

Doubly charmed baryon decays $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ in the quark model

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In this work we study the doubly charmed baryon decays $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ within the framework of the nonrelativistic quark model (NRQM). Factorizable amplitudes are expressed in terms of transition form factors, while nonfactorizable amplitudes arising from the inner W emission are evaluated using current algebra and the pole model and expressed in terms of baryonic matrix elements and axial-vector form factors. Nonperturbative parameters are then calculated using the NRQM. They can be expressed in terms of the momentum integrals of baryon wave functions, which are in turn expressed in terms of the harmonic oscillator parameters α_ρ and α_λ for ρ - and λ -mode excitation. The measured ratio R of the branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$ relative to $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ can be accommodated in the NRQM with $\alpha_{\rho 1}$ and $\alpha_{\rho 2}$ being in the vicinity of 0.51 and 0.19, respectively, where $\alpha_{\rho 1}$ is the α_ρ parameter for Ξ_{cc}^{++} and $\alpha_{\rho 2}$ for $\Xi_c^{(\prime)+}$. Decay asymmetries are predicted to be -0.78 and -0.89 for $\Xi_c^+ \pi^+$ and $\Xi_c^{+\prime} \pi^+$ modes, respectively, which can be tested in the near future. We compare our results with other works and point out that although some other models can accommodate the ratio R , they tend to lead to a branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ too large compared to that inferred from the LHCb measurement of its rate relative to $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$.

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I. INTRODUCTION

Hadronic decays of the doubly charmed baryon Ξ_{cc}^{++} have been measured through the decays $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ [1] and $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ [2]. Recently, the decay $\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$ was first observed by LHCb [3] and its branching fraction relative to that of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ was also reported

$$R \equiv \frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)} = 1.41 \pm 0.17 \pm 0.10, \quad (1.1)$$

while the branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ relative to $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ was measured to be [2]

$$\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) \times \mathcal{B}(\Xi_c^+ \rightarrow p K^- \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+) \times \mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+)} = 0.035 \pm 0.009(\text{stat}) \pm 0.003(\text{syst}). \quad (1.2)$$

Both two-body decay modes $\Xi_c^+ \pi^+$ and $\Xi_c^{+\prime} \pi^+$ proceed through the topological diagrams, external W -emission T and inner W -emission C' (see Fig. 1).¹ Many early studies focused only on the factorizable contribution from T [4–7]. It turns out that light-front quark model [4,6] and QCD sum rules [7] lead to a rate of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ larger than that of $\Xi_{cc}^{++} \rightarrow \Xi_c^{+\prime} \pi^+$. This implies that factorizable contributions alone will yield $R < 1$. Nonfactorizable inner W -emission C' has been considered in Refs. [8–11] and partially in Ref. [12]. In Ref. [11], nonfactorizable effects were estimated based on the final-state rescattering. The interference between T and C' in $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ was found to be destructive in Refs. [8–10] but constructive in Ref. [11].

¹Color-suppressed internal W -emission diagram is denoted by C .

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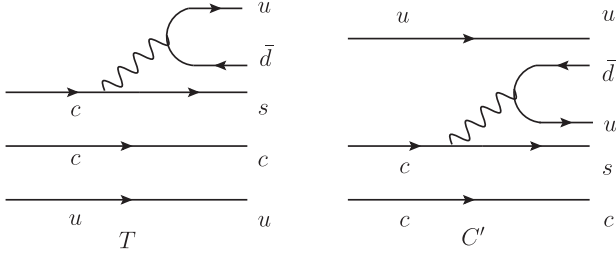


FIG. 1. Topological diagrams contributing to $\Xi_{cc}^{++} \rightarrow \Xi_c^{(l)+} \pi^+$ decays: external W -emission T and inner W -emission C' .

On the contrary, a large constructive interference in the P -wave amplitude was obtained in Ref. [12], while non-factorizable corrections to the S -wave one were not considered (see Table III below).

We have mentioned that factorizable contributions alone will usually lead to $R < 1$. A possibility of accounting for the observation of $R > 1$ is to consider the $\Xi_c^+ - \Xi_c'^+$ mixing

$$\begin{aligned} |\Xi_c^+\rangle &= \cos\theta |\Xi_c^{\bar{3}}\rangle + \sin\theta |\Xi_c^6\rangle, \\ |\Xi_c'^+\rangle &= -\sin\theta |\Xi_c^{\bar{3}}\rangle + \cos\theta |\Xi_c^6\rangle, \end{aligned} \quad (1.3)$$

where Ξ_c and Ξ_c' are physical states, and $\Xi_c^{\bar{3}(6)}$ are antitriplet (sextet) charmed baryons. As pointed out in Ref. [13], the ratio $R = 0.56$ predicted in Ref. [6] can be enhanced to 1.41 when the mixing angle θ is either 16.27° or 85.54° . However, we have to keep in mind that the effect of inner W emission needs to be taken into account eventually. Recently, the effect of $\Xi_c - \Xi_c'$ mixing was studied in Ref. [14] in an attempt of resolving the tension between the experimental measurements and theoretical expectations in $\Xi_c^0 \rightarrow \Xi_c^- e^+ \nu_e$. The mixing angle was found to be $\theta = \pm 0.137(5)\pi = \pm(24.7 \pm 0.9)^\circ$.

Most of the studies in the literature aim at the accommodation of the ratio R . Although the absolute branching fractions have not been directly measured, we nevertheless can get some information on $\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)$. Using the measurements $\mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+) = (6.28 \pm 0.32)\%$ and $\mathcal{B}(\Xi_c^+ \rightarrow p K^- \pi^+) = (0.62 \pm 0.30)\%$ [15], it follows from Eq. (1.2) that

$$\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+)} = 0.35 \pm 0.20. \quad (1.4)$$

As pointed out in Ref. [9], it is plausible to assume that $\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+) \approx \frac{2}{3} \mathcal{B}(\Xi_{cc}^{++} \rightarrow \Sigma_c^{++} \bar{K}^{*0})$. Since $\Xi_{cc}^{++} \rightarrow \Sigma_c^{++} \bar{K}^{*0}$ is a purely factorizable process, its rate can be reliably estimated once the relevant form factors are determined. Taking the latest prediction

$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Sigma_c^{++} \bar{K}^{*0}) = 5.61\%$ from [16] as an example, we obtain²

$$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)_{\text{expt}} \approx (1.33 \pm 0.74)\%. \quad (1.5)$$

Therefore, there exist two constraints: the ratio R and the absolute branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ inferred from the LHCb measurement of its rate relative to $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$.

In Ref. [9] we have considered the two-body decays of doubly charmed baryons within the framework of the MIT bag model. The branching fractions of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ and $\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$ were found to be 3.60% and 4.65%, respectively. At this level, $R = 1.29$. Because of a large destructive interference between T and C' occurred in the former mode, its branching fraction is reduced from 3.60% to 0.69%, whereas the latter mode is almost not affected by the internal W emission owing to the Pati-Woo theorem [18]. Although the final branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ is consistent with Eq. (1.5), the ratio R is enhanced from 1.29 to 6.74, which is evidently too large compared to the experiment. Since the interference is destructive in $\Xi_c^+ \pi^+$ and negligible in $\Xi_c'^+ \pi^+$, this means that in order to account for the measured value of R , one should have $R < 1$ before the inner W emission is turned on.

Very recently, it was pointed out in Ref. [19] that the difficulty with the bag model calculation can be overcome by considering the $\Xi_c^+ - \Xi_c'^+$ mixing. At $\theta = -24.7^\circ$, one will have branching fractions 2.24% and 3.25%, respectively, for $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ and $\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$. Hence $R = 1.45$ is accommodated nicely and the rate of $\Xi_c^+ \pi^+$ is consistent with Eq. (1.5).

To explore the possibility of accounting for both the ratio R and the absolute branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ inferred from Eq. (1.2) within a phenomenological model, in this work we shall focus on the nonrelativistic quark model (NRQM) to see if we can achieve both aforementioned goals. This paper is organized as follows. In Sec. II we follow Ref. [9] to express the factorizable and non-factorizable amplitudes of $\Xi_{cc}^{++} \rightarrow \Xi_c^{(l)+} \pi^+$ decays in terms of the form factors and baryonic matrix elements that in turn are evaluated using the NRQM. Numerical results are presented in Sec. III. We summarize our results in Sec. IV. Appendix A recapitulates the essences of the NRQM. Derivations of the nonperturbative parameters in the quark model are shown in Appendix B.

II. FORMALISM

The amplitude of the two-body baryonic weak decay $\mathcal{B}_i \rightarrow \mathcal{B}_f P$ is given by

$$M(\mathcal{B}_i \rightarrow \mathcal{B}_f P) = i\bar{u}_f (A - B\gamma_5) u_i, \quad (2.1)$$

²Our previous number $(1.83 \pm 1.01)\%$ given in Ref. [9] is modified as the world average of the branching fraction of $\Xi_c^+ \rightarrow p K^- \pi^+$ has been updated due to a new measurement from LHCb [17].

where $\mathcal{B}_i(\mathcal{B}_f)$ is the initial (final) baryon and P is a pseudoscalar meson. The decay width and up-down decay symmetry have the expressions

$$\Gamma = \frac{p_c}{8\pi} \left[\frac{(m_i + m_f)^2 - m_P^2}{m_i^2} |A|^2 + \frac{(m_i - m_f)^2 - m_P^2}{m_i^2} |B|^2 \right],$$

$$\alpha = \frac{2\kappa \text{Re}(A^*B)}{|A|^2 + \kappa^2 |B|^2}, \quad (2.2)$$

with $\kappa = p_c/(E_f + m_f)$, where p_c is the c.m. momentum in the rest frame of the mother baryon. The S - and P -wave amplitudes of the two-body decay generally receive both factorizable and nonfactorizable contributions

$$A = A^{\text{fac}} + A^{\text{nf}}, \quad B = B^{\text{fac}} + B^{\text{nf}}. \quad (2.3)$$

For doubly charmed baryon decays $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$, the relevant effective Hamiltonian reads

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cs} (c_1 O_1 + c_2 O_2) + \text{H.c.},$$

$$O_1 = (\bar{s}c)(\bar{u}d), \quad O_2 = (\bar{u}c)(\bar{s}d),$$

$$(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2. \quad (2.4)$$

Factorizable amplitudes read

$$A^{\text{fac}} = \frac{G_F}{\sqrt{2}} a_{1,2} V_{ud}^* V_{cs} f_P (m_{\mathcal{B}_{cc}} - m_{\mathcal{B}_c}) f_1(q^2),$$

$$B^{\text{fac}} = -\frac{G_F}{\sqrt{2}} a_{1,2} V_{ud}^* V_{cs} f_P (m_{\mathcal{B}_{cc}} + m_{\mathcal{B}_c}) g_1(q^2), \quad (2.5)$$

where f_1 and g_1 are the form factors defined by

$$\langle \mathcal{B}_c(p_2) | \bar{c} \gamma_\mu (1 - \gamma_5) u | \mathcal{B}_{cc}(p_1) \rangle$$

$$= \bar{u}_2 [f_1(q^2) \gamma_\mu - g_1(q^2) \gamma_\mu \gamma_5 + \dots] u_1. \quad (2.6)$$

For nonfactorizable contributions we follow Ref. [9] to evaluate them using current algebra and the pole model. The expressions are

$$A^{\text{nf}}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) = \frac{1}{f_\pi} (-a_{\Xi_c^+ \Xi_{cc}^{++}}),$$

$$A^{\text{nf}}(\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+) = \frac{1}{f_\pi} (-a_{\Xi_c'^+ \Xi_{cc}^{++}}),$$

$$B^{\text{nf}}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) = \frac{1}{f_\pi} \left(a_{\Xi_c^+ \Xi_{cc}^{++}} \frac{m_{\Xi_{cc}^{++}} + m_{\Xi_c^+}}{m_{\Xi_c^+} - m_{\Xi_{cc}^{++}}} g_{\Xi_c^+ \Xi_{cc}^{++}}^{A(\pi^+)} \right),$$

$$B^{\text{nf}}(\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+) = \frac{1}{f_\pi} \left(a_{\Xi_c'^+ \Xi_{cc}^{++}} \frac{m_{\Xi_{cc}^{++}} + m_{\Xi_c'^+}}{m_{\Xi_c'^+} - m_{\Xi_{cc}^{++}}} g_{\Xi_c'^+ \Xi_{cc}^{++}}^{A(\pi^+)} \right), \quad (2.7)$$

where $a_{\mathcal{B}_f \mathcal{B}_i} \equiv \langle \mathcal{B}_f | \mathcal{H}_{\text{eff}}^{\text{PC}} | \mathcal{B}_i \rangle$ are baryonic matrix elements with $\mathcal{H}_{\text{eff}}^{\text{PC}}$ being the parity-conserving part of the effective Hamiltonian and $g_{\mathcal{B}' \mathcal{B}}^{A(P)}$ are axial-vector form factors. The matrix element can be recast to the form

$$a_{\mathcal{B}_f \mathcal{B}_i} = \frac{G_F}{2\sqrt{2}} V_{ud}^* V_{cs} c_\pm \langle \mathcal{B}_f | O_\pm^{\text{PC}} | \mathcal{B}_i \rangle, \quad (2.8)$$

where $c_\pm = c_1 \pm c_2$ and $O_\pm = (\bar{s}c)(\bar{u}d) \pm (\bar{s}d)(\bar{u}c)$.

In the NRQM, the nonperturbative parameters f_1 , g_1 , $g_{\mathcal{B}' \mathcal{B}}^{A(P)}$, and $\langle \mathcal{B}_f | O_- | \mathcal{B}_i \rangle$ can be expressed in terms of the momentum integrals of baryon wave functions X , Y , and Z given in Eq. (B11) (see Appendix B for details)

$$f_1 = \langle \mathcal{B}_f \uparrow | b_u^\dagger b_c | \mathcal{B}_i \uparrow \rangle X,$$

$$g_1 = \langle \mathcal{B}_f \uparrow | b_u^\dagger b_c \sigma_z | \mathcal{B}_i \uparrow \rangle X,$$

$$g_{\mathcal{B}' \mathcal{B}}^{A(P)} = \langle \mathcal{B}_f \uparrow | b_d^\dagger b_u \sigma_z | \mathcal{B}_i \uparrow \rangle Y,$$

$$\langle \mathcal{B}_f | (\bar{q}_1 q_2)(\bar{q}_3 q_4) | \mathcal{B}_i \rangle = \langle \mathcal{B}_f \uparrow | (b_{q_1}^\dagger b_{q_2})_1 (b_{q_3}^\dagger b_{q_4})_2$$

$$\times (1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) | \mathcal{B}_i \uparrow \rangle Z, \quad (2.9)$$

where the subscripts 1 and 2 appearing in the last line indicate that the quark operator acts only on the first and second quarks, respectively, in the baryon wave function. The coefficients $\langle \mathcal{B}_f \uparrow | \dots | \mathcal{B}_i \uparrow \rangle$ depend on the spin-flavor functions of baryons and they are displayed in Table I. The momentum integrals can be expressed in terms of the harmonic oscillator parameters α_ρ and α_λ for ρ - and λ -mode excitation, respectively. We shall use $\alpha_{\rho 1}$, $\alpha_{\lambda 1}$ for Ξ_{cc}^{++} , $\alpha_{\rho 2}$, $\alpha_{\lambda 2}$ for both Ξ_c^+ and $\Xi_c'^+$, and $\alpha_{\rho 3}$, $\alpha_{\lambda 3}$ for Ξ_c^+ . Explicitly (see Appendix B),

$$X = \left(\frac{16(m_s + m_u)^2 \alpha_{\lambda 1} \alpha_{\lambda 2} \alpha_{\rho 1} \alpha_{\rho 2}}{D_1 + D_2} \right)^{3/2},$$

$$Y = 8 \left(\frac{\alpha_{\lambda 1} \alpha_{\lambda 3} \alpha_{\rho 1} \alpha_{\rho 3}}{(\alpha_{\lambda 1}^2 + \alpha_{\lambda 3}^2)(\alpha_{\rho 1}^2 + \alpha_{\rho 3}^2)} \right)^{3/2},$$

$$Z = 128 \sqrt{2} \pi^{3/2} \left[\frac{\alpha_{\lambda 2} \alpha_{\lambda 3} \alpha_{\rho 2} \alpha_{\rho 3}}{4\alpha_{\lambda 2}^2 + \alpha_{\lambda 3}^2 + 4\alpha_{\rho 3}^2} \right]^{3/2}, \quad (2.10)$$

TABLE I. Nonperturbative parameters relevant for $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ decays in the NRQM.

	$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$	$\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$
f_1	$\frac{\sqrt{6}}{2} X$	$\frac{\sqrt{2}}{2} X$
g_1	$\frac{1}{\sqrt{6}} X$	$\frac{5\sqrt{2}}{6} X$
$g_{\Xi_c^+ \Xi_{cc}^{++}}^{A(\pi^+)}$	$-\frac{1}{3} Y$	$-\frac{1}{3} Y$
$\langle \mathcal{B}_f O_- \mathcal{B}_i \rangle$	$4\sqrt{6} Z$	0

TABLE II. Form factors f_1 , g_1 , the axial-vector form factor $g_{\mathcal{B}'\mathcal{B}}^{A(\pi)}$ and baryonic matrix elements $\langle \mathcal{B}_f | O_- | \mathcal{B}_i \rangle$ calculated in the NRQM with the specified harmonic oscillator parameters $(\alpha_{\rho_1}, \alpha_{\rho_2})$ in units of GeV. The results of Ref. [9] obtained from the bag model are also shown here for comparison.

	$(\alpha_{\rho_1}, \alpha_{\rho_2})$	$f_1(m_p^2)$	$g_1(m_p^2)$	$g_{\mathcal{B}'\mathcal{B}}^{A(\pi)}$	$\langle \mathcal{B}_f O_- \mathcal{B}_i \rangle$
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$					
Case 1	(0.50, 0.21)	0.709	0.236	-0.333	0.0310
Case 2	(0.51, 0.19)	0.574	0.191	-0.333	0.0247
Case 3	(0.53, 0.17)	0.425	0.141	-0.333	0.0191
Cheng <i>et al.</i> [9]		0.577	0.222	-0.217	0.0214
$\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$					
Case 1	(0.50, 0.21)	0.397	0.662	-0.333	0
Case 2	(0.51, 0.19)	0.323	0.538	-0.333	0
Case 3	(0.53, 0.17)	0.240	0.400	-0.333	0
Cheng <i>et al.</i> [9]		0.386	0.703	-0.217	8.4×10^{-5}

where

$$D_1 = (m_s + m_u)^2 [4\alpha_{\rho_1}^2 \alpha_{\rho_2}^2 + \alpha_{\lambda_1}^2 (\alpha_{\rho_1}^2 + 4\alpha_{\rho_2}^2)],$$

$$D_2 = \alpha_{\lambda_2}^2 [(2m_s + m_u)^2 \alpha_{\lambda_1}^2 + 4(m_u^2 \alpha_{\rho_1}^2 + [m_s + m_u]^2 \alpha_{\rho_2}^2)],$$
(2.11)

and

$$\alpha_{\lambda_1} = \left[\frac{16m_u}{3(2m_c + m_u)} \right]^{\frac{1}{4}} \alpha_{\rho_1}, \quad \alpha_{\lambda_3} = \left[\frac{16m_d}{3(2m_c + m_d)} \right]^{\frac{1}{4}} \alpha_{\rho_3},$$

$$\alpha_{\lambda_2} = \left[\frac{4m_c(m_s + m_u)^2}{3m_s m_u (m_s + m_u + m_c)} \right]^{\frac{1}{4}} \alpha_{\rho_2}.$$
(2.12)

Thus only α_{ρ_1} , α_{ρ_2} , and α_{ρ_3} are independent.

III. NUMERICAL RESULTS

In the NRQM we shall take the parameters as follows: $m_s = 0.45$ GeV, $m_u = m_d = 0.33$ GeV for light quark masses [20] and $m_c = 1.6$ GeV for the charm quark mass. The parameter α_{ρ_1} is a harmonic oscillator parameter in the spatial wave function of the ρ -mode excitation between the two charm quarks. It has been taken to be $\alpha_{\rho_1} = 0.47$ GeV as in the charmonium system [21].³ The parameter α_{ρ_2} was determined to be 0.25 GeV in Ref. [22]. Notice that for Λ_c^+ , α_ρ ranges from 0.26 to 0.32 GeV [20,23,24]. For α_{ρ_3} , we shall take $\alpha_{\rho_3} = \alpha_{\rho_1}$ as it should be the same as α_{ρ_1} in the isospin limit.

³It should be stressed that a set of the Jacobi coordinate defined in Eq. (A16) has been adopted in Ref. [21] so that the harmonic oscillator parameters $\tilde{\alpha}_\rho$ and $\tilde{\alpha}_\lambda$ respect the relation $\tilde{\alpha}_\lambda = [3m_q/(2m_Q + m_q)]^{1/4} \tilde{\alpha}_\rho$. The tilde and untild harmonic oscillator parameters are related through Eq. (A18). We have translated the result of $\tilde{\alpha}_{\rho_1} = 0.66$ GeV in Ref. [21] into $\alpha_{\rho_1} = 0.47$ GeV.

Form factors f_1 and g_1 , the axial-vector form factor $g_{\mathcal{B}'\mathcal{B}}^{A(\pi)}$, and baryonic matrix elements $\langle \mathcal{B}_f | O_- | \mathcal{B}_i \rangle$ are calculated in the NRQM using Eq. (2.9) and Table I. The numerical results are exhibited in Table II with several specified harmonic oscillator parameters $(\alpha_{\rho_1}, \alpha_{\rho_2})$ to be discussed below. We also show the bag model results obtained in Ref. [9] for comparison. Notice that the matrix element for $\Xi_{cc}^{++} \rightarrow \Xi_c'^+$ transition receives contributions only from the small component of the quark wave function in the bag model and hence it vanishes in the NRQM. In the bag model the matrix element for $\Xi_{cc}^{++} \rightarrow \Xi_c'^+$ transition is nonzero, but it is quite suppressed relative to the matrix element $a_{\Xi_c^+ \Xi_{cc}^{++}}$.

We plot in Fig. 2 the allowed regions for the harmonic oscillator parameters α_{ρ_1} and α_{ρ_2} constrained by the ratio of branching fractions R [see Eq. (1.1)] and the absolute branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ inferred from Eq. (1.5). It is clear that the allowed range of 0.505–0.545 GeV for α_{ρ_1} is compatible with the value of 0.47 GeV inferred from the charmonium system. However, the preferred range of α_{ρ_2} , (0.145–0.195) GeV, is somewhat smaller than the naive

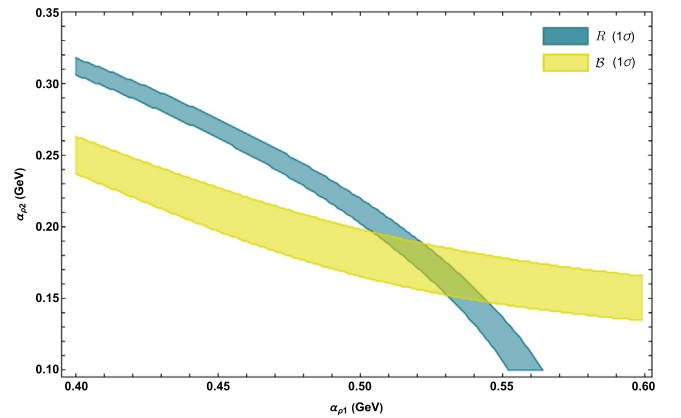


FIG. 2. Allowed regions for the harmonic oscillator parameters α_{ρ_1} and α_{ρ_2} constrained by the ratio R and the absolute branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ inferred from Eq. (1.5).

TABLE III. Comparison of the predicted S - and P -wave amplitudes (in units of $10^{-2}G_F \text{ GeV}^2$) of $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ decays, their branching fractions (in units of 10^{-2}) and the decay asymmetry parameter α in various approaches with only the central values being cited. For the predictions of Sharma and Dhir [12], we quote only the flavor-independent pole amplitudes for both NRQM and heavy quark effective theory (HQET). Two bag models are considered in the work of Liu and Geng [19]: the static bag (SB) and homogeneous bag (HB) models.

	A^{fac}	A^{nf}	A^{tot}	B^{fac}	B^{nf}	B^{tot}	\mathcal{B}	α	R
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$									
This work									
Case 1	9.1	-15.6	-6.5	-16.0	27.4	11.4	3.01	-0.78	
Case 2	7.4	-12.4	-5.0	-13.0	21.8	8.8	1.83	-0.78	
Case 3	5.5	-9.6	-4.1	-9.6	16.8	7.2	1.20	-0.78	
Cheng <i>et al.</i> [9]	7.4	-10.8	-3.4	-15.1	18.9	3.8	0.69	-0.41	
Gutsche <i>et al.</i> [8]	-8.1	11.5	3.4	13.0	-18.5	-5.6	0.71	-0.57	
Sharma and Dhir [12]									
NRQM	7.38	0	7.38	-16.77	-24.95	-41.72	6.64	-0.99	
HQET	9.52	0	9.52	-19.45	-24.95	-44.40	9.19	-0.99	
Shi <i>et al.</i> [10]									
LCSR + HQET (light cone sum rules)	9.52	-16.67	-7.18	-19.45	-20.47	-39.92	6.22	+0.99	
Ke and Li [13]									
$\theta = 16.27^\circ$							2.14	-0.09	
$\theta = 85.54^\circ$							2.14	-0.95	
Liu and Geng [19] ^a									
SB ($\theta = -24.7^\circ$)	4.83	-9.99	-5.16	5.16	13.6	18.8	2.24	-0.93	
HB ($\theta = 24.7^\circ$)	7.08	-20.3	-13.2	-22.1	33.0	10.9	10.3	-0.30	
$\Xi_{cc}^{++} \rightarrow \Xi_c^{\prime+} \pi^+$									
This work									
Case 1	4.6	0	4.6	-45.6	0	-45.6	4.32	-0.89	1.44
Case 2	3.7	0	3.7	-37.1	0	-31.0	2.86	-0.89	1.56
Case 3	2.8	0	2.8	-27.6	0	-27.6	2.16	-0.89	1.32
Cheng <i>et al.</i> [9]	4.5	-0.04	4.5	-48.5	-0.06	-48.4	4.65	-0.84	6.74
Gutsche <i>et al.</i> [8]	-4.3	-0.1	-4.4	37.6	1.4	39.0	3.39	-0.93	4.33
Sharma and Dhir [12]									
NRQM	4.29	0	4.29	-53.65	0	-53.65	5.39	-0.78	0.81
HQET	5.10	0	5.10	-62.37	0	-62.37	7.34	-0.79	0.80
Shi <i>et al.</i> [10]									
LCSR + HQET	5.10	-0.83	4.27	-62.37	-8.86	-71.23	8.85	-0.64	1.42
Ke and Li [13]									
$\theta = 16.27^\circ$							3.02	-0.99	1.41
$\theta = 85.54^\circ$							3.02	-0.51	1.41
Liu and Geng [19]									
SB ($\theta = -24.7^\circ$)	7.38	-4.82	2.56	-51.0	7.26	-43.7	3.25	-0.63	1.45
HB ($\theta = 24.7^\circ$)	0.61	9.65	10.3	-28.1	-17.4	-45.5	8.91	-0.96	0.87
LHCb [3]									1.41 ± 0.20

^aWe wish to thank C. W. Liu and C. Q. Geng for providing us the numerical values of the S - and P -wave amplitudes in their work.

expectation of 0.25 GeV. Accordingly, in Table II we choose three sets of harmonic oscillator parameters denoted by cases 1, 2, and 3 with $(\alpha_{\rho_1}, \alpha_{\rho_2})$ being in the vicinity of 0.51 and 0.19 GeV, respectively.

With the input for various parameters from Table II we are ready to compute the factorizable and nonfactorizable amplitudes for both S and P waves using Eqs. (2.5) and (2.7). The numerical results of individual S - and P -wave amplitudes, branching fractions of $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ decays and their decay asymmetries are shown in Table III for three different sets of the harmonic oscillator parameters α_{ρ_1} and

α_{ρ_2} given in Table II. Evidently, the interference between the factorizable diagram T and the nonfactorizable C' is destructive in $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$. On the contrary, the decay $\Xi_{cc}^{++} \rightarrow \Xi_c^{\prime+} \pi^+$ does not receive nonfactorizable contributions. This is consistent with the so-called Pati-Woo theorem [18], which results from the facts that the $(V - A) \times (V - A)$ structure of weak interactions is invariant under the Fierz transformation and that the baryon wave function is color antisymmetric. As a consequence of this theorem, the quark pair in a baryon produced by weak interactions be antisymmetric in flavor. Since the sextet Ξ'_c

is symmetric in light quark flavor, it cannot contribute to C' . Because the form factor g_1 in the $\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$ mode is larger than that in $\Xi_c^+ \pi^+$ (see Table II), the P -wave amplitude of the former is much higher than that of the latter. Consequently, the branching fraction of the $\Xi_c^+ \pi^+$ mode is larger than the $\Xi_c^+ \pi^+$ one. Taking case 2 as an example, we have the results

$$\begin{aligned} \mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) &= 1.83\%, & \alpha(\Xi_c^+ \pi^+) &= -0.78, \\ \mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+) &= 2.86\%, & \alpha(\Xi_c'^+ \pi^+) &= -0.89, \end{aligned} \quad (3.1)$$

and hence $R = 1.56$.

In Table III we also compare our results with other approaches. The nonfactorizable effects have been evaluated in two entirely different approaches: current algebra and the pole model in Ref. [9] and the covariant confined quark model in Ref. [8]. It is interesting to notice that both approaches yielded a large destructive interference in $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ and obtained similar branching fractions of order 0.70%. Although this is consistent with the experimental value of Eq. (1.5) to the lower end, the predicted ratios 6.74 in Ref. [9] and 4.33 in Ref. [8] are too large compared to the LHCb value of 1.41 ± 0.20 [3]. In the work of Sharma and Dhir [12], a large constructive interference in the P -wave amplitude was found, while nonfactorizable corrections to the S -wave one were not considered. From Table III we see that the P -wave amplitude in this model is much larger than other works. $\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)$ of order (7–9)% [(13–16)%] for flavor-independent (flavor-dependent) pole amplitudes was obtained in this work, which is obviously too large compared to Eq. (1.5).

In the recent work of Shi *et al.* [10], nonfactorizable internal W -emission contributions to $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ decays were evaluated using light-cone sum rules, see A^{nf} and B^{nf} terms shown in Table III. Factorizable contributions were then taken from the work of Sharma and Dhir [12] under ‘‘HQET.’’ The sizable nonfactorizable contribution to the P wave of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ seems to be an issue in view of the Pati-Woo theorem. From Table III we see that the S -wave amplitude denoted by A^{tot} for $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ is modified from 9.52 to -7.18 owing to the presence of a destructive nonfactorizable contribution.⁴ Consequently, S - and P -wave amplitudes are of the same sign in this model and yield a positive decay asymmetry $\alpha = 0.99$, which is in sharp contrast to the other works where the decay asymmetry is always predicted to be negative. Hence, even a sign measurement of $\alpha(\Xi_c^+ \pi^+)$ will allow to discriminate the model of Shi *et al.* from others.

⁴Recall that the relative sign convention between S and P waves is defined in Eq. (2.1).

As discussed in the Introduction, the external W -emission diagram T alone usually leads to $R < 1$. It was first pointed out by Ke and Li [13] that the observation of $R > 1$ can be accommodated by considering the $\Xi_c^+ - \Xi_c'^+$ mixing. Two mixing angles were found, $\theta = 16.27^\circ$ or 85.54° (see Table III). However, when the nonfactorizable effect due to internal W emission is turned on, the mixing angle will be affected. As noticed in passing, when the $\Xi_c^+ - \Xi_c'^+$ mixing effect is applied to the static bag model calculation performed in Ref. [9], Liu and Geng [19] have shown that at the mixing angle $\theta = -0.137\pi$, the ratio R is well accommodated and the branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ is consistent with the constraint derived from Eq. (1.5).

However, there is one issue with the static bag model, namely, a static bag is not invariant under space translation and it is impossible for a static bag to be at rest. The unwanted c.m. motion of the bag model is an issue and it should be removed for a consistent treatment [25]. For example, the bag model calculation for the heavy-flavor-conserving decays are improved by removing c.m. motion corrections. The predictions for $\Xi_c^0 \rightarrow \Lambda_c^+ \pi^-$ and $\Xi_b^- \rightarrow \Lambda_b^0 \pi^-$ are both in good agreement with experiment [26]. It is clear from Table III that nonfactorizable S - and P -wave amplitudes of $\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$ are no longer subject to the constraint from the Pati-Woo theorem because of the contribution from $\Xi_{cc}^{++} \rightarrow (\Xi_c^3)^+ \pi^+$. Unfortunately, the same bag model without c.m. motion will lead to an even smaller ratio, $R = 0.19$ [19]. When the $\Xi_c^+ - \Xi_c'^+$ mixing is included, R is increased to 0.90 at $\theta = 24.7^\circ$, but the branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$ becomes 10.3%, which is too large compared to the constraint inferred from Eq. (1.5).

IV. CONCLUSIONS

In this work we have studied the doubly charmed baryon decays $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ within the framework of the NRQM. Factorizable amplitudes are expressed in terms of transition form factors, while nonfactorizable amplitudes arising from the inner W emission are evaluated using current algebra and the pole model and expressed in terms of baryonic matrix elements and axial-vector form factors.

We draw some conclusions from our analysis:

- (1) Nonperturbative parameters are calculated in the NRQM. They can be expressed in terms of the momentum integrals of baryon wave functions, which are in turns expressed in terms of the harmonic oscillator parameters α_ρ and α_λ for ρ - and λ -mode excitation, respectively.
- (2) Denoting the harmonic oscillator parameters $\alpha_{\rho 1}, \alpha_{\lambda 1}$ for Ξ_{cc}^{++} and Ξ_{cc}^+ , $\alpha_{\rho 2}, \alpha_{\lambda 2}$ for Ξ_c^+ and $\Xi_c'^+$, we found that the measured ratio R of the branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$ relative to $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ can be accommodated in the NRQM with $\alpha_{\rho 1}$ and $\alpha_{\rho 2}$ being in the vicinity of 0.51 and 0.19 GeV, respectively.

- (3) We have compared our results with other approaches. While the ratio R has been accommodated in some other models, the predicted branching fraction of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ is often too large compared to that inferred from the LHCb measurement of its rate relative to $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$.
- (4) Decay asymmetries are predicted to be -0.78 and -0.89 for $\Xi_c^+ \pi^+$ and $\Xi_c'^+ \pi^+$ modes, respectively, which can be tested in the near future.
- (5) Although the static bag model fails to account for the ratio R , it is interesting to notice that, when the $\Xi_c^+ - \Xi_c'^+$ mixing effect is taken into account in the bag model calculations, data can be nicely accommodated with the mixing angle $\theta = -24.7^\circ$.

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APPENDIX A: CONVENTION AND EXPRESSION OF THE WAVE FUNCTION

The quark and antiquark fields are expanded in the convention adopted in [24]

$$\begin{aligned} q(x) &= \int \frac{d\mathbf{p}}{(2\pi)^3} \left(\frac{m}{p^0}\right)^{\frac{1}{2}} \sum_s [u_s(\mathbf{p}) b_s(\mathbf{p}) e^{ip \cdot x} \\ &\quad + v_s(\mathbf{p}) d_s^\dagger(\mathbf{p}) e^{-ip \cdot x}], \\ \bar{q}(x) &= \int \frac{d\mathbf{p}}{(2\pi)^3} \left(\frac{m}{p^0}\right)^{\frac{1}{2}} \sum_s [\bar{u}_s(\mathbf{p}) b_s^\dagger(\mathbf{p}) e^{-ip \cdot x} \\ &\quad + \bar{v}_s(\mathbf{p}) d_s(\mathbf{p}) e^{ip \cdot x}], \end{aligned} \quad (\text{A1})$$

associated with anticommutation relations between the creation and annihilation operators

$$\{b_s(\mathbf{p}), b_{s'}^\dagger(\mathbf{p}')\} = \{d_s(\mathbf{p}), d_{s'}^\dagger(\mathbf{p}')\} = \delta_{ss'} \delta^3(\mathbf{p} - \mathbf{p}'), \quad (\text{A2})$$

and the normalization relations of spinor

$$u_s^\dagger(\mathbf{p}) u_{s'}(\mathbf{p}) = v_s^\dagger(\mathbf{p}) v_{s'}(\mathbf{p}) = \left(\frac{p^0}{m}\right) \delta_{ss'}. \quad (\text{A3})$$

Then the baryon state in momentum space can be expressed in terms of mock states,

$$\begin{aligned} |\mathcal{B}(\mathbf{P}_c)_{J,M}\rangle &= \sum_{S_z, M_L; c_i} \langle L, M_L; S, S_z | J, M \rangle \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_c) \Psi_{N,L,M_L}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \\ &\quad \times \chi_{s_1, s_2, s_3}^{S, S_z} \frac{\epsilon_{c_1 c_2 c_3}}{\sqrt{6}} \phi_{i_1, i_2, i_3} b_{c_1, i_1, s_1, \mathbf{p}_1}^\dagger b_{c_2, i_2, s_2, \mathbf{p}_2}^\dagger b_{c_3, i_3, s_3, \mathbf{p}_3}^\dagger |0\rangle, \end{aligned} \quad (\text{A4})$$

which is normalized by

$$\langle \mathcal{B}(\mathbf{P}'_c)_{J,M} | \mathcal{B}(\mathbf{P}_c)_{J,M} \rangle = \delta^3(\mathbf{P}'_c - \mathbf{P}_c). \quad (\text{A5})$$

In particular, with the quantum numbers defined as

$$N = 2(n_\rho + n_\lambda) + l_\rho + l_\lambda, \quad L = l_\rho + l_\lambda, \quad (\text{A6})$$

the baryon spatial wave function in Eq. (A4) is

$$\Psi_{LM_L n_\rho l_\rho n_\lambda l_\lambda}(\mathbf{P}, \mathbf{p}_\rho, \mathbf{p}_\lambda) = \delta^3(\mathbf{P} - \mathbf{P}_c) \sum_m \langle LM_L | l_\rho m, l_\lambda M_L - m \rangle \psi_{n_\rho l_\rho m}(\mathbf{p}_\rho) \psi_{n_\lambda l_\lambda (M_L - m)}(\mathbf{p}_\lambda), \quad (\text{A7})$$

associated with quark wave function in momentum space

$$\psi_{nLm}(\mathbf{p}) = (i)^l (-1)^n \left[\frac{2n!}{(n+L+\frac{1}{2})!} \right]^{\frac{1}{2}} \frac{1}{\alpha^{L+\frac{1}{2}}} e^{-\frac{p^2}{2\alpha^2}} L_n^{L+\frac{1}{2}} \left(\frac{\mathbf{p}^2}{\alpha^2} \right) \mathcal{Y}_{Lm}(\mathbf{p}). \quad (\text{A8})$$

To describe baryon state, the Jacobi coordinate has been introduced in NRQM. In general, the Jacobi coordinates \mathbf{x}_j for the N -body system are defined as

$$\mathbf{x}_j = \frac{1}{m_{0j}} \sum_{k=1}^j m_k \mathbf{r}_k - \mathbf{r}_{j+1}, \quad \{j = 1, 2, \dots, N-1\},$$

$$\mathbf{x}_k = \frac{1}{m_{0N}} \sum_{k=1}^N m_k \mathbf{r}_k, \quad (\text{A9})$$

where $m_{0j} = \sum_{k=1}^j m_k$. For the baryon system we have $j = 2$ and the coordinates can be chosen as

$$\mathbf{R}_c = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + m_3 \mathbf{r}_3}{m_1 + m_2 + m_3},$$

$$\boldsymbol{\rho} = \mathbf{r}_1 - \mathbf{r}_2,$$

$$\boldsymbol{\lambda} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} - \mathbf{r}_3. \quad (\text{A10})$$

By introducing masses of ρ - and λ -mode excitation $m_\rho = \frac{m_1 m_2}{m_1 + m_2}$ and $m_\lambda = \frac{(m_1 + m_2) m_3}{(m_1 + m_2 + m_3)}$, together with baryon mass $M = m_1 + m_2 + m_3$, the corresponding momentums are

$$\mathbf{p} = M \dot{\mathbf{R}}_c = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3,$$

$$\mathbf{p}_\rho = m_\rho \dot{\boldsymbol{\rho}} = \frac{m_2}{m_1 + m_2} \mathbf{p}_1 - \frac{m_1}{m_1 + m_2} \mathbf{p}_2,$$

$$\mathbf{p}_\lambda = m_\lambda \dot{\boldsymbol{\lambda}} = \frac{m_3(\mathbf{p}_1 + \mathbf{p}_2) - (m_1 + m_2)\mathbf{p}_3}{(m_1 + m_2 + m_3)}. \quad (\text{A11})$$

Then the Hamiltonian to describe a particle interacting in the harmonic oscillator potential, given by

$$H = \sum_{i=1}^3 \frac{\mathbf{p}_i^2}{2m_i} + \frac{1}{2} K \sum_{i < j} (\mathbf{r}_i - \mathbf{r}_j)^2, \quad (\text{A12})$$

with K being a spring constant, becomes the form in terms of Jacobi coordinate

$$H = \frac{\mathbf{p}^2}{2M} + \frac{\mathbf{p}_\rho^2}{2m_\rho} + \frac{\mathbf{p}_\lambda^2}{2m_\lambda} + \frac{1}{2} m_\rho \omega_\rho^2 \boldsymbol{\rho}^2 + \frac{1}{2} m_\lambda \omega_\lambda^2 \boldsymbol{\lambda}^2. \quad (\text{A13})$$

The harmonic oscillator strengths of the two modes can be further defined as

$$\alpha_\rho^2 = m_\rho \omega_\rho = \sqrt{\frac{3K m_1 m_2}{2(m_1 + m_2)}},$$

$$\alpha_\lambda^2 = m_\lambda \omega_\lambda = \sqrt{\frac{2K m_3 (m_1 + m_2)}{m_1 + m_2 + m_3}} \quad (\text{A14})$$

for the purpose of convenience. Hence a useful connection between the two strengths,

$$\alpha_\lambda = \left[\frac{4m_3(m_1 + m_2)^2}{3m_1 m_2 (m_1 + m_2 + m_3)} \right]^{\frac{1}{4}} \alpha_\rho, \quad (\text{A15})$$

can be found evidently. Notice we have an unity Jacobi determinant between the Jacobi coordinate and the ordinary one in current convention.

In literature, there is another set of conventions for the Jacobi coordinate giving

$$\tilde{\mathbf{R}}_c = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + m_3 \mathbf{r}_3}{m_1 + m_2 + m_3}, \quad \tilde{\mathbf{p}} = M \dot{\tilde{\mathbf{R}}}_c = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3,$$

$$\tilde{\boldsymbol{\rho}} = \frac{1}{\sqrt{2}} (\mathbf{r}_1 - \mathbf{r}_2), \quad \tilde{\mathbf{p}}_\rho = \tilde{m}_\rho \dot{\tilde{\boldsymbol{\rho}}} = \sqrt{2} \left(\frac{m_2}{m_1 + m_2} \mathbf{p}_1 - \frac{m_1}{m_1 + m_2} \mathbf{p}_2 \right),$$

$$\tilde{\boldsymbol{\lambda}} = \sqrt{\frac{2}{3}} \left(\frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} - \mathbf{r}_3 \right), \quad \tilde{\mathbf{p}}_\lambda = \tilde{m}_\lambda \dot{\tilde{\boldsymbol{\lambda}}} = \sqrt{\frac{3}{2}} \left[\frac{m_3(\mathbf{p}_1 + \mathbf{p}_2) - (m_1 + m_2)\mathbf{p}_3}{(m_1 + m_2 + m_3)} \right], \quad (\text{A16})$$

with ρ - and λ -type masses $\tilde{m}_\rho = \frac{2m_1 m_2}{m_1 + m_2}$ and $\tilde{m}_\lambda = \frac{3(m_1 + m_2) m_3}{2(m_1 + m_2 + m_3)}$. Then one can derive the harmonic oscillator strengths

$$\tilde{\alpha}_\rho^2 = \tilde{m}_\rho \tilde{\omega}_\rho = \sqrt{\frac{6K m_1 m_2}{(m_1 + m_2)}}, \quad \alpha_\rho = \frac{1}{\sqrt{2}} \tilde{\alpha}_\rho, \quad \alpha_\lambda = \sqrt{\frac{2}{3}} \tilde{\alpha}_\lambda, \quad (\text{A18})$$

$$\tilde{\alpha}_\lambda^2 = \tilde{m}_\lambda \tilde{\omega}_\lambda = 3 \sqrt{\frac{K m_3 (m_1 + m_2)}{2(m_1 + m_2 + m_3)}}. \quad (\text{A17})$$

in the tilde convention. The relations of α parameters between the two conventions,

is helpful in the analysis.

APPENDIX B: MATRIX ELEMENTS OF QUARK OPERATORS

In the pole model calculation of doubly charmed baryon decays, nonperturbative quantities such as form factors and four-quark operator matrix elements play an essential role. Here the derivations of these nonperturbative parameters in the NRQM are shown in detail in this section.

1. Form factors

The form factors are defined to parametrize the baryon matrix element of the bilinear quark operator

$\bar{q}'_1 \gamma_\mu (1 - \gamma_5) q'_2$ have been given in Eq. (2.6). For a further calculation of form factors, we follow the treatment in Ref. [27] and obtain

$$\begin{aligned} f_1(q^2) &= \langle \mathcal{B}_f(\mathbf{P}_f) | \bar{q}'_1 \gamma_0 q'_2 | \mathcal{B}_i(\mathbf{P}_i) \rangle, \\ g_1(q^2) &= \langle \mathcal{B}_f(\mathbf{P}_f) | \bar{q}'_1 \gamma_3 \gamma_5 q'_2 | \mathcal{B}_i(\mathbf{P}_i) \rangle, \end{aligned} \quad (\text{B1})$$

in which the Breit frame is adopted. Now by employing the baryon wave functions in NRQM given by Eq. (A4), we have

$$\begin{aligned} f_1(q^2) &= (-1) \times \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 d\mathbf{p}_5 d\mathbf{p}_6 d\mathbf{p}'_1 d\mathbf{p}'_2 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_i) \delta^3(\mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 - \mathbf{P}_f) \\ &\quad \times \Psi_f^*(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \delta^3(\mathbf{p}'_1 - \mathbf{p}'_2) \langle \mathcal{B}_f \uparrow | b_{q'_1}^\dagger b_{q'_2} | \mathcal{B}_i \uparrow \rangle \langle 0 | b_6 b_5 b_4 b_1^\dagger b_2^\dagger b_3^\dagger | 0 \rangle, \\ &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 d\mathbf{p}_5 d\mathbf{p}_6 d\mathbf{p}'_1 d\mathbf{p}'_2 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_i) \delta^3(\mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 - \mathbf{P}_f) \\ &\quad \times \delta^3(\mathbf{p}_1 - \mathbf{p}'_2) \delta^3(\mathbf{p}_4 - \mathbf{p}'_1) \delta^3(\mathbf{p}_2 - \mathbf{p}_6) \delta^3(\mathbf{p}_3 - \mathbf{p}_5) \delta^3(\mathbf{p}'_1 - \mathbf{p}'_2) \\ &\quad \times \Psi_f^*(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \langle \mathcal{B}_f \uparrow | b_{q'_1}^\dagger b_{q'_2} | \mathcal{B}_i \uparrow \rangle, \end{aligned} \quad (\text{B2})$$

where the baryon wave function defined in Eq. (A7) are denoted as $\Psi_{i,f}$ concisely. Some details are presented in above two equations. In the first equation, the two matrix elements are calculated in spin-flavor space and momentum space, respectively, while the factor -1 results from a product of two color wave functions. Four-momentum δ functions have been produced after considering the

anticommutation relations, corresponding to explicit initial and final state baryons, in the second equation. A more compact form of f_1 hence can be expressed as

$$f_1(q^2) = \langle \mathcal{B}_f \uparrow | b_{q'_1}^\dagger b_{q'_2} | \mathcal{B}_i \uparrow \rangle X, \quad (\text{B3})$$

with

$$\begin{aligned} X &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 d\mathbf{p}_5 d\mathbf{p}_6 d\mathbf{p}'_1 d\mathbf{p}'_2 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_i) \delta^3(\mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 - \mathbf{P}_f) \\ &\quad \times \delta^3(\mathbf{p}_1 - \mathbf{p}'_2) \delta^3(\mathbf{p}_4 - \mathbf{p}'_1) \delta^3(\mathbf{p}_2 - \mathbf{p}_6) \delta^3(\mathbf{p}_3 - \mathbf{p}_5) \delta^3(\mathbf{p}'_1 - \mathbf{p}'_2) \Psi_f^*(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3). \end{aligned} \quad (\text{B4})$$

To be specific, the convention for matrix element in Eq. (B3) in spin-flavor space keeps the same as our previous work [9]. A similar derivation leads to

$$g_1(q^2) = \langle \mathcal{B}_f \uparrow | b_{q'_1}^\dagger b_{q'_2} \sigma_z | \mathcal{B}_i \uparrow \rangle X, \quad (\text{B5})$$

in which a common spatial wave function integral X has been shared with the form factor $f_1(q^2)$. As for the axial-vector form factor $g_{B'B}^{A(P)}$, it differs g_1 from its spatial wave function integral Y , giving

$$g_{B'B}^{A(P)}(q^2) = \langle \mathcal{B}_i \uparrow | b_{q'_1}^\dagger b_{q'_2} \sigma_z | \mathcal{B}_f \uparrow \rangle Y, \quad (\text{B6})$$

with

$$\begin{aligned} Y &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 d\mathbf{p}_5 d\mathbf{p}_6 d\mathbf{p}'_1 d\mathbf{p}'_2 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_i) \delta^3(\mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 - \mathbf{P}_f) \\ &\quad \times \delta^3(\mathbf{p}_3 - \mathbf{p}'_2) \delta^3(\mathbf{p}_6 - \mathbf{p}'_1) \delta^3(\mathbf{p}_1 - \mathbf{p}_4) \delta^3(\mathbf{p}_2 - \mathbf{p}_5) \delta^3(\mathbf{p}'_1 - \mathbf{p}'_2) \Psi_f^*(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3). \end{aligned} \quad (\text{B7})$$

Though the two integrals X and Y formally share the same structure, their difference exists in the wave functions of two baryons and hence brings a further difference in δ functions. In fact, f_1 and g_1 depict the transition between the initial and final state baryons while $g_{B'B}^{A(P)}$ is devoted to the one between initial and intermediate (or intermediate and final) baryons. Therefore such a difference between X and Y is a consequence of the pole model.

2. Matrix elements of four-quark operators

We continue working in the NRQM to calculate the matrix element of four-quark operator $(\bar{q}'_1 q'_2)(\bar{q}'_3 q'_4)$. With the help of baryon wave function Eq. (A4), it can be expanded as

$$\begin{aligned} \langle \mathcal{B}_f(\mathbf{P}_f) | (\bar{q}'_1 q'_2)(\bar{q}'_3 q'_4) | \mathcal{B}_i(\mathbf{P}_i) \rangle &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 d\mathbf{p}_5 d\mathbf{p}_6 d\mathbf{p}'_1 d\mathbf{p}'_2 d\mathbf{p}'_3 d\mathbf{p}'_4 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_i) \delta^3(\mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 - \mathbf{P}_f) \\ &\quad \times \Psi_f^*(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \delta^3(\mathbf{p}'_1 + \mathbf{p}'_3 - \mathbf{p}'_2 - \mathbf{p}'_4) \\ &\quad \times \langle \mathcal{B}_f \uparrow | (b_{q'_1}^\dagger b_{q'_2})_1 (b_{q'_3}^\dagger b_{q'_4})_2 (1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) | \mathcal{B}_i \uparrow \rangle \langle 0 | b_6 b_5 b_4 b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_1^\dagger b_2^\dagger b_3^\dagger | 0 \rangle, \\ &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 d\mathbf{p}_5 d\mathbf{p}_6 d\mathbf{p}'_1 d\mathbf{p}'_2 d\mathbf{p}'_3 d\mathbf{p}'_4 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_i) \delta^3(\mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 - \mathbf{P}_f) \\ &\quad \times \Psi_f^*(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \delta^3(\mathbf{p}'_1 + \mathbf{p}'_3 - \mathbf{p}'_2 - \mathbf{p}'_4) \delta^3(\mathbf{p}'_2 - \mathbf{p}_1) \delta^3(\mathbf{p}'_4 - \mathbf{p}_3) \delta^3(\mathbf{p}_4 - \mathbf{p}'_1) \\ &\quad \times \delta^3(\mathbf{p}_5 - \mathbf{p}'_3) \delta^3(\mathbf{p}_6 - \mathbf{p}_2) \langle \mathcal{B}_f \uparrow | (b_{q'_1}^\dagger b_{q'_2})_1 (b_{q'_3}^\dagger b_{q'_4})_2 (1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) | \mathcal{B}_i \uparrow \rangle. \end{aligned} \quad (\text{B8})$$

Two equations are presented with the similar treatment to the form factor in Eq. (B3), then a compact form can be achieved as

$$\langle \mathcal{B}_f(\mathbf{P}_f) | (\bar{q}'_1 q'_2)(\bar{q}'_3 q'_4) | \mathcal{B}_i(\mathbf{P}_i) \rangle = \langle \mathcal{B}_f \uparrow | (b_{q'_1}^\dagger b_{q'_2})_1 (b_{q'_3}^\dagger b_{q'_4})_2 (1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) | \mathcal{B}_i \uparrow \rangle Z \quad (\text{B9})$$

with

$$\begin{aligned} Z &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 d\mathbf{p}_5 d\mathbf{p}_6 d\mathbf{p}'_1 d\mathbf{p}'_2 d\mathbf{p}'_3 d\mathbf{p}'_4 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_i) \delta^3(\mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 - \mathbf{P}_f) \delta^3(\mathbf{p}'_2 - \mathbf{p}_1) \delta^3(\mathbf{p}'_4 - \mathbf{p}_3) \\ &\quad \times \delta^3(\mathbf{p}_4 - \mathbf{p}'_1) \delta^3(\mathbf{p}_5 - \mathbf{p}'_3) \delta^3(\mathbf{p}_6 - \mathbf{p}_2) \delta^3(\mathbf{p}'_1 + \mathbf{p}'_3 - \mathbf{p}'_2 - \mathbf{p}'_4) \Psi_f^*(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3). \end{aligned} \quad (\text{B10})$$

Here the spatial integral Z is taken between initial (final) and intermediate baryons with four quark fields involved, which brings in one more δ function compared with the case in integral Y .

3. Momentum integrals of baryon wave functions

After integrating all the δ functions in Eqs. (B3), (B7), and (B8), keeping the momentum conservation, we have concise expressions of X , Y , Z in terms of momentum integrals of baryon wave functions,

$$\begin{aligned} X &= \delta^3(\mathbf{P}_i - \mathbf{P}_f) \int d\mathbf{p}_2 d\mathbf{p}_3 \Psi_f^*((\mathbf{P}_i - \mathbf{p}_2 - \mathbf{p}_3), \mathbf{p}_3, \mathbf{p}_2) \Psi_i((\mathbf{P}_i - \mathbf{p}_2 - \mathbf{p}_3), \mathbf{p}_2, \mathbf{p}_3), \\ Y &= \delta^3(\mathbf{P}_i - \mathbf{P}_f) \int d\mathbf{p}_2 d\mathbf{p}_3 \Psi_f^*((\mathbf{P}_i - \mathbf{p}_2 - \mathbf{p}_3), \mathbf{p}_2, \mathbf{p}_3) \Psi_i((\mathbf{P}_i - \mathbf{p}_2 - \mathbf{p}_3), \mathbf{p}_2, \mathbf{p}_3), \\ Z &= \delta^3(\mathbf{P}_i - \mathbf{P}_f) \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_4 \Psi_f^*(\mathbf{p}_4, (\mathbf{P}_i - \mathbf{p}_2 - \mathbf{p}_4), \mathbf{p}_2) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, (\mathbf{P}_i - \mathbf{p}_1 - \mathbf{p}_2)). \end{aligned} \quad (\text{B11})$$

The integrals X and Y are similar except the interchange of \mathbf{p}_2 and \mathbf{p}_3 in the wave function Ψ_f . Then the remaining task is to evaluate them. Before proceeding to a detailed calculation, it is useful to firstly deal with the product of two wave functions as it is the common part in all the three integrals. Taking the one in X as an example, a direct calculation leads to

$$\Psi_f^*(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6) \Psi_i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \frac{1}{\pi^3} (\alpha_{\rho_1} \alpha_{\lambda_1} \alpha_{\rho_2} \alpha_{\lambda_2})^{-\frac{3}{2}} e^{-\frac{1}{2} \left(\frac{p_{4i}^2}{a_{\rho_1}^2} + \frac{p_{4i}^2}{a_{\lambda_1}^2} + \frac{p_{6i}^2}{a_{\rho_2}^2} + \frac{p_{6i}^2}{a_{\lambda_2}^2} \right)}, \quad (\text{B12})$$

which is based on the relations between two coordinates

$$\begin{aligned} \mathbf{p}_{\rho i} &= \frac{m_2}{m_1 + m_2} \mathbf{p}_1 - \frac{m_1}{m_1 + m_2} \mathbf{p}_2, & \mathbf{p}_{\rho f} &= \frac{m_5}{m_4 + m_5} \mathbf{p}_4 - \frac{m_4}{m_4 + m_5} \mathbf{p}_5, \\ \mathbf{p}_{\lambda i} &= \frac{m_3(\mathbf{p}_1 + \mathbf{p}_2) - (m_1 + m_2)\mathbf{p}_3}{(m_1 + m_2 + m_3)}, & \mathbf{p}_{\lambda f} &= \frac{m_6(\mathbf{p}_4 + \mathbf{p}_5) - (m_4 + m_5)\mathbf{p}_6}{(m_4 + m_5 + m_6)}. \end{aligned} \quad (\text{B13})$$

A replacement of the index $2 \rightarrow 3$ in Eq. (B12) gives the one in Y while $1 \rightarrow 3$ provides the corresponding one to Z . Taking a static limit of initial baryon $\mathbf{P}_i = 0$, a further calculation for X yields

$$X = \frac{1}{\pi^3} (\alpha_{\rho 1} \alpha_{\lambda 1} \alpha_{\rho 2} \alpha_{\lambda 2})^{-\frac{3}{2}} \left(\frac{4\pi^2}{a_X b_X - \frac{c_X^2}{4}} \right)^{3/2} = d_X (\alpha_{\rho 1} \alpha_{\lambda 1} \alpha_{\rho 2} \alpha_{\lambda 2})^{-\frac{3}{2}}, \quad (\text{B14})$$

associated with the auxiliary parameters, giving

$$d_X = 8 \left(a_X b_X - \frac{c_X^2}{4} \right)^{-3/2}, \quad (\text{B15})$$

with

$$a_X = \frac{1}{\alpha_{\rho 1}^2} + \frac{1}{\alpha_{\lambda 2}^2} + \frac{m_u^2}{(m_s + m_u)^2 \alpha_{\rho 2}^2}, \quad b_X = \frac{1}{\alpha_{\rho 2}^2} + \frac{1}{\alpha_{\lambda 1}^2} + \frac{1}{4\alpha_{\rho 1}^2}, \quad c_X = \frac{1}{\alpha_{\rho 1}^2} + \frac{2m_u}{(m_s + m_u)\alpha_{\rho 2}^2}.$$

Likewise, Y and Z can be derived as

$$Y = d_Y (\alpha_{\rho 1} \alpha_{\lambda 1} \alpha_{\rho 3} \alpha_{\lambda 3})^{-\frac{3}{2}}, \quad Z = d_Z (\alpha_{\rho 3} \alpha_{\lambda 3} \alpha_{\rho 2} \alpha_{\lambda 2})^{-\frac{3}{2}}, \quad (\text{B16})$$

together with

$$\begin{aligned} d_Y &= 8 \left(a_Y b_Y - \frac{c_Y^2}{4} \right)^{-3/2}, & d_Z &= 8 (2\pi \alpha_{\rho 2}^2)^{3/2} \left(a_Z b_Z - \frac{c_Z^2}{4} \right)^{-3/2}, \\ a_Y &= \frac{1}{\alpha_{\rho 1}^2} + \frac{1}{\alpha_{\rho 3}^2}, & a_Z &= \frac{1}{4\alpha_{\rho 3}^2} + \frac{1}{\alpha_{\lambda 3}^2}, \\ b_Y &= \frac{1}{\alpha_{\lambda 3}^2} + \frac{1}{\alpha_{\lambda 1}^2} + \frac{1}{4\alpha_{\rho 1}^2} + \frac{1}{4\alpha_{\rho 3}^2}, & b_Z &= \frac{1}{4\alpha_{\rho 3}^2} + \frac{1}{\alpha_{\lambda 3}^2} + \frac{1}{\alpha_{\lambda 2}^2}, \\ c_Y &= \frac{1}{\alpha_{\rho 1}^2} + \frac{1}{\alpha_{\rho 3}^2}, & c_Z &= -2 \left(\frac{1}{4\alpha_{\rho 3}^2} - \frac{1}{\alpha_{\lambda 3}^2} \right). \end{aligned} \quad (\text{B17})$$

Equations (B14) and (B16) give the final expressions of the momentum integrals.

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