatively clean channel to study the $f_0(980)$ resonance. It is interesting that BESIII Collaboration also measured the resonance signal of $f_0(500)$ in $D_s^+ \to \pi^+\pi^0\pi^0$ decay and found that the significance of $f_0(500)$ was less than 2σ while there was a pronounced peak for $f_0(980)$. Furthermore the chiral unitary approach can be used to explain this phenomenon by the final state interaction of pseudoscalar mesons, which can naturally lead to the $f_0(980)$ resonance. In the whole process, we do not introduce $f_0(500)$ or $f_0(980)$ resonance artificially, but it is dynamically generated by chiral unitary approach. This approach is an unitary extension of the chiral perturbation theory [29–32], characterized by the Bethe-Salpeter equation for the meson-meson interaction in coupled channel, which can provide a complete meson-meson rescattering amplitude and predict the invariant mass distribution with a minimum input.

In our work, we develop a model containing a few free parameters to fit the $M_{\pi^0\pi^0}$ and $M_{\pi^+\pi^0}$ distributions simultaneously.[1] Besides the $f_0(980)$ resonance calculated by using the chiral unitary approach, we also consider the other possible contributions via the intermediate mesons in p-waves and d-waves. We reproduce the peak of the $f_0(980)$ resonance in the $M_{\pi^0\pi^0}$ distribution without the appearance of the $f_0(500)$ resonance, focusing on the external W-emission mechanism and the final state interaction in the coupled channel to prove the chiral unitary approach is applicable in this process. We also provide an argument for the necessity of amplitudes in the model; when both $f_2(1430)$ and $f_2(1270)$ contributions are removed from the full model, there is an obvious influence on the fit effect. The final result shows good agreement with the experimental data, which indirectly revealed the nature of the $f_0(980)$ resonance.

Our work is organized as follows. In Sec. II, we introduce the formalism of each amplitude in our model, especially the production of the $f_0(980)$ resonance within the chiral unitary approach. The combined fit to $M_{\pi^0\pi^0}$ and $M_{\pi^+\pi^0}$ distributions for $D_s^+ \to \pi^+\pi^0\pi^0$ decay as well as discussions are presented in Sec. III. Finally, a short conclusion is made in Sec. IV.

[1]Recently, a work [33] focusing on the same process appeared in the literature.

II. FORMALISM

In this process, the D_s^+ meson decays into the final state through a weak decay of a charm quark, which includes three steps. First, the c quark in the D_s^+ meson is converted to an s quark, with the $u\bar{d}$ pair produced by external W-emission, which is depicted in Fig. 1. Note that the internal W-emission contributions have been ignored in this work since they are color suppressed. We need $\pi^+\pi^0\pi^0$ in the final state; therefore, the $u\bar{d}$ pair forms the π^+ meson, and the $s\bar{s}$ pair hadronizes with a $q\bar{q}(=\bar{u}u + \bar{d}d + \bar{s}s)$ pair from the vacuum. Then, the pseudoscalar meson pairs produced by the hadronization will lead to dynamically generated resonances. The weak decay of D_s^+ can be expressed as

$$\begin{aligned} D_s^+ &\Rightarrow V_{cs}V_{ud}((u\bar{d} \to \pi^+)[s\bar{s} \to s\bar{s} \cdot (u\bar{u} + d\bar{d} + s\bar{s})]) \\ &\Rightarrow V_{cs}V_{ud}(u\bar{d} \to \pi^+)[M_{33} \to (M \cdot M)_{33}], \end{aligned} \tag{1}$$

where M is the matrix consisting of $q\bar{q}$ quarks,

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}. \tag{2}$$

We can also rewrite the M matrix with pseudoscalar mesons,

$$\Phi = M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \tag{3}$$

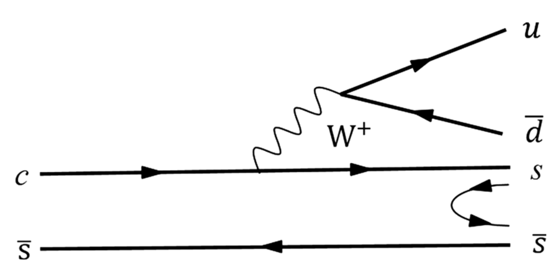


FIG. 1. External W-emission for $D_s^+ \to \pi^+s\bar{s}$, where $s\bar{s}$ hadronizes into the final meson pair through the $q\bar{q}$ pair from the vacuum.

where $\eta' = \eta_1$ will be removed since it is unnecessary in chiral perturbation theory, and we only take $\eta = \eta_8$ into account. Thus, the hadronization process is shown as

$$s\bar{s}(u\bar{u} + d\bar{d} + s\bar{s}) = (\Phi \cdot \Phi)_{33} = K^- K^+ + K^0\bar{K}^0 + \frac{2}{3}\eta\eta. \tag{4}$$

Then, we obtain all the final states from the hadronization of the $s\bar{s}$ pair, and $\pi^0\pi^0$ can be obtained from the final state interaction. Thus, we have

$$H = V_p V_{cs} V_{ud}\left(K^- K^+ + K^0\bar{K}^0 + \frac{2}{3}\eta\eta\right)\pi^+, \tag{5}$$

where V_P is the production vertex of weak decay containing all the dynamical factors, and V_{cs}, V_{ud} are the elements of the CKM matrix.

In Eq. (5), there is no final state $\pi^0\pi^0$ produced for the D_s^+ decay. Therefore, we take the final state interaction into consideration. As depicted in Fig. 2, one can see that the rescattering of the final states in the s-wave, which dynamically generates the $f_0(980)$ resonance, will eventually produce the $\pi^0\pi^0$ final state. Thus, obtain the full amplitude of the D_s^+ decay in the s-wave,

$$\begin{aligned} t(s_{23}) &= V_p V_{cs} V_{ud}\bigg(G_{K^-K^+}T_{K^-K^+\to\pi^0\pi^0} \\ &\quad + G_{K^0\bar{K}^0}T_{K^0\bar{K}^0\to\pi^0\pi^0} + \frac{2}{3}\cdot 2\cdot\frac{1}{2}G_{\eta\eta}T_{\eta\eta\to\pi^0\pi^0}\bigg) \\ &= \mathcal{D}\bigg(G_{K^-K^+}T_{K^-K^+\to\pi^0\pi^0} \\ &\quad + G_{K^0\bar{K}^0}T_{K^0\bar{K}^0\to\pi^0\pi^0} + \frac{2}{3}\cdot 2\cdot\frac{1}{2}G_{\eta\eta}T_{\eta\eta\to\pi^0\pi^0}\bigg), \end{aligned} \tag{6}$$

where we labeled π^+, π^0, π^0 as 1, 2, 3 final states and $s_{ij} = (p_i + p_j)^2$. Here, $\mathcal{D}$ is the parameter that contains the production vertex V_P and the elements of the CKM matrix, V_{cs}, V_{ud}. To confirm the experimental data, the normalization factor is also included in $\mathcal{D}$. In Eq. (6), there is a factor of 2 in the $G_{\eta\eta}T_{\eta\eta\to\pi^0\pi^0}$ term due to the two $\eta\eta \to \pi^0\pi^0$ processes producing the final state. The other factor of $\frac{1}{2}$ is a result of integrating the loop function involving the indistinguishability of $\eta\eta$ mesons. As elements of the diagonal matrix, G_{ii} are calculated by two intermediate meson loop functions, given by

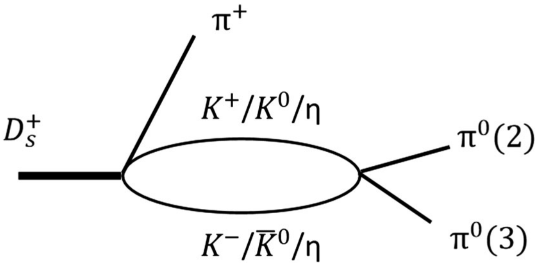


FIG. 2. Diagrammatic representation of the final state interaction, leading to the production of the $\pi^0\pi^0$ final state, via the rescattering of K^-K^+, $K^0\bar{K}^0$, and $\eta\eta$.

$$G_{ii}(s) = i\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_1^2 + i\epsilon}\frac{1}{(p_1 + p_2 - q)^2 - m_2^2 + i\epsilon}, \tag{7}$$

where p_1, p_2 are the four-momentum of the two initial particles, q is one of the loop momentums of intermediate mesons, and m_1, m_2 are the masses of two intermediate mesons. Instead of using the three-momentum cutoff method, we use the dimensional regularization method [34] to solve this logarithmically divergent integral,

$$\begin{aligned} G_{kk}(s) = \frac{1}{16\pi^2}\bigg\{ & a_\mu + \ln\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\ln\frac{m_2^2}{m_1^2} \\ & + \frac{q_{cmk}(s)}{\sqrt{s}}[\ln(s - (m_2^2 - m_1^2) + 2q_{cmk}(s)\sqrt{s}) \\ & + \ln(s + (m_2^2 - m_1^2) + 2q_{cmk}(s)\sqrt{s}) \\ & - \ln(-s - (m_2^2 - m_1^2) + 2q_{cmk}(s)\sqrt{s}) \\ & - \ln(-s + (m_2^2 - m_1^2) + 2q_{cmk}(s)\sqrt{s})]\bigg\}, \end{aligned} \tag{8}$$

with $q_{cmk}(s)$ the modulus of the three-momentum in the center-of-mass frame corresponding to the coupled channel,

$$q_{cmk}(s) = \frac{\lambda^{\frac{1}{2}}(s, m_1^2, m_2^2)}{2\sqrt{s}}, \tag{9}$$

where μ is the dimensional regularization scale and a_μ is the subtraction constant. Following Ref. [34], we take the values of the subtraction constants in different channels as $a_{K^+K^-} = -1.66$, $a_{K^0\bar{K}^0} = -1.66$, and $a_{\eta\eta} = -1.71$. Note that λ is the Källén function, represented as $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$.

In addition, the elements $t_{i\to j}$ are the scattering amplitudes making up the scattering matrix. In the chiral unitary approach, we can obtain them by calculating the coupled channel Bethe-Salpeter equation [35]

$$T = [1 - VG]^{-1}V, \tag{10}$$

where V is a 5×5 symmetric matrix for the scattering potential in the s-wave, with the elements taken from Ref. [36], corresponding to the coupled channels. Here, the coupled channels for $I = 0$ are usually numbered as 1 for

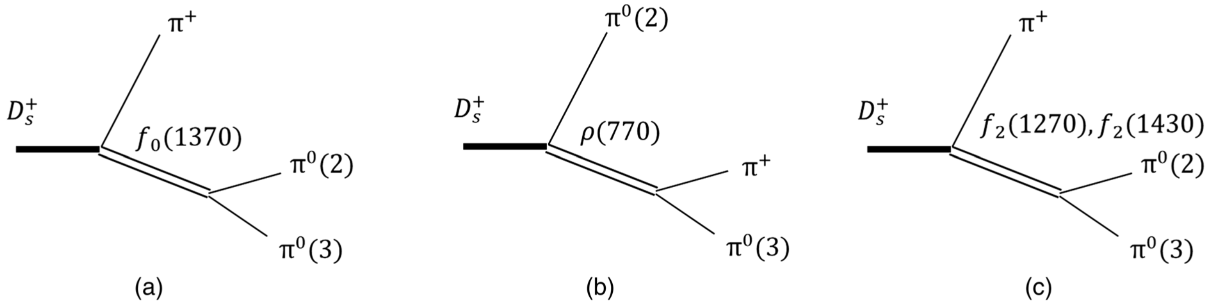


FIG. 3. Decay of $D_s^+ \to \pi^+\pi^0\pi^0$ via meson resonances: (a) via the $f_0(1370)$ meson in the s-wave; (b) via the $\rho(770)$ meson in the p-wave; (c) via the $f_2(1270)$ or $f_2(1430)$ meson in the d-wave.

$\pi^+\pi^-$, 2 for $\pi^0\pi^0$, 3 for K^+K^-, 4 for $K^0\bar{K}^0$, and 5 for $\eta\eta$. Thus, the V matrix can be expressed as

$$V_{11} = -\frac{1}{2f^2}s, \quad V_{12} = -\frac{1}{\sqrt{2}f^2}(s-m_\pi)^2, \quad V_{13} = -\frac{1}{4f^2}s,$$
$$V_{14} = -\frac{1}{4f^2}s, \quad V_{15} = -\frac{1}{3\sqrt{2}f^2}m_\pi^2, \quad V_{22} = -\frac{1}{2f^2}m_\pi^2,$$
$$V_{23} = -\frac{1}{4\sqrt{2}f^2}s, \quad V_{24} = -\frac{1}{4\sqrt{2}f^2}s, \quad V_{25} = -\frac{1}{6f^2}m_\pi^2,$$
$$V_{33} = -\frac{1}{2f^2}s, \quad V_{34} = -\frac{1}{4f^2}s,$$
$$V_{35} = -\frac{1}{12\sqrt{2}f^2}(9s-6m_\eta^2-2m_\pi^2), \quad V_{44} = -\frac{1}{2f^2}s,$$
$$V_{45} = -\frac{1}{12\sqrt{2}f^2}(9s-6m_\eta^2-2m_\pi^2),$$
$$V_{55} = -\frac{1}{18f^2}(16m_K^2-7m_\pi^2), \tag{11}$$

where $f = 93$ MeV is the pion decay constant, and m_π, m_K, and m_η are the averaged masses of the pion, kaon, and η mesons, respectively.

Note that the chiral unitary approach has a limit in that we can only make reliable predictions up to 1.1–1.2 GeV. Since we are only interested in the region below 1.1 GeV, we can use Eq. (19) in Ref. [37] to smoothly extrapolate the GT amplitude above the energy cut $\sqrt{s_{\mathrm{cut}}} = 1.1$ GeV,

$$G(s)T(s) = G(s_{\mathrm{cut}})T(s_{\mathrm{cut}})e^{-\alpha(\sqrt{s}-\sqrt{s_{\mathrm{cut}}})}, \tag{12}$$

where α is a parameter that we can determine by fitting to the experimental data.

The $\pi^0\pi^0$ pair can be produced directly in the s-wave, and the amplitude of the process $D_s^+ \to \pi^+(\pi^0\pi^0)_S$ can be simply written as

$$\mathcal{M}_{(\pi^0\pi^0)_S} = \mathcal{D}_{(\pi^0\pi^0)_S}e^{i\alpha_{(\pi^0\pi^0)_S}}, \tag{13}$$

where $\mathcal{D}_{(\pi^0\pi^0)_S}$ and $\alpha_{(\pi^0\pi^0)_S}$ are parameters of the normalization constant and phase, respectively, and will be determined by fitting to the experimental data.

In the s-wave, we also take into account the possibility that the $s\bar{s}$ pair can form the $f_0(1370)$, and the $f_0(1370)$ will decay into the $\pi^0\pi^0$ pair. The process is depicted in Fig. 3(a), and the amplitude can be described by the formula

$$\mathcal{M}_{f_0(1370)}(s_{23}) = \frac{\mathcal{D}_{f_0(1370)}e^{i\alpha_{f_0(1370)}}}{s_{12} - M_{f_0(1370)}^2 + iM_{f_0(1370)}\Gamma_{f_0(1370)}}, \tag{14}$$

where $\mathcal{D}_{f_0(1370)}$, $\alpha_{f_0(1370)}$ are parameters of the process $D_s^+ \to \pi^+ f_0(1370) \to \pi^+\pi^0\pi^0$ and $\Gamma_{f_0(1370)}$ is the total width of $f_0(1370)$, taken as $\Gamma_{f_0(1370)} = 300$ MeV in Ref. [38].

Except for the production in the s-wave, the $\pi^0\pi^0$ pair can also be produced by the decay of a vector meson in the p-wave. We consider the amplitude for $D_s^+ \to \rho(770)^+\pi^0 \to \pi^+\pi^0\pi^0$, as depicted in Fig. 3(b), which is given by [24]

$$\mathcal{M}_\rho(s_{12}, s_{23}) = \mathcal{D}_\rho e^{i\alpha_\rho}\left[\frac{s_{12}-s_{23}}{s_{13}-M_\rho^2+iM_\rho\Gamma_\rho} + \frac{s_{13}-s_{23}}{s_{12}-M_\rho^2+iM_\rho\Gamma_\rho}\right], \tag{15}$$

where $\mathcal{D}_\rho$ and α_ρ are parameters, and $\Gamma_\rho = 150.2$ MeV. Although there are three s_{ij} variables, only two of them are independent and fulfill the following relation,

$$s_{12} + s_{13} + s_{23} = M_{D_s^+}^2 + m_{\pi^+}^2 + m_\pi^2 + m_\pi^2. \tag{16}$$

Similarly, the $\pi^0\pi^0$ pair can be produced directly in the d-wave via the process $D_s^+ \to \pi^+(\pi^0\pi^0)_D$ with the amplitude written as

$$\mathcal{M}_{(\pi^0\pi^0)_D}(s_{12}, s_{23}) = \mathcal{D}_{(\pi^0\pi^0)_D}e^{i\alpha_{(\pi^0\pi^0)_D}}\left[\left(\frac{s_{12}-2m_\pi^2}{2}\right)^2 + \left(\frac{s_{13}-2m_\pi^2}{2}\right)^2\right], \tag{17}$$

where $\mathcal{D}_{(\pi^0\pi^0)_D}$ and $\alpha_{(\pi^0\pi^0)_D}$ are parameters.

In the d-wave, the $\pi^0\pi^0$ production will proceed via the decay of $f_2(1270)$ and $f_2(1430)$, as shown in Fig. 3(c). The decay amplitude is

$$\mathcal{M}_{f_2}(s_{12}, s_{23}) = \frac{\mathcal{D}_{f_2} e^{i\alpha_{f_2}}}{s_{23} - M_{f_2}^2 + iM_{f_2}\Gamma_{f_2}} \frac{1}{12M_{f_2}^2} \{M_{f_2}^2[-16m_\pi^4 + 4m_\pi^2 s_{23} + 3(s_{12} - s_{13})^2] + (4m_\pi^2 - s_{23})(-4m_\pi^2 + s_{12} + s_{13})^2\}, \tag{18}$$

where $\mathcal{D}_{f_2}$ and α_{f_2} are the parameters of the decay of $f_2(1270)$ or $f_2(1430)$. In addition, Γ_{f_2} is the total width, taken as $\Gamma_{f_2(1270)} = 1275.5$ MeV and $\Gamma_{f_2(1430)} = 1430$ MeV.

With the considerations above, we obtain the total amplitude of $D_s^+ \to \pi^+\pi^0\pi^0$ as

$$t'(s_{12}, s_{23}) = t(s_{23}) + \mathcal{M}_{(\pi^0\pi^0)_S}(s_{12}, s_{23}) + \mathcal{M}_{f_0(1370)}(s_{23}) + \mathcal{M}_{\rho(770)^+}(s_{12}, s_{23}) + \mathcal{M}_{(\pi^0\pi^0)_D}(s_{12}, s_{23}) + \mathcal{M}_{f_2(1270)}(s_{12}, s_{23}) + \mathcal{M}_{f_2(1430)}(s_{12}, s_{23}). \tag{19}$$

Finally, considering that there are only two independent invariant masses, we use the formula of double differential width for three-body decay [38] as

$$\frac{d^2\Gamma}{dM_{12}dM_{23}} = \frac{1}{(2\pi)^3} \frac{1}{2} \frac{M_{12}M_{23}}{8m_{D_s^+}^3} |t'(s_{12}, s_{23})|^2. \tag{20}$$

Then, one can obtain the single differential mass distribution by integrating over another invariant mass. To integrate over M_{23}, the limits of integration in PDG [38] are given by

$$(M_{23})^2_{\max} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2}\right)^2, \tag{21}$$

$$(M_{23})^2_{\min} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2}\right)^2, \tag{22}$$

where E_2^* and E_3^* are energies in the $\pi^+\pi^0$ rest frame,

$$E_2^* = \frac{s_{12} - m_1^2 + m_2^2}{2\sqrt{s_{12}}}, \tag{23}$$

$$E_3^* = \frac{m_{D_s^+}^2 - s_{12} - m_3^2}{2\sqrt{s_{12}}}. \tag{24}$$

To integrate over M_{12}, the limits of integration are given by

$$(M_{12})^2_{\max} = (E_2^* + E_1^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_1^{*2} - m_1^2}\right)^2, \tag{25}$$

$$(M_{12})^2_{\min} = (E_2^* + E_1^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_1^{*2} - m_1^2}\right)^2, \tag{26}$$

where E_2^* and E_1^* are energies in the $\pi^0\pi^0$ rest frame,

$$E_2^* = \frac{s_{23} - m_3^2 + m_2^2}{2\sqrt{s_{23}}}, \tag{27}$$

$$E_1^* = \frac{m_{D_s^+}^2 - s_{23} - m_1^2}{2\sqrt{s_{23}}}. \tag{28}$$

III. RESULTS

In our calculations above, we introduced seven amplitudes with a total of 14 parameters. In Fig. 4, we determine the parameters by simultaneously fitting the $M_{\pi^0\pi^0}$ and $M_{\pi^+\pi^0}$ distributions of $D_s^+ \to \pi^+\pi^0\pi^0$ from the BESIII data using the model above, and the result is shown in Table I. In contrast to the experiment, two additional contributions, $\rho(770)^+$ and $f_2(1430)$, have been introduced in our model. Their branching ratios are smaller than that of the main contributions and are ignored in the experimental figures. For a more meaningful comparison with the experimental data, we smear the theoretical curve with bin width and choose the Gaussian function for smearing. For the original function $f(x)$, the smeared function $\tilde{f}(x)$ is

$$\tilde{f}(x) = \int_{y_{\min}}^{y_{\max}} dy\, g(|x - y|) f(y), \tag{29}$$

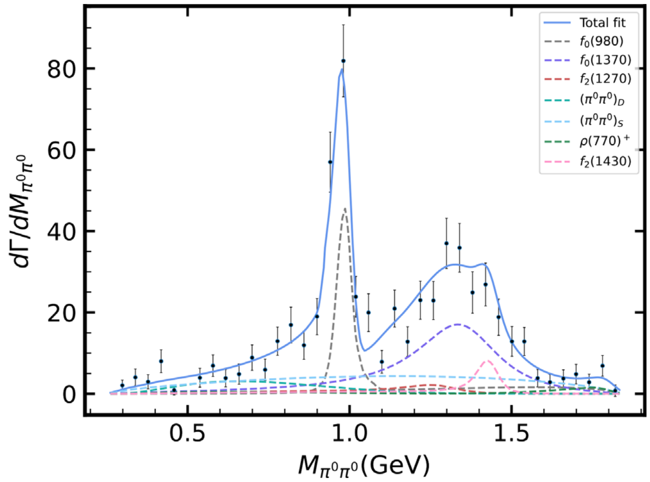
with

$$g(x) = \frac{1}{\sqrt{\pi}\sigma} \exp\left(-\frac{x^2}{\sigma^2}\right), \tag{30}$$

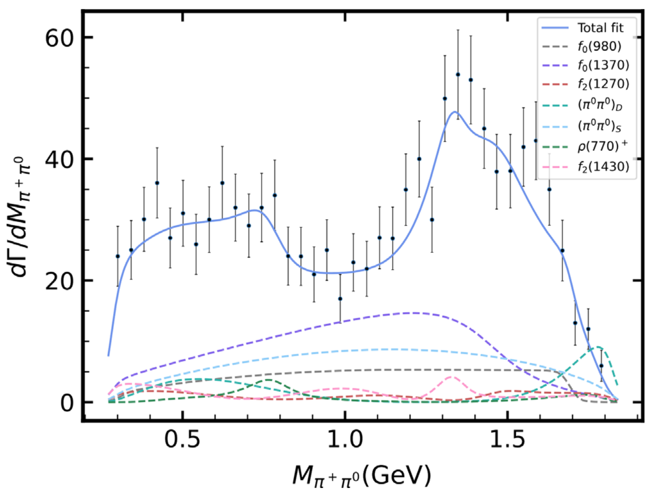
where $\sigma = \frac{\sqrt{2}}{2}\Delta$, with Δ the bin width. The ranges of integration are (after checking convergence)

$$y_{\min} \sim x - 3\sigma, \quad y_{\max} \sim x + 3\sigma. \tag{31}$$

For the region above 1.1 GeV in the s-wave, we smoothly extrapolate the amplitude above the energy cut instead of the cutoff directly. For the impact of the model on the uncertainty below 1.1 GeV in the s-wave, see the discussions in Refs. [37,39], where the influence on



(a) $M_{\pi^0\pi^0}$ distribution for $D_s^+ \to \pi^+\pi^0\pi^0$ decay with $\chi^2/dof. = 54.63/38 = 1.44$.



(b) $M_{\pi^+\pi^0}$ distribution for $D_s^+ \to \pi^+\pi^0\pi^0$ decay with $\chi^2/dof. = 27.33/38 = 0.72$.

FIG. 4. Combined fit to (a) $M_{\pi^0\pi^0}$ distribution and (b) $M_{\pi^+\pi^0}$ distribution for $D_s^+ \to \pi^+\pi^0\pi^0$ decay. The curve labeled as "Total fit" shows the result of the full model. The dashed curves are the results of the component contributions. The data are from Ref. [27].

$f_0(980)$ below 1.1 GeV is very small. Therefore, we can obtain the smoothing extrapolation parameter by fitting to the experimental data.

In Fig. 4(a), we present the $M_{\pi^0\pi^0}$ distribution of the D_s^+ decay. One can clearly see the peak at 0.98 GeV, corresponding to the dynamically generated resonance $f_0(980)$. On the basis of the original model, we take into account the experimental resolution in energy, so the value of the $f_0(980)$ peak is in better agreement with the experiment. Another noteworthy point is that the contribution of $f_0(1370)$ dominates at $M_{\pi^0\pi^0}$ higher than 1.1 GeV, while the peak of $f_2(1270)$ is not as sharp. Except for a discrepancy around 1.43 GeV, the overall shape of the invariant mass distribution is in good agreement with experimental data since the additional $f_2(1430)$ contribution leads to a small peak at 1.43 GeV in the $M_{\pi^0\pi^0}$ distribution. Then, in Fig. 4(b), we depict the $M_{\pi^+\pi^0}$ distribution. Obviously, $f_0(1370)$ is the main contribution, and $f_2(1430)$ has a certain impact on the peak of the total fit curve around 1.35 GeV.

TABLE I. Parameters and fit values for the full model obtained from fitting the BESIII data [27].

Parameters	Fit values	Parameters	Fit values
$\mathcal{D}$	783.1	α	-0.974 GeV^{-1}
$\mathcal{D}_{f_0(1370)}$	253.4 GeV^2	$\alpha_{f_0(1370)}$	3.535
$\mathcal{D}_{(\pi^0\pi^0)_S}$	306.5	$\alpha_{(\pi^0\pi^0)_S}$	-4.700
$\mathcal{D}_\rho$	-12.15	α_ρ	9.507
$\mathcal{D}_{f_2(1270)}$	150.0 GeV^{-2}	$\alpha_{f_2(1270)}$	-8.803
$\mathcal{D}_{f_2(1430)}$	166.7 GeV^{-2}	$\alpha_{f_2(1430)}$	4.368
$\mathcal{D}_{(\pi^0\pi^0)_D}$	-29.75 GeV^{-4}	$\alpha_{(\pi^0\pi^0)_D}$	1.002

In Fig. 5, we present the $M_{\pi^+\pi^0}$ distribution for the full model with the $f_2(1430)$ or $f_2(1270)$ contribution removed. In the region of 1.3–1.6 GeV, the curve is significantly lower than the experimental data point if the $f_2(1430)$ contribution is removed from the full model, and the fit effect of the curve near the edge of the phase space is worse than that with the $f_2(1430)$ contribution added. Therefore, the $f_2(1430)$ contribution is necessary in our model. For the contribution of $f_2(1270)$, the amplitude is very small in both Figs. 4(a) and 4(b). However, if we remove the $f_2(1270)$ contribution, one can see that, although the total fit curve is very close to the experimental data at about 1.35 GeV, it is not ideal in the region of 1.4–1.7 GeV. Thus, in our model, we choose

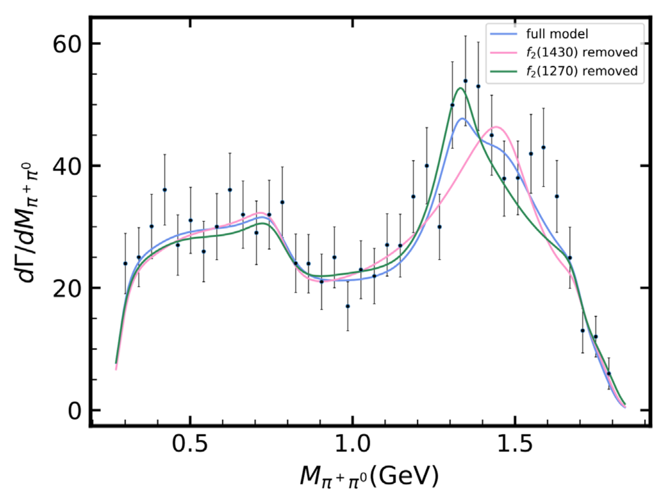


FIG. 5. The $M_{\pi^+\pi^0}$ distribution from combined fits for different models. The blue curve is for the full model, the pink curve is for the full model with the $f_2(1430)$ contribution removed, and the green curve is for the full model with the $f_2(1270)$ contribution removed. The data are from Ref. [27].

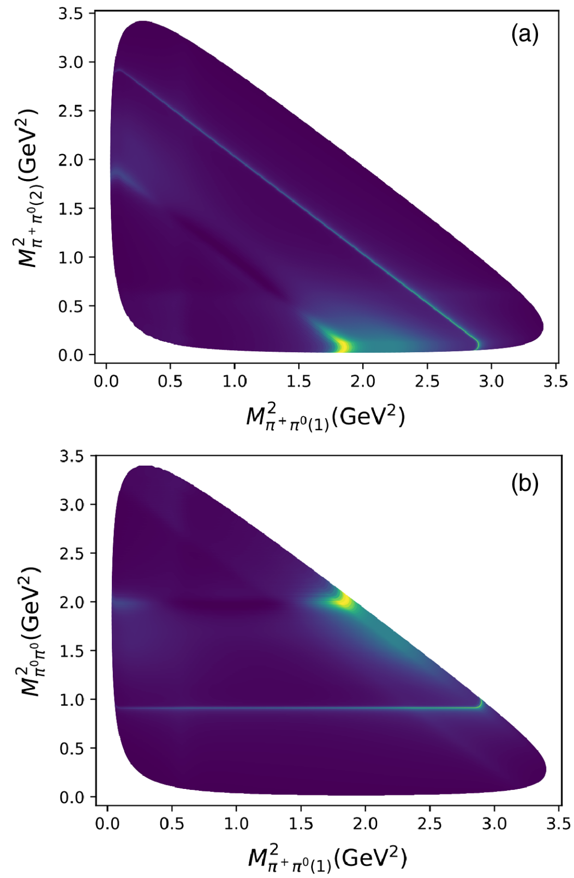


FIG. 6. The Dalitz plots for the $D_s^+ \to \pi^+\pi^0\pi^0$ decay. (a) $M^2_{\pi^+\pi^0(2)}$ vs $M^2_{\pi^+\pi^0(1)}$ and (b) $M^2_{\pi^0\pi^0}$ vs $M^2_{\pi^+\pi^0(1)}$.

to add the $f_2(1270)$ contribution to make the fit effect more consistent with the experimental data. Other contributions also appeared in the experiment—$(\pi^+\pi^0)_D$, $\rho(1450)$, and $f_0(1500)$—which we have tested, but the impact on the fit effect is so slight that we do not add them into our model.

We also show the Dalitz plots of "$M^2_{\pi^0\pi^0}$" versus "$M^2_{\pi^+\pi^0(1)}$" and "$M^2_{\pi^+\pi^0(2)}$" versus "$M^2_{\pi^+\pi^0(1)}$" in the D_s^+ process, depicted in Fig. 6, which can be used to test the model.

IV. CONCLUSION

We have given a theoretical study of $D_s^+ \to \pi^+\pi^0\pi^0$. In our model, we considered the external W-emission at quark level and hadronized it with $q\bar{q}$ pairs to produce pseudoscalar mesons, which finally decayed into the $\pi^+\pi^0\pi^0$ state. According to the chiral unitary approach, we focused on the interaction of pseudoscalar mesons, where $f_0(980)$ can be explained as a dynamically generated state in the s-wave. We also used contributions from the p-wave and d-wave resonances. In order to explore the consistency of our approach and experimental measurements, we investigated the $M_{\pi^0\pi^0}$ and $M_{\pi^+\pi^0}$ invariant mass distributions using the parameters mentioned in the formalism to fit the experimental data, and we found a clear peak of $f_0(980)$ in the $M_{\pi^0\pi^0}$ invariant mass distribution with no contribution of $f_0(500)$, which conformed to the theoretical prediction. Therefore, the chiral unitary approach is well applicable to this kind of process, providing support for explaining the nature of $f_0(980)$. Furthermore, we compared the full model with the model with the $f_2(1430)$ or $f_2(1270)$ contribution removed to verify the necessity of amplitudes in our model; we found that the full model fit the experimental data best.

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