Decays of the fully open flavor state $T^0_{c\bar{s}0}$ in a D^*K^* molecule scenario

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Inspired by the recent observations of $T^{0/++}_{c\bar{s}0}$ in the processes $B^0 \to \bar{D}^0 D_s^+ \pi^-$ and $B^+ \to D^- D_s^+ \pi^+$ by LHCb Collaboration, we investigate the decay properties of the $T^0_{c\bar{s}0}$ in a D^*K^* molecule scenario, and the widths of $T^0_{c\bar{s}0} \to D^0 K^0$, $D_s^+\pi^-$, $D_s^{*+}\rho^-$, $D_{s1}^{(\prime)+}\pi^-$, and $D^{*0}(D\pi)^0$ are estimated. Our estimations indicate that the width of $T^0_{c\bar{s}0} \to D_s^+\pi^-$ is sizable to be observed and the dominant decay mode of $T^0_{c\bar{s}0}$ is D^0K^0. Considering the isospin symmetry, we proposed to search $T_{c\bar{s}0}(2900)^{++}$ in the D^+K^+ invariant mass distributions of the process $B^+ \to D^+D^-K^+$, where some preliminary experimental hints have been observed by LHCb Collaboration.

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I. INTRODUCTION

Searching for the fully open-flavor tetraquark states are particular interesting since their quark components are obviously different with the traditional mesons, which makes them much easier to be identified as a tetraquark states. The first instance of hadronic state with valence quarks of four different flavors, $X(5568)$, was reported in the mass spectrum of $B_s^0\pi^\pm$ based on 10.4 fb^{-1} of $p\bar{p}$ collision data by the D0 experiment at the Fermilab Tevatron collider in 2016 [1,2]. After the observation of $X(5568)$ by the D0 Collaboration, the LHCb [3], CMS [4], CDF [5], and ATLAS [6] Collaborations searched the $X(5568)$ in the same final states successively, but no signal was observed. On the theoretical side, there are different views in the existence of $X(5568)$, for example, almost all the investigations in Refs. [7–23] supported the existence of $X(5568)$ and could be considered as a BK molecular state or $\bar{b}s\bar{q}q$ compact tetraquark state, while the authors in Refs. [24–32] had just the opposite opinions.

Another two additional fully open-flavor tetraquark states are $X_0(2900)$ and $X_1(2900)$, which were first observed in the D^-K^+ invariant mass spectrum of the process $B^+ \to D^+D^-K^+$ in 2020 [33,34]. Their most possible quark components are $ud\bar{c}\,\bar{s}$, and the fully open-flavor property of these two states has attracted the theorists' great interest and various interpretations have been proposed by different groups. The estimations in the constituent quark model [35] and QCD sum rule [36,37] indicate that $X_0(2900)$ could be a compact tetraquark state with $I(J^P) = 0(0^+)$, while the estimations in Ref. [38] supported the diquark-antidiquark tetraquark interpretation for $X_1(2900)$, but for $X_0(2900)$, they found it could be a D^*K^* molecular state. The estimations of QCD two-point sum rule method [39] and potential model [40–45] supported $X_0(2900)$ as $D^*\bar{K}^*$ molecular state. Moreover, in Ref. [46], the authors investigated the decay properties of the $X_0(2900)$ in the D^*K^* molecular scenario based on an effective Lagrangian approach.

Very recently, the LHCb Collaboration reports two resonances $T_{c\bar{s}0}(2900)^{0/++}$ (abbreviated to $T^{0/++}_{c\bar{s}0}$ here and after), which are two of the isospin triplets, in the $D_s\pi$ invariant mass spectrum of the processes $B^0 \to \bar{D}^0 D_s^+\pi^-$ and $B^+ \to D^-D_s^+\pi^+$ with a significance of 9σ. The analysis indicated that the spin parity is preferred to be $J^P = 0^+$. The resonance parameters of $T^{0/++}_{c\bar{s}0}$ are measured to be [47,48]

$$m_{T^0_{c\bar{s}0}} = (2892 \pm 14 \pm 15)\ \text{MeV},$$
$$\Gamma_{T^0_{c\bar{s}0}} = (119 \pm 26 \pm 12)\ \text{MeV}, \tag{1}$$

and

$$m_{T^{++}_{c\bar{s}0}} = (2921 \pm 17 \pm 19)\ \text{MeV},$$
$$\Gamma_{T^{++}_{c\bar{s}0}} = (137 \pm 32 \pm 14)\ \text{MeV}, \tag{2}$$

respectively. The quark components of $T^0_{c\bar{s}0}$ are $c\bar{s}\,\bar{u}\,d$, which is also a fully open-flavor state. In Ref. [49], the

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authors investigated the open-flavor tetraquark states by using QCD sum rule and suggested searching for exotic doubly-charged tetraquark state $[sd][\bar{u}\,\bar{c}]$ in the $D_s^{(*)-}\pi^-$ channel. The estimations in the constituent quark model [50] and multiquark color flux-tube model [51] suggested that $T_{c\bar{s}0}$ can be assigned to be a tetraquark state. Noted that the thresholds of $D^{*0}K^{*0}$ and $D_s^*\rho$ are about 2902 MeV and 2887 MeV, respectively, which are both in the vicinity of T_{cs0}^0. Thus, the authors in Ref. [52] interpreted the $T_{c\bar{s}0}$ as a threshold effect from the interaction of the D^*K^* and $D_s^*\rho$ channels. Moreover, one can find that the center value of the mass of T_{cs0}^0 is above the threshold of $D_s^+\rho^-$ and several MeV below the one of $D^{*0}K^{*0}$. More importantly, the width of ρ meson is about 150 MeV, and much larger than the one of K^*, which indicates that ρ meson are much more difficult to form a bound state than the K^* meson. Thus, $T_{c\bar{s}0}$ seems to be a good candidate of molecular state composed of D^* and K^*. In Ref. [53], the interactions between $D^{(*)}K^{(*)}$ were investigated by using the one-boson exchange model and the estimation indicate that the $T_{c\bar{s}0}$ could be interpreted as a D^*K^* molecular state with $I(J^P) = 1(0^+)$. Along this way, in the present work we further inspect the possibility of the D^*K^* molecular interpretation by investigating the decay properties of $T_{c\bar{s}0}$, and try to find its dominant decay modes, which may be helpful for searching $T_{c\bar{s}0}$ in further experiments at the Belle and LHCb Collaborations.

This work is organized as follows. The hadronic molecular structures of $T_{c\bar{s}0}$ is discussed in the following section and the possible decay channel, including the two-body and three-body decays are estimated in Sec. III. The numerical results and the relevant discussions are presented in Sec. IV, and the last section is dedicated to a short summary.

II. HADRONIC MOLECULAR STRUCTURE

In the present work, we consider $T_{c\bar{s}0}$ as a molecular state composed of D^*K^* with $I(J^P) = 1(0^+)$, and here we take the neutral one, $T_{c\bar{s}0}^0$, as an example. The effective Lagrangian between $T_{c\bar{s}0}^0$ and its components is

$$\mathcal{L} = g_T T_{c\bar{s}0}^0(x)\int dy \Phi(y^2) D^{*0\mu}(x+\omega_{K^{*0}}y) K_\mu^{*0}(x-\omega_{D^{*0}}y), \tag{3}$$

where $\omega_{K^{*0}} = m_{K^{*0}}/(m_{K^{*0}}+m_{D^{*0}})$, $\omega_{D^{*0}} = m_{D^{*0}}/(m_{K^{*0}}+m_{D^{*0}})$. $\Phi(y^2)$ is the correlation function, which is introduced to describe the inner distributions of the component. The Fourier transformation of $\Phi(y^2)$ is

$$\Phi(y^2) = \int \frac{d^4p}{(2\pi)^4} e^{-ipy}\tilde{\Phi}(-p^2). \tag{4}$$

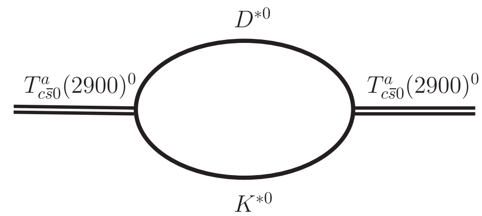

FIG. 1. The mass operator of the $T_{c\bar{s}0}^0$.

The choices of the $\tilde{\Phi}(-p^2)$ should satisfy the conditions that can describe the interior structure of the molecular state and fall fast enough in the ultraviolet region. Here we use the correlation function in the Gaussian form, which is [54–59],

$$\tilde{\Phi}(p_E^2) = \exp(-p_E^2/\Lambda_T^2), \tag{5}$$

where Λ_T is a model parameter related to the distribution of components in the molecular state, P_E is the Jacobi momentum used to describe the relative motion.

The coupling constant g_T introduced in Eq. (3) could be determined by the compositeness condition, which was first proposed to study the deuteron as a bound state of proton and neutron in Refs. [60,61] and then applied to investigate the properties of exotic states in the molecular frame in recent two decades [23,54–59]. In the present work, we consider the $T_{c\bar{s}0}^0$ as a pure molecular state composed of $D^{*0}K^{*0}$, which implies that the renormalization constant of the hadron wave function is zero, i.e., [54,55],

$$Z = 1 - \Pi'(m_T^2) \equiv 0, \tag{6}$$

where $\Pi'(m_T^2)$ is the derivative of the $T_{c\bar{s}0}^0$ mass operator described by the diagram in Fig. 1 and m_T is the mass of $T_{c\bar{s}0}^0$. Based on the effective Lagrangian in Eq. (3), the particular form of mass operator can be written as

$$\begin{aligned}\Pi(m_T^2) = g_T^2 \int \frac{d^4q}{(2\pi)^4}\tilde{\Phi}^2[-(q-\omega_{D^*}p)^2, \Lambda_T^2] \\ \times \frac{-g^{\mu\nu} + q^\mu q^\nu/m_{D^*}^2}{q^2 - m_{D^*}^2} \\ \times \frac{-g_{\mu\nu} + (p_\mu - q_\mu)(p_\nu - q_\nu)/m_{K^*}^2}{(p-q)^2 - m_{K^*}^2},\end{aligned} \tag{7}$$

where p is the momentum of $T_{c\bar{s}0}$ with $p^2 = m_T^2$. With Eqs. (6) and (7), one can estimate the coupling constant g_T depending on the model parameter Λ_T.

III. STRONG DECAYS OF $T_{c\bar{s}0}^0$

In the present work, we further inspect the possibility of $T_{c\bar{s}}$ as a D^*K^* molecular state by investigating the decay behavior of $T_{c\bar{s}}$. As for $T_{c\bar{s}0}^0$, we find that the possible

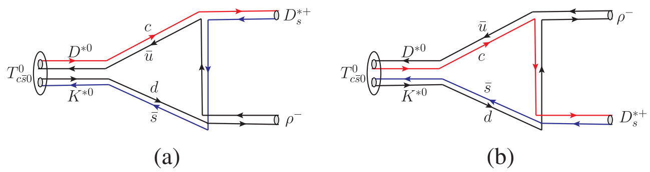

FIG. 2. Typical diagrams of the decay $T^0_{cs0} \to D_s^+\rho^-$ in the quark level.

two-body decay channels include $T^0_{c\bar{s}0} \to D^0K^0$, $D_s^+\pi^-$, $D_s^{*+}\rho^-$, $D_{s1}^{(\prime)+}\pi^-$, and three body decay channel $T^0_{c\bar{s}0} \to D^{*0}(K\pi)^0$. Taking $T^0_{cs0} \to D_s^+\rho^-$ as an example of two-body decay process, we present the typical quark level diagrams of the decay mechanism in Fig. 2. From the figure one can find the T^0_{cs0} decay into $D_s^{*+}\rho^-$ by exchanging a $s\bar{u}$ [diagram (a)] or a $c\bar{d}$ [diagram (b)] quark-antiquark pair. Since the initial $D^{*0}K^{*0}$ and final $D_s^*\rho$ are color singlets, thus the exchange quark pair $s\bar{u}$ and $c\bar{d}$ are color singlets, too. Thus, one can approximate $s\bar{u}$ quark-antiquark pair as a strange meson and $c\bar{d}$ quark-antiquark pair as a charmed meson, and then, we can estimated the two-body decays of $T^0_{c\bar{s}0}$ in hadron level. It should be clarified that all the possible exchange mesons should be considered without loss of generality, however, the contributions from the mesons with large masses, such as charmed mesons and excited strange meson, are negligible compared to those from low-lying strange mesons. Thus, in the present calculations, only the contributions from light-meson exchange diagrams are included, and the typical diagrams related to these decay processes are collected in Figs. 3 and 4.

A. Effective Lagrangians

In the present work, we estimate the decay process in the hadron level and the interaction between hadrons are described by effective Lagrangians. Considering the heavy quark limit and chiral symmetry, the relevant phenomenological Lagrangians are [62–66]

$$
\begin{aligned}
\mathcal{L}_{\mathcal{D}^*\mathcal{D}\mathcal{P}} &= ig_{\mathcal{D}^*\mathcal{D}\mathcal{P}}(\mathcal{D}^{*\mu}\partial_\mu\mathcal{P}\bar{\mathcal{D}} - \mathcal{D}\partial_\mu\mathcal{P}\bar{\mathcal{D}}^{*\mu}),\\
\mathcal{L}_{\mathcal{D}^*\mathcal{D}\mathcal{V}} &= -2f_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu\mathcal{V}^\nu)^i_j(\mathcal{D}_i^\dagger\overleftrightarrow{\partial}_\alpha\mathcal{D}^{*\beta j} - \mathcal{D}_i^{*\beta\dagger}\overleftrightarrow{\partial}_\alpha\mathcal{D}^j),\\
\mathcal{L}_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}} &= \frac{1}{2}g_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}}\varepsilon_{\mu\nu\alpha\beta}\mathcal{D}_i^{*\mu}\partial^\nu\mathcal{P}^{ij}\overleftrightarrow{\partial}^\alpha\mathcal{D}_j^{*\beta\dagger},\\
\mathcal{L}_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} &= ig_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_i^{*\nu\dagger}\overleftrightarrow{\partial}_\mu\mathcal{D}_\nu^{*j}(\mathcal{V}^\mu)^i_j\\
&\quad + 4if_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_{i\mu}^{*\dagger}(\partial^\mu\mathcal{V}^\nu - \partial^\nu\mathcal{V}^\mu)^i_j\mathcal{D}_\nu^{*j} + \text{H.c.},\\
\mathcal{L}_{\mathcal{D}_1\mathcal{D}^*\mathcal{P}} &= g_{\mathcal{D}_1\mathcal{D}^*\mathcal{P}}[3\mathcal{D}_1^\mu(\partial_\mu\partial_\nu\mathcal{P})\mathcal{D}^{*\nu\dagger} - \mathcal{D}_1^\mu(\partial^\nu\partial_\nu\mathcal{P})\mathcal{D}_\mu^{*\dagger}]\\
&\quad + \text{H.c.},\\
\mathcal{L}_{\mathcal{D}_1'\mathcal{D}^*\mathcal{P}} &= ig_{\mathcal{D}_1'\mathcal{D}^*\mathcal{P}}(\mathcal{D}_1'^\mu\overleftrightarrow{\partial}_\nu\mathcal{D}_\mu^{*\dagger})\partial^\nu\mathcal{P} + \text{H.c.},
\end{aligned}
\tag{8}
$$

where $\mathcal{D}^{(*)\dagger} = (\bar{D}^{(*)0}, D^{(*)-}, D_s^{(*)-})$. The symbols $\mathcal{V}$ and $\mathcal{P}$ are the matrixes form of vector nonet and pseudoscalar nonet, respectively, their concrete form are

$$
\mathcal{V} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho^0+\omega) & \rho^+ & K^{*+} \\ \rho^- & \frac{1}{\sqrt{2}}(-\rho^0+\omega) & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix},
$$

$$
\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + +\alpha\eta + \beta\eta' & K^0 \\ K^- & \bar{K}^0 & \gamma\eta + \delta\eta' \end{pmatrix},
\tag{9}
$$

where α, β, γ and δ are the parameters related to the mixing angle, which are

$$
\begin{aligned}
\alpha &= \frac{\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}}, &\qquad \beta &= \frac{\sin\theta + \sqrt{2}\cos\theta}{\sqrt{6}},\\
\gamma &= \frac{-2\cos\theta - \sqrt{2}\sin\theta}{\sqrt{6}}, &\qquad \delta &= \frac{-2\sin\theta + \sqrt{2}\cos\theta}{\sqrt{6}},
\end{aligned}
\tag{10}
$$

where the mixing angle θ is determined to be 19.1° [67,68].

Considering SU(3) symmetry, the effective Lagrangians between light pseudoscalar mesons and vector mesons can be [46,63,69,70]

$$
\begin{aligned}
\mathcal{L}_{K^*KP} &= -ig_{K^*KP}(\bar{K}\partial^\mu P - \partial^\mu\bar{K}P)K_\mu^* + \text{H.c.},\\
\mathcal{L}_{K^*K^*P} &= -g_{K^*K^*P}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha\bar{K}_\beta^*P\partial_\mu K_\nu^*,\\
\mathcal{L}_{K^*KV} &= -g_{K^*KV}\epsilon^{\eta\tau\rho\sigma}\partial_\rho\bar{K}_\sigma^*\partial_\eta V_\tau K + \text{H.c.},\\
\mathcal{L}_{K^*K^*V} &= -ig_{K^*K^*V}[(\partial^\mu\bar{K}_\nu^*V^\nu - \bar{K}_\nu^*\partial^\mu V^\nu)K_\mu^*\\
&\quad + \bar{K}_\mu^*(\partial^\mu V^\nu K_\nu^* - V^\nu\partial^\mu K_\nu^*)\\
&\quad \times + (\bar{K}_\nu^*V^\mu\partial_\mu K^{*\nu} - \partial_\mu\bar{K}_\nu^*V^\mu K^{*\nu})],
\end{aligned}
\tag{11}
$$

where P refers to the triplet of π, η and η' from pseudoscalar nonet, and the vector meson V stands for ρ triplet and ω from vector nonet. The doublet $K^{(*)}$ is $K^{(*)} = (K^{(*)-}, \bar{K}^{(*)0})$. The relevant coupling constants in Eqs. (9) and (11) will be discussed in the following section.

B. Two-body decay process

According to the effective Lagrangians listed above, we can obtain the amplitude for $T^0_{c\bar{s}0} \to D^0K^0$, $D_s^+\pi^-$, $D_s^{*+}\rho^-$, $D_{s1}^{(\prime)+}\pi^-$ corresponding to diagrams in Fig 3, which are

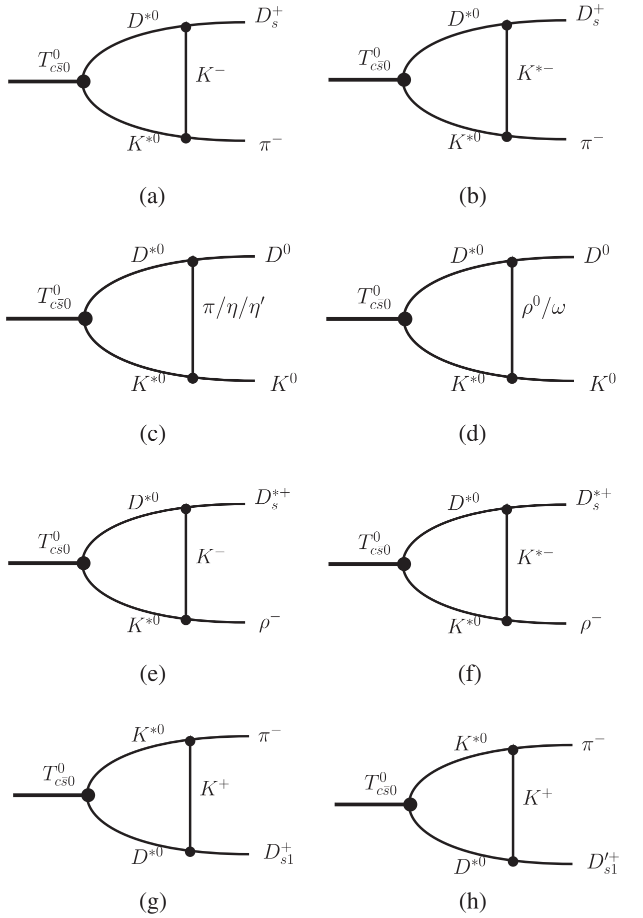

FIG. 3. The hadron level typical diagrams contributing to $T^0_{c\bar{s}0} \to D_s^+\pi^-$ [diagrams (a) and (b)], $T^0_{c\bar{s}0} \to D^0K^0$ [diagrams (c) and (d)], $T^0_{c\bar{s}0} \to D_s^{*+}\rho^-$ [diagrams (e) and (f)], $T^0_{c\bar{s}0} \to D_{s1}^{(\prime)+}\pi^-$ [diagrams (g) and (h)], the D_{s1} and D_{s1}' refer to $D_{s1}(2460)$ and $D_{s1}(2536)$, respectively.

$$\begin{aligned} i\mathcal{M}_a = i^3 \int \frac{d^4q}{(2\pi)^4} & [g_T\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)g^{\phi\tau}] \\ & \times [ig_{D^*DP}(iq^\mu)g^{\mu\delta}][-ig_{K^*KP}(ip_4^\nu + iq^\nu)g^{\nu a}] \\ & \times \frac{-g^{\phi\delta}+p_1^\phi p_1^\delta/m_1^2}{p_1^2-m_1^2}\frac{-g^{\tau a}+p_2^\tau p_2^a/m_2^2}{p_2^2-m_2^2}\frac{1}{q^2-m_q^2} \\ & \times \mathcal{F}^2(m_q,\Lambda), \end{aligned}$$

$$\begin{aligned} i\mathcal{M}_b = i^3 \int \frac{d^4q}{(2\pi)^4} & [g_T\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)g^{\phi\tau}] \\ & \times [-2f_{D^*DV}\epsilon_{\mu\nu\alpha\beta}(iq^\mu)g^{\nu\lambda}(-ip_1^\alpha - ip_3^\alpha)g^{\beta\delta}] \\ & \times [-g_{K^*K^*P}\epsilon_{\omega\theta\rho\sigma}(-ip_2^\omega)(-iq^\rho)g^{\theta\xi}g^{\sigma a}] \\ & \times \frac{-g^{\phi\delta}+p_1^\phi p_1^\delta/m_1^2}{p_1^2-m_1^2}\frac{-g^{\tau a}+p_2^\tau p_2^a/m_2^2}{p_2^2-m_2^2} \\ & \times \frac{-g^{\lambda\xi}+q^\lambda q^\xi/m_q^2}{q^2-m_q^2}\mathcal{F}^2(m_q,\Lambda), \end{aligned}$$

$$\begin{aligned} i\mathcal{M}_c = i^3 \int \frac{d^4q}{(2\pi)^4} & [g_T\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)g^{\phi\tau}] \\ & \times [-ig_{D^*DP}(iq^\mu)g^{\mu\delta}][-ig_{K^*KP}(-iq^\nu - ip_4^\nu)g^{\nu a}] \\ & \times \frac{-g^{\phi\delta}+p_1^\phi p_1^\delta/m_1^2}{p_1^2-m_1^2}\frac{-g^{\tau a}+p_2^\tau p_2^a/m_2^2}{p_2^2-m_2^2}\frac{1}{q^2-m_q^2} \\ & \times \mathcal{F}^2(m_q,\Lambda), \end{aligned}$$

$$\begin{aligned} i\mathcal{M}_d = i^3 \int \frac{d^4q}{(2\pi)^4} & [g_T\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)g^{\phi\tau}] \\ & \times [2f_{D^*DV}\epsilon_{\mu\nu\alpha\beta}(iq^\mu)g^{\nu\lambda}(ip_3^\alpha + ip_1^\alpha)g^{\beta\delta}] \\ & \times [g_{K^*KV}\epsilon_{\omega\theta\rho\sigma}(-iq)^\omega(-ip_2)^\rho g^{\theta\xi}g^{\sigma a}] \\ & \times \frac{-g^{\phi\delta}+p_1^\phi p_1^\delta/m_1^2}{p_1^2-m_1^2}\frac{-g^{\tau a}+p_2^\tau p_2^a/m_2^2}{p_2^2-m_2^2} \\ & \times \frac{-g^{\lambda\xi}+q^\lambda q^\xi/m_q^2}{q^2-m_q^2}\mathcal{F}^2(m_q,\Lambda), \end{aligned}$$

$$\begin{aligned} i\mathcal{M}_e = i^3 \int \frac{d^4q}{(2\pi)^4} & [g_T\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)g^{\phi\tau}] \\ & \times \left[-\frac{1}{2}g_{D^*D^*P}\epsilon_{\mu\nu\alpha\beta}g^{\mu\delta}(iq)^\nu(ip_3^\alpha + ip_1^\alpha)g^{\beta\theta}\varepsilon^b(p_3)\right] \\ & \times [-g_{K^*KV}\epsilon_{\eta mn\sigma}(-ip_2)^n g^{\sigma a}(ip_4)^\eta g^{m\rho}\varepsilon^c(p_4)] \\ & \times \frac{-g^{\phi\delta}+p_1^\phi p_1^\delta/m_1^2}{p_1^2-m_1^2}\frac{-g^{\tau a}+p_2^\tau p_2^a/m_2^2}{p_2^2-m_2^2}\frac{1}{q^2-m_q^2} \\ & \times \mathcal{F}^2(m_q,\Lambda), \end{aligned}$$

$$\begin{aligned} i\mathcal{M}_f = i^3 \int \frac{d^4q}{(2\pi)^4} & [g_T\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)g^{\phi\tau}] \\ & \times [ig_{D^*D^*V}(-ip_1^\mu - ip_3^\mu)g^{\mu\lambda}g^{\nu\delta}g^{\nu\theta} \\ & + 4if_{D^*D^*V}g^{\mu\theta}(iq^\mu g^{\nu\lambda} - iq_\nu g^{\mu\lambda})g^{\nu\delta}\epsilon^b(p_3)] \\ & \times [-ig_{K^*K^*V}(((-ip_2)^\mu g^{\nu a}g^{\nu\rho} - (ip_4)^\mu g^{\nu a}g^{\nu\rho})g^{\mu\xi} \\ & + g^{\mu a}((ip_4^\mu)g^{\nu\rho}g^{\nu\xi} - g^{\nu\rho}(-iq)^\mu g^{\nu\xi}) \\ & + g^{\nu a}g^{\mu\rho}(-iq)^\mu g^{\nu\xi} - (-ip_2)^\mu g^{\nu a}g^{\mu\rho}g^{\nu\xi})\epsilon^c(p_4)] \\ & \times \frac{-g^{\phi\delta}+p_1^\phi p_1^\delta/m_1^2}{p_1^2-m_1^2}\frac{-g^{\tau a}+p_2^\tau p_2^a/m_2^2}{p_2^2-m_2^2} \\ & \times \frac{-g^{\lambda\xi}+q^\lambda q^\xi/m_q^2}{q^2-m_q^2}\mathcal{F}^2(m_q,\Lambda), \end{aligned}$$

$$\begin{aligned} i\mathcal{M}_g = i^3 \int \frac{d^4q}{(2\pi)^4} & [g_T\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)g^{\phi\tau}] \\ & \times [-ig_{K^*KP}i(p_3^\mu - q^\mu)g^{\mu\delta}][g_{D_1D^*P}(3g^{\omega\rho}(-iq)^\omega \\ & \times (-iq^\nu)g^{\nu a} - g^{\omega\rho}(-iq)^\nu(-iq)^\nu g^{\omega a})\varepsilon^t(p_4)] \\ & \times \frac{-g^{\phi\delta}+p_1^\phi p_1^\delta/m_1^2}{p_1^2-m_1^2}\frac{-g^{\tau a}+p_2^\tau p_2^a/m_2^2}{p_2^2-m_2^2} \\ & \times \frac{1}{q^2-m_q^2}\mathcal{F}^2(m_q,\Lambda), \end{aligned}$$

$$i\mathcal{M}_h = i^3 \int \frac{d^4q}{(2\pi)^4}[g_T\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)g^{\phi\tau}] \times [-ig_{K^*KP}i(p_3^\mu - q^\mu)g^{\mu\delta}] \times [g_{D_1'D^*P}(-ip_2^\nu - ip_4^\nu)g^{\omega a}g^{\omega\rho}(-iq)^\nu\epsilon^t(p_4)] \times \frac{-g^{\phi\delta}+p_1^\phi p_1^\delta/m_1^2}{p_1^2-m_1^2}\frac{-g^{\tau a}+p_2^\tau p_2^a/m_2^2}{p_2^2-m_2^2} \times \frac{1}{q^2-m_q^2}\frac{1}{q^2-m_q^2}\mathcal{F}^2(m_q,\Lambda). \quad (12)$$

In the above amplitudes, we introduce a form factor in monopole form to represent the inner structure and off-shell effect of the exchanging mesons, which is

$$\mathcal{F}(m_q,\Lambda) = \frac{m_q^2-\Lambda^2}{q^2-\Lambda^2}, \quad (13)$$

where Λ is of order of 1 GeV.

The total amplitudes of $T_{c\bar{s}0}^0 \to D_s^+\pi^-$, D^0K^0, $D_s^{*+}\rho^-$, $D_{s1}^+\pi^-$, and $D_{s1}'^+\pi^-$ are

$$\mathcal{M}_{T_{c\bar{s}0}^0\to D_s^+\pi^-} = \mathcal{M}_a + \mathcal{M}_b,$$
$$\mathcal{M}_{T_{c\bar{s}0}^0\to D^0\pi^0} = \mathcal{M}_c^{\pi^0} + \mathcal{M}_c^{\eta} + \mathcal{M}_c^{\eta'} + \mathcal{M}_d^{\rho^0} + \mathcal{M}_d^{\omega},$$
$$\mathcal{M}_{T_{c\bar{s}0}^0\to D_s^{*+}\rho^-} = \mathcal{M}_e + \mathcal{M}_f,$$
$$\mathcal{M}_{T_{c\bar{s}0}^0\to D_{s1}^+\pi^-} = \mathcal{M}_g,$$
$$\mathcal{M}_{T_{c\bar{s}0}^0\to D_{s1}'^+\pi^-} = \mathcal{M}_h, \quad (14)$$

respectively.

With the total amplitudes defined in Eq. (14), one can estimate the partial width of the above decay processes by

$$\Gamma_{T_{c\bar{s}0}^0\to\ldots} = \frac{1}{8\pi}\frac{|\vec{p}|}{m_{T_{c\bar{s}0}^0}^2}|\overline{\mathcal{M}_{T_{c\bar{s}0}^0\to\ldots}}|^2, \quad (15)$$

where $|\vec{p}|$ is the momentum of the final state in the initial state rest frame.

C. Three-body decay process

In addition to the two-body decays, we also considered the contribution of the possible three-body decay process. The dominant three-body decay should be $T_{c\bar{s}0}^0 \to D^{*0}K^{*0} \to D^{*0}K\pi$, which is shown in Fig. 4. The corresponding amplitude of the three-body decay is

$$\mathcal{M} = [g^{\phi\tau}\epsilon^\phi(p)\tilde{\Phi}(-p_{12}^2,\Lambda_T^2)][ig_{K^*KP}g^{\mu\lambda}i(p_3^\mu - p_2^\mu)] \times \frac{-g^{\tau\lambda}+q^\lambda q^\tau/m_{K^*}^2}{p_2^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}}, \quad (16)$$

and then the partial width is

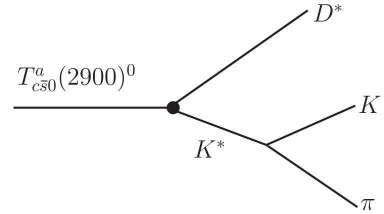

FIG. 4. The three body decay of the $T_{c\bar{s}0}^0$.

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}\overline{|\mathcal{M}|^2}dm_{12}^2dm_{23}^2. \quad (17)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

Before we estimate the widths of the considered channels, the relevant coupling constants should be clarified. Considering the heavy quark limit and chiral symmetry, the coupling constants between the light mesons and charmed meson pairs are [71–75]

$$g_{\mathcal{D}^*\mathcal{D}\mathcal{P}} = \frac{2g}{f_\pi}\sqrt{m_{\mathcal{D}}m_{\mathcal{D}}^*} = g_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}}\sqrt{m_{\mathcal{D}}m_{\mathcal{D}^*}},$$
$$f_{\mathcal{D}^*\mathcal{D}\mathcal{V}} = \frac{\lambda g_V}{\sqrt{2}} = \frac{f_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}}{m_{\mathcal{D}^*}},$$
$$g_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} = \frac{\beta_0 g_V}{\sqrt{2}},$$
$$g_{\mathcal{D}_1\mathcal{D}^*P} = -2\sqrt{\frac{2}{3}}\frac{h'}{\Lambda_\chi f_\pi}\sqrt{m_{\mathcal{D}_1}m_{\mathcal{D}^*}},$$
$$g_{\mathcal{D}_1'\mathcal{D}^*P} = -\frac{h}{f_\pi}, \quad (18)$$

where $g = 0.59$ [73] is determined by the measured width of $\Gamma(D^* \to D\pi)$. The values of the other parameters are $g_V = m_\rho/f_\pi$, $f_\pi = 132$ MeV, $\Lambda_\chi = 1$ GeV, $\beta_0 = 0.9$ and $\lambda = 0.56\ \mathrm{GeV}^{-1}$ [62]. The gauge couplings h and h' are estimated to be $h = 0.56 \pm 0.04$ and $h' = 0.43 \pm 0.01$ [76,77].

The coupling constants relevant to the light mesons are [46,69]

$$g_{K^*KP} = g_{K^*K^*V} = \frac{1}{4}g,$$
$$g_{K^*KV} = g_{K^*K^*P} = \frac{1}{4}\frac{g^2N_c}{16\pi^2 f_\pi}, \quad (19)$$

where the parameter $g = 0.78$ are determined via measured width of the process $K^* \to K\pi$. $N_c = 3$ is the color degree of freedom.

Moreover, the coupling constant of $T_{c\bar{s}0}^0$ to its components, g_T, can be estimated by the compositeness condition as given in Eq. (6). The phenomenological parameter Λ_T should be of the order of 1 GeV. In the present work, we

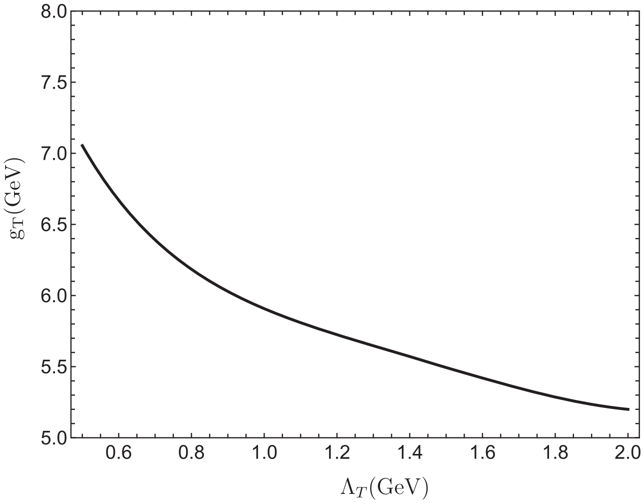

FIG. 5. The coupling constant g_T depending on model parameter Λ_T.

varies Λ_T in a sizable range from 0.5 GeV to 2.0 GeV. The numerical results of the g_T depending on the parameter Λ_T is presented in Fig. 5. In the considered parameter range, we can find the coupling constant g_T decreases from 7.05 GeV to 5.20 GeV.

With the above preparations, we can estimate the partial widths of $T^0_{c\bar{s}0} \to D^0 K^0$, $D_s^+\pi^-$, $D_s^{*+}\rho^-$, $D_{s1}^{(\prime)+}\pi^-$, and $D^{*0}(K\pi)^0$ depending on the model parameters Λ_T and Λ, which were introduced by the correlation function of molecular state and the form factor in the amplitudes. These two parameters are all of order of 1 GeV. In the present estimations, we varies Λ_T from 0.5 GeV to 2 GeV and take several typical values of Λ, which is 1.6, 1.8, and 2.0 GeV, respectively. In Fig. 6, we present our estimations of

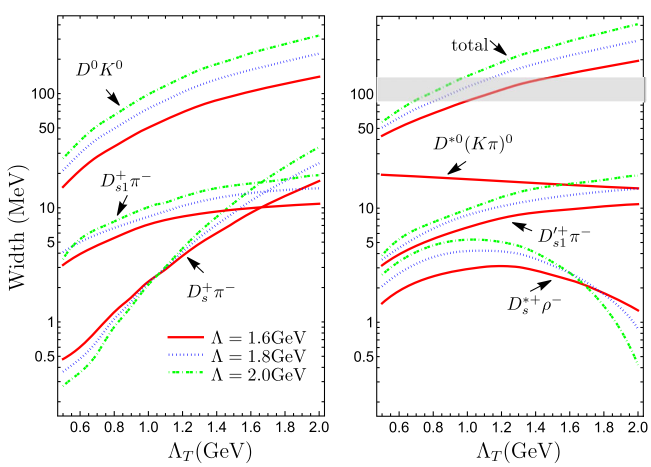

FIG. 6. The partial widths of $T^0_{c\bar{s}0} \to D^0K^0$, $D_s^+\pi^-$, $D_s^{*+}\rho^-$, $D_{s1}^{(\prime)+}\pi^-$, and $D^{*0}(K\pi)^0$ depending on the model parameter Λ_T and Λ. The total width are the sum of the partial widths of the considered channels. The gray band are the width of $T^0_{c\bar{s}0}$ reported by LHCb Collaboration [47,48].

TABLE I. The predicted partial widths of the considered channels.

Channel	Width (MeV)
$T^0_{c\bar{s}0} \to D^0K^0$	52.6–101.7
$T^0_{c\bar{s}0} \to D_s^+\pi^-$	0.55–8.35
$T^0_{c\bar{s}0} \to D_s^{*+}\rho^-$	2.96–5.3
$T^0_{c\bar{s}0} \to D_{s1}^+\pi^-$	6.63–10.29
$T^0_{c\bar{s}0} \to D_{s1}^{\prime+}\pi^-$	6.63–10.3
$T^0_{c\bar{s}0} \to D^{*0}(K\pi)^0$	16.11–18.96

the widths of the considered processes. With the assumption that $T^0_{c\bar{s}0}$ dominantly decay into these final states, we can compare the sum of partial widths of the considered channels with the measured width, which is $(119 \pm 26 \pm 12)$ MeV. From Fig. 6, we can find in the considered parameters space, our estimations of the total width can overlap with the experimental measurement from the LHCb Collaboration, in particular, the determined Λ_T range are 1.03–1.56 GeV, 0.83–1.19 GeV, and 0.71–1.01 GeV for $\Lambda = 1.6$, 1.8, and 2.0 GeV, respectively. In these parameter ranges the partial widths of the considered channels are presented in Table I. From the table, one can find the branching ratio of the observed channel $D_s^+\pi^-$ is $(0.65 \sim 5.38)\%$, which should be sizable to be observed experimentally. It should be emphasize again that the above results are obtained with the assumption that $T^0_{c\bar{s}0}$ is a pure $D^{*0}K^{*0}$ molecular state. As we have mention at the beginning of this work, the thresholds of both $D_s^{*+}\rho^-$ and $D^{*0}K^{*0}$ are in the vicinity of $T^0_{c\bar{s}0}$, thus the mixing between $D^{*0}K^{*0}$ and $D_s^{*+}\rho^-$ may be non-negligible, which could further influence on the decay properties of $T^0_{c\bar{s}0}$.

Moreover, our estimations indicate that $T^0_{c\bar{s}0}$ dominantly decays into D^0K^0. Considering the isospin symmetry, D^+K^+ should be the dominant decay channel of $T^{++}_{c\bar{s}0}$. In Refs. [33,34], the LHCb Collaboration have measured the decay process $B^+ \to D^+D^-K^+$, where two new structure $X_0(2900)$ and $X_1(2900)$ were observed in the D^-K^+ invariant mass distributions. Moreover, the LHCb Collaboration also reported their measurements of the D^+K^+ invariant mass distributions as shown in Fig. 7. From the figure one can find the fitted curve can not describe the experimental data well around 2.9 GeV, which indicates that there should exist an addition resonance. Since $T^{++}_{c\bar{s}0}$ dominantly decays into D^+K^+ and its mass also well conform the one of the additional resonance, we believe that this structure may comes form the contribution of $T^{++}_{c\bar{s}0}$, which could be tested by further experimental analysis by LHCb Collaboration.

V. SUMMARY

Very recently, two new resonances $T^{0/++}_{c\bar{s}0}$ were reported in $D_s\pi$ invariant mass spectrum of the processes

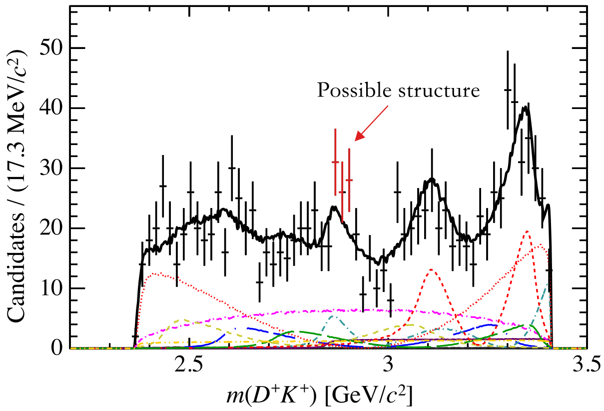

FIG. 7. The D^+K^+ invariant mass distributions of the process $B^+ \to D^+D^-K^+$ reported by the LHCb Collaboration [33,34], where one can find a structure around 2.9 GeV, which may correspond to the contributions of $T^{++}_{c\bar{s}0}$.

$B^0 \to \bar{D}^0 D_s^+ \pi^-$ and $B^+ \to D^- D_s^+ \pi^+$, by the LHCb Collaboration. These two states are two of the isospin triplet, and the most possible $I(J^P)$ quantum numbers are $1(0^+)$. The observed mass of $T_{c\bar{s}0}$ is very close to the threshold of D^*K^*, which indicate that $T_{c\bar{s}0}$ could be a good candidate of hadronic molecular state composed of D^*K^*.

In the present work, we investigate the decay behavior of $T^0_{c\bar{s}0}$ in the D^*K^* molecular scenario by using an effective Lagrangian approach. Six possible dominant-decay channels have been considered, which are $T^0_{c\bar{s}0} \to D^0K^0$, $D_s^+\pi^-$, $D_s^{*+}\rho^-$, $D_{s1}^{(\prime)+}\pi^-$, and $D^{*0}(K\pi)^0$. Our estimations indicate that the branching ratio of $T^0_{c\bar{s}0} \to D_s^+\pi^-$ can reach up to 5.38%, which should be sizable enough to be observed.

Moreover, our estimations indicate that $T^0_{c\bar{s}0}$ dominantly decays into D^0K^0. Considering the isospin symmetry, one can expect that D^+K^+ is the dominant decay mode of $T^{++}_{c\bar{s}0}$. By checking the D^+K^+ invariant-mass distributions of the process $B^+ \to D^+D^-K^+$, we find the fitted curve can not describe the experimental data well around 2.9 GeV, which may indicate the signal of $T^{++}_{c\bar{s}0}$ in the process $B^+ \to D^+D^-K^+$.

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