Toward discovering the excited Ω baryons through nonleptonic weak decays of Ω_c

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The nonleptonic weak decay processes $\Omega_c \to \Omega\pi^+/\Omega(1P)\pi^+/\Omega(1D)\pi^+/\Omega(2S)\pi^+$ are studied using the constituent quark model. The branching fraction of $\Omega_c \to \Omega\pi^+$ is predicted to be 1.05%. Considering the newly observed $\Omega(2012)$ resonance as a conventional $1P$-wave Ω excited state with spin parity $J^P = 3/2^-$, the newly measured ratio $\mathcal{B}[\Omega_c \to \Omega(2012)\pi^+ \to (\Xi\bar{K})^-\pi^+]/\mathcal{B}[\Omega_c \to \Omega\pi^+]$ at Belle can be well understood. Besides, the production rates for the missing $1P$-wave state $\Omega(1^2P_{1/2^-})$, two spin quartet $1D$-wave states $\Omega(1^4D_{1/2^+})$ and $\Omega(1^4D_{3/2^+})$, and two $2S$-wave states $\Omega(2^2S_{1/2^+})$ and $\Omega(2^4S_{3/2^+})$ are also investigated. It is expected that these missing excited Ω baryons should have large potentials to be discovered through the nonleptonic weak decays of Ω_c in forthcoming experiments by Belle II and/or LHCb.

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I. INTRODUCTION

Establishing a relatively complete hadron spectrum and understanding the properties of hadrons are important topics in hadron physics. Knowledge about the Ω baryon spectrum is very scarce. So far, the ground state $\Omega(1672)$ and its four possible excited states $\Omega(2012)$, $\Omega(2250)$, $\Omega(2380)$, and $\Omega(2470)$ have been observed in experiments [1]. The unambiguous discovery of $\Omega(1672)$ in both production and decay was by Barnes *et al.* in 1964 using the K^--meson beam at the Brookhaven National Laboratory [2,3]. In 1985, the $\Omega(2250)$ and $\Omega(2380)$ resonances decaying into $\Xi^-\pi^+K^-$ were observed in an experiment at the CERN Super Proton Synchrotron charged hyperon beam using incident Ξ^- [4]. In 1987, the $\Omega(2250)$ resonance was produced in K^-p interactions at SLAC [5]. In 1988, the $\Omega(2470)$ resonance was observed in the $\Omega^-\pi^+\pi^-$ invariant mass spectrum with a signal significance claimed to be at least 5.5 standard deviations by using the K^-p scattering at SLAC [6]. Since then, there was no progress toward searching for Ω resonances for as long as 30 years due to no effective production mechanisms. In order to promote the experiment, people proposed to produce Ω states on a proton target in CLAS12 through the photoproduction processes [7] or produce them by using a secondary kaon beam from the photoproduction processes at JLab, etc. [8,9].

In 2018, the first low-lying $\Omega(2012)$ resonance was observed by the Belle Collaboration in the $K^-\Xi^0$ and $K_S^0\Xi^-$ invariant mass distributions by using a data sample of e^+e^- annihilations [10]. The $\Omega(2012)$ resonance may favor the low-lying P-wave excited Ω state with $J^P = 3/2^-$ [11–15], although it may be a candidate of a hadronic molecule state as discussed in the literature [16–23]. Recently, the Belle Collaboration also discovered the $\Omega(2012)$ resonance by using the Ω_c weak decay process $\Omega_c \to \Omega(2012)\pi^+$ [24]. The measured branching fraction ratio $\mathcal{B}[\Omega_c \to \Omega(2012)\pi^+ \to (\Xi\bar{K})^-\pi^+]/\mathcal{B}[\Omega_c \to \Omega\pi^+]$ is $0.220 \pm 0.059(\text{stat}) \pm 0.035(\text{syst})$ [24]. Such a large relative ratio indicates that the weak decay processes

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TABLE I. The predicted mass spectrum (MeV) of Ω baryons with principal quantum number $N \le 2$ in various quark models. The baryon states are denoted as $|N_6, {}^{2S+1}N_3, N, L, J^P\rangle$, where N_6 stands for the irreducible representation of spin-flavor SU(6) group, N_3 stands for the irreducible representation of flavor SU(3) group, and N, S, L, and J^P stand for the principal, spin, total orbital angular momentum, and spin-parity quantum numbers, respectively. In the $L-S$ coupling scheme, the Ω states are also denoted by $n^{2S+1}L_{J^P}$.

$n^{2S+1}L_{J^P}$	$\lvert N_6, {}^{2S+1}N_3, N, L, J^P\rangle$	Ref. [36]	Ref. [30]	Ref. [31]	Ref. [26]	Ref. [27]	Ref. [25]	Ref. [34]	Ref. [11]	Observed mass
$1^4S_{\frac{3}{2}^+}$	$\lvert 56, {}^4 10, 0, 0, \frac{3}{2}^+\rangle$	1694	1635	1678	1675	1673	1656	1642(17)	1672	1672.45
$1^2P_{\frac{1}{2}^-}$	$\lvert 70, {}^2 10, 1, 1, \frac{1}{2}^-\rangle$	1837	1950	1941	2020	2015	1923	1944(56)	1957	
$1^2P_{\frac{3}{2}^-}$	$\lvert 70, {}^2 10, 1, 1, \frac{3}{2}^-\rangle$	1978	2000	2038	2020	2015	1953	2049(32)	2012	2012.5
$2^2S_{\frac{1}{2}^+}$	$\lvert 70, {}^2 10, 2, 0, \frac{1}{2}^+\rangle$	2140	2220	2301	2190	2182	2191	2350(63)	2232	
$2^4S_{\frac{3}{2}^+}$	$\lvert 56, {}^4 10, 2, 0, \frac{3}{2}^+\rangle$		2165	2173	2065	2078	2170		2159	
$1^2D_{\frac{3}{2}^+}$	$\lvert 70, {}^2 10, 2, 2, \frac{3}{2}^+\rangle$	2282	2345	2304	2265	2263	2194	2470(49)	2245	
$1^2D_{\frac{5}{2}^+}$	$\lvert 70, {}^2 10, 2, 2, \frac{5}{2}^+\rangle$		2345	2401	2265	2260	2210		2303	
$1^4D_{\frac{1}{2}^+}$	$\lvert 56, {}^4 10, 2, 2, \frac{1}{2}^+\rangle$	2140	2255	2301	2210	2202	2175	2481(51)	2141	
$1^4D_{\frac{3}{2}^+}$	$\lvert 56, {}^4 10, 2, 2, \frac{3}{2}^+\rangle$	2282	2280	2304	2215	2208	2182	2470(49)	2188	
$1^4D_{\frac{5}{2}^+}$	$\lvert 56, {}^4 10, 2, 2, \frac{5}{2}^+\rangle$		2280	2401	2225	2224	2178		2252	
$1^4D_{\frac{7}{2}^+}$	$\lvert 56, {}^4 10, 2, 2, \frac{7}{2}^+\rangle$		2295	2332	2210	2205	2183		2321	

$\Omega_c \to \Omega^*(X)\pi^+$ may provide a new and ideal platform to investigate the low-lying excited states $\Omega^*(X)$ both theoretically and experimentally.[1]

Theoretical studies on the $\Omega^*(X)$ resonances mainly focus on the mass spectrum within various approaches, such as nonrelativistic quark models [11,25–29], relativistic quark models [30–33], lattice QCD [34,35], and the Skyrme model [36]. The predicted mass spectrum for the conventional Ω baryons are collected in Table I as a reference. It can be seen that most of the predicted masses for the $1P$-, $2S$-, and $1D$-wave states lie in the mass ranges $\sim 2000 \pm 50$, $\sim 2200 \pm 50$, and $\sim 2300 \pm 50$ MeV, respectively. Additionally, in Refs. [37–39], the authors investigated the low-lying five-quark Ω configurations with negative parity and further considered their mixing combined with the corresponding low-lying three-quark Ω configurations. Recently, stimulated by the newly observed resonance $\Omega(2012)$ at Belle, the strong decay behaviors of some low-lying $1P$-, $2S$-, and $1D$-wave Ω resonances were also systematically investigated using the chiral quark model [11,12] and ${}^3P_0$ model [40]. The results suggest that the $1P$-, $2S$-, and $1D$-wave Ω baryons have relatively narrow decay widths of less than 50 MeV, and they may be discovered in the $\Xi\bar{K}$ and/or $\Xi(1530)\bar{K}$ final states. Some previous studies of the decays can be found in Refs. [41,42].

On the other hand, there are only a few studies on the production of Ω and its excited states through the weak decays of Ω_c in theory. For example, the production of the ground state $\Omega(1672)$ has been studied via semileptonic decays of Ω_c using a constituent quark model [43] and the nonleptonic two-body decays of Ω_c by the covariant confined quark model [44,45] and the light-front quark model [46]. In Ref. [43], the author also studied the production of the $1P$-wave excited states $\Omega^*(1P)$, which are considered via the Ω_c semileptonic weak decay processes using a quark model. On the other hand, the newly observed $\Omega(2012)$ resonance as a dynamically generated state was theoretically studied in the nonleptonic weak decays of $\Omega_c \to \pi^+\Omega(2012) \to (\Xi\bar{K})^-\pi^+$ and $(\Xi\bar{K}\pi)^-\pi^+$ in Ref. [47]. So far, the production of the $1P$-, $2S$-, and $1D$-wave excited states $\Omega(X)$ via the Ω_c nonleptonic weak decay processes are not systematically studied in theory.

In this work, we systematically study the production of the low-lying $1P$-, $2S$-, and $1D$-wave resonances $\Omega^*(X)$ via the hadronic weak decays of $\Omega_c \to \Omega^{(*)}(X)\pi^+$ using the constituent quark model. Recently, this model has been developed to study the hadronic weak decays of Λ_c, the heavy quark conserving weak decays of Ξ_Q, and hyperon weak radiative decay by Niu *et al.* [48–50]. This model is similar to that developed to deal with the semileptonic decays of heavy Λ_Q and Ω_Q baryons in Refs. [43,51].

This paper is organized as follows. We perform the detailed formalism of two-body nonleptonic weak decays of Ω_c in Sec. II. Then, the theoretical numerical results and discussions are presented in Sec. III. Finally, a short summary is given in Sec. IV.

II. FRAMEWORK

A. The model

A unique feature of $\Omega_c \to \Omega^{(*)}(X)\pi^+$ is that this decay proceeds only via external W-emission diagram [52], which is displayed in Fig. 1. We consider the simple quark-level transition $c \to su\bar{d}$, which is relevant for the Cabibbo-favored

[1]Here and after, we denote the Ω excited state as $\Omega^*(X)$ with mass X in the unit of MeV.

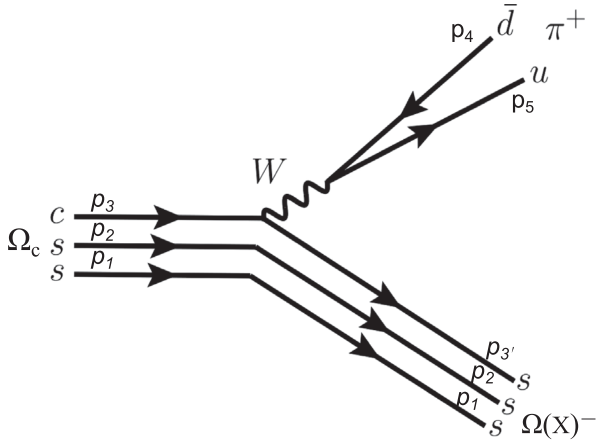

FIG. 1. The nonleptonic weak decay Feynman diagram for the processes of $\Omega_c \to \Omega(X)^- \pi^+$.

decay process of $\Omega_c \to \Omega(X)^- \pi^+$. The effective Hamiltonian for $c \to su\bar{d}$ can be given by [53]

$$H_W = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2), \tag{1}$$

where $G_F = 1.1663787 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant [1] and $C_1 = 1.26$ and $C_2 = -0.51$ are the Wilson coefficients taken at the m_c scale [53]. The Cabibbo-Kobayashi-Maskawa matrix elements $V_{cs} = 0.987$ and $V_{ud} = 0.974$ are taken from the Review of Particle Physics (RPP) [1], and the current-current operators are

$$\mathcal{O}_1 = \bar{\psi}_{\bar{s}_a} \gamma_\mu (1 - \gamma_5) \psi_{c_a} \bar{\psi}_{\bar{u}_b} \gamma^\mu (1 - \gamma_5) \psi_{d_b}, \tag{2}$$

$$\mathcal{O}_2 = \bar{\psi}_{\bar{s}_a} \gamma_\mu (1 - \gamma_5) \psi_{c_b} \bar{\psi}_{\bar{u}_b} \gamma^\mu (1 - \gamma_5) \psi_{d_a}, \tag{3}$$

with ψ_{j_δ} ($j = u/d/s/c, \delta = a/b$) representing the jth quark field in a meson or baryon and a and b being color indices.

According to its parity behavior, H_W can be separated into a parity-conserving part (H_W^{PC}) and a parity-violating part (H_W^{PV}) [48]:

$$H_W = H_W^{PC} + H_W^{PV}. \tag{4}$$

With a nonrelativistic expansion, the two operators can be approximately expressed as [48]

$$\begin{aligned} H_W^{PC} \simeq{}& \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \frac{C_i \phi_c^i \gamma}{(2\pi)^3} \delta^3(\mathbf{p}_3 - \mathbf{p}_3' - \mathbf{p}_4 - \mathbf{p}_5) \Big\{ \langle s_3' | I | s_3 \rangle \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle \left(\frac{\mathbf{p}_5}{2m_5} + \frac{\mathbf{p}_4}{2m_4} \right) - \left[\left(\frac{\mathbf{p}_3'}{2m_3'} + \frac{\mathbf{p}_3}{2m_3} \right) \langle s_3' | I | s_3 \rangle \right. \\ & \left. - i \langle s_3' | \boldsymbol{\sigma} | s_3 \rangle \times \left(\frac{\mathbf{p}_3}{2m_3} - \frac{\mathbf{p}_3'}{2m_3'} \right) \right] \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle - \langle s_3' | I | s_3 \rangle \left[\left(\frac{\mathbf{p}_5}{2m_5} + \frac{\mathbf{p}_4}{2m_4} \right) \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle - i \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle \right. \\ & \left. \times \left(\frac{\mathbf{p}_4}{2m_4} - \frac{\mathbf{p}_5}{2m_5} \right) \right] + \langle s_3' | I | s_3 \rangle \left(\frac{\mathbf{p}_3'}{2m_3'} + \frac{\mathbf{p}_3}{2m_3} \right) \langle s_5 \bar{s}_4 | I | 0 \rangle \Big\} \hat{\alpha}_3^-, \end{aligned} \tag{5}$$

$$\begin{aligned} H_W^{PV} \simeq{}& \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \frac{C_i \phi_c^i \gamma}{(2\pi)^3} \delta^3(\mathbf{p}_3 - \mathbf{p}_3' - \mathbf{p}_4 - \mathbf{p}_5) \{ - \langle s_3' | I | s_3 \rangle \\ & \langle s_5 \bar{s}_4 | I | 0 \rangle - \langle s_3' | \boldsymbol{\sigma} | s_3 \rangle \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle \} \hat{\alpha}_3^-. \end{aligned} \tag{6}$$

In the above equations, $\mathbf{p}_j$ and m_j stand for the momentum and mass of the jth quark, respectively, as shown in Fig. 1. The ϕ_c^i ($i = 1, 2$ and $\phi_c^1 = 1, \phi_c^2 = \frac{1}{3}$) are color factors, I is the dimension-two unit matrix, and $\hat{\alpha}_3^-$ is the flavor operator which transforms a c quark to an s quark. The s_j and $\bar{s}_4$ stand for the spin of the jth quark and the fourth antiquark, respectively. γ is a symmetry factor and equals to one for the direct pion emission process considered in the present work.

In order to evaluate the spin matrix element $\langle s_5 \bar{s}_4 | I | 0 \rangle$ and $\langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle$ including an antiquark, the particle-hole conjugation [54] should be employed. Within the particle-hole conjugation relation

$$|j, -m\rangle \to (-1)^{j+m} |j, m\rangle, \tag{7}$$

the antiquark spin transforms as follows: $|\bar{\uparrow}\rangle \to |\downarrow\rangle$ and $|\bar{\downarrow}\rangle \to -|\uparrow\rangle$. For instance,

$$\begin{aligned} \langle \frac{1}{\sqrt{2}} (\uparrow_5 \bar{\downarrow}_4 - \downarrow_5 \bar{\uparrow}_4) | I | 0 \rangle &= \frac{1}{\sqrt{2}} (\langle \uparrow_5 | I | - \uparrow_4 \rangle - \langle \downarrow_5 | I | \downarrow_4 \rangle) \\ &= -\sqrt{2}. \end{aligned} \tag{8}$$

For a given decay process $A \to BC$, the transition amplitude $\mathcal{M}$ is calculated by

$$\begin{aligned} \mathcal{M}_{J_f, J_f^z; J_i, J_i^z} &= \langle C(\mathbf{P}_f; J_f, J_f^z) B(\mathbf{q}) | H_W | A(\mathbf{P}_i; J_i, J_i^z) \rangle \\ &= \langle C(\mathbf{P}_f; J_f, J_f^z) B(\mathbf{q}) | H_W^{PC} | A(\mathbf{P}_i; J_i, J_i^z) \rangle \\ &\quad + \langle C(\mathbf{P}_f; J_f, J_f^z) B(\mathbf{q}) | H_W^{PV} | A(\mathbf{P}_i; J_i, J_i^z) \rangle \\ &= \mathcal{M}^{PC}_{J_f^z, J_i^z} + \mathcal{M}^{PV}_{J_f^z, J_i^z}, \end{aligned} \tag{9}$$

where $A(\mathbf{P}_i; J_i, J_i^z)$, $B(\mathbf{q})$, and $C(\mathbf{P}_f; J_f, J_f^z)$ stand for the wave functions of the initial baryon A, final meson B, and final baryon C, respectively. $(\mathbf{P}_i, \mathbf{P}_f)$, (J_i, J_f), and (J_i^z, J_f^z)

are the momentum, the total angular momentum, and the third component of the total angular momentum of the initial baryon A and the final baryon C, respectively. $\mathbf{q}$ is the three-momentum of the final state meson in the initial state rest frame.

Then, the partial decay width for a given decay process $A \to BC$ can be expressed as

$$\Gamma = \frac{\Phi(ABC)}{2J_A + 1} \sum_{\text{spins}} |\mathcal{M}|^2, \tag{10}$$

where $\Phi(ABC)$ is the phase-space factor for the decay.

The choice of phase space is not clear. For the phase-space factor $\Phi(ABC)$, there are three typical options adopted in the literature [55–58]. The usual option is the relativistic phase-space factor (RPF)

$$\Phi(ABC) = 8\pi^2 \frac{|\mathbf{q}| E_B E_C}{M_A}, \tag{11}$$

where M_A is the mass of the initial hadron A while E_B and E_C stand for the energies of final hadrons B and C, respectively.

To match the transition matrix element calculated nonrelativistically, a fully nonrelativistic phase-space factor (NRPF) is used, that is,

$$\Phi(ABC) = 8\pi^2 \frac{|\mathbf{q}| M_B M_C}{M_A}, \tag{12}$$

where M_B and M_C are the mass of the final hadron B and C, respectively.

However, in many cases, the momenta of the final hadrons are quite large so that the relativistic phase space is significantly different from the nonrelativistic limit. In Ref. [55], Kokoski and Isgur suggested a "mock-hadron" phase-space factor (MHPF),

$$\Phi(ABC) = 8\pi^2 \frac{|\mathbf{q}| \tilde{M}_B \tilde{M}_C}{\tilde{M}_A}, \tag{13}$$

in their calculation of meson decay widths. The $\tilde{M}_A$, $\tilde{M}_B$, and $\tilde{M}_C$ are effective hadron masses of hadron A, B, and C, respectively. They are evaluated with a spin-independent interquark interaction. In the weak-binding limit, the mass of the π meson is degenerate with that of the ρ meson.

B. Wave functions

To work out the decay amplitude $\mathcal{M}$, we need the wave functions of the initial and final states. Here, the initial state is the ground Ω_c baryon, and the final states are the π^+ meson and the $\Omega^{(*)}(X)$ states. These wave functions are constructed within the nonrelativistic constituent quark model. For simplicity, the spatial wave functions of the baryons and mesons are adopted to the harmonic oscillator form in our calculations.

The spatial wave function for a baryon with principal quantum number N and total orbital angular momentum quantum numbers L and M_L is a product of the ρ-oscillator part and the λ-oscillator part. In momentum space, the baryon spatial wave function is given by [11]

$$\Psi^{\sigma}_{NLM_L}(\mathbf{p}_\rho, \mathbf{p}_\lambda) = \sum_{N, M_L} C^{n_\rho l_\rho m_\rho}_{n_\lambda l_\lambda m_\lambda} [\psi_{n_\rho l_\rho m_\rho}(\mathbf{p}_\rho) \psi_{n_\lambda l_\lambda m_\lambda}(\mathbf{p}_\lambda)]^{\sigma}_{NLM_L}, \tag{14}$$

with $N = 2(n_\rho + n_\lambda) + l_\rho + l_\lambda$, $M_L = m_\rho + m_\lambda$, and

$$\psi^{\alpha}_{nlm}(\mathbf{p}) = (i)^l (-1)^n \left[\frac{2n!}{(n+l+1/2)!}\right]^{1/2} \frac{1}{\alpha^{l+3/2}} \exp\left(-\frac{\mathbf{p}^2}{2\alpha^2}\right) L_n^{l+1/2}(\mathbf{p}^2/\alpha^2) \mathcal{Y}_{lm}(\mathbf{p}). \tag{15}$$

Here, $\mathcal{Y}_{lm}(\mathbf{p}) = |\mathbf{p}|^l Y_{lm}(\hat{\mathbf{p}})$ is the lth solid harmonic polynomial. $\mathbf{p}_\rho$ and $\mathbf{p}_\lambda$ are the internal momenta of the ρ- and λ-oscillator wave functions, respectively. They can be expressed as functions of the quark momenta $\mathbf{p}_j$ ($j = 1, 2, 3$):

$$\mathbf{p}_\rho = \frac{\sqrt{2}}{2}(\mathbf{p}_1 - \mathbf{p}_2), \tag{16}$$

$$\mathbf{p}_\lambda = \frac{\sqrt{6}}{2} \frac{m_3(\mathbf{p}_1 + \mathbf{p}_2) - (m_1 + m_2)\mathbf{p}_3}{m_1 + m_2 + m_3}. \tag{17}$$

The n_ρ and n_λ are the principal quantum numbers of the ρ- and λ-mode oscillators, respectively. (l_ρ, m_ρ) and (l_λ, m_λ) are the orbital angular momentum quantum numbers of the ρ- and λ-mode oscillators, respectively. $\sigma = s, \rho, \lambda, a, \ldots$ stand for different excitation modes with different permutation symmetries. α_ρ and α_λ are two oscillator parameters. For the Ω baryons we have $\alpha_\rho = \alpha_\lambda$, while for the charmed Ω_c baryons we have

$$\alpha_\lambda = \left(\frac{3m_c}{2m_s + m_c}\right)^{1/4} \alpha_\rho, \tag{18}$$

where m_s and m_c stand for the masses of the strange and charmed quarks, respectively. The flavor and spin wave functions of the Ω_c and Ω baryons have been given in our previous works [59,60]. The product of spin, flavor, and spatial wave functions of the heavy baryons must be symmetric, since the color wave function is antisymmetric. The details about the quark model classifications for the Ω_c spectrum can be found in Refs. [59,61,62], while those for the Ω baryon spectrum can be found in Refs. [11,60].

Finally, the wave function of the π^+ meson is constructed by

$$\varphi(\mathbf{p}_4, \mathbf{p}_5) = \phi_{\pi^+}\chi^a\psi(\mathbf{p}_4, \mathbf{p}_5), \tag{19}$$

where the spin wave function χ^a is

$$\chi^a = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \tag{20}$$

and the flavor wave function ϕ_{π^+} is

$$\phi_{\pi^+} = u\bar{d}. \tag{21}$$

The spatial wave function in the momentum space is adopted the simple harmonic oscillator form

$$\psi(\mathbf{p}_4, \mathbf{p}_5) = \frac{1}{\pi^{3/4}\beta^{3/2}}\exp\left[-\frac{(\mathbf{p}_4 - \mathbf{p}_5)^2}{8\beta^2}\right], \tag{22}$$

where β is a size parameter of the meson wave function. The $\mathbf{p}_4$ and $\mathbf{p}_5$ stand for the quark momenta of the π^+ meson as shown in Fig. 1.

C. Parameters

For self-consistency, the quark model parameters are taken the same as those adopted in our previous work [59]. The constituent masses for the u/d, s, and c quarks are taken to be $m_{u/d} = 330$ MeV, $m_s = 450$ MeV, and $m_c = 1480$ MeV, respectively. For the initial state Ω_c, the harmonic oscillator parameter α_ρ is taken to be $\alpha_\rho = 440$ MeV, and the other harmonic oscillator parameter α_λ is related to α_ρ by $\alpha_\lambda = [3m_c/(2m_s + m_c)]^{1/4}\alpha_\rho$. For the final state $\Omega^{(*)}(X)$, a unified harmonic oscillator parameter is adopted, i.e., $\alpha_\lambda = \alpha_\rho = 440$ MeV. For the π^+ meson, the size parameter is taken to be $\beta = 280$ MeV as that adopted in Ref. [48]. The masses for the π^+, Ω, and Ω_c are taken the RPP average values 140, 1672, and 2695 MeV, respectively [1]. In the MHPF defined in Eq. (13), we need to determine the effective masses of the mock hadrons. For the process $\Omega_c \to \Omega^{(*)}\pi$, we adopt $\tilde{M}_\pi = 0.72$ GeV, consistent with Kokoski and Isgur [55], $\tilde{M}_{\Omega_c} = M_{\Omega_c}$, and $\tilde{M}_{\Omega_c^{(*)}} = M_{\Omega_c^{(*)}}$.

III. NUMERICAL RESULTS AND DISCUSSION

In this work, considering the uncertainties from the relativistic effect, we perform our calculations with the three typical phase-space options RPF, NRPF, and MHPF. Our results are listed in Table III. It is seen that the nonleptonic weak decay properties of Ω_c have a significance dependence on the options of the phase-space factor. The results from RPF and MHPF are comparable with each other. However, the predicted partial widths with NRPF are a factor of ~2–6 smaller those calculated with RPF and MHPF.

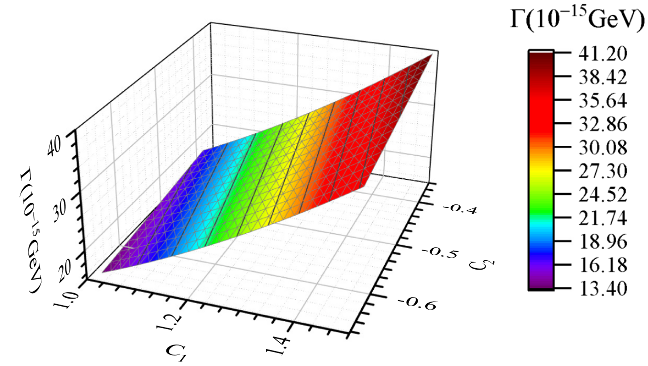

FIG. 2. The dependencies of the partial decay width of $\Omega_c \to \Omega^-\pi^+$ on the parameters C_1 and C_2. The results are obtained by adopting the RPF.

The Wilson coefficients C_1 and C_2 are usually taken to be $C_1 = 1.26$ and $C_2 = -0.51$ at the m_c scale [53]. These coefficients have some uncertainties due to their scale dependencies. To see the effects of the uncertainties of C_1 and C_2 on our results, as an example in Fig. 2, we plot the partial width of $\Gamma[\Omega_c^0 \to \Omega^-\pi^+]$ as a function of the C_1 and C_2 in the range of $C_1 \in (1.0, 1.5)$ and $C_2 \in (-0.64, -0.38)$. From the figure, one can see that, considering a 20% uncertainty for the Wilson coefficients $C_1 = 1.26$ and $C_2 = -0.51$ at the m_c scale, the partial decay width of $\Gamma[\Omega_c^0 \to \Omega^-\pi^+]$ lies in the range of $(1.3, 4.0) \times 10^{-14}$ GeV, which shows a sizable decadency on the Wilson coefficients.

A. $\Omega_c^0 \to \Omega^-\pi^+$

First, we study the weak decay process $\Omega_c^0 \to \Omega^-\pi^+$. This weak decay process, as an important process, has been widely studied by the Belle, *BABAR*, CLEO, SELEX, and FOCUS Collaborations [63–69]. With the RPF, the partial decay width of $\Omega_c^0 \to \Omega^-\pi^+$ is predicted to be

$$\Gamma[\Omega_c^0 \to \Omega^-\pi^+] \simeq 2.6 \times 10^{-14} \text{ GeV}. \tag{23}$$

By using the measured lifetime $\tau = 2.68 \times 10^{-13}$ s of Ω_c^0 [1], we further predict the branching fraction

$$\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+] \simeq 1.05\%. \tag{24}$$

If adopting the MHPF, there is a ~20% correction to the results of RPF. However, when adopting the NRPF, the results are about a factor of ~6.6 smaller than that predicted with RPF. From Table II, it is found that our predicted branching fraction with both RPF and MHPF is close to the predictions in Refs. [46,70]. While, if adopting the NRPF, our predicted branching fraction $\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+] \simeq 0.16\%$ is consistent with that from the covariant confined quark model [45].

TABLE II. Predicted branching fraction for the $\Omega_c^0 \to \Omega^- \pi^+$ precess compared with that of other theoretical works.

RPF/MHPF/NRPF	Ref. [45]	Ref. [70]	Ref. [46]	Ref. [44]
1.05%/0.82%/0.16%	0.2%	1.0%	0.5%	2.3%

Furthermore, combining the predicted branching fraction of $\mathcal{B}[\Omega_c^0 \to \Omega^- \pi^+]$ with the measured relative branching ratios $\frac{\Gamma[\Omega_c^0 \to \Xi^0 \bar{K}^- \pi^+]}{\Gamma[\Omega_c^0 \to \Omega^- \pi^+]} = 1.20 \pm 0.24$ and $\frac{\Gamma[\Omega_c^0 \to \Xi^- \bar{K}^0 \pi^+]}{\Gamma[\Omega_c^0 \to \Omega^- \pi^+]} = 2.12 \pm 0.38$, the branching fractions for the three-body weak decay processes $\Omega_c^0 \to \Xi^0 \bar{K}^- \pi^+ / \Xi^- \bar{K}^0 \pi^+$ can be obtained easily. With the RPF, we have

$$\mathcal{B}[\Omega_c^0 \to \Xi^0 \bar{K}^- \pi^+] \simeq (1.26 \pm 0.27) \times 10^{-2}, \quad (25)$$

$$\mathcal{B}[\Omega_c^0 \to \Xi^- \bar{K}^0 \pi^+] \simeq (2.23 \pm 0.43) \times 10^{-2}. \quad (26)$$

When adopting the NRPF, we have small branching fractions

$$\mathcal{B}[\Omega_c^0 \to \Xi^0 \bar{K}^- \pi^+] \simeq (0.19 \pm 0.04) \times 10^{-2}, \quad (27)$$

$$\mathcal{B}[\Omega_c^0 \to \Xi^- \bar{K}^0 \pi^+] \simeq (0.33 \pm 0.06) \times 10^{-2}, \quad (28)$$

due to the small nonrelativistic phase-space factor.

B. $\Omega_c^0 \to \Omega^-(1P)\pi^+$

In the Ω family, there are two $1P$-wave states $\Omega(1^2P_{1/2^-})$ and $\Omega(1^2P_{3/2^-})$ with spin parity $J^P = 1/2^-$ and $J^P = 3/2^-$, respectively. The newly observed $\Omega(2012)$ resonance may favor the assignment of $\Omega(1^2P_{3/2^-})$ state, since both the measured mass and width are consistent with the quark model predictions [11–15]. The masses of the unestablished $\Omega^*(X)$ states are taken with the predictions in Ref. [11], which have been collected in Table I. However, the $\Omega(1^2P_{1/2^-})$ classified in the quark model is still missing.

Considering $\Omega(2012)$ as the $\Omega(1^2P_{3/2^-})$ assignment, we have studied the $\Omega_c^0 \to \Omega^-(2012)\pi^+$ process, and the results are listed in Table III. It is found that the Ω_c baryon has a fairly large decay rate into $\Omega(2012)^-\pi^+$; with the RPF or MHPF, the branching fraction is predicted to be

$$\mathcal{B}[\Omega_c^0 \to \Omega(2012)^- \pi^+] \simeq 2.2 \times 10^{-3}. \quad (29)$$

Combining it with the branching fraction of $\mathcal{B}[\Omega_c^0 \to \Omega^- \pi^+]$ obtained in Eq. (24), we predict the relative ratio

$$R_1^{\mathrm{Th}} = \frac{\mathcal{B}[\Omega_c^0 \to \Omega(2012)^- \pi^+]}{\mathcal{B}[\Omega_c^0 \to \Omega^- \pi^+]} \simeq 0.22, \quad (30)$$

which is in good agreement with experimental value $R_1^{\mathrm{Exp}} = 0.220 \pm 0.059(\mathrm{stat}) \pm 0.035(\mathrm{syst})$ that was recently measured by the Belle Collaboration [24]. According to the strong decay properties of $\Omega(2012)$ predicted using the constituent quark model in Refs. [11,12], branching fractions of $\Omega(2012)$ decaying into $\Xi^0 K^-$ and $\Xi^- \bar{K}^0$ are predicted to be $\mathcal{B}[\Omega_c(2012) \to \Xi^0 K^-] \simeq 52\%$ and $\mathcal{B}[\Omega_c(2012) \to \Xi^- \bar{K}^0] \simeq 48\%$, respectively. Combining these strong branching fractions of $\Omega(2012)$ with our predicted branching fractions for the weak decay processes $\mathcal{B}[\Omega_c^0 \to \Xi^0 \bar{K}^- \pi^+ / \Xi^- \bar{K}^0 \pi^+ / \Omega(2012)^- \pi^+]$ in Eqs. (25), (26), and (29), one can obtain

TABLE III. Predicted decay properties of the $\Omega_c \to \Omega^{(*)}(X)^- \pi^+$ processes within three options of the phase space RPF, NRPF, and MHPF, respectively. Γ_i stands for the partial decay width, $\mathcal{B}$ stands for the branching fraction, and M_f stands for the mass of the final state $\Omega^{(*)}(X)$. The total width of Ω_c is $\Gamma = 2.47 \times 10^{-12}$ GeV (corresponding to lifetime $\tau = 2.68 \times 10^{-13}$ s [1]). The units for decay width Γ_i and branching ratio $\mathcal{B}$ are 10^{-15} GeV and 10^{-3}, respectively.

		RPF			NRPF			MHPF		
Final state	M_f (MeV)	Γ_i	$\mathcal{B}$	$\frac{\Gamma[\Omega_c^0 \to \Omega^{(*)}(X)^- \pi^+]}{\Gamma[\Omega_c^0 \to \Omega^- \pi^+]}$	Γ_i	$\mathcal{B}$	$\frac{\Gamma[\Omega_c^0 \to \Omega^{(*)}(X)^- \pi^+]}{\Gamma[\Omega_c^0 \to \Omega^- \pi^+]}$	Γ_i	$\mathcal{B}$	$\frac{\Gamma[\Omega_c^0 \to \Omega^{(*)}(X)^- \pi^+]}{\Gamma[\Omega_c^0 \to \Omega^- \pi^+]}$
$\Omega(1^4S_{\frac{3}{2}^+})\pi^+$	1672	26	10.5	1.0	3.8	1.6	1.0	21	8.2	1
$\Omega(1^2P_{\frac{1}{2}^-})\pi^+$	1957	9.5	3.8	0.38	2.0	0.80	0.50	8.7	3.6	0.44
$\Omega(1^2P_{\frac{3}{2}^-})\pi^+$	2012	5.4	2.2	0.22	1.2	0.49	0.31	5.2	2.1	0.26
$\Omega(2^2S_{\frac{1}{2}^+})\pi^+$	2232	1.2	5.0×10^{-1}	0.05	3.9×10^{-1}	0.16	0.01	1.5	6.3×10^{-1}	0.08
$\Omega(2^4S_{\frac{3}{2}^+})\pi^+$	2159	3.0	1.2	0.12	0.8	0.34	0.21	3.3	1.4	0.17
$\Omega(1^2D_{\frac{3}{2}^+})\pi^+$	2245	2.1×10^{-1}	8.4×10^{-2}	0.008	6.7×10^{-2}	2.7×10^{-2}	0.002	2.6×10^{-1}	1.1×10^{-1}	0.01
$\Omega(1^2D_{\frac{5}{2}^+})\pi^+$	2303	1.3×10^{-2}	5.0×10^{-3}	5.0×10^{-4}	5.4×10^{-3}	2.0×10^{-3}	1×10^{-3}	1.9×10^{-2}	7.7×10^{-3}	9.4×10^{-4}
$\Omega(1^4D_{\frac{1}{2}^+})\pi^+$	2141	3.3	1.3	0.13	8.8×10^{-1}	0.36	0.23	3.6	1.5	0.18
$\Omega(1^4D_{\frac{3}{2}^+})\pi^+$	2188	2.3	0.95	0.09	6.8×10^{-1}	0.28	0.18	2.7	1.1	0.13
$\Omega(1^4D_{\frac{5}{2}^+})\pi^+$	2252	3.3×10^{-3}	1.3×10^{-3}	1.3×10^{-4}	1.2×10^{-3}	4.5×10^{-4}	2.8×10^{-4}	4.2×10^{-3}	1.7×10^{-3}	2.1×10^{-4}
$\Omega(1^4D_{\frac{7}{2}^+})\pi^+$	2321	3.2×10^{-3}	1.3×10^{-3}	1.3×10^{-4}	1.3×10^{-3}	5.1×10^{-4}	3.2×10^{-4}	4.7×10^{-3}	1.9×10^{-3}	2.3×10^{-4}

$$R_2^{\rm Th} = \frac{\mathcal{B}[\Omega_c^0 \to \Omega(2012)\pi^+]\mathcal{B}[\Omega_c(2012) \to \Xi^0 K^-]}{\mathcal{B}[\Omega_c^0 \to \Xi^0 \bar{K}^- \pi^+]} \simeq 0.09, \tag{31}$$

$$R_3^{\rm Th} = \frac{\mathcal{B}[\Omega_c^0 \to \Omega(2012)\pi^+]\mathcal{B}[\Omega_c(2012) \to \Xi^- \bar{K}^0]}{\mathcal{B}[\Omega_c^0 \to \Xi^- \bar{K}^0 \pi^+]} \simeq 0.05, \tag{32}$$

which are also consistent with the experimental values $R_2^{\rm Exp} = 0.096 \pm 0.032({\rm stat}) \pm 0.018({\rm syst})$ and $R_3^{\rm Exp} = 0.055 \pm 0.028({\rm stat}) \pm 0.007({\rm syst})$ recently measured by the Belle Collaboration [24], respectively. It should be mentioned that these predicted relative ratios $R_i^{\rm Th}$ ($i = 1$, 2, 3) are nearly independent on the options of phase-space factor in the calculations.

Then we consider the weak decay rate of Ω_c into the other $1P$-wave state $\Omega(1^2P_{1/2^-})$ by emitting a π^+ meson. The mass of $\Omega(1^2P_{1/2^-})$ is predicted to be $\sim$1950 MeV within the lattice QCD [34] and the relativized quark models [30,31]. Experimentally, there seems to be a weak enhancement around 1950 MeV in the $\Xi\bar{K}$ invariant mass distributions from the Belle observations [10,24], which may be a hint of $\Omega(1^2P_{1/2^-})$. Hence, in the calculations the mass of $\Omega(1^2P_{1/2^-})$ is taken to be 1957 MeV. If adopting the RPF or MHPF, the branching fraction is predicted to be

$$\mathcal{B}[\Omega_c^0 \to \Omega(1^2P_{1/2^-})\pi^+] \simeq 3.8 \times 10^{-3}, \tag{33}$$

which is about a factor of 5 larger than that predicted with NRPF. The predicted branching fraction $\mathcal{B}[\Omega_c^0 \to \Omega(1^2P_{1/2^-})\pi^+]$ should be slightly larger than that of the $\Omega(2012)^-\pi^+$ final states. The branching fraction ratio between $\Omega_c^0 \to \Omega(1^2P_{1/2^-})\pi^+$ and $\Omega_c^0 \to \Omega^-\pi^+$ is predicted to be

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^2P_{1/2^-})\pi^+]}{\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+]} \simeq 0.38\text{–}0.50, \tag{34}$$

which is insensitive to options of the phase-space factor. Such a large relative branching ratio indicates that the other missing $1P$-wave state $\Omega(1^2P_{1/2^-})$ has a good potential to be observed in the weak decay process $\Omega_c^0 \to \Omega(1^2P_{1/2^-})\pi^+$.

According to the strong decay analysis in Refs. [11,12,40], the decays of $\Omega(1^2P_{1/2^-})$ should be nearly saturated by the Ξ^0K^- and $\Xi^-\bar{K}^0$ channels. Combining the strong decay properties predicted within the chiral quark model in Refs. [11,12], we can estimate the ratios

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^2P_{1/2^-})\pi^+]\mathcal{B}[\Omega(1^2P_{1/2^-}) \to \Xi^0 K^-]}{\mathcal{B}[\Omega_c^0 \to \Xi^0 K^- \pi^+]} \simeq 16\%, \tag{35}$$

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^2P_{1/2^-})\pi^+]\mathcal{B}[\Omega(1^2P_{1/2^-}) \to \Xi^- \bar{K}^0]}{\mathcal{B}[\Omega_c^0 \to \Xi^- \bar{K}^0 \pi^+]} \simeq 8\%, \tag{36}$$

which may provide useful references for future experiments.

To further explain the results of the $\Omega(1^2P_{1/2^-})$ and $\Omega(1^2P_{3/2^-})$ states, we fit the $(\bar{K}\Xi)^-$ invariant mass spectrum of the process $\Omega_c \to \pi^+\Omega^*(X) \to \pi^+(\bar{K}\Xi)^-$ measured by the Belle Collaboration [24]. In our analysis, we adopt a relativistic Breit-Wigner function to describe the event distribution [1,71–73]

$$\frac{dN}{dM_{(\bar{K}\Xi)^-}} = f_{BG} + C_R \sum_R \frac{M^2_{(\bar{K}\Xi)^-}\Gamma_{\pi^+\Omega^*(X)}(M_{(\bar{K}\Xi)^-})\Gamma_{(\Xi\bar{K})^-}(M_{(\bar{K}\Xi)^-})}{|M^2_{(\bar{K}\Xi)^-} - m_R^2 + im_R\Gamma_R|^2}, \tag{37}$$

where $M_{(\bar{K}\Xi)^-)}$ and m_R stand for the invariant mass of $(\bar{K}\Xi)^-$ and the resonance mass of $\Omega^*(X)$, respectively. $\Gamma_{\pi^+\Omega^*(X)}(M_{(\bar{K}\Xi)^-})$ and $\Gamma_{(\Xi\bar{K})^-}(M_{(\bar{K}\Xi)^-})$ are the partial decay widths of $\Omega_c^0 \to \Omega^*(X)\pi^+$ and $\Omega^*(X) \to (\Xi K)^-$, respectively. The total decay width Γ_R is adopted as the predictions obtained in Ref. [11], while f_{BG} stands for the background contributions. In this work, a linear background $f_{BG} = 18.5$ $({\rm MeV}/c^2)^{-1}$ is adopted, which is determined by fitting the backgrounds taken in Ref. [24]. Finally, C_R is a global parameter related to the resonance production rates.

In Fig. 3, we show our theoretical results for the $(\bar{K}\Xi)^-$ invariant mass distributions of the decay $\Omega_c \to \pi^+\Omega^*(X) \to \pi^+((\bar{K}\Xi)^-)$. The red curve has been adjusted to the strength of the experimental data of the Belle Collaboration [24] at the peak around 2012 MeV by taking $C_R = 0.064$. Furthermore, the dashed curve stands for the resonance contribution of $\Omega(1^2P_{3/2^-})$ with $M_R = 2012$ MeV and $\Gamma_R = 5.7$ MeV, while the dash-dotted curve stands for the $\Omega(1^2P_{1/2^-})$ contribution with $M_R = 1957$ MeV and $\Gamma_R = 12.4$ MeV. From Fig. 3, one can easily find that the $\Omega(1^2P_{3/2^-})$ state has a significant contribution around 2012 MeV and the experimental data around that energy can be well reproduced. However, the contribution of the $\Omega(1^2P_{1/2^-})$ state is overestimated compared with the experimental data around 1957 MeV.

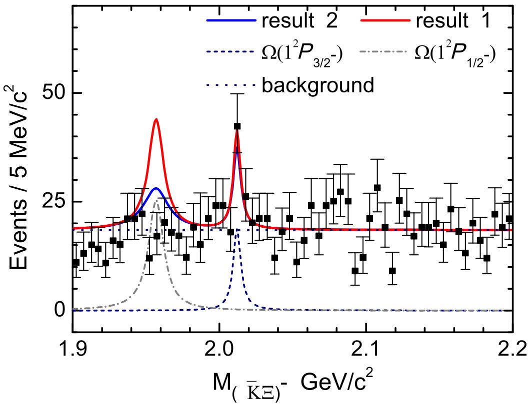

FIG. 3. The $(\bar{K}\Xi)^-$ invariant mass spectrum measured of the decay $\Omega_c \to \pi^+\Omega^*(X) \to \pi^+((\bar{K}\Xi)^-)$ by the Belle Collaboration [24] (solid squares) compared to the theoretical description with two possible $\Omega^-(1P)$-wave states, $\Omega(1^2P_{1/2^-})$ and $\Omega(1^2P_{3/2^-})$. Results 1 and 2 are fitting results of $\Omega(1^2P_{1/2^-})$ with widths about 12.4 and 20.0 MeV, respectively.

Yet, the quark model predicted widths for the $\Omega(1^2P_{1/2^-})$ and $\Omega(1^2P_{3/2^-})$ states have uncertainties; we perform a new calculation with a slightly larger width $\Gamma_R = 20.0$ MeV for $\Omega(1^2P_{1/2^-})$ state, while we take the experimental value of 6.4 MeV for $\Omega(1^2P_{3/2^-})$. The new theoretical results are also shown in Fig. 3 with a blue curve, where we see that the signal of the $\Omega(1^2P_{1/2^-})$ is much suppressed. It is expected that more precise experimental data can be used to pin down the contribution of the $\Omega(1^2P_{1/2^-})$ state in the future.

On the other hand, the $\Omega_c^0 \to \pi^+\Omega(2012) \to \pi^+K^-\Xi^0$ decay was investigated within the picture that the $\Omega(2012)$ is a molecular state in Ref. [74], where the numerical results are also consistent with the experimental data. Indeed, we need further efforts to understand the nature of the $\Omega(2012)$ state [75,76].

C. $\Omega_c^0 \to \Omega^-(1D)\pi^+$

There are six $1D$-wave states, $\Omega(1^2D_{3/2^+,5/2^+})$ and $\Omega(1^4D_{1/2^+,3/2^+,5/2^+,7/2^+})$, according to the quark quark model classification. Most of the predicted masses for the $1D$-wave states lies in the mass range $\sim 2200 \pm 50$ MeV in various quark models. Taking the mass recently predicted in Ref. [11], we calculate the weak decay properties for the $\Omega_c^0 \to \Omega^-(1D)\pi^+$ processes. Our results are listed in Table III. It is seen that Ω_c has significant branching fractions decaying into the spin quartet states $\Omega(1^4D_{1/2^+})$ and $\Omega(1^4D_{3/2^+})$. The predicted branching fractions $\mathcal{B}[\Omega_c^0 \to \Omega(1^4D_{1/2^+,3/2^+})\pi^+]$ can reach up to the order of $\sim O(10^{-4})$–$O(10^{-3})$. With the RPF, their relative ratios to $\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+]$ are predicted to be

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^4D_{1/2^+})\pi^+]}{\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+]} \simeq 0.13, \tag{38}$$

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^4D_{3/2^+})\pi^+]}{\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+]} \simeq 0.09, \tag{39}$$

which are close to the results predicted with MHPF. The predicted branching fractions and ratios are comparable with those of Ω_c decaying into the $\Omega(2012)\pi^+$ and $\Omega(1^2P_{1/2^-})\pi^+$ channels. However, the decay rates of Ω_c into the other four $1D$-wave states $\Omega(1^2D_{3/2^+,5/2^+})$ and $\Omega(1^4D_{5/2^+,7/2^+})$ are ∼1–3 orders of magnitude smaller. The relatively large decay rates indicate that both $\Omega(1^4D_{1/2^+})$ and $\Omega(1^4D_{3/2^+})$ have good potential to be established by using the weak decay processes $\Omega_c^0 \to \Omega(1^4D_{1/2^+,3/2^+})\pi^+$.

We further analyze the reasons of the small decay rates of $\Omega_c^0 \to \Omega(1^4D_{5/2^+,7/2^+})\pi^+/\Omega(1^2D_{3/2^+,5/2^+})\pi^+$ compared with that of $\Omega_c^0 \to \Omega(1^4D_{1/2^+,3/2^+})\pi^+$ as follows. We note that the helicity transition amplitudes

$$\mathcal{M}_{JJ_z;\frac{1}{2}-\frac{1}{2}} \propto \sum_{M_L+S_z=J_z} \langle LM_LSS_z|JJ_z\rangle\langle\Psi^\sigma_{NLM_L}\chi^\sigma_{S_z}|\hat{O}|\Psi_{\Omega_c}\chi^\lambda_{-\frac{1}{2}}\rangle, \tag{40}$$

where Ψ_{Ω_c} (Ψ_{NLM_L}) and $\chi^\lambda_{-\frac{1}{2}}$ ($\chi^\sigma_{S_z}$) are the spatial and spin wave functions of the initial (final) baryons, respectively. For the decay processes involving the spin quartet states $\Omega(1^4D_{1/2^+,3/2^+,5/2^+,7/2^+})$, the decay amplitude is the sum of $c_1\langle\Psi^S_{221}\chi^S_{-3/2}|\hat{O}|\Psi_{\Omega_c}\chi^\lambda_{-\frac{1}{2}}\rangle$ and $c_2\langle\Psi^S_{220}\chi^S_{-1/2}|\hat{O}|\Psi_{\Omega_c}\chi^\lambda_{-\frac{1}{2}}\rangle$. These two terms have strong constructive and destructive interference for $\Omega_c^0 \to \Omega(1^4D_{1/2^+,3/2^+})\pi^+$ and $\Omega_c^0 \to \Omega(1^4D_{5/2^+,7/2^+})\pi^+$, respectively. Thus, the decay rates of $\Omega_c^0 \to \Omega(1^4D_{5/2^+,7/2^+})\pi^+$ are strongly suppressed by the destructive interference between the two terms of the helicity transition amplitude, while for the decay processes involving the spin doublet $\Omega(1^2D_{3/2^+,5/2^+})$, the decay amplitudes are proportional to $\langle\Psi^{\rho,\lambda}_{220}\chi^{\rho,\lambda}_{-1/2}|\hat{O}|\Psi_{\Omega_c}\chi^\lambda_{-\frac{1}{2}}\rangle$. In this term, the contribution from the part of the spin wave functions is about a factor of 2–4 smaller than that for the spin quartet states. Thus, the decay rates of $\Omega_c^0 \to \Omega(1^2D_{3/2^+,5/2^+})\pi^+$ is suppressed by the relative small overlapping of the spin wave functions of the initial and final states.

According to the analysis of the strong decay properties [11,12], the $\Omega(1^4D_{1/2^+})$ state has a width of $\Gamma \simeq 42$ MeV and dominantly decays into the $\Xi\bar{K}$ channel with a branching fraction ∼94%, while the $\Omega(1^4D_{3/2^+})$ has a width of $\Gamma \simeq 31$ MeV and dominantly decays into $\Xi\bar{K}$ with a branching fraction ∼64%. Thus, the Ξ^0K^- and $\Xi^-\bar{K}^0$ final states can be used to look for the $\Omega(1^4D_{1/2^+})$ and

$\Omega(1^4D_{3/2^+})$ states if they are produced by the Ω_c weak decays. For the $\Omega(1^4D_{1/2^+})$ state, by combining the results of RPF we can estimate the following ratios:

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^4D_{1/2^+})\pi^+]\mathcal{B}[\Omega(1^4D_{1/2^+}) \to \Xi^0 K^-]}{\mathcal{B}[\Omega_c^0 \to \Xi^0 \bar{K}^- \pi^+]} \simeq 5\%, \tag{41}$$

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^4D_{1/2^+})\pi^+]\mathcal{B}[\Omega(1^4D_{1/2^+}) \to \Xi^- \bar{K}^0]}{\mathcal{B}[\Omega_c^0 \to \Xi^- \bar{K}^0 \pi^+]} \simeq 3\%, \tag{42}$$

while for the $\Omega(1^4D_{3/2^+})$ state we can estimate the following ratios:

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^4D_{3/2^+})\pi^+]\mathcal{B}[\Omega(1^4D_{3/2^+}) \to \Xi^0 K^-]}{\mathcal{B}[\Omega_c^0 \to \Xi^0 \bar{K}^- \pi^+]} \simeq 2\%, \tag{43}$$

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(1^4D_{3/2^+})\pi^+]\mathcal{B}[\Omega(1^4D_{3/2^+}) \to \Xi^- \bar{K}^0]}{\mathcal{B}[\Omega_c^0 \to \Xi^- \bar{K}^0 \pi^+]} \simeq 1\%. \tag{44}$$

The above predicted ratios are less dependent on the options of the phase-space factor.

Finally, it should be pointed out that the predicted masses of the $1D$-wave Ω states have some model dependencies. To see the effects from the mass uncertainties of the $1D$-wave Ω states on our predicted weak decay properties, we plot the weak branching fractions of $\Omega_c^0 \to \pi^+\Omega^*(X)$ as functions of the masses of the $1D$-wave Ω excited state in their possible range $M \in (2.1\text{–}2.3)$ GeV in Fig. 4. It is seen that, in the most possible mass range $\sim 2200 \pm 50$ MeV, the upper limit of our predicted partial widths is about a factor of 2 larger than that of the lower limit.

D. $\Omega_c^0 \to \Omega(2S)\pi^+$

In the constituent quark model, there are two $2S$-wave states $\Omega(2^2S_{1/2^+})$ and $\Omega(2^4S_{3/2^+})$. There are large uncertainties in the predictions of their masses in various quark models. The predicted masses scatter in the range of ∼2.10–2.30 GeV. In Fig. 4, by using the RPF we plot the weak decay widths of the $\Omega_c^0 \to \Omega(2^2S_{1/2^+}/2^4S_{3/2^+})\pi^+$ processes as functions of the masses of the $2S$-wave Ω states. It is seen that in the mass range 2100–2300 MeV, for the weak decay process $\Omega_c^0 \to \Omega(2^2S_{1/2^+})\pi^+$, the partial decay width is predicted to be $\Gamma[\Omega_c^0 \to \Omega(2^2S_{1/2^+})\pi^+] \simeq (1.2 \pm 0.45) \times 10^{-15}$ GeV, and the branching fraction can reach up to

$$\mathcal{B}[\Omega_c^0 \to \Omega(2^2S_{1/2^+})\pi^+] \simeq (0.50 \pm 0.18) \times 10^{-3}. \tag{45}$$

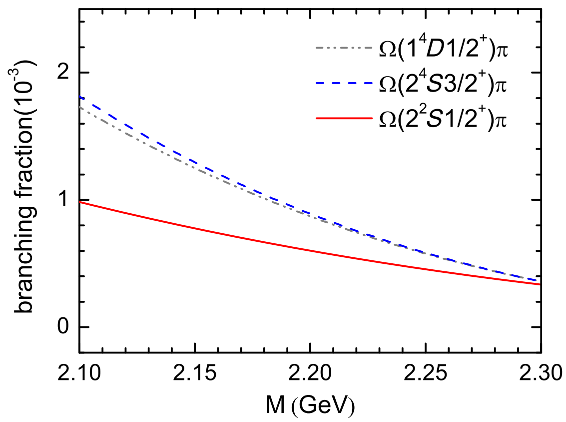

FIG. 4. The branching fraction of the $\Omega_c^0 \to \Omega(1^4D_{1/2^+}, 1^4D_{3/2^+}, 2^2S_{1/2^+}, 2^4S_{3/2^+})\pi^+$ as a function of the mass of the final state $\Omega^*(X)$. It should be noted that, since the results of $1^4D_{1/2^+}$ and $1^4D_{3/2^+}$ are the same, we omit the results of $1^4D_{3/2^+}$ here.

Combined with the predicted branching fraction $\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+] \simeq 10\%$, we obtain the relative branching ratio

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(2^2S_{1/2^+})\pi^+]}{\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+]} \simeq 0.05 \pm 0.02. \tag{46}$$

The production rate of $\Omega(2^2S_{1/2^+})$ via the Ω_c weak decay is about a factor of 5–6 smaller than that of Ω(2012). Because of the large decay rate into the $\Xi(1530)\bar{K}$ channel [11,12], the $\Omega(2^2S_{1/2^+})$ state is suggested to be searched for in the decay chain $\Omega_c^0 \to \Omega(2^2S_{1/2^+})\pi^+ \to (\Xi(1530)K)^-\pi^+ \to (\Xi\pi K)^-\pi^+$ in future experiments.

For the other weak decay process $\Omega_c^0 \to \Omega(2^4S_{3/2^+})\pi^+$, by using the RPF the partial decay width is predicted to be $\Gamma[\Omega_c^0 \to \Omega(2^4S_{3/2^+})\pi^+] \simeq (3.0 \pm 1.6) \times 10^{-15}$ GeV, and the branching fraction can reach up to

$$\mathcal{B}[\Omega_c^0 \to \Omega(2^4S_{3/2^+})\pi^+] \simeq (1.2 \pm 0.6) \times 10^{-3}. \tag{47}$$

Similarly, the relative branching ratio is predicted to be

$$\frac{\mathcal{B}[\Omega_c^0 \to \Omega(2^4S_{3/2^+})\pi^+]}{\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+]} \simeq 0.12 \pm 0.06. \tag{48}$$

The production rate of $\Omega(2^4S_{3/2^+})$ via the Ω_c weak decay is comparable with that of $\Omega(2^2S_{1/2^+})$. The dominant decay mode of $\Omega(2^4S_{3/2^+})$ is the $\Xi(1530)\bar{K}$ channel, and one can look for it in the decay chain $\Omega_c^0 \to \Omega(2^4S_{3/2^+})\pi^+ \to (\Xi(1530)\bar{K})^-\pi^+ \to (\Xi\pi\bar{K})^-\pi^+$.

IV. SUMMARY

In this work, we calculate the Cabibbo-favored weak decay processes $\Omega_c \to \Omega^{(*)}(X)\pi^+$ within a constituent quark model. Our predicted branching fraction $\mathcal{B}[\Omega_c^0 \to \Omega^-\pi^+] \simeq 1.05\%$, which is in agreement with the early predictions in orders in Refs. [44,70]. Considering the newly observed $\Omega(2012)$ resonance as the conventional $\Omega(1^2P_{3/2^-})$ state, it is found that the measured ratio $\mathcal{B}[\Omega_c \to \Omega(2012)\pi^+ \to (\Xi\bar{K})^-\pi^+]/\mathcal{B}[\Omega_c \to \Omega\pi^+] = 0.220 \pm 0.059(\text{stat}) \pm 0.035(\text{syst})$ at Belle can be well understood within our model calculations here. The production potentials of the missing low-lying $1P$-, $2S$-, and $1D$-wave resonances $\Omega^*(X)$ via the hadronic weak decays of Ω_c are discussed as well. Our main conclusions are summarized as follows.

(i) The missing $1P$-wave state $\Omega(1^2P_{1/2^-})$ has a large potential to be observed in the decay chain $\Omega_c^0 \to \Omega(1^2P_{1/2^-})\pi^+ \to (\Xi\bar{K})^-\pi^+$. The production rate of $\Omega(1^2P_{1/2^-})$ via the hadronic weak decays of Ω_c is even slightly larger than that of $\Omega(2012)$.

(ii) For the $1D$-wave Ω states, we find that both $\Omega(1^4D_{1/2^+})$ and $\Omega(1^4D_{3/2^+})$ have fairly large production rates via the $\Omega_c \to \Omega(1^4D_{1/2^+})\pi^+$ and $\Omega_c \to \Omega(1^4D_{3/2^+})\pi^+$ processes, respectively. Their production rates via the hadronic weak decays of Ω_c are comparable with those of the $1P$-wave Ω states. Both $\Omega(1^4D_{1/2^+})$ and $\Omega(1^4D_{3/2^+})$ are most likely to be observed in the process $\Omega_c^0 \to \Omega(1^4D_{1/2^+,3/2^+})\pi^+ \to (\Xi\bar{K})^-\pi^+$.

(iii) The $2S$ states $\Omega(2^2S_{1/2^+})$ and $\Omega(2^4S_{3/2^+})$ also have fairly large production rates via the hadronic weak decays of Ω_c. Their production rates are about a factor of 5–6 smaller than that of $\Omega(2012)$. Both $\Omega(2^2S_{1/2^+})$ and $\Omega(2^4S_{3/2^+})$ dominantly decay into the $\Xi(1530)K$ channel; thus, they can be looked for in the decay chains $\Omega_c^0 \to \Omega(2^2S_{1/2^+})\pi^+/\Omega(2^4S_{3/2^+})\pi^+ \to (\Xi(1530)\bar{K})^-\pi^+ \to (\Xi\pi\bar{K})^-\pi^+$.

Finally, it should be mentioned that our predicted partial widths for the weak decay processes $\Omega_c \to \Omega^{(*)}\pi^+$ may have large uncertainties due to relativistic effects. To roughly see the uncertainties from the relativistic corrections, we perform our calculations with the three typical phase-space options: the relativistic phase space, the nonrelativistic phase space, and the "mock-hadron" phase space. The predicted partial widths with the nonrelativistic phase space are a factor of ∼2–6 smaller those calculated with the usual relativistic phase space and the mock-hadron phase space.

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