Glitches due to quasineutron-vortex scattering in the superfluid inner crust of a pulsar

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We revisit the mechanism of vortex unpinning caused by the neutron-vortex scattering [B. Layek and P. R. Yadav, Mon. Not. R. Astron. Soc. **499**, 455 (2020)] in the inner crust of a pulsar. The strain energy released by the crustquake is assumed to be absorbed in some part of the inner crust and causes pair-breaking quasineutron excitations from the existing free neutron superfluid in the bulk of the inner crust. The scattering of these quasineutrons with the vortex core normal neutrons unpins a large number of vortices from the thermally affected regions and results in pulsar glitches. We consider the geometry of a cylindrical shell of the affected pinning region to study the implications of the vortex unpinning in the context of pulsar glitches. We find that a pulsar can release about $\sim 10^{11}$–10^{13} vortices by this mechanism. These numbers are equivalent to the glitch size of orders $\sim 10^{-11}$–10^{-9} for Vela-like pulsars with the characteristic age $\tau \simeq 10^4$ years. We also suggest a possibility of a vortex avalanche triggered by the movement of the unpinned vortices. A rough estimate of the glitch size caused by an avalanche shows an encouraging result.

I. INTRODUCTION

After the discovery of a pulsar glitch in the Vela pulsar in 1969 [1], a large number of glitches have been observed [2] and reported to date [3]. The size of glitches lies in the range $\sim (10^{-5} - 10^{-12})$, with an interglitch time of a few years. A sudden change of moment of inertia (MI) of a pulsar caused by a crustquake was initially proposed [4] to be responsible for such events. However, it is now realized [5] that though the crustquake model can account for small size Crab-like glitches $\Delta\Omega/\Omega \leq 10^{-8}$, the model is not compatible with Vela-like glitches ($\Delta\Omega/\Omega \simeq 10^{-6}$). Presently, the models associated with pinning and unpinning of superfluid vortices [6,7] are considered to be the most popular models for glitches. Although the crustquake model [4] is not compatible with large size glitches, the crustquake event is believed to be a regular phenomenon of a rotating neutron star. Hence, there were suggestions to relate crustquake as a source of other astronomical events, viz., the giant magnetic flare in magnetars [8,9], observed gamma-ray burst [10], the emission of gravitational waves [11,12] from isolated pulsars, etc. From the glitch perspective, there were attempts to unify the crustquake model with the model of superfluid vortices [13,14]. For example, the authors of Refs. [13,14] proposed that the crustquake might act as a trigger mechanism for vortex avalanches, which is responsible for the release of a large number of vortices ($\sim 10^{18}$) from the inner crust of the star and hence can produce large size glitches. Similarly in Ref. [15], it was suggested that the motion of vortices attached to the broken crustal plate caused by the crustquake might be responsible for the glitches. There was also a study [16] of glitches through thermal creep theory, where the pulsar glitch was suggested to be driven by sudden energy deposition in the inner crust. The crustquake has been assumed to be one such resource for energy deposition. The deposited energy propagates as thermal waves throughout some parts of the inner crust and raises the temperature locally. This affects the coupling between the neutron superfluid and the rigid outer crust, causing the star to spin up.

The above discussion suggests that besides explaining small size glitches or attempting to understand a few astronomical events, the crustquake is also believed to have a role in the superfluid vortex model. In view of this, some of us [17] earlier proposed a mechanism of vortex unpinning initiated by a crustquake. The purpose of the unification of the crustquake with the superfluid vortex model is to explain large size glitches without changing the interglitch time. Note that in the crustquake model, a large glitch requires a longer interglitch time, contrary to the observation. Thus, the model alone is not sufficient for explaining large size glitches. On the other hand, though the superfluid model is compatible with the larger glitches, it requires a dynamical mechanism to unpin the vortices from the inner crust of the star. The occurrence of a crustquake followed by vortex unpinning may resolve the above important issue associated

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with the size and frequency of glitches from an individual pulsar. With this aim, we will further explore the unpinning mechanism as proposed in Ref. [17] by supplementing a few more qualitative arguments in favor of the unpinning process. In Ref. [17], the study was initiated by taking a simple cubical geometry of the affected pinning site to test mainly the new vortex unpinning mechanism. Here we will extend our analysis by considering a cylindrical geometry of the affected pinning region. Such geometry is taken, partly encouraged by the studies of thermal glitches through creep theory in Ref. [16], where authors assumed an energy deposition in a cylindrical shell in the inner crust region. However, the shell was taken in an arbitrary location in that study. As the location of the pinning site (i.e., the place where the thermal energy is presumably absorbed) was unknown, we will study the effect by taking the shell at different locations in the inner crust. In this case, depending on the affected region's location, vortex unpinning through avalanche is further possible. In contrast, the possibility of vortex avalanches was ineffective in the earlier study in Ref. [17]. As vortex avalanches may be responsible for large-size glitches, it is worth exploring the such possibility in the new proposed picture of vortex unpinning. Here, we should mention that the energy absorption by neutron star matter is an important issue, and the absorption should depend on the properties of the matter in the inner crust, as noted by the authors in Ref. [18]. However, the author in that work assumed a simplistic polytropic equation of the state of the matter. So, working in this direction is essential to settle the question of energy absorption by taking a more realistic equation of state. However, such a study needs particular attention, and we would like to explore it in our future work.

We assume that the strain energy released by a crustquake is absorbed in the inner crust of the pulsars. This breaks a significant fraction of the neutron Cooper pairs from the existing free neutron superfluid in the inner crust. The creation of *free electrons* and holes by breaking up the Cooper pairs have been often mentioned in the literature [19,20] in the context of electrical superconductivity. The free electrons resulting from the pair breaking are referred to as quasiparticles. The breaking (and formation) of Cooper pairs from the neutron superfluid has also been extensively discussed in the context of the cooling mechanism of neutron stars through neutrino emission [20–22]. The existence of the superfluid gap sets the minimum energy requirement for breaking up Cooper pairs. For the case of neutron superfluid, the quasiparticles are the neutrons, and the corresponding gap parameter for the bulk neutron superfluid matter will be denoted by Δ_f.

Here we should mention that the above picture of quasiparticle excitations created through the breaking of Cooper pairs is different from the excitations (i.e., phonon, roton, kelvon, etc., depending on the nature of the superfluidity) as suggested through Landau's phenomenological

two-component superfluid model [23]. Here we will take the picture of quasineutron excitations produced from pair breaking of neutron superfluid. These quasineutrons scatter with the vortex core *normal* neutrons and share their energy with the pinned vortices. (Here, the quasineutrons should be treated like normal neutrons. See the comments made in [19]). This causes unpinning of a large number of pinned vortices ($\sim 10^{13}$) from the affected region in the inner crust and results in the glitch event. In the previous work [17], the volume of the pinned region, the number of vortices, and eventually the glitch size was estimated for a fixed value of Fermi momentum (k_f). In this work, all the above quantities have been evaluated by varying the Fermi momentum of the bulk superfluid neutrons, which can be achieved by taking the shell at a different location. We will see an important implication if the affected shell is taken into the deeper region of the inner crust. In that case, the vortex unpinning from that region may trigger a vortex avalanche and results in Vela-like large-size glitches. We will explore this feasibility by taking a picture of proximity knock-on as described in the literature [24,25].

The paper is organized in the following manner. After the initial setup of the formulations in Sec. II, we will describe the geometry of the affected pinned vortex region in Sec. III. There, we will develop the mathematical tools for estimating various quantities, namely, thickness (δ_s) of the shell, number of unpinned vortices (N_v), and size of pulsar glitches $\Delta\Omega/\Omega$ etc. The mechanism of vortex unpinning by neutron-vortex scattering will be presented in Sec. IV. We will present our results in Sec. V. The feasibility of the vortex avalanche through the process of proximity knock-on will be discussed in Sec. VI. We will comment and conclude our work in Sec. VII.

II. PULSAR GLITCHES THROUGH VORTEX UNPINNING: THE BASIC FORMULATION

First, we will mention the neutron star model, based on which we will implement the vortex unpinning mechanism [17] for studying pulsar glitches. Since the possibility of the existence of neutron stars was hypothesized in 1934 [26,27], there have been attempts to understand various properties of the neutron star. For a given mass of a neutron star, the internal structure (such as the inner crust thickness, its moment of inertia, etc.) and the bulk properties (such as the radius, total moment of inertia, etc.) depend on the equation of state (EOS) of the nuclear matter [28–31]. However, extracting the precise values of neutron star parameters is still challenging, mainly because of nonadequate knowledge of EOS for the highly dense matter prevailing inside the star. For our purpose, we will take a broad picture of the neutron star's internal structure that has emerged after several studies and is taken in the studies of various pulsar phenomena. A neutron star of mass about $1.4 M_\odot$ and radius of order (10–12) km consists of the inner crust of thickness (1–2) km sandwiched between

(0.3–0.5) km thick rigid outer crust and the core of radius ~ 9 km [29–31]. The baryon density in the inner crust for such a stellar mass neutron star lies in the range $(10^{11}$–$10^{14})$ g-cm^{-3}. The matter at such high baryon density exists in the form of neutron superfluid [6,32], even though the internal temperature can be in the sub-MeV scale [16]. The denser inner core consists of quantum liquid forming neutron and proton superfluid. There is a conjecture for the existence of an even more exotic form of matter inside the core of a more massive and compact neutron star [33]. For such a case, the values of various parameters are expected to deviate from those mentioned above. As our focus will be mainly on Vela-like pulsars, we will assume the neutron stars of mass $\sim 1.4 M_\odot$ with radius ~ 11 km [29] and study the pulsar glitches implementing the vortex unpinning picture of [17] in the standard superfluid vortex model [6]. So, we will also fix the other neutron star parameters, most importantly, the thickness of the inner crust, based on the above neutron star model only. For more precise calculations and consistency checks, one should extract the information about the neutron star parameters by choosing the EOS correctly. We will discuss the issue of parameter-dependent sensitivity of our results in an appropriate place (Sec. V).

For a rotating neutron star, the rigid part of the star's crust (i.e., outer crust) and the inner core are assumed to corotate with a moment of inertia I_c. The neutron superfluid component of a moment of inertia I_f in the inner crust acts as an angular momentum reservoir in the form of pinned vortices. The time $t = 0$ is set when the superfluid vortices are pinned in the inner crust region. After a crustquake, a fraction of these vortices get unpinned [17] at $t = t_p$ and results in pulsar glitches. We will see that the unpinning occurs from a localized region of the inner crust, and we call it *local unpinning* to distinguish the unpinning occurring through the avalanche process (Sec. VI). The geometry of the affected region and the unpinning mechanism will be discussed in Secs. III and IV, respectively. The time t_p is the interglitch time and assumed (such assumption will be justified later) to be of the same order as the frequency of the successive crustquakes. We denote Ω_p as the angular velocity of the superfluid component in the inner crust, which remains fixed during $t = 0$ to $t = t_p$. $\Omega_c(t)$ is the angular velocity of the corotating crust-core system with $\Omega_c(0) = \Omega_p$. The development of differential angular velocity $\delta\Omega = \Omega_p - \Omega_c(t)$ between the inner crust superfluid and the rest of the star follows the time evolution of the star. The differential angular velocity at $t = t_p$ can be expressed as

$$\frac{\Omega_p - \Omega_c(t_p)}{\Omega_c(t_p)} \equiv \left(\frac{\delta\Omega}{\Omega}\right)_{t_p} = \frac{t_p}{2\tau}, \qquad (1)$$

where $\tau = -(\Omega/2\dot{\Omega})$ is the characteristic age of the pulsar and we assume, $t_p \ll \tau$. For the ease of notation,

from now onward, we denote $(\delta\Omega/\Omega)_{t_p}$ as $\delta\Omega/\Omega$. Applying the standard superfluid vortex model [6,7] (see also the review [34] for various models of glitches), the glitch size can be written as

$$\frac{\Delta\Omega}{\Omega} = \left(\frac{I_f}{I_c}\right)\left(\frac{\delta\Omega}{\Omega}\right)\left(\frac{N_v}{N_{vt}}\right) = \left(\frac{I_f}{I_c}\right)\left(\frac{t_p}{2\tau}\right)\left(\frac{N_v}{N_{vt}}\right). \qquad (2)$$

Here I_f/I_c is the MI ratio of the bulk neutron superfluid in the inner crust to the rest of the star. The quantities N_v and N_{vt} are the number of pinned vortices in the affected region and the total number of pinned vortices in the equatorial plane in the inner crust [Eqs. (7) and (8)], respectively. The ratio N_v/N_{vt} takes care of the fact that only a fraction of the pinned vortices is affected by the local unpinning.

III. MATHEMATICAL TOOLS FOR ESTIMATING VARIOUS QUANTITIES RELATED TO GLITCH

As proposed in Ref. [17], the thermally excited neutrons are responsible for unpinning the superfluid vortices. In turn, the number of unpinned vortices depends on the release of energy in a crustquake event. Here we will briefly describe the crustquake model [4], mainly focusing on the model's essential features relevant to our study (for more details, see the Ref. [5]). The oldest theoretical model for pulsar glitches, namely, the crustquake model, assumes the existence of a solid deformed crust of a pulsar. The oblateness parameter ϵ can characterize the deformation as $\epsilon = \frac{I_{zz} - I_{xx}}{I_0}$. Where I_{zz}, I_{xx} and I_0 are the moment of inertia about the z-axis (rotation axis), x-axis, and the spherical star, respectively [5]. At an early stage of formation, with a very high rotational frequency, the crust solidified with an initial larger value of oblateness. As the star slows down due to electromagnetic radiation loss, the oblateness $\epsilon(t)$ keeps decreasing, causing crustal strain development in the star's outer crust. Once the critical strain is reached, the crustquake occurs to achieve a new equilibrium. This results in sudden change (decrease) of oblateness $\Delta\epsilon$. Henceforth, we will take $\Delta\epsilon$ to be positive. The decrease in oblateness causes the star's moment of inertia (MI) to decrease, increasing its rotational frequency (following angular momentum conservation). The glitch size $(\frac{\Delta\Omega}{\Omega})$ of a pulsar is directly related to $\Delta\epsilon$ through $\frac{\Delta\Omega}{\Omega} = -\frac{\Delta I}{I_0} = \Delta\epsilon$. Immediately after the proposal, it was realized [5] that the interglitch time or the waiting time of two successive glitches is also determined by the change of oblateness and is proportional to $\Delta\epsilon$. As noted in Ref. [5], 10^{-8} size Crab-like glitches requires an average one-year waiting time [35]. Thus the glitch size being proportional to $\Delta\epsilon$, a larger glitch needs a longer waiting time, contrary to the observations. For example, for two successive glitch events, Vela pulsar (of glitch size $\simeq 10^{-6}$) requires about 100 years. This is the most critical problem the crustquake model has encountered since its inception. As mentioned before,

unifying the crustquake with the superfluid vortex model may explain large-size glitches with compatible interglitch time. Now, for determining the energy release in a single crustquake event, we will take $\Delta\epsilon = 10^{-8}$. This value is consistent with a typical one-year observed waiting time between successive glitch events. As the crustquake occurs, it releases strain energy of magnitude $\Delta E = B\Delta\epsilon$. Here the constant B ($\sim 10^{48}$ erg) is related to the modulus of rigidity of the crust. As mentioned earlier in Sec. II, the released energy by the crustquake is assumed to be thermally absorbed in a local region in the inner crust of the star. This results in breaking up neutron-neutron Cooper pairs from the bulk neutron superfluid and creates free quasineutrons (excited neutron) in the inner crust. The sharing of energy through the scattering of these excited neutrons with the vortex core normal neutrons causes the unpinning of vortices (Sec. IV) and results the glitch event. For the estimate of the number of unpinned vortices N_v, the affected pinning region is taken to be of a cylindrical shell of height h_s and thickness δ_s. Such geometry was also taken in Ref. [16] in their study of pulsar glitches through thermal creep theory. However, barring the geometry, our formulation of generation of pulsar glitches is completely different compared to the approach taken in Ref. [16]. The reason for choosing the affected region around the equatorial plane is motivated by the crustquake picture of [5]. Also, in the studies of the crustal strain [36], the authors have found the strain to be maximum at the equatorial plane, which makes it the most likely place for the quake site.

As we will see, the thickness (δ_s) of the cylindrical shell that determines the number of pinned vortices therein crucially depends on the Fermi momentum k_f of the bulk superfluid neutrons, which in turn depends on the baryon density of the specific region. We will follow the work of Pastore *et al.* [37], where the authors have studied the properties of the neutron superfluid in the inner crust of the star (see also the seminal work of Negele and Vautherin [28]). In that work, the Fermi momentum was computed for the baryon density region ranging from $\rho \simeq 10^{12}$ gm cm^{-3} to $\rho \simeq 10^{14}$ gm cm^{-3}. As mentioned by the authors, the spherical Wigner-Seitz approximation, which was used to calculate the various quantities, can reproduce well ground-state properties of the outermost regions of the inner crust. However, as the methodology breaks down beyond $\rho \simeq 10^{14}$ gm cm^{-3}; we will restrict our study up to the baryon density $\rho = 10^{14}$ gm cm^{-3}. The corresponding Fermi momentum is given as $k_f = 1.2$ fm^{-1} (Table 1 of Ref. [37]). As the Fermi momentum depends on the local mass density, or equivalently on the depth of the crust, the value of δ_s should also depend on the location of the shell. As the precise location of the thermally affected region is unknown to us, we will vary the distance of the shell R_s (as measured from the center of the star) from 10.3 km to 9.9 km for the estimating of various quantities associated with the glitches. The above values of

R_s are in accordance with the Fermi momentum in the range (0.2–1.2) fm^{-1} and local mass density in the range $(10^{12}$–$10^{14})$ gm cm^{-3} (Fig. 1 of [16] and Table 1 of [37]). Here we should mention that the authors [37] have calculated a finite discrete set of Fermi momentum at various values of the mass density in the inner crust of the star. However, we have assumed a continuous distribution of k_f within a particular range as mentioned above. We hope that the interpolation of k_f in between two successive data will not make any significant changes to our results. Note that it is due to the uncertainty on the location of the shell, the various quantities are calculated at various Fermi momentum. However, it serves a useful purpose of testing the sensitivity of the results due to the uncertainty in the affected region. As mentioned earlier, it also allows exploring the feasibility of avalanches through the picture of proximity knock-on (Sec. VI) triggered by the unpinned vortices.

We will now calculate the thickness of the shell δ_s, the number of the unpinned vortices N_v, and the glitch size $\Delta\Omega/\Omega$ at various values of the Fermi momentum. The volume V_s of the affected cylindrical region can be estimated by energy balance [17,38]

$$B\Delta\epsilon = N_e\Delta_f = \frac{\Delta_f^2}{E_f}n_f V_s, \tag{3}$$

i.e.,

$$V_s = \frac{B\Delta\epsilon E_f}{n_f \Delta_f^2}, \tag{4}$$

where n_f, Δ_f and E_f denote the number density of the bulk superfluid neutrons, the energy gap parameter, and the Fermi energy, respectively. N_e is the number of excited neutrons produced from the Cooper pair break up. For a cylindrical geometry, the volume V_s within which the energy is deposited can be written as $V_s = 2\pi R_s h_s \delta_s$. The number of pinned vortices in the enclosed area $A_s (= 2\pi R_s \delta_s)$ in the equatorial plane can be estimated from the vortex areal density $n_v = 4m_n\Omega/h \simeq 10^7$ m^{-2} (Ω/s^{-1}). The height of the cylinder is determined by defining the affected pinning region such that all the vortex lines enclosed in an equatorial area A_s will contribute to the vortex unpinning. For this, the cylinder is cut at an approximate height h_s, where the vortex lines terminate on the boundary of the inner crust with the outer crust. Here we should mention that because of the sphericity of the inner crust, the affected region is not expected to be in the form of a perfect right circular cylinder. However, considering the number of affected vortices to be enormous, we assume the geometry to be a right circular cylinder for simplicity. Thus the height of the cylinder is given by (see Fig. 1 for illustration),
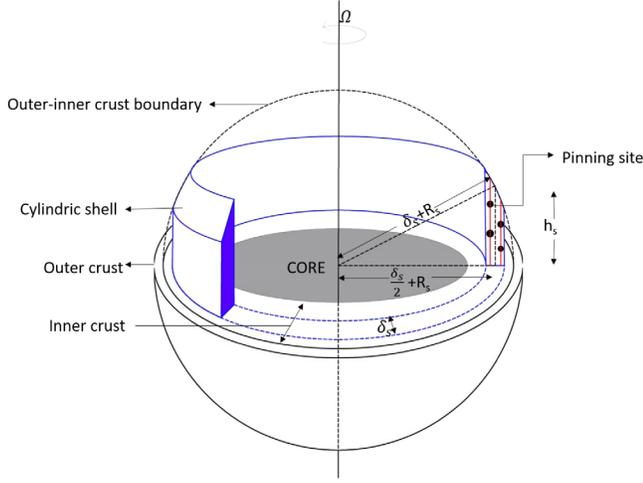
FIG. 1. The cylindrical shell of thickness δ_s and height h_s represents the affected pinning site (blue colored region) in the inner crust. The vortex lines (red color) terminating on the outer-inner crust boundary defines the (average) height of the cylinder.

$$h_s = [(R_s + \delta_s)^2 - (R_s + \delta_s/2)^2]^{1/2} \simeq (R_s\delta_s)^{1/2}. \quad (5)$$

Here we have taken the approximation $\delta_s \ll R_s$. Using the above equation, the volume V_s can now be expressed in terms of the thickness of the shell as

$$V_s = 2\pi(R_s\delta_s)^{3/2} = \frac{B\Delta\epsilon E_f}{n_f\Delta_f^2}. \quad (6)$$

Using the above equation, we can now determine the shell thickness δ_s by choosing the Fermi momentum k_f in the range 0.2 fm^{-1}–1.2 fm^{-1}. Note that neutron density n_f, Fermi energy E_f, and the gap parameter Δ_f all are functions of k_f. In the above, we assumed the Fermi momentum to be uniform within a shell. However, the vortex lines emanating from the equatorial plane pass through various baryon density regions; hence, k_f should vary along the height of the cylinder. Including this factor in the calculation requires a detailed density profile of the inner crust region, which depends on EOS for a given neutron star mass. We expect some uncertainty in our estimate of δ_s and the other associated quantities by assuming a fixed k_f (and n_f, Δ_f, E_f, etc.). Within this limitation, the number of pinned vortices enclosed in A_s can be estimated as

$$N_v = A_s n_v = \frac{V_s n_v}{h_s} = \frac{B\Delta\epsilon E_f n_v}{n_f\Delta_f^2(R_s\delta_s)^{1/2}}. \quad (7)$$

We have used Eq. (6) for the last equality in the above equation. The vortex lines cannot terminate in the inner crust superfluid region for topological reasons. These can form either a close loop or should end on the boundary of the inner crust. Thus, the number of vortex lines within the affected region is the same as that of vortices enclosed in A_s (ignoring the loop). Once we fix the thickness of the cylindrical shell from Eq. (6) (by choosing the appropriate values of the parameters), we can estimate the number of unpinned vortices using Eq. (7). We will use Eqs. (1) and (2) for the estimate of size of the glitches. The total number of vortices N_{vt} in the whole inner crust is obtained as

$$N_{vt} \simeq (2\pi R\Delta R)n_v, \quad (8)$$

where, $R(\simeq 10$ km) is the average distance of the inner crust from the center of the star and $\Delta R(\simeq 1$ km) is the thickness of the inner crust. In the above, we assumed a uniform vortex density n_v throughout the crust while estimating the number of vortices. As the vortex density, n_v is proportional to the rotational frequency Ω; in principle, one can relax such an assumption. One can take instead a local vortex density $n_v(r)$, which depends on the distance (r) from the rotation axis. This is equivalent to assuming the radial distance-dependent rotational frequency of the various superfluid region. In fact, the thermal creep theory [39] considers such radial distance dependence vortex density $n_v(r) = \frac{2\Omega(r)}{\kappa} + r\frac{\partial\Omega(r)}{\partial r}$ (see Refs. [16,39] for the detailed formulation of creep theory). Here $\kappa = h/2m_n$ is the quantum vorticity with m_n being the mass of a neutron. As in creep theory, the dynamics and the steady state behavior of the superfluid-crust couple system are realized through the motion of vortices. In such a scenario, if the coupling is assumed to happen locally, taking the radial dependence of angular frequency and hence the vortex density is essential. However, for explaining large size glitches ($\sim 10^{-6}$) within the standard superfluid vortex model [7], a very large fraction of the pinned vortices must be released simultaneously. In that case, assuming a common angular frequency throughout a significant region of the superfluid components in the inner crust is natural. The glitch size is now obtained from Eq. (2) as

$$\frac{\Delta\Omega}{\Omega} = \left(\frac{I_f}{I_c}\right)\left(\frac{t_p}{2\tau}\right)\left(\frac{R_s}{R}\right)\left(\frac{\delta_s}{\Delta R}\right). \quad (9)$$

We will use the above equation to present the results in Sec. V. Now, we will revisit the vortex unpinning mechanism, supplementing a few more physical arguments in favor of our earlier proposal [17].

IV. VORTEX UNPINNING THROUGH NEUTRON-VORTEX SCATTERING

The strain energy released in a crustquake event is taken as $\Delta E = B\Delta\epsilon \simeq 10^{40}$ erg. We assume this energy is partially absorbed in the cylindrical shell and thermally excites the neutrons from the bulk neutron superfluid by breaking some fraction of Cooper pairs. The excited quasineutrons,

in turn, unpin the vortices from the shell through neutron-vortex scattering. In the volume V_s, the absorbed energy can excite about N_e ($\sim \frac{\Delta_f}{E_f} n_f V_s$) number of neutrons from the bulk neutron superfluid. Each of these excited neutrons, on average, has an energy E_f. We will see that this energy is more than the pinning energy E_p (per site) in the cylindrical shell, irrespective of the shell's location. We also note that, if N_p ($\sim \frac{V_s}{d_v^3}$) denotes the number of pinned vortices in the shell, then the number of excited neutrons per pinned vortex (i.e., N_e/N_P) is of order $\sim (d_v k_f)^3 \sim 10^{30}$. Here d_v ($\sim 10^{10}$ fm) is the intervortex distance and the Fermi momentum k_f is of order fm^{-1}. Thus, each pinned vortex, on average, is surrounded by approximately 10^{30} excited neutrons. So these neutrons are expected to scatter with the vortex core normal neutrons to unpin the vortex. In the picture of unpinning through scattering, the pinning energy E_p should be treated as the binding energy of the vortex-nucleus system arising due to the vortex-nucleus interaction. The sharing of energy by the excited neutrons with the vortex core neutrons increases the latter's energy. Here the energy of the excited neutron acts as activation energy helping to overcome the pinning barrier. The neutron-vortex scattering can be represented as [17].

$$\text{excited neutron}\,(\sim E_f) + \text{pinned vortex}\,(-E_P)$$
$$\rightarrow \text{de-excited neutron}\,(E_f - E_p) + \text{free vortex}.$$

In above, the energy of various objects are denoted in bracket. The negative sign in front of $E_p (> 0)$ conventionally signifies the binding energy of a pinned vortex.

We now compare the average energy of the excited neutron, i.e., Fermi energy with the pinning energy. The pinning energy per site depends on the local mass density, or equivalently, on the Fermi momentum of the neutron superfluid and it is given by [39,40]

$$E_p = \frac{3}{8} \gamma \frac{\Delta_f^2}{E_f} n_f V, \qquad (10)$$

where $V = \frac{4}{3} \pi \xi^3$ is the overlap volume between the vortex and the nucleus. The size of a vortex core $\xi (\simeq 10$ fm$)$ is of the same order as the nuclear radius. The numerical value of γ is of order unity [39,41]. The value of E_p, which can be determined from Eq. (10), should now be compared with the average energy (i.e., the Fermi energy) of each excited neutron. As per the results (see Fig. 2), the energy required to overcome the pinning barrier is satisfied comfortably throughout the region of interest.

Macroscopically, the possibility of vortex unpinning through neutron-neutron scattering can also be realized from the following arguments. While proposing the superfluid vortex model for glitches, it was [6] mentioned that the magnitude of the frictional force arises due to the
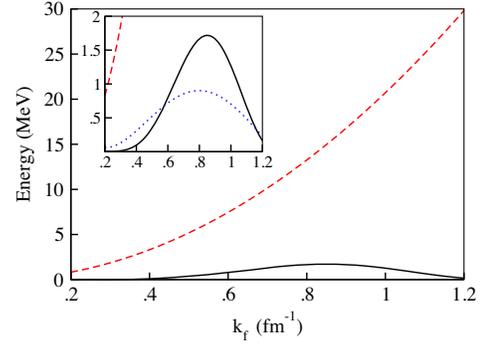

FIG. 2. Excitation energy E_f (red dashed) and pinning energy per site E_p (black solid) as a function of Fermi momentum. Inset shows the same plot with smaller energy range for demonstration of E_p more clearly. Blue dotted line shows the gap parameter $\Delta_f(k_f)$ (The expression of $\Delta_f(k_f)$ is taken from [42]).

scattering of electrons with the vortex core neutrons (e-n scattering) is too small compared to the pinning force to unpin the vortices. However, as we see below, in comparison, the frictional force that arises due to neutron-neutron scattering can be pretty large. To understand this, let us write the expression of the frictional force (per unit volume) caused by n-n scattering as (following Ref. [6])

$$F_{nn} \simeq \frac{\rho_n R \delta \Omega}{\tau_{nn}} \simeq \left(\frac{\Delta_f}{E_f} \rho_f \right) \left(\frac{R \delta \Omega}{\tau_{nn}} \right), \qquad (11)$$

where $\rho_n (\simeq \frac{\Delta_f}{E_f} \rho_f)$ and ρ_f are the mass density of excited neutrons and superfluid neutrons, respectively. The timescale τ_{nn} is set by the scattering events of the excited neutrons with the normal vortex neutron (neutron-neutron scattering). Note that for the frictional force due to electron-neutron scattering, ρ_n and τ_{nn} will be replaced by electron mass density ρ_e and the relevant timescale τ_{en}, respectively. The numerical value of τ_{nn} is expected [17] to be quite small of order 10^{-5} s compared to e-n scattering timescale τ_{en}. Note, τ_{en} is the typical spin-up decay time of order few months [6,43]. The small value of τ_{nn} arises solely due to larger strength of neutron-neutron magnetic moment interaction relative to the strength of e-n interaction [17,43]. Now for comparison with the frictional force caused by e-n scattering, we take the free electron mass density [6] in the inner crust $\rho_e \simeq 10^7$ g-cm^{-3} and $\tau_{en} \simeq 10^7$ s. Similarly, for n-n scattering the mass density of excited neutron can be approximately taken as $\rho_n \simeq 10^{12}$ g-cm^{-3}. For an order of magnitude estimate, Fermi momentum k_f, for example, is taken as 0.8 fm^{-1} and the values of other parameters are fixed accordingly. Thus there is about 10^{17} order of magnitude enhancement of the frictional force due to n-n scattering in comparison to e-n scattering. The frictional force for n-n scattering turns out to be quite large because of larger values of $(1/\tau_{nn}) \simeq 10^{12} (1/\tau_{en})$ and $\rho_n \simeq 10^5 \rho_e$.

This makes the n-n scattering more effective for vortex unpinning.

Note that after successful unpinning, the vortices move radially outward. These unpinned vortices encounter a large number of vortices during their motion and should trigger an avalanche through the so-called "proximity knock-on" process [24,25]. Thus, the vortex avalanche can be a viable process to produce large size glitches in this picture of local unpinning. We will explore the feasibility of an avalanche later in Sec. VI.

V. RESULTS: NUMERICAL VALUES OF UNPINNED VORTICES AND THE GLITCH SIZE

A. Unpinned vortices

The volume of the pinned region depends on the availability of energy for deposition and a few other parameters such as k_f (or, n_f), E_f and Δ_f. The value of the neutron superfluid Fermi momentum k_f depends on the local mass density ρ, which increases with the depth of the inner crust [16,37], and lies in the range $\sim(10^{11}–10^{14})$ gm-cm^{-3}. Thus, the neutron density $n_f = k_f^3/3\pi^2$ also varies accordingly. We will use the results of [37], where the Fermi momenta have been calculated at various values of the local mass density. For superfluid gap parameter $\Delta_f(k_f)$, we used the analysis of [42], where the authors have numerically fitted their data to find the analytical expression of the gap parameter as a function of k_f (Fig. 2). Thus, knowing the variation of N_v with k_f, supplemented with the information about the variation of k_f with ρ, and the variation of ρ with the depth of the inner crust, allow us to calculate the number of unpinned vortices at different location of the cylindrical shell.

We have taken the values of k_f from 0.2 fm^{-1} to 1.2 fm^{-1} with the corresponding mass density ρ in the range $\sim(10^{12}–10^{14})$ gm-cm^{-3}. Accordingly, the distance R_s of the shell from the star's center is taken from 10.3 km to 9.9 km. The results are shown in Figs. 3 and 4. The plots show that as k_f increases from 0.2 fm^{-1} to 1.2 fm^{-1}, the
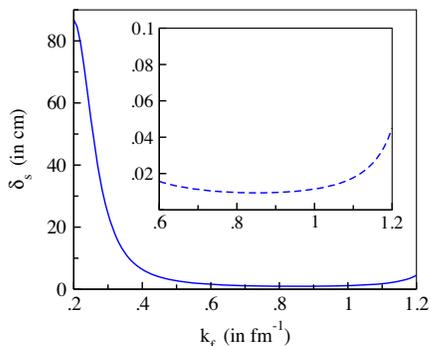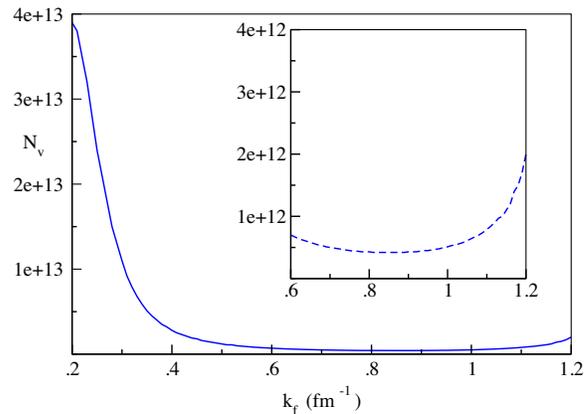

FIG. 4. The number of unpinned vortices (N_v) caused by the neutron-vortex scattering in the cylindrical shell of thickness δ_s versus the Fermi momentum k_f. For clarity, inset shows only the lower part of N_v.

thickness δ_s of the shell, and hence the number of unpinned vortices N_v decreases almost monotonically. The values of δ_s and N_v lie in the range (87–1) cm and $(10^{13}–10^{12})$, respectively. Since k_f maps the depth of the crust, the variation of δ_s and N_v are due to the location of the shell in our cylindrical geometry. For illustration, $k_f = 0.2$ corresponds to relatively outer layer of the inner crust ($R_s \simeq 10.3$ km) with mass density $\rho \simeq 10^{12}$ gm-cm^{-3}. Therefore, the energy deposition around this region leads to the release of about 10^{13} vortices due to the scattering by the excited neutrons. Similarly, if the shell is located around the region with $k_f = 1.2$ ($R_s \simeq 9.9$ km), the number of released vortices will be reduced to 10^{12}. Although the number is reduced with the depth of the crust, this has an important implication in the context of vortex avalanche. We explore this possibility in Sec. VI.

B. Glitches

The glitch size can be determined using Eq. (9). The interglitch time t_p is set by the frequency of occurrence of crustquake events and is proportional to the change of oblateness $\Delta\epsilon$. We choose $\Delta\epsilon = 10^{-8}$ to be consistent with the observed typical interglitch time $t_p \simeq 1$ year. It is now evident from the Eq. (9) that the glitch size of an individual pulsar of characteristic age τ depends on various neutron star parameters, such as R, R_s, and ΔR, etc. The values of these parameters are taken by following the model of neutron star structure as mentioned in Sec. II. The glitch size also depends on the ratio of the moment of inertia of the superfluid component in the inner crust to the rest of the star I_f/I_c. This ratio depends on a specific glitch model. For example, the author in Ref. [7] suggested that a normal liquid layer can exist between the inner crust neutron superfluid and the core superfluid, resulting in a larger MI ratio. Without such a transition layer, the ratio can be of

FIG. 3. Thickness δ_s of the shell versus the Fermi momentum k_f. Inset shows the same plot with smaller scale for demonstration of the lower part of δ_s more clearly.

order $\sim 10^{-2}$ [7]. There was also an attempt to fix this ratio through statistical analysis of glitches of Vela, Crab, and a few other pulsars (see Ref. [30]). Using the observed values of glitch parameters, the authors constrained the ratio as $I_{res}/I_c \geq 1.4\%$ for Vela. Here I_{res} is the MI contribution by the angular momentum reservoir components. In our case, the whole inner crust superfluid is assumed to be an angular momentum reservoir, i.e., $I_{res} \equiv I_f$. It is then reasonable to take $I_f/I_c \simeq 10^{-2}$ for the estimate of glitch size for Vela. For consistency, note that by setting $\delta_s = \Delta R$ and $R \sim R_s$, the above value of MI ratio results in maximum glitch size of order 10^{-6} for Vela pulsar ($\tau \simeq 10^4$ years). This is expected as the above glitch size corresponds to the release of all vortices. As noted in Ref. [30], the analysis of Crab pulsar reveals a tiny MI ratio $I_f/I_c \simeq 10^{-5}$. It was suggested that losing angular momentum between glitches through vortex creep may be responsible for the lower value of the MI ratio. However, one should note that putting $I_f/I_c \simeq 10^{-5}, \delta_s = \Delta R$ (For the case when all the vortices are released.), $R_s \simeq R$, and $\tau \simeq 10^3$ years in Eq. (9), the smaller value of the MI ratio of Crab does not contradict the observed glitch size $\sim 10^{-8}$ of this pulsar. Thus, the above discussion suggests that the values of I_f/I_c, as mentioned above, are compatible with Vela and Crab pulsars. Now, the glitch size through local unpinning, where only a fraction of total vortices are released, depends on the value of δ_s and ΔR. In terms of these quantities, we get the glitch size of the Vela pulsar (with $\tau \simeq 10^4$ years) from Eq. (9) as

$$\left(\frac{\Delta\Omega}{\Omega}\right) \simeq 10^{-6}\left(\frac{\delta_s}{\Delta R}\right). \qquad (12)$$

Thus, for k_f in the range $(0.2$–$1.2)$ fm^{-1}, the glitch size for the Vela pulsar lies in the range $(10^{-9}$–$10^{-11})$ as shown in Table I and in Fig. 5. The above results follow from the local unpinning, without the effects of an avalanche. In the next section, we will discuss the implication of local unpinning on vortex avalanche.

Before concluding this section, we should comment on our results' sensitivity to the neutron star model. In this study, we have taken a generic model for the internal structure of a neutron star and fixed the values of various

TABLE I. Fermi momentum (k_f), distance of the cylindrical shell from the center (R_s), thickness of the shell (δ_s), the number of unpinned vortices (N_v), and the order of magnitude of the glitch size ($\Delta\Omega/\Omega$) for the Vela pulsar through local unpinning by excited neutrons.

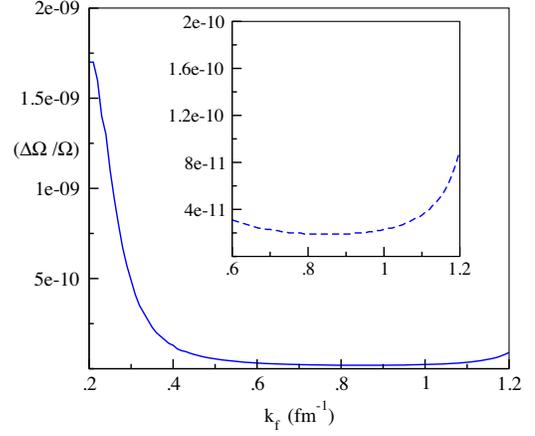| k_f (fm^{-1})	R_s (in km)	δ_s (in meter)	N_v	$\left(\frac{\Delta\Omega}{\Omega}\right)$
0.2	10.3	0.87	3.9×10^{13}	$\sim 10^{-9}$
0.8	10.2	0.01	4.3×10^{11}	$\sim 10^{-11}$
1.2	9.9	0.05	2.0×10^{12}	$\sim 10^{-9}$

FIG. 5. The typical glitch size for a Vela like pulsar caused by the local unpinning from the cylindrical shell of thickness δ_s versus the Fermi momentum k_f. For clarity, inset shows only the lower part of $\Delta\Omega/\Omega$.

neutron star parameters accordingly. The parameter-dependent sensitivity of our results can be understood from Eq. (9). As mentioned earlier, the MI ratio I_f/I_c depends on the specific glitch model. Assuming the standard super-fluid vortex picture (i.e., the inner crust superfluid component is the angular momentum reservoir responsible for the glitch event), the above ratio can be fixed following the statistical analysis of glitch events [30]. Once I_f/I_c is specified, the glitch size should depend only on δ_s and ΔR. Now $\delta_s(k_f)$ is a function of Fermi momentum k_f, which depends on the local baryon density in the inner crust region. The density-dependent Fermi momentum is now taken from Ref. [37]. As the methodology breaks down beyond $\rho \simeq 10^{14}$ gm cm^{-3}, we restricted our study up to the baryon density 10^{14} gm cm^{-3} and the corresponding Fermi momentum $k_f = 1.2$ fm^{-1}. Note these calculations rely on the methodology used to study the properties of matter's superfluid at a high baryon density regime. Though the purpose of such a study is to understand the properties of the inner crust of a neutron star, the author [37] did not assume any specific model for neutron star structure. We adopt their data of density-dependent Fermi momentum for our purpose. For the inner crust density profile, we have followed the work of Ref. [16], where such a profile is provided for a neutron star of mass $1.4M_\odot$ (see Ref. [16] and the references therein for details). The value of δ_s is quite sensitive to the Fermi momentum and varies by about two orders of magnitudes, as k_f changes from 0.2 fm^{-1} to 1.2 fm^{-1} (see Table I). So finally, relying on the works of Refs. [16,37], the model-dependent uncertainty in our results may arise only due to the value of the inner crust thickness ΔR. For a given neutron star mass, the value of ΔR and the other neutron star parameters depend on the equation of state of the neutron star matter. For an of mass $\simeq 1.4M_\odot$, the value of ΔR lies in the range $(1$–$2)$ km [16,29–31]. Accordingly, we

have taken $\Delta R = 1$ km while estimating the number of pinned vortices in the inner crust and the glitch size. Following the above discussion, we can safely say that our results are trustworthy for the pulsars of mass $\simeq 1.4 M_\odot$, i.e., the EOS-dependent sensitivity should not affect the order of magnitude estimate of glitch size.

VI. THE VORTEX AVALANCHE THROUGH PROXIMITY KNOCK-ON

Instantaneous release of a large number of vortices ($\sim 10^{18}$) is necessary for explaining large size glitches ($\Delta\Omega/\Omega \sim 10^{-6}$) of the pulsars. This seems to be feasible through the mechanism of vortex avalanche as suggested in the literature [13,15,24]. For this to happen, a trigger mechanism is required to initiate the process like in any other natural avalanche event. In our model of local unpinning, the movement of the unpinned vortices may trigger an avalanche through the proximity knock-on process [25]. We will see below that the usefulness of such events from the context of explaining large size glitches depends on the position of the shell. The scope of knock-on is reduced if the shell is located very close to the outer part of the inner crust (Recall the statement in Sec. I regarding the energy deposition in the inner crust). For a qualitative understanding of the process, we follow the basic picture of proximity knock-on as described in Ref. [25]. In the proximity picture, when an unpinned vortex comes closer to a pinned vortex from an intervortex distance d_v to a distance ηd_v ($\eta < 1$), it effectively reduces the pinning barrier. Here, the difference in angular frequency $\delta\Omega = (t_p/2\tau)\Omega$ [Eq. (1)] between the superfluid and the crust is modeled as a pinning barrier, which is expressed as

$$\gamma = \left(1 - \frac{\delta\Omega}{\delta\Omega_{cr}}\right) = 1 - \left(\frac{t_p}{2\tau}\right)\left(\frac{\Omega}{\delta\Omega_{cr}}\right). \tag{13}$$

The critical value of $\delta\Omega$ at which the magnus force balances the pinning force is denoted by $\delta\Omega_{cr}$, and is given by [39]

$$\delta\Omega_{cr} = \frac{E_p}{\rho\kappa R b\xi}, \tag{14}$$

where, $\kappa = h/2m_n$ is the quantum vorticity with m_n being the mass of a neutron. The coherence length of the bulk superfluid is denoted by $\xi(\simeq 10$ fm$)$. The nucleus-nucleus distance is denoted by $b(\simeq 100$ fm$)$, and R ($\simeq 10$ km) is the (average) distance of the inner crust from the center of the star. For a given value of pinning energy E_p, the numerical value of $\delta\Omega_{cr}$ can be determined using Eq. (14). The unpinning rate for a single vortex from a region characterized by the pinning energy E_p is given by [25]

$$\lambda_0 = \Gamma_0 e^{-\beta E_p\gamma}. \tag{15}$$

The factor $\Gamma_0 = 10^{22}$ s^{-1} is calculated by assuming the decay process to be of quantum origin (see Ref. [25] and the references therein). The quantity $\beta(= 1/kT)$ characterizes the star's temperature. As an unpinned vortex gets closer to a pinned vortex, the pinning barrier is reduced by

$$\Delta\gamma = \left(\frac{\kappa}{2\pi d_v R\delta\Omega_{cr}}\right)\left(\frac{1-\eta}{\eta}\right), \tag{16}$$

and the unpinning rate is increased to

$$\lambda = \lambda_0 e^{\beta E_p\Delta\gamma}. \tag{17}$$

Note, Eq. (15) suggests that the unpinning rate from a specific region of the inner crust having a fixed value of E_p is completely determined by the ratio of $\delta\Omega$ to $\delta\Omega_{cr}$. As the star slows down, $\delta\Omega$ gets close to $\delta\Omega_{cr}$. As a result, the γ factor, which is interpreted as a pinning barrier, is decreased, causing the unpinning rate to increase with time. However, one should note that with a typical few years interglitch time t_p, γ is approximately fixed. Also, as λ varies with E_p, it is quite impossible that all the pinned vortices spread across the inner crust of the star get unpinned simultaneously. The vortex avalanche, therefore, seems to be a quite natural mechanism for unpinning of so many vortices. In our case, there are already a large number of unpinned vortices ($\sim 10^{14}$–10^{12}) moving radially outward. These can act as triggers to "knock-on" the pinned vortices in the inner crust of the star.

Now, Eq. (17) provides the vortex unpinning rate triggered by a single vortex. In our case, this will be modified due to the presence of a large number of triggers caused by local unpinning. For a qualitative understanding of the process, let us exploit the azimuthal symmetry of the vortex motion and focus only in one radial direction. We can estimate the effective number of triggers at the onset of the knock-on process across the shell of thickness δ_s with vortex density n_v as $N_{tr}^i \simeq \delta_s\sqrt{n_v}$. As δ_s lies in the range $\sim(87$–$1)$ cm (depending on the location of the shell), N_{tr}^i should lie in the range $\sim(10^3$–$10^2)\sqrt{\Omega}$. Here, Ω should be taken in units of s^{-1}. The value of N_{tr}^i, which is the effective numbers of triggers in one dimension sets the lower limit of triggers initiating the knock-on process. Eventually, these vortices may unpin the other vortices, which in turn should participate in the process. This increase the triggers N_{tr} cumulatively. The above picture should eventually lead to a few orders of magnitude enhancement of unpinning rate of a single vortex. The effects of these multiple triggers can be incorporated by modifying Eq. (17) as

$$\lambda_{tr} = N_{tr}\lambda = \Gamma_0 N_{tr} e^{-\beta E_p\gamma} e^{\beta E_p\Delta\gamma}. \tag{18}$$

TABLE II. A Few values of the Fermi momentum k_f at which the unpinning probability for a single vortex is enhanced due to multiple triggers.

k_f (fm^{-1})	$\lambda \tau_{tr}$	$\lambda_{tr} \tau_{tr}$
0.52	$\sim 10^{-3}$	~ 1.0
1.14	$\sim 10^{-3}$	~ 1.0

An average time τ_{tr} for a single unpinning event can be taken as $\tau_{tr} \simeq d_v/v_r$. Where, $v_r \simeq R_s \delta \Omega \simeq (t_p/2\tau)\Omega R_s$ [see Eq. (1)] is the radial velocity of a vortex, and $d_v \sim 1/\sqrt{n_v}$ is the intervortex distance. For the Vela pulsar with $t_p \sim 1$ year, τ_{tr} is about 10^{-7} s. For conservative estimate, let us set $N_{tr} = N_{tr}^i$. In this case, the unpinning probability for a single vortex can be written as

$$\lambda_{tr} \tau_{tr} = \left(\frac{2\tau}{t_p \Omega R_s \sqrt{n_v}} \right) N_{tr} \Gamma_0 e^{-\beta E_p \gamma} e^{\beta E_p \Delta \gamma}. \quad (19)$$

In the picture of proximity knock-on, unpinning occurs provided the condition $\lambda_{tr} \tau_{tr} \simeq 1$ is satisfied [25]. In Table II, we have listed a few values of the Fermi momentum k_f around which the above condition is fulfilled in the presence of multiple triggers. For example, in the region around $k_f = 1.14$ fm^{-1}, $\lambda \tau_{tr} \simeq 10^{-3}$ and $\lambda_{tr} \tau_{tr} \simeq 1$. Thus, there is about three order of magnitude enhancement in the unpinning probability for a single vortex in the presence of triggers. Similar effects are also observed around $k_f \simeq 0.52$ fm^{-1}. The multiple triggers could therefore significantly enhance the chances of unpinning from a few specific regions. However, it should be mentioned that there are also a few regions, where $\lambda_{tr} \tau_{tr}$ is still too low to meet the condition for unpinning.

We have described above a qualitative picture of the knock-on process. To properly understand the implications of avalanche, one has to resort to dynamical simulation. Also, one should note that the unpinning rate being depend on the exponential function of various parameters [Eq. (17) and/or Eq. (18)], the results are quite sensitive to the values of those parameters. These factors should be taken into account in the proper analysis of vortex avalanche.

We can make a rough estimate for the expected glitch size, if we assume that the knock-on causes unpinning from a region of total thickness δ_a. Here δ_a is the sum of the thickness of various regions where the unpinning condition is satisfied. This also includes the region, which were already affected by the local unpinning. So, the glitch size for the Vela pulsar will be modified as

$$\left(\frac{\Delta \Omega}{\Omega} \right) \simeq 10^{-6} \left(\frac{\delta_a}{\Delta R} \right). \quad (20)$$

The maximum value of δ_a is expected to occur, if the cylindrical shell is located around $R_s = 9.9$ km

$(k_f = 1.2$ fm$^{-1})$. In this case, there are about $10^2 \sqrt{\Omega}$ triggers available (along a particular direction) for the process. These vortices while moving, unpin the vortices from the region whenever the quantity $\lambda_{tr} \tau_{tr}$ is approximately closed to unity. If all the vortices beyond $R_s = 9.9$ km get unpinned, the value of δ_a will be approximately equal to 400 meter (from $R_s = 9.9$ km to 10.3 km). For this case, the glitch size will be of order 10^{-6} for a Vela-like pulsars (with $\tau \sim 10^4$ years). However, analyzing the unpinning probability at various regions (a few regions have too low probability), only a fraction $f_a = (\delta_a/400$ m) of the region beyond $R_s = 9.9$ km seems to be affected by the trigger mechanism. For proper estimate of f_a, a proper numerical algorithm is required to implement the above process (see Ref. [25] for one such algorithm). It will be then interesting to see if the local unpinned vortices trigger an avalanche to produce a large size glitches without affecting the typical one year interglitch time. Note, in a crustquake model, the larger size glitches need a larger interglitch time [5] contrary to the observations.

VII. COMMENT AND CONCLUSION

We have observed that the crustquake followed by vortex unpinning from a cylindrical shell may produce glitches of size 10^{-9}–10^{-11} for relatively old pulsars with characteristic age $\tau \sim 10^4$ years. There is also scope for the generation of larger size glitches through avalanche triggered by the unpinned vortices as observed by the Vela pulsar. Although not regular, the Crab pulsar also exhibited a large size glitch ($\Delta \Omega / \Omega \sim 0.52 \times 10^{-6}$) as reported in 2017 [44]. It is impossible [44] to explain such a large glitch from a younger pulsar through the standard crustquake model. A partial avalanche (i.e., affecting only a few fractions of total vortices in the inner crust) triggered by the locally unpinned vortices can account for such occasional glitch activity of a young pulsar.

Besides the occurrence through vortex unpinning, the crustquake itself also produces glitches due to a sudden change of the pulsar's shape. The nonobservation of successive glitches within a short span can be explained if the time interval between these two events turns out to be small compared to the current observational limit of individual glitches [45]. Now we estimate the time difference between these two glitch events. For that, we assume the energy deposition to the inner crust is almost instantaneous (See Ref. [16] for the time evolution of the temperature profile in the inner crust after energy absorption.) and ignore the timescale associated with the vortex unpinning (n-n scattering timescale $\tau_{nn} \simeq 10^{-5}$ s only). Thus the time interval between two glitch events is determined by the time t_v taken by the vortex to reach the outer crust. This can be obtained from the radial velocity v_r of the unpinned vortices. For $v_r \simeq R(\delta \Omega) = (t_p/2\tau)\Omega R \simeq 10^4$ cm-s^{-1}, the time $t_v \simeq v_r/\Delta R$ turns out to be ~ 0.1 s for the Vela pulsar.

The above time difference is associated with the glitches occurring through crustquake followed by the glitch produced through local unpinning. For the case of avalanche, a single unpinning event takes an average time $\tau_{tr} \simeq d_v/v_r \sim 10^{-7}$ s and the whole process is expected to be completed within $\sim 10^{-6}$ s (see [17] and the reference therein). Thus we see that the time interval between the glitches produced by the crustquake and the vortex unpinning is of order tenth of a second, i.e., they seem to overlap. The short time interval also justifies our earlier assumption (in Sec. II) that the waiting time between two successive crustquake events almost overlaps with the time duration $t = 0$ to $t = t_p$, i.e., the time during which vortices remain pinned to the sites.

Thus, unifying crustquakes with the superfluid-vortex model can consistently produce regular glitches with a typical frequency of once every few years (set by the time interval of successive crustquake events). In case of an avalanche, unifying the models can produce large size Vela or Vela-like glitches without affecting the waiting time of crustquakes. Recall that the crustquake model alone is not compatible with such glitches. For Crab-like younger pulsars, partial avalanche may be responsible for the occasional larger size glitch activity [44].

From the observational perspective, as we mentioned earlier, with the current resolution [45], it is impossible to resolve the subsecond time interval between the glitches produced by the crustquake followed by a glitch through unpinning. For the case of vortex avalanche, though, it is the larger size glitch that is expected to dominate the glitch feature. Hence the source of the larger glitch (i.e., vortex unpinning) will be easily identifiable. However, if the avalanche mechanism is ineffective (for example, if the energy is deposited at the outer part of the inner crust), it will be an observational challenge to resolve two successive, almost identical size glitches. This is a common feature of all the proposed models, where both the crustquake and vortex model are involved in the generation of glitches (see, for example, the Refs. [15,16,46]). For such cases, the improved glitch observation time of individual pulsars can help identify the precise source of the glitches.

To conclude, we studied the vortex unpinning mechanism by scattering excited neutrons with the vortex core neutrons in the inner crust of the pulsars. The strain energy released by the crustquake is assumed to be absorbed in some part of the inner crust around the equatorial plane of the star and causes excitation of the free superfluid neutrons surrounding the vortices. The excited neutrons unpin a large number of vortices from that region and result in pulsar glitches. We considered a cylindrical shell around the equatorial plane and studied the effects of the thermally excited neutrons in unpinning the superfluid vortices from the affected site. As the affected pinned region's precise location is unknown, we took the shell at various depths (equivalently, at different local mass density regions) of the inner crust. This was achieved by varying the Fermi momentum k_f of the superfluid neutrons from 0.2 fm^{-1} to 1.2 fm^{-1}. We then determined the shell thickness δ_s, which has been observed to lie in the range (85–1) cm. The corresponding values of unpinned vortices caused by the neutron-vortex scattering were found to lie in the range (10^{13}–10^{11}), which are equivalent to glitch size in the range (10^{-9}–10^{-11}) for the Vela pulsars.

We suggested the possibility of a vortex avalanche triggered by the vortices, which were already unpinned from the cylindrical shell. In the picture of proximity knock-on, the presence of multiple triggers can enhance the unpinning probability and hence the glitch size. A rough estimate of glitch sizes has been presented. The result is quite encouraging from the perspective of producing a large size glitch without affecting the waiting time of successive crustquakes. The various timescales associated with our model are compatible with the observed features of glitches.

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