Faddeev fixed-center approximation to the $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$ systems

Qing-Hua Shen[1,2,*] and Ju-Jun Xie[1,2,3,†]

[1]*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*
[2]*School of Nuclear Sciences and Technology, University of Chinese Academy of Sciences, Beijing 101408, China*
[3]*Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, Lanzhou University, Lanzhou, Gansu 730000, China*

(Received 15 November 2022; accepted 6 February 2023; published 22 February 2023)

The three-body $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$ systems are investigated within the framework of fixed-center approximation to the Faddeev equations, where $K^*\bar{K}^*$ is treated as the scalar meson $f_0(1710)$. The interactions between π, η, K, and K^* are taken from the chiral unitary approach. By scattering the η meson on the clusterized $(K^*\bar{K}^*)_{f_0(1710)}$ system, we find a peak in the modulus squared of the three-body scattering amplitude and it can be associated as a bound state with quantum numbers $I^G(J^{PC}) = 0^+(0^{-+})$. Its mass and width are around 2054 and 60 MeV, respectively. This state could be associated with the $\eta(2100)$ meson. For the $\pi(K^*\bar{K}^*)_{f_0(1710)}$ scattering, we find a bump structure around 1900–2000 MeV with quantum numbers $1^-(0^{-+})$, while for the $K(K^*\bar{K}^*)_{f_0(1710)}$ system, there are three structures. One of them is quite stable and its mass is about 2130 MeV. It is expected that these theoretical predictions here could be tested by future experimental measurements, such as by the BESIII, BelleII, and LHCb Collaborations.

DOI: 10.1103/PhysRevD.107.034019

I. INTRODUCTION

It is known that quark models have achieved great success in studying the properties of hadrons, especially for these ground states. Within the quark models, it is commonly accepted that mesons are composed of quark and antiquark ($q\bar{q}$) and baryons are composed of three quarks (qqq). Recently, the topic of meson-meson and meson-baryon states, with hadrons and hadrons governed by strong interactions, has been well developed by the combination of the chiral effective Lagrangians with non-perturbative unitary techniques in coupled channels, which has been a very fruitful scheme to study the nature of many hadronic states, both on light and heavy sectors [1–4]. Some of them are not easily explained by the classical quark models. For example, a state with exotic quantum numbers $J^{PC} = 1^{-+}$, denoted as $\eta_1(1855)$, is observed by the BESIII Collaboration [5,6], which cannot be explained by traditional $q\bar{q}$ picture. However, it can be easily obtained in the molecular picture. In Refs. [7,8], the $\eta_1(1855)$ is interpreted as a $\bar{K}K_1(1400)$ molecular state. In Ref. [9], the $\eta_1(1855)$ state was investigated by the method of QCD sum rules and the results support the interpretation of the $\eta_1(1855)$ as a $\bar{s}sg$ hybrid meson.

On the other hand, those states with exotic quantum numbers can be dynamically generated from the three-body interactions. In Ref. [10], the meson $\pi_1(1600)$, with exotic quantum numbers $I^G(J^{PC}) = 1^-(1^{-+})$, is interpreted as a dynamically generated state of the $\pi K^*\bar{K}$ system by using the fixed-center approximation (FCA), where the two-body $\bar{K}K^*$ is fixed as $f_1(1285)$ state [11–15]. Similarly, in Ref. [16], possible exotic states with mass around 1700 MeV and quantum numbers $I^G(J^{PC}) = 0^+(1^{-+})$ in the $\eta K^*\bar{K}$ system were predicted.

Indeed, there is growing evidence that some existing and new observed hadronic states could be interpreted in terms of resonances or bound states of three hadrons [17–19]. Furthermore, some new states are also predicted in three-body systems [20–22]. For example, DKK and $DK\bar{K}$ systems were studied in Ref. [23] within FCA, where the evidence of a state with mass of about 2833–2858 MeV was found in the $DK\bar{K}$ system. In Ref. [22], $D^*D^*\bar{K}^*$ is studied by FCA, and they obtain bound states with isospin $I = 1/2$ in total spin $J = 0$, 1, and 2. These states are very narrow. In Ref. [24], the $KD\bar{D}$ system is studied using the Gaussian expansion method, minimizing the energy of the

*shenqinghua@impcas.ac.cn
†xiejujun@impcas.ac.cn

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP^3.

system. A new hidden-charm $K_{c\bar{c}}(4180)$ excited state was predicted. The same system is also studied in Ref. [25] within the framework of FCA and the results obtained are very similar. Besides, the FCA method have many achievements on the study of three-body systems [26–38]. Those successes give us confidence in the FCA method that we use in the present work to study the $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$ systems. In the framework of FCA, there is a cluster of two bound particles ($K^*\bar{K}^*$ in this work) and the third one (η, π, or K in this work) collides with the components of this cluster without modifying its wave function. Certainly, if the third particle is lighter than the constituents of the cluster, the approximation is better. Note that the η, π, and K mesons are much lighter than the $K^*/\bar{K}^*$ meson. Whatever it is, one does not know the accuracy of the FCA until a comparison is made with the results by the full Faddeev calculations.

For the $K^*\bar{K}^*$ subsystem, it has strong attraction between K^* and $\bar{K}^*$. In fact, the well-established scalar meson $f_0(1710)$ was proposed to be a state dynamically generated from the vector meson-vector meson interactions in coupled channels [39–43]. Within this picture, the $f_0(1710)$ resonance can be interpreted as a $K^*\bar{K}^*$ molecular state, and the properties of $f_0(1710)$ resonance and its production have been investigated in Refs. [44–50].

In this work, we investigate the three-body $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$ systems by considering the interactions of the three components among themselves, keeping in mind the expected strong correlations of the $K^*\bar{K}^*$ subsystem to make the $f_0(1710)$ state. Then, in terms of two-body pseudoscalar meson-vector meson scattering amplitudes, which were obtained from the chiral unitary approach [11,12], we solve the Faddeev equations for the η, π, and K scattering on the $K^*\bar{K}^*$ cluster in S wave by using the fixed-center approximation. In this way, it is quite probable to be able to generate pseudoscalar mesons with three-body nature. The above three-body systems have quantum numbers $J^{PC} = 0^{-+}$ and isospin $I = 0$, 1, and 1/2, respectively.

The paper is organized as follows. In the next section, we present the FCA method to the three-body $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$ systems. In Sec. III, our theoretical results and discussions are presented. Finally, a short summary follows.

II. FORMALISM AND INGREDIENTS

We are interested in three different systems constituted by three mesons: $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$. In order to study the dynamics of these systems we will solve the Faddeev equations using the fixed-center approximation to obtain the total scattering amplitudes. One basic feature of the FCA is that one has a cluster bound of two particles and one allows the multiple scattering of the third particle with this cluster, which is supposed not to be changed by the interaction of the third particle. In this section we will summarize the derivation of the three-body scattering amplitudes within the framework of the FCA.

A. Fixed-center approximation

We are going to use the fixed-center approximation formalism to study the three-body $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$ systems. In this framework, we consider the $K^*\bar{K}^*$ as a cluster, which we take as the $f_0(1710)$ state, and η, π, or K interacts with it. The corresponding diagrams are shown in Fig. 1. In the following we will label K^*, $\bar{K}^*$, and η (π or K) as particle 1, 2, and 3, respectively.

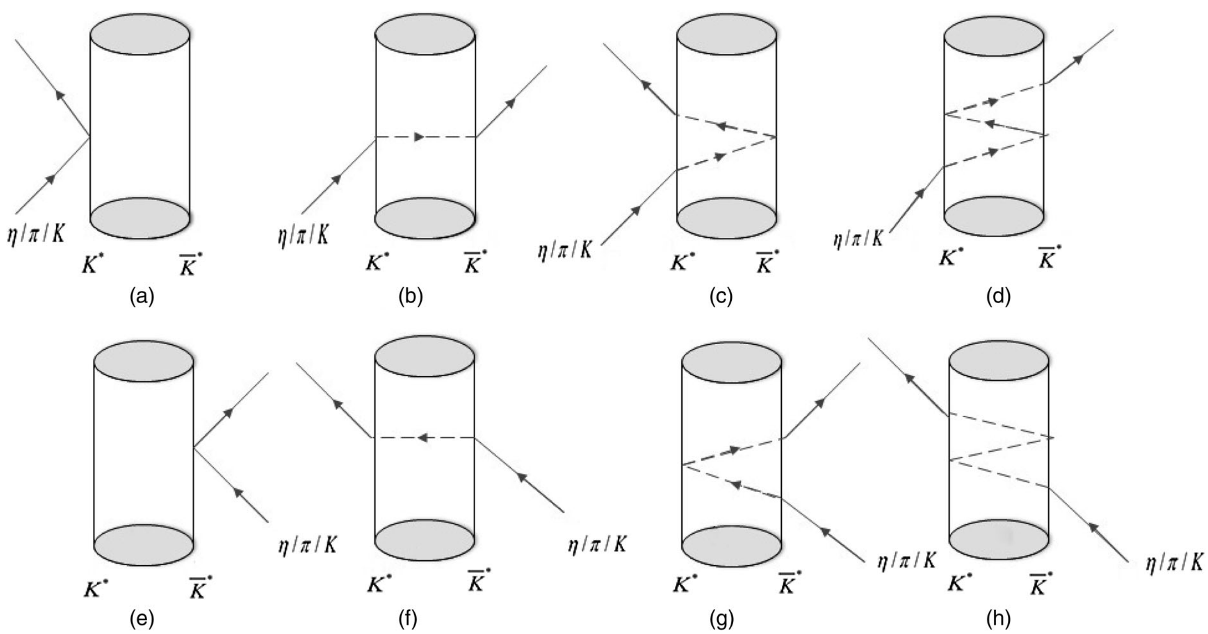

FIG. 1. Diagrammatic representation of FCA for the Faddeev equations for the three-body $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$ systems.

Following the formalism of Ref. [51], the three-body scattering amplitude T for η (π or K) collisions with the $K^*\bar{K}^*$ can be obtained by the sum of the partition functions T_1 and T_2:

$$T_1 = t_1 + t_1 G_0 T_2, \tag{1}$$

$$T_2 = t_2 + t_2 G_0 T_1, \tag{2}$$

$$T = T_1 + T_2 = \frac{t_1 + t_2 + 2t_1 t_2 G_0}{1 - t_1 t_2 G_0^2}, \tag{3}$$

where T_1 stands for the sum of all the diagrams in Fig. 1 where the third particle collides firstly with the particle K^* in the cluster, while T_2 is the sum of all the diagrams where the third particle collides firstly with particle $\bar{K}^*$ of the cluster. The t_1 and t_2 represent the unitary two-body scattering amplitudes in coupled channels for the interactions of particle 3 with particle 1 and 2, respectively. These two-body scattering amplitudes will be discussed later on.

Besides, in the above equations, G_0 is the loop function for the particle 3 propagating between the K^* and $\bar{K}^*$ of the cluster, which can be written as

$$G_0(s) = \frac{1}{2M_{f_0(1710)}} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{F_{f_0(1710)}(q)}{q^{0^2} - |\vec{q}|^2 - m_3^2 + i\epsilon}, \tag{4}$$

where s is the invariant mass square of the whole three-body system, and $M_{f_0(1710)}$ is the mass of the bound state $f_0(1710)$. Besides, $F_{f_0(1710)}(q)$ is the form factor of the $K^*\bar{K}^*$ subsystem, which is treated as the $f_0(1710)$ state. The q^0 is the energy of particle 3 with mass m_3 in the center-of-mass frame of particle 3 and the $(K^*\bar{K}^*)_{f_0(1710)}$ cluster, which is given by

$$q^0(s) = \frac{s + m_3^2 - M_{f_0(1710)}^2}{2\sqrt{s}}. \tag{5}$$

Following the approach of Refs. [51–53], one can easily get the expression of the form factor $F_{f_0(1710)}$ for the S-wave $K^*\bar{K}^*$ bound state $f_0(1710)$ as

$$\begin{aligned} F_{f_0(1710)}(q) = &\frac{1}{N}\int_{|\vec{p}|\le\Lambda, |\vec{p}-\vec{q}|\le\Lambda} d^3\vec{p} \frac{1}{2\omega_1(\vec{p})} \frac{1}{2\omega_2(\vec{p})} \\ &\times \frac{1}{M_{f_0(1710)} - \omega_1(\vec{p}) - \omega_2(\vec{p})} \\ &\times \frac{1}{2\omega_1(\vec{p}-\vec{q})} \frac{1}{2\omega_2(\vec{p}-\vec{q})} \\ &\times \frac{1}{M_{f_0(1710)} - \omega_1(\vec{p}-\vec{q}) - \omega_2(\vec{p}-\vec{q})}, \end{aligned} \tag{6}$$

where $\omega_1(\vec{p}) = \omega_2(\vec{p}) = \sqrt{|\vec{p}|^2 + m_{K^*}^2}$, and the normalization factor N is given by

$$N = \int_{|\vec{p}|\le\Lambda} d^3\vec{p} \left(\frac{1}{4\omega_1^2(\vec{p})} \frac{1}{M_{f_0(1710)} - 2\omega_1(\vec{p})} \right)^2. \tag{7}$$

The cutoff parameter Λ is needed to regularize the vector meson–vector meson loop functions in the chiral unitary approach [39,40]. In the present work, the upper-integration limit of Λ has the same value of the cutoff used in Refs. [39,40]. With the values of Λ, one can get the $f_0(1710)$ state in the vector meson–vector meson interactions in coupled channels.

In Fig. 2, we show numerical results of the form factor $F_{f_0(1710)}$ as a function of $q = |\vec{q}|$, where the solid and dashed curves are obtained with $\Lambda = 945$ and 1125 MeV, respectively. Note that with these values for cutoff parameter Λ, one can get the $f_0(1710)$ state in the vector-vector interactions in coupled channels. Besides, the integration condition $|\vec{p} - \vec{q}| < \Lambda$ implies that the form factor of $F_{f_0(1710)}$ is exactly zero when $q > 2\Lambda$.

B. Unitarized $K^*\bar{K}^*$ interaction for the $f_0(1710)$ state and the loop function G_0

The $K^*\bar{K}^*$ interaction in S wave has been analyzed within the formalism developed in Ref. [39] for studying the interaction of the nonet of vector mesons among themselves. In Fig. 3, the module square of $t_{K^*\bar{K}^*\to K^*\bar{K}^*}$ obtained from the chiral unitary approach in $\rho\rho$, $\omega\omega$, $\omega\phi$, $\phi\phi$, and $K^*\bar{K}^*$ coupled channels for isospin $I = 0$ sector, are shown, where one can see that there is a clear peak for $f_0(1710)$ state around 1704 MeV with $\Lambda = 1125$ MeV, while the peak moves to 1733 MeV if one takes $\Lambda = 945$ MeV. Note that in the calculations of these

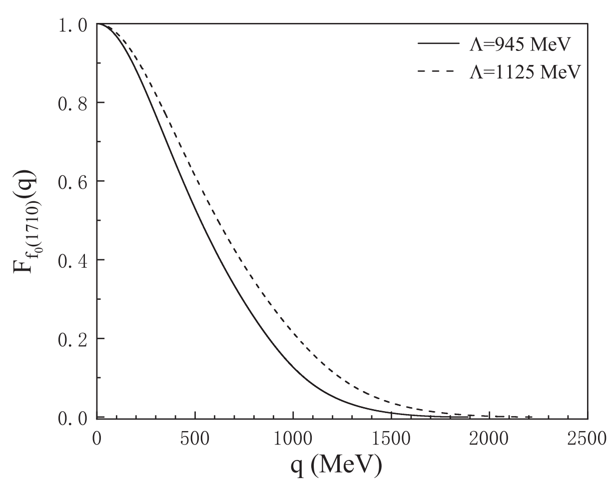

FIG. 2. Form factor of $(K^*\bar{K}^*)_{f_0(1710)}$ cluster with $\Lambda = 945$ MeV (solid line) and $\Lambda = 1125$ MeV (dashed line).

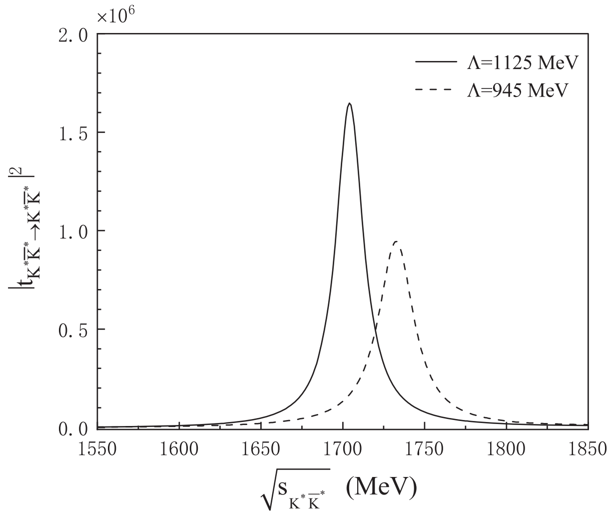

FIG. 3. Modulus squared of $t_{K^*\bar{K}^*\to K^*\bar{K}^*}$ as a function of the invariant mass of $K^*\bar{K}^*$ system with $\Lambda = 945$ MeV (dashed line) and $\Lambda = 1125$ MeV (solid line).

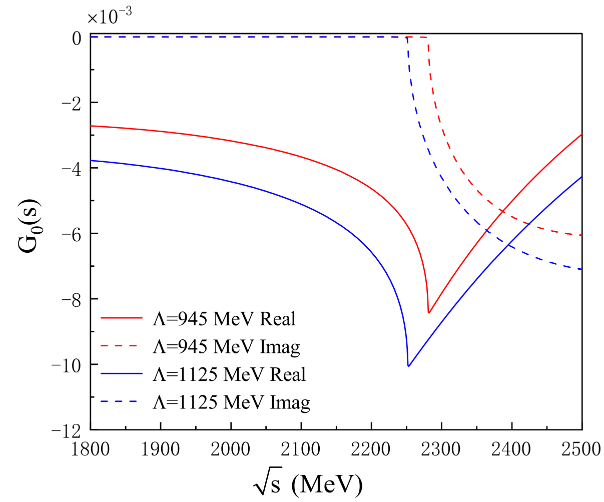

FIG. 4. Real (solid line) and imaginary (dashed line) parts of the loop function G_0 for $\eta K^*\bar{K}^*$ system with the cutoff parameter $\Lambda = 945$ MeV (red line) and $\Lambda = 1125$ MeV (blue line).

two-body loop functions of $G_{\rho\rho}$ and $G_{K^*\bar{K}^*}$, the widths of vector mesons ρ and K^* are taken into account, since they have large total decay widths. On the other hand, in the calculations for the form factor of $F_{f_0(1710)}(q)$, the mass of cluster $f_0(1710)$ is taken as 1704 MeV when $\Lambda = 1125$ MeV, while $M_{f_0(1710)} = 1733$ if one takes $\Lambda = 945$ MeV. As pointed out in Ref. [39], the $f_0(1710)$ resonance is dynamically generated from the vector-vector coupled interactions, with a small decay width. But, these two pseudoscalars are its decay channels. By including these effects of pseudoscalar-pseudoscalar channels, the obtained width of $f_0(1710)$ resonance is compatible with the experimental result. Meanwhile, the obtained mass of it does not change much (see more details in Ref. [39]). It is worthy to mention that the "our evaluation" value in the 2020 version and 2021 updated of the *Review of Particle Physics* (RPP) [54] for the mass of $f_0(1710)$ resonance is 1704 ± 12 MeV, while the "our average" value for it is 1733^{+8}_{-7} MeV. In fact, even though the $f_0(1710)$ resonance is well established in the RPP, there are still many doubts about its nature [55]. We refer to the *Review of Particle Physics* [54] for more details.

Next, we present the numerical results for the loop function G_0, which is the η (π and K) propagator between the K^* and $\bar{K}^*$ of the cluster $f_0(1710)$. Note that G_0 is dependent on the invariant mass $\sqrt{s}$ of the three-body system. In this work, there are three G_0 functions for η-$(K^*\bar{K}^*)_{f_0(1710)}$, π-$(K^*\bar{K}^*)_{f_0(1710)}$, and K-$(K^*\bar{K}^*)_{f_0(1710)}$ systems. In Fig. 4, we show the real and imaginary parts of the G_0 function for $I_{K^*\bar{K}^*} = 0$ as a function of the total three-body η-K^*-$\bar{K}^*$ system invariant mass. The other two cases can be easily obtained by the replacements of $\eta \to \pi$ and $\eta \to K$, respectively. They are shown in Figs. 5 and 6.

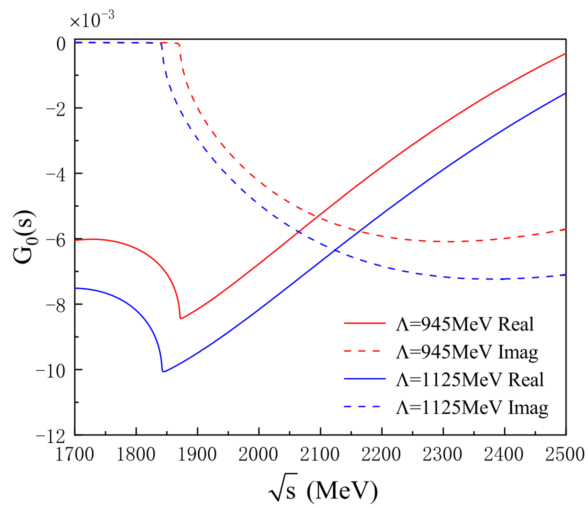

FIG. 5. As in Fig. 4 but for the $\pi K^*\bar{K}^*$ system.

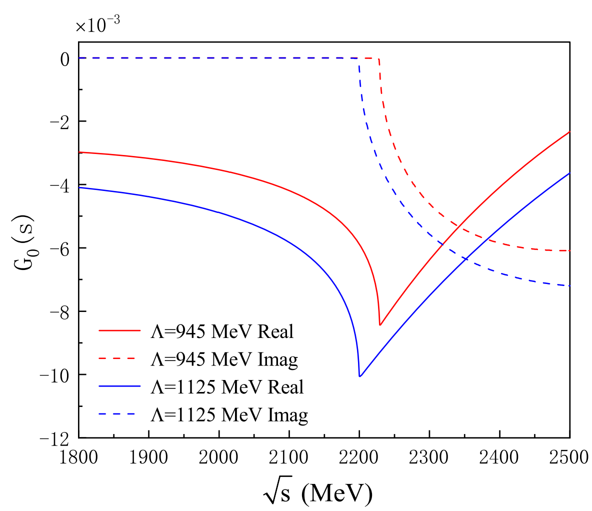

FIG. 6. As in Fig. 4 but for the $KK^*\bar{K}^*$ system.

C. Contributions for the η, π, and K interaction with the $K^*\bar{K}^*$ system

According to Figs. 1(a) and 1(e), the single-scattering contribution from t_1 and t_2 are appropriate for the combination of the two-body unitarized scattering amplitudes. For example, let us firstly consider the $\eta K^*\bar{K}^*$ system with the $K^*\bar{K}^*$ cluster in $I=0$ for the $f_0(1710)$ state. The $K^*\bar{K}^*$ isospin state is written as

$$|K^*\bar{K}^*\rangle_{I=0} = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2},-\frac{1}{2}\right\rangle - \left|-\frac{1}{2},\frac{1}{2}\right\rangle\right), \tag{8}$$

where the kets on the right side of the above equation represent $|I_z^{K^*}, I_z^{\bar{K}^*}\rangle$ for $K^*\bar{K}^*$.

Then, the single-scattering contributions to the total amplitude of $\langle \eta K^*\bar{K}^*|\hat{t}|\eta K^*\bar{K}^*\rangle$ can be easily obtained in terms of unitary two-body transition amplitudes $t_{\eta K^*\to\eta K^*}$ and $t_{\eta\bar{K}^*\to\eta\bar{K}^*}$ derived in Ref. [12]. Here, we write explicitly the case of $\eta(K^*\bar{K}^*)_{f_0(1710)} \to \eta(K^*\bar{K}^*)_{f_0(1710)}$,

$$\begin{aligned}
&\langle \eta(K^*\bar{K}^*)|\hat{t}|\eta(K^*\bar{K}^*)\rangle \\
&\quad = (\langle A_1| + \langle A_2|)(\hat{t}_{31} + \hat{t}_{32})(|A_1\rangle + |A_2\rangle) \\
&\quad = \langle A_1|\hat{t}_{31}|A_1\rangle + \langle A_2|\hat{t}_{32}|A_2\rangle,
\end{aligned} \tag{9}$$

where $|A_1\rangle$ stands for the state combined with η and K^*, while $|A_2\rangle$ is the state of η and $\bar{K}^*$. They are given by

$$\begin{aligned}
|A_1\rangle &= \frac{1}{\sqrt{2}}\left|\frac{1}{2}\frac{1}{2},-\frac{1}{2}\right\rangle - \frac{1}{\sqrt{2}}\left|\frac{1}{2}-\frac{1}{2},\frac{1}{2}\right\rangle, \\
|A_2\rangle &= \frac{1}{\sqrt{2}}\left|\frac{1}{2}-\frac{1}{2},\frac{1}{2}\right\rangle - \frac{1}{\sqrt{2}}\left|\frac{1}{2}\frac{1}{2},-\frac{1}{2}\right\rangle.
\end{aligned} \tag{10}$$

Thus, we have

$$\begin{aligned}
t_1 &= \langle A_1|\hat{t}_{31}|A_1\rangle \\
&= \frac{1}{2} t^{I=\frac{1}{2}}_{\eta K^*\to\eta K^*} + \frac{1}{2} t^{I=\frac{1}{2}}_{\eta K^*\to\eta K^*}, \\
&= t^{I=\frac{1}{2}}_{\eta K^*\to\eta K^*} \\
t_2 &= \langle A_2|\hat{t}_{32}|A_2\rangle \\
&= \frac{1}{2} t^{I=\frac{1}{2}}_{\eta\bar{K}^*\to\eta\bar{K}^*} + \frac{1}{2} t^{I=\frac{1}{2}}_{\eta\bar{K}^*\to\eta\bar{K}^*} \\
&= t^{I=\frac{1}{2}}_{\eta\bar{K}^*\to\eta\bar{K}^*}.
\end{aligned} \tag{11}$$

Similarly, for the case of $\pi K^*\bar{K}^*$, we have

$$\begin{aligned}
t_1 &= \langle A_1|\hat{t}_{31}|A_1\rangle = \frac{2}{3} t^{I=\frac{3}{2}}_{\pi K^*\to\pi K^*} + \frac{1}{3} t^{I=\frac{1}{2}}_{\pi K^*\to\pi K^*}, \\
t_2 &= \langle A_2|\hat{t}_{32}|A_2\rangle = \frac{2}{3} t^{I=\frac{3}{2}}_{\pi\bar{K}^*\to\pi\bar{K}^*} + \frac{1}{3} t^{I=\frac{1}{2}}_{\pi\bar{K}^*\to\pi\bar{K}^*}.
\end{aligned} \tag{12}$$

Next, for the case of $KK^*\bar{K}^*$, we can get the following results:

$$t_1 = \langle A_1|\hat{t}_{31}|A_1\rangle = \frac{3}{4} t^{I=1}_{KK^*\to KK^*} + \frac{1}{4} t^{I=0}_{KK^*\to KK^*}, \tag{13}$$

$$t_2 = \langle A_2|\hat{t}_{32}|A_{32}\rangle = \frac{3}{4} t^{I=1}_{K\bar{K}^*\to K\bar{K}^*} + \frac{1}{4} t^{I=0}_{K\bar{K}^*\to K\bar{K}^*}. \tag{14}$$

On the other hand, following the approach developed in Refs. [34,51], we need to give a weight to t_1 and t_2 such that we have the right normalization for the fields of mesons. This is achieved by replacing

$$\begin{aligned}
t_1 &\to \tilde{t}_1 = \frac{M_{f_0(1710)}}{m_{K^*}} t_1, \\
t_2 &\to \tilde{t}_2 = \frac{M_{f_0(1710)}}{m_{\bar{K}^*}} t_2.
\end{aligned} \tag{15}$$

Finally, we have the three-body scattering amplitude:

$$T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2 G_0}{1 - \tilde{t}_1\tilde{t}_2 G_0^2}. \tag{16}$$

It is worth to mention that the three-body total scattering amplitude T is a function of the total invariant mass $\sqrt{s}$ of the three-body system, while the two-body scattering amplitudes t_1 and t_2 depend on the invariant masses $\sqrt{s_1}$ and $\sqrt{s_2}$, which are the invariant masses of η (π or K) and particle K^* ($\bar{K}^*$) inside the cluster of $f_0(1710)$. The arguments s_1 and s_2 are obtained as

$$\begin{aligned}
s_1 &= m^2_{\eta/\pi/K} + m^2_{K^*} + \frac{s - m^2_{\eta/\pi/K} - M^2_{f_0(1710)}}{2} \\
s_2 &= m^2_{\eta/\pi/K} + m^2_{\bar{K}^*} + \frac{s - m^2_{\eta/\pi/K} - M^2_{f_0(1710)}}{2},
\end{aligned} \tag{17}$$

where we have used $m_{K^*} = m_{\bar{K}^*}$.

III. NUMERICAL RESULTS

In this section we show the theoretical numerical results obtained for the scattering-amplitude square of the three-body $\eta K^*\bar{K}^*$, $\pi K^*\bar{K}^*$, and $KK^*\bar{K}^*$ systems with total isospin $I=0$, 1, and 1/2, respectively. We evaluate the three-body scattering amplitude T and associate the peaks in the modulus squared $|T|^2$ resonances. In addition, the masses of the particles considered in this work are shown in Table I. These values are taken from the *Review of Particle Physics* [54]. For the mass of $f_0(1710)$ state,

TABLE I. Particle masses (in MeV) used in this work.

m_K	m_π	m_η	$m_{\eta'}$	Γ_{K^*}	$M_{f_0(1710)}$ (eva.)
495.6455	138.04	547.862	957.78	49.1	1704
$m_{K^*/\bar{K}^*}$	m_ρ	m_ϕ	m_ω	Γ_ρ	$M_{f_1(1710)}$ (ave.)
893.1	775.26	1019.461	782.66	149.1	1733

both “evaluation mass“ (eva.) and “average mass” (ave.) are shown.

A. Three-body $\eta K^*\bar{K}^*$ system

In order to obtain two-body system scattering amplitude t_1 and t_2, we need the scattering amplitude of ηK^* and $\eta\bar{K}^*$, which were studied in Refs. [12,56]. In this work, we have considered also the effect of η' meson as done in Refs. [57,58]. Within the model parameters as used in Refs. [12,56]: $\mu = 900$ MeV and $\alpha_\mu = -1.85$, one can easily obtain the modulus squared of $t_{\eta K^*\to\eta K^*}$, which is shown in Fig. 7. One can see that there is a clear peak for the $K_1(1270)$ state [56], and the interaction of ηK^* is strong. Note that the widths of these vector mesons are taken into account in the calculations for the loop functions [39], and the effects of those channels including the η' meson are very small and could be safely neglected.

On the other hand, one can also get the transition amplitude $t_{\eta\bar{K}^*\to\eta\bar{K}^*}$. Since the $K^*\bar{K}^*$, ηK^*, and $\eta\bar{K}^*$ interactions are attractive and strong enough to generate bound states, it is natural to expect the existence of multihadron states composed of $K^*\bar{K}^*$ and η.

Then, we obtain the total three-body scattering amplitude $T_{\eta(K^*\bar{K}^*)_{f_0(1710)}\to\eta(K^*\bar{K}^*)_{f_0(1710)}}$. In Fig. 8 we show the modulus squared $|T_{\eta(K^*\bar{K}^*)_{f_0(1710)}\to\eta(K^*\bar{K}^*)_{f_0(1710)}}|^2$ for the $\eta(K^*\bar{K}^*)_{f_0(1710)} \to \eta(K^*\bar{K}^*)_{f_0(1710)}$ scattering amplitude. The picture shows a clear peak around 2100 MeV with cutoff parameter $\Lambda = 945$ MeV. If one takes $\Lambda = 1125$ MeV, the peak moves to 2054 MeV. This peak with quantum numbers $I^G(J^{PC}) = 0^+(0^{-+})$ could be interpreted as $\eta(2100)$ that has mass 2050^{+30+75}_{-24-26} MeV and width $250^{+36+181}_{-30-164}$ MeV. In addition, it is found that the peak is not sensitive to the cutoff parameter Λ.

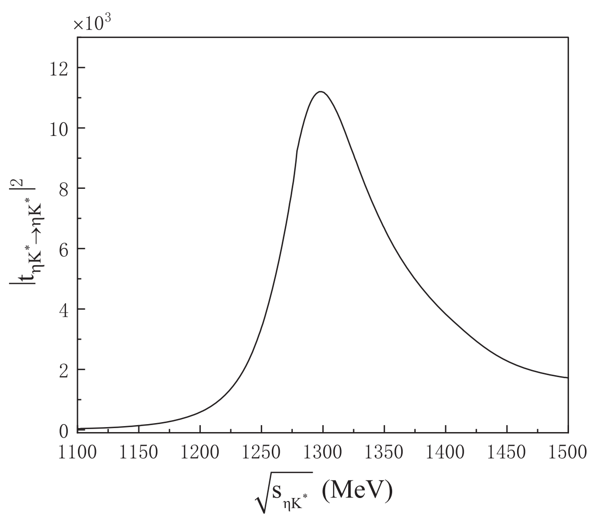

FIG. 7. Modulus squared of $t_{\eta K^*\to\eta K^*}$ as a function of the invariant mass of ηK^* system.

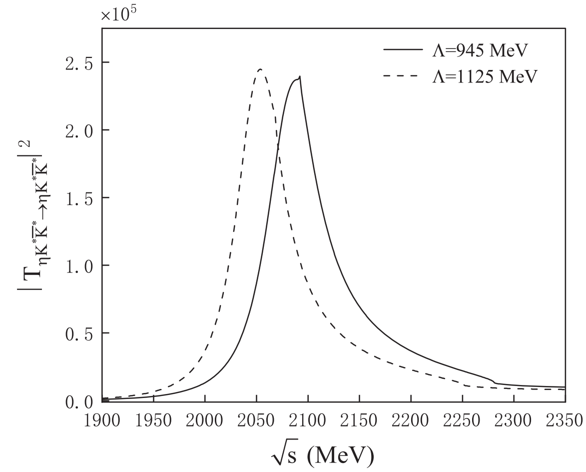

FIG. 8. Modulus squared of scattering amplitude for $\eta K^*\bar{K}^*$ three-body system with $\Lambda = 945$ MeV (solid line) and $\Lambda = 1125$ MeV (dashed line).

On the experimental side, the $\eta(2100)$ meson was observed firstly by the DM2 Collaboration in 1988 [59]. Then, it was also observed by the BESIII Collaboration in a partial wave analysis of the $J/\psi \to \gamma\phi\phi$ decay with 22σ significance [60]. However, this state is not listed in the summary meson tables of the RPP [54], which indicates more analysis of it is needed. Besides, in Ref. [61], the $\eta(2100)$ meson is interpreted as a candidate for $\eta(4S)$ of the fourth pseudoscalar meson nonet.

B. Three-body $\pi K^*\bar{K}^*$ system

In the case of $\pi K^*\bar{K}^*$ system, we need the two-body $\pi K^* \to \pi K^*$ scattering amplitude as inputs. Within the formula and theoretical parameters of Ref. [12], one can easily get the two-body scattering amplitude $t_{\pi K^*\to\pi K^*}$ in coupled channels. Note that we use the cutoff regularization scheme for the loop functions of the intermediate vector–pseudoscalar mesons, and the widths of the vector mesons are also considered. Furthermore, we take cutoff parameter $\Lambda_{\pi K^*} = 1000$ MeV to regularize the loop function containing a vector and a pseudoscalar meson.

The modulus squared of $t_{\pi K^*\to\pi K^*}$ in isospin $I = 1/2$ is shown in Fig. 9, from where one can see that there is a clear peak around 1150 MeV. As discussed in Ref. [12], it could be the lower pole of the two $K_1(1270)$ states.

The interaction of πK^* in isospin $I = 3/2$ is much weaker than the one with $I = 1/2$. The numerical results of $t_{\pi K^*\to\pi K^*}$ with $I = 3/2$ are shown in Fig. 10. One can see that the strength of $t^{I=3/2}_{\pi K^*\to\pi K^*}$ is much smaller than the one of $t^{I=1/2}_{\pi K^*\to\pi K^*}$ and there is no clear bump structure of the $|t^{I=3/2}_{\pi K^*\to\pi K^*}|^2$ in a wide energy region.

In Fig. 11, we show the obtained modulus squared of scattering amplitude $|T_{\pi(K^*\bar{K}^*)_{f_0(1710)}\to\pi(K^*\bar{K}^*)_{f_0(1710)}}|^2$ of the three-body $\pi K^*\bar{K}^*$ system. It is found that there is a wide bump structure around 1900–2000 MeV, which may be

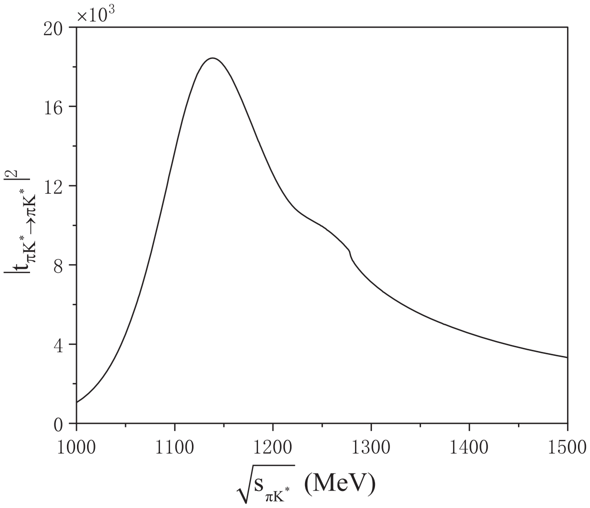

FIG. 9. Modulus squared of scattering amplitude of $t_{\pi K^*\to\pi K^*}$ in isospin $I = 1/2$ sector as the function of πK^* system.

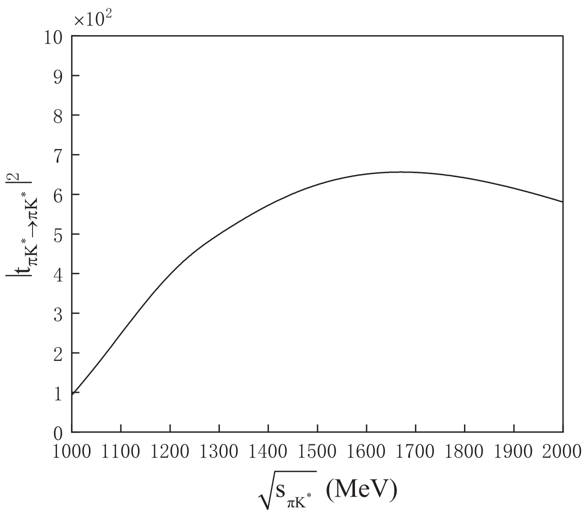

FIG. 10. Modulus squared of scattering amplitude of $t_{\pi K^*\to\pi K^*}$ in isospin $I = 3/2$ sector as the function of πK^* system.

associated with the $\pi(2070)$ state, which was not quoted in the RPP [54]. Around that energy region, the $\pi(2070)$ state was needed in a combined partial wave analysis of $\bar{p}p$ annihilation channels in Ref. [62], and its mass and width are about 2070 ± 35 MeV and 310^{+100}_{-50} MeV, respectively. The obtained width of $\pi(2070)$ state is rather wide. Thus, the mass of $\pi(2070)$ state needs to be precisely measured by further experiment since its large width will lead to difficulties for the measurements.

C. Three-body $KK^*\bar{K}^*$ system

The interaction between K and $\bar{K}^*$ is strong, and the $f_1(1285)$ state is dynamically generated from the $K\bar{K}^*$ interaction in S wave and isospin $I = 0$ sector [12], while in the $I = 1$ sector, the $a_1(1260)$ and $b_1(1235)$ can be

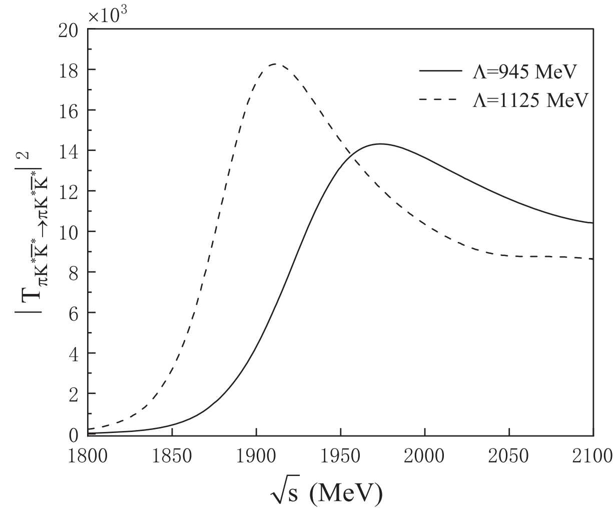

FIG. 11. Modulus squared of scattering amplitude for $\pi K^*\bar{K}^*$ three-body system with $\Lambda = 945$ MeV (solid line) and $\Lambda = 1125$ MeV (dashed line).

obtained also [12]. Within the same parameters as used in Ref. [12], the obtained modulus squared of $t_{K\bar{K}^*\to K\bar{K}^*}$ in $I = 0$ and $I = 1$ are shown in Figs. 12 and 13, respectively. From these figures, one can see that there are clearly the peaks for the $f_1(1285)$ and $a_1(1260)/b_1(1235)$ states, and the former peak is much narrower; yet, the later one is wide and the peak is superposition of $a_1(1260)$ and $b_1(1235)$, since in the calculations, we do not distinguish their G parity. Furthermore, the interaction of $K\bar{K}^*$ in $I = 1$ sector is much weaker than that in the $I = 0$ sector.

On the other hand, the interaction between K and K^* is even weaker, and the two-body scattering amplitude of $t_{KK^*\to KK^*}$ in $I = 0$ is zero. In the $I = 1$ sector, the scattering amplitude squared of $t_{KK^*\to KK^*}$ is shown in Fig. 14. The strength of $|t_{KK^*\to KK^*}|^2$ is much smaller than those shown in Figs. 12 and 13 for the case of $K\bar{K}^*$ scattering.

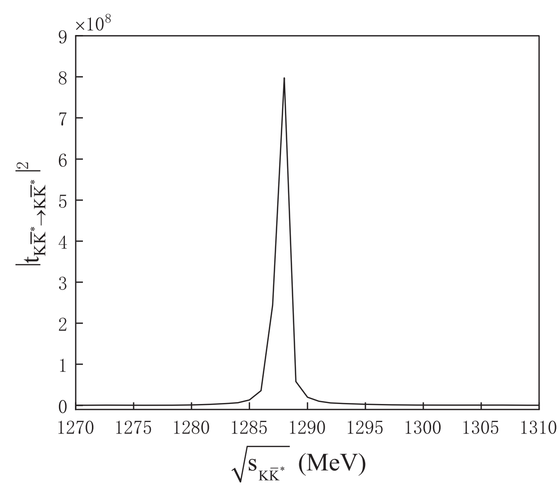

FIG. 12. Modulus squared of $t_{K\bar{K}^*\to K\bar{K}^*}$ in $I = 0$ as a function of the invariant mass of $K\bar{K}^*$ system.

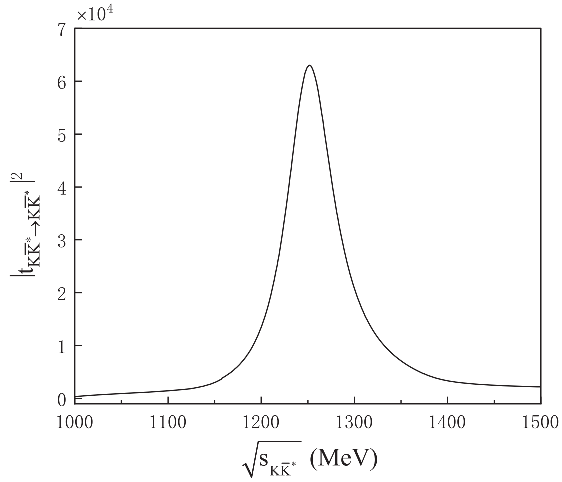

FIG. 13. Modulus squared of $t_{K\bar{K}^*\to K\bar{K}^*}$ in $I = 1$ as a function of the invariant mass of $K\bar{K}^*$ system.

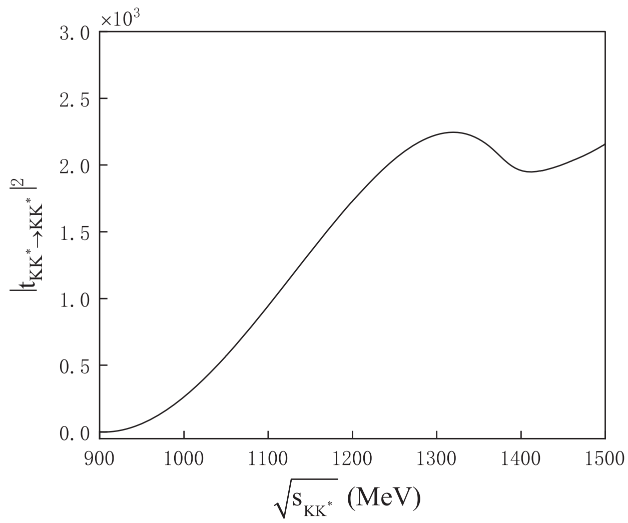

FIG. 14. Modulus squared of $t_{KK^*\to KK^*}$ in $I = 1$ as a function of the invariant mass of KK^* system.

In Fig. 15, the modulus squared of scattering amplitude of three-body $KK^*\bar{K}^*$ system is shown, where there are three peaks. The one around 2130 MeV is not sensitive to the cutoff parameter, while the other two are much dependent on the value of cutoff parameter Λ. The lowest one is located at about 2072 MeV, and the highest one is about 2219 MeV.

On the experimental side, there is a meson with strangeness around 2062 MeV and named $X(2075)$. It was observed by the BESII Collaboration in the $p\bar{\Lambda}$ invariant mass spectrum of the decay $J/\psi \to K^- p\bar{\Lambda}$ [63]. Its Breit-Wigner mass and width are $2075 \pm 12(\text{stat.}) \pm 5(\text{syst.})$ MeV and $90 \pm 35(\text{stat.}) \pm 9(\text{syst.})$ MeV, respectively. The spin of $X(2075)$ state could be 0 or 1. Note that as pointed out in Ref. [63], its interpretation as a conventional K^* meson would be disfavored. In addition, a similar

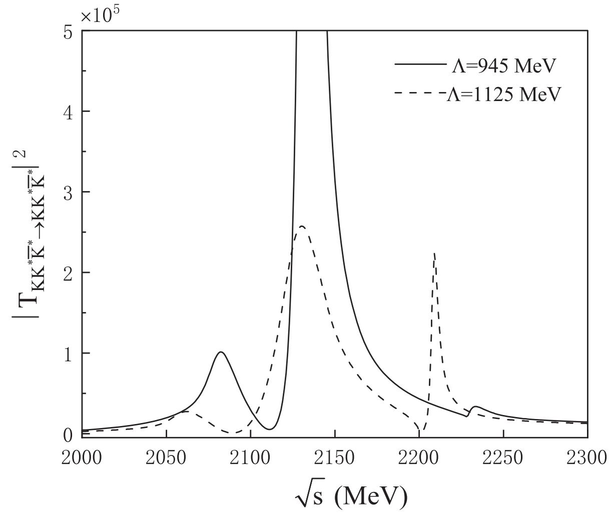

FIG. 15. Modulus squared of scattering amplitude for $KK^*\bar{K}^*$ three-body system with $\Lambda = 945$ MeV (solid line) and $\Lambda = 1125$ MeV (dashed line).

near-threshold enhancement in the $p\bar{\Lambda}$ system was also observed in $B^+ \to p\bar{\Lambda}D^0$ by the Belle Collaboration [64]. In fact, one can notice that there is a large blank space between 2000–3000 MeV in RPP for those strangeness states [54]. We look forward to further experiment that can give us more information about those mesons with a strange quark in this energy region.

In the FCA, we keep the cluster wave function unchanged by the presence of the third particle. In order to estimate uncertainties of the FCA due to this frozen condition we admit that the wave function of the cluster could be modified by the presence of the third particle, which is the normal situation in a full Faddeev calculation. In fact, the $\eta K^*\bar{K}^*$ and $\pi K^*\bar{K}^*$ systems may couple to each other or other three-body channels, while the $KK^*\bar{K}^*$ system could be mixed with, for example, $K\rho\bar{K}^*$ and $K\omega\bar{K}^*$ channels. However, the mass thresholds of such channels are far from the energy region we considered. Furthermore, including such contributions, the three-body scattering amplitude would become more complex due to additional parameters from the nondiagonal transitions, and we cannot determine or constrain these parameters. Hence, we will leave these contributions to future studies when more experimental data become available. For the sake of simplicity we do not include other channels in the present work.

IV. SUMMARY

In the framework of fixed-center approximation, we have performed a calculation for the three-body $\eta(K^*\bar{K}^*)_{f_0(1710)}$, $\pi(K^*\bar{K}^*)_{f_0(1710)}$, and $K(K^*\bar{K}^*)_{f_0(1710)}$ systems treating $f_0(1710)$ as a $K^*\bar{K}^*$ bound state as found in previous studies of the vector-vector interactions. The total three-body scattering amplitude can be obtained in terms of the

TABLE II. Peak positions of squared of three-body scattering amplitude corresponding to different value of cutoff parameter Λ. Unit in MeV.

Λ		855	945	1035	1125	1215
$M_{f_0(1710)}$		1747	1733	1718	1704	1690
$M_{\eta K^* \bar{K}^*}$		2104	2092	2070	2054	2038
$M_{\pi K^* \bar{K}^*}$		1981	1963	1948	1929	1913
	State 1	2092	2083	2072	2062	2053
$M_{K K^* \bar{K}^*}$	State 2	2140	2134	2130	2130	2134
	State 3	⋯	2233	2219	2209	2211

two-body interactions which are taken from the chiral unitary approach. We associate the peaks in the modulus squared of the three-body scattering amplitude with hadronic states.

Our theoretical results for the obtained mass of $f_0(1710)$ state and these peak positions of squared three-body scattering amplitudes with different values of the cutoff parameter Λ are listed in Table II. The peak positions are associated with the mass of the three-body state. The state found in the $\eta K^* \bar{K}^*$ system could be interpreted as $\eta(2100)$ meson. For the $\pi K^* \bar{K}^*$ system, we find a bump structure between 1900 and 2000 MeV, while for the $K K^* \bar{K}^*$ system, three peaks appeared. One of them is not sensitive to the cutoff parameter, and its mass is about 2130 MeV. The third one disappeared if the cutoff parameter was less than 855 MeV. It is expected the theoretical calculations here could be tested by future experiments.

ACKNOWLEDGMENTS

We thank Xu Zhang for useful discussions. This work is partly supported by the National Natural Science Foundation of China under Grants No. 12075288, No. 11735003, and No. 11961141012. It is also supported by the Youth Innovation Promotion Association CAS.

[1] E. Oset *et al.*, Int. J. Mod. Phys. E **25**, 1630001 (2016).
[2] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Rev. Mod. Phys. **90**, 015004 (2018); **94**, 029901(E) (2022).
[3] X.-K. Dong, F.-K. Guo, and B.-S. Zou, Prog. Phys. **41**, 65 (2021).
[4] X.-K. Dong, F.-K. Guo, and B.-S. Zou, Commun. Theor. Phys. **73**, 125201 (2021).
[5] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **106**, 072012 (2022).
[6] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **129**, 192002 (2022).
[7] F. Yang, H. Q. Zhu, and Y. Huang, Nucl. Phys. **A1030**, 122571 (2023).
[8] X.-K. Dong, Y.-H. Lin, and B.-S. Zou, Sci. China Phys. Mech. Astron. **65**, 261011 (2022).
[9] H.-X. Chen, N. Su, and S.-L. Zhu, Chin. Phys. Lett. **39**, 051201 (2022).
[10] X. Zhang, J.-J. Xie, and X. Chen, Phys. Rev. D **95**, 056014 (2017).
[11] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. **A730**, 392 (2004).
[12] L. Roca, E. Oset, and J. Singh, Phys. Rev. D **72**, 014002 (2005).
[13] Y. Zhou, X.-L. Ren, H.-X. Chen, and L.-S. Geng, Phys. Rev. D **90**, 014020 (2014).
[14] L.-S. Geng, X.-L. Ren, Y. Zhou, H.-X. Chen, and E. Oset, Phys. Rev. D **92**, 014029 (2015).
[15] J.-J. Xie, G. Li, and X.-H. Liu, Chin. Phys. C **44**, 114104 (2020).
[16] X. Zhang and J.-J. Xie, Chin. Phys. C **44**, 054104 (2020).
[17] A. Martinez Torres, K. P. Khemchandani, L. Roca, and E. Oset, Few-Body Syst. **61**, 35 (2020).
[18] T.-W. Wu, Y.-W. Pan, M.-Z. Liu, and L.-S. Geng, Sci. Bull. **67**, 1735 (2022).
[19] B. B. Malabarba, K. P. Khemchandani, and A. M. Torres, Eur. Phys. J. A **58**, 33 (2022).
[20] S.-Q. Luo, T.-W. Wu, M.-Z. Liu, L.-S. Geng, and X. Liu, Phys. Rev. D **105**, 074033 (2022).
[21] S.-Q. Luo, L.-S. Geng, and X. Liu, Phys. Rev. D **106**, 014017 (2022).
[22] N. Ikeno, M. Bayar, and E. Oset, Phys. Rev. D **107**, 034006 (2023).
[23] V. R. Debastiani, J. M. Dias, and E. Oset, Phys. Rev. D **96**, 016014 (2017).
[24] T.-W. Wu, M.-Z. Liu, and L.-S. Geng, Phys. Rev. D **103**, L031501 (2021).
[25] X. Wei, Q.-H. Shen, and J.-J. Xie, Eur. Phys. J. C **82**, 718 (2022).
[26] A. Martinez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale, and E. Oset, Phys. Rev. D **78**, 074031 (2008).
[27] X.-L. Ren, B. B. Malabarba, L.-S. Geng, K. P. Khemchandani, and A. Martínez Torres, Phys. Lett. B **785**, 112 (2018).
[28] J. M. Dias, V. R. Debastiani, L. Roca, S. Sakai, and E. Oset, Phys. Rev. D **96**, 094007 (2017).
[29] M. Bayar, J. Yamagata-Sekihara, and E. Oset, Phys. Rev. C **84**, 015209 (2011).
[30] C. W. Xiao, M. Bayar, and E. Oset, Phys. Rev. D **84**, 034037 (2011).
[31] M. Bayar, W. H. Liang, T. Uchino, and C. W. Xiao, Eur. Phys. J. A **50**, 67 (2014).
[32] W. Liang, C. W. Xiao, and E. Oset, Phys. Rev. D **88**, 114024 (2013).
[33] B. Durkaya and M. Bayar, Phys. Rev. D **92**, 036006 (2015).

[34] J. Yamagata-Sekihara, L. Roca, and E. Oset, Phys. Rev. D **82**, 094017 (2010); **85**, 119905(E) (2012).
[35] A. Martinez Torres, K. P. Khemchandani, and L.-S. Geng, Phys. Rev. D **99**, 076017 (2019).
[36] M. Sanchez Sanchez, L.-S. Geng, J.-X. Lu, T. Hyodo, and M. P. Valderrama, Phys. Rev. D **98**, 054001 (2018).
[37] J.-J. Xie, A. Martinez Torres, and E. Oset, Phys. Rev. C **83**, 065207 (2011).
[38] J.-J. Xie, A. Martinez Torres, E. Oset, and P. Gonzalez, Phys. Rev. C **83**, 055204 (2011).
[39] L. S. Geng and E. Oset, Phys. Rev. D **79**, 074009 (2009).
[40] L. S. Geng, E. Oset, R. Molina, and D. Nicmorus, *Proc. Sci.*, EFT09 (2009) 040 [arXiv:0905.0419].
[41] M.-L. Du, D. Gülmez, F.-K. Guo, U.-G. Meißner, and Q. Wang, Eur. Phys. J. C **78**, 988 (2018).
[42] C. García-Recio, L. S. Geng, J. Nieves, L. L. Salcedo, E. Wang, and J.-J. Xie, Phys. Rev. D **87**, 096006 (2013).
[43] Z.-L. Wang and B.-S. Zou, Phys. Rev. D **104**, 114001 (2021).
[44] H. Nagahiro, L. Roca, E. Oset, and B. S. Zou, Phys. Rev. D **78**, 014012 (2008).
[45] T. Branz, L. S. Geng, and E. Oset, Phys. Rev. D **81**, 054037 (2010).
[46] L. S. Geng, F. K. Guo, C. Hanhart, R. Molina, E. Oset, and B. S. Zou, Eur. Phys. J. A **44**, 305 (2010).
[47] A. Martinez Torres, K. P. Khemchandani, F. S. Navarra, M. Nielsen, and E. Oset, Phys. Lett. B **719**, 388 (2013).
[48] J.-J. Xie and E. Oset, Phys. Rev. D **90**, 094006 (2014).
[49] L.-R. Dai, J.-J. Xie, and E. Oset, Phys. Rev. D **91**, 094013 (2015).
[50] R. Molina, L. R. Dai, L. S. Geng, and E. Oset, Eur. Phys. J. A **56**, 173 (2020).
[51] L. Roca and E. Oset, Phys. Rev. D **82**, 054013 (2010).
[52] D. Gamermann, J. Nieves, E. Oset, and E. Ruiz Arriola, Phys. Rev. D **81**, 014029 (2010).
[53] J. Yamagata-Sekihara, J. Nieves, and E. Oset, Phys. Rev. D **83**, 014003 (2011).
[54] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[55] X. Zhu, D.-M. Li, E. Wang, L.-S. Geng, and J.-J. Xie, Phys. Rev. D **105**, 116010 (2022).
[56] L. S. Geng, E. Oset, L. Roca, and J. A. Oller, Phys. Rev. D **75**, 014017 (2007).
[57] F.-K. Guo, R.-G. Ping, P.-N. Shen, H.-C. Chiang, and B.-S. Zou, Nucl. Phys. **A773**, 78 (2006).
[58] B.-X. Sun, Y.-Y. Fan, and Q.-Q. Cao, arXiv:2206.02961.
[59] D. Bisello *et al.* (DM2 Collaboration), Phys. Rev. D **39**, 701 (1989).
[60] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **93**, 112011 (2016).
[61] L.-M. Wang, S.-Q. Luo, Z.-F. Sun, and X. Liu, Phys. Rev. D **96**, 034013 (2017).
[62] A. V. Anisovich, C. A. Baker, C. J. Batty, D. V. Bugg, V. A. Nikonov, A. V. Sarantsev, V. V. Sarantsev, and B. S. Zou, Phys. Lett. B **517**, 261 (2001).
[63] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **93**, 112002 (2004).
[64] P. Chen *et al.* (Belle Collaboration), Phys. Rev. D **84**, 071501 (2011).