Collective transitions of two entangled atoms near a Schwarzschild black hole

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We study the collective transitions of a pair of static atoms near a Schwarzschild black hole. The two atoms are assumed to be placed along the radial direction of the black hole and be coupled with fluctuating massless scalar fields in the Hartle-Hawking and Unruh vacua. When the two-atom system is prepared in the symmetric or antisymmetric entangled state, the average rate of change of energy of the two-atom system is oblivious of the vacuum fluctuations and solely a result of the radiation reaction. Consequently, the average rate, contrary to the existing results in the literature, is finite near the event horizon and is not affected by the Hawking radiation of the black hole.

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I. INTRODUCTION

Spontaneous emission is a phenomenon in which an atom transitions from a higher energy state to a lower one and emits a photon, which can be attributed to vacuum fluctuations [1], radiation reaction [2,3], or a combination of them [4,5]. The indeterminacy of physical interpretation arises from different choices of ordering of the operators of the atoms and the quantum field, and this ambiguity is settled by Dalibard, Dupont-Roc, and Cohen-Tannoudji (DDC) [6,7], who proposed that the symmetric operator ordering between the variables of the atoms and the field should be adopted in order to ensure that the effective Hamiltonians originating from vacuum fluctuations and radiation reaction are separately Hermitian. Then, the effects of vacuum fluctuations and radiation reaction have independent physical meanings. Ever since, the DDC approach has been widely used to study various phenomena concerning atom-field interaction, such as the radiative properties of atoms near a conducting plane [8] and the energy-level shifts of atoms in a cavity [9].

The radiative behaviors are richer for atoms in noninertial motion or in curved spacetimes. With the help of the DDC approach, it has been shown that, for uniformly accelerated atoms in vacuum, the balance between the contributions of vacuum fluctuations and radiation reaction to the rate of change of the mean atomic energy that exists for inertial atoms is broken, so spontaneous excitation becomes possible [10]. In accordance with equivalence principle, spontaneous excitation is also found to be possible for static atoms outside a Schwarzschild black hole [11]. These studies establish an interesting relationship between quantum effects in curved spacetimes, such as the Unruh effect [12–14] and the Hawking radiation [15,16], and the radiative properties of atoms. In Refs. [10,11], the atoms are assumed to be coupled with the fluctuating vacuum scalar field. Later, these studies have been generalized to the cases of the electromagnetic field [17,18], the Dirac field [19], and as well as the gravitational field [20].

Recently, there has been growing interest in the interplay between quantum entanglement and the radiative properties of a multiatom system. On the one hand, the dynamics of quantum entanglement of the atom system is crucially dependent on the radiative properties of the atoms [21]. For example, a pair of initially entangled atoms may get completely disentangled within a finite time due to spontaneous emission, known as entanglement sudden death [22]. On the other hand, the radiative properties of entangled atoms are also significantly different from those of separable ones [23–25]. Therefore, a number of recent studies have been focusing on the radiative properties of the atoms due to the interplay between quantum entanglement and the quantum effects in noninertial frames and curved spacetimes [26–30]. In particular, in Ref. [27], based on a generalization of the DDC formalism [6,7], the rate of variation of the atomic energy of a pair of entangled atoms at fixed positions outside a Schwarzschild black hole has been investigated. It has been argued that, as the atoms approach the event horizon of the black hole, the average rate of variation of the atomic energy grows rapidly due to the large proper acceleration of the atoms, and the Hawking

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radiation plays a significant role in the collective transitions of two entangled atoms. However, as has been pointed out in Ref. [30], the general expression for the contribution of vacuum fluctuations derived in Ref. [27] is incorrect, and so the total rate of change of the energy of the two-atom system obtained there is questionable. Therefore, it is unclear whether the conclusions in Ref. [27] are still valid.

In this paper, we reinvestigate the collective transitions of a pair of static atoms outside a Schwarzschild black hole. As we will show in detail, when the atoms are prepared in the symmetric or antisymmetric entangled state, the average rate of change of the atomic energy comes solely from the contribution of radiation reaction, and consequently, it is not affected by the Hawking radiation of the black hole and the result is finite in the vicinity of the event horizon. This is sharply different from what has been found in Ref. [27]. The structure of the paper is as follows. In Sec. II, we review the basic procedures of the generalized DDC formalism for the study of the average rate of change of the energy of a two-atom system shown in Ref. [30]. In Sec. III, we investigate the average rate of change of the energy of a static two-atom system near a Schwarzschild black hole. Since the geometry near the horizon of a Schwarzschild black hole can be approximated with the Rindler spacetime, we also investigate the rate of change of the energy of a static two-atom system in the Rindler spacetime, and check if the two situations are equivalent in Sec. IV. We summarize our results in Sec. V.

II. THE DDC FORMALISM

We study the collective transitions of a pair of entangled atoms near a Schwarzschild black hole. The system consists of two identical two-level atoms A and B, which are coupled with a fluctuating massless scalar field. The ground and excited states of the two-level atoms are labeled as $|g\rangle$ and $|e\rangle$ respectively, and the corresponding eigenenergies are $-\frac{\omega_0}{2}$ and $\frac{\omega_0}{2}$ respectively. The Hamiltonian of the two-atom system is

$$H_S = \omega_0 R_3^A(\tau) + \omega_0 R_3^B(\tau), \tag{1}$$

where the atomic operator R_3 takes the form

$$R_3 = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|). \tag{2}$$

The Hamiltonian of the scalar field is,

$$H_F(\tau) = \int d^3k\, \omega_k a_k^\dagger a_k \frac{dt}{d\tau}, \tag{3}$$

where t and τ represent the coordinate time and the proper time respectively, and a_k and $a_k^\dagger$ are the creation and annihilation operators of the field mode k respectively. The Hamiltonian describing the atom-field interaction is taken in analogy to the electric dipole interaction as [10],

$$H_I(\tau) = \lambda R_2^A(\tau)\phi(x_A(\tau)) + \lambda R_2^B(\tau)\phi(x_B(\tau)), \tag{4}$$

where

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \frac{1}{\sqrt{2\omega_k}} [a_k(t)e^{ik\cdot x} + a_k^\dagger(t)e^{-ik\cdot x}] \tag{5}$$

is the operator of the massless scalar field, and $R_2 = \frac{i}{2}(R_- - R_+)$, with $R_+ = |g\rangle\langle e|$ and $R_- = |e\rangle\langle g|$ being the raising and lowering operators of the atoms respectively.

In the following, we investigate the collective transitions of two entangled atoms near a Schwarzschild black hole following Ref. [30], which is a generalization of the original DDC formalism [6,7] to the case of two atoms. In this approach, the contributions from vacuum fluctuations and radiation reaction to the collective transition rates can be separately identified. For atom A, the average rate of change of the atomic energy due to vacuum fluctuations and radiation reaction can be obtained as [30],

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{n,vf} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x_A(\tau), x_A(\tau')) \frac{d}{d\tau}\chi_n^A(\tau,\tau'), \tag{6}$$

$$\begin{aligned}\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{n,rr} &= 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x_A(\tau), x_A(\tau')) \frac{d}{d\tau} C_n^{AA}(\tau,\tau') \\ &\quad + 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x_A(\tau), x_B(\tau')) \\ &\quad \times \frac{d}{d\tau} C_n^{AB}(\tau,\tau').\end{aligned} \tag{7}$$

Here,

$$\chi_n^\xi(\tau,\tau') = \frac{1}{2}\langle \psi_n | [R_2^\xi(\tau), R_2^\xi(\tau')] | \psi_n \rangle, \tag{8}$$

$$C_n^{\xi\xi'}(\tau,\tau') = \frac{1}{2}\langle \psi_n | \{R_2^\xi(\tau), R_2^{\xi'}(\tau')\} | \psi_n \rangle, \tag{9}$$

are the statistical functions of the atoms, and

$$\chi^F(x_\xi(\tau), x_{\xi'}(\tau')) = \frac{1}{2}\langle 0 | [\phi(x_\xi(\tau)), \phi(x_{\xi'}(\tau'))] | 0 \rangle, \tag{10}$$

$$C^F(x_\xi(\tau), x_{\xi'}(\tau')) = \frac{1}{2}\langle 0 | \{\phi(x_\xi(\tau)), \phi(x_{\xi'}(\tau'))\} | 0 \rangle, \tag{11}$$

are the statistical functions of the quantum field, where $\xi, \xi' = A, B$. Exchanging the labels A and B in Eqs. (6) and (7), one obtains the average rate of change of energy for atom B. Summing the contributions from atoms A and B, the average rate of change of energy of the two-atom system due to vacuum fluctuations and radiation reaction can be respectively expressed as

$$\left\langle \frac{dH_S(\tau)}{d\tau} \right\rangle_{n,vf} = \sum_{\xi=A,B} 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x_\xi(\tau), x_\xi(\tau')) \times \frac{d}{d\tau}\chi_n^\xi(\tau,\tau'), \tag{12}$$

$$\left\langle \frac{dH_S(\tau)}{d\tau} \right\rangle_{n,rr} = \sum_{\xi,\xi'=A,B} 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x_\xi(\tau), x_{\xi'}(\tau')) \times \frac{d}{d\tau} C_n^{\xi\xi'}(\tau,\tau'). \tag{13}$$

Here let us note that the formula for the contribution of vacuum fluctuations Eq. (12) is composed of two terms depending on atoms A and B, respectively, which is different from the corresponding formula [i.e., Eq. (9)] in Ref. [27] containing four terms, with two redundant cross terms depending on both of the two atoms. These redundant cross terms originate from the erroneous expressions of the source parts of the atomic dynamical variables. In the DDC formalism [6,7], dynamical variables are divided into two parts, i.e., the free part that exists even when the atoms and the quantum field are decoupled, and the source part which is induced by the interaction between the atoms and the quantum field. As shown in Eqs. (15)–(16) in Ref. [30], the source part of the dynamical variables of one of the atoms is independent of the other atom. However, in Eq. (15) in Ref. [26], a precursor work of Ref. [27], the source part of the dynamical variable of one of the atoms is related to both atoms. This leads to the erroneous expression of the contribution of vacuum fluctuations.

In the present paper, we assume that the two-atom system is prepared in the symmetric or antisymmetric entangled state $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|ge\rangle \pm |eg\rangle)$. Then, direct calculations show that $\chi^A(\tau,\tau') = \chi^B(\tau,\tau') = 0$, and

$$C^{\xi\xi'}(\tau,\tau') = \begin{cases} \frac{1}{8}(e^{i\omega_0(\tau-\tau')} + e^{-i\omega_0(\tau-\tau')}), & \xi = \xi', \\ \pm\frac{1}{8}(e^{i\omega_0(\tau-\tau')} + e^{-i\omega_0(\tau-\tau')}), & \xi \neq \xi', \end{cases} \tag{14}$$

where $\pm$ refer to the cases of the symmetric state and the antisymmetric state respectively. Therefore, for a two-atom system prepared in the symmetric or antisymmetric entangled state, the average rate of change of the atomic energy comes solely from the contribution of radiation reaction [30], and the result can be rewritten as,

$$\left\langle \frac{dH_S(\tau)}{d\tau} \right\rangle = \sum_{\xi,\xi'=A,B} \left\langle \frac{dH_S(\tau)}{d\tau} \right\rangle^{\xi\xi'}, \tag{15}$$

where

$$\left\langle \frac{dH_S(\tau)}{d\tau} \right\rangle^{\xi\xi'} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x_\xi(\tau), x_{\xi'}(\tau')) \frac{d}{d\tau} C_n^{\xi\xi'}(\tau,\tau'). \tag{16}$$

III. RATE OF CHANGE OF ENERGY OF A TWO-ATOM SYSTEM OUTSIDE A SCHWARZSCHILD BLACK HOLE

The line element of the Schwarzschild spacetime is

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{1-\frac{2M}{r}} - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{17}$$

where M is the mass of the Schwarzschild black hole. We assume that the two atoms are static and are aligned along the radial direction of the Schwarzschild black hole. So, the trajectories of the two atoms can be described as,

$$t_A = t, \quad r_A = r + L/2, \quad \theta_A = \theta, \quad \varphi_A = \varphi, \tag{18}$$

$$t_B = t, \quad r_B = r - L/2, \quad \theta_B = \theta, \quad \varphi_B = \varphi, \tag{19}$$

where $L = r_A - r_B$ is the coordinate distance between the two atoms, and $r = (r_A + r_B)/2$ is the average distance of the two atoms to the center of the black hole. Note that $r - 2M > L/2$ should be satisfied so as to ensure that the closer atom is outside the event horizon. As the two atoms are located at different radial positions, their proper times are different with $\tau_A = t(1-2M/r_A)^{1/2}$ and $\tau_B = t(1-2M/r_B)^{1/2}$ respectively. Here, for simplicity, we assume that the interatomic separation L is extremely small such that $L \ll 2(r-2M)$. Then, $\tau_A \approx \tau_B \approx t(1-2M/r)^{1/2}$.

In the exterior region of the Schwarzschild black hole, a complete set of normalized field modes reads [31],

$$\overrightarrow{u}_{\omega lm}(x) = (4\pi\omega)^{-1/2} e^{-i\omega t} \overrightarrow{R}_{\omega l}(r) Y_{lm}(\theta,\varphi), \tag{20}$$

$$\overleftarrow{u}_{\omega lm}(x) = (4\pi\omega)^{-1/2} e^{-i\omega t} \overleftarrow{R}_{\omega l}(r) Y_{lm}(\theta,\varphi), \tag{21}$$

in which $Y_{lm}(\theta,\varphi)$ is the spherical harmonic function, and $\overrightarrow{R}_{\omega l}(r)$ and $\overleftarrow{R}_{\omega l}(r)$ are the radial functions. Here $\overrightarrow{u}$ denotes the outgoing field mode emerging from the event horizon, and $\overleftarrow{u}$ the ingoing field mode coming in from infinity.

In the Schwarzschild spacetime, three different vacuum states, i.e., the Boulware [32], Unruh [14] and Hartle-Hawking [33] vacua, can be defined, each corresponding to a different choice of time coordinate. Let us note that, the Boulware vacuum corresponds to our familiar concept of a vacuum state of a massive spherical body at large radii. The Unruh vacuum is supposed to be the vacuum state best approximating the state following the gravitational collapse of a massive body to a black hole, since it corresponds to an outgoing flux of blackbody radiation at the Hawking temperature into empty space. The Hartle-Hawking vacuum, however, describes a black hole in equilibrium with a sea of thermal radiation. In this paper, we are interested in the scenario in which the two-atom system is placed outside a Schwarzschild black hole with Hawking radiation, so we focus on the cases of the Hartle-Hawking vacuum and the Unruh vacuum in the following.

A. The Hartle-Hawking vacuum

Let us begin with the case of the Hartle-Hawking vacuum. The Wightman function of the massless scalar field in the Hartle-Hawking vacuum is [34],

$$G_H^+(x_A(\tau),x_B(\tau')) = \frac{1}{16\pi^2}\sum_l(2l+1)\int_{-\infty}^{\infty}\frac{d\omega}{\omega} \times\left[e^{-i\omega(t-t')}\frac{\vec{R}_{\omega l}(r_A)\vec{R}^*{}_{\omega l}(r_B)}{1-e^{-\omega/T_H}} + e^{i\omega(t-t')}\frac{\overleftarrow{R}_{\omega l}(r_A)\overleftarrow{R}^*{}_{\omega l}(r_B)}{e^{\omega/T_H}-1}\right]. \quad (22)$$

Here $T_H=\kappa/2\pi$ is the Hawking temperature, where $\kappa=1/4M$ is the surface gravity of the black hole. Note that the radial functions satisfy $\vec{R}_{-\omega l}(r)=\vec{R}^*{}_{\omega l}(r)$, and $\overleftarrow{R}_{-\omega l}(r)=\overleftarrow{R}^*{}_{\omega l}(r)$, so the statistical function of the scalar field can be further simplified as,

$$\chi_H^F(x_A(\tau),x_B(\tau')) = \frac{1}{32\pi^2}\sum_l(2l+1)\int_0^{\infty}\frac{d\omega}{\omega} \times[e^{-i\omega(t_A-t_B)}(\vec{R}_{\omega l}(r_A)\vec{R}^*{}_{\omega l}(r_B) + \overleftarrow{R}_{\omega l}(r_A)\overleftarrow{R}^*{}_{\omega l}(r_B)) - e^{i\omega(t_A-t_B)}(\vec{R}_{\omega l}(r_B)\vec{R}^*{}_{\omega l}(r_A) + \overleftarrow{R}_{\omega l}(r_B)\overleftarrow{R}^*{}_{\omega l}(r_A))]. \quad (23)$$

That is, although the Wightman function in the Hartle-Hawking vacuum G_H^+ is dependent on the Hawking temperature T_H, the antisymmetric correlation function χ_H^F is not.

Now, we calculate the average rate of change of energy Eq. (15). We begin with $\langle\frac{dH_S(\tau)}{d\tau}\rangle^{AB}$. Taking Eqs. (14) and (23) into Eq. (16), we have

$$\left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle^{AB} = \mp\frac{i\mu^2\omega_0}{128\pi^2}\sum_l(2l+1) \times\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\left(\frac{1}{\omega_0+\frac{\omega}{\sqrt{g_{00}}}-i\varepsilon}+\frac{1}{\omega_0-\frac{\omega}{\sqrt{g_{00}}}+i\varepsilon}\right) \times(\vec{R}_{\omega l}(r_A)\vec{R}^*{}_{\omega l}(r_B)+\overleftarrow{R}_{\omega l}(r_A)\overleftarrow{R}^*{}_{\omega l}(r_B)). \quad (24)$$

So far, the result is expressed as the sum of the radial functions $\vec{R}_{\omega l}(r)$ and $\overleftarrow{R}_{\omega l}(r)$, whose exact forms at the position r is unknown. However, they take the following asymptotic forms near the horizon and at infinity [31],

$$\vec{R}_{\omega l}(r)\sim\begin{cases} r^{-1}e^{i\omega r^*}+\vec{A}_l(\omega)r^{-1}e^{-i\omega r^*}, & r\to 2M,\\ B_l(\omega)r^{-1}e^{i\omega r^*}, & r\to\infty,\end{cases} \quad (25)$$

$$\overleftarrow{R}_{\omega l}(r)\sim\begin{cases} B_l(\omega)r^{-1}e^{-i\omega r^*}, & r\to 2M,\\ r^{-1}e^{-i\omega r^*}+\overleftarrow{A}_l(\omega)r^{-1}e^{i\omega r^*}, & r\to\infty,\end{cases} \quad (26)$$

in which $r^*=r+2M\ln(\frac{r}{2M}-1)$. Here we are interested in the case near the event horizon, i.e., $r\to 2M$. In this region, the sum of the radial functions takes the following forms,

$$\sum_l(2l+1)\vec{R}_{\omega l}(r_A)\vec{R}^*{}_{\omega l}(r_B)\sim-\frac{2i\omega}{L}(f^{4iM\omega}-f^{-4iM\omega}), \quad (27)$$

and

$$\sum_l(2l+1)\overleftarrow{R}_{\omega l}(r_A)\overleftarrow{R}^*{}_{\omega l}(r_B)\sim\frac{27\omega^2}{4}e^{-\frac{i\omega L}{g_{00}}}, \quad (28)$$

where $f\equiv f(r,L)=\frac{\sqrt{1-\frac{2M}{r}+\frac{L}{2r}}}{\sqrt{1-\frac{2M}{r}-\frac{L}{2r}}}$. The derivation of Eq. (27) is shown in the Appendix, and the geometrical optics approximation [31]

$$B_l(\omega\sqrt{g_{00}})\sim\theta\left(\sqrt{27M\omega\sqrt{g_{00}}}-l\right), \quad (29)$$

has been used in the derivation of Eq. (28).

Plugging Eqs. (27) and (28) into Eq. (24), one obtains

$$\left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle^{AB} = \mp\frac{\mu^2\omega_0\sqrt{g_{00}}}{16\pi L} \times\sin\left(2M\omega_0\sqrt{g_{00}}\ln\frac{1-\frac{2M}{r}+\frac{L}{2r}}{1-\frac{2M}{r}-\frac{L}{2r}}\right). \quad (30)$$

Repeating the procedures above, we obtain $\langle\frac{dH_S(\tau)}{d\tau}\rangle^{BA}=\langle\frac{dH_S(\tau)}{d\tau}\rangle^{AB}$, and

$$\left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle^{AA}=-\frac{\mu^2\omega_0^2M}{8\pi r_A}, \qquad \left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle^{BB}=-\frac{\mu^2\omega_0^2M}{8\pi r_B}. \quad (31)$$

Thus, for a two-atom system near a Schwarzschild black hole $(r_A,r_B\to 2M)$, the total average rate of change of energy is,

$$\left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle=-\frac{\mu^2\omega_0^2}{8\pi}\mp\frac{\mu^2\omega_0}{8\pi L_0}\sin\left(\frac{\omega_0}{2a}\ln\frac{1+aL_0}{1-aL_0}\right). \quad (32)$$

Here $L_0=L/\sqrt{g_{00}}$ is the proper distance between the two atoms, and $a=\kappa/\sqrt{g_{00}}$ is the proper acceleration of a static observer outside the black hole. The first term in Eq. (32) is the same as the corresponding term in the case of two inertial atoms in the Minkowski vacuum (See Eq. (44) in Ref. [30]). The second term is dependent on the proper interatomic separation L_0 and the proper acceleration a. The opposite signs of the second term reflect the interference effect of the radiative fields of the two atoms in the symmetric and antisymmetric entangled states, respectively. In the limit $L_0 \to 0$, the average rate of change of the atomic energy is twice that of a single excited atom when the two-atom system is in the symmetric state, while it vanishes when the two-atom system is in the antisymmetric state. This is the well-known phenomena of superradiance and subradiance [35].

B. The Unruh vacuum

We now move on to the case of the Unruh vacuum. From the Wightman function of the massless scalar field in the Unruh vacuum [34],

$$G_U^+(x_A(\tau), x_B(\tau'))$$
$$= \frac{1}{16\pi^2}\sum_l (2l+1)\int_{-\infty}^{\infty}\frac{d\omega}{\omega}e^{-i\omega(t-t')}$$
$$\times\left[\frac{\vec{R}_{\omega l}(r_A)\vec{R}^*{}_{\omega l}(r_B)}{1-e^{-\omega/T_H}} + \theta(\omega)\overleftarrow{R}_{\omega l}(r_A)\overleftarrow{R}^*{}_{\omega l}(r_B)\right], \quad (33)$$

it is direct to obtain that the statistical function of the scalar field $\chi_U^F(x_A(\tau), x_B(\tau'))$ in the Unruh vacuum takes exactly the same form as that in the Hartle-Hawking vacuum shown in Eq. (23), so the average rate of change of atomic energy coincides with Eq. (32). Again, we find that the rate of change of energy is oblivious of the Hawking radiation at temperature T_H. In fact, the same result can also be obtained when the atoms are assume to be coupled with a fluctuating massless scalar field in the Boulware vacuum, which corresponds to the vacuum state of a massive spherical body without Hawking radiation.

Now we compare our result with what has been obtained in Ref. [27]. First, from the calculations above, it is clear that, when the two-atom system is prepared in the symmetric or antisymmetric entangled state, the average rate of change of energy comes solely from the contribution of radiation reaction [30]. However, in Ref. [27], the general expression for the contribution of vacuum fluctuations is erroneous, so the total rate of change of the energy of the two-atom system depends both on the contributions from vacuum fluctuations and radiation reaction, and is also incorrect. Second, as shown above, when the two-atom system is prepared in the symmetric or antisymmetric entangled state, the average rates of change of energy in the cases of the Boulware, Unruh and Hartle-Hawking vacua are the same. That is, although the two-atom system is placed outside a black hole, the rate of change of energy is actually the same as that in the case outside a massive body without Hawking radiation. This is in sharp contrast to the conclusion in Ref. [27], in which three different results are obtained when the atoms are assumed to be in the three different vacua, and the rate of change of atomic energy is claimed to be significantly affected by the Hawking radiation. Third, in the vicinity of the event horizon, the rate of change of energy of the two-atom system is finite instead of divergent as found in Ref. [27]. Note that the interatomic separation L is assumed to be extremely small such that $L \ll 2(r-2M)$, which implies that $aL_0 \ll 1$ is satisfied here.

Actually, it can be expected that, for a two-atom system outside a Schwarzschild black hole prepared in the symmetric or antisymmetric entangled state, the contribution of vacuum fluctuations to the average rate of change of the atomic energy is absent. From Eq. (12), we see that the contribution of vacuum fluctuations is composed of two terms depending on the two atoms individually. As shown in Ref. [11], for a single two-level atom outside a Schwarzschild black hole, the spontaneous excitation rate for the ground state and the spontaneous deexcitation rate for the excited state are the same when only the effects of vacuum fluctuations are taken into account. Moreover, for two-atom systems prepared in the symmetric/antisymmetric state $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|ge\rangle \pm |eg\rangle)$, the populations of the atoms in the ground and excited states are equal. Therefore, the contribution of vacuum fluctuations from the ground state cancels out that from the excited state. If the two-atom system is prepared in a more general state such that the populations of the ground state and the excited state are not the same, the contribution of vacuum fluctuations would exist, and the rate of change of atomic energy would be affected by the Hawking radiation in general.

IV. RATE OF CHANGE OF ENERGY OF A TWO-ATOM SYSTEM IN RINDER SPACETIME

It is well known that the geometry near the horizon of a Schwarzschild black hole can be approximated by the Rindler spacetime. See, e.g., Ref. [36]. In this section, we investigate the rate of change of energy of a quantum system composed of two atoms which are static in the Rindler spacetime, and check if the result agrees with what has been obtained directly in the vicinity of the horizon of a Schwarzschild black hole.

Let us begin with the geometry of the spacetime near the horizon of a Schwarzschild black hole. After introducing a parameter z satisfying

$$r - 2M = \frac{z^2}{8M}, \quad (34)$$

the Schwarzschild metric Eq. (17) near the event horizon can be written as,

$$ds^2 = (az)^2 d\tau^2 - dz^2 - 4M^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (35)$$

which is a Rindler metric with acceleration parameter $a = \kappa/\sqrt{g_{00}}$. Here, τ is the proper time. Taking

$$z = \frac{1}{a}e^{a\xi}, \quad x = 2M\sin\theta\cos\varphi, \quad y = 2M\sin\theta\sin\varphi, \quad (36)$$

into Eq. (35), the metric takes the usual form of the Rindler metric, i.e.,

$$ds^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - dx^2 - dy^2. \quad (37)$$

We assume that the two atoms are static, and are located at ξ_A and ξ_B respectively. In the Rindler coordinates, the wave equation for the scalar field takes the form [32],

$$\left[e^{-2a\xi}\left(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\xi^2}\right) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right]\phi(x) = 0, \quad (38)$$

and the positive frequency Rindler modes v_{ω,k_x,k_y} can be solved as,

$$v_{\omega,k_x,k_y} = \frac{1}{2\pi^2\sqrt{a}}\sinh^{1/2}\left(\frac{\pi\omega}{a}\right)K_{i\omega/a}\left(\frac{k_\perp}{a}e^{a\xi}\right)e^{ik_x + ik_y - i\omega\tau}, \quad (39)$$

where $k_\perp = \sqrt{k_x^2 + k_y^2}$, and $K_\nu(x)$ is the Bessel function of imaginary argument. Then, a vacuum state of the field can be defined, which is known as the Rindler vacuum. It is the vacuum state perceived by a uniformly accelerated observer, and is analogous to the Boulware vacuum in the Schwarzschild spacetime. In this paper, we are interested in the case when the atoms are subjected to the Hawking radiation, so we assume that the environment the two atoms immersed in is a thermal bath of Rindler particles at a temperature T. In the Rindler spacetime, the Wightman function of the massless scalar field at a finite temperature T is given by [37],

$$\begin{aligned} G^+(x_A(\tau), x_B(\tau')) &= \int_0^\infty d\omega \int_{-\infty}^\infty dk_x \int_{-\infty}^\infty dk_y \frac{\sinh(\frac{\pi\omega}{a})}{4\pi^4 a} \\ &\times K_{i\omega/a}\left(\frac{k_\perp}{a}e^{a\xi_A}\right)K_{i\omega/a}\left(\frac{k_\perp}{a}e^{a\xi_B}\right) \\ &\times \left[\frac{e^{\omega/T}}{e^{\omega/T} - 1}e^{-i\omega(\tau-\tau')} + \frac{1}{e^{\omega/T} - 1}e^{i\omega(\tau-\tau')}\right]. \end{aligned} \quad (40)$$

With the help of the following integral (c.f. Eq. 6.521-3 in Ref. [38]),

$$\int_0^\infty xK_\nu(ax)K_\nu(bx)dx = \frac{\pi(ab)^{-\nu}(a^{2\nu} - b^{2\nu})}{2\sin(\nu\pi)(a^2 - b^2)}, \quad (41)$$

the statistical functions of the scalar field can be obtained as

$$\begin{aligned} \chi^F(x_A(\tau), x_B(\tau')) = &-\frac{a}{8i\pi^2(e^{2a\xi_A} - e^{2a\xi_B})} \\ &\times \int_0^\infty d\omega \frac{e^{2i\omega\xi_A} - e^{2i\omega\xi_B}}{e^{i\omega(\xi_A+\xi_B)}} \\ &\times [e^{i\omega(\tau-\tau')} - e^{-i\omega(\tau-\tau')}]. \end{aligned} \quad (42)$$

Taking Eqs. (14) and (42) into Eq. (16), one obtains

$$\left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle^{AB} = \mp\frac{\mu^2 a\omega_0}{8\pi}\frac{\sin[\omega_0(\xi_A - \xi_B)]}{e^{2a\xi_A} - e^{2a\xi_B}}. \quad (43)$$

Further calculations show that, $\langle\frac{dH_S(\tau)}{d\tau}\rangle^{AB} = \langle\frac{dH_S(\tau)}{d\tau}\rangle^{BA}$, and

$$\left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle^{AA} = -\frac{\mu^2\omega_0^2}{16\pi e^{2a\xi_A}}, \quad \left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle^{BB} = -\frac{\mu^2\omega_0^2}{16\pi e^{2a\xi_B}}. \quad (44)$$

Then, the total rate of change of the energy of the two-atom system is,

$$\begin{aligned} \left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle = &-\frac{\mu^2\omega_0^2}{16\pi}\left(\frac{1}{e^{2a\xi_A}} + \frac{1}{e^{2a\xi_B}}\right) \\ &\mp\frac{\mu^2 a\omega_0}{4\pi}\frac{\sin[\omega_0(\xi_A - \xi_B)]}{e^{2a\xi_A} - e^{2a\xi_B}}, \end{aligned} \quad (45)$$

which is expressed in the Rindler coordinates ξ_A and ξ_B. Putting Eqs. (34) and (36) into Eq. (45), and allowing for the relations $r_A = r + \frac{L}{2}$, $r_B = r - \frac{L}{2}$, and $L_0 = L/\sqrt{g_{00}}$, the total rate of change of the energy of the two-atom system near a Schwarzschild can be rewritten with the proper distance between the two atoms L_0, which takes exactly the same form as Eq. (32). Again, it is shown that the rate of change of the energy of the two-atom system does not rely on the temperature T, which verifies our conclusion that the result is independent of the Hawking radiation.

Note that in accordance with the scenario that the two atoms are placed along the radial direction of the Schwarzschild spacetime, here we assume that the two atoms are aligned in the same direction of the acceleration. In a recent work [30], the average rate of change of energy for a pair of uniformly accelerated atoms with the interatomic separation perpendicular to acceleration has been investigated. We quote the result obtained in Ref. [30] as follows,

$$\left\langle\frac{dH_S(\tau)}{d\tau}\right\rangle = -\frac{\mu^2\omega_0^2}{8\pi} \mp \frac{\mu^2\omega_0}{8\pi L_0}\frac{\sin\left(\frac{2\omega_0}{a}\sinh^{-1}\frac{aL_0}{2}\right)}{\sqrt{1 + \frac{1}{4}a^2L_0^2}}. \quad (46)$$

Comparing the results Eqs. (32) and (46), it is obvious that the average rates of change of atomic energy in the parallel and perpendicular cases are different. This shows that the relative direction between the interatomic separation and the acceleration plays an important role in the collective transitions of two entangled atoms.

V. SUMMARY

In this paper, we have investigated the average rate of change of the atomic energy of a pair of static atoms near a Schwarzschild black hole with the help of the DDC formalism. We assume that the two atoms are placed along the radial direction of the black hole and are coupled with fluctuating massless scalar fields in the Hartle-Hawking and Unruh vacua. Different from the result obtained in the recent work [27], we have shown that the average rate of change of the atomic energy for a two-atom system prepared in the symmetric or antisymmetric entangled state is finite and is not affected by the Hawking radiation of the black hole, since the result comes solely from the contribution of radiation reaction. In addition, we have also verified our result by approximating the geometry in the vicinity of the Schwarzschild black hole by the Rindler spacetime.

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APPENDIX: CALCULATION OF THE SUMMATION OF THE RADIAL FUNCTIONS NEAR THE HORIZON

The appendix is devoted to the calculation of the summation of the radial functions $\sum_l (2l+1)\vec{R}_{\omega l}(r_A)\vec{R}^*_{\omega l}(r_B)$ and $\sum_l (2l+1)\overleftarrow{R}_{\omega l}(r_A)\overleftarrow{R}^*_{\omega l}(r_B)$ in statistical function of the field Eq. (23) near the horizon. For a detailed derivation, please refer to Appendix B in Ref. [39]. Here we summarize the main steps in the following.

After the introduction of a new parameter $\zeta = (\frac{r}{2M} - 1)^{1/2}$, the general solution of the differential equation satisfied by the radial function can be written as

$$\vec{R}_l(\zeta) = a_l K_{iq}(2l\zeta) + b_l I_{-iq}(2L\zeta), \tag{A1}$$

where $q = 4M\omega$, and a_l and b_l are two coefficients to be determined. Here, b_l should be an exponential small function of l, since the effective potential in the differential equation the radial function satisfies tends to be infinitely large. On the other hand, the coefficient a_l can be determined by comparing Eq. (A1) with the approximate analytical expression Eq. (25), and the result is

$$a_l = \frac{e^{i\frac{q}{2}} l^{-iq}}{M\Gamma(iq)}. \tag{A2}$$

Then,

$$\begin{aligned}\sum_l (2l+1)\vec{R}_{\omega l}(r_A)\vec{R}^*{}_{\omega l}(r_B) &\sim \frac{1}{M^2\Gamma(iq)\Gamma(-iq)}\sum_l (2l+1)K_{iq}(2l\zeta_A)K_{iq}(2l\zeta_B) \\ &= \frac{2}{M^2\Gamma(iq)\Gamma(-iq)}\int_0^\infty dl\, l K_{iq}(2l\zeta_A)K_{iq}(2l\zeta_B). \end{aligned}\tag{A3}$$

With the help of the integral shown in Eq. (41) and

$$\Gamma(i\xi)\Gamma(-i\xi) = \frac{\pi}{\xi\sinh(\pi\xi)}, \tag{A4}$$

the equation above can be simplified as

$$\sum_l (2l+1)\vec{R}_{\omega l}(r_A)\vec{R}^*{}_{\omega l}(r_B) \sim -\frac{2i\omega}{L}(f^{4iM\omega} - f^{-4iM\omega}), \tag{A5}$$

with

$$f \equiv f(r, L) = \frac{\sqrt{1 - \frac{2M}{r} + \frac{L}{2r}}}{\sqrt{1 - \frac{2M}{r} - \frac{L}{2r}}}. \tag{A6}$$

In the limit $L \to 0$, Eq. (A5) becomes

$$\sum_l (2l+1)\vec{R}_{\omega l}(r)\vec{R}^*{}_{\omega l}(r) \sim \frac{8M\omega^2}{r g_{00}}. \tag{A7}$$

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